

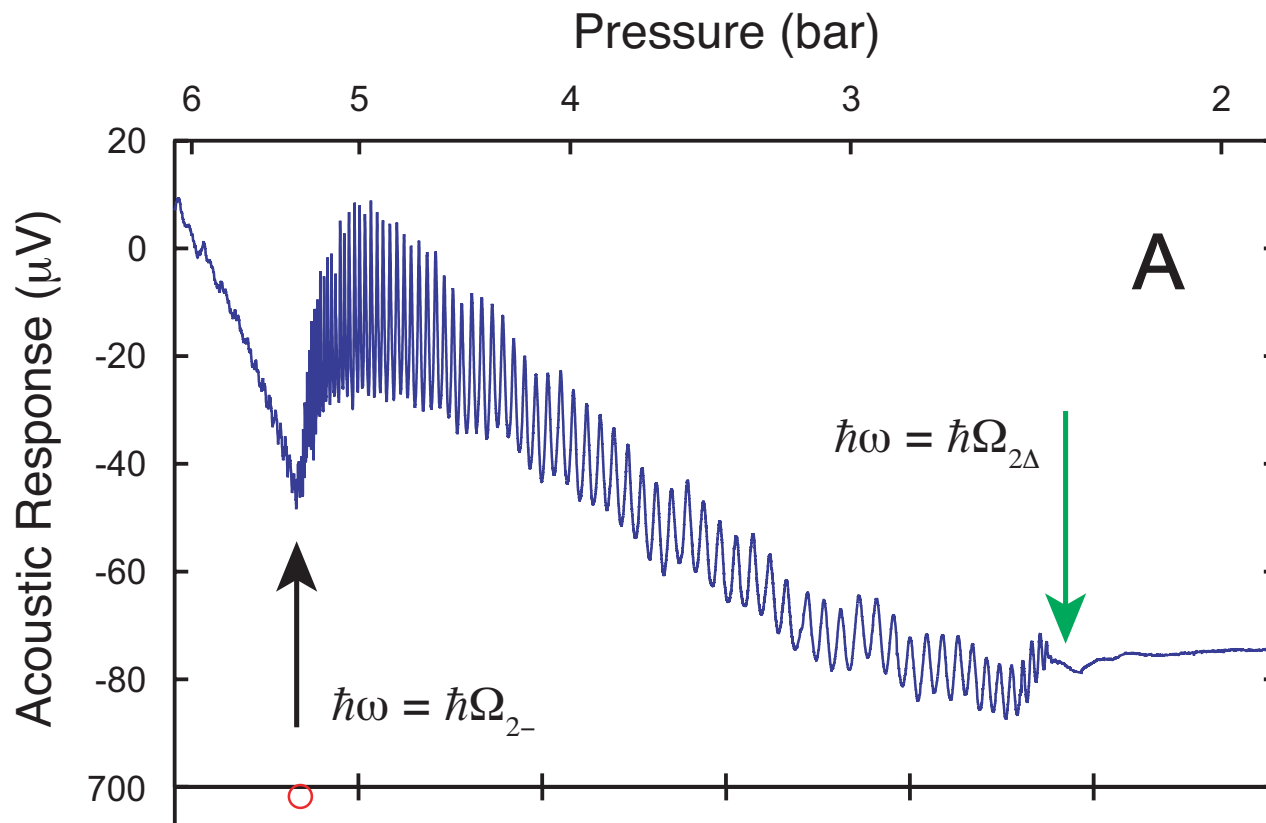
Sounds of Broken Symmetry

in

Superfluid ^3He

Jim Sauls

Northwestern University



Transverse Sound
Cavity Modes in $^3\text{He-B}$

John Davis et al. (2007)
Bill Halperin (ULT @ NU)

Collisionless Sound Modes in Normal ^3He

Deformation of the Fermi Surface

$$\delta\Phi(\hat{p}) = \sum_{lm} \delta\Phi_{lm} P_l^m(\cos\vartheta) e^{im\varphi}$$

$$\delta\varepsilon(\hat{p}) = \sum_{lm} F_l \delta\Phi_{lm} P_l^m(\cos\vartheta) e^{im\varphi}$$

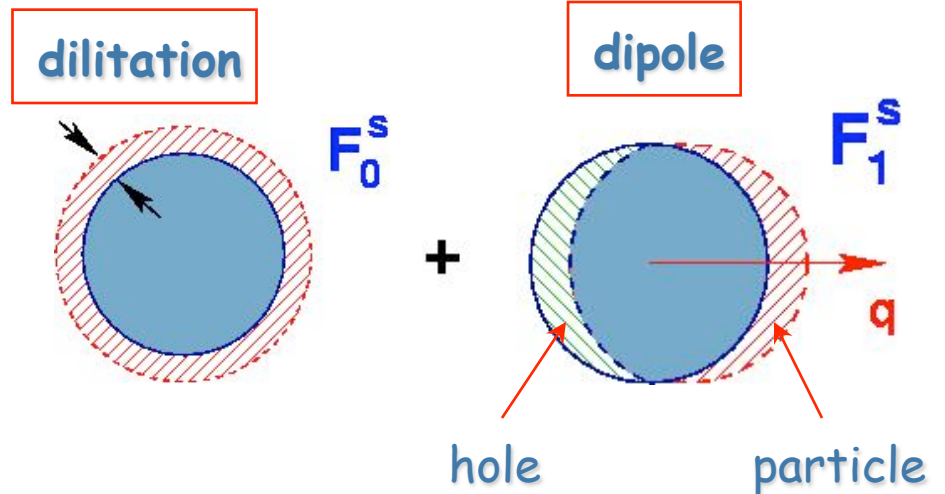
$$\omega\tau(T) > 1$$

$$\delta\Phi_{00} \propto \delta n$$

$$\delta\Phi_{10} \propto \hat{q} \times \delta \vec{j}$$

Longitudinal (zero) sound

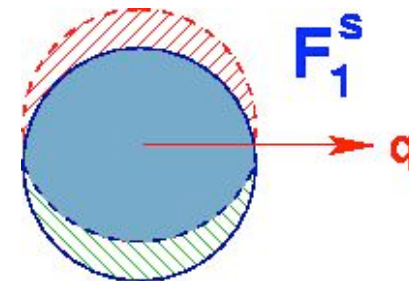
$$\frac{c_0 - c_1}{c_1} \simeq \frac{4}{5} \left(\frac{1}{F_0^s} \right)$$



Transverse (zero) sound

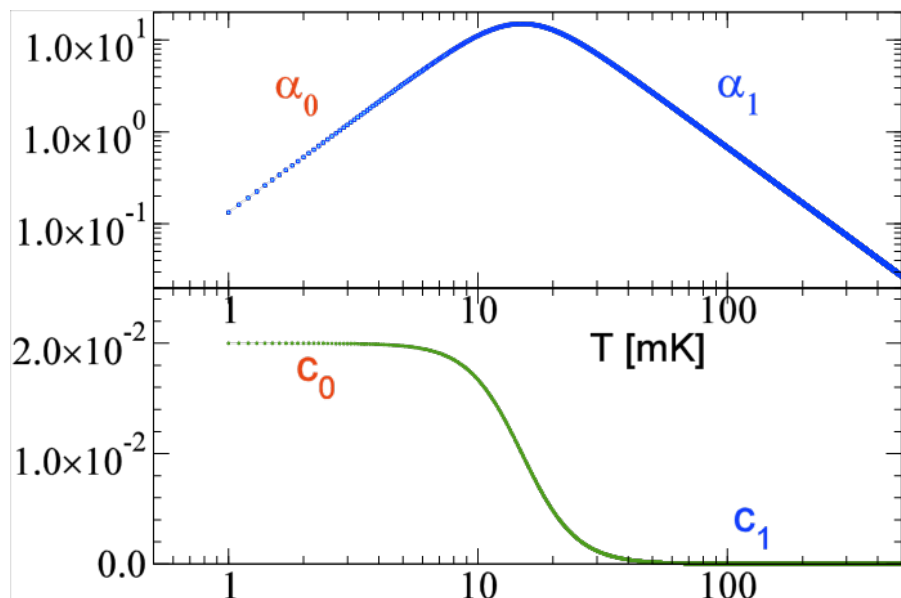
$$\frac{c_t}{v_f} \simeq \frac{F_1^s}{15}$$

$$\left. \begin{aligned} \delta\Phi_{1,+1} &\propto \delta j_{\text{RCP}} \\ \delta\Phi_{1,-1} &\propto \delta j_{\text{LCP}} \end{aligned} \right\}$$



Landau (1958)

Longitudinal (zero) sound



Abel, Anderson & Wheatley (1966)

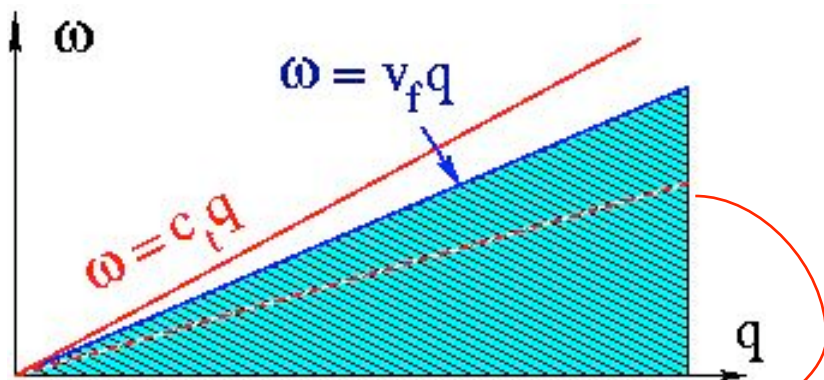
$$\alpha_0 ; \frac{1}{c_0 \tau} \propto T^2$$

$$\alpha_1 ; \omega^2 \eta \propto T^{-2}$$

$$\frac{c_0 - c_1}{c_1} ; \frac{4}{5} \left(\frac{1}{F_0^s} \right) \approx 10^{-2}$$

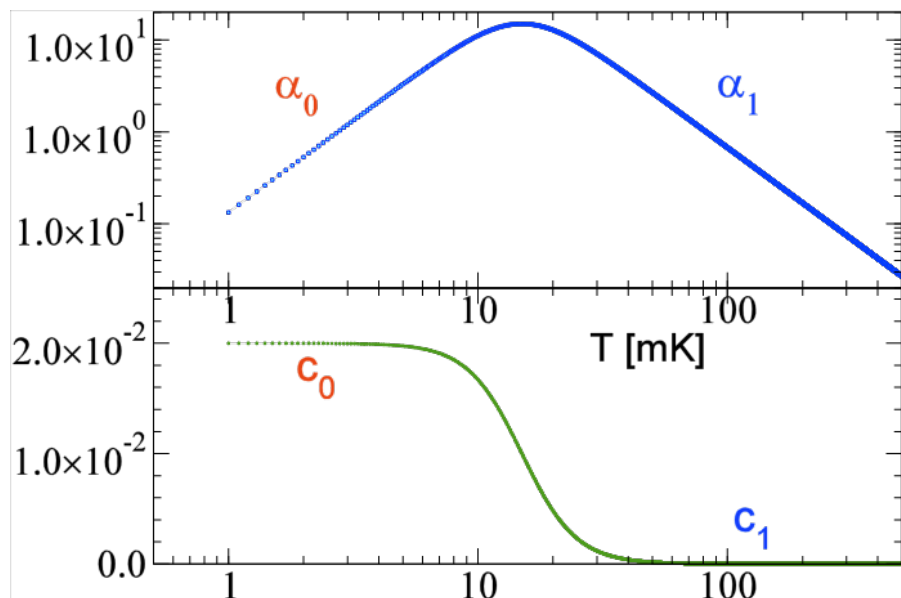
Transverse (zero) sound

No conclusive Observation of TZS in **normal** ^3He so far.



particle-hole continuum

Longitudinal (zero) sound



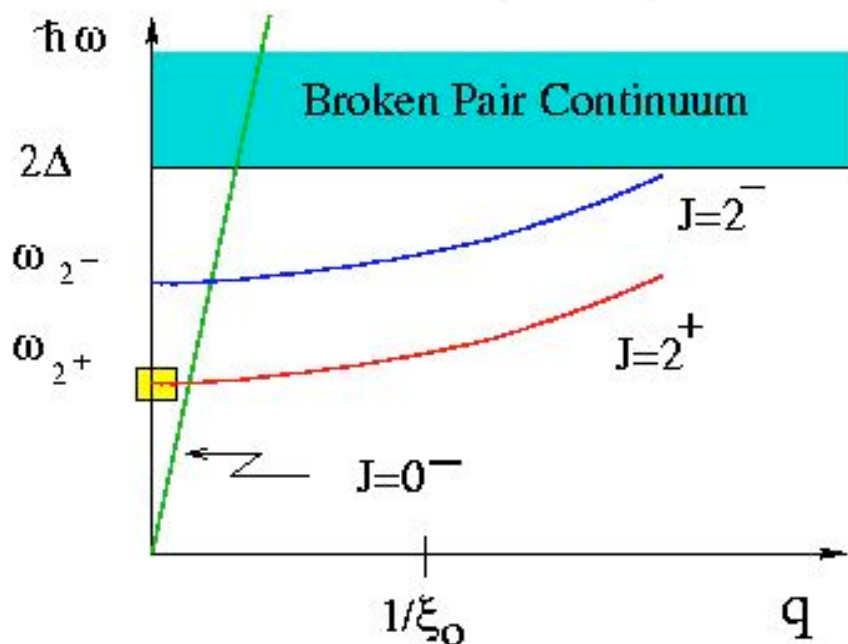
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Transverse (zero) sound



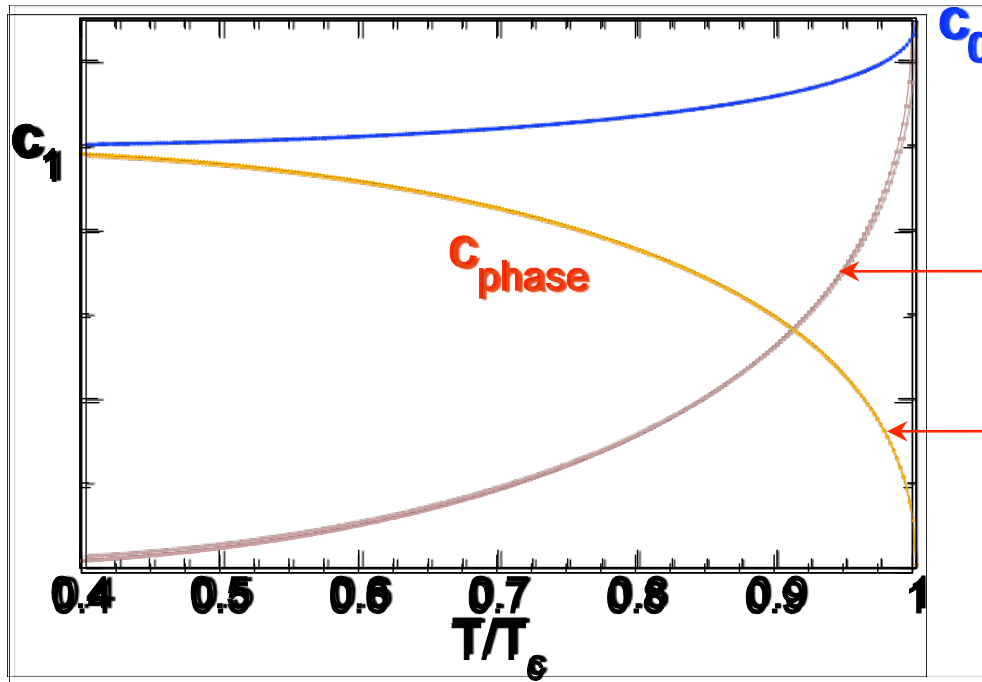
TZS in **superfluid $^3\text{He-B}$**

Restoring Force - coupling to
Collective mode - "pair exciton"

(G. Moores and JAS 1992)

Observed (Y. Lee et al. 1999)

What is the Fate of Zero Sound for $T < T_c$?



qp restoring force

$$E_p \geq \Delta$$

Energy Gap

Collective Mode

(Anderson-Bogoliubov)

$$\Delta(\mathbf{R}, t) = \Delta_0 e^{i\varphi(\mathbf{R}, t)}$$

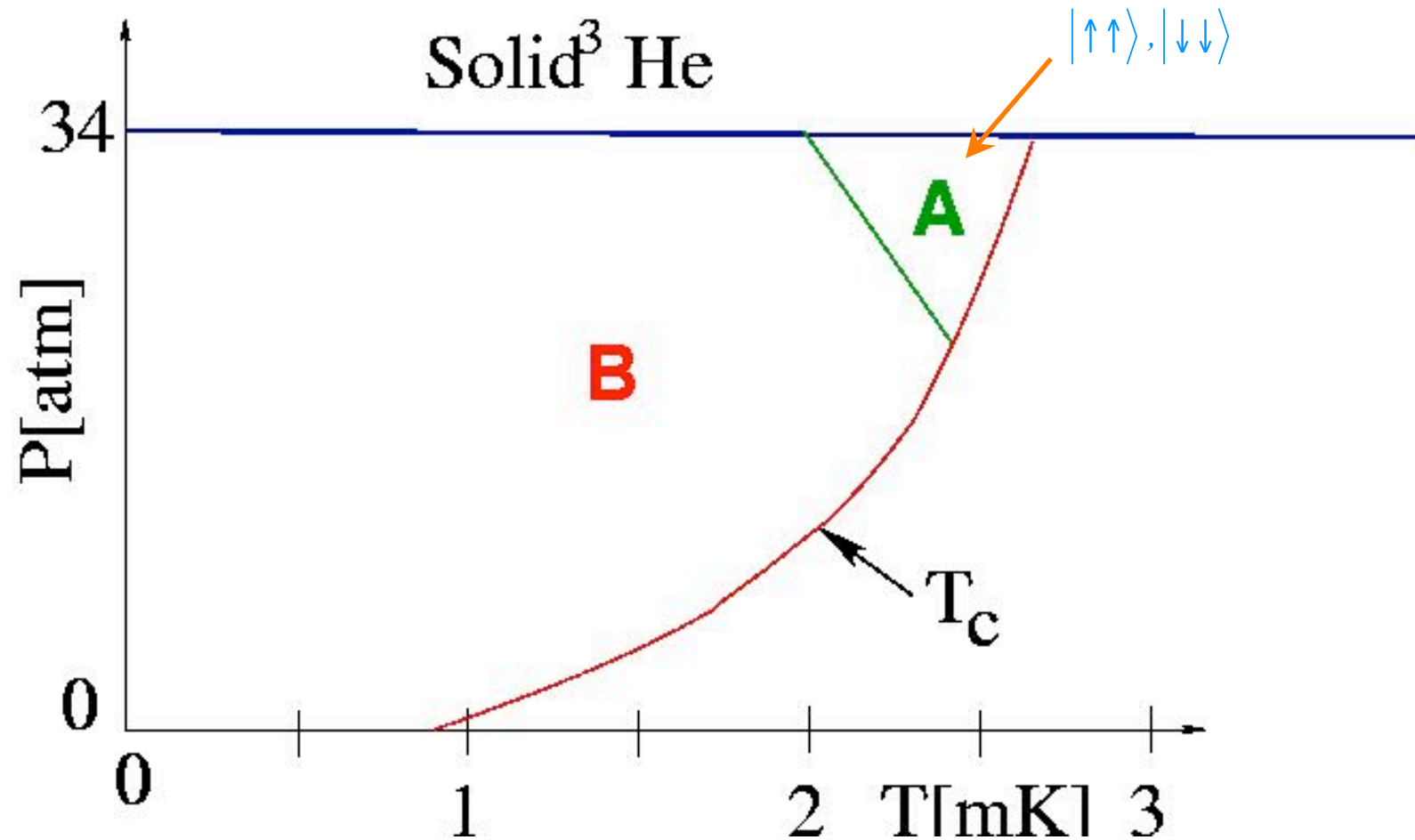


$$\delta\varphi(\mathbf{q}, \omega) = \frac{\omega}{\omega^2 - \frac{1}{3}q^2 v_f^2} \delta\mu$$

$$\vec{\nabla} \cdot \vec{j} + \partial_t \rho = 0$$

$$\vec{j} = \rho \vec{\nabla} \varphi$$

Superfluid Phases of ^3He



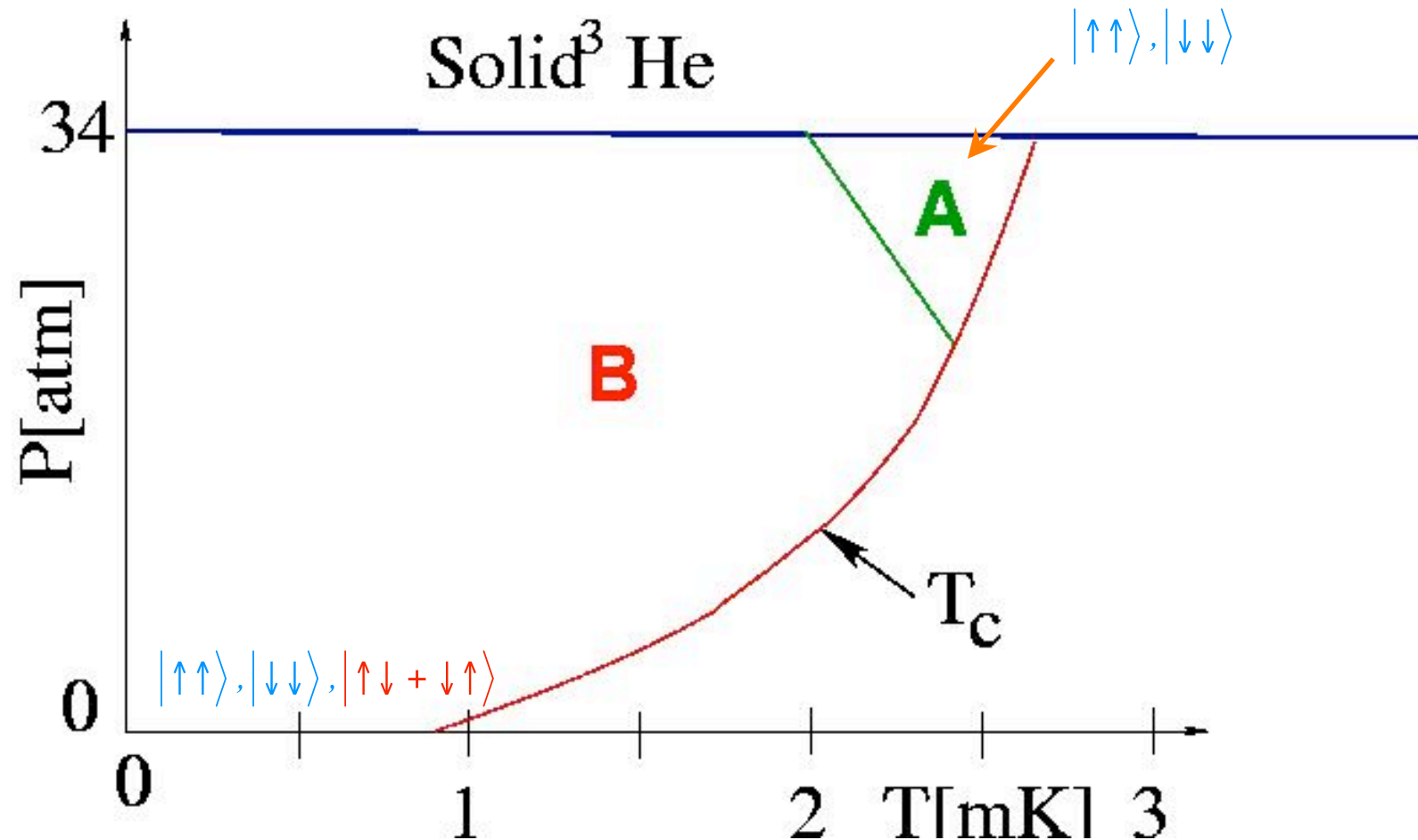
BCS pair condensates with:

$$\mathbf{S}_{\text{pair}} = \hbar \text{ ("triplet")}$$

&

$$\mathbf{L}_{\text{pair}} = \hbar \text{ ("p-wave")}$$

Superfluid Phases of ^3He



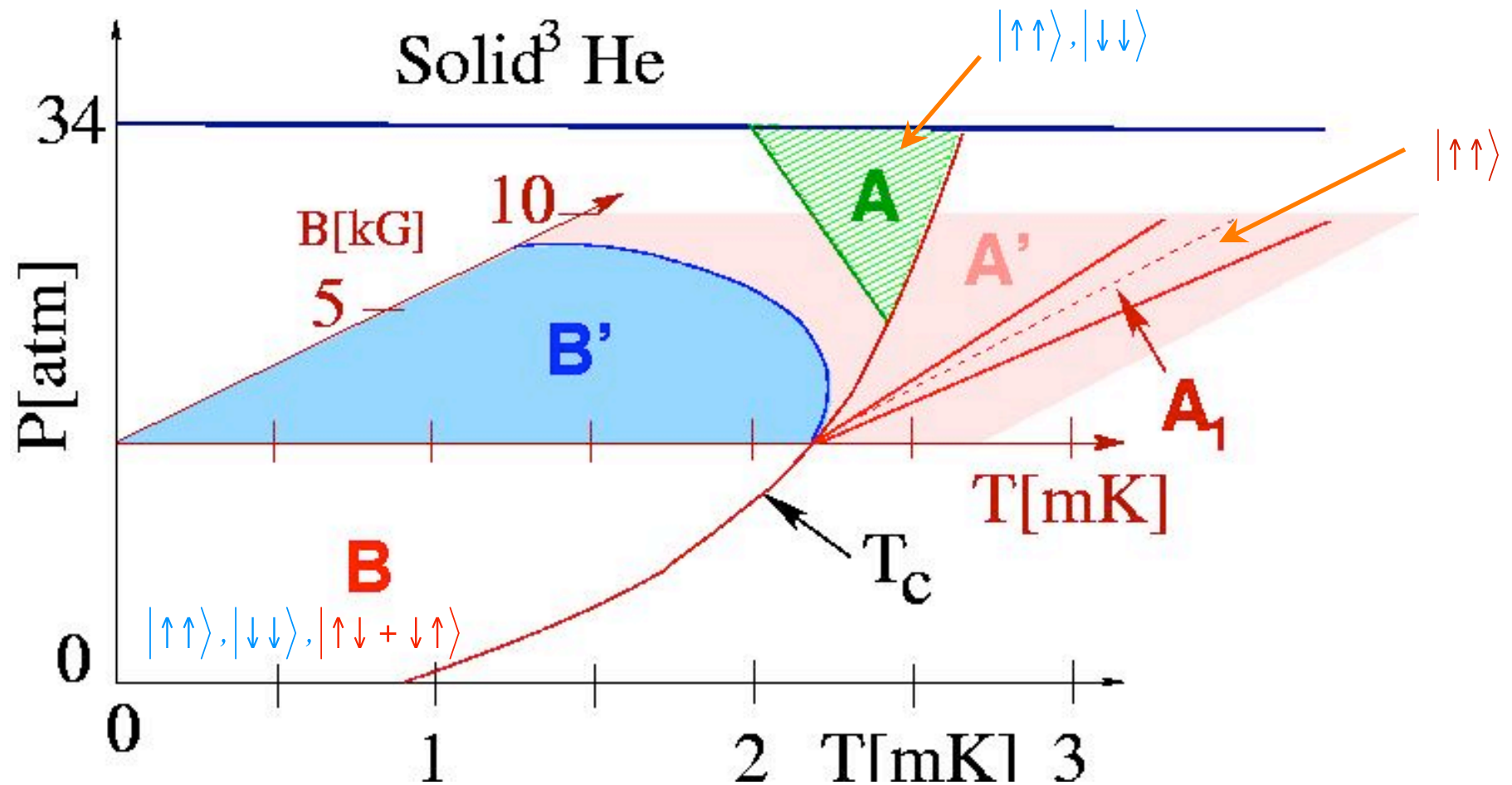
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Superfluid Phases of ^3He



BCS pair condensates with:

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Superfluid $^3\text{He-B}$

$$|\mathbf{B}\rangle = \Delta_B \left\{ \frac{\hat{p}_x + i\hat{p}_y}{\sqrt{2}} \right\} |\downarrow\downarrow\rangle + \left\{ \frac{\hat{p}_x - i\hat{p}_y}{\sqrt{2}} \right\} |\uparrow\uparrow\rangle + \hat{p}_z |\uparrow\downarrow + \downarrow\uparrow\rangle$$

➤ Balian & Werthamer (1963)

$$\left. \begin{array}{l} \mathbf{L} = 1 \\ \mathbf{S} = 1 \end{array} \right\} \Rightarrow \mathbf{J} = 0$$

$$\mathbf{G} = \mathbf{SO}(3)_S \times \mathbf{SO}(3)_L \times \mathbf{U}(1)_{\text{gauge}} \times \mathbf{P} \times \mathbf{T} \times \mathbf{C}$$

$$\mathbf{G}_B = \mathbf{SO}(3)_{L+S} \times \mathbf{T}$$

$$\vec{\mathbf{J}} = \vec{\mathbf{L}} + \vec{\mathbf{S}}$$

Approximate **particle-hole** symmetry

violation:

$$\left(\mathbf{T}/E_f \right)^2 \gg 10^{-5}$$

Neglect Nuclear Dipole Energy

violation:

$$E_{\text{dip}}/E_f \gg 10^{-7}$$

Broken relative **spin-orbit** symmetry

Acoustic Birefringence

NMR frequency shifts

A. Leggett

Collective Mode Dynamics in $^3\text{He-B}$

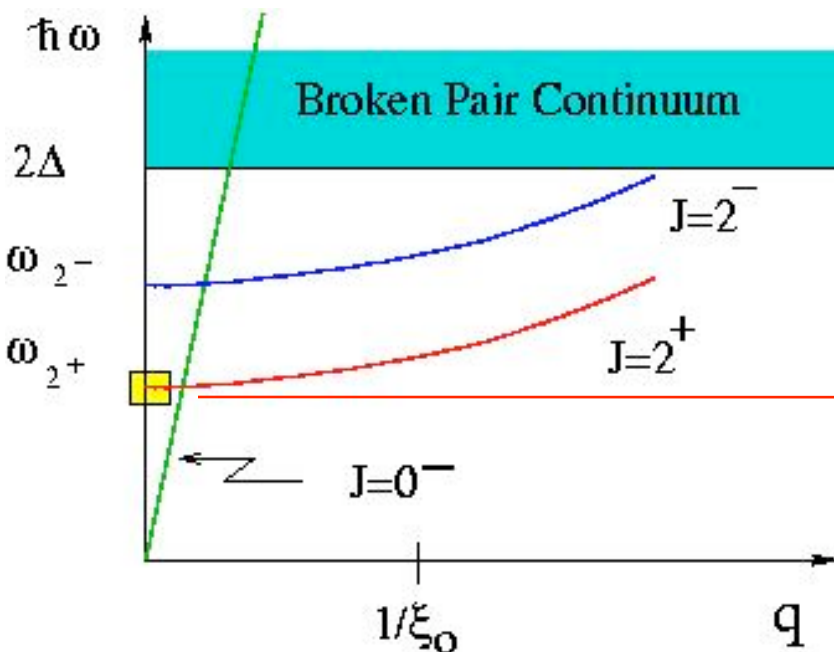
Goldstone Mode w/ $J=0^-$ \longrightarrow $D_{00}^{(-)} = i|\Delta| \underbrace{\varphi(\mathbf{q}, \omega)}_{\text{phase mode}}$

$$\left(\omega^2 - \frac{1}{3} q^2 v_f^2 \right) D_{00}^{(-)} = \dots$$

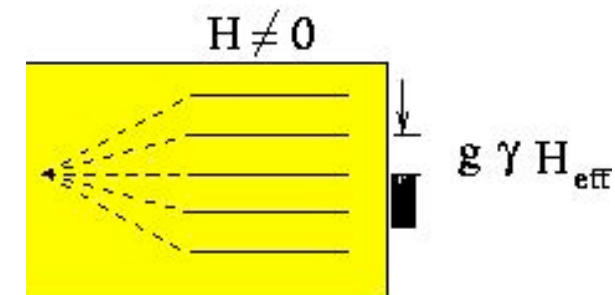
Pair Excitons w/ $J=2^{\pm}$

$$\left(\omega^2 - \omega_{J^{\text{CM}}}^2(q) \right) D_{JM}^{(C)} = \dots$$

coupling to internal & external fields



Nuclear Zeeman levels



Schopohl

Serene & JAS

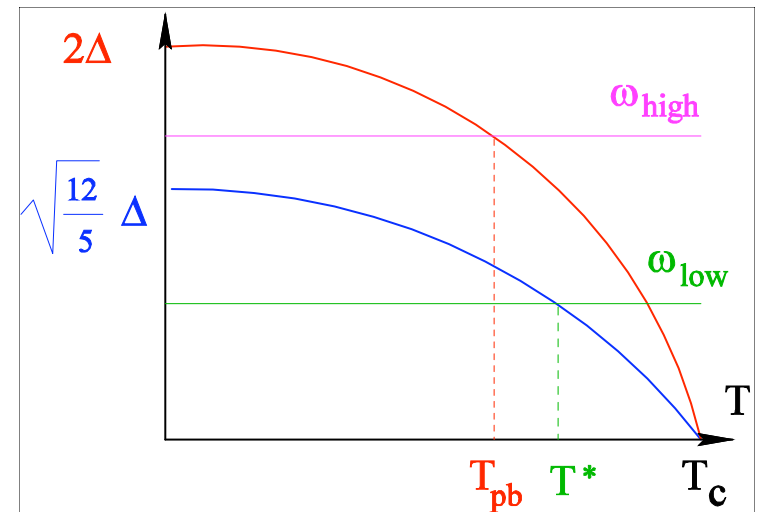
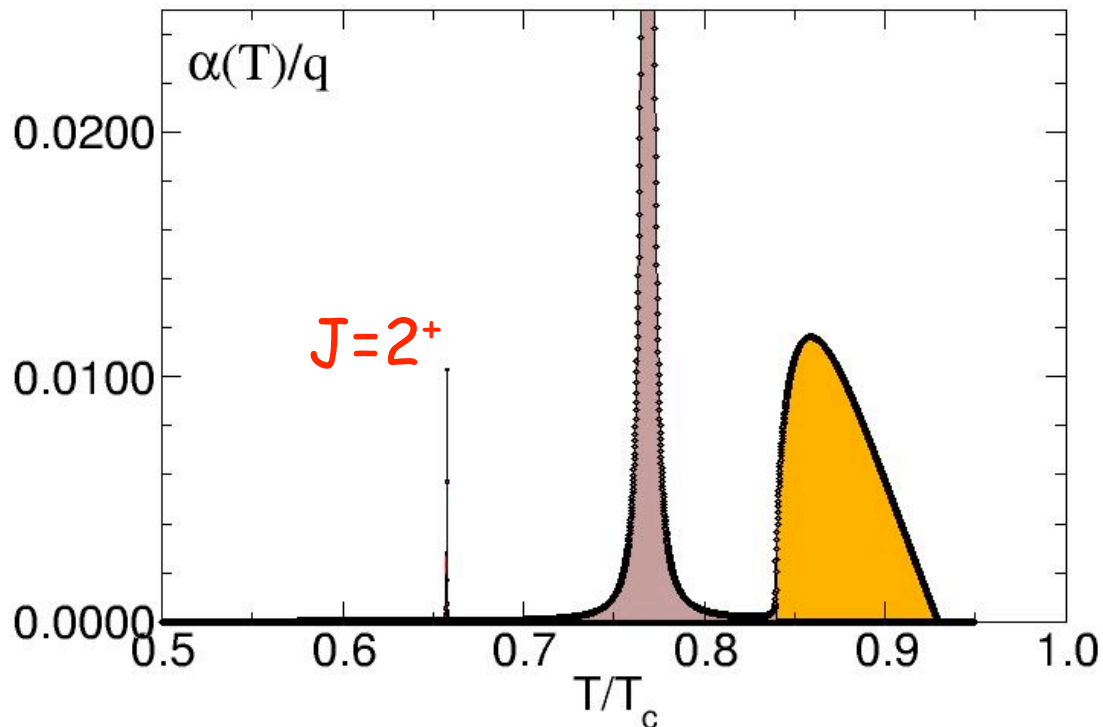
Absorption of Longitudinal Sound in $^3\text{He-B}$

Pairbreaking Absorption $\alpha_{pb} \propto \sqrt{\omega - 2\Delta} \Theta(\omega - 2\Delta)$

Resonant absorption by $J=2^{\pm}$ modes; Linewidth $\Gamma \approx \Gamma_N e^{-\Delta/T} = \Delta$

Particle-hole asymmetry $\int \alpha_{2^+} dT / \int \alpha_{2^-} dT = a^2 \approx 10^{-4}$ (Koch & Wolfle)

$J=2^-$



What is the Fate of Transverse
Zero Sound for $T < T_c$?

Transverse Zero Sound in $^3\text{He-B}$

$$\frac{\partial \vec{j}_t}{\partial t} + \vec{\nabla} \cdot \vec{\Pi} = 0 \longrightarrow \text{Dispersion Relation}$$

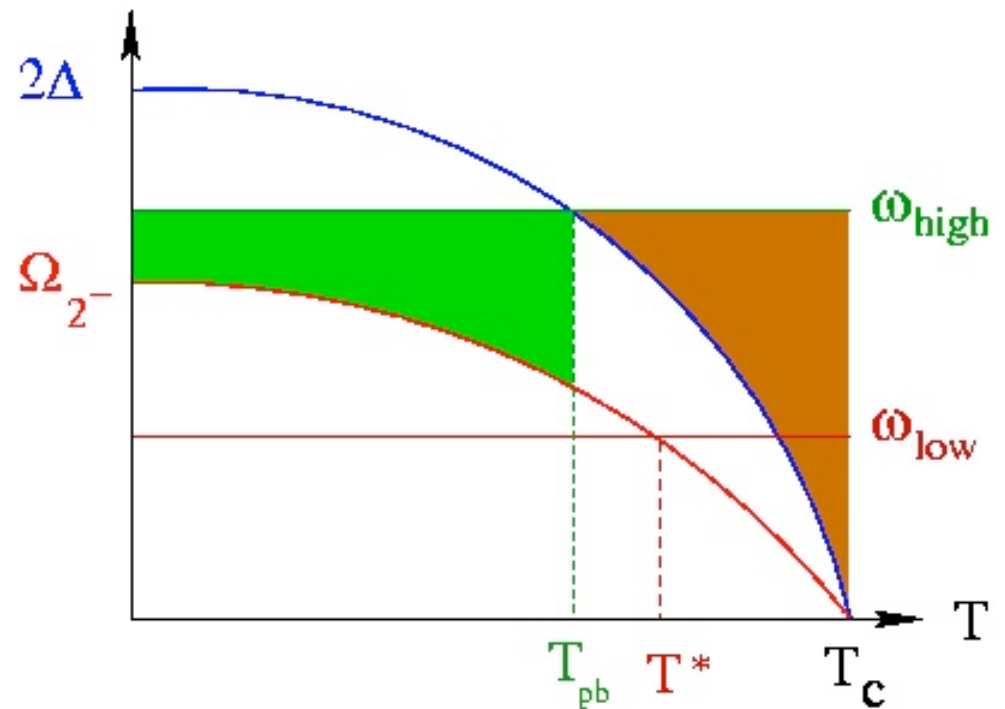
$$\left(\frac{\omega}{qv_f} \right)^2 = \underbrace{\left(\frac{c_t}{v_f} \right)^2 \rho_n(\omega)}_{\text{quasiparticle stiffness}} + \underbrace{\frac{2}{5} \left(\frac{c_t}{v_f} \right)^2 \rho_s(\omega)}_{\text{condensate stiffness}} \left\{ \frac{\omega^2}{(\omega + i\Gamma)^2 - \Omega_{2^-}^2} \right\}$$

quasiparticle
stiffness

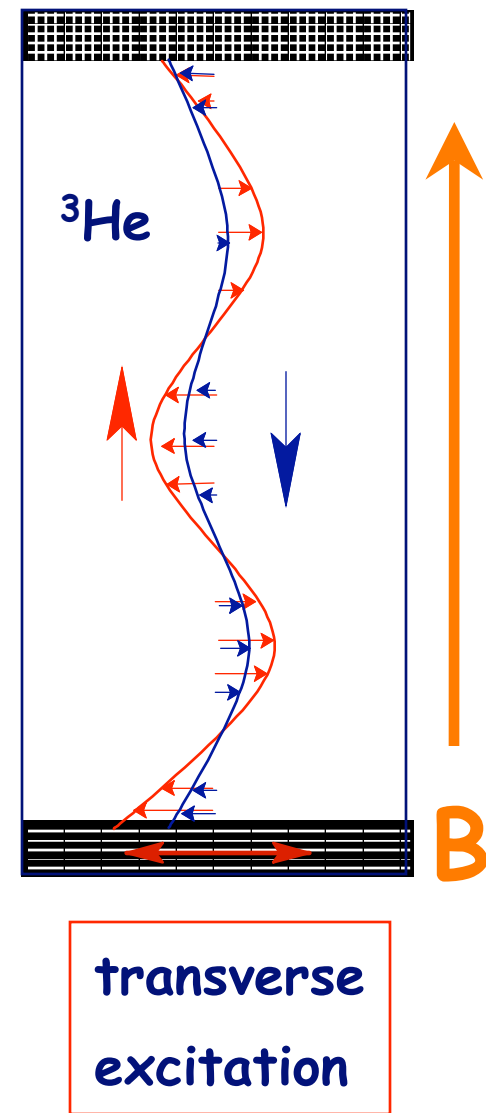
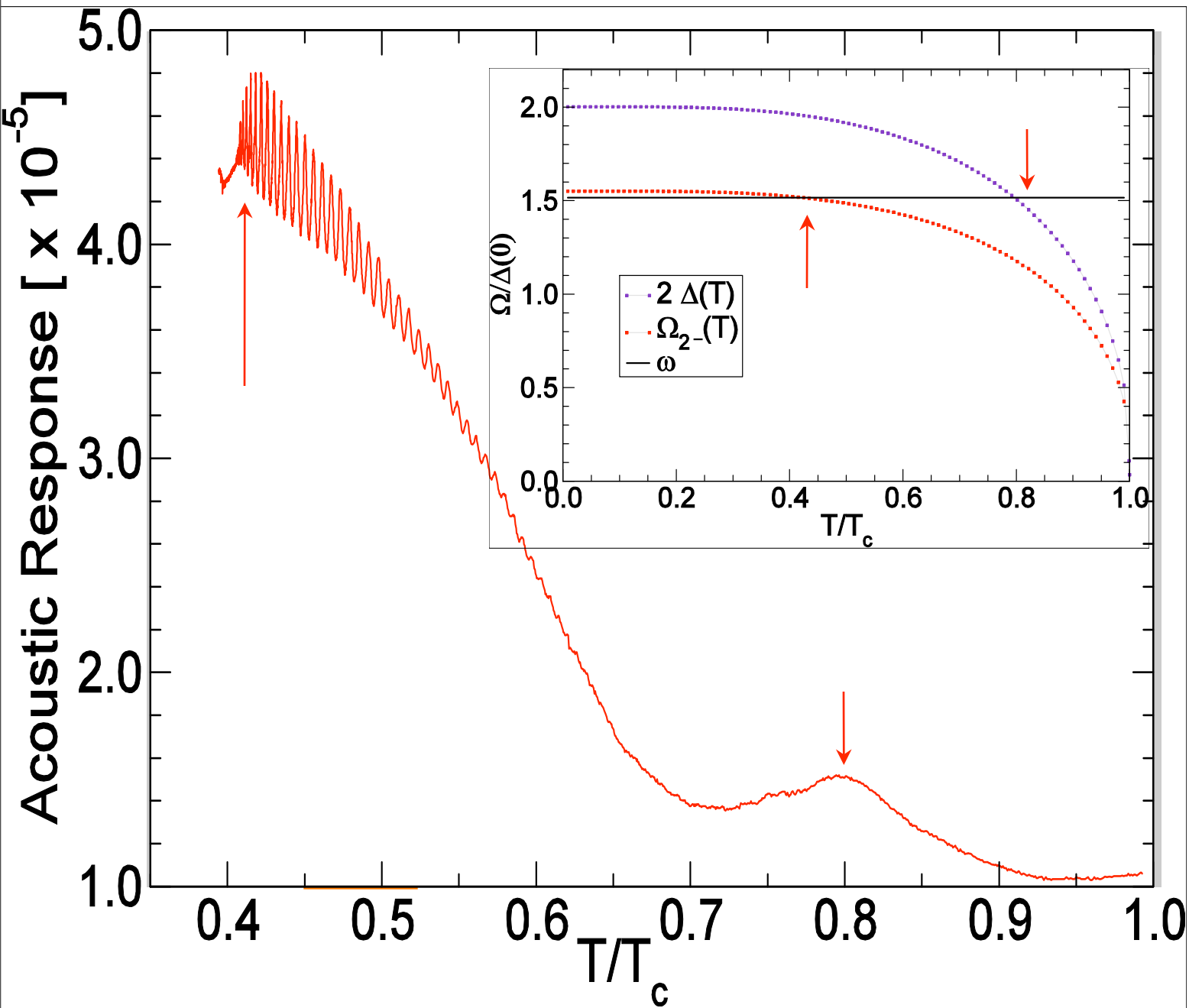
condensate
stiffness

Propagation for $\Omega_{2^-} < \omega < 2\Delta$

Extinction for $\omega \leq \Omega_{2^-}$



Transverse Cavity Waves in $^3\text{He-B}$



Y. Lee et al., Nature 400, 431 (1999)

Optical Faraday Effect

$$\vartheta = V \mathbf{B}_0 l$$

Verdet's constant

$$V = 2^\circ \text{ cm}^{-1} \text{ T}^{-1}$$

e.g. in glass

$$V = \frac{e}{2mc^2} \omega \left(\frac{dn}{d\omega} \right)$$

Becquerel (1897)
(Rosenfeld, Kramers, Heisenberg)

Acoustic Faraday Effect

$$\vec{E} \leftrightarrow \vec{j} \quad (\text{transverse mass current})$$

$$V_A = \underbrace{g_{2^-}}_{g\text{-factor}} \underbrace{\gamma_{^3\text{He}}}_{\text{Nuclear gyro-magnetic ratio}} \underbrace{f(T)}_{\text{Fermi-liquid factor}} \omega \frac{d}{d\omega} \left(\underbrace{\frac{1}{C_t}}_{\text{transverse velocity}} \right)$$

JAS (1999)

$$V_A = 10^4 \text{ deg} \cdot \text{cm}^{-1} \text{ T}^{-1} \quad \text{in } ^3\text{He}$$

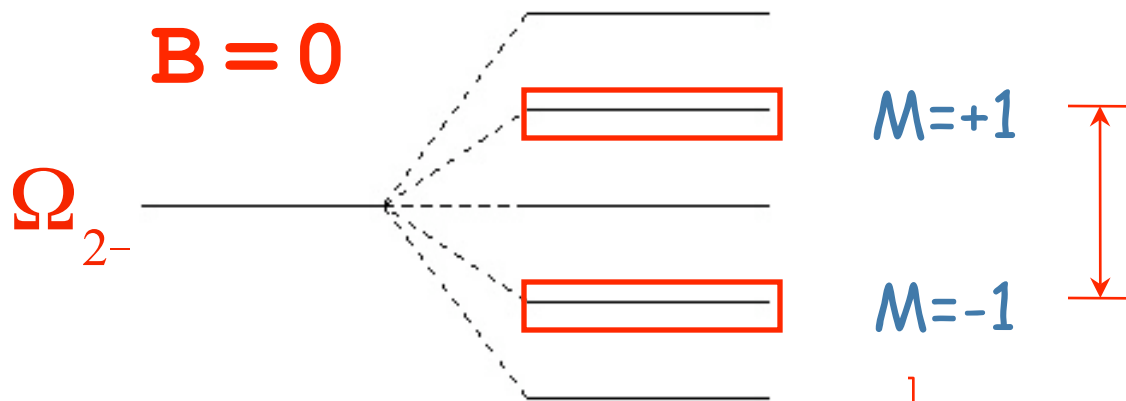
Field-induced acoustic birefringence in $^3\text{He-B}$

$$j_{\text{RCP}} (+) \leftrightarrow \mathbf{D}_{M=+1}^{(-)} \\ \text{LCP} (-) \quad \quad \quad 2, M=-1$$

$$\vec{\mathbf{B}} \neq 0 \\ \vec{\mathbf{B}} \parallel \vec{\mathbf{q}}$$

Time-inversion (broken)

Axial symmetry (preserved)



$$\Delta\omega = 2 \underbrace{g_{2-}}_{g\text{-factor}} \underbrace{\gamma_{^3\text{He}}}_{\text{Nuclear gyro-magnetic ratio}} \mathbf{B}$$

$$\frac{C_{\text{RCP}} - C_{\text{LCP}}}{\bar{C}} \approx \frac{g_{2-} \gamma_{^3\text{He}} \mathbf{B}}{\omega}$$

Acoustic Faraday Rotation

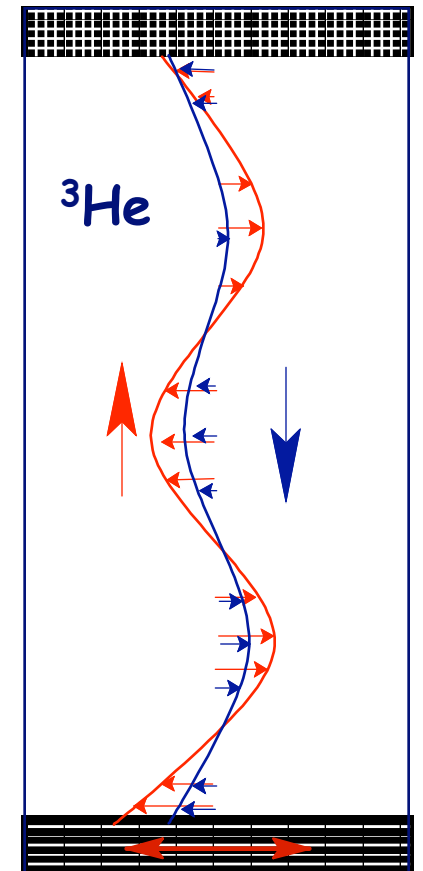
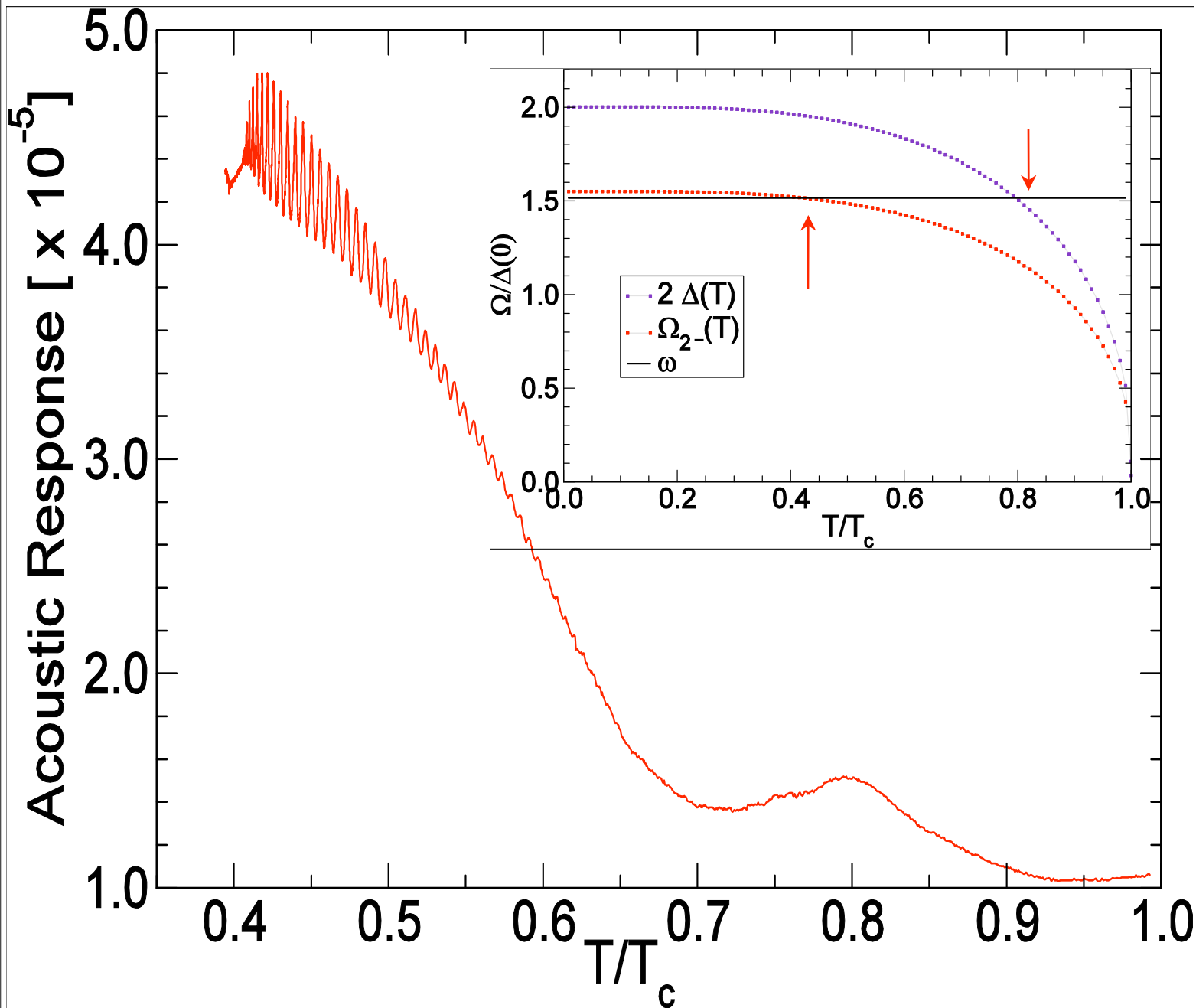
circular birefringence

linear polarization

$$\vartheta = V_A \mathbf{B} l$$

$$V_A = g_{2-} \gamma_{^3\text{He}} \underbrace{f(T)}_{\text{Fermi-liquid factor}} \omega \underbrace{\frac{d}{d\omega} \left(\frac{1}{C_t} \right)}_{\text{transverse velocity}} = 10^4 \text{ deg-cm}^{-1} \text{ T}^{-1}$$

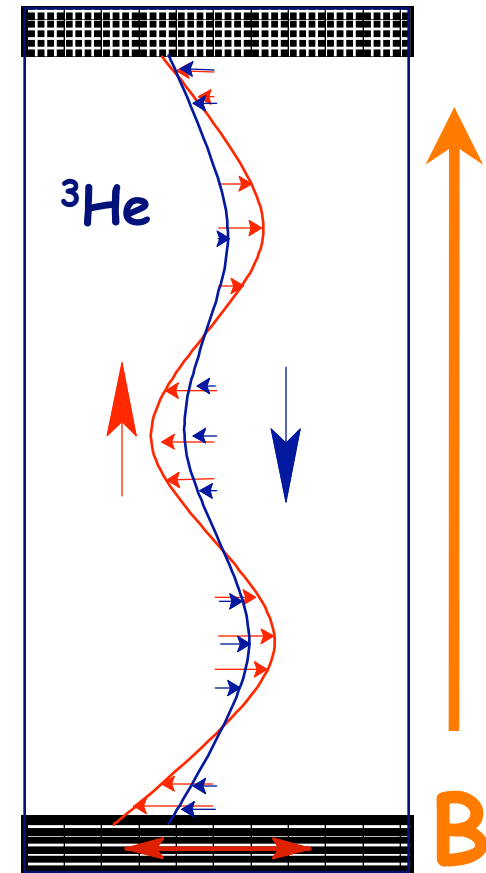
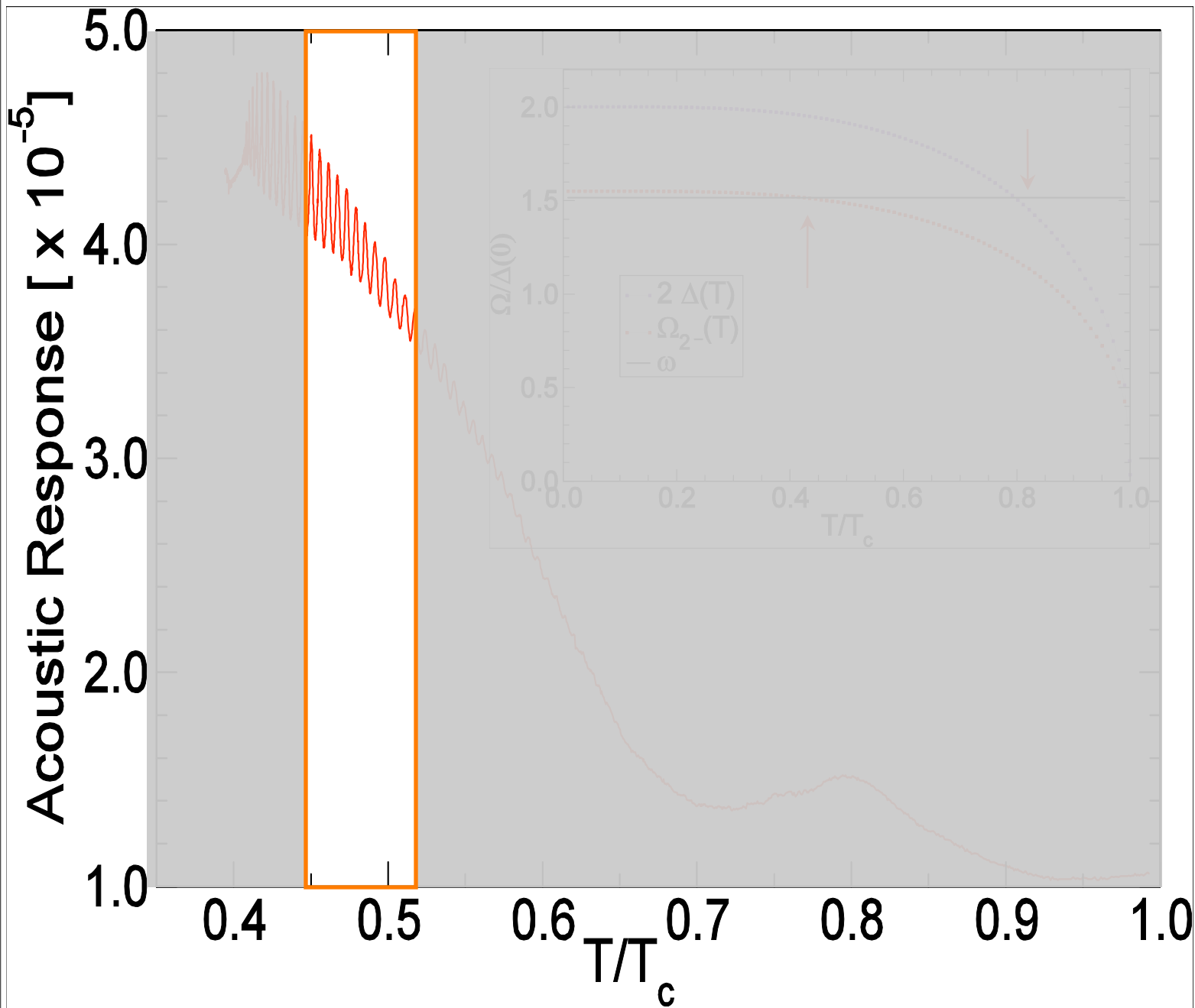
Transverse Cavity Waves in $^3\text{He-B}$



transverse
excitation

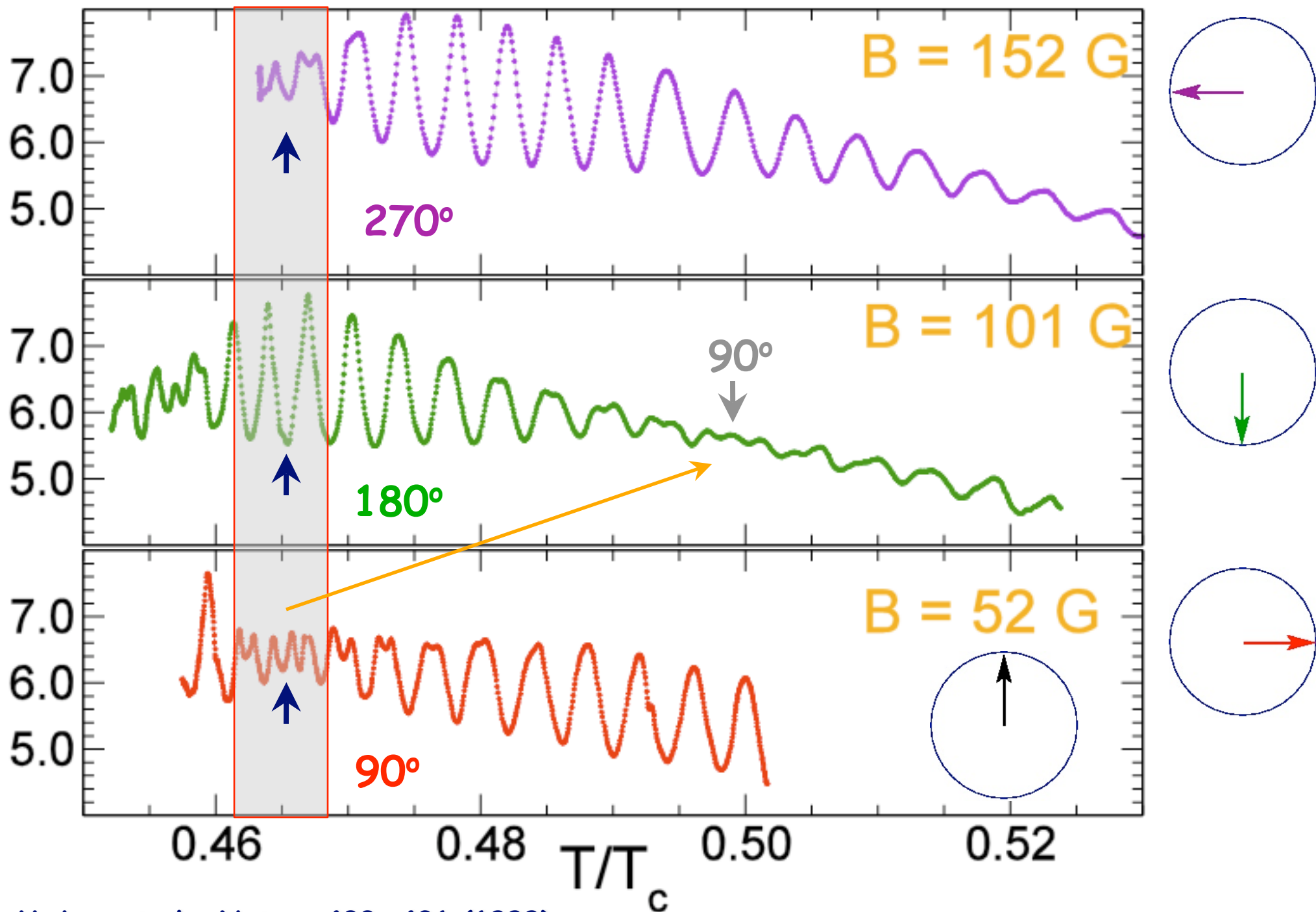
Y. Lee et al., Nature 400, 431 (1999)

Transverse Cavity Waves in $^3\text{He-B}$

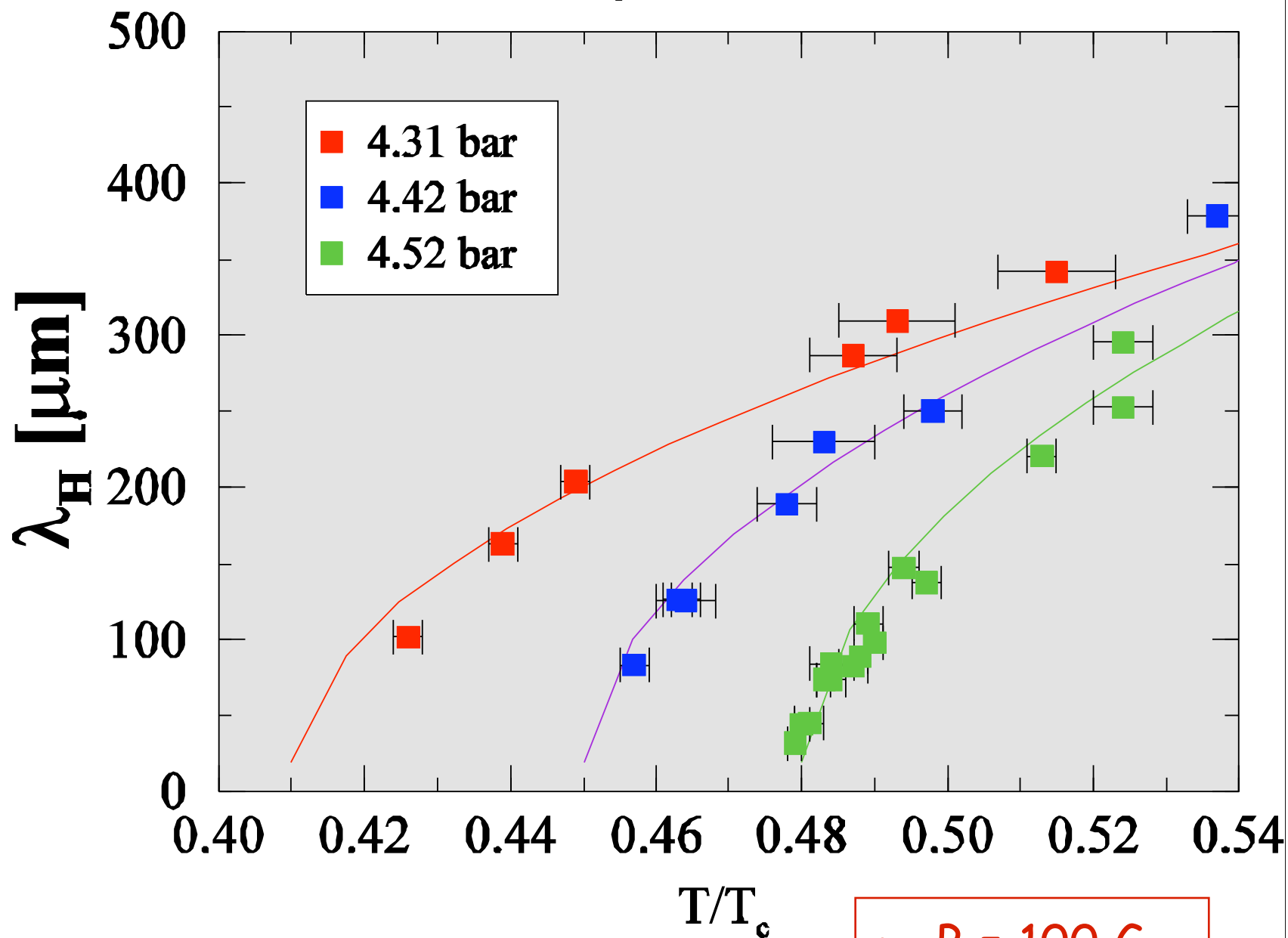


Y. Lee et al., Nature 400, 431 (1999)

Faraday "Beats" in the Transverse Impedance



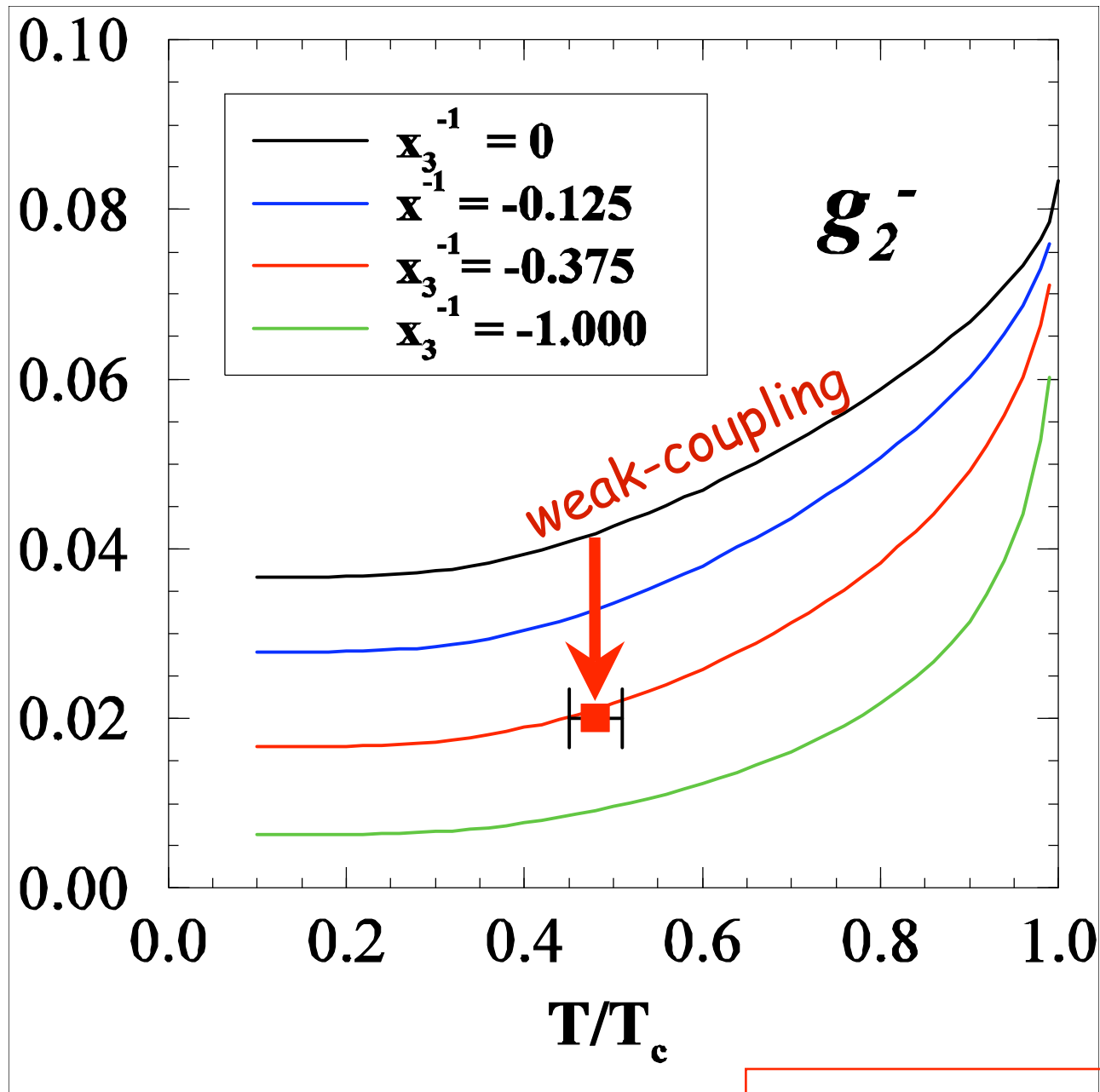
Faraday Rotation Period



• $B = 100 \text{ G}$

• $g(T_+) = 0.02$

g-factor for the J=2- modes



$$g(T_+) = 0.02$$

f-wave pairing

$$x_3^{-1} = -0.375$$

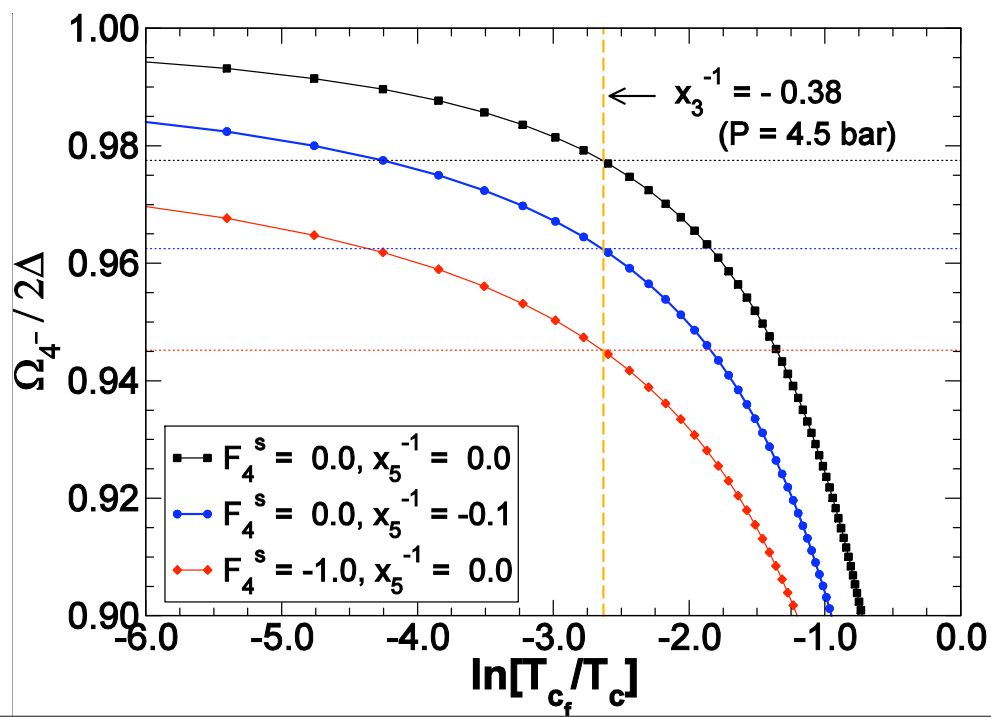
$$T_{cf} = 0.07 T_c$$

Implications of f-wave pairing

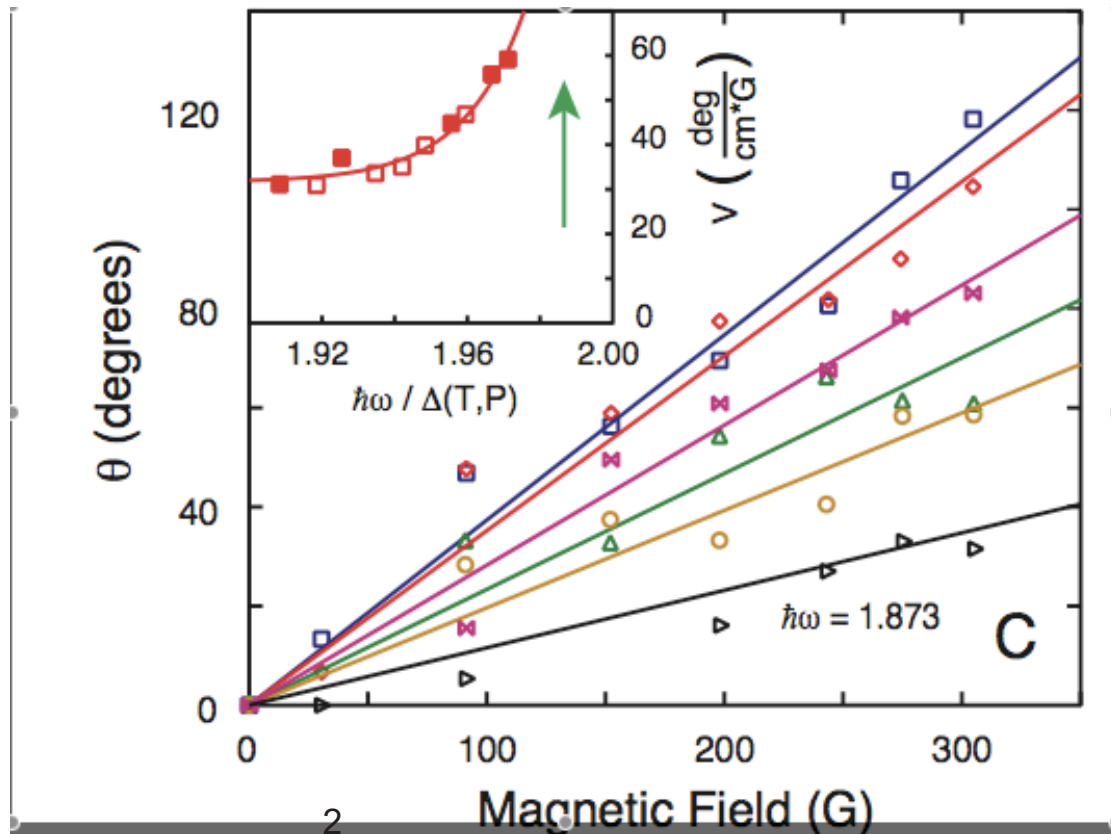
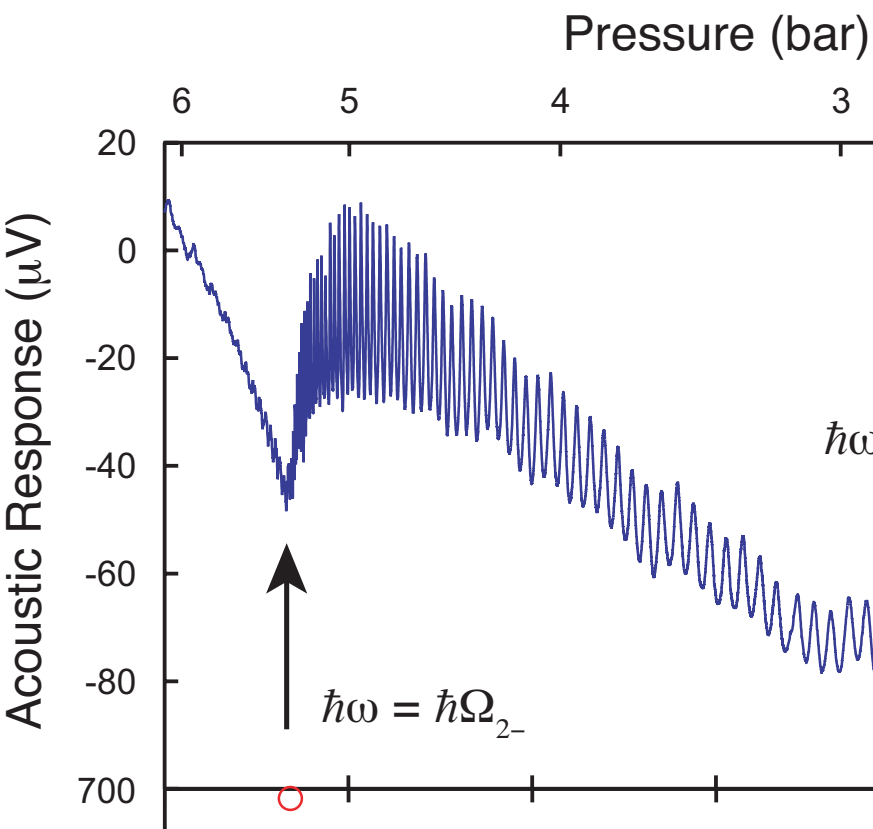
Superfluid $^3\text{He-A}$: is p-wave/f-wave mixture

- f-wave pairing & strong-coupling effects on A-B transition
- Inhomogeneous phases (vortices) with f-wave “cores”
- Surface phases with f-wave pairing correlations

Superfluid $^3\text{He-B}$: new collective modes



$$\left. \begin{array}{l} L = 3 \\ S = 1 \end{array} \right\} \Rightarrow J = 4$$



Transverse Sound
Cavity Modes in $^3\text{He-B}$

New Collective Mode
- Circular Birefringence
- $J=4^-, M=\pm 1$

John Davis et al.
(submitted 2007)

What about the A-phase?

Superfluid $^3\text{He-A}$

$$|\mathbf{A}\rangle = \frac{1}{2} \Delta_A (\hat{p}_x \pm i\hat{p}_y) \left\{ |\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle \right\}$$

➤ Anderson & Morel (1962) $\left. \begin{array}{l} \mathbf{L} = 1 \\ \mathbf{S} = 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} L_z = \pm 1 \\ S_z = 0 \end{array} \right.$

$$G = \text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_{\text{gauge}} \times \text{P} \times \cancel{\text{T}} \times \text{C}$$

$$G_{\text{AM}} = \text{U}(1)_{S_z} \times \text{U}(1)_{N-L_z}$$

$S_z \quad N - L_z$

Broken relative **spin-orbit** symmetry

Broken relative **gauge-orbit** symmetry

Broken **time-reversal** symmetry

Ground state orbital current

Spontaneous Acoustic Birefringence