Stability of Half-Quantum Vortices in $p_x + ip_y$ Superconductors

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Work done in collaboration with

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Energetics: Parameters and Settings

$$f_{\rm grad}^{\rm 2D} = \frac{1}{2}\left(\frac{\hbar}{2m}\right)^2\left[\rho_{\rm s}\left(\nabla_\perp\phi - \frac{2e}{\hbar c}\mathbf{A}\right)^2 + \rho_{\rm sp}\left(\nabla_\perp\alpha\right)^2\right] + \frac{1}{8\pi}(\nabla\times\mathbf{A})^2$$

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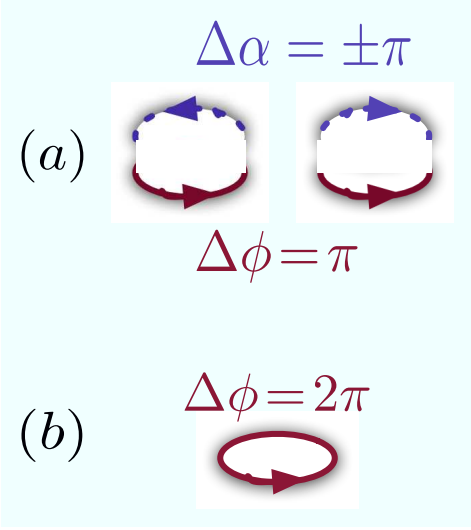
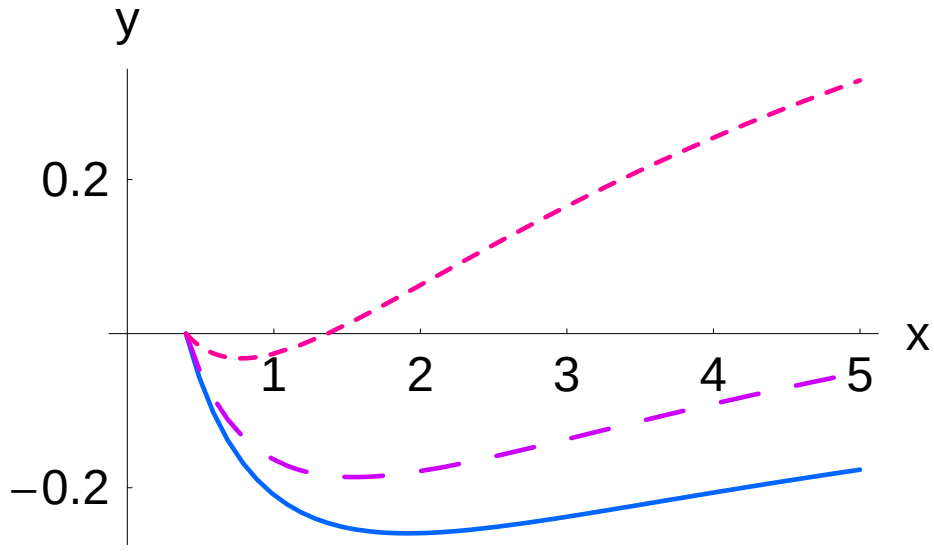
- Two problems:
  - Pair energetics in bulk
  - Mesoscopic cylinder

Pair energetics in bulk





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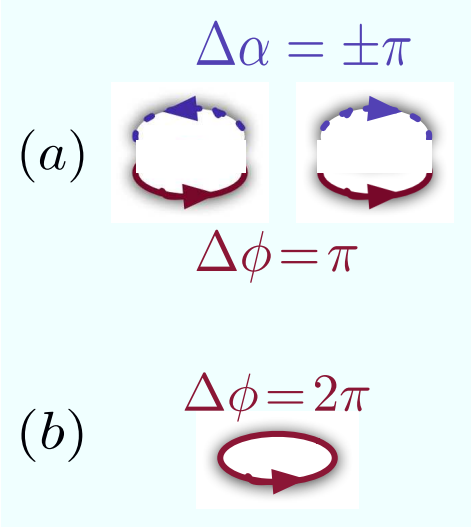
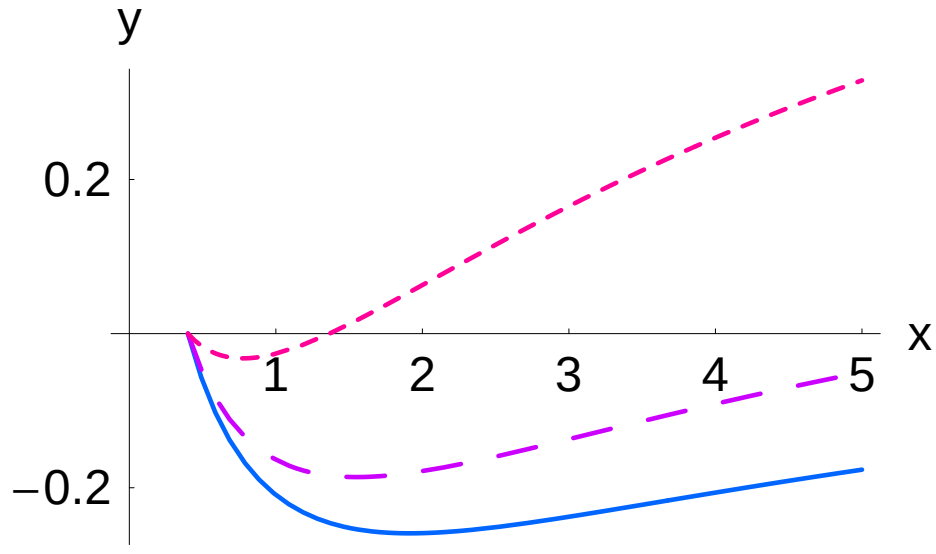
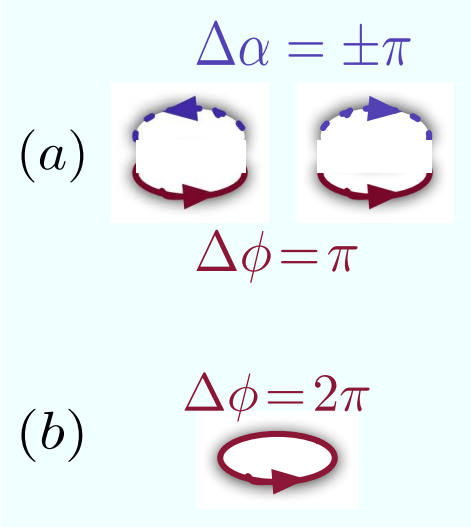
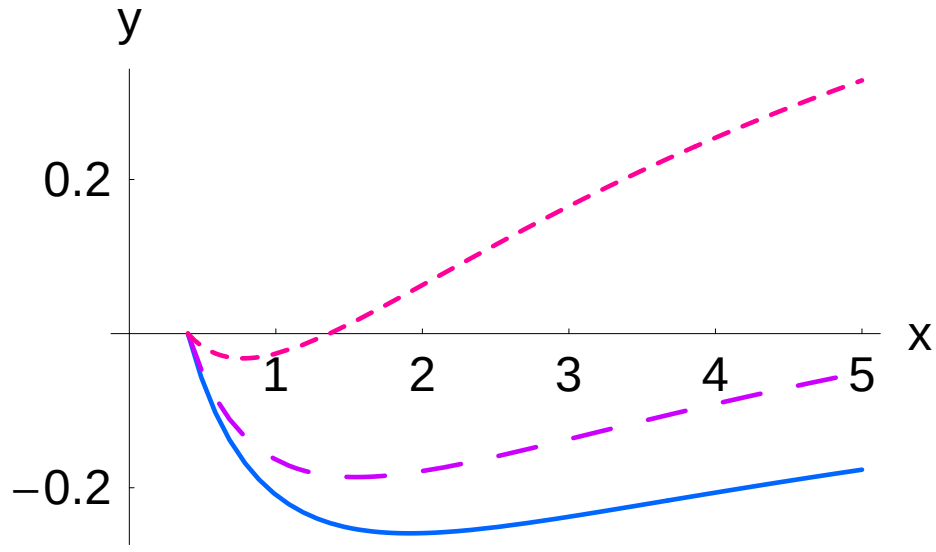




- $E_{\rm pair}^{\rm half} - E_{\rm full} = \frac{1}{2}\frac{\Phi_0^2}{16\pi^2\lambda^2}\left[K_0\left(\frac{r_{12}}{\lambda}\right) + \frac{\rho_{\rm sp}}{\rho_{\rm s}}\ln\left(\frac{r_{12}}{\xi}\right) - \ln\left(\frac{\lambda}{\xi}\right)\right]$

Pair energetics in bulk





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- $\kappa \equiv \lambda/\xi = 2.5$ ($\lambda \sim 150{\rm nm}$ and $\xi \sim 60{\rm nm}$ in Sr_2RuO_4)
  $\rho_{\rm sp}/\rho_{\rm s} = 0.3, 0.4, 0.7$ (blue, purple, red)

Why Mesoscopic Samples?

- No half-quantum vortex observed in bulk (limitation of London approx.)
- Need to consider finite-size geometry to
  - Cure spin current energy divergence
  - Reduce supercurrent screening
- Slab, cylinder, hollow cylinder

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- Φ range ratio for 'half-quantum' to 'full-quantum':

$$\left[1 - \frac{\rho_\mathrm{sp}}{\rho_\mathrm{s}} (1+\beta) \right] : \left[1 + \frac{\rho_\mathrm{sp}}{\rho_\mathrm{s}} (1+\beta) \right]$$

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- $\rho_{\rm sp}/\rho_{\rm s}$ of Sr_2RuO_4 smaller than ~ 1

  $*$ $m/m^* \lesssim 0.3$ near $T = 0$ for Sr_2RuO_4

  $*$ $\rho_{\rm sp}/\rho_{\rm s} \sim 1$ require strong interaction between like spin and very weak interaction between opposite spins

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- What's signature of half-quantum vortex?