One- and two-particle microrheology in solutions of actin, fd-virus and inorganic rods

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Collaborators:

Frederick MacKintosh, Frederick Gittes, Peter Olmstedt, Bernhard Schnurr , all over the place

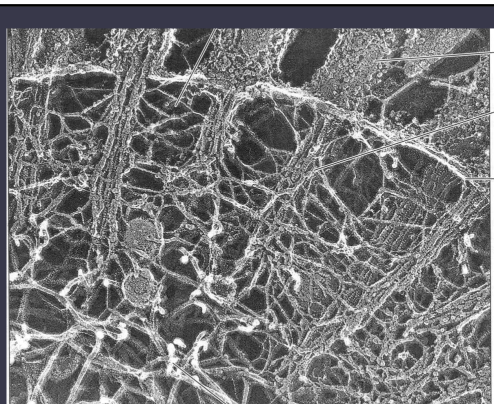
Jay Tang, Karim Addas, Indiana University

Alex Levine, UC Santa Barbara

Gijsje Koenderink, Albert Philipse, Universiteit Utrecht

Cytoskeleton of cells

QuickTime™ and a
Grafik decompressor
are needed to see this picture.



Complex dynamic machinery, based on polymer networks and membranes

Single filament dynamics

QuickTime™ and a Sorenson Video decompressor are needed to see this picture.

QuickTime™ and a Sorenson Video decompressor are needed to see this picture.

semiflexible: $l_p >> a$

l_P = persistence length,
a = monomer radius

$$l_p = \frac{EI}{k_b T}$$

E = Young's modulus,
I = area moment of inertia

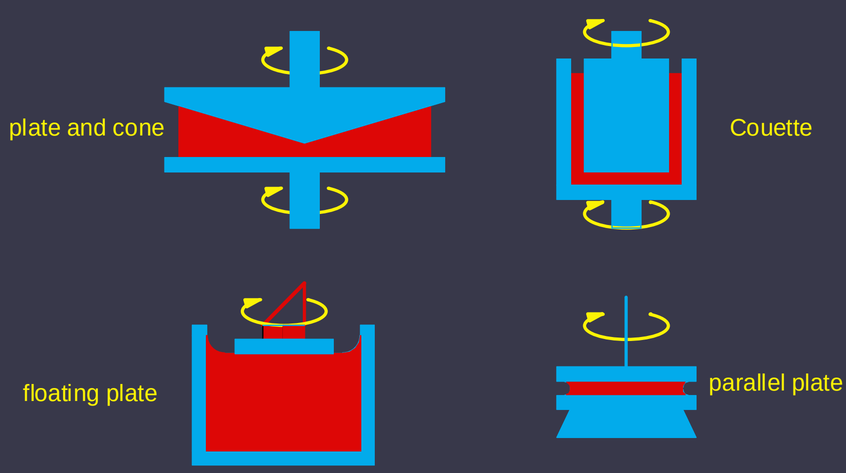
(for solid rod) $I = \frac{\pi}{4} a^4$

➔ aspect ratio: $\frac{l_p}{a} \propto \frac{a^4}{a} = a^3$

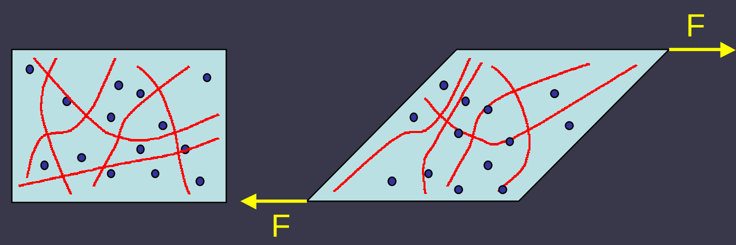
with E - 1 Gpa, a - 20 – 50 Å,

$$l_p \approx 10 - 2500 \mu m$$

Macrorheology



Shear deformation of a viscoelastic object

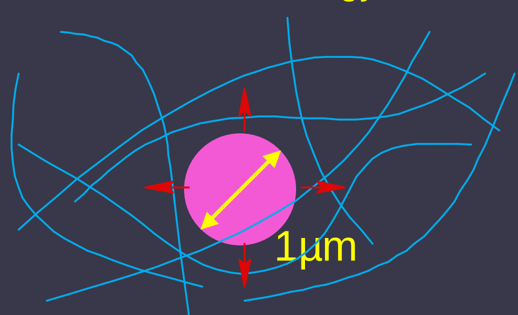


stress = G x strain

complex shear modulus: $G^*(\omega) = G'(\omega) + i\,G''(\omega)$

storage modulus   loss modulus

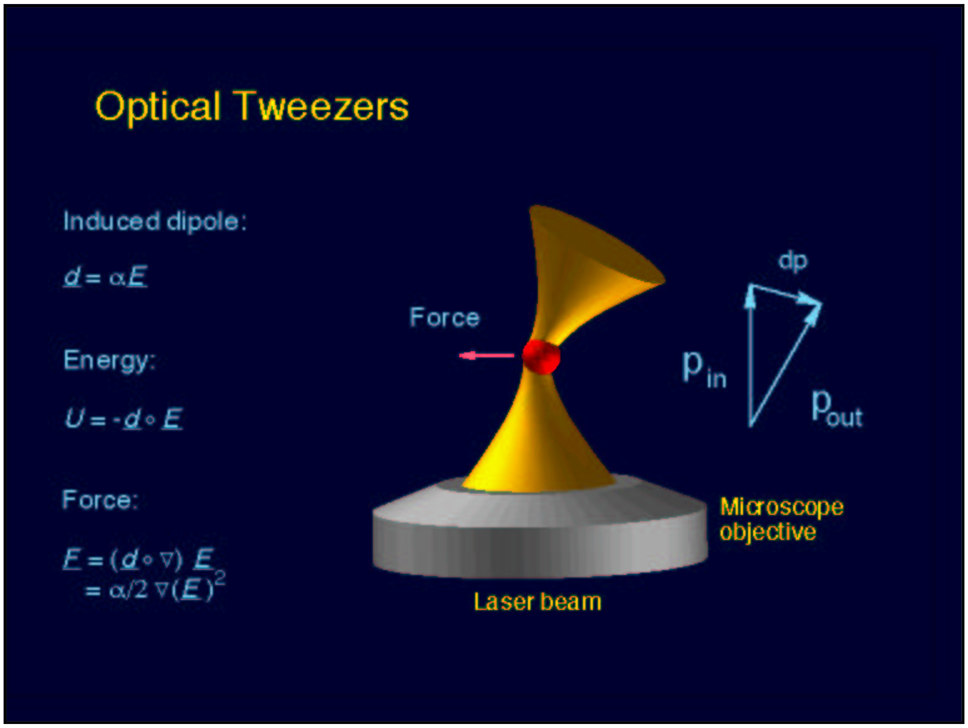
Microrheology

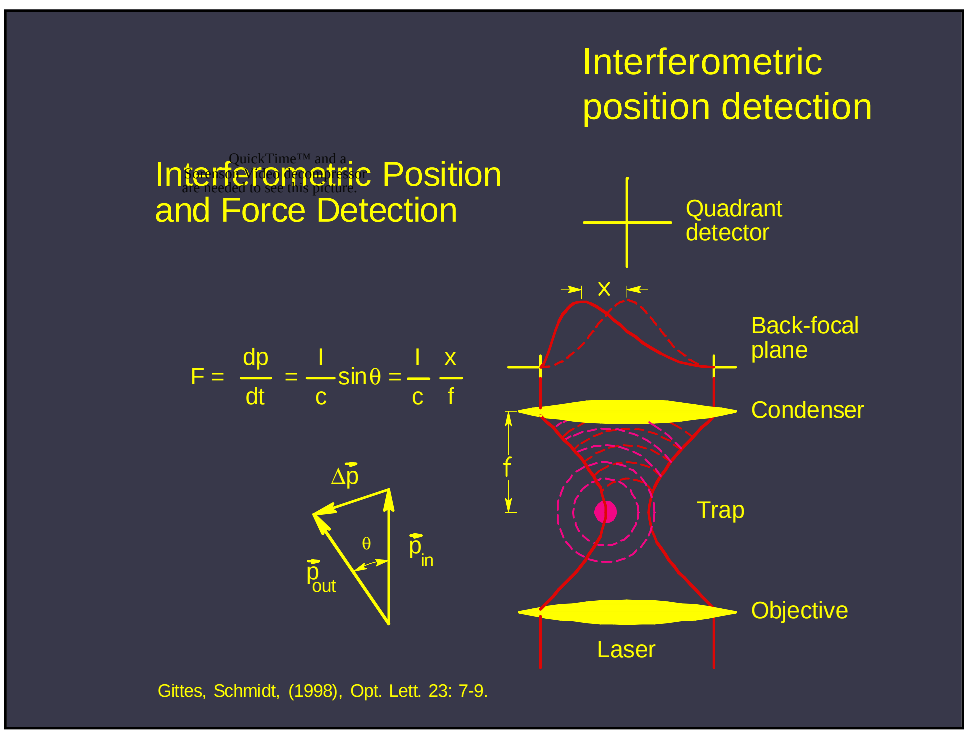


QuickTime™ and a Sorenson Video decompressor are needed to see this picture.

Advantages:

— study inhomogeneities
— study small samples, biological cells
— reach high frequencies (-> MHz) w/o inertial effects
— probe scale dependent material properties
by varying probe size
— active vs. passive, single bead vs multiple bead (Weitz &Co.)


Optical Tweezers
Induced dipole:
d = αE
Energy:
U = -d ∘ E
Force:
F = (d ∘ ∇) E
= α/2 ∇(E)²
Force
dp
p_in
p_out
Microscope objective
Laser beam


Interferometric position detection
QuickTime™ and a
are needed to see this picture.
Interferometric Position and Force Detection
F = dp/dt = I/c sinθ = I/c x/f
Δp
θ
p_in
p_out
Quadrant detector
x
Back-focal plane
Condenser
f
Trap
Objective
Laser
Gittes, Schmidt, (1998), Opt. Lett. 23: 7-9.

Experiments with semidilute semiflexible polymer networks

Parameters:

Bead diameter >> • • • • • • • • •
Entanglement length > mesh size
Persistence length > mesh size
0.1 Hz < ω/2π < 20 kHz

For larger beads and higher frequencies:

generalized Stokes-Einstein:

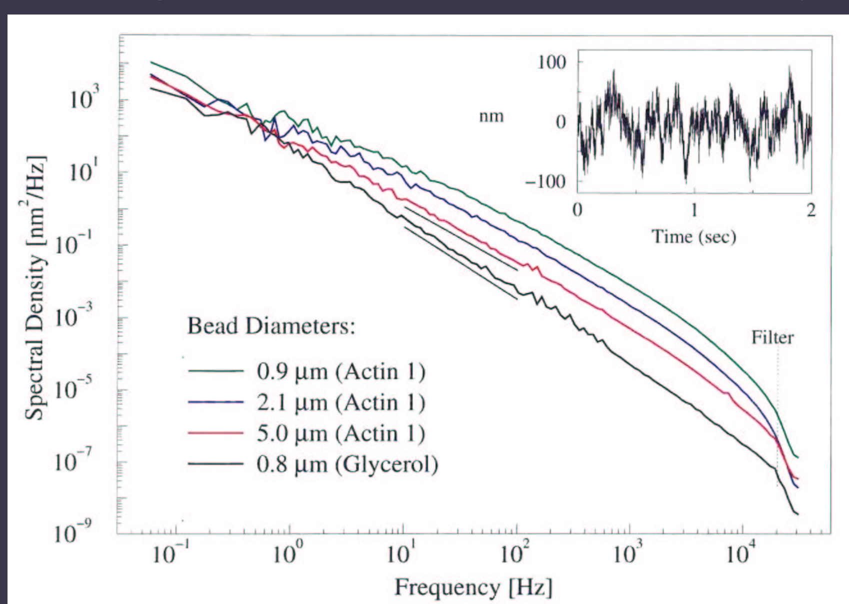
$$x_\omega = \alpha^*(\omega) f(\omega)$$

$$\alpha^*(\omega) = \frac{1}{6\pi G^*(\omega) a}$$

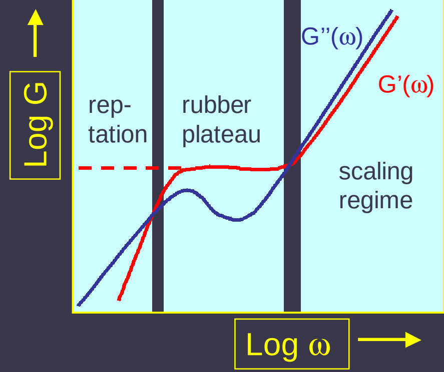
Data processing:

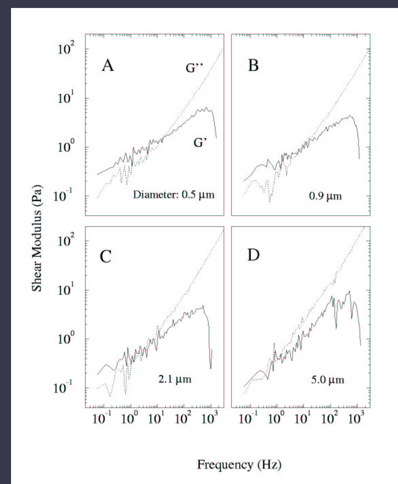
time-series data, x(t) → power spectral density, $\langle x_\omega^2 \rangle$ → fluctuation-dissipation: $\langle x_\omega^2 \rangle = \frac{4 k_B T \alpha''(\omega)}{\omega}$ → Kramers-Kronig: $\alpha'(\omega) = \frac{2}{\pi} P \int_0^\infty \frac{\zeta \alpha''(\zeta)}{\zeta^2 - \omega^2}$ → $G'(\omega), G''(\omega)$

Power spectra of thermal bead motion in actin gel



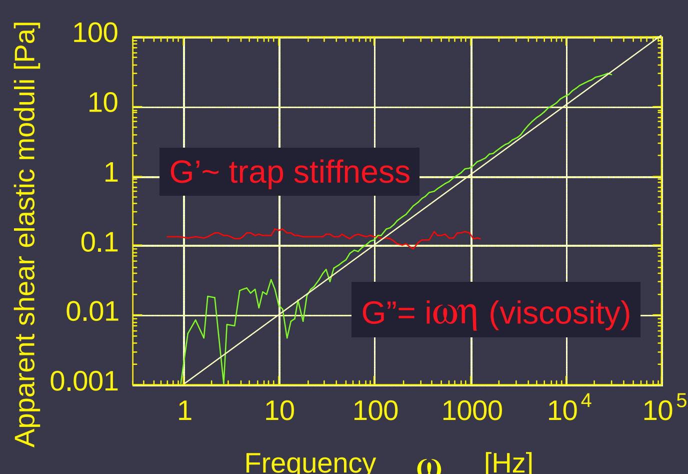
Single bead microrheology with actin solutions

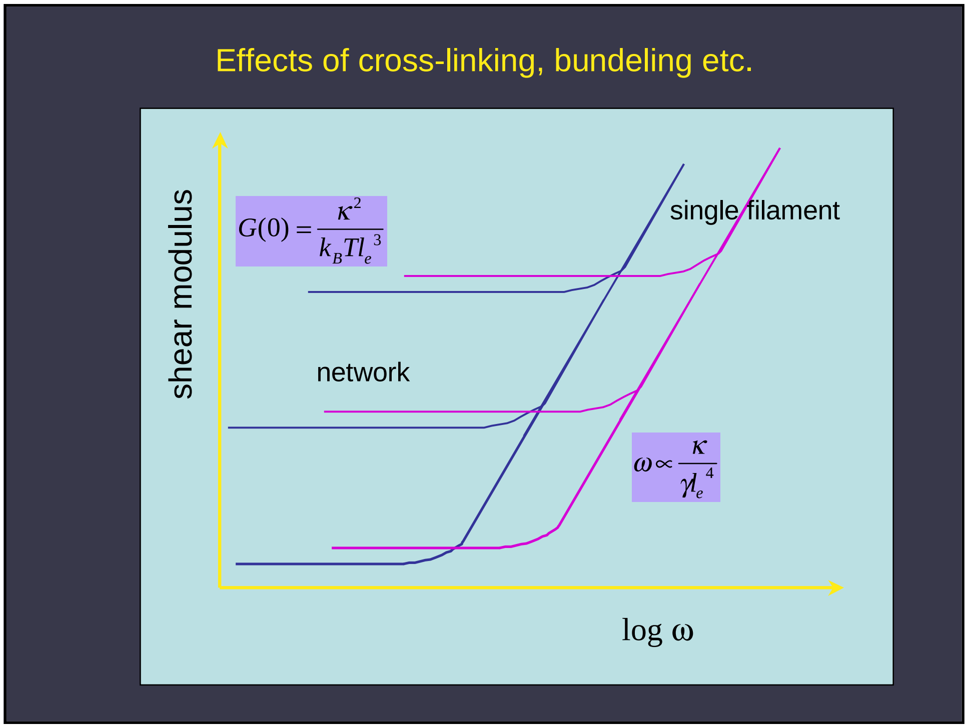


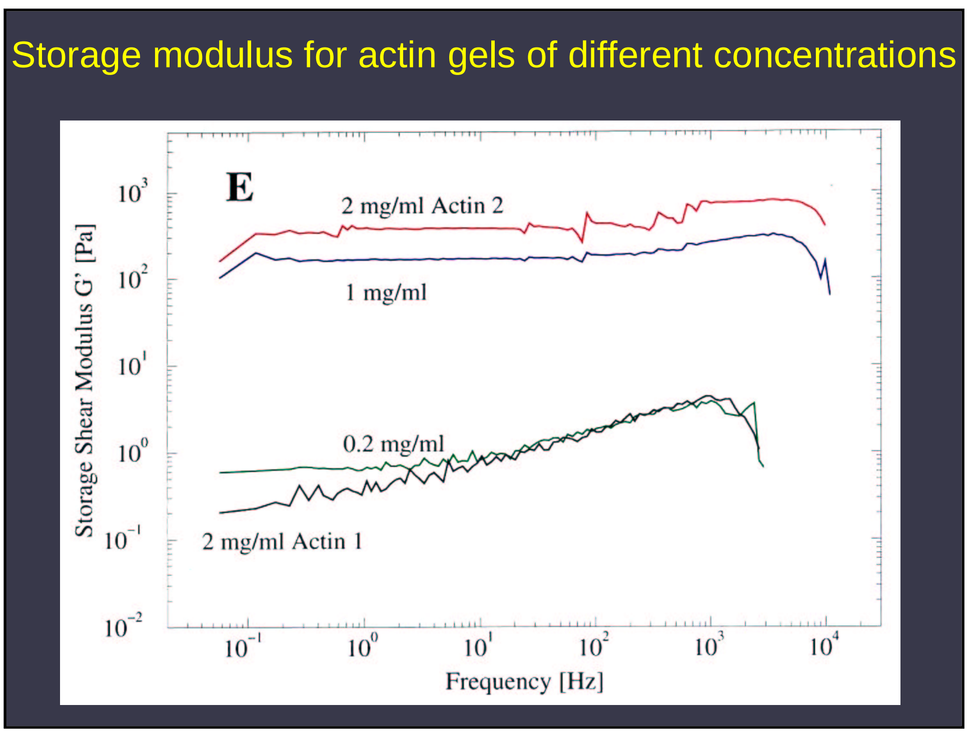


Gittes, F., B. Schnurr, P. D. Olmsted, F. C. MacKintosh and C. F. Schmidt (1997). "Microscopic viscoelasticity: shear moduli of soft materials determined from thermal fluctuations." Physical Review Letters **79**(17): 3286-9.
Schnurr, B., F. Gittes, F. C. MacKintosh and C. F. Schmidt (1997). "Determining microscopic viscoelasticity in flexible and semiflexible polymer networks from thermal fluctuations." Macromolecules **30**: 7781-7792.

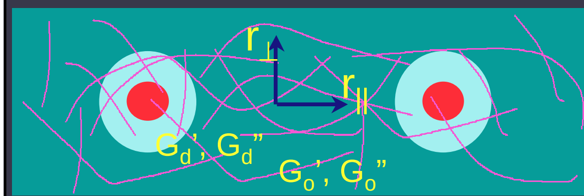
Apparent shear elastic moduli of trapped bead in water




Effects of cross-linking, bundeling etc.
shear modulus
$G(0) = \frac{\kappa^2}{k_B T l_e^3}$
single filament
network
$\omega \propto \frac{\kappa}{\gamma l_e^4}$
log ω


Storage modulus for actin gels of different concentrations
E
2 mg/ml Actin 2
1 mg/ml
0.2 mg/ml
2 mg/ml Actin 1
Storage Shear Modulus G' [Pa]
Frequency [Hz]
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{4}

Artifacts with 1-bead microrheology/2-bead microrheology



QuickTime™ and a Sorenson Video decompressor are needed to see this picture.

Depletion layer ~ r, l_p

2-bead microrheology:

Mutual compliance:

$$x_\omega^{m,i} = \alpha_{ij}^{nm}(\omega) f_\omega^{n,j}$$

n,m: particle index,
i,j = x,y,z

Relation to Lamé-coefficients

$$\alpha_{\parallel}^{(1,2)}(\omega) = \frac{1}{4\pi r \mu_0(\omega)} \quad , \quad \mu_0(\omega) = G(\omega)$$

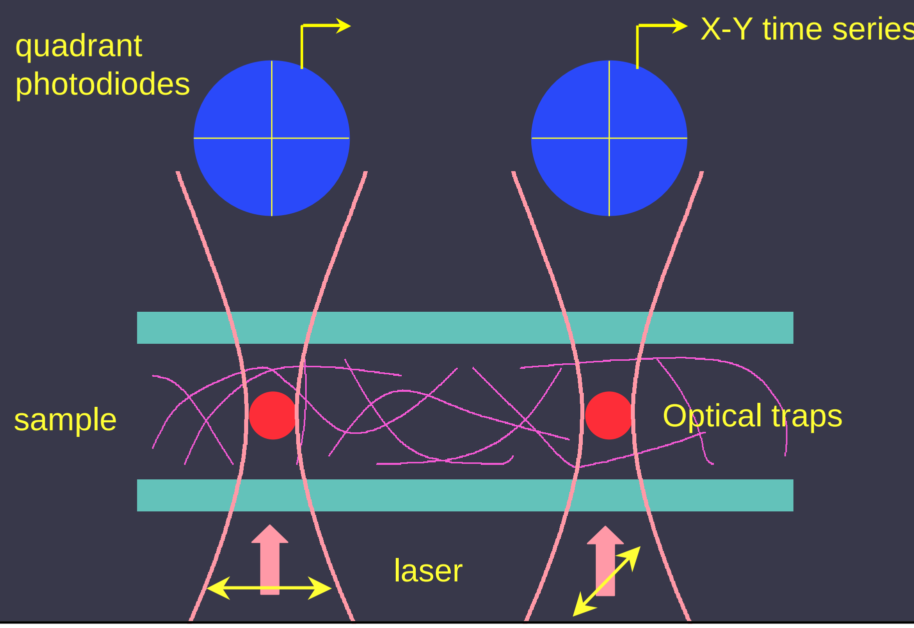
$$\alpha_{\perp}^{(1,2)}(\omega) = \frac{1}{8\pi r \mu_0(\omega)} \left[\frac{\lambda_0(\omega) + 3\mu_0(\omega)}{\lambda_0(\omega) + 2\mu_0(\omega)} \right]$$

In incompressible limit:

$$\frac{\alpha_{\parallel}^{(1,2)}(\omega)}{\alpha_{\perp}^{(1,2)}(\omega)} = 2$$

See: A. J. Levine, T.C. Lubensky, PRL, 85: 1774 (2000)

1-bead/2-bead microrheology



Data evaluation for 2-bead microrheology

Time series position data: $r_i^1(t)\ ,\ r_i^2(t)\ ,\ i=x,y$

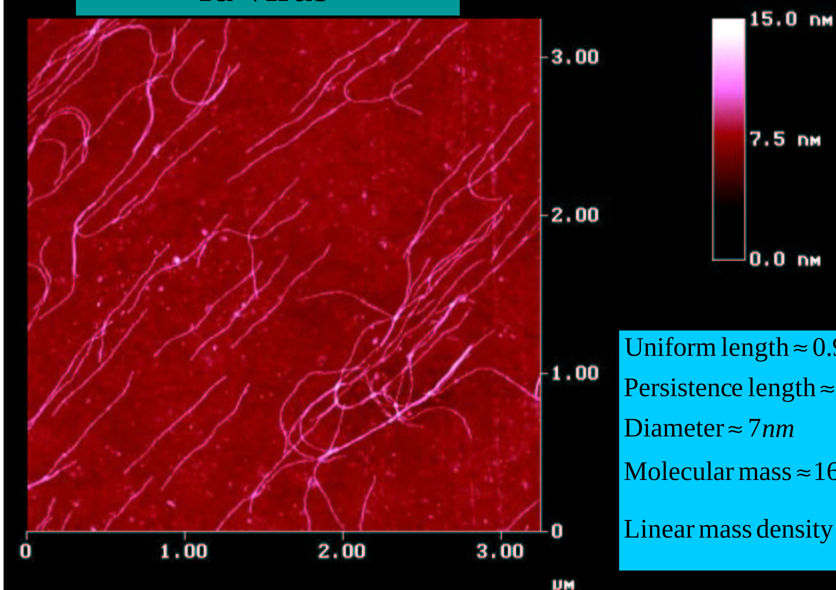
Power spectral density of position cross-correlation: $S_{x_{ij}^{12}}(\omega)=R_i^1(\omega)R_j^2(\omega)^\dagger$

Fluctuation-dissipation: $\left\langle R_i^1(\omega)R_j^2(\omega)^\dagger\right\rangle=\frac{4kT}{\omega}\alpha_{ij}^{12''}(\omega)$

Kramers-Kronig: $\alpha_{ij}^{12'}(\omega)$

Elastic coefficients: $\mu(\omega)=G(\omega),\lambda(\omega)$

fd virus



Uniform length $\approx 0.9\mu m$

Persistence length $\approx 2.2\mu m$

Diameter $\approx 7nm$

Molecular mass $\approx 16.4\times10^6\ Da$

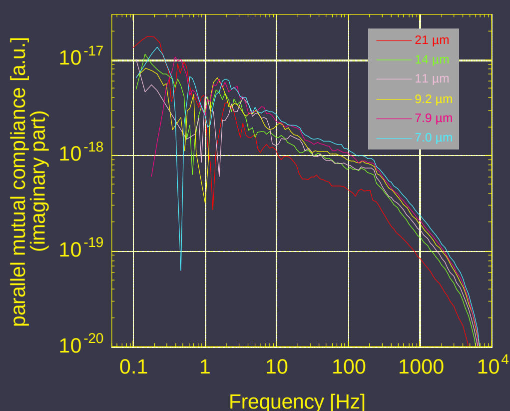
Linear mass density $\approx 18{,}200\frac{kD}{\mu m}$

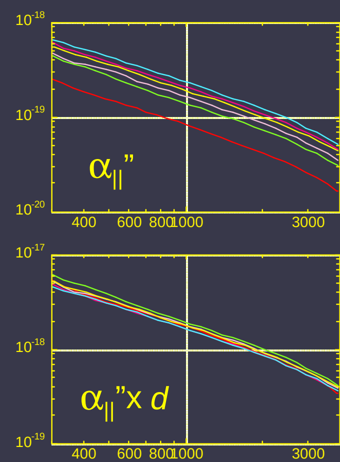
Collaboration with Jay Tang, Karim Addas

Mutual compliance as function of bead distance *d*, $\alpha_{||}''$, fd solution 18 mg/ml

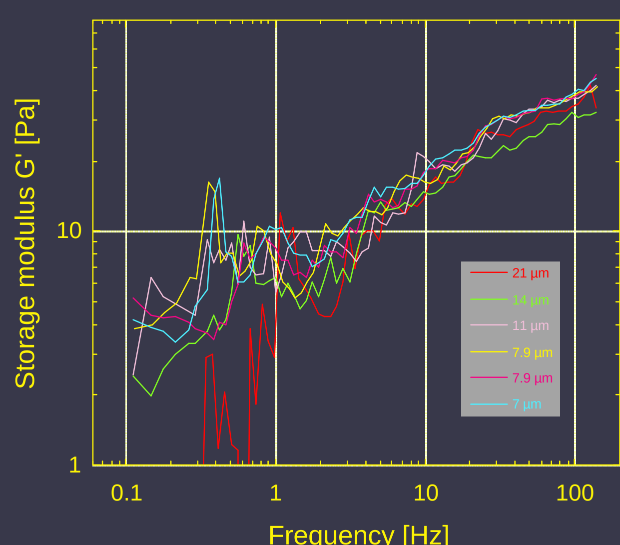
$$\alpha_{\parallel}^{(1,2)}(\omega)=\frac{1}{4\pi r\mu_0(\omega)} \quad , \quad \mu_0(\omega)=G(\omega)$$

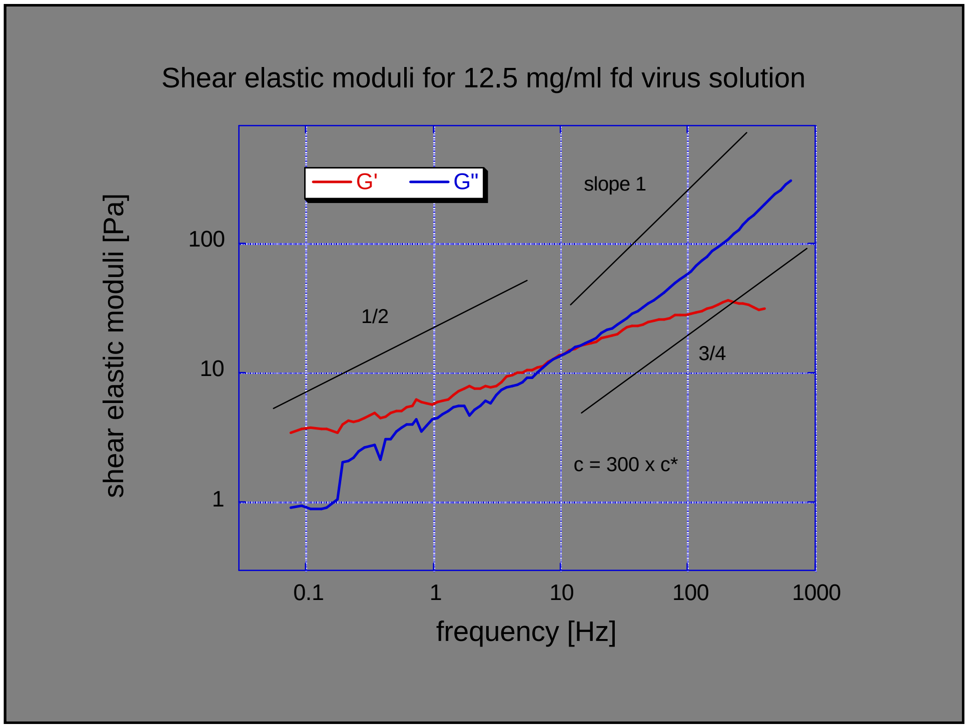
$$\alpha_{\perp}^{(1,2)}(\omega)=\frac{1}{8\pi r\mu_0(\omega)}\left[\frac{\lambda_0(\omega)+3\mu_0(\omega)}{\lambda_0(\omega)+2\mu_0(\omega)}\right]$$

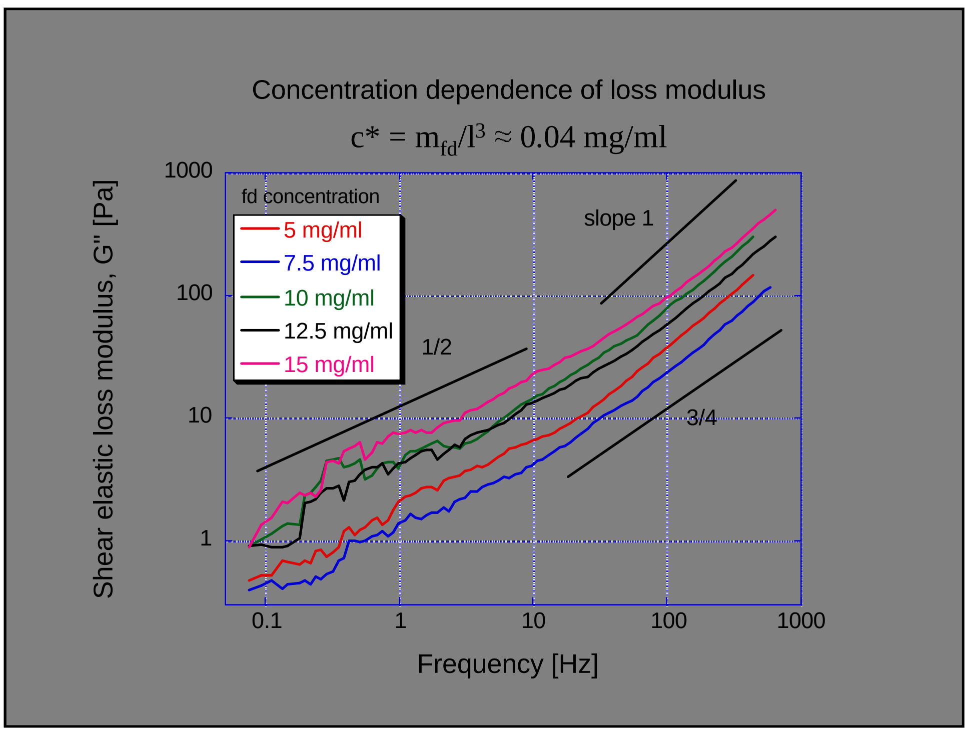


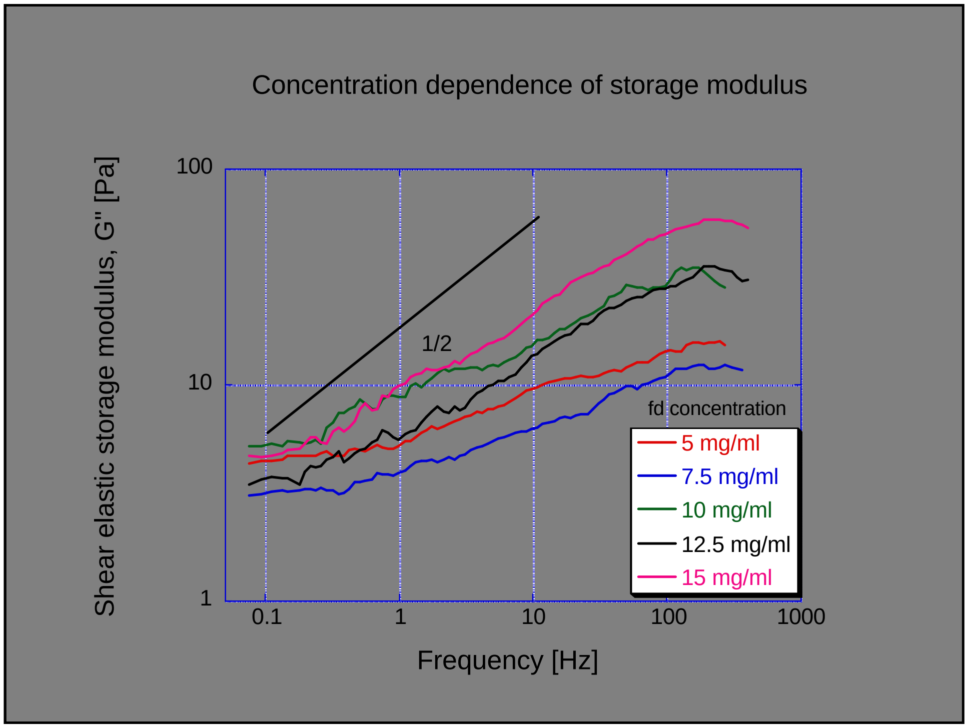


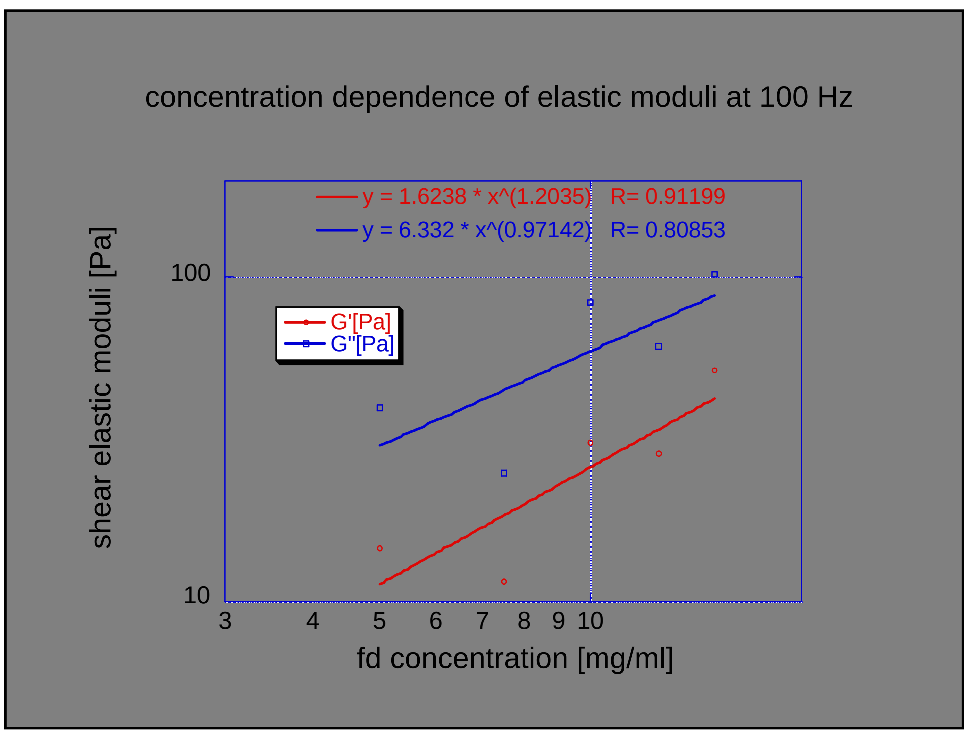
Shear elastic storage modulus for fd solution 18 mg/ml, measured by two-bead microrheology



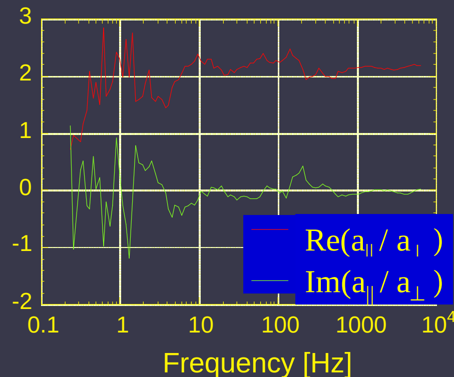

Shear elastic moduli for 12.5 mg/ml fd virus solution
G'
G''
slope 1
1/2
3/4
c = 300 x c*
100
10
1
0.1
1
10
100
1000
shear elastic moduli [Pa]
frequency [Hz]


Concentration dependence of loss modulus
$c^* = m_{fd}/l^3 \approx 0.04$ mg/ml
fd concentration
5 mg/ml
7.5 mg/ml
10 mg/ml
12.5 mg/ml
15 mg/ml
slope 1
1/2
3/4
1000
100
10
1
0.1
1
10
100
1000
Shear elastic loss modulus, G'' [Pa]
Frequency [Hz]


Concentration dependence of storage modulus
Shear elastic storage modulus, G'' [Pa]
100
10
1
0.1
1
10
100
1000
Frequency [Hz]
1/2
fd concentration
5 mg/ml
7.5 mg/ml
10 mg/ml
12.5 mg/ml
15 mg/ml


concentration dependence of elastic moduli at 100 Hz
y = 1.6238 * x^(1.2035) R= 0.91199
y = 6.332 * x^(0.97142) R= 0.80853
shear elastic moduli [Pa]
100
10
G'[Pa]
G"[Pa]
3
4
5
6
7
8
9
10
fd concentration [mg/ml]

Gel (in)compressibility measured by 2-bead microrheology
fd solution 18 mg/ml



$$\alpha_{\parallel}^{(1,2)}(\omega) = \frac{1}{4\pi r \mu_0(\omega)} \quad , \quad \mu_0(\omega) = G(\omega)$$

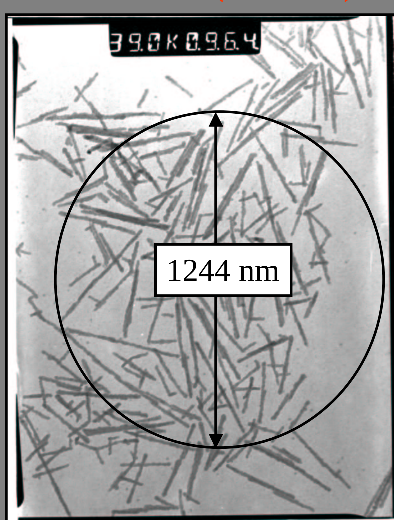
$$\alpha_{\perp}^{(1,2)}(\omega) = \frac{1}{8\pi r \mu_0(\omega)} \left[\frac{\lambda_0(\omega) + 3\mu_0(\omega)}{\lambda_0(\omega) + 2\mu_0(\omega)} \right]$$

In incompressible limit:

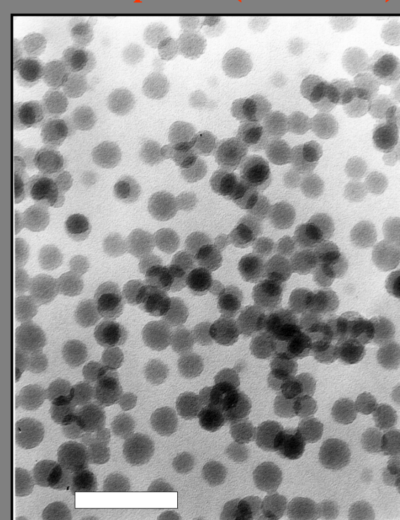
$$\frac{\alpha_{\parallel}^{(1,2)}(\omega)}{\alpha_{\perp}^{(1,2)}(\omega)} = 2$$

Translational Brownian motion: optical tweezers

Host = rods (L/D = 11)



Host = spheres (R = 15 nm)



Translational Brownian motion: optical tweezers

Host = rods (L/D = 10)

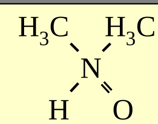
L = 202.7 nm
D = 17.8 nm
L/D = 11.4
$D_{probe}/L = 6$
$D_{probe}/D = 70$
ρ = 1.7845 g/mL

Core = AlOOH
Shell = silica (4 nm)
Negative charge from Si-OH

Host = spheres (R = 15 nm)

D ƒ 15 nm (DLS, SLS)
$D_{probe}/L = 83$
ρ = 2 g/mL

Material = silica
Negative charge from Si-OH

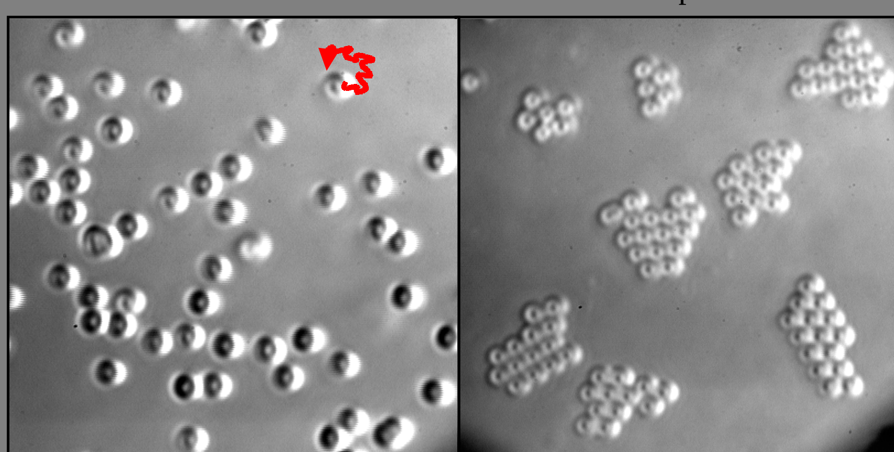


$\varepsilon_r = 36.7$
n = 1.4305
$\eta_0 = 0.796$ mPa.s
$pK_{auto} \sim 29$

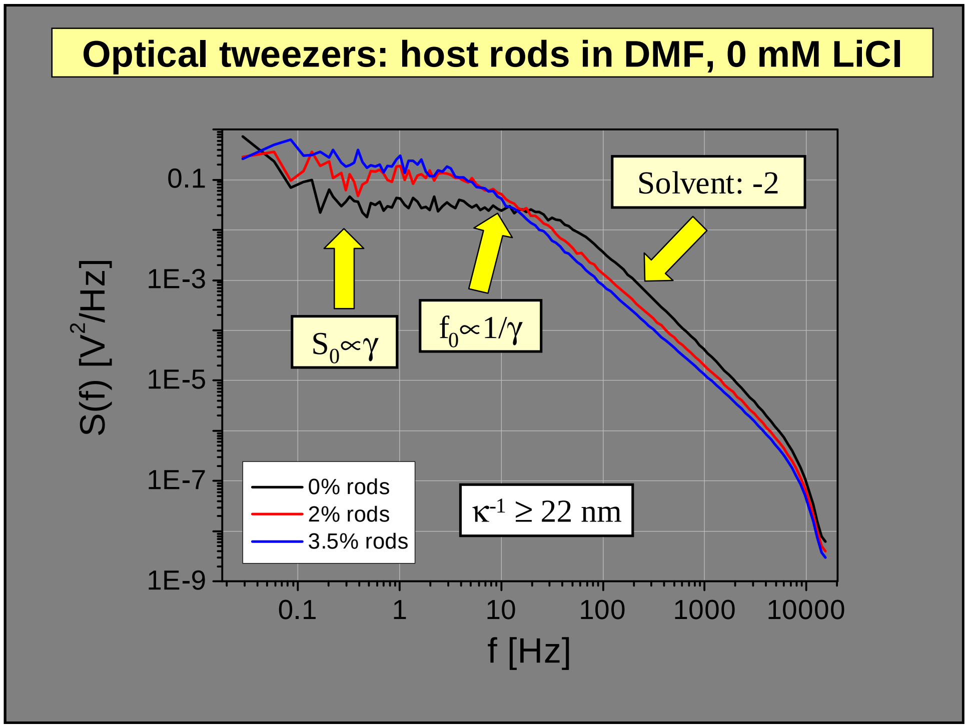
Solvent = DMF (N,N-dimethylformamide
Salt = LiCl (κ^{-1} = 2 - 22 nm)

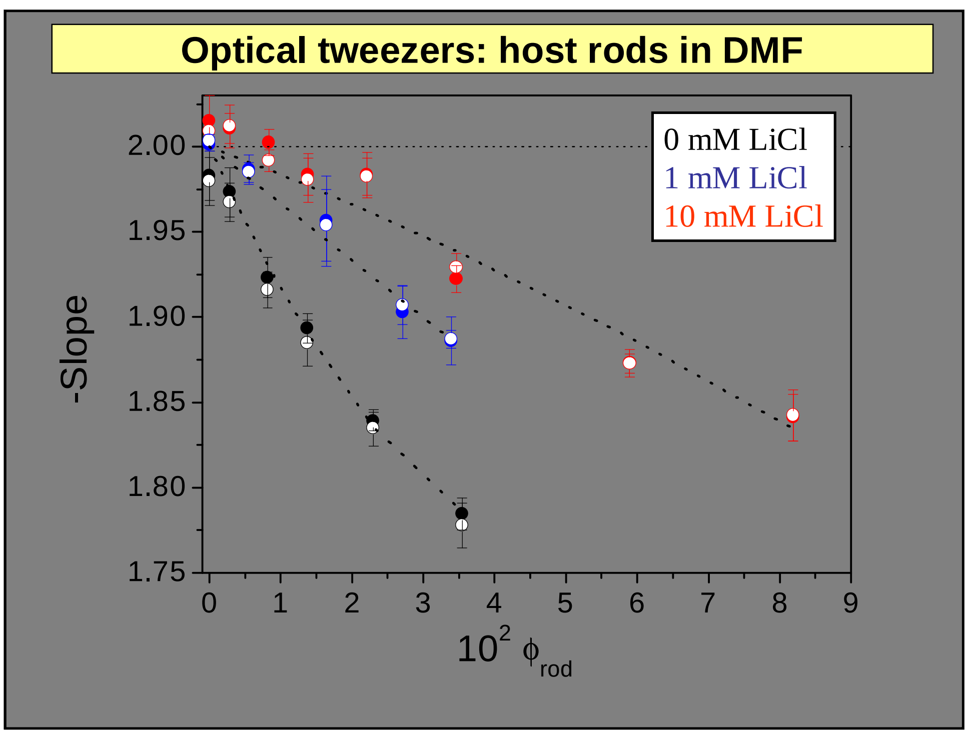
Depletion attraction

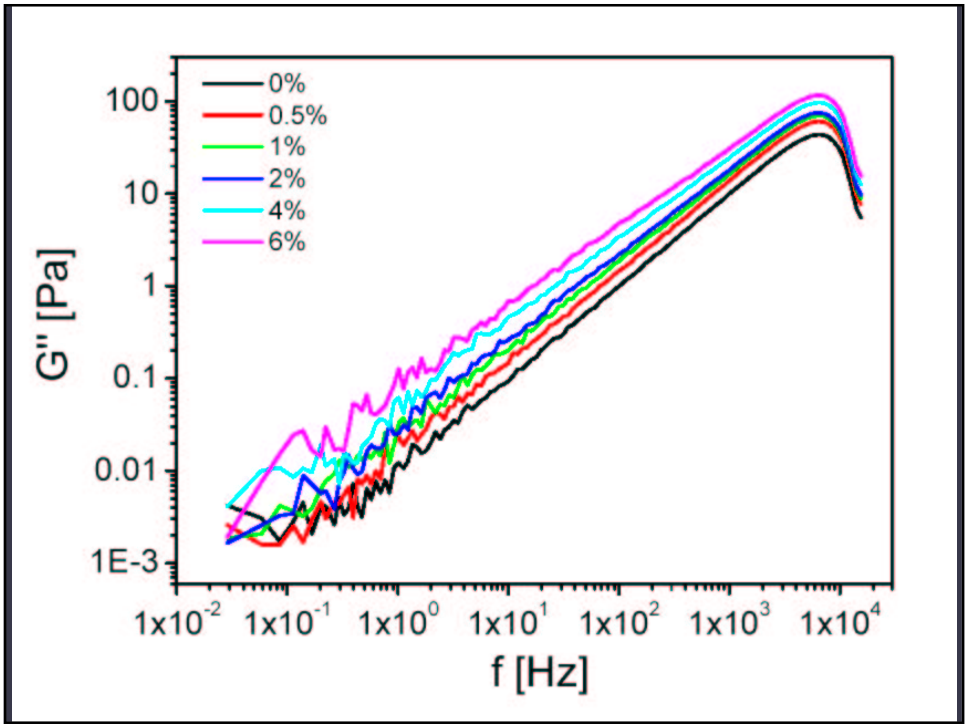
Solvent — Rod dispersion

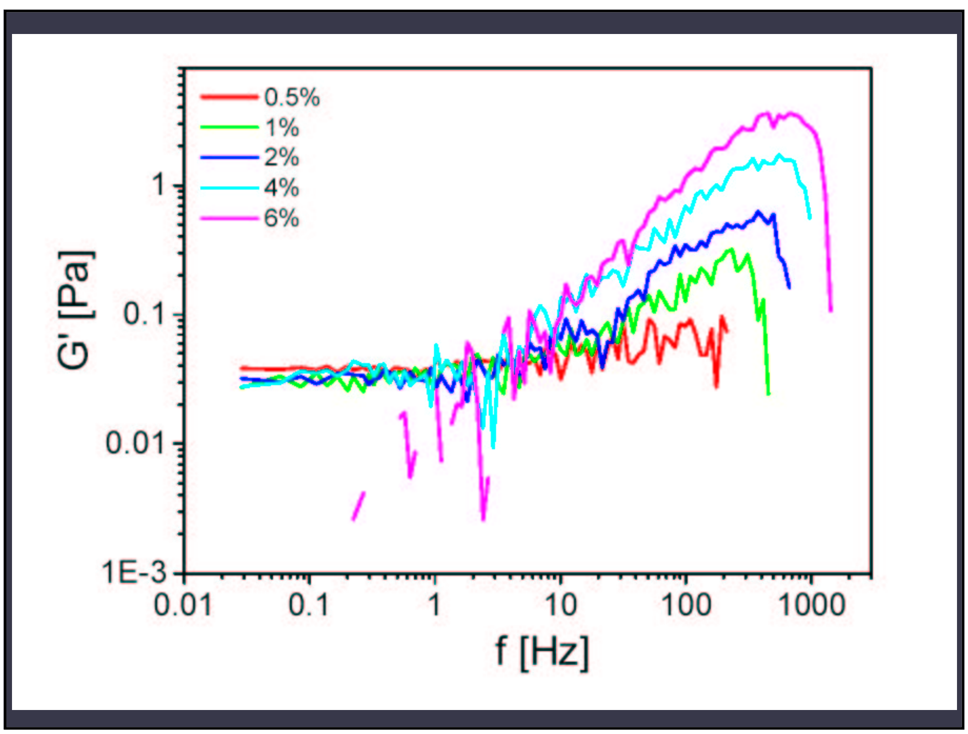


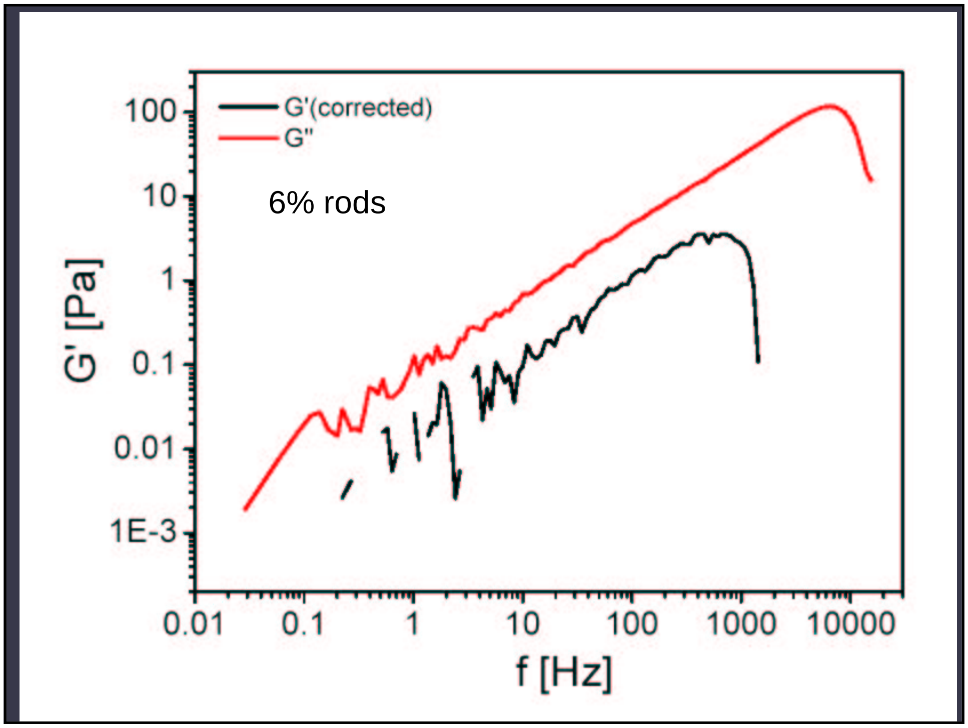
Koenderink et al., Depletion induced crystallization in colloidal rod-sphere mixtures, Langmuir **15**(97)4693


Optical tweezers: host rods in DMF, 0 mM LiCl
Solvent: -2
S0∝γ
f0∝1/γ
κ-1 ≥ 22 nm
0% rods
2% rods
3.5% rods
S(f) [V2/Hz]
f [Hz]
0.1
1E-3
1E-5
1E-7
1E-9
0.1
1
10
100
1000
10000


Optical tweezers: host rods in DMF
0 mM LiCl
1 mM LiCl
10 mM LiCl
-Slope
10² φrod
2.00
1.95
1.90
1.85
1.80
1.75
0
1
2
3
4
5
6
7
8
9


0%
0.5%
1%
2%
4%
6%
G'' [Pa]
f [Hz]
100
10
1
0.1
0.01
1E-3
1x10^-2
1x10^-1
1x10^0
1x10^1
1x10^2
1x10^3
1x10^4


0.5%
1%
2%
4%
6%
G' [Pa]
f [Hz]
1
0.1
0.01
1E-3
0.01
0.1
1
10
100
1000



Conclusions

— sampling of µm volumes is possible with microrheology

— frequency range can be extended to 10s of KHz and even MHz (DWS)

— wide frequency range makes it possible to probe different dynamic regimes

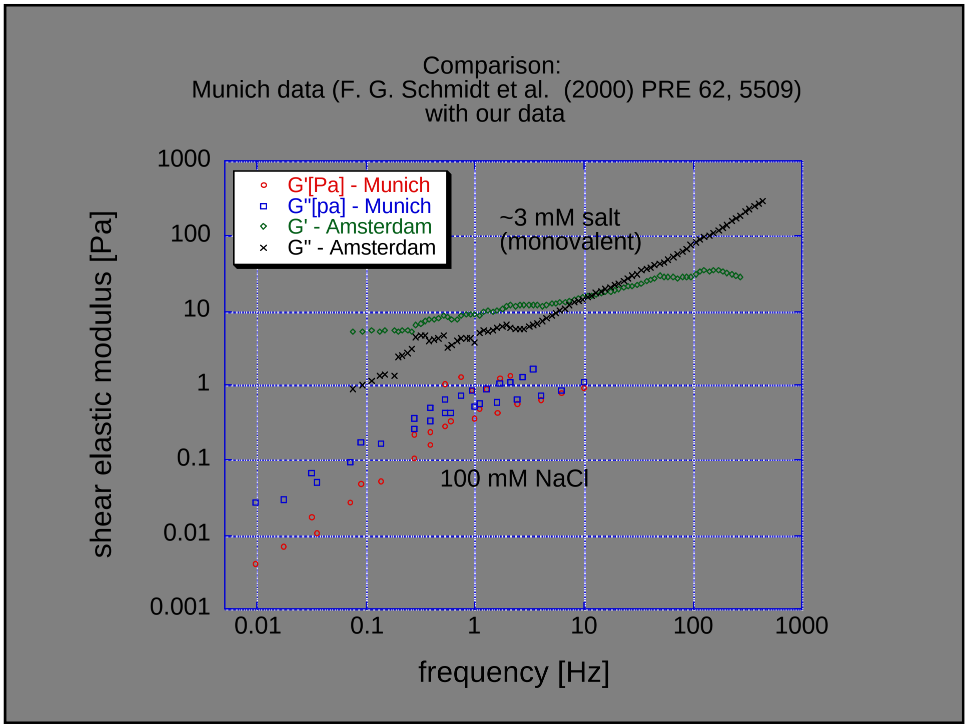
— especially interesting for semiflexible polymers, complex transitions, $\omega^{3/4}$scaling

— effect of electrostatics, not understood.

— data interpretation has to be done with caution: depletion layers, different dynamics at low frequencies etc.

— two-bead microrheology avoids depletion layer effects and measures compressibility directly

— open questions: plateau values,subtleties of transitions regimes, cross-linking, non-linearities, single-filament dynamics


Comparison:
Munich data (F. G. Schmidt et al. (2000) PRE 62, 5509)
with our data
shear elastic modulus [Pa]
frequency [Hz]
G'[Pa] - Munich
G"[pa] - Munich
G' - Amsterdam
G" - Amsterdam
~3 mM salt
(monovalent)
100 mM NaCl
1000
100
10
1
0.1
0.01
0.001
0.01
0.1
1
10
100
1000