A new idea to detect spin-triplet superconductor based on unusual proximity effect via odd-frequency pairing amplitude

Yukio Tanaka
Department of Applied Physics
Nagoya University

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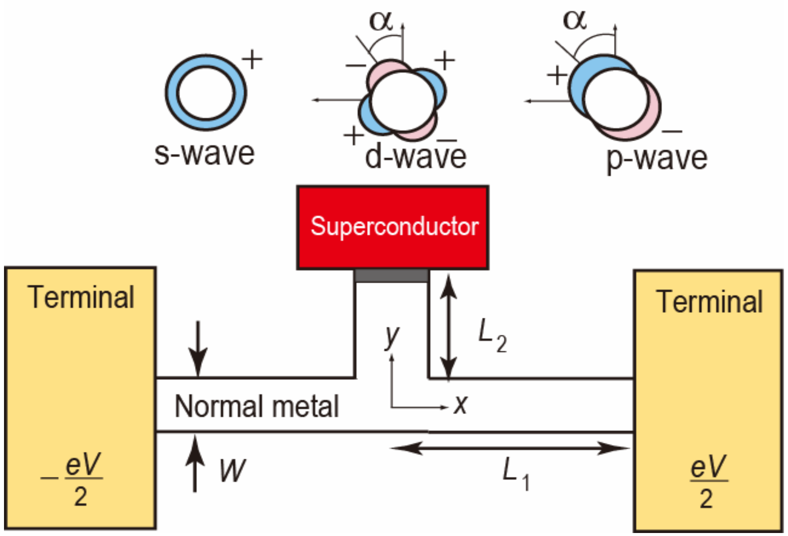
Main Collaborators

A. A. Golubov	Twente University
Y. Asano	Hokkaido University
S. Kashiwaya	NIAIST (Tsukuba)
M. Ueda	Tokyo Institute of Technology
Y. V. Nazarov	Delft University



We propose **3T**: **T**-shaped junction to **T**est **T**riplet

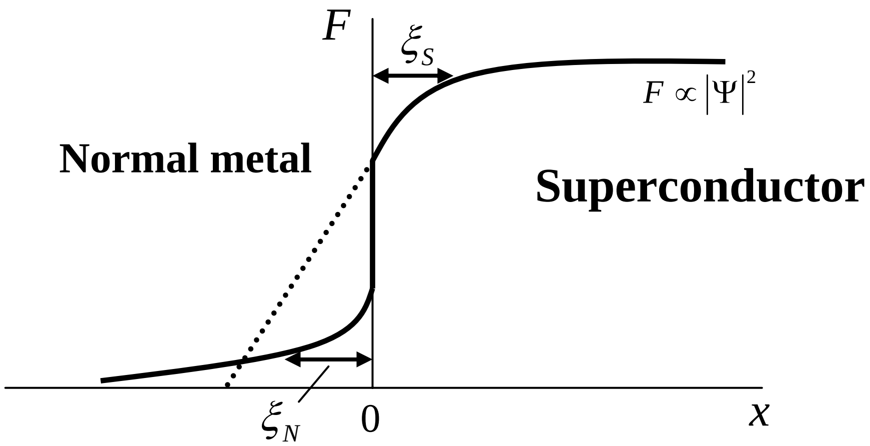
Spin-singlet : zero-bias dip
Spin-triplet : zero-bias peak

Y. Asano, Y. Tanaka, A. A. Golubov, and S. Kashiwaya, PRL 99, 067005 (2007).

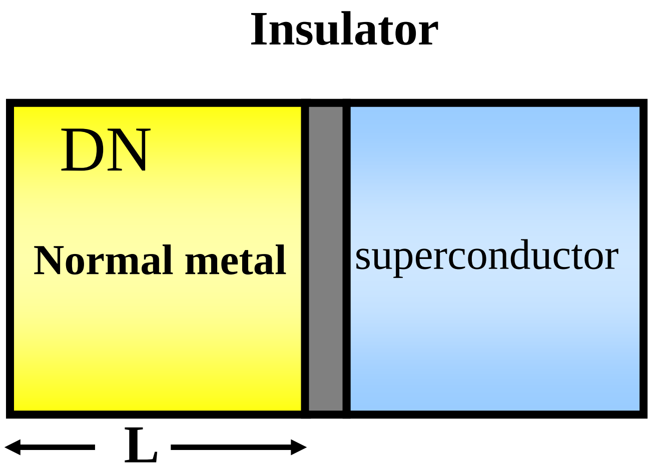
Superconducting proximity effect

Pair amplitude $F = \langle \Psi_{\uparrow} \Psi_{\downarrow} \rangle$

Spatial dependence in DN $F \propto \exp\left(-\frac{|x|}{\xi_N} \right)$



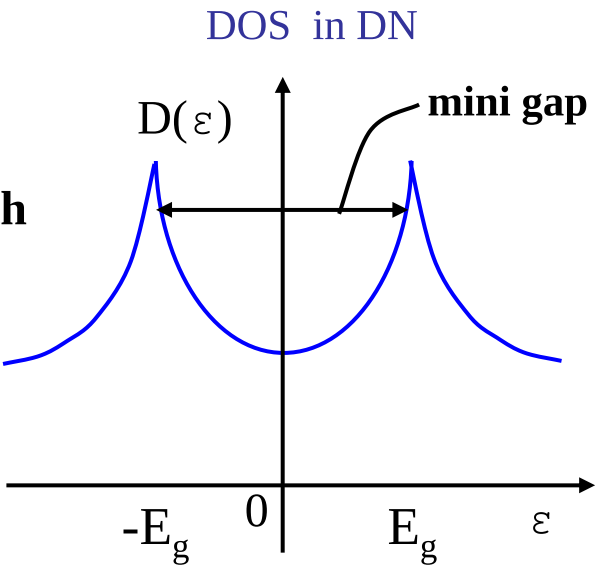
Proximity effect Penetration of Cooper pair into DN
diffusive normal metal（DN）



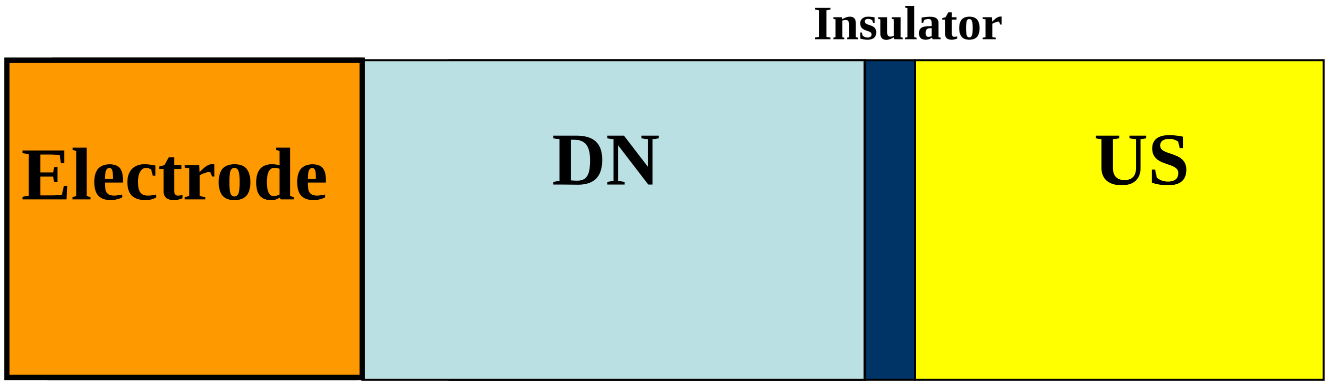
ξ **：localization length**

l **：mean free path**

$l \ll L \ll \xi$



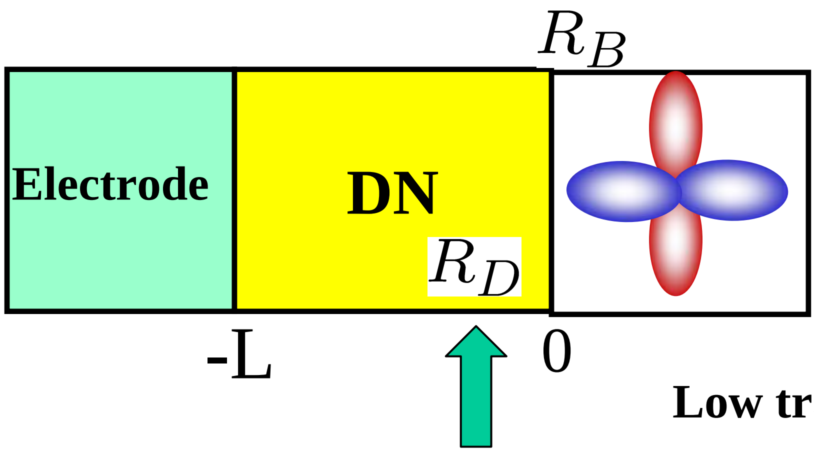
Diffusive normal metal (DN)/ unconventional superconductor (US) junctions

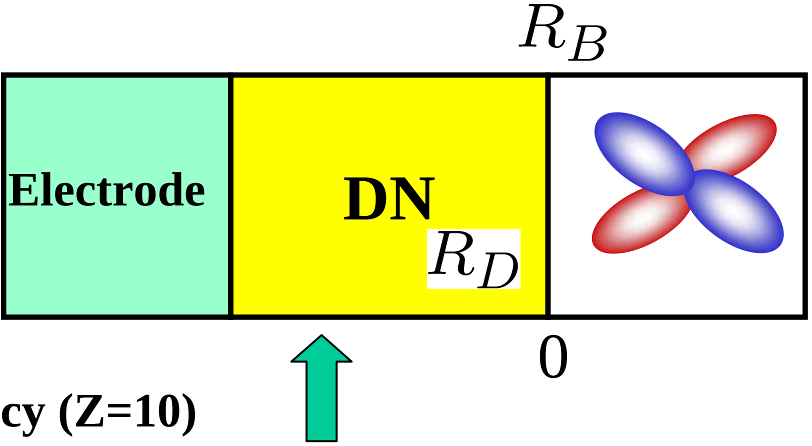


PRL 90 167003 (2003)
PRB 70 012507 (2004)

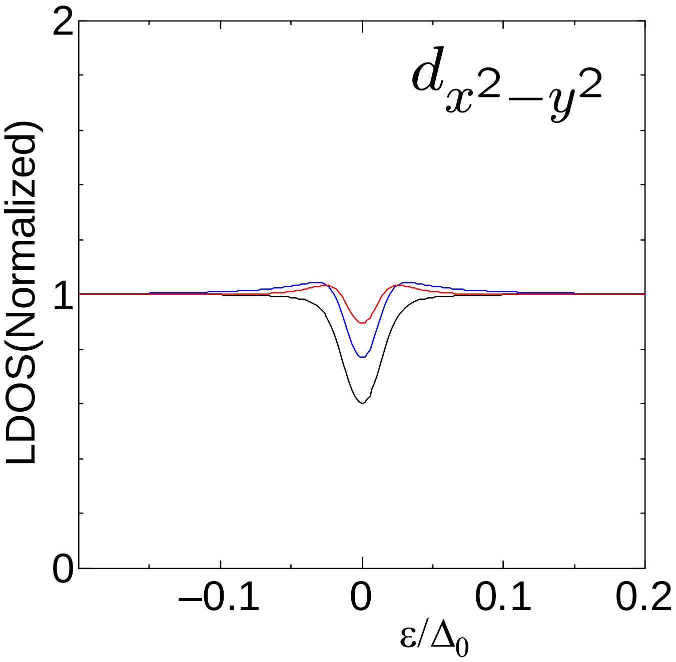
We must make a proper boundary condition at the interface of Usadel Green’s function in DN. (T=0K)

Local density of states in DN (d-wave)





Low transparency (Z=10)



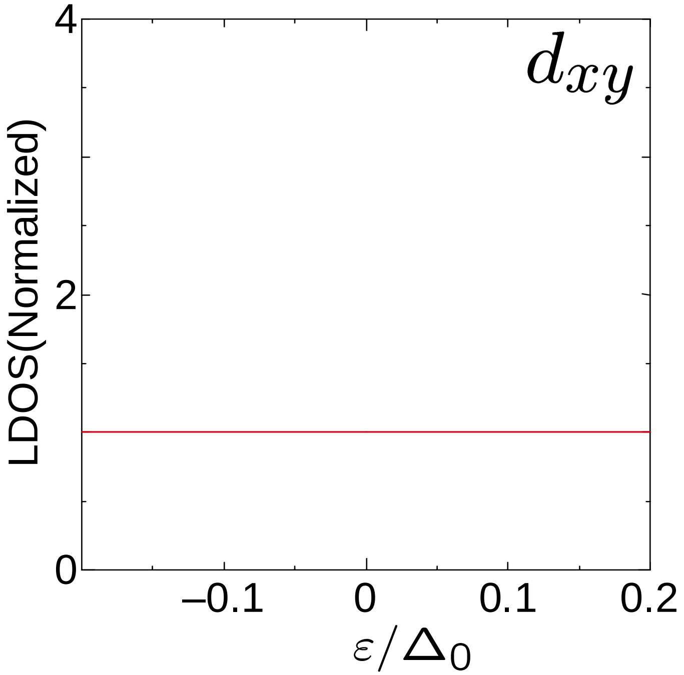
$$L/\xi = 18$$

$$E_{th}/\Delta_0 = 0.01$$

$$\xi = \sqrt{D/(2\pi T_C)}$$

$$E_{Th} = D/L^2$$

x=0
x= –L/4
x= –L/2

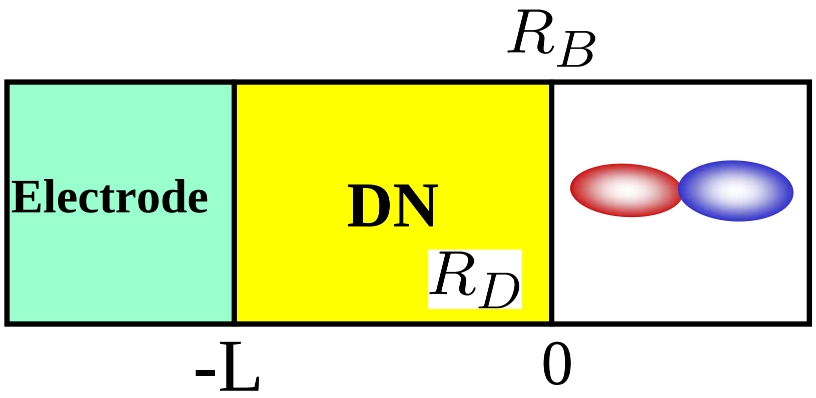


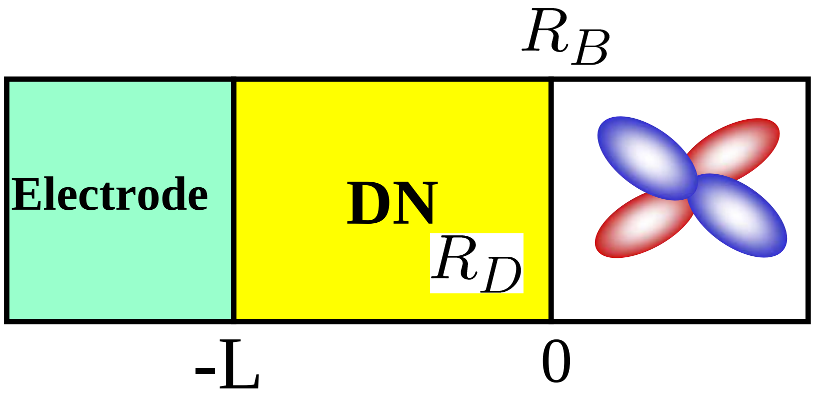
Energy gap due to proximity effect

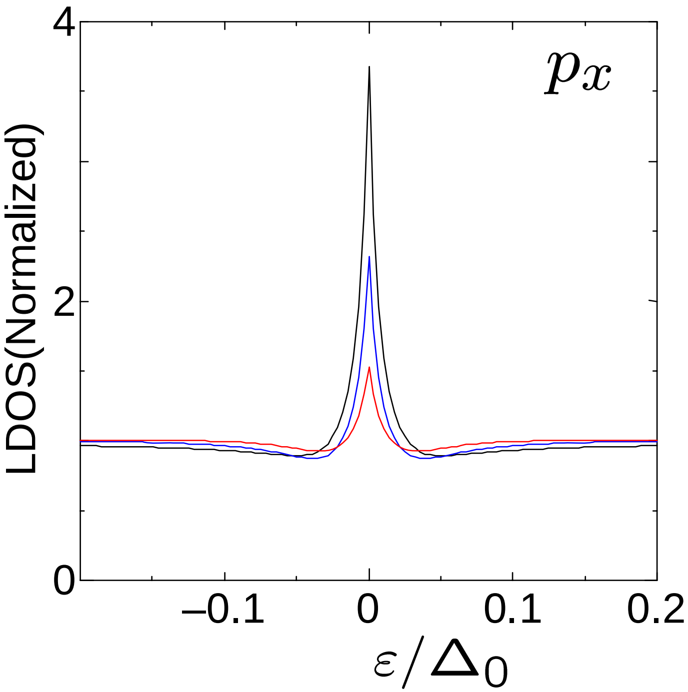
No proximity effect

PRB 70 012507 (2004)

Local density of states in DN







Z=1.5

$R_D/R_B = 0.5$

$E_{th}/\Delta_0 = 0.02$

$L/\xi = 13$

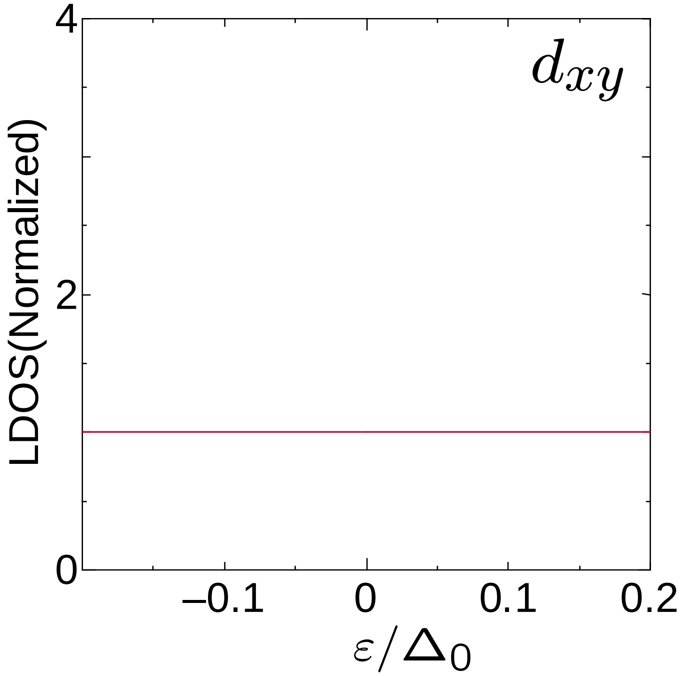
$\xi = \sqrt{D/(2\pi T_C)}$

$E_{Th} = D/L^2$

x=0

x=–L/4

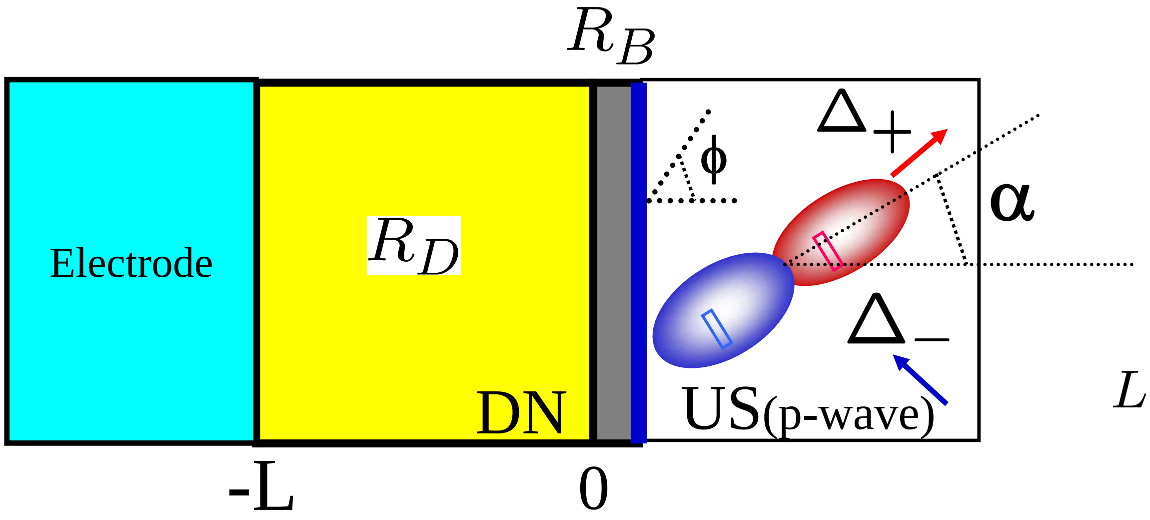
x=–L/2



Zero energy peak (ZEP) is expected only for triplet junctions!!

PRB 70 012507 (2004)

Local density of states in DN for p-wave

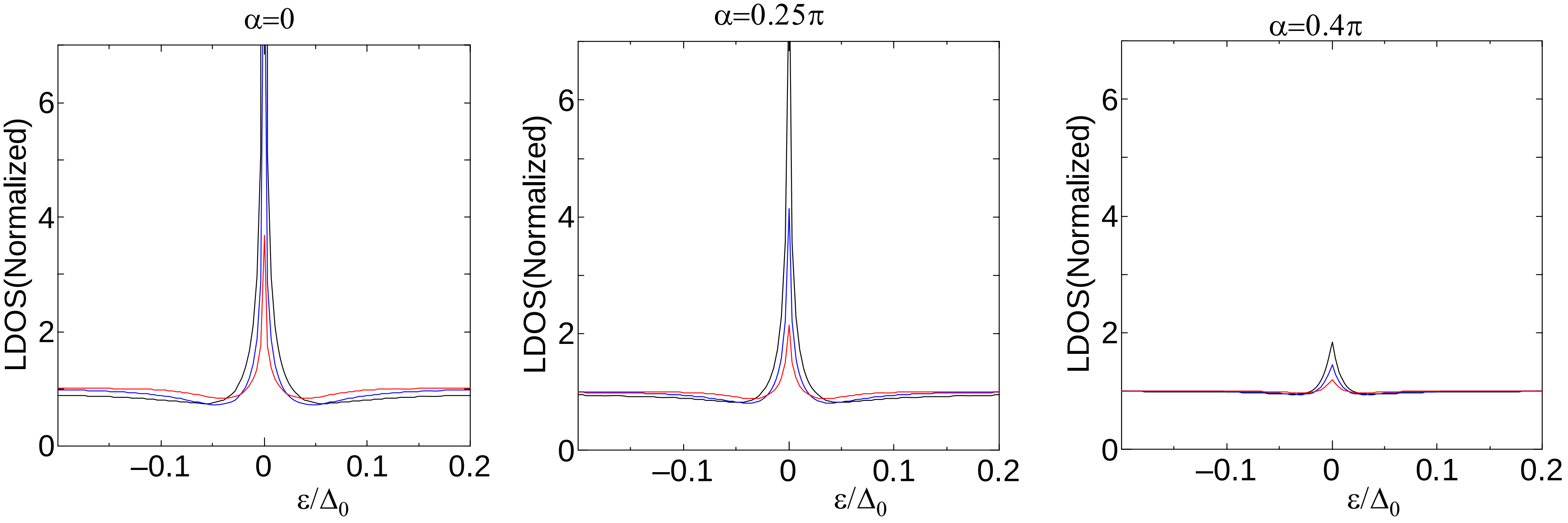


Z=1.5

$R_D/R_B = 1$ $\quad E_{Th} = D/L^2$

$E_{Th}/\Delta_0 = 0.02$

$L/\xi = 13$ $\quad \xi = \sqrt{D/(2\pi T_C)}$

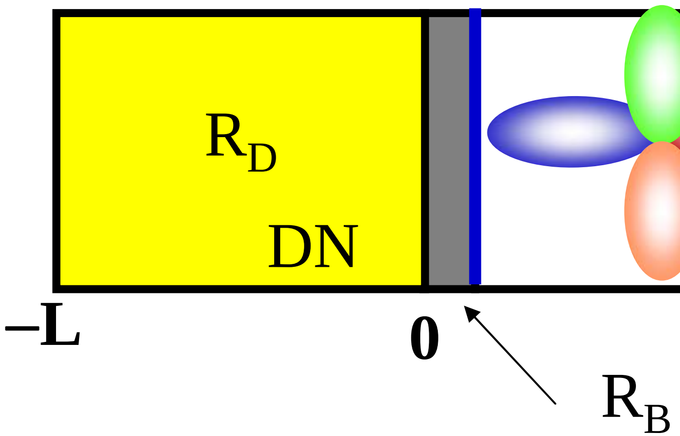


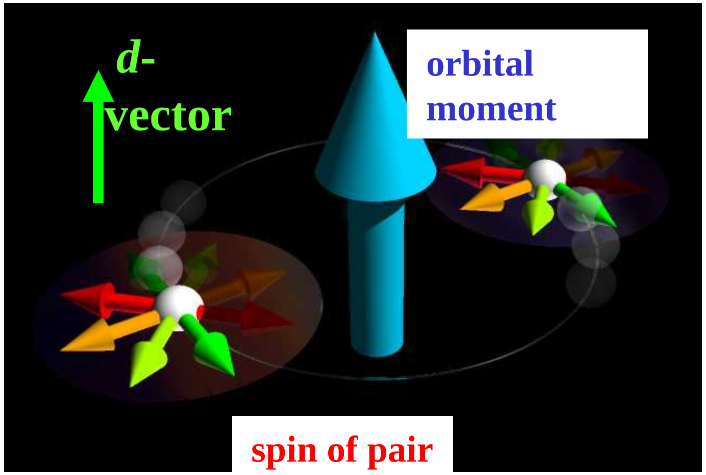
x=0 **x= –L/4** **x= –L/2**

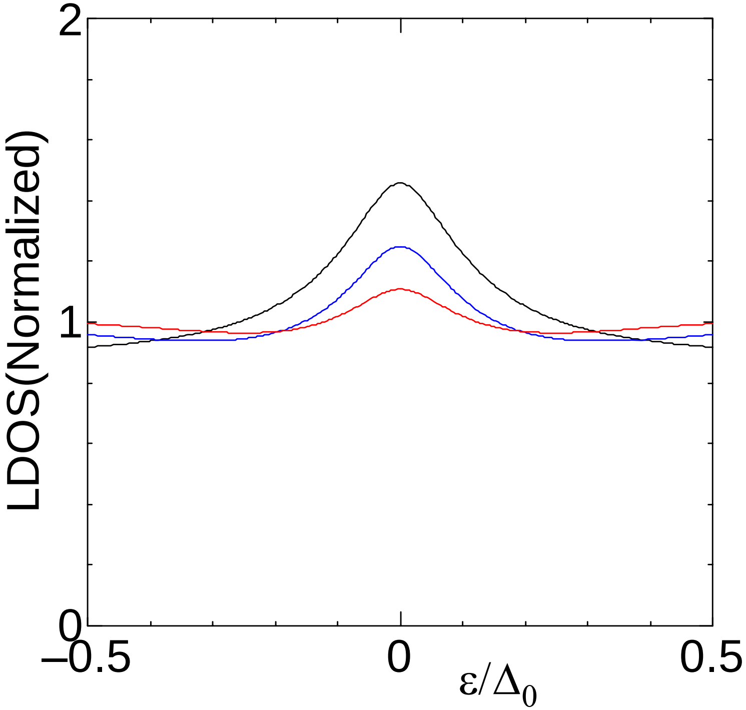
ZEP in LDOS for various α

PRB 70 012507 (2004)

Local density of states in DN (p_x+ip_y-wave)







x=0

x=–L/4

x=–L/2

$R_D/R_B = 1$ **Z=5**

$E_{th}/\Delta_0 = 0.1$ $E_{Th} = D/L^2$

$\xi = \sqrt{D/(2\pi T_C)}$

$L/\xi = 6$

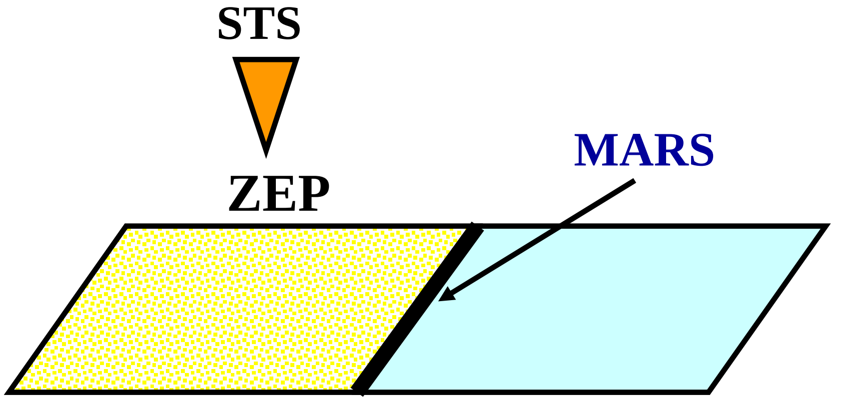
ZEP of LDOS in DN is expected for p_x+ip_y-wave junction

PRB **71**, 094513 (2005)

New idea to detect spin-triplet superconductor

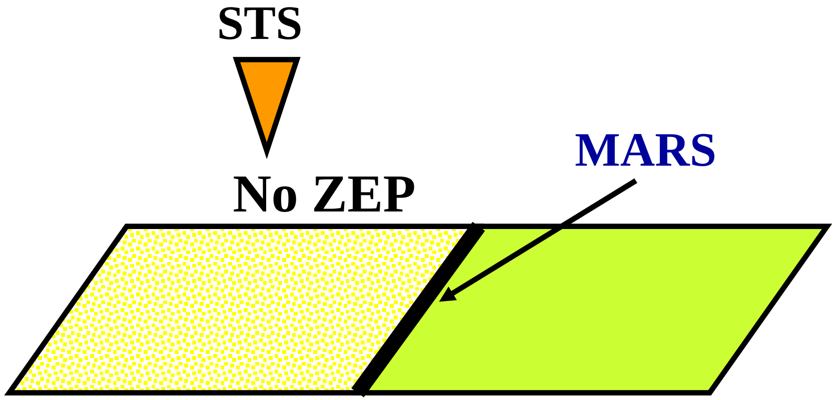
PRB 70 012507 (2004)

MARS (Mid gap Andreev resonance state) can penetrate into DN by proximity effect only for triplet superconductor junctions



Diffusive normal Metal (DN)

Triplet superconductor



Diffusive normal Metal (DN)

Singlet superconductor

LDOS in DN has a zero energy peak!!

LDOS in DN has a zero energy dip!!

The underling physics

Odd-frequency pairing amplitude induced at the interface and normal metal

Tanaka and Golubov, Phys. Rev. Lett. 98, 037003 (2007)

Pair amplitude
(pair correlation)

$$F_{\alpha,\beta}(\boldsymbol{r}_1 t_1, \boldsymbol{r}_2 t_2) = -i \left\langle \mathsf{T} \psi_\alpha(\boldsymbol{r}_1 t_1) \psi_\beta(\boldsymbol{r}_2 t_2) \right\rangle$$

$$= -i\theta(t_1 - t_2) \left\langle \psi_\alpha(\boldsymbol{r}_1 t_1) \psi_\beta(\boldsymbol{r}_2 t_2) \right\rangle + i\theta(t_2 - t_1) \left\langle \psi_\beta(\boldsymbol{r}_2 t_2) \psi_\alpha(\boldsymbol{r}_1 t_1) \right\rangle$$

Exchange of two electrons

$$F_{\alpha,\beta}(\boldsymbol{r}_1 t_1, \boldsymbol{r}_2 t_2) = -F_{\beta,\alpha}(\boldsymbol{r}_2 t_2, \boldsymbol{r}_1 t_1)$$

Fermi-Dirac statistics

Pair amplitude
(pair correlation)

Exchange of time

Even-frequency pairing

$$F_{\alpha,\beta}(\boldsymbol{r}_1 t_1, \boldsymbol{r}_2 t_2) = F_{\alpha,\beta}(\boldsymbol{r}_1 t_2, \boldsymbol{r}_2 t_1)$$

Odd-frequency pairing

$$F_{\alpha,\beta}(\boldsymbol{r}_1 t_1, \boldsymbol{r}_2 t_2) = -F_{\alpha,\beta}(\boldsymbol{r}_1 t_2, \boldsymbol{r}_2 t_1)$$

Symmetry of the pair function

+ symmetric, – anti-symmetric

	Frequency (time)	**Spin**	**Orbital**	**Total**
ESE	**+(even)**	**– (singlet)**	**+(even)**	**–**
ETO	**+(even)**	**+ (triplet)**	**–(odd)**	**–**
OTE	**–(odd)**	**+ (triplet)**	**+(even)**	**–**
OSO	**–(odd)**	**– (singlet)**	**–(odd)**	**–**

ESE (Even-frequency spin-singlet even-parity)
ETO (Even-frequency spin-triplet odd-parity)
OTE (Odd-frequency spin-triplet even-parity)
OSO (Odd-frequency spin-singlet odd-parity)

Previous studies about odd-frequency pairing

Bulk state

Berezinskii (1974)

Balatsky Abrahams Schrieffer Scalapino(1992-1993)

Zachar Kievelson Emery (1996)

Coleman Mirranda Tsvelik (1997)

Fuseya Kohno Miyake (2003)

Ferromanget / s-wave spin-singlet junctions

Bergeret, Efetov, Volkov, (2001)

Ubiquitous presence of the odd frequency pairing state

⇩

Even in the conventional even-frequency superconductors, odd-frequency pairing state can be expected near the interface (surface).

Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
Phys. Rev. Lett. 99, 037005 (2007)

M. Eschrig, T. Lofwander, Th. Champel, J.C. Cuevas and G. Schon
J. Low Temp. Phys 147 457(2007)

Quasi-classical Green’s functions

Eilenberger’s equation

$$\mp i v_{Fx} \partial_x f_{1\pm} = 2\omega_n f_{2\pm} - 2\bar{\Delta}_{\pm}(x) g_{\pm}$$

$$\mp i v_{Fx} \partial_x g_{\pm} = 2\bar{\Delta}_{\pm}(x) f_{1\pm},$$

$$\mp i v_{Fx} \partial_x f_{2\pm} = -2\omega_n f_{1\pm},$$

$$f_{1\pm}^2 + f_{2\pm}^2 + g_{\pm}^2 = 1,$$

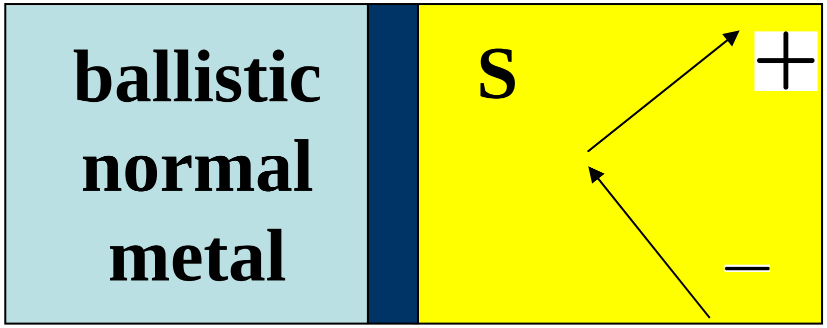
$$\bar{\Delta}_{\pm}(x) = \Delta(x)\Phi_{\pm}(\theta).$$

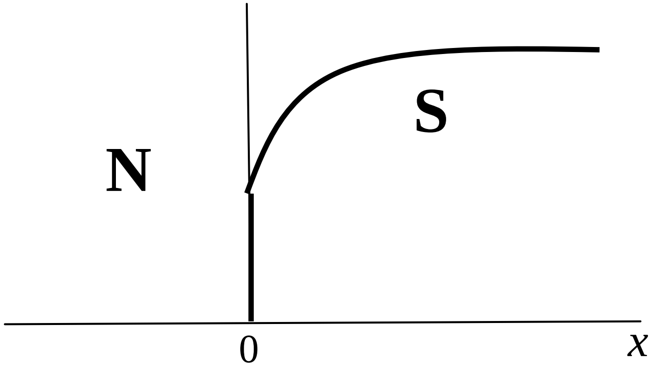
Quasiparticle function	$g_{\pm}$
Pair amplitude	$f_{1\pm},\ f_{2\pm}$

$\Phi_{\pm}(\theta)$

Bulk state ⇒ $f_{2\pm}$

$\bar{\Delta}_{\pm}(x)$ Form factor





General properties（frequency)

Pair potential has even-frequency symmetry.

$$f_{2\pm}(\omega_n,\theta)=f_{2\pm}(-\omega_n,\theta)$$

Even-frequency (real), bulk-state

Spatial change of the pair potential

$$f_{1\pm}(\omega_n,\theta)=-f_{1\pm}(-\omega_n,\theta)$$

Odd-frequency (imaginary)

Interface-induced state

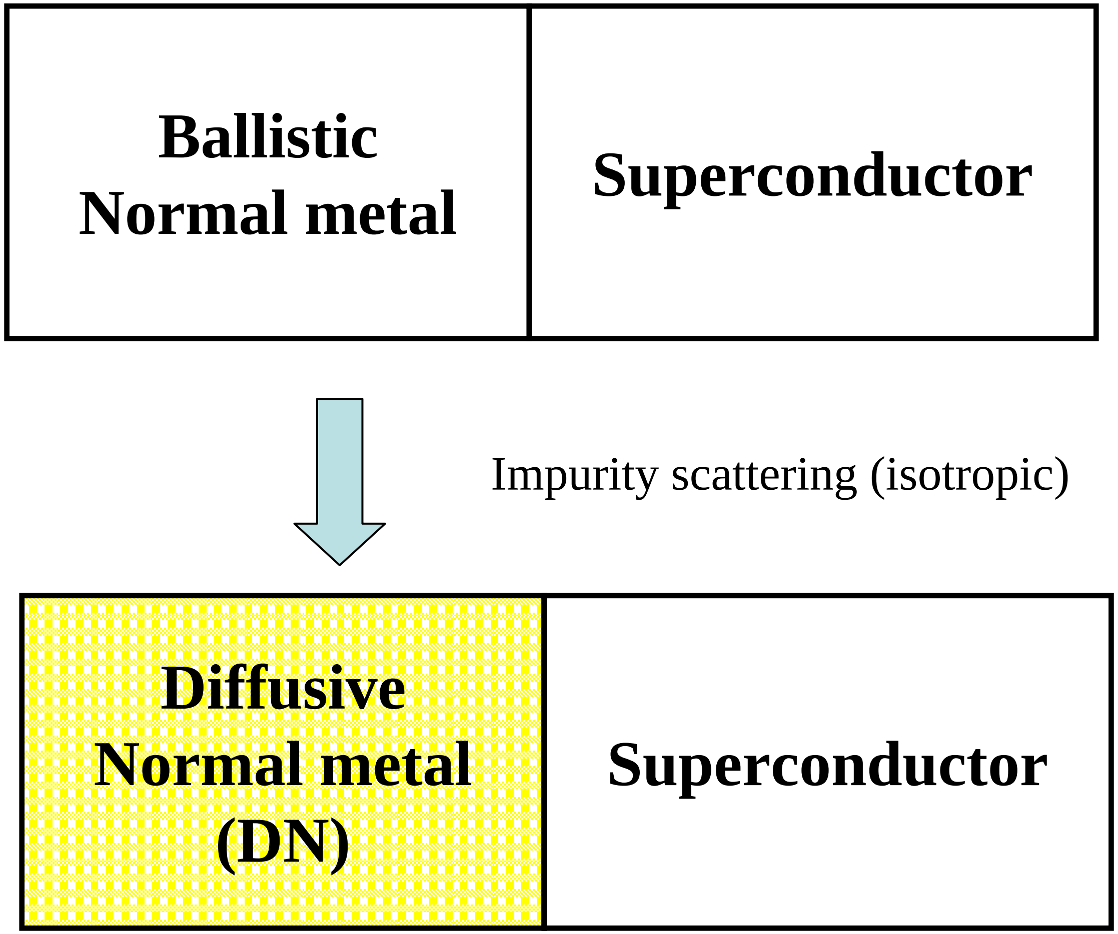
Summary of proximity effect (No spin flip)

	Bulk state	Sign change **(MARS)**	Interface-induced state (subdominant component)
(1)	ESE (s,d_{x2-y2} -wave)	No	**ESE** + (OSO)
(2)	ESE (d_{xy}-wave)	Yes	**OSO** +(ESE)
(3)	ETO (p_x-wave)	Yes	**OTE** + (ETO)
(4)	ETO (p_y-wave)	No	**ETO** + (OTE)

- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity**

Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
Phys. Rev. Lett. 99 037005 (2007)

Impurity scattering effect



Ballistic Normal metal

Superconductor

Impurity scattering (isotropic)

Diffusive Normal metal (DN)

Superconductor

Only s-wave pairing amplitude is possible in DN

Tanaka and Golubov, Phys. Rev. Lett. 98, 037003 (2007)

Summary of proximity effect (No spin flip)

	Bulk state	Sign change	Interface-induced state (subdominant)	Proximity into DN
(1)	ESE(s,dx2-y2 -wave)	No	**ESE** + (OSO)	**ESE**
(2)	ESE (d_{xy}-wave)	Yes	**OSO** +(ESE)	No
(3)	ETO (p_x-wave)	Yes	**OTE** + (ETO)	**OTE**
(4)	ETO (p_y-wave)	No	**ETO** + (OTE)	No

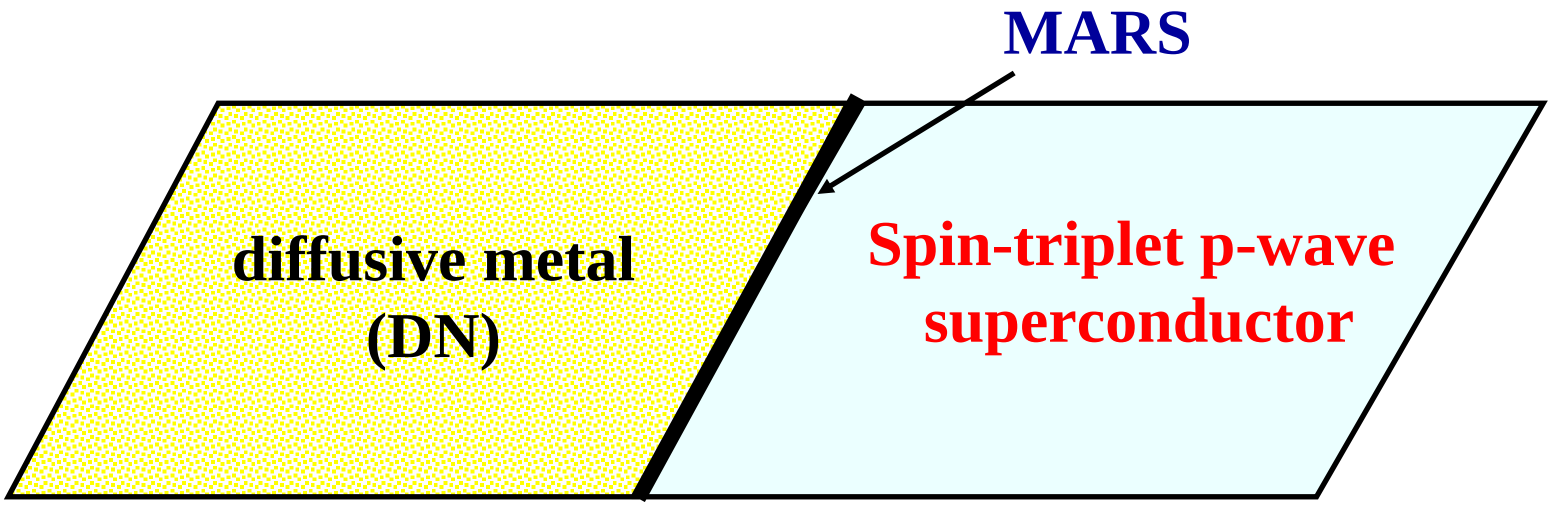
- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity)**

Proximity into DN (Diffusive normal metal)

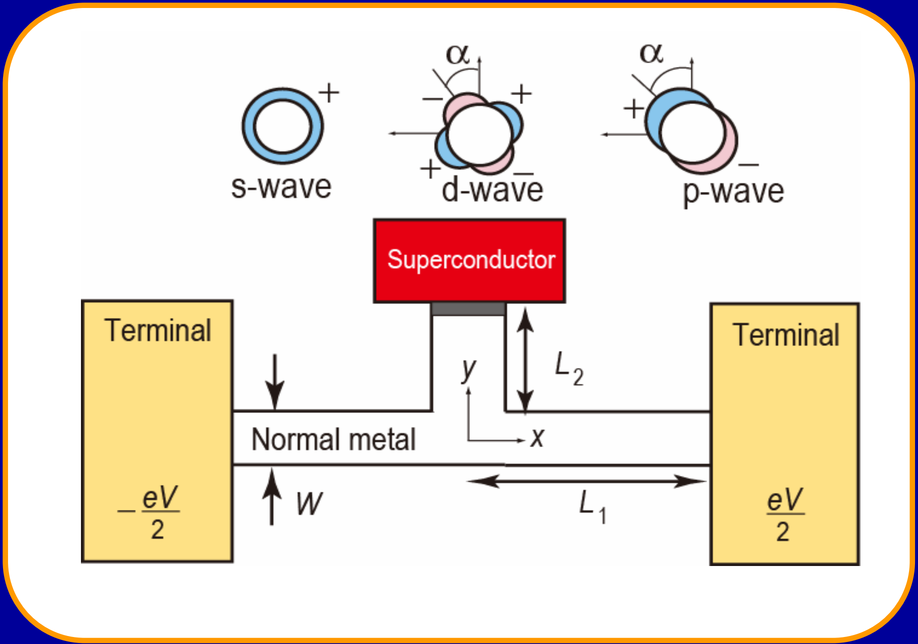
even-parity (s-wave)○ Odd-parity ×

Y. Tanaka, et al Phys. Rev. Lett. 99, 037005 (2007)
Phys. Rev. Lett. 98, 037003 (2007)

Odd-frequency spin-triplet s-wave pairing state is induced in diffusive normal metal



Mid gap Andreev state (MARS) can penetrate into DN as an odd-frequency pairing state.



We propose **3T**: **T**-shaped junction to **T**est **T**riplet

Spin-singlet : zero-bias dip
Spin-triplet : zero-bias peak

Y. Asano, Y. Tanaka, A. A. Golubov, and S. Kashiwaya, PRL 99, 067005 (2007).

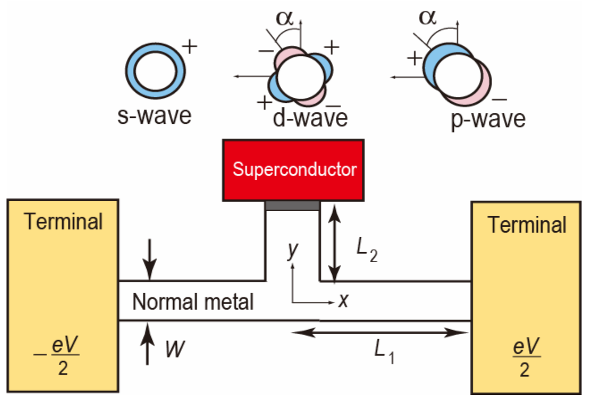
Results

p-wave

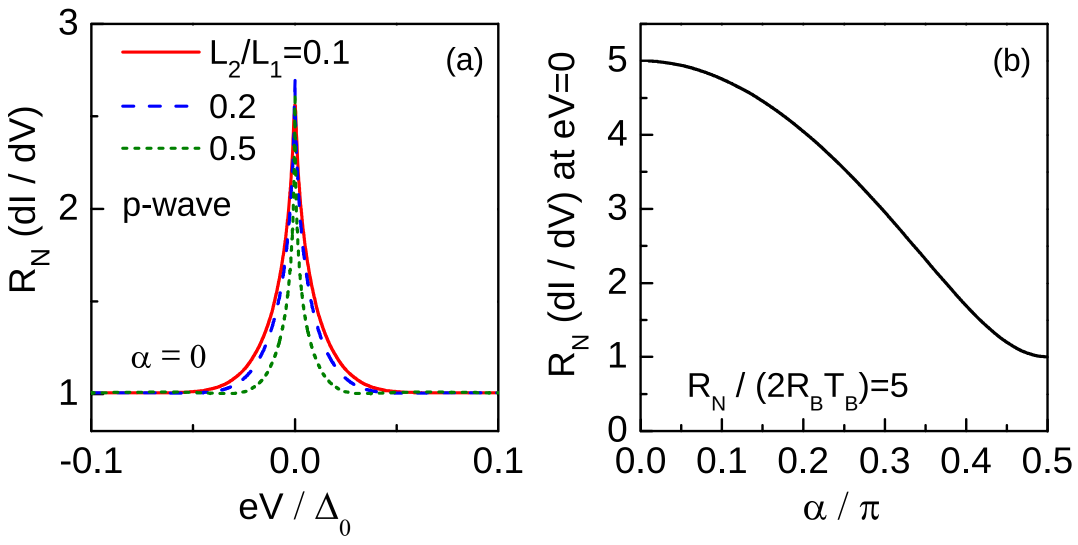
$$E_{Th} = \frac{\hbar D}{L^2}$$

$$L = L_1 + L_2$$

$$R_N \left.\frac{dI}{dV}\right|_{eV=0} = \frac{\frac{R_N \cos\alpha}{2R_B T_B}}{\tanh\left(\frac{R_N \cos\alpha}{2R_B T_B}\right)}.$$



zero-bias peak

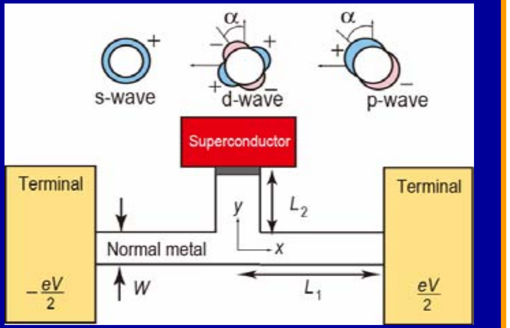


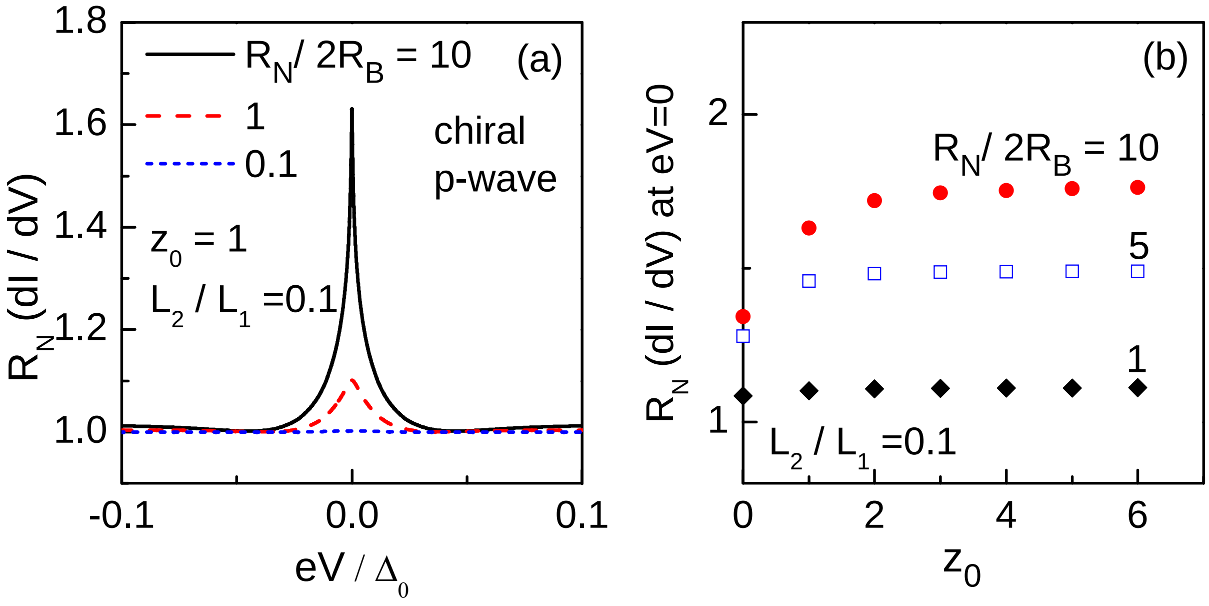
No proximity effect at $\alpha=\pi/2$: Asano, JPSJ 71, 905 (2002).

PRL 99, 067005 (2007).

Results

chiral p-wave (ruthenates)



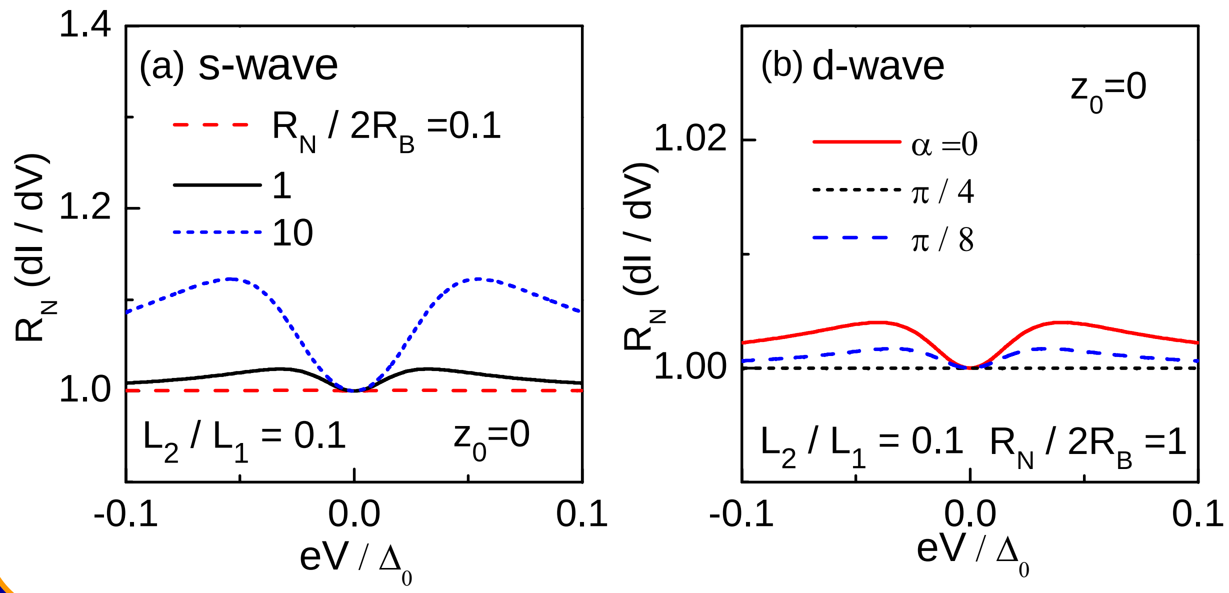


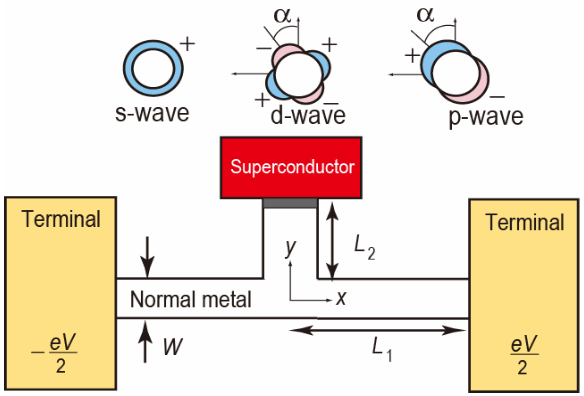
zero-bias peak for large R_N / R_B

PRL 99, 067005 (2007)

Results

Spin-singlet

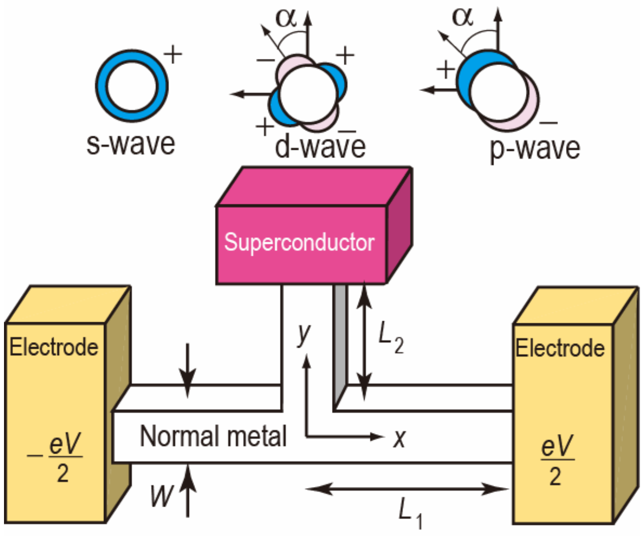


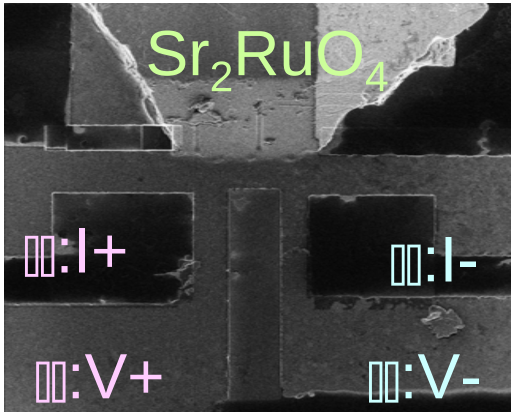


zero-bias dip

Theoretical prediction to detect odd-frequency paring state

Asano Tanaka Golubov Kashiwaya, Phys. Rev. Lett. **99**, 067005 (2007).



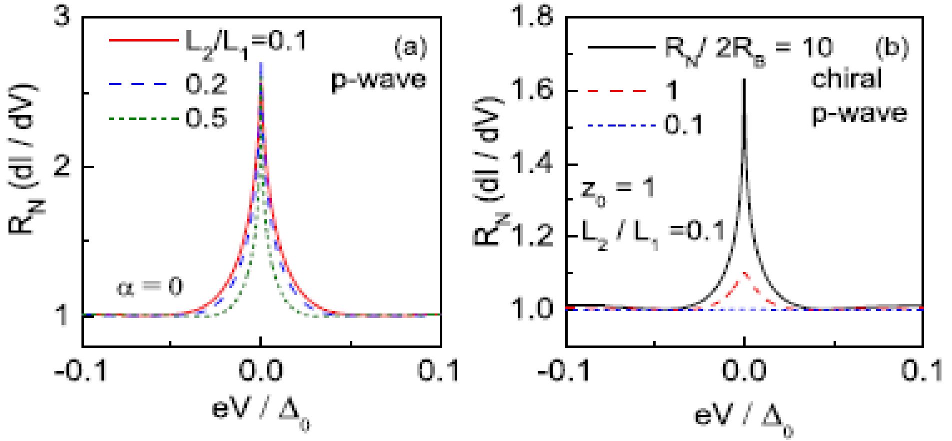


Kashiwaya, Maeno 2007

OTE (Odd-frequency spin-triplet even-parity)

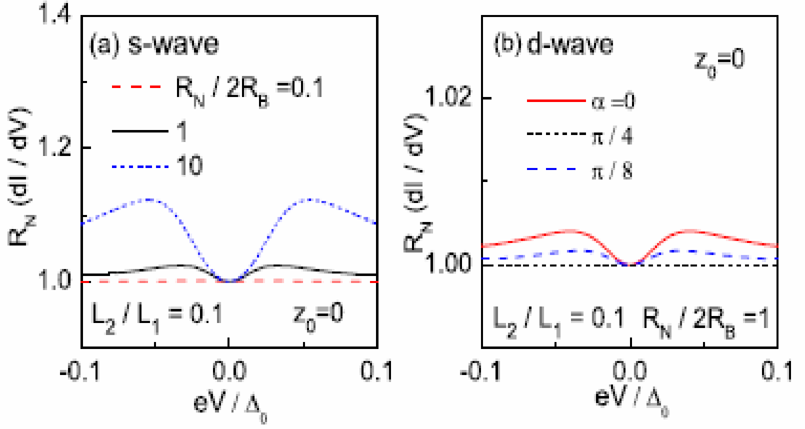
ESE (Even-frequency spin-singlet even-parity)

OTE state



Zero energy peak

ESE state



No Zero energy peak

Summary of calculation

Spin-singlet	zero-bias dip
Spin-triplet	zero-bias peak

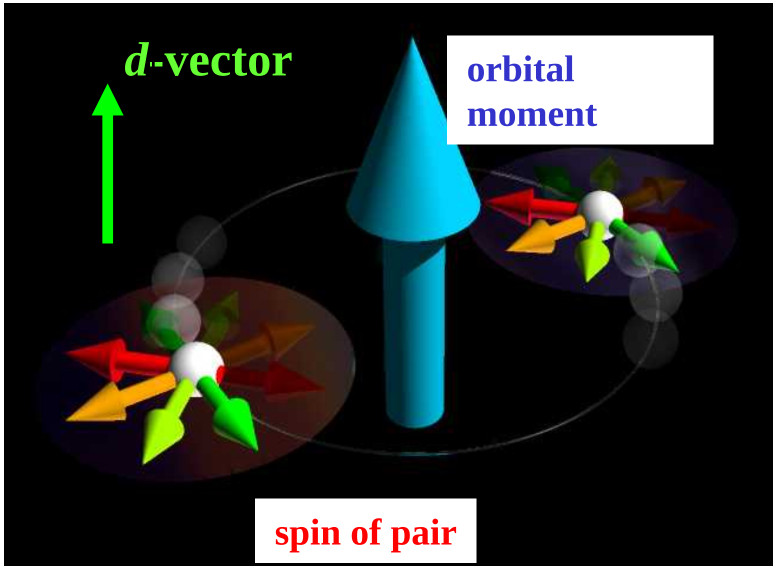
Symmetry of Cooper pairs in a normal metal

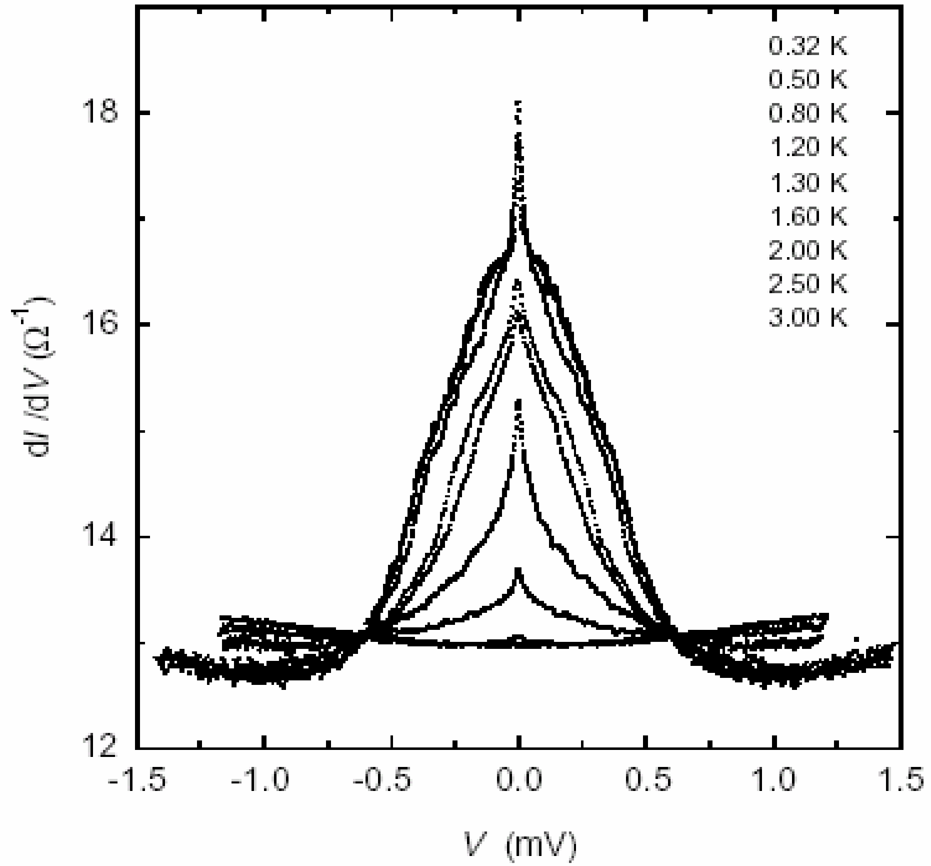
Spin-singlet	even-frequency (DOS dip)
Spin-triplet	odd-frequency (DOS peak)

MARS observed in triplet superconductor Sr_2RuO_4

Y. Maeno, G. Bednorz et al.
Nature 372 532 (1994)

(p-wave chiral)





Mao, Nelson, Jin, Liu and Maeno Phys. Rev. Lett. 87, 037003 (2001)

Kawamura, Yaguchi, Kikugawa Maeno Takayanagi
J. Phys. Soc. Jpn. 74 531 (2005)

Advantages of T-shaped geometry

Yes / No experiment

singlet : zero-bias dip
triplet: zero-bias peak

To peak or not to peak

No current through NS interface

effects of unexpected interface states is suppressed
high quality NS interface is not necessary

Possible by the accessible technique

microfabrication only on the normal segment

No applied magnetic field

test UGe_2

Requirements for measurement

[1] enough low temperature $k_B T < E_{Th} = \hbar D/L^2$

Au wire $E_F = 6.4 \times 10^4 [\mathrm{K}]$

$$\rho = 2.2 \times 10^{-6} [\Omega \mathrm{cm}]$$

$$E_{Th} = 1.4 \times 10^{-1}/L^2 [\mathrm{K}] \text{ at } L = 1\mu\mathrm{m}$$

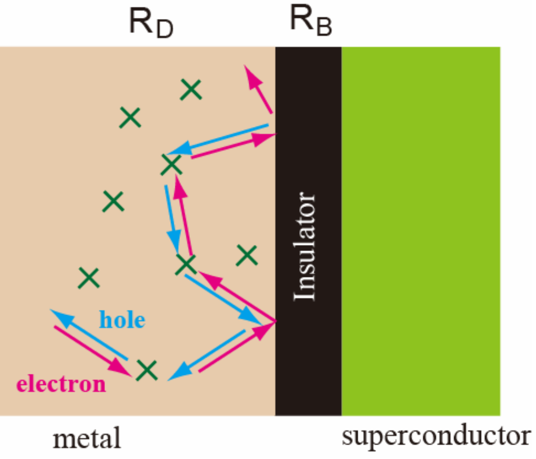
[2] microfabrication technique

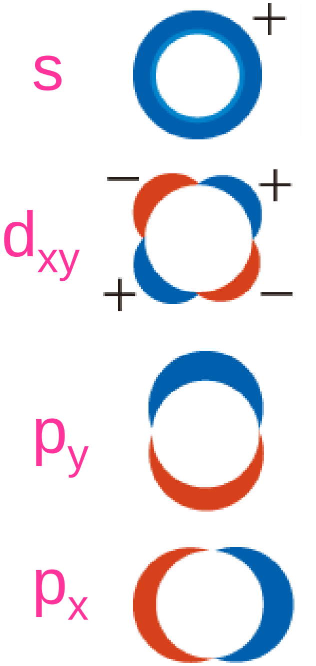
[3] technique to measure very small resistance

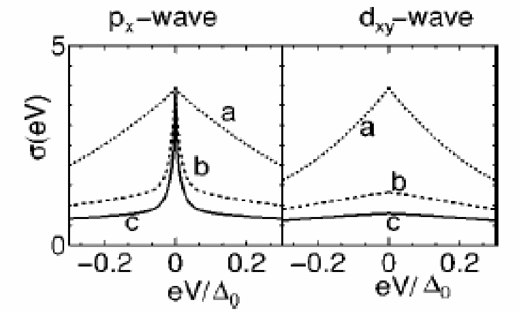
[4] optimistic personality for having NS contact

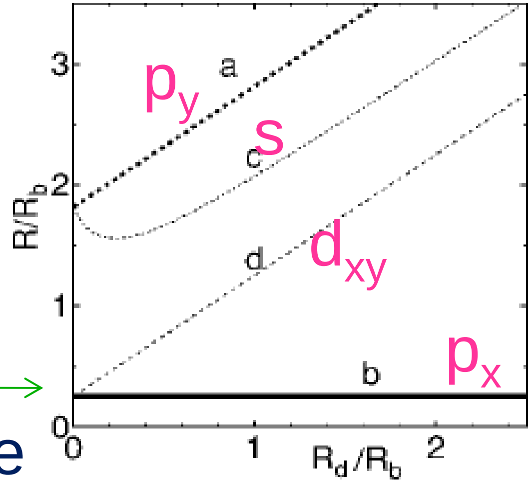
Don’t give up

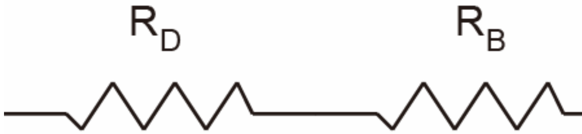
Anomalous Proximity Effect I









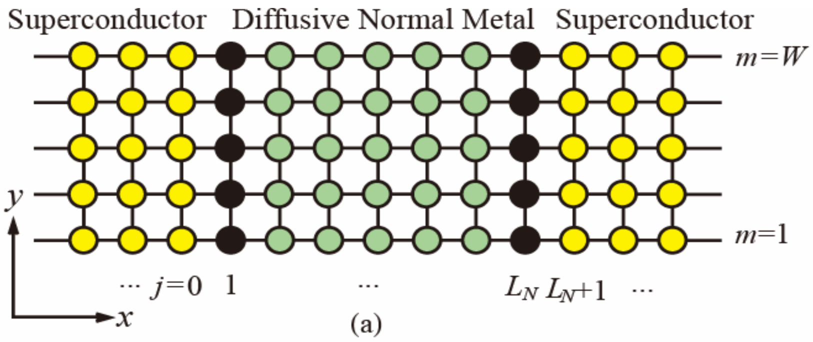


$R = R_D + R_B$?

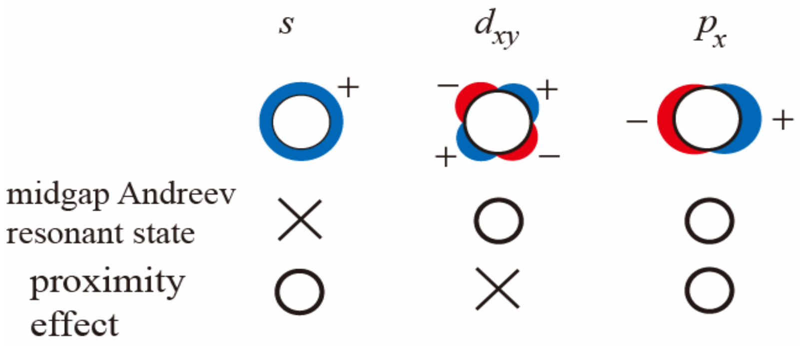
Sharvin resistance

Y. Tanaka and S. Kashiwaya, PRB **70**, 012507 (2004)

Anomalous Proximity Effect I I

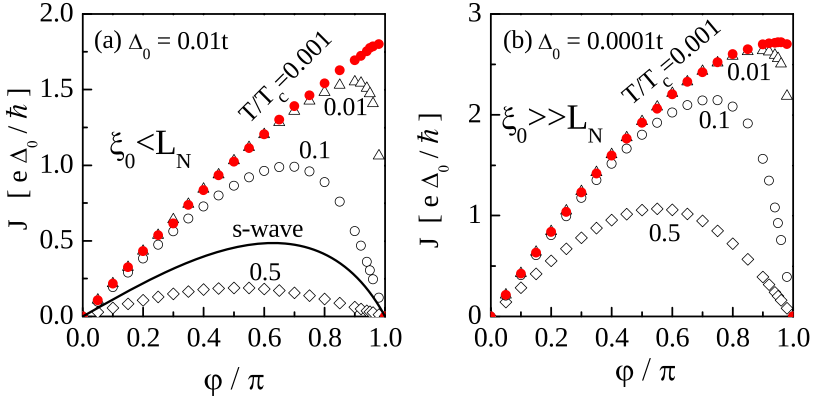


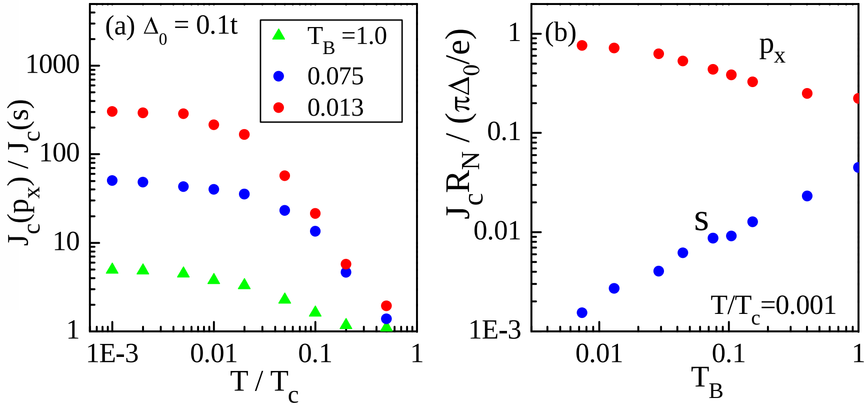
(a)



	s	d_{xy}	p_x
midgap Andreev resonant state	×	○	○
proximity effect	○	×	○

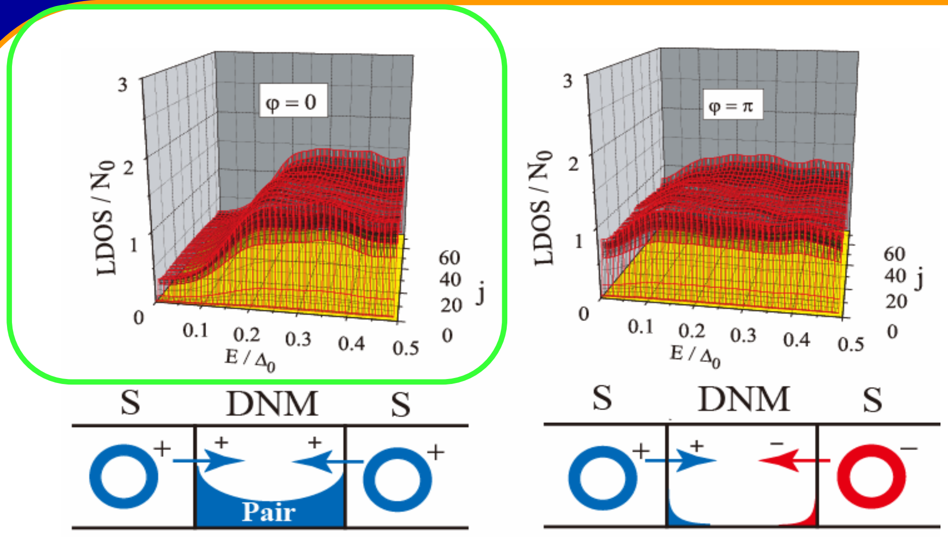
(b)





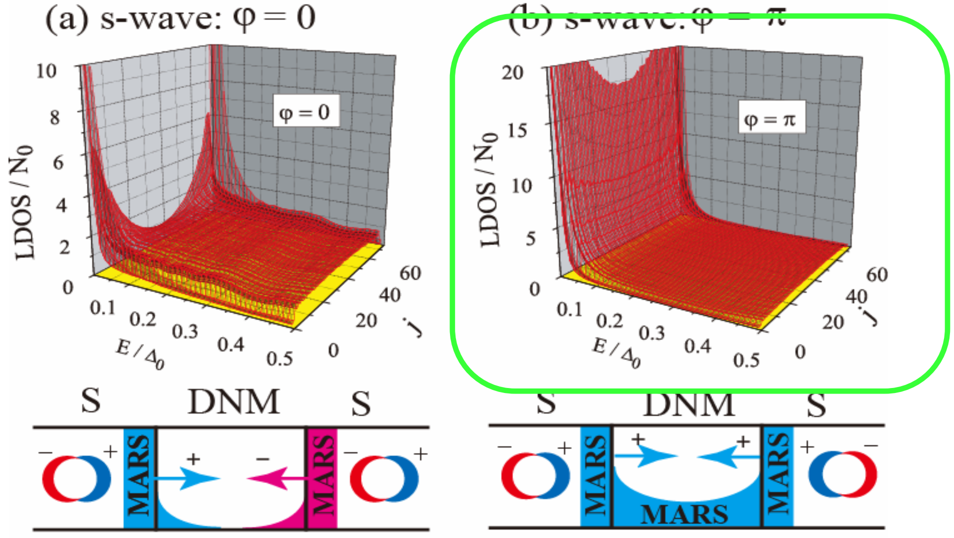
Y.Asano, Y.Tanaka, and S.Kashiwaya, PRL 96, 097007 (2006)

Origin of Anomaly



(a) s-wave: φ = 0

(b) s-wave: φ = π



(c) p_x-wave: φ = 0

(d) p_x-wave: φ = π

Zero-energy quasiparticle

↕

Odd-frequency pairs

singlet : dip at E=0
triplet : peak at E=0

Two aspects of proximity effect

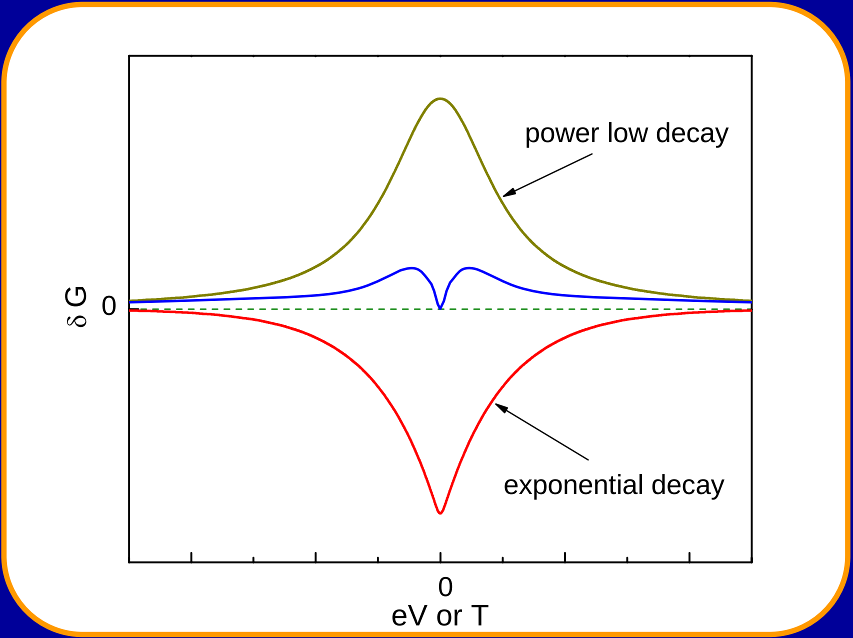
Spin-singlet

Proximity effect assists charge transport (positive)

Proximity effect suppresses DOS (negative) $\sigma = e^2 N_0 D$

Two effects exactly cancel at eV=T=0

Yu. V. Nazarov and T. H. Stoof,
PRL76, 823 (1996).
A. F. Volkov and H. Takayanagi,
PRL76, 4026 (1996).



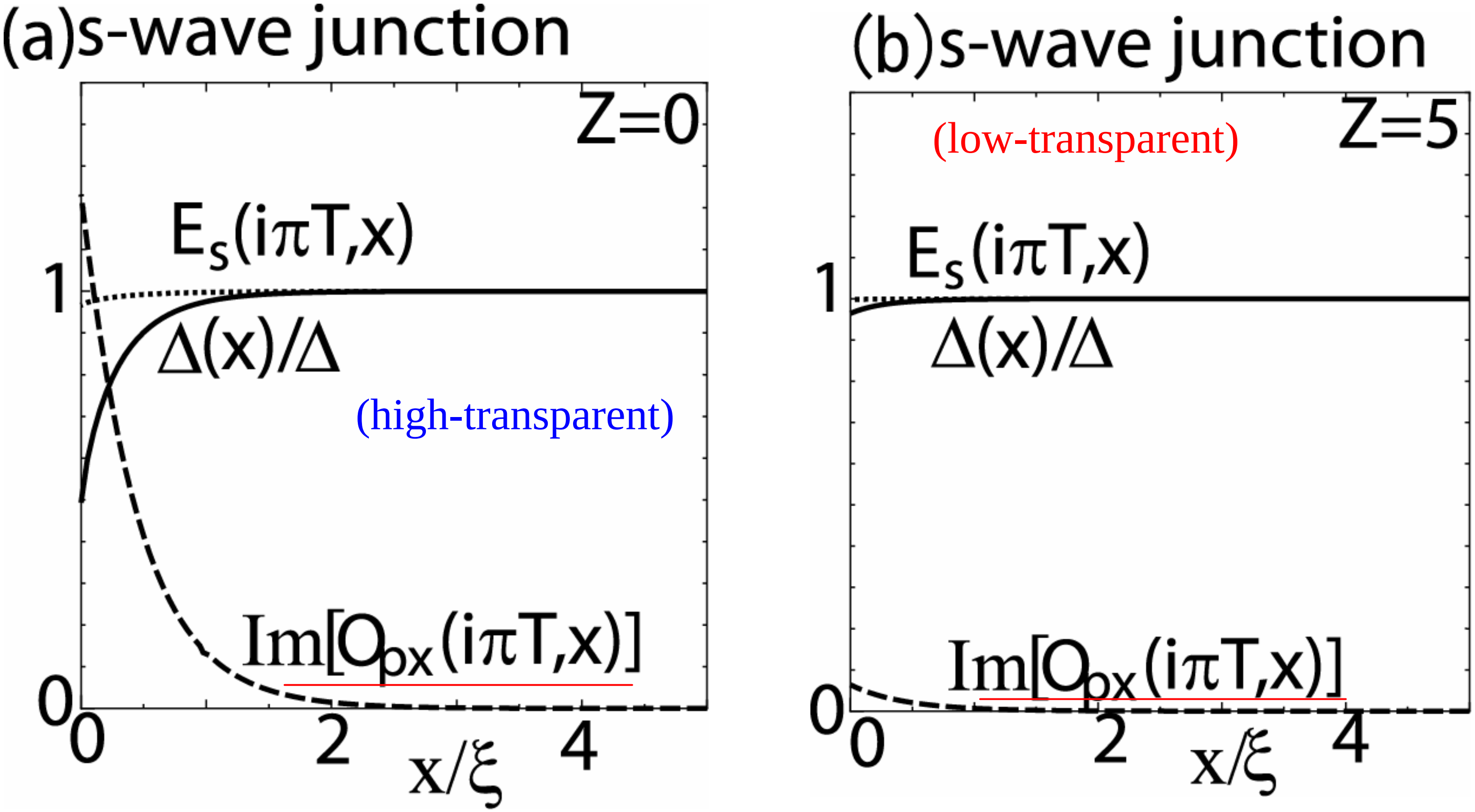
Spin-triplet

Proximity effect assist charge transport (positive)

Proximity effect enhances DOS at E=0 (positive)

Why zero-energy peak?

Even-frequency Spin-singlet even parity (ESE) superconductor

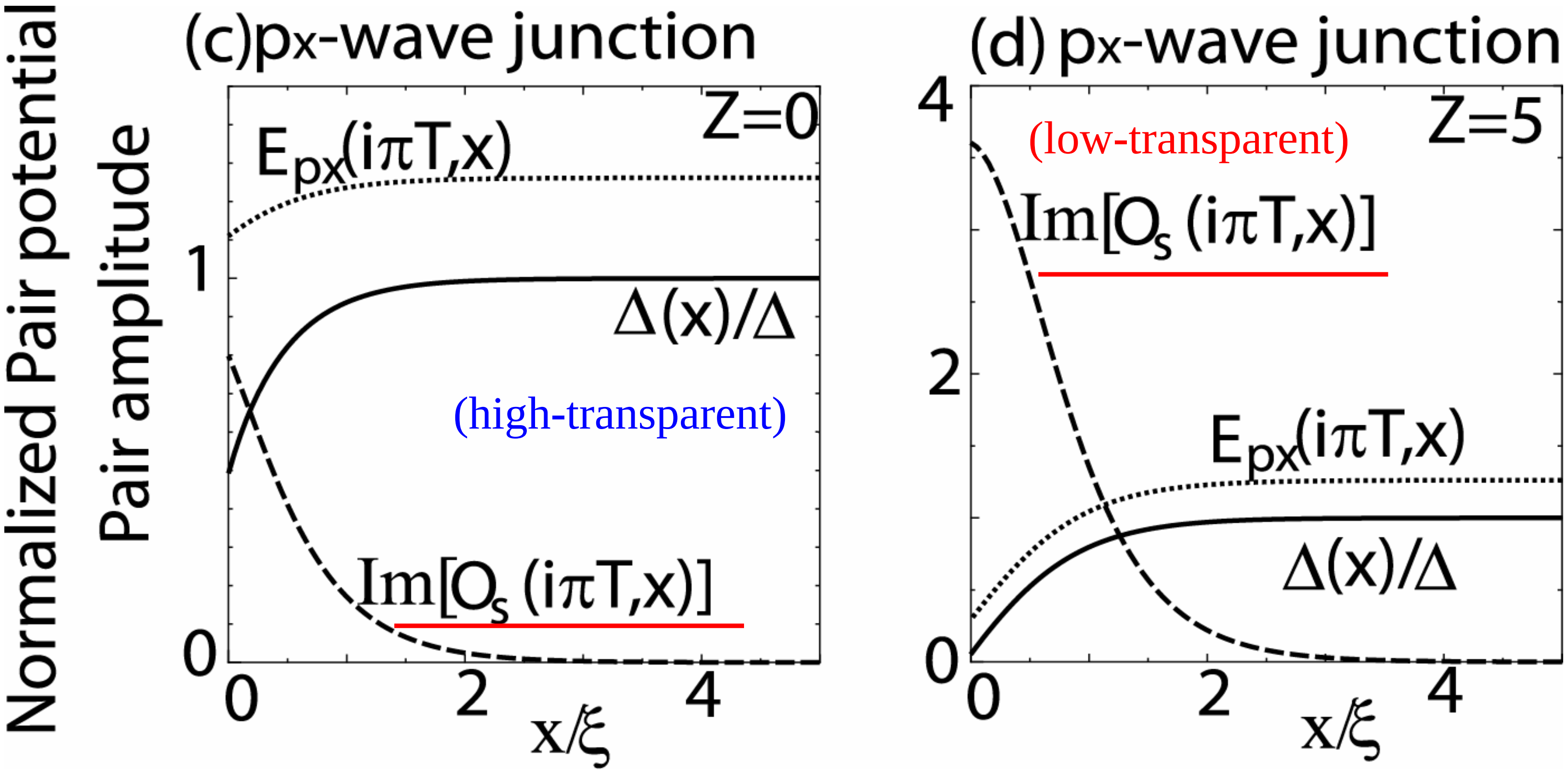


$E_s(i\omega_n, x)$ **even-frequency s-wave component**

$O_{px}(i\omega_n, x)$ **odd-frequency p_x-wave component**

Y. Tanaka, et al PRL 99 037005 (2007)

Even-frequency Spin-triplet odd-parity (ETO) superconductor

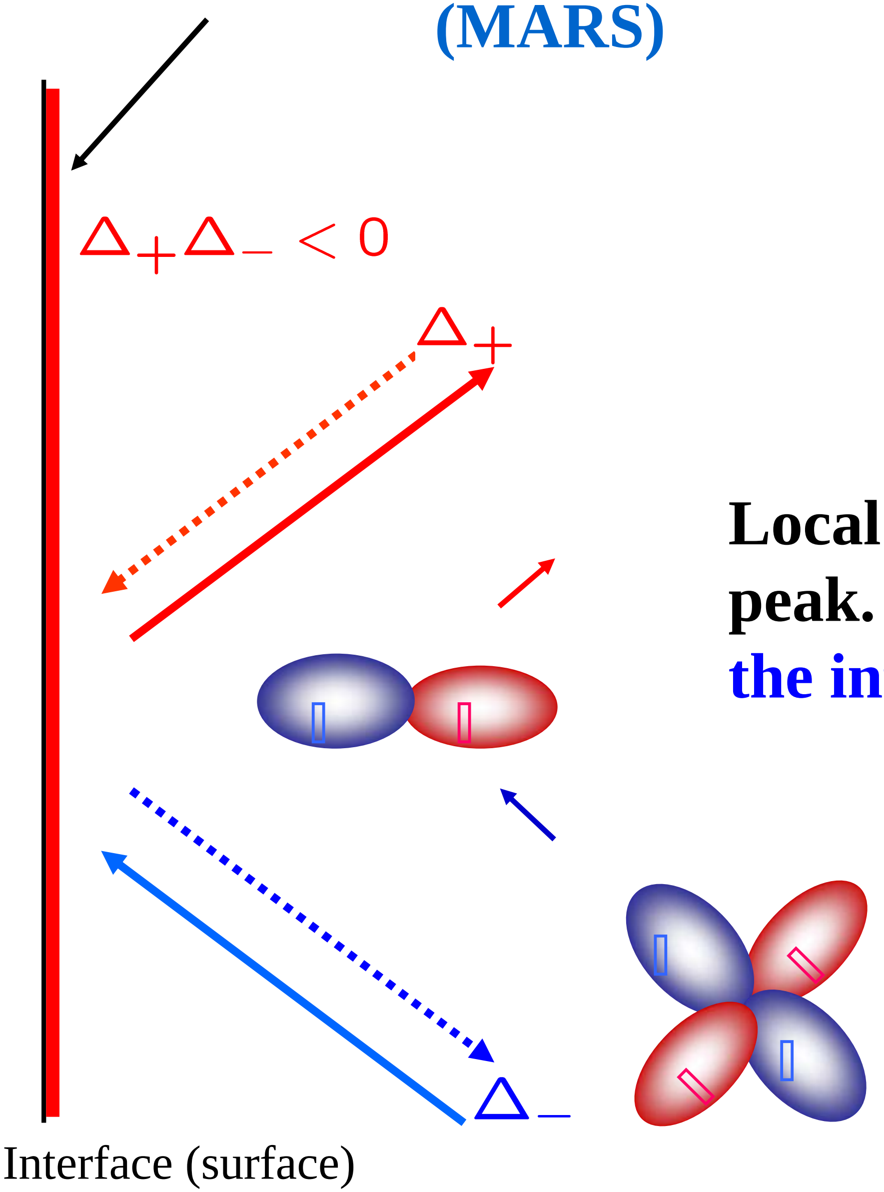


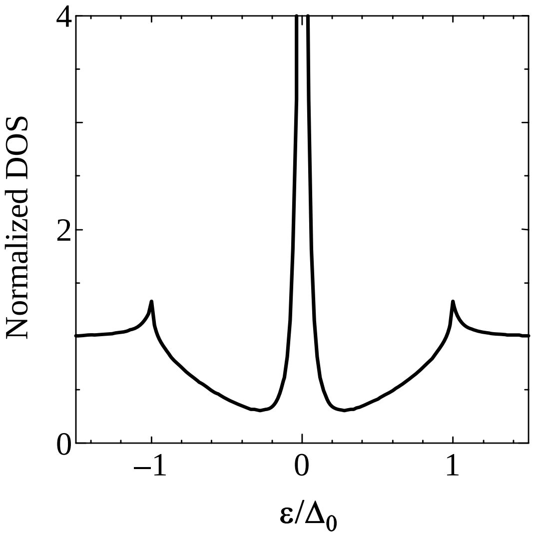
$E_{px}(i\omega_n, x)$ **even-frequency p_x-wave component**

$O_s(i\omega_n, x)$ **odd-frequency s-wave component**

Y. Tanaka, et al PRL 99 037005 (2007)

Mid gap Andreev resonant (bound) state (MARS)





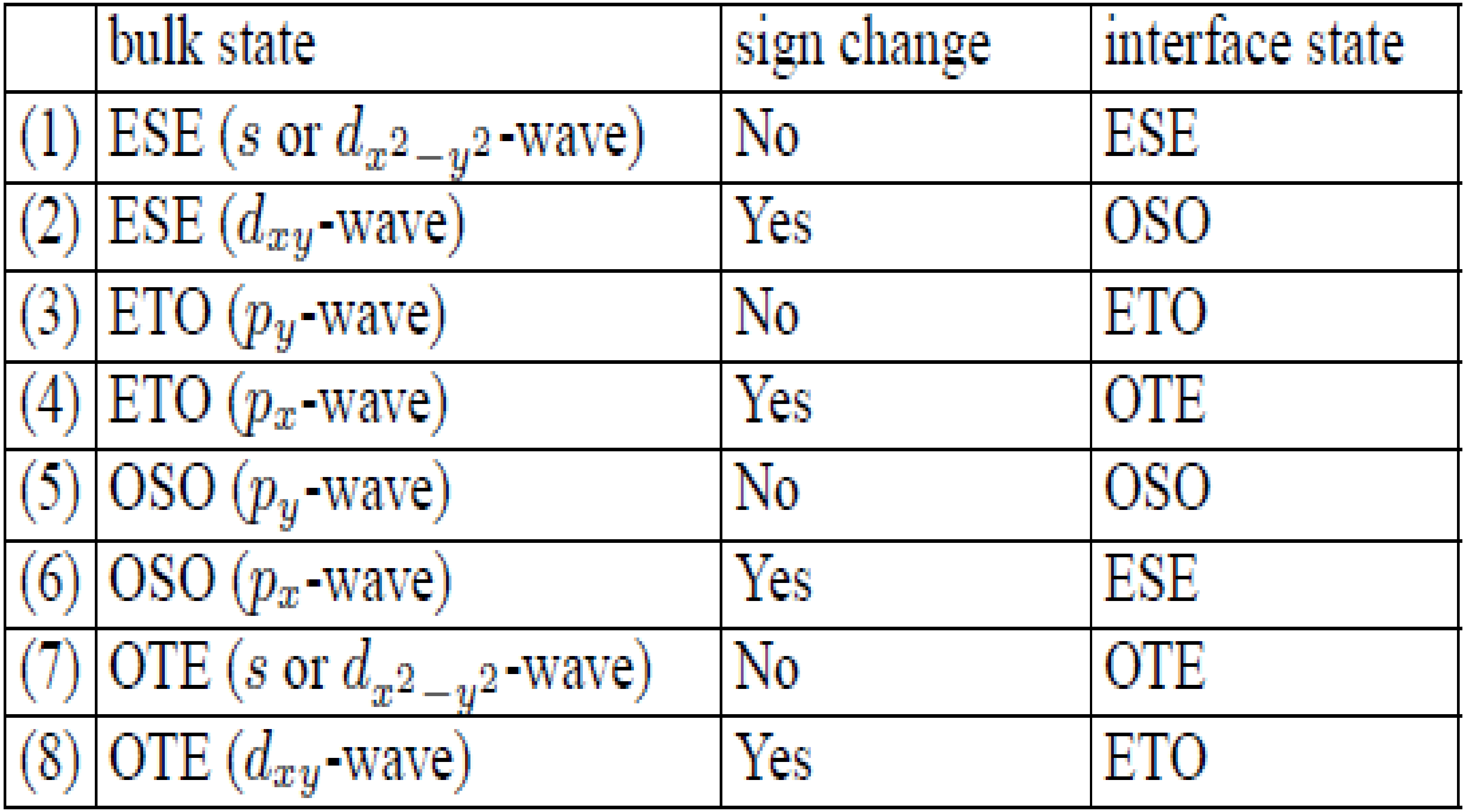
Local density of state has a zero energy peak. Sign change of the pair potential at the interface.

Tanaka Kashiwaya PRL 74 3451 (1995),
Rep. Prog. Phys. 63 1641 (2000)
Buchholz(1981) Hara Nagai(1986)
Hu(1994) Matsumoto Shiba(1995)
Ohashi Takada(1995)
Hatsugai and Ryu (2002)

Extensions to odd-frequency superconductors

Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
Phys. Rev. Lett. 99 037005 (2007)

Low transparency limit

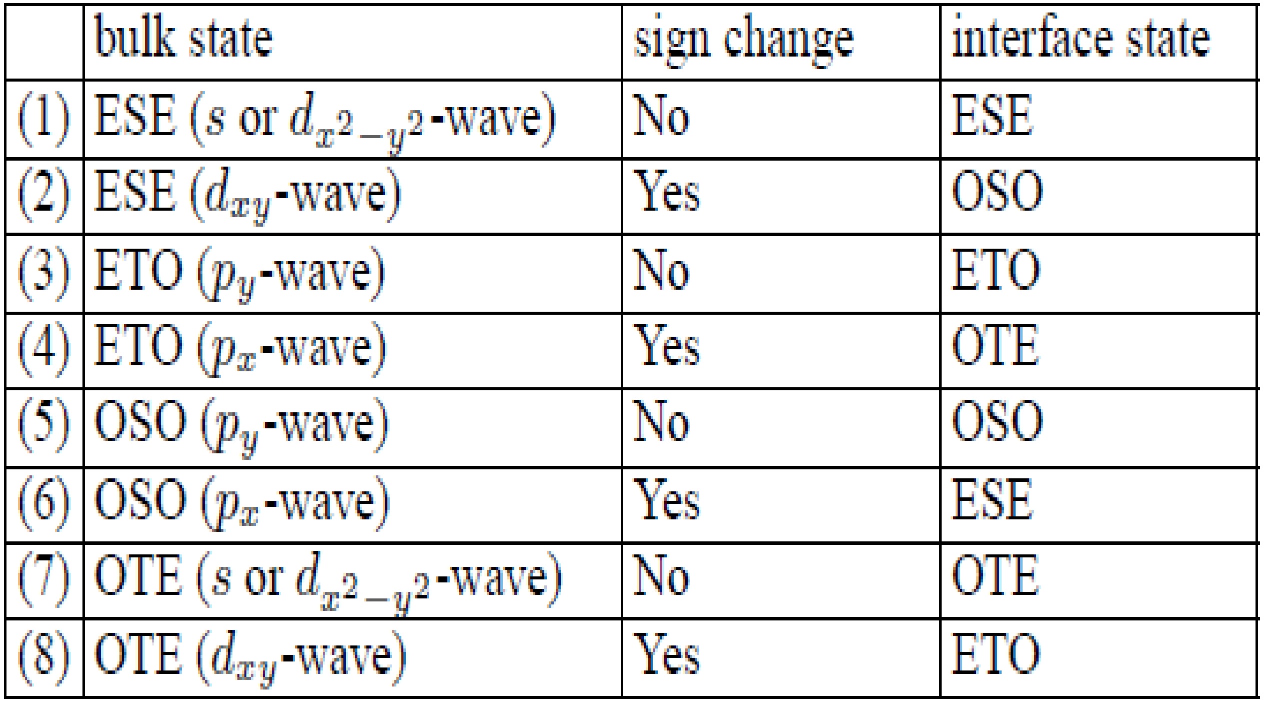


	bulk state	sign change	interface state
(1)	ESE (s or $d_{x^2-y^2}$-wave)	No	ESE
(2)	ESE (d_{xy}-wave)	Yes	OSO
(3)	ETO (p_y-wave)	No	ETO
(4)	ETO (p_x-wave)	Yes	OTE
(5)	OSO (p_y-wave)	No	OSO
(6)	OSO (p_x-wave)	Yes	ESE
(7)	OTE (s or $d_{x^2-y^2}$-wave)	No	OTE
(8)	OTE (d_{xy}-wave)	Yes	ETO

- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity**

PRL 99 037005 (2007)

Josephson couplings between even-frequency superconductor and odd-frequency one



	bulk state	sign change	interface state
(1)	ESE (s or $d_{x^2-y^2}$-wave)	No	ESE
(2)	ESE (d_{xy}-wave)	Yes	OSO
(3)	ETO (p_y-wave)	No	ETO
(4)	ETO (p_x-wave)	Yes	OTE
(5)	OSO (p_y-wave)	No	OSO
(6)	OSO (p_x-wave)	Yes	ESE
(7)	OTE (s or $d_{x^2-y^2}$-wave)	No	OTE
(8)	OTE (d_{xy}-wave)	Yes	ETO

1. (1) and (6)
2. (2) and (5)
3. (3) and (8)
4. (4) and (7)

Presence of the Lowest order Josephson coupling

PRL 99 037005 (2007)

Previous theory

Abrahams, Balatsky, Scalapino, and Schrieffer

Phys. Rev. B 52, 1271 - 1278 (1995)

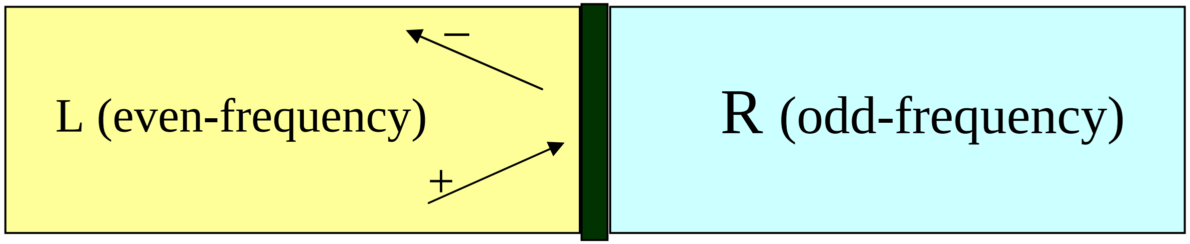
There is no lowest-order Josephson coupling between odd- and even-frequency superconductors.

Interface induced state is neglected!!

Josephson current

$$R_N I(\varphi) = \frac{\pi}{2e}\sum_{\sigma} k_B T \sum_{\omega}\{< f_{1L+} f_{1R+} + f_{2L+} f_{2R+} > \sin\varphi + < f_{1L+} f_{2R+} - f_{2L+} f_{1R+} > \cos\varphi\}$$

(Lowest Order coupling)



φ

(Macroscopic phase difference between two superconductors)

f_{1L+} f_{1R+} **Interface state**

(1)L-side (Even-frequency superconductor)

f_{1L+} Odd function of Matsubara

f_{2L+} Even function of Matsubara

(2)R-side (odd-frequency superconductor)

f_{1R+} Even function of Matsubara

f_{2R+} Odd function of Matsubara

→ $\cos\varphi$

Anomalous current phase relation

PRL 99 037005 (2007)

Underlying physics (1)

Near the interface, even and odd-parity pairing states can mix due to the breakdown of the translational symmetry.

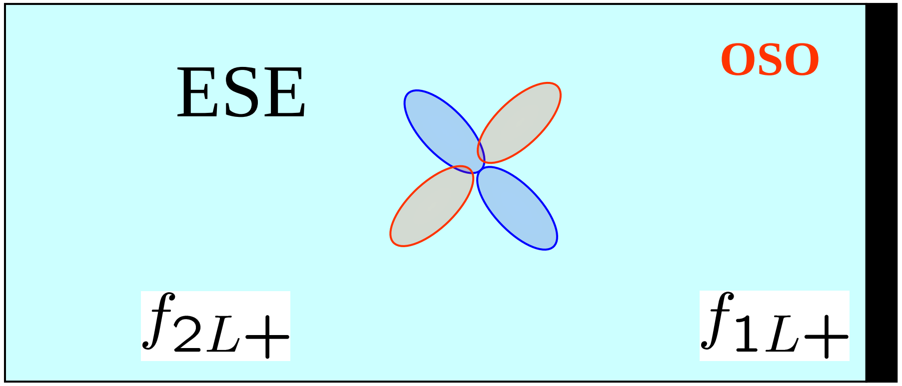
↓

The Fermi-Dirac statistics then dictates that the induced pair amplitude at the interface should be odd (even) in frequency where the bulk pair potential has an even (odd)-frequency component.

PRL 99 037005 (2007)

Underlying physics (2)

To be compatible with the time reversal invariance in each superconductor, the phase of the interface induced pair amplitude undergoes a π/2 shift from that of the bulk one.

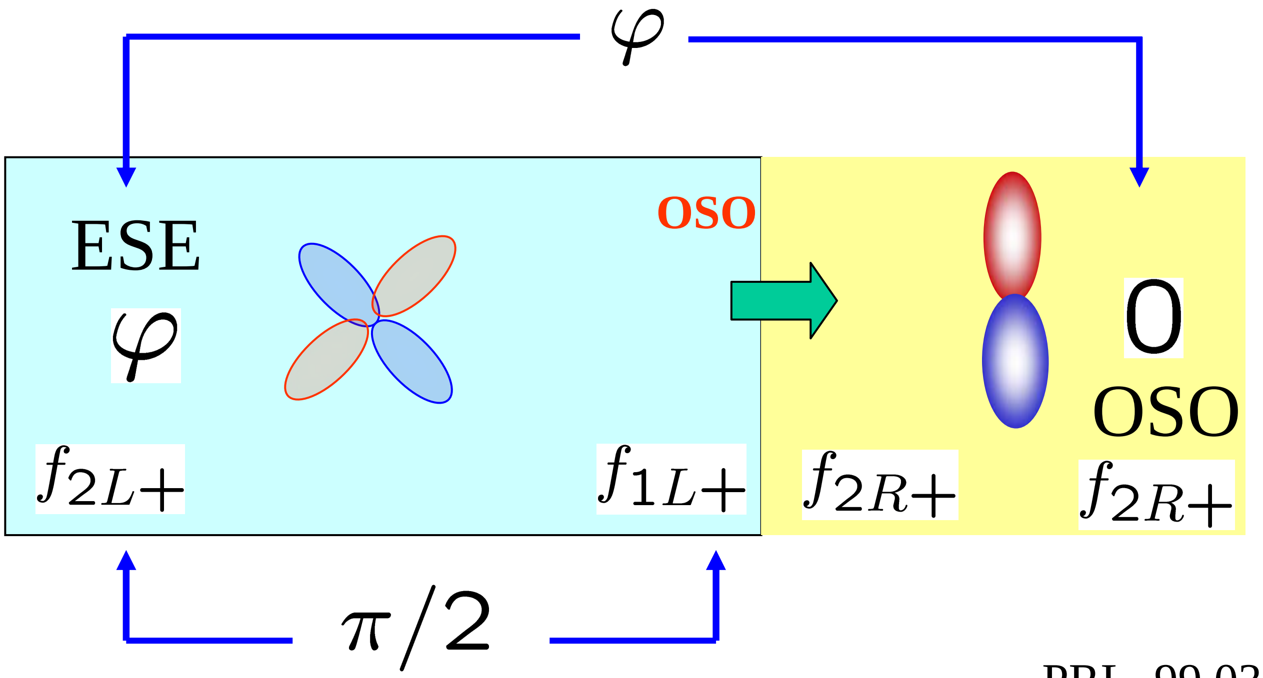


The phase of f_{1L+} has a π/2 shift from that of f_{2L+}.

PRL 99 037005 (2007)

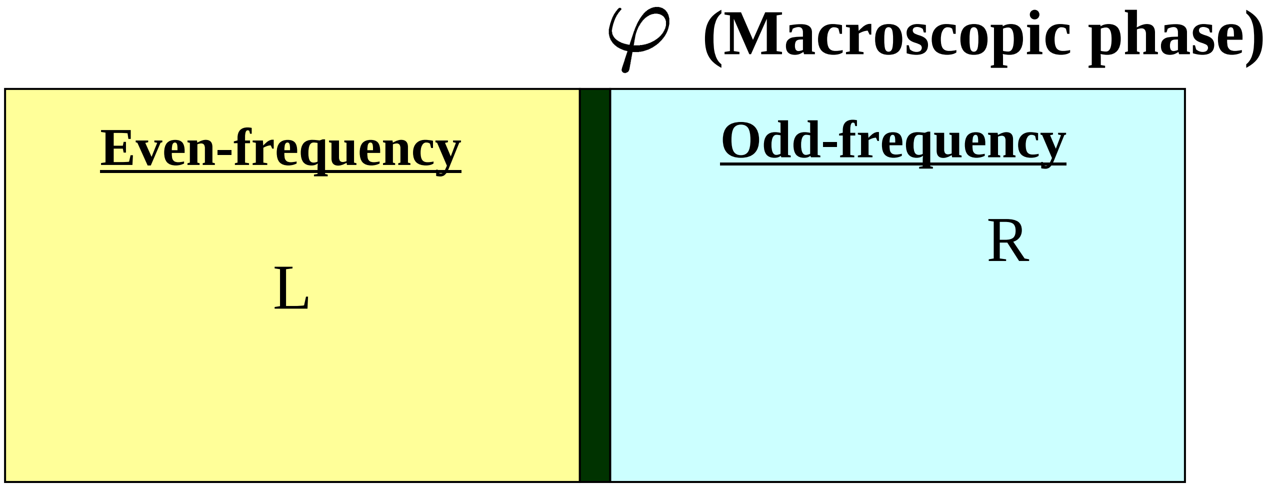
Underlying physics (3)

This twist of the phase of the pair amplitude gives rise to an anomalous Josephson current, where the current phase relation is proportional cosφ.



PRL 99 037005 (2007)

Breakdown of the time reversal symmetry when we make a junction



φ **(Macroscopic phase)**

Even-frequency

L

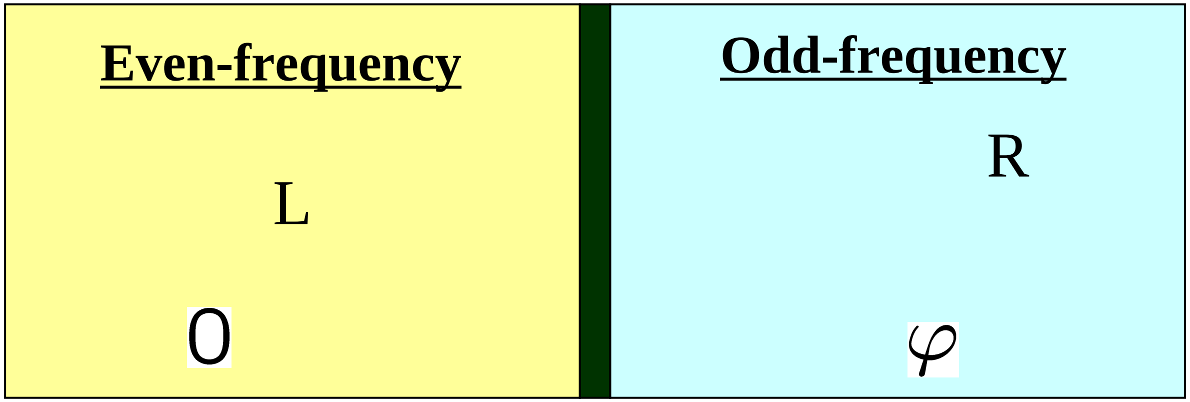
Odd-frequency

R

Although both superconductors **do not break the time reversal symmetry themselves**, the resulting Josephson coupling **breaks the time reversal symmetry** since the parities with respect to frequency dependence in even- and odd-frequency superconductors differ from each other.

PRL 99 037005 (2007)

Anomalous Josephson effect between even- and odd-frequency superconductors



Current-phase relation becomes $\cos\varphi$

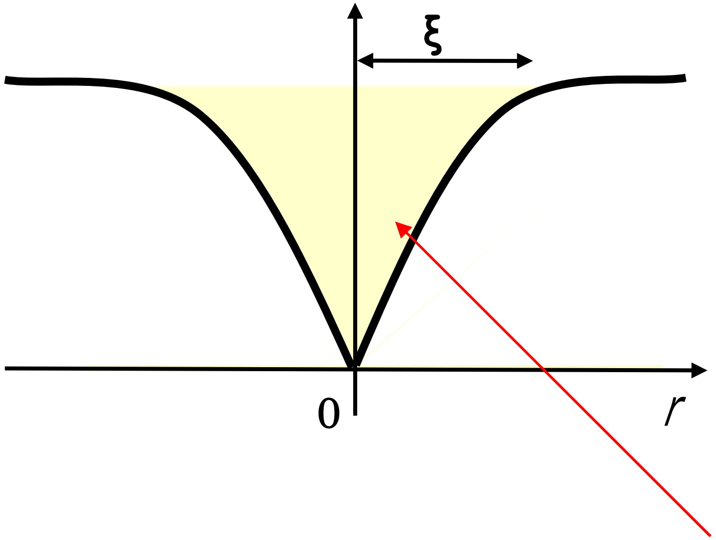
Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
PRL 99 037005 (2007)

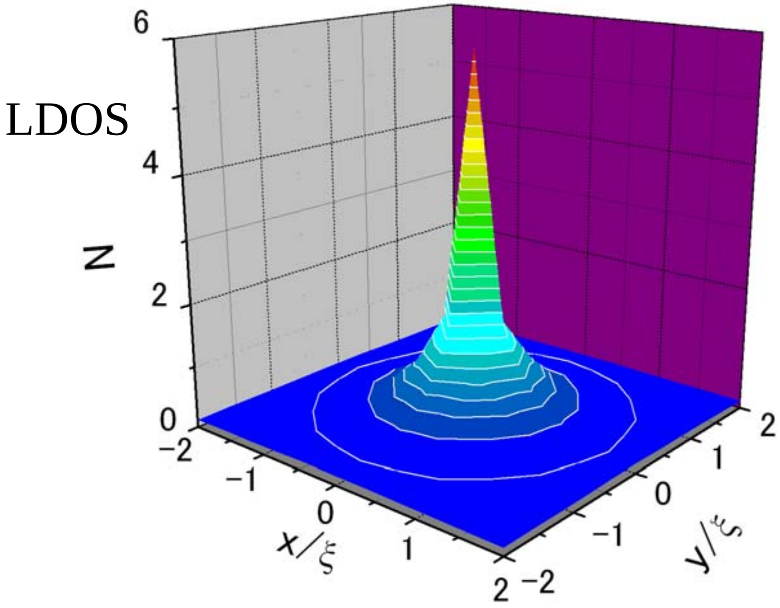
Vortex core state (s-wave)

$$\Delta(\mathbf{r}) = \Delta_0 \tanh\left(\frac{\sqrt{x^2+y^2}}{\xi}\right)\frac{x+iy}{\sqrt{x^2+y^2}}$$

Core center sits at the origin
□□□□□0□ in s-wave
superconductor

Pair potential

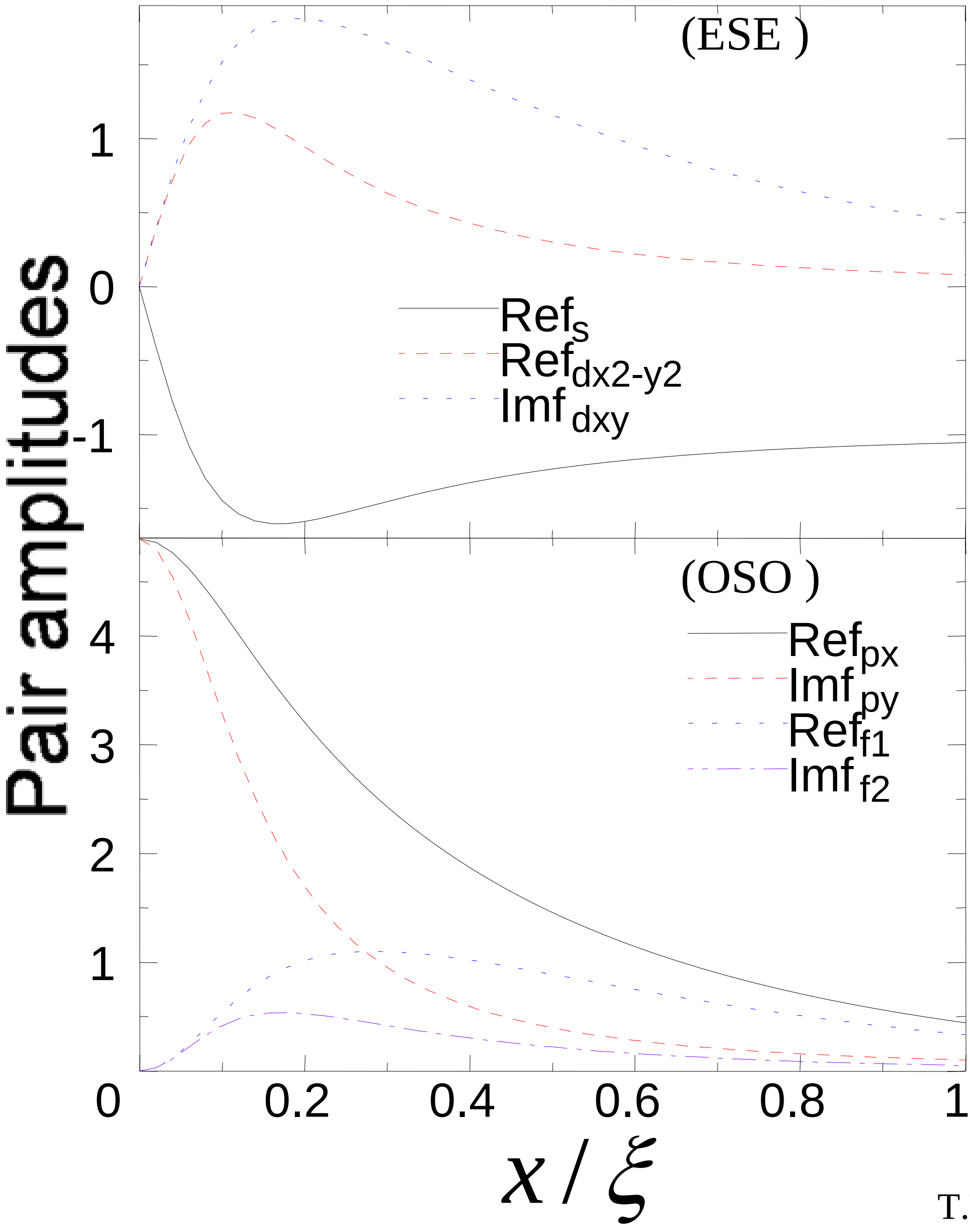




How about the symmetry of the pairing amplitude
around the core?

T.Yokoyama, Y. Tanaka, A.A. Golubov **arXiv: 0710.2967**

Spatial dependence of pair amplitude



□y□E=0□

$$f \cong f_s + \underline{f_{px}\cos\theta + f_{py}\sin\theta}$$

$$+ f_{dx^2-y^2}\cos 2\theta + f_{dxy}\sin 2\theta$$

$$\underline{+ f_{f1}\cos 3\theta + f_{f2}\sin 3\theta}$$

$$\underline{\mathrm{Re}\ f_{px} = \mathrm{Im}\ f_{py}}$$

$$\underline{\mathrm{Im}\ f_{px} = -\mathrm{Re}\ f_{py}}$$

(core center)

At the core center only odd-frequency singlet chiral p-wave pairing amplitude can survive

T.Yokoyama, Y. Tanaka, A.A. Golubov **arXiv: 0710.2967**

Symmetry of the vortex core

$$\Delta(\mathbf{r}) = \Delta_0 \exp(il\varphi) \tanh\left(\frac{\sqrt{x^2+y^2}}{\xi}\right)\left(\frac{x+iy}{\sqrt{x^2+y^2}}\right)^m$$

l angular momentum

m vorticity

l	*m*	bulk	Center of the vortex core
Even	Even	ESE (s-wave..)	ESE
Even	Odd	ESE (s-wave..)	**OSO**
Odd	Even	ETO (chiral p-wave)	ETO
Odd	Odd	ETO (chiral p-wave)	**OTE**

ESE (Even-frequency spin-singlet even-parity)
ETO (Even-frequency spin-triplet odd-parity)
OTE (Odd-frequency spin-triplet even-parity)
OSO (Odd-frequency spin-singlet odd-parity

Angular momentum at the center of core
l+m

Yokoyama Tanaka Golubov(2007)