

Center for  
Electronic Correlations and Magnetism  
University of Augsburg

## Realistic Investigations of Materials with Strong Electronic Correlations

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Supported by Deutsche Forschungsgemeinschaft through SFB 484

### 1. Introduction:

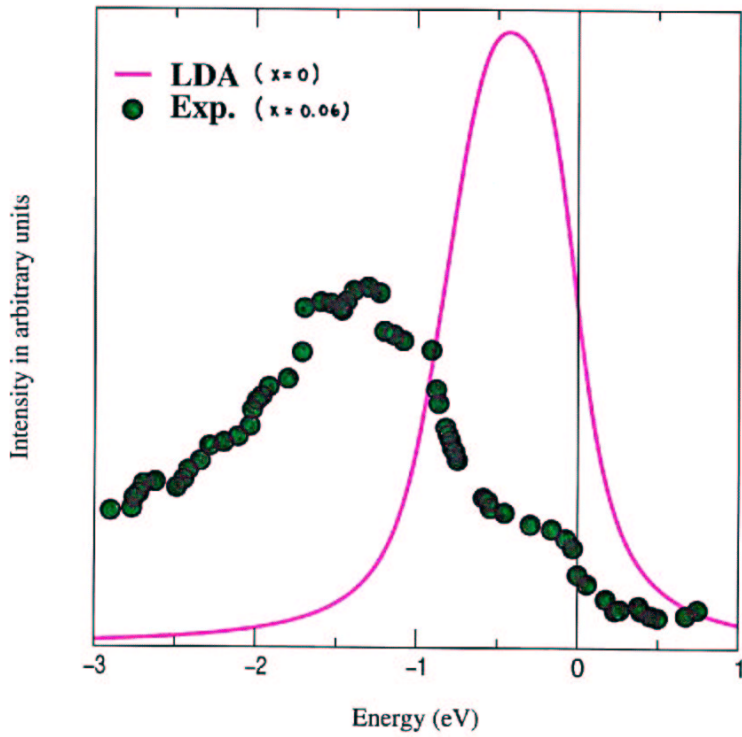
- DFT/LDA vs. model approaches
- Dynamical mean-field theory (DMFT)
- Single-impurity physics
- LDA + DMFT

### 2. Applications:

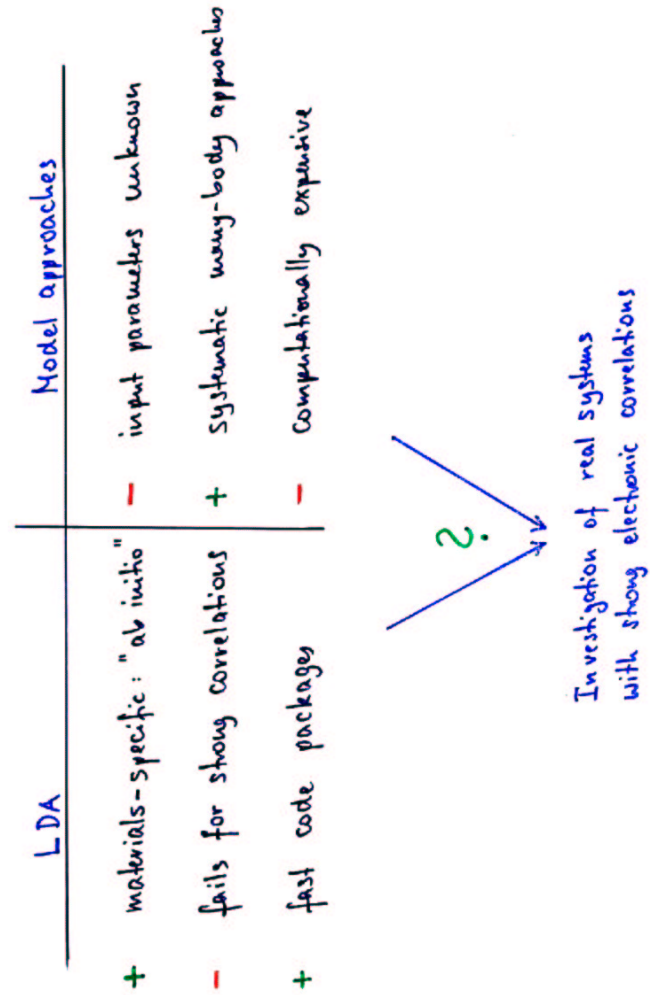
- $(La, Sr)TiO_3$
- $V_2O_3$
- $(Ca, Sr)VO_3$
- [ •  $LiV_2O_4$  ]
- How much LDA-input is necessary?

### 3. Questions

Photoemission on  $\text{La}_{1-x}\text{Sr}_x\text{TiO}_3$ ,  $x = 0.06$



Comparison of the experimental photoemission spectrum [Fujimori et al., PRL **69**, 1796 (1992)] to the LDA result.



How to include correlations ?

LDA + U

Anisimov, Zaanen, Andersen (1991)

But: genuine correlation effects are due to many-body dynamics!  
 $\Sigma(\omega)$

LDA + U<sup>2</sup>

Steiner, Albers, Sham (1992)

⋮

LDA + DMFT

Anisimov, Poteryaev, Korotin, Anokhin, Kotliar (1997)

LDA<sup>++</sup>

Lichtenstein, Katsnelson (1998)

GW + DMFT

Biermann, Aryasetiawan, Georges ('02)

⋮

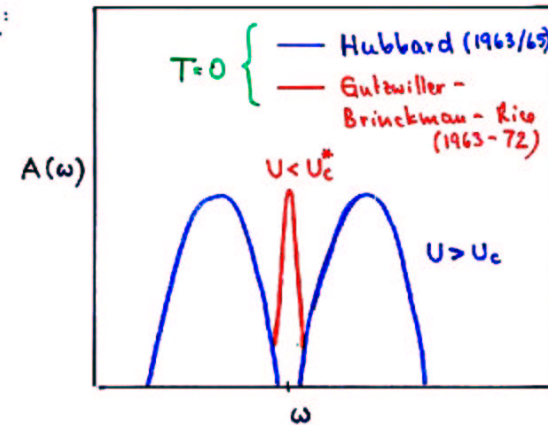
DFT + multi-band Gutzwiller WF

Bünemann, Gebhard, Weber ('99)

## Models Approach

$$H = \sum_{ij,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

n=1:

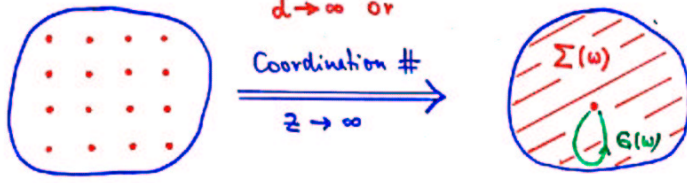


Coherent, thermodynamically consistent framework ?

From the  $d \rightarrow \infty$  limit to dynamical mean-field theory (DMFT)

Metzner, Vollhardt (1989)

Real lattice system



Self-energy  $\Sigma_{ij}(\omega) \rightarrow \Sigma(\omega) \delta_{ij}$   
 Propagator  $G_{ij}(\omega) \rightarrow G(\omega) \delta_{ij}$

- Dynamics fully included
  - Spatial dependence simplified
- Müller-Hartmann (1989)

$$1) G(\omega) = \int d\epsilon \frac{N^o(\epsilon)}{\omega + \mu - \epsilon - \Sigma(\omega)} = G^o(\omega - \Sigma(\omega))$$

$$2) G(\omega) = -\frac{1}{Z} \int \mathcal{D}[\psi, \psi^*] \psi \psi^* e^A$$

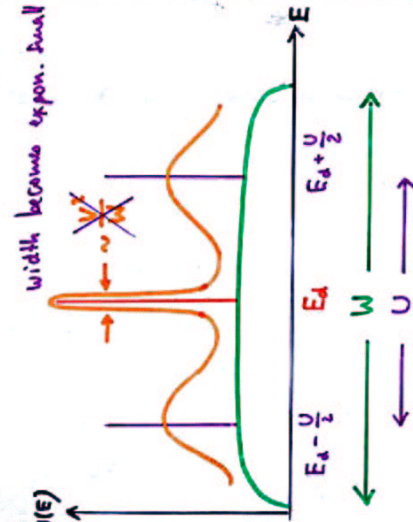
Braut, Hildich (1989)  
 Janis (1991)

single-site action:  $A \sim \underbrace{\text{kinetic energy}}_{\epsilon^{-1}} - \underbrace{\text{potential energy}}_{U n_p n_d - \Sigma}$

$$= \text{tr} \psi^* \underbrace{[\epsilon^{-1} + \Sigma]}_{\mathcal{G}^{-1}} \psi - U \int_0^{\beta} d\tau n_p(\tau) n_d(\tau)$$

$\Rightarrow$  Single-impurity ("Anderson") problem Georges, Kotliar (1992)

Abrikosov-Suhl resonance:



one orbital/site

$$H = \sum_{\langle ij \rangle, \sigma} t_{ij} c_{i\sigma} c_{j\sigma} + E_d \sum_{\sigma} d_{\sigma} d_{\sigma} + V \sum_{i, \sigma} c_{i\sigma} d_{\sigma} + \text{h.c.}$$

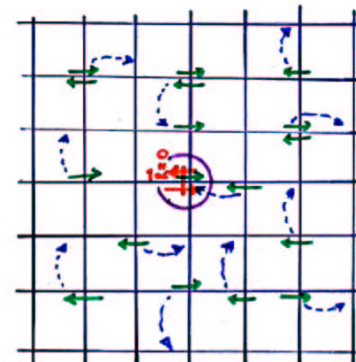
additional orbital at  $Z=0$  (d-elect)

hybridization

- characteristic (three-)peak structure
- physics well-understood

$$+ U n_p^d n_d^d$$

Single-impurity physics



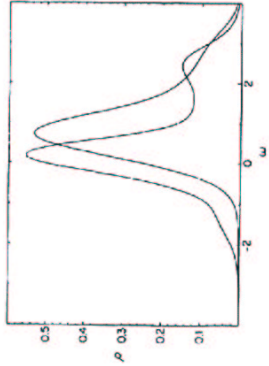


FIG. 1. Local spectral density for  $U=2.5$  and densities  $n=0.6$ ,  $n=0.2$  (from left to right), obtained by the procedure described in the text.

cal moment regime for large  $U$  is likely to arise only quite close to  $n=1$  for large  $U$ . Most densities are then in the mixed-valence regime, with the dilute regime setting in roughly near the Hartree-Fock boundary.

Finally, we present results obtained by solving (3)-(5) using second-order perturbation theory for the single-site dynamics (3). A previous weak-coupling study was carried out by Müller-Hartmann,<sup>10</sup> using self-consistent perturbation theory, i.e., inserting the full propagator  $G$  into the calculation of the second-order proper irreducible self-energy. Our method and results differ significantly from this approach. Indeed, it has been shown by Yosida and Yamada<sup>11</sup> that perturbation theory in  $U$  is quite well behaved for the Anderson model, provided the expansion is made around the nonmagnetic Hartree-Fock solution, i.e., that  $U(n_1 - n/2)(n_1 - n/2)$  is treated as a perturbation. Only this procedure is able to handle correctly the

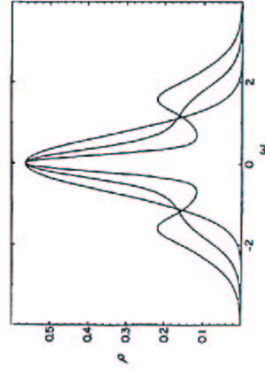
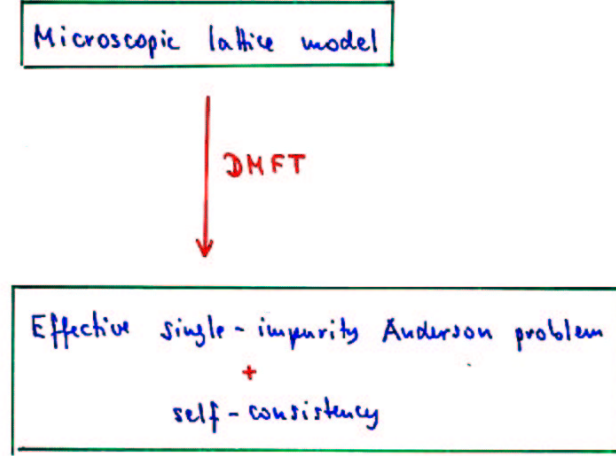


FIG. 2. Local spectral density of the paramagnetic solution at half filling ( $n=1$ ) for  $U=0$ ,  $U=1.5$ , and  $U=2.5$ .

approaches, in contrast to direct perturbation theory. The Hartree shift is responsible for the appearance of the upper-band satellite peaks, as also pointed out recently by Schweitzer and Czycholl at half-filling in a related context [Ref. 14 has results qualitatively close to ours; the effect of satisfying (5) is to push down the upper band to lower energies and to somewhat reduce  $Z$  further].

As a conclusion, we believe that the infinite-dimensional Hubbard model provides a very natural mean-field description of strongly correlated Fermi liquids. It captures both the itinerant and atomic aspects and their interplay, which is at the heart of the strong correlation problem. The connection to a single-impurity problem clarifies the analytic structure of the perturbation theory in the weak- and strong-coupling regimes, it explains how the Hubbard bands emerge in the spectral function of the Hubbard model, and suggests useful approximation schemes to extract the essential physics of the Hubbard

$$\sum_{\mathbf{x}} \chi(\omega) : \text{"local" theory}$$



Set-up for LDA + many-body theory

Anisimov et al. (1997)  
Lichtenstein, Kubel'son (1998)

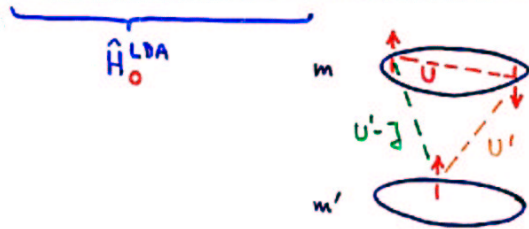
$$\hat{H} = \underbrace{\hat{H}^{LDA} \{t_{ilm,jl'm'}, \epsilon_{ilm}\}}_{\text{LDA-LMTO}} + \underbrace{\hat{H}_{\text{corr}} \{U_{mm'}, J_{mm'}\}}_{\text{local correl.}} - \underbrace{\hat{H}_U^{LDA}}_{\text{Coulomb Correl. in } \hat{H}^{LDA}}$$

$\downarrow$   $E^{LDA}$  (total LDA energy)       $\downarrow$   $E_U^{LDA}$  (average Correl. energy in LDA)

$$\epsilon_{ilm}^0 := \frac{d}{dn_{ilm}} (E^{LDA} - E_U^{LDA})$$

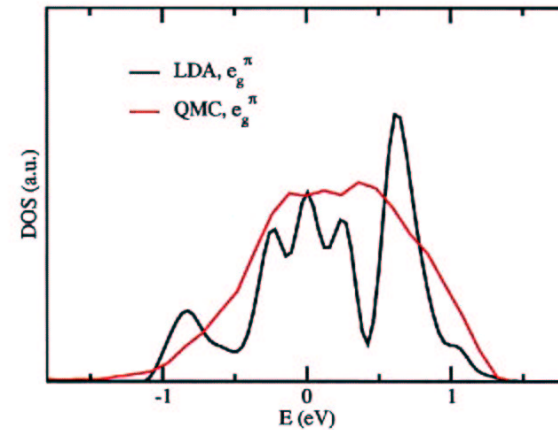
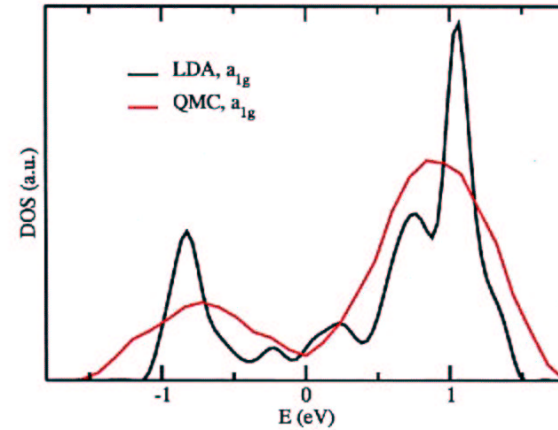
⇒ "ab initio Hamiltonian"

$$\hat{H}_{LDA+corr} = \hat{H}^{LDA} \{t_{ilm,jl'm'}, \epsilon_{ilm}^0\} + \hat{H}_{\text{corr}} \{U_{mm'}, J_{mm'}\}$$



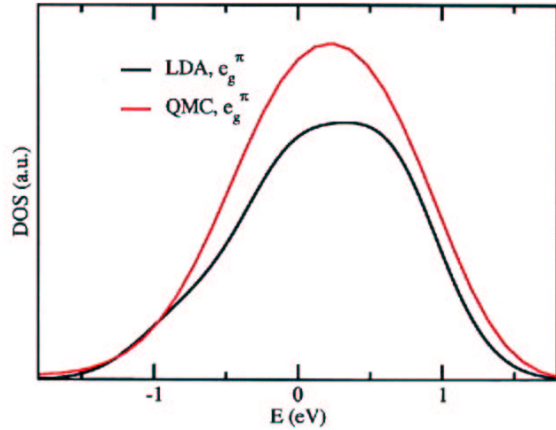
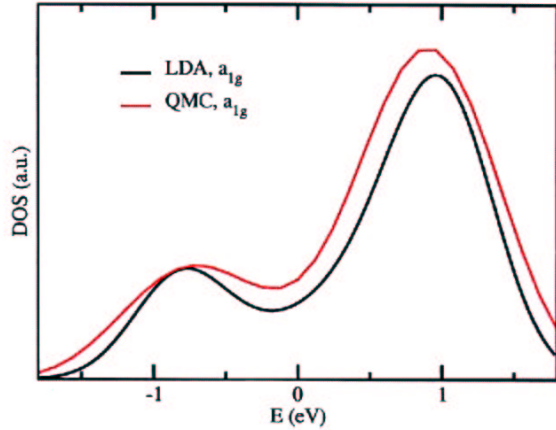
G. Keller et al. (2002)

Comparison between LDA and LDA+DMFT(QMC) at  $U = 0$ ;  $V_1 O_3$

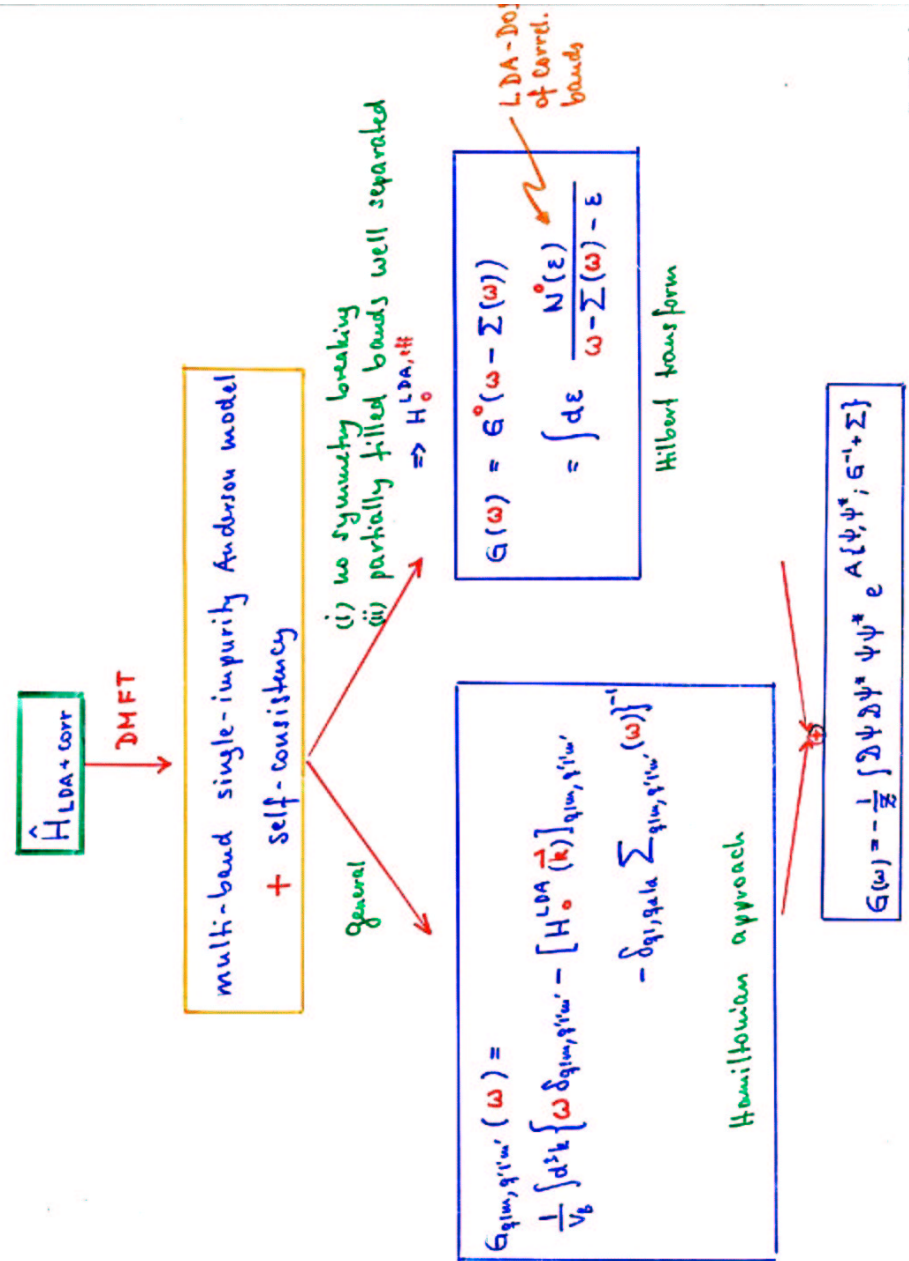


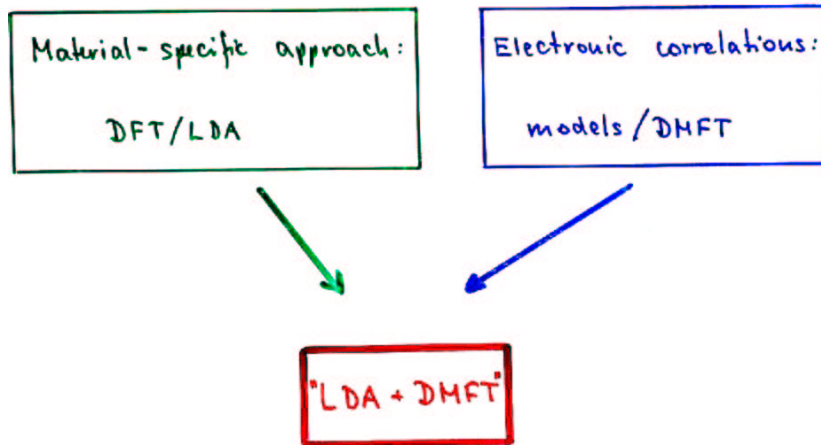
QMC-Simulations at  $T = 1100$  K and  $U = 0$  eV

Comparison between LDA and LDA+DMFT(QMC) at  $U = 0$  (with broadening)



QMC-Simulations at  $T = 1100$  K and  $U = 0$  eV, assumed broadening for LDA and LDA+DMFT 0.3 eV





Dynamical mean-field theory (DMFT) for  $\hat{H}_{\text{LDA+corr}}$



LDA + DMFT (X)

X: single-impurity solver

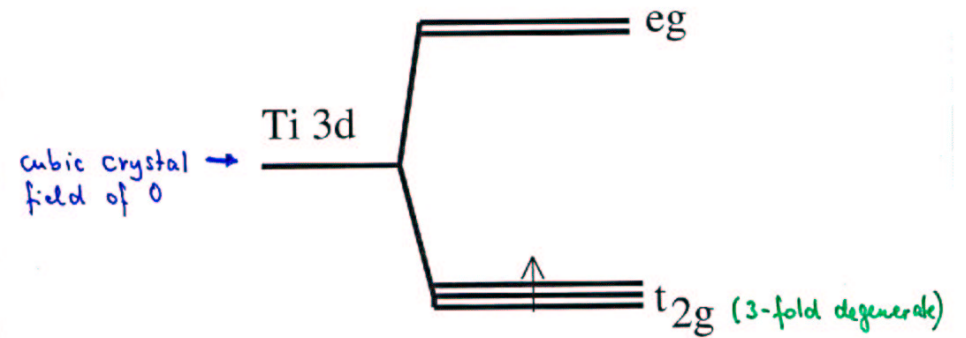
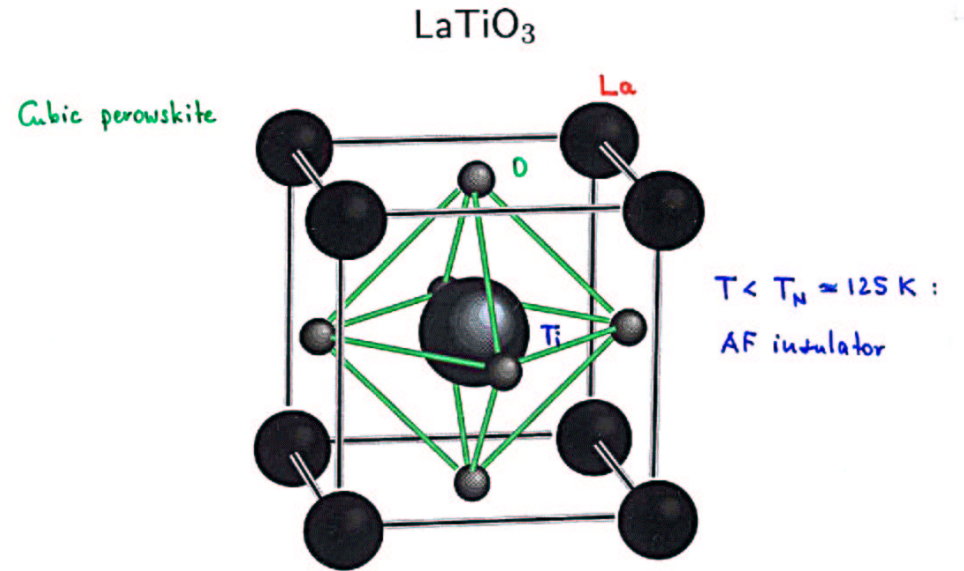
IPT, NCA, QMC

Alternatives ?

- NRG
- Flow equ. approach

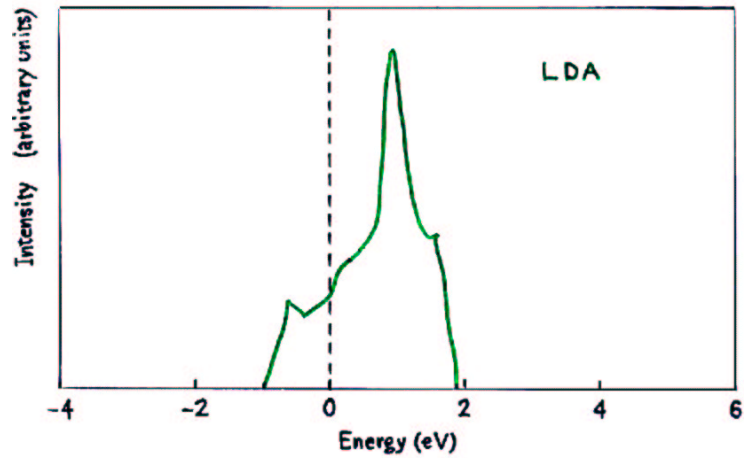


Application to  $\text{La}_{1-x}\text{Sr}_x\text{TiO}_3$



Sr - doping  $x \geq 5\%$  : strongly correlated paramagnetic metal

LaTiO<sub>3</sub> : partial t<sub>2g</sub> density of states



Input to DMFT

$x=6\%: \text{La}_{1-x}\text{Sr}_x\text{TiO}_3 :$

$$U^{\text{LDA}} = 4.2 \pm 1 \text{ eV}$$

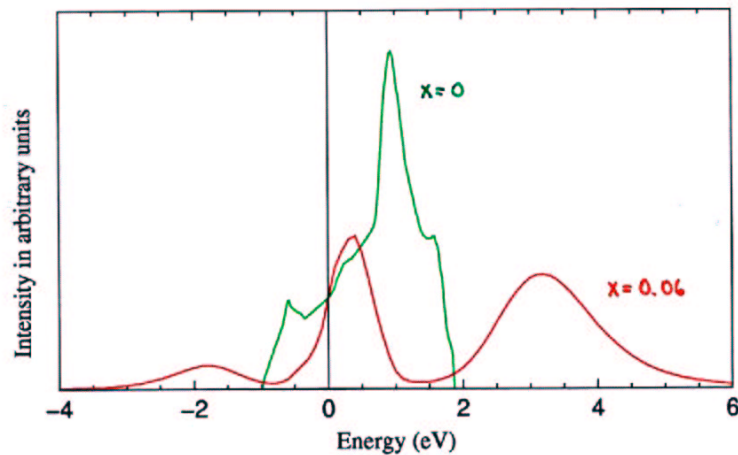
Sensitive dependence on orthogon. of WF  
+ choice of orbitals : TB-LMTO-ASA  
ASA-LMTO  
⋮

Need "best possible" LDA values for  $U, J, \dots$

La<sub>1-x</sub>Sr<sub>x</sub>TiO<sub>3</sub>: Results from LDA+DMFT(QMC)

Nekrasov, Held, Blümer, Poteryaev, Anisimov, Vollhardt  
Eur. Phys. J. B **18**, 55 (2000)

Partial  $t_{2g}$  densities of states



LDA (—)

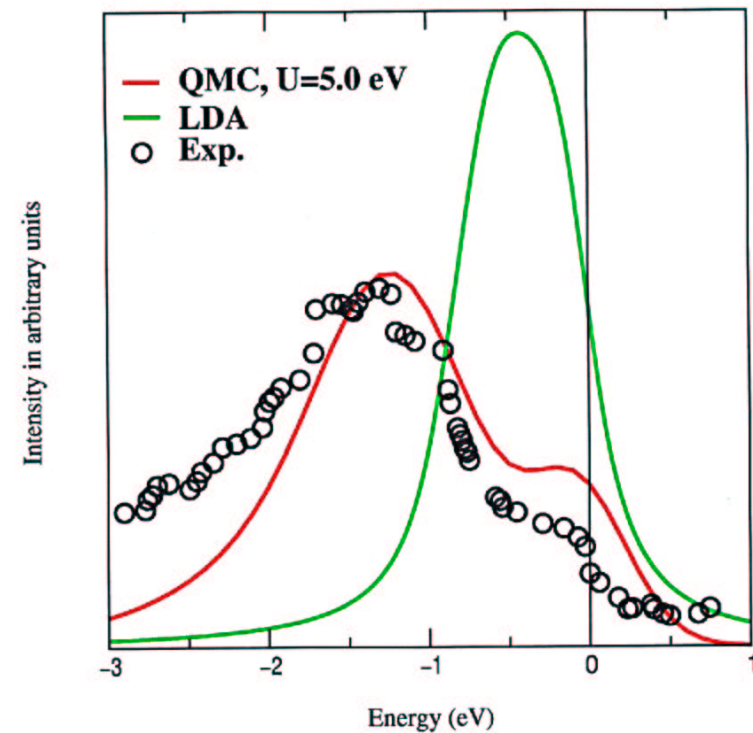
LDA+DMFT(QMC) (—)

Parameters for DMFT:

$x = 0.06$ ,  $U = 4.0$  eV,  $T = 0.1$  eV  $\sim 1000$  K, 3 degenerate bands

## Comparison with experiment

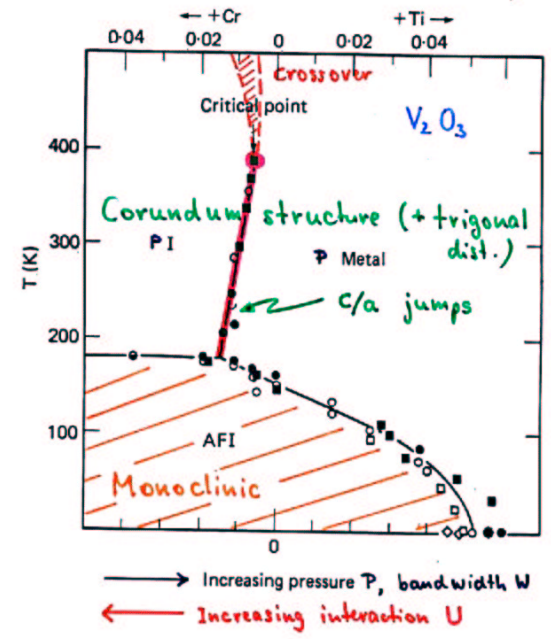
Nekrasov, Held, Blümer, Poteryaev, Anisimov, Vollhardt  
Eur. Phys. J. B **18**, 55 (2000)



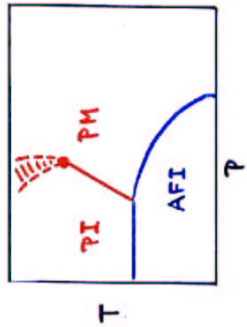
Comparison of the experimental photoemission spectrum ( $T = 80$  K, resolution  $\Delta\varepsilon = 0.3$  eV) [Fujimori et al., PRL **69**, 1796 (1992)], the LDA result ( $T = 0$ ), and the LDA+DMFT(QMC) calculation ( $T = 1000$  K) for LaTiO<sub>3</sub> with 6% hole doping.

Application to  $V_2O_3$

Mott-Hubbard Metal-Insulator Transition in  $V_2O_3$



- PI → PM:
- 1) 1. order transition, but no change in crystal symmetry
  - 2)  $\frac{dU}{dT} < 0 \implies S_I > S_M \leftrightarrow$  "Pomeranchuk effect"

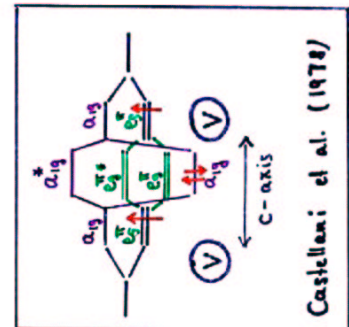
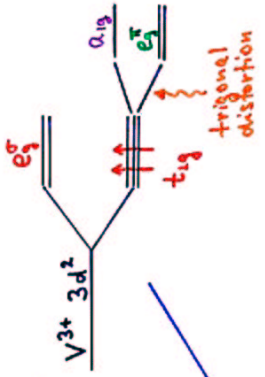


⇒ MIT driven by electronic interactions

Models approach

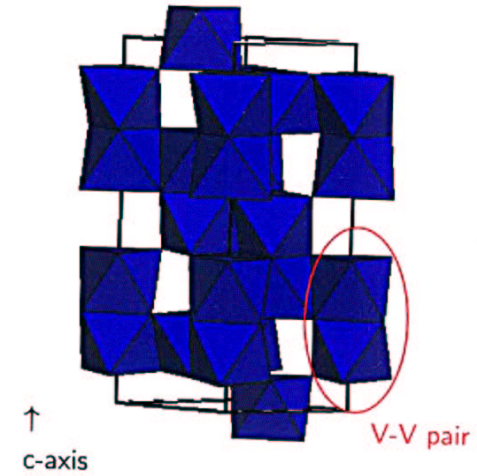
Material-specific input

nc-band Hubbard model  
 ( $S = \frac{1}{2}$ ,  $n=1$ , "frustrated" AF)  
 effective  $S = \frac{1}{2}$  model,  
 2-fold degeneracy

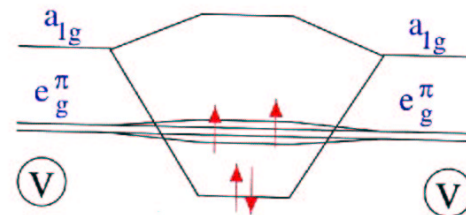


### Spin state of $V_2O_3$

#### Crystal structure



#### $a_{1g}$ -splitting



Castellani et al. '78 → Spin  $S = \frac{1}{2}$

Castellani et al. (1978) : 1 spin / V in  $e_g^{\uparrow}$  orbital  
 $\Rightarrow S = \frac{1}{2}$  model

**But:**

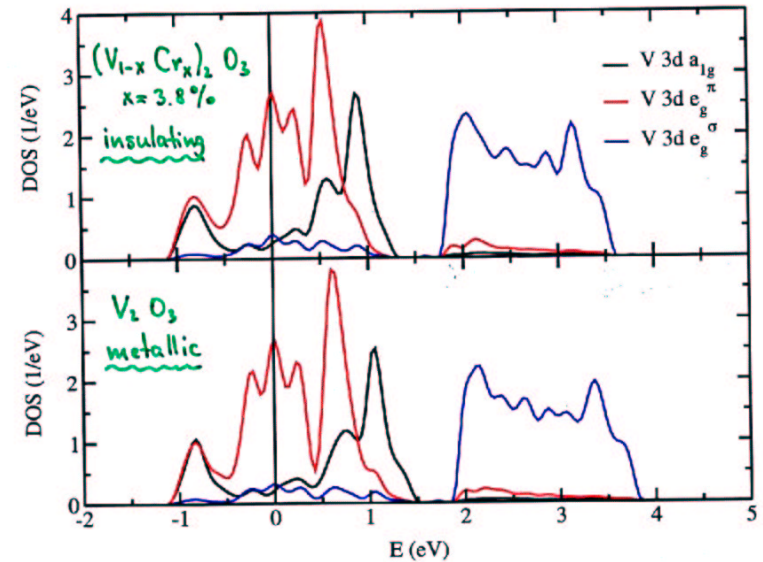
Park et al. (2000) : polarized x-ray spectroscopy  
 $S=1$  state,  
 $e_g^{\uparrow} e_g^{\uparrow} \oplus e_g^{\uparrow} a_{1g}$  confg.

Eehov et al. (1999) }  
 LDA + U }  
 Mila et al. (2000) }  
 (AFI)  $\rightarrow$   $S=1$  state  
 orbital degeneracy

$\rightarrow$  theory for the paramagnetic phase?

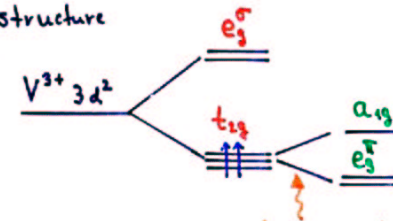
(iv) LDA results for  $V_2O_3$

Held, Keller, Eyert, Vollhardt, Anisimov (2001)



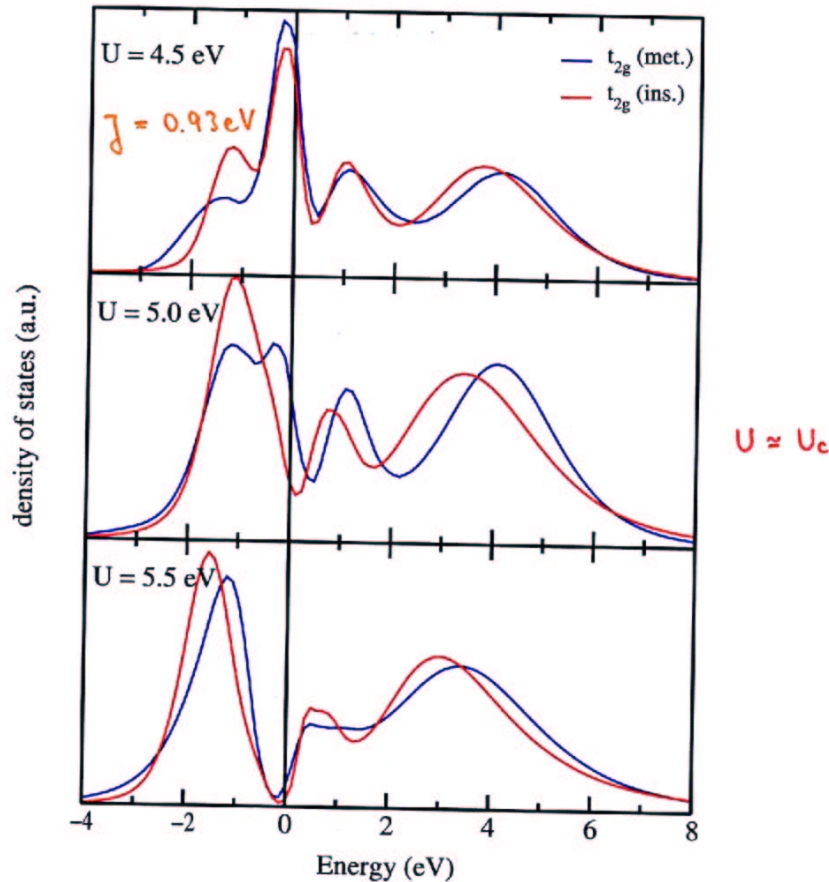
Total and partial densities of states (DOS) of  $V_2O_3$  (metallic paramagnetic phase) and  $(V_{0.962}Cr_{0.038})_2O_3$  (insulating paramagnetic phase) per unit cell

PI/PM : corundum structure



(ii) Introduce electronic correlations by LDA+DMFT(QMC)

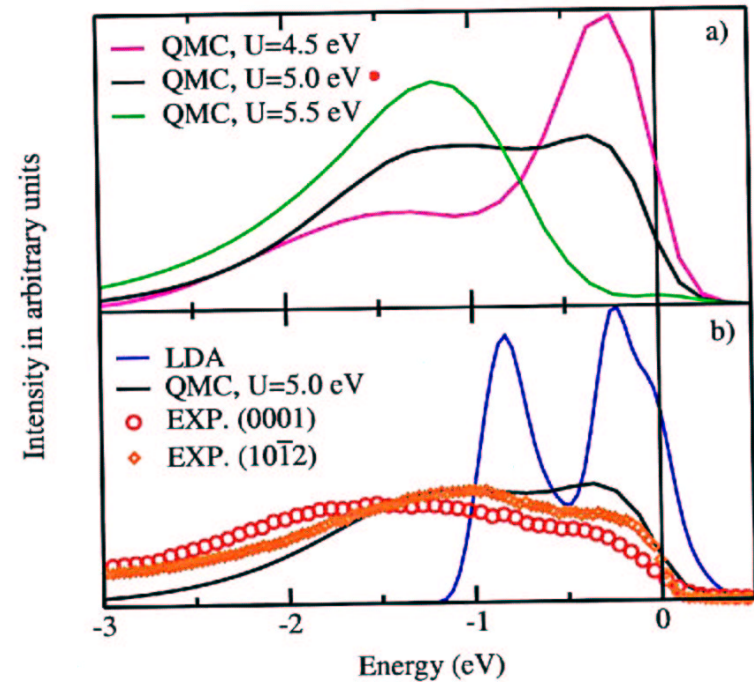
Held, Keller, Eyert, Vollhardt, Anisimov, PRL 2001



LDA+DMFT(QMC) spectra for paramagnetic  $(V_{0.962}Cr_{0.038})_2O_3$  ("ins.") and  $V_2O_3$  ("met.") for  $U = 4.5, 5, 5.5$  eV;  $T = 1100$  K.

$V_2O_3$

Held, Keller, Eyert, Vollhardt, Anisimov [PRL **86**, 5345 (2001)]



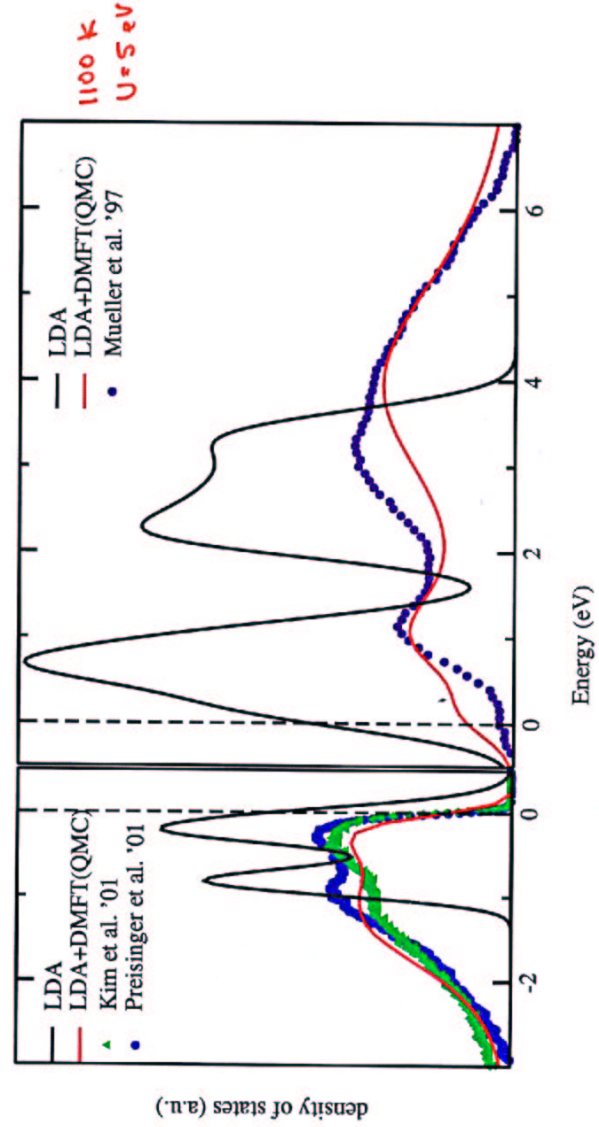
a) Comparison of LDA+DMFT(QMC) calculations for  $U = 4.5, U = 5.0$  and  $U = 5.5$  eV  
 b) Comparison of the experimental photoemission spectrum [Schramme et al.], the LDA result [Eyert], and the LDA+DMFT(QMC) calculation for  $V_2O_3$  and Coulomb interaction 5.0 eV (assumed experimental resolution 0.05 eV).

Correlation parametrized by, e.g.,

$\epsilon/C$  ← less screening at surface  
 ← smaller coord. # ⇒ smaller band width

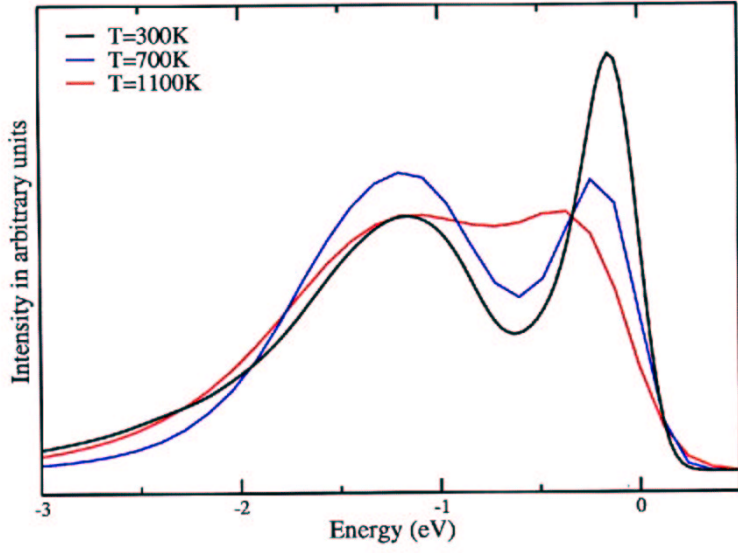
↓  $\epsilon/C$  larger at surface

$V_2O_3$ : Comparison with photoemission spectroscopy (PES) and X-ray absorption data (XAS)



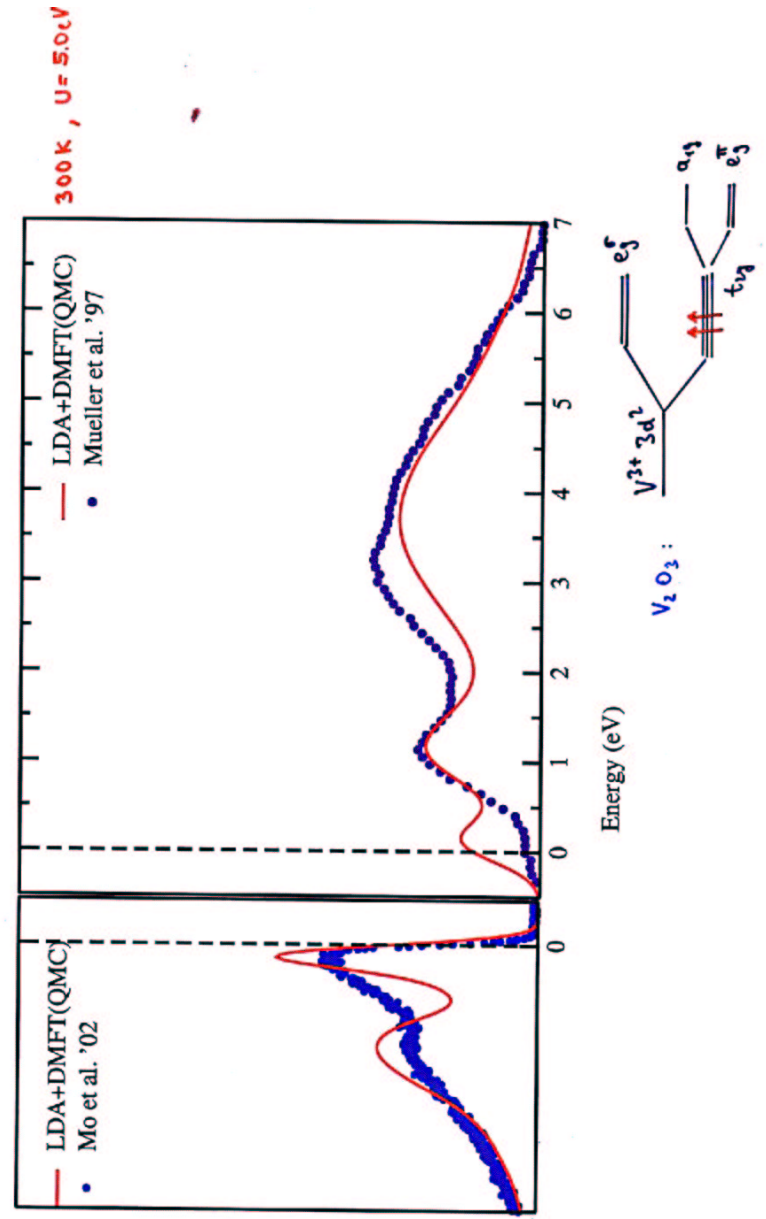


LDA+DMFT(QMC) results for metallic  $V_2O_3$



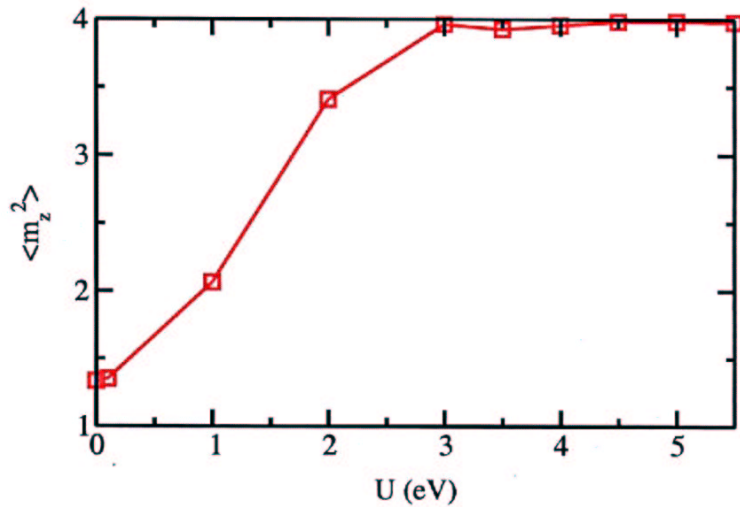
Calculations at  $T = 300$  K,  $T = 700$  K and  $T = 1100$  K for  $U = 5.0$  eV

$V_2O_3$ : Comparison with photoemission spectroscopy (PES) and X-ray absorption data (XAS)



### Spin state of V<sub>2</sub>O<sub>3</sub>

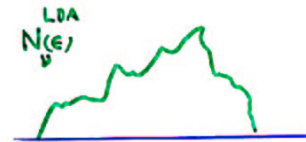
Held, Keller, Eyert, Vollhardt, Anisimov [PRL **86**, 5345 (2001)]



$$\langle m_s^2 \rangle = \left\langle \left[ \sum_{\nu=1}^3 (\hat{n}_{\nu\uparrow} - \hat{n}_{\nu\downarrow}) \right]^2 \right\rangle \xrightarrow{U > 3\text{eV}} 4$$

**S=1 state**

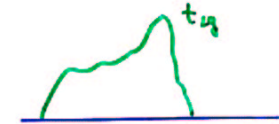
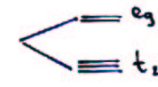
LDA+DMFT: How much material-specific (LDA) input is required?



$$M_{\nu}^{(n)} = \int d\epsilon \epsilon^n N_{\nu}^{\text{LDA}}(\epsilon) : \begin{cases} n=0 & \text{density} \\ n=1 & \bar{\epsilon} \\ n=2 & (\text{bandwidth})^2 \end{cases}$$

1) Degenerate bands

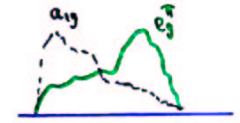
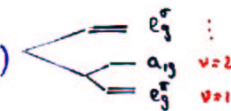
e.g. LaTiO<sub>3</sub>



→ M<sup>(2)</sup>

2) Non-degenerate bands

e.g. V<sub>2</sub>O<sub>3</sub> (trig. splitting)



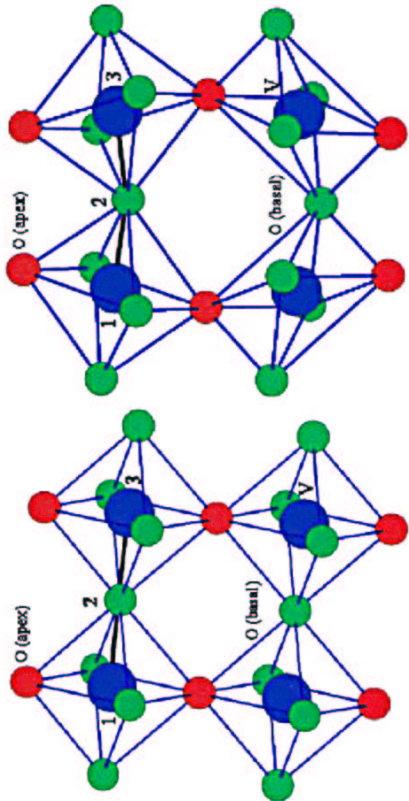
→ M<sub>ν</sub><sup>(1)</sup>, M<sub>ν</sub><sup>(2)</sup>, (M<sub>ν</sub><sup>(3)</sup>)

3) General case

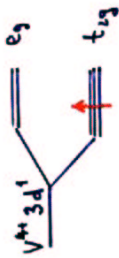
- Strong hybridization of bands
- very diff. bond strengths, e.g. NaV<sub>2</sub>O<sub>5</sub>
- ...

→ beyond LDA DOS: Hamiltonian approach

SrVO<sub>3</sub> vs. CaVO<sub>3</sub>: crystal structure



• cubic perovskite



• paramagnetic metals  
thermodyn. + transport  
quantities of Sr<sub>1-x</sub>Ca<sub>x</sub>VO<sub>3</sub>  
almost indep. of x

• natural oxygen vacancies  
(~1%)

SrVO <sub>3</sub> is ideal Fm $\bar{3}$ m $\angle 123 = 180^\circ$	CaVO <sub>3</sub> is orthorhombically distorted Pbnm $\angle 123 = \cancel{147^\circ} 160^\circ$
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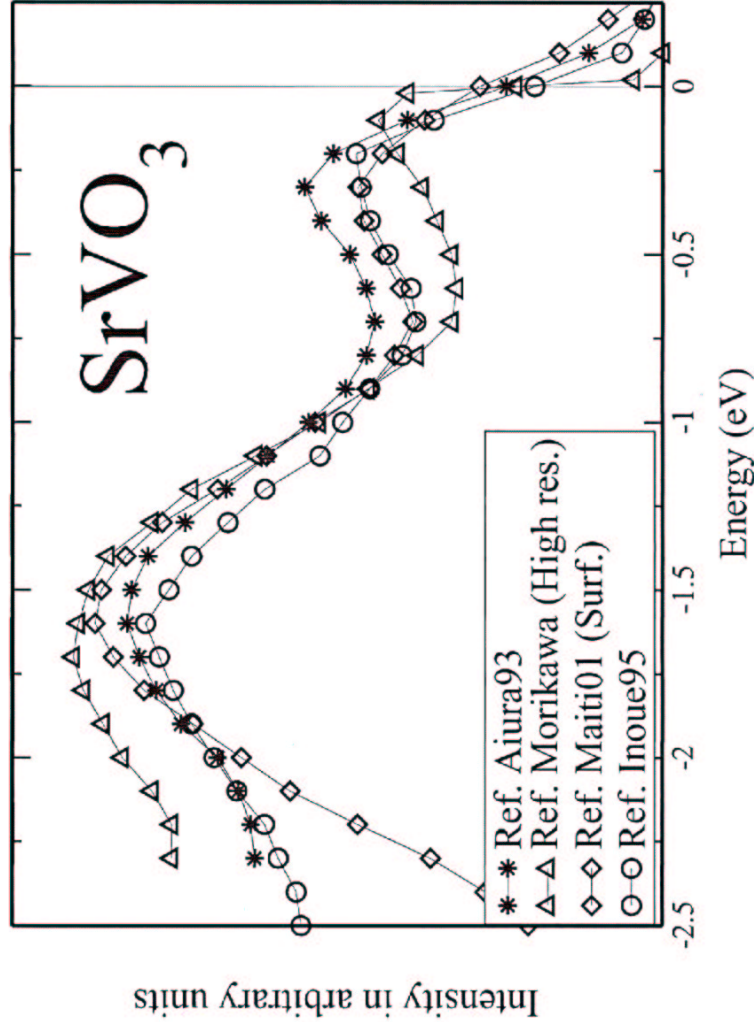


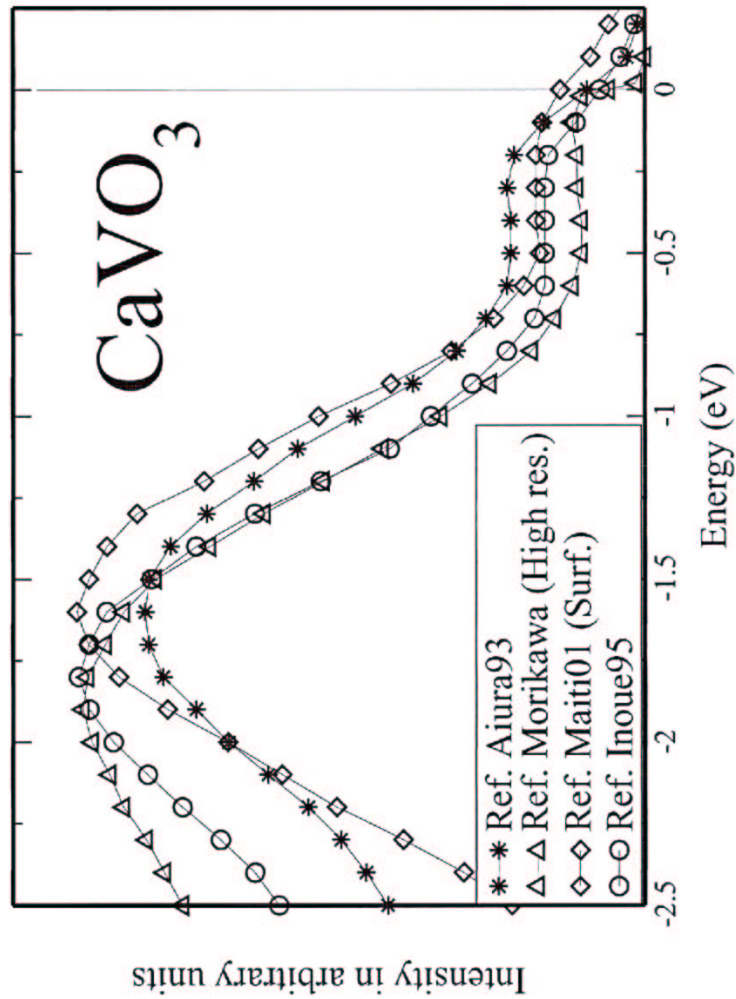
main effect: reduction of  $\angle 123$

$|\cos \theta| = 0.94$

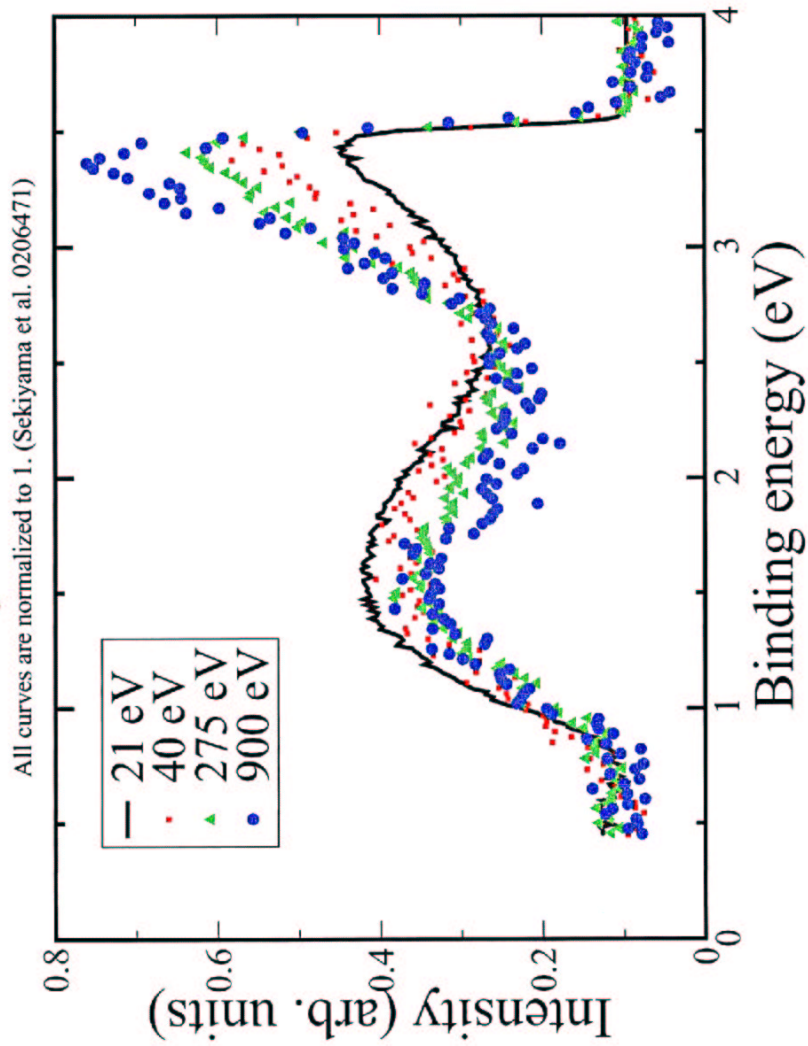
$\Rightarrow W_{Sr} > W_{Ca}$

1

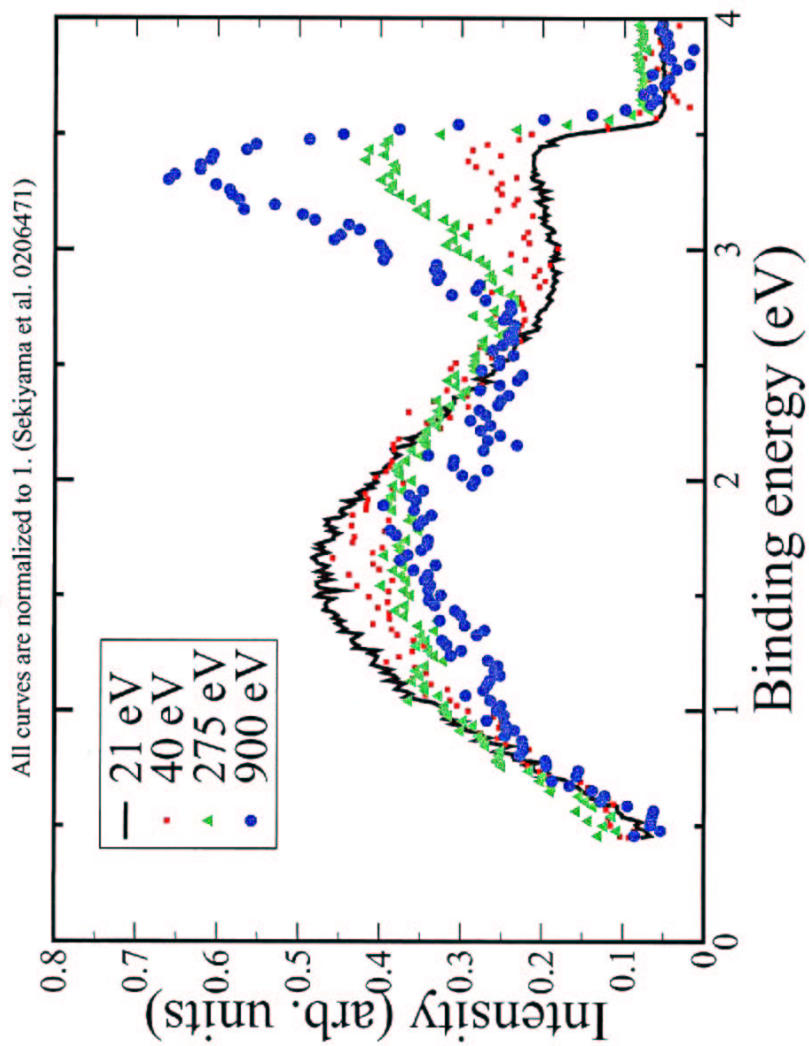




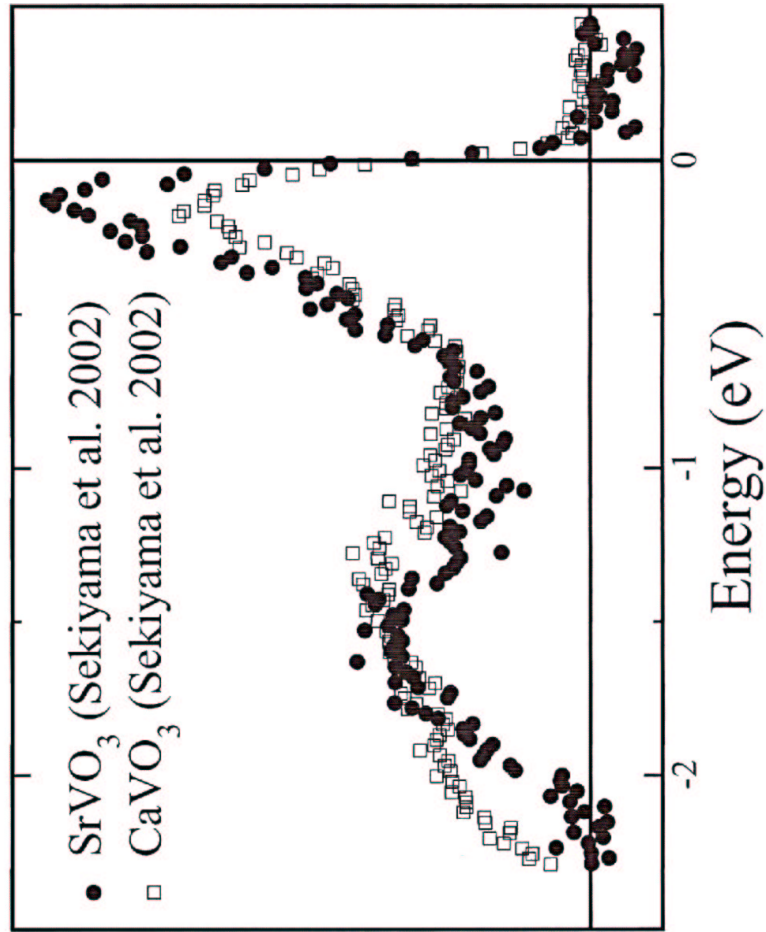
Comparison of SrVO<sub>3</sub> spectra for different photon energies.



Comparison of  $\text{CaVO}_3$  spectra for different photon energies.



Intensity (arbitrary units)



Bulk:

$\text{SrVO}_3$  and  $\text{CaVO}_3$  very similar :  $|\cos\theta|$ -effect

Surface:

$\text{CaVO}_3$  considerably stronger correlated

- than in bulk
- than  $\text{SrVO}_3$

?

vicinity of MIT ?

Questions:

- Can one "optimize" the LDA-calculated values of  $U, J, \dots$  needed as input to DMFT ?
- Is there a fast and reliable impurity solver at low  $T$  ?
- Suitable acronym for  $X + \text{DMFT}$  ?       $\text{RDHF}$  ?