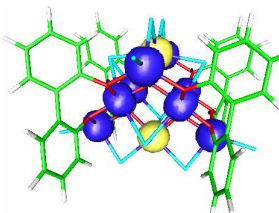
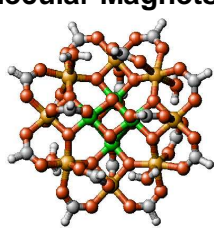


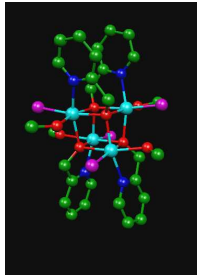
**Density-Functional Based Simulation of Molecular Magnets**



**Mark R. Pederson**  
Naval Research Laboratory  
Washington DC



Collaborators: J. Kortus (MPI), T. Baruah (Georgetown/NRL), S. Hellberg (NRL), K. Park (Howard/NRL) and N. Bernstein (NRL), S.N. Khanna (VCU).



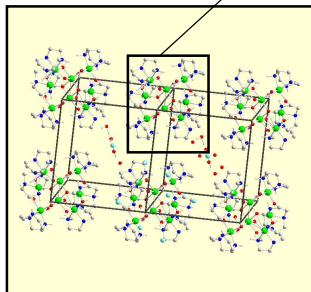
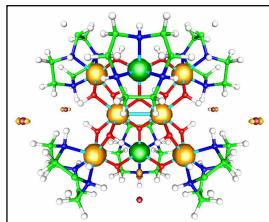
Supported by ONR, DOD HPCMP

**Outline:**

- Motivation: Practical considerations plus neat experiments.
- Method: DFT, NRLMOL, L.S Coupling
- Background on 2nd-order Barrier
- Examples:  $Mn_{12}$ ,  $Mn_{10}$ ,  $Fe_4$ ,  $Fe_8$ ,  $Cr_1$ ,  $Co_4$  (Good Agreement with Expt)
- What determines Axes and Barriers?
  - Pairing dependencies?
  - Global vs Local?
  - Total Moment?
  - Ligand Variation?
- Why does it work?
  - If GGA is exact.
  - If many-electron hamiltonian is starting point.
- 4th-order MAE via spin-orbit vibron interaction.

**Molecular Magnetic Crystals -Type 1 : Anatomy and Challenges**

From NRLMOL



- Individual molecular magnets are “isolated” magnetically and electronically.
- Six-Fold coordinated transition metal centers.
- Metal-metal superexchange interactions are strong compared to magnetic reorientation temperature.
- Well defined spin-ordering for each molecule (so far).

From Cambridge Crystal Data Bank

**EARLY EXPERIMENTS ON RESONANT TUNNELING OF  
MAGNETIZATION**

J.R. Friedman et al, Phys. Rev. Lett. 76, 3830 (1996)

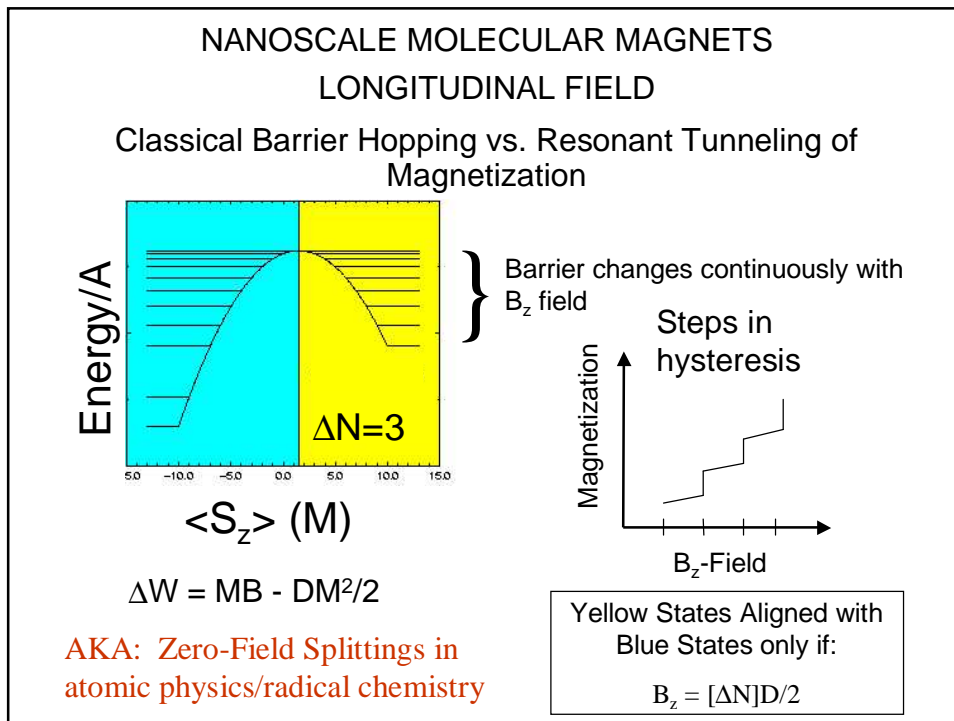
L. Thomas et al, Nature 383, 245 (1996)

SOME MOLECULAR CRYSTALS OF  
CURRENT INTEREST:

“ $\text{Mn}_{12}\text{O}_{12}$ -Acetate” - Longitudinal Tunneling

“ $\text{Fe}_8\text{O}_2$ -tacn” - Transverse Tunneling

Different Physics than Spin Dependent Transport and/or Tunneling!



Calculation of Anisotropy Hamiltonian within DFT, DFT+U, DFT-SIC or DMFT?

### Electronic and Atomic Equations of Motion -DFT

$$E = \sum_{i\sigma} \langle \Psi_{i\sigma} | -\frac{\nabla^2}{2} + V_{nuc}(\vec{r}) | \Psi_{i\sigma} \rangle + \frac{1}{2} \int d\vec{r} d\vec{r}' \frac{\rho(\vec{r})\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} + \int d\vec{r} g(\rho_{\uparrow}, \rho_{\downarrow}, \nabla\rho_{\uparrow}, \nabla\rho_{\downarrow}, \dots) + \frac{1}{2} \sum'_{nm} \frac{Z_n Z_m}{|R_n - R_m|}$$

$$\sum_k \left\langle \frac{d\Psi_{i\sigma}}{dq^k} \left| -\frac{\nabla^2}{2} + V_{eff}(\vec{r}) - \lambda_{i\sigma} \right| \Psi_{i\sigma} \right\rangle = 0$$

$$\Psi_{i\sigma}(\vec{r}) = \sum_{j\vec{A}} C_{j\vec{A}}^{i\sigma} \cdot P_{l_j}(\vec{r} - \vec{A}) \cdot \exp[-\alpha_j(r - A)^2]$$

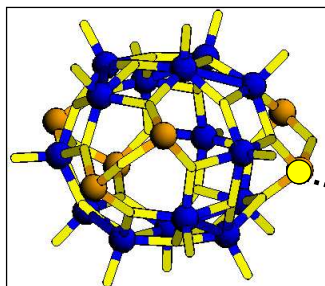
$$\{q^k\} = \{\vec{A}, C_{j\vec{A}}^{i\sigma}, \alpha_j\}$$

$$\nabla_n E = 0 \quad (\text{Hellmann-Feynman Force})$$

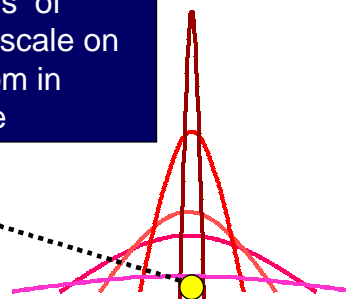
### Vary Single-Electron Orbitals to Minimize Energy

$$\phi_a(\tau) = \sum_{nlm, R} C_{nlm, R}^{(a)} P_{lm}(\mathbf{r}-R) R_{nl}[(\mathbf{r}-R)^2] \chi(\sigma_a)$$

$\swarrow$   $C_{nj}$        $\searrow$   $u_j(r)$



Place Trial Functions of variable scale on each atom in molecule



Many Length Scales Required!

**NRLMOL: Computational Tool Developed at NRL**

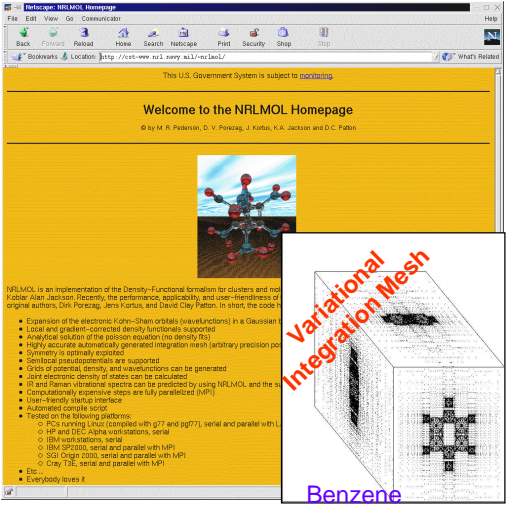
Recent Review:  
MR Pederson, DV Porezag, J Kortus and D.C. Patton, Phys. Stat. Solidi B **217**, 197 (2000).

- Added Pseudopotentials [Porezag, Pederson and Liu, PRB **60**, 14132 (1999)].
- $Z^{10/3}$  theorem on gaussian basis set completeness, [Porezag and Pederson, PRA **60**, 2840 (1999).]

**Study electronic, magnetic, structural and vibrational effects in molecules and clusters**

**HONEY BEE ALGORITHMS**

**Automatic Dynamical Load Balancing on Homo- or Hetero-geneous Parallel Platforms**

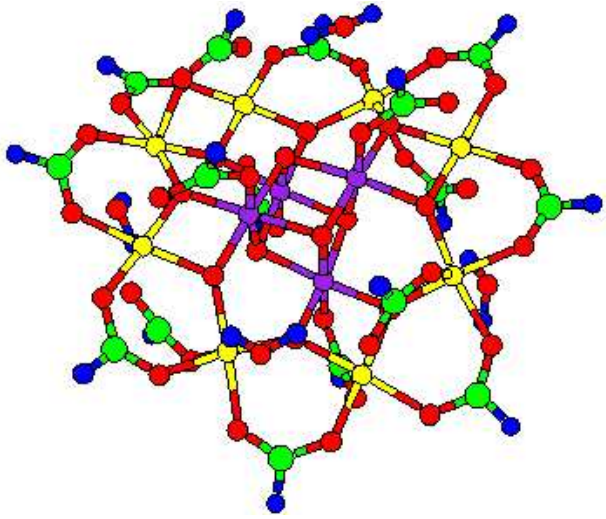


Variational Integration Mesh

Benzene

Massively Parallel NRLMOL: distributed to research community via CHSSI

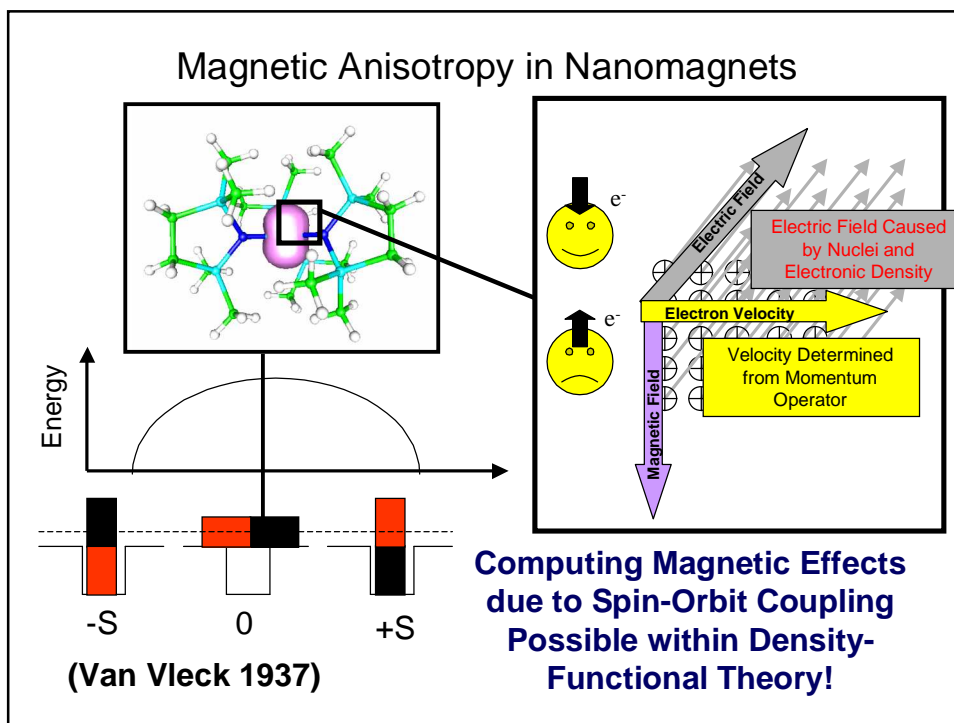
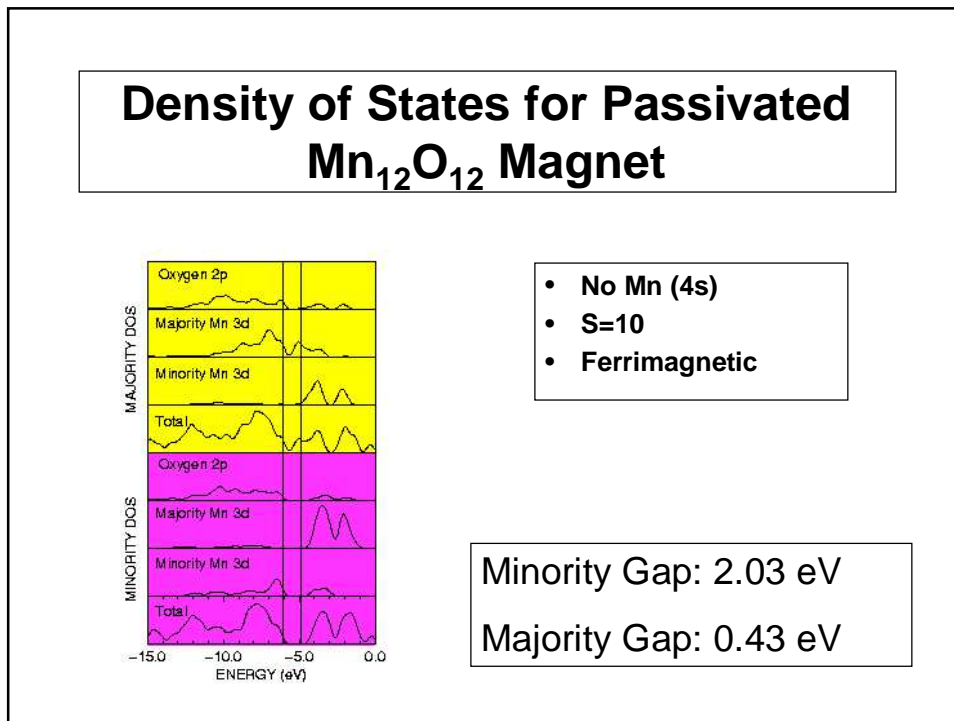
**Mn<sub>12</sub>O<sub>12</sub>-Acetate molecule**



**Mn<sub>12</sub>O<sub>12</sub><sup>2-</sup>(COOCH<sub>3</sub>)<sub>16</sub><sup>1-</sup>(H<sub>2</sub>O)<sub>4</sub>**

**Ligands:**

- Separate Magnet Cores
- Six Fold Coordination of Mn Atoms
- Prevent Reconstruction of Magnetic Core.
- “Steal” Mn 3d and 4s electrons?



### Anisotropy Hamiltonian

Pederson and Khanna PRB 1999  
Effect on total energy due to spin-orbit L.S term  
Dependent on axis of spin quantization

$$|\chi_1\rangle = \cos(\theta/2)|\uparrow\rangle + e^{i\beta}\sin(\theta/2)|\downarrow\rangle$$

$$|\chi_2\rangle = -e^{i\beta}\sin(\theta/2)|\uparrow\rangle + \cos(\theta/2)|\downarrow\rangle$$

To lowest order:  
(2nd order perturbation in  $L \cdot S$ )

Determine  $\gamma_{ab}$  from DFT

$$\Delta_2 = \sum_{ab} \gamma_{ab} \langle S_a \rangle \langle S_b \rangle \rightarrow$$

$$\Delta E_2 = -D S_z S_z - E (S_x S_x - S_y S_y)$$

$\langle S_z \rangle$  (M)

### MAGNETOMOLECULAR ANISOTROPY ENERGY

$\Delta E \sim (1/4C^4) M^2$

E =

DFT Energy

+

⊖

Spin Orbit Energy

⊖

=

ME's

$C^4$

$(3.5 \times 10^8)$

### Accuracy of Non-Relativistic DFT for Spin-Orbit?

NRLMOL
DIRAC
EXPT

Kr 3d	1.282	1.303	
Kr 3p	7.551	7.883	
Kr 2p	50.97	53.43	
Mn 2p	10.3		11-12
Ru 2p	121		125

GAS PHASE

CRYSTALLINE PHASE

M.R. Pederson, A.Y. Liu, T. Baruah, E.Z. Kurmaev, A. Moewes, S Chiuazbidian, M. Neuman, C.R. Kmety, K.L. Stevenson and D.Ederer, To appear in *Phys. Rev. B* 2002

### SPIN ORBIT COUPLING: CARTESIAN VS. L.S?

$$U(r, \mathbf{p}, \mathbf{S}) = -\frac{1}{2c^2} \mathbf{S} \cdot \mathbf{p} \times \nabla \Phi(r) \quad \text{😊}$$

$$U(r, \mathbf{L}, \mathbf{S}) = \frac{1}{2c^2} \mathbf{S} \cdot \mathbf{L} \frac{1}{r} \frac{d\Phi(r)}{dr}$$

**Most (all?) computational methods have used the L.S representation**



WAVEFUNCTIONS ARE EXPANDED IN  
TERMS OF SOME BASIS SET

(GAUSSIANS, PLANEWAVES, MTOS, APWS ETC...)

$$\Psi_{is}(\mathbf{r}) = \sum_{j\sigma} C_{j\sigma}^{is} f_j(\mathbf{r}) \chi_{\sigma}$$

$$\langle f_j \chi_{\sigma} | U(\mathbf{r}, \mathbf{p}, \mathbf{S}) | f_k \chi_{\sigma'} \rangle = ?$$

MATRIX ELEMENTS FOR SPIN-ORBIT COUPLING  
OPERATOR

[M. R. Pedersen - Submitted to PRRB (1999)]

WAVEFUNCTIONS EXPANDED IN TERMS OF  
SPIN-ORBITAL PRODUCTS

$$\psi_{is}(\mathbf{r}) = \sum_{j\sigma} C_{j\sigma}^{is} f_j(\mathbf{r}) \chi_{\sigma} \quad (1)$$

$$\begin{aligned} U_{j\sigma, k\sigma'} &= \langle f_j \chi_{\sigma} | U(\mathbf{r}, \mathbf{p}, \mathbf{S}) | f_k \chi_{\sigma'} \rangle \\ &= \sum_{\alpha} \frac{-1}{2c^2} \langle f_j | [\nabla \times \nabla \Phi(\mathbf{r})]_{\alpha} | f_k \rangle \langle \chi_{\sigma} | S_{\alpha} | \chi_{\sigma'} \rangle \\ &= \sum_{\alpha} \frac{1}{2} \langle f_j | V_{\alpha} | f_k \rangle \langle \chi_{\sigma} | S_{\alpha} | \chi_{\sigma'} \rangle \end{aligned} \quad (2)$$

"STANDARD" CARTESIAN REPRESENTATION OF  
SPATIAL OPERATOR

$$\langle f_i | V_z | f_j \rangle = \frac{1}{2c^2} \langle f_i | \left( \frac{d\Phi}{dy} \frac{d}{dz} - \frac{d\Phi}{dz} \frac{d}{dy} \right) | f_j \rangle \quad (3)$$

INTEGRATION BY PARTS:

$$\begin{aligned} \langle f_i | \frac{d\Phi}{dy} \frac{d}{dz} | f_j \rangle &= \int d^3r \frac{d}{dy} \left[ f_i \Phi \frac{df_j}{dz} \right] \\ &- \langle \frac{df_i}{dy} | \Phi | \frac{df_j}{dz} \rangle - \langle f_i | \Phi | \frac{d^2 f_j}{dz dy} \rangle \end{aligned} \quad (4)$$

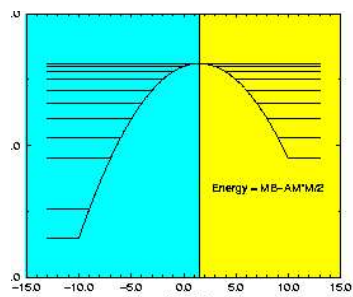
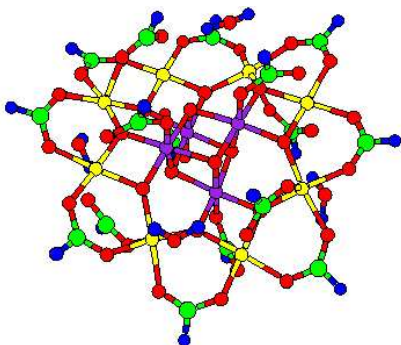
SIMPLIFIED REPRESENTATION FOR SPIN-ORBIT  
MATRIX ELEMENTS

$$\langle f_i | V_z | f_j \rangle = \frac{1}{2c^2} \left( \langle \frac{df_i}{dz} | \Phi | \frac{df_j}{dy} \rangle - \langle \frac{df_i}{dy} | \Phi | \frac{df_j}{dz} \rangle \right) \quad (5)$$

## Second Order Molecular Magnetic Anisotropy Barrier Theory vs. Experiment

All Electron GGA (NRLMOL): 55.7 K

Expt. (Barra et al, Fort et al) : 55.6 K



What about correlations and multideterminantal contributions to anisotropies?

What about spin contaminated wavefunctions?

What are the conditions for which existing approximations to DFT should be good enough?

### THE ANATOMY OF A SLATER DETERMINANT

Many Electron Determinant

$$\begin{matrix} \phi_a(\tau_1) & \phi_a(\tau_2) & \phi_a(\tau_3) & \phi_a(\tau_4) & \dots & \phi_a(\tau_N) \\ \phi_b(\tau_1) & \phi_b(\tau_2) & \phi_b(\tau_3) & \phi_b(\tau_4) & \dots & \phi_b(\tau_N) \\ \phi_c(\tau_1) & \phi_c(\tau_2) & \phi_c(\tau_3) & \phi_c(\tau_4) & \dots & \phi_c(\tau_N) \\ & & & & \dots & \\ & & & & & \phi_N(\tau_1) & \phi_N(\tau_2) & \phi_N(\tau_3) & \phi_N(\tau_4) & \dots & \phi_N(\tau_N) \end{matrix}$$

$$\Phi(\tau_1, \tau_2, \tau_3 \dots) = \frac{1}{(N!)^{1/2}}$$

$\phi_a(\tau_1) = \psi_\alpha(r_1)\chi(\sigma_1)$

“Slater Determinant” Ensures Wavefunction is Antisymmetric, Normalized and *Managable*

**IFF:**

$$\langle \phi_a(\tau) | \phi_b(\tau) \rangle = \delta_{ab}$$

Single Electron Spin Orbital

Spatial part

Spin part

### Configuration Interaction

$$|\Psi\rangle = C_0|\Phi_0\rangle + C_1|\Phi_1\rangle + C_2|\Phi_2\rangle + C_3|\Phi_3\rangle + \dots$$

$$|\Phi_I\rangle = |\phi_a\phi_b\phi_c\dots\rangle = \frac{1}{(N!)^{1/2}}$$

$$\begin{matrix} \phi_a(\tau_1) & \phi_a(\tau_2) & \phi_a(\tau_3) & \phi_a(\tau_4) & \dots & \phi_a(\tau_N) \\ \phi_b(\tau_1) & \phi_b(\tau_2) & \phi_b(\tau_3) & \phi_b(\tau_4) & \dots & \phi_b(\tau_N) \\ \phi_c(\tau_1) & \phi_c(\tau_2) & \phi_c(\tau_3) & \phi_c(\tau_4) & \dots & \phi_c(\tau_N) \\ & & & & \dots & \\ & & & & & \phi_N(\tau_1) & \phi_N(\tau_2) & \phi_N(\tau_3) & \phi_N(\tau_4) & \dots & \phi_N(\tau_N) \end{matrix}$$

$(\Phi_I | [H - E] | \Psi) = (\Phi_I | [(\sum_i f_i + \sum_{ij} g_{ij}) - E] | \Psi) = 0$

$\sum_J (\Phi_I | [(\sum_i f_i + \sum_{ij} g_{ij}) - E] | \Phi_J) C_J = 0$

Many Electron  
Secular Eqn.

### Multiconfigurational Contributions to Magnetic Anisotropy

$$|\Psi\rangle = C_0|\Phi_0\rangle + C_1|\Phi_1\rangle + C_2|\Phi_2\rangle + C_3|\Phi_3\rangle + \dots$$

$$\langle \Psi | V_{L.S} | \Psi \rangle = \sum_{\nu} C_{\nu}^* C_{\nu} \langle \Phi_{\nu} | V_{L.S} | \Phi_{\nu} \rangle + \sum'_{\nu\mu} C_{\nu}^* C_{\mu} \langle \Phi_{\nu} | V_{L.S} | \Phi_{\mu} \rangle$$

$$V_{L.S} = \sum_i f_i + \sum'_{ij} g_{ij}$$

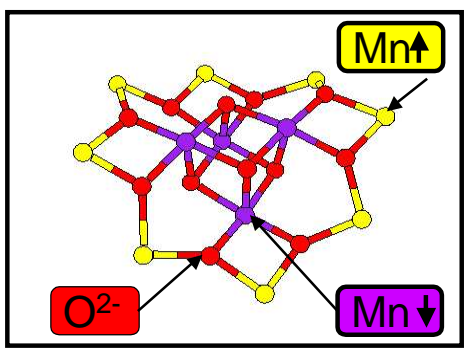
(See J.C. Slater)

Interaction between electric fields due to nuclei with each moving electron. (1 electron operator)

Interaction between electric fields due to all electrons with each moving electron. (2 electron operator)

Try: (1) All diagonal terms the same or  $|C_{\nu}|^2$  very small.  
 (2) Off diagonal terms small due to zero overlap or small  $C_{\nu}$

### Wannier Description of Magnetic Core of Molecule



**Mn↑**

**Mn↓**

**O<sup>2-</sup>**

	<b>Mn↑</b>	<b>Mn↓</b>
E <sub>g</sub>	—	—
T <sub>2g</sub>	≡	≡
E <sub>g</sub>	—	—
T <sub>2g</sub>	≡	≡

Lowest Electronic Excitation: **O<sup>2-</sup>**

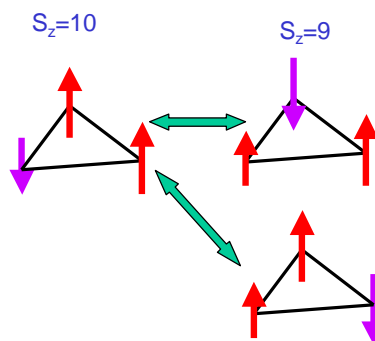
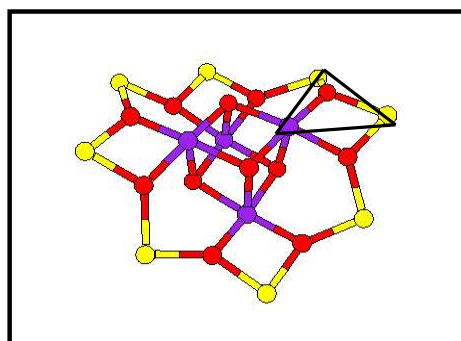
"band-like" d(eg) - d(eg)  
majority spin excitation

2p

≡

### Excited Configurations of the Mn<sub>12</sub>-Acetate Molecule

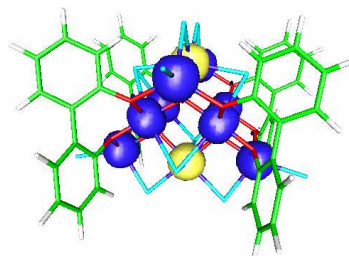
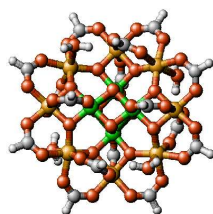
Mn(3d) - Mn(3d) Majority Excitations	>0.44 eV
Localized single spin flip	>1.00 eV
Concerted local moment flips (3 or 4 3d e <sup>-</sup> at once):	~0.05 eV
Charge-Transfer	~6.00 eV (?)



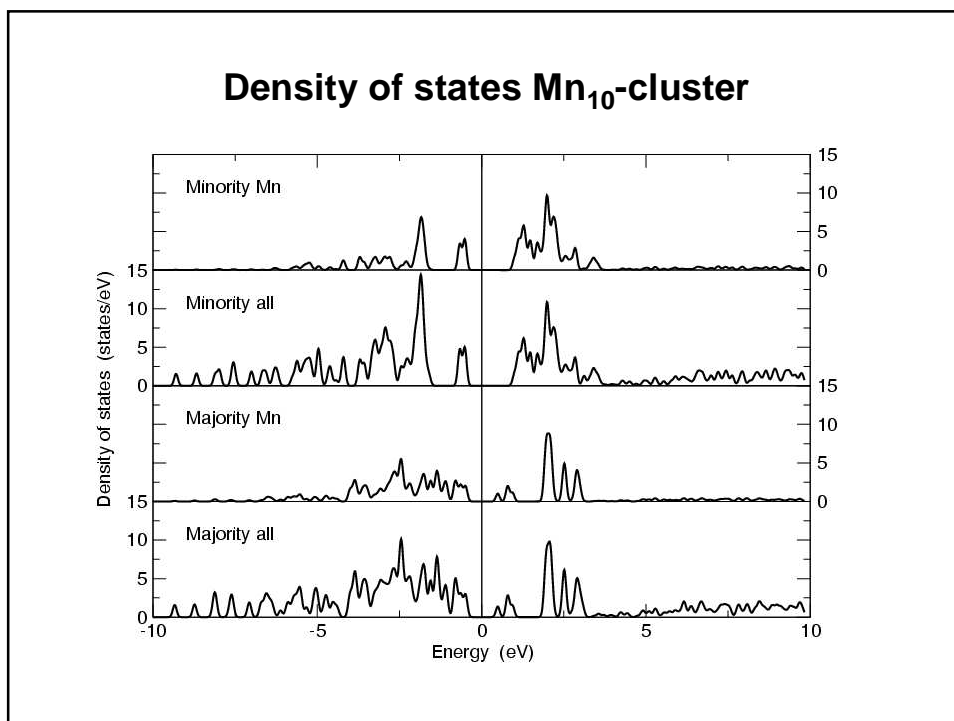
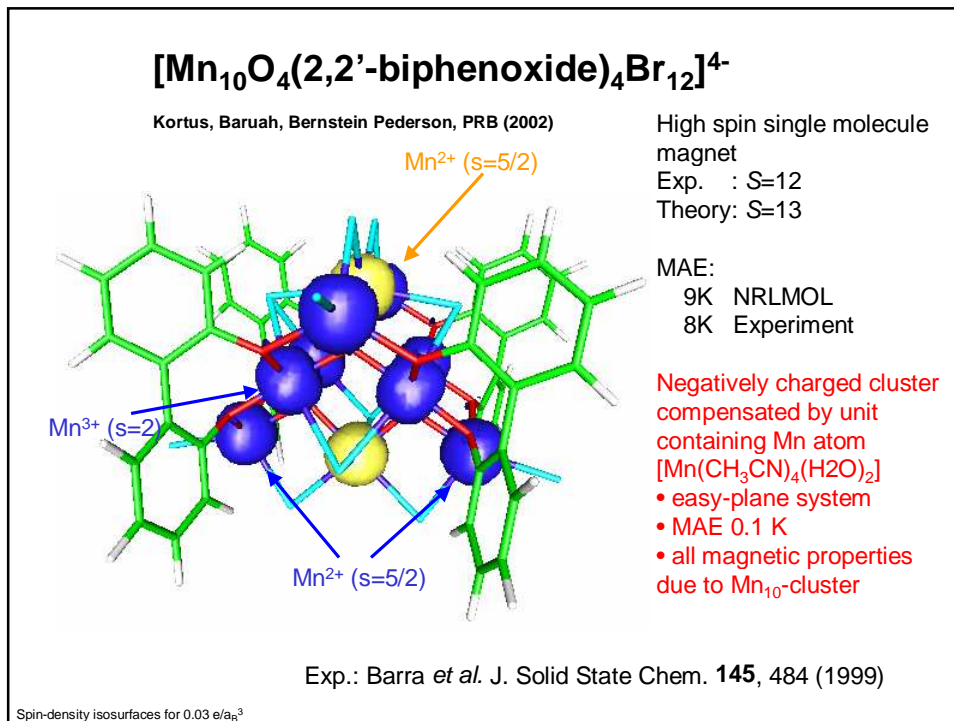
Self Consistency Shows that Local Charges and Moment Size Unchanged

### Mn<sub>12</sub> vs Mn<sub>10</sub>?

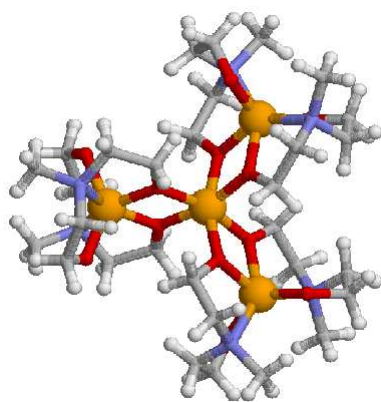
	Total Spin	Local Spins	Interference?	Anisotropy
Mn <sub>12</sub> :	S=10	S=3/2 and S=2	Constructive	Large
Mn <sub>10</sub> :	S=12/13	S=5/2	Destructive	Small



# Molecular Magnets within DFT: Tunnel Splittings and Magnetic Anisotropies



### Ferric star Kortus et al, (to be published)



The cluster ground state is ferrimagnetic with  $S = 5$ .

The three outer Fe(III) ions ( $s = 5/2$ ) couple antiferromagnetic to the inner Fe(III) ion.

Fe-Fe(center) distances of 3.2 Å.

Theory	$D = -0.56$ K	$ E  = 0.064$ K
Exp.	$D = -0.57$ K	$ E  = 0.056$ K

Exp.: S. Schromm, O. Waldmann, P. Müller (Uni Erlangen)

### Co<sub>4</sub> Based Molecular Magnet

Baruah and Pederson, CPL 2002

Magnetic moment:  $12 \mu_B$

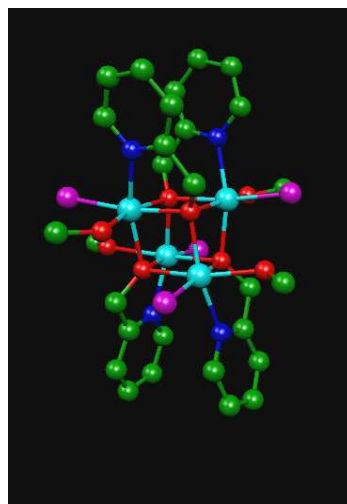
Local Co moment:  $3 \mu_B$

Addition of 4 hydrogens reduced moment by  $4 \mu_B$

Anisotropy varies strongly with molecular distortions (23-60K)

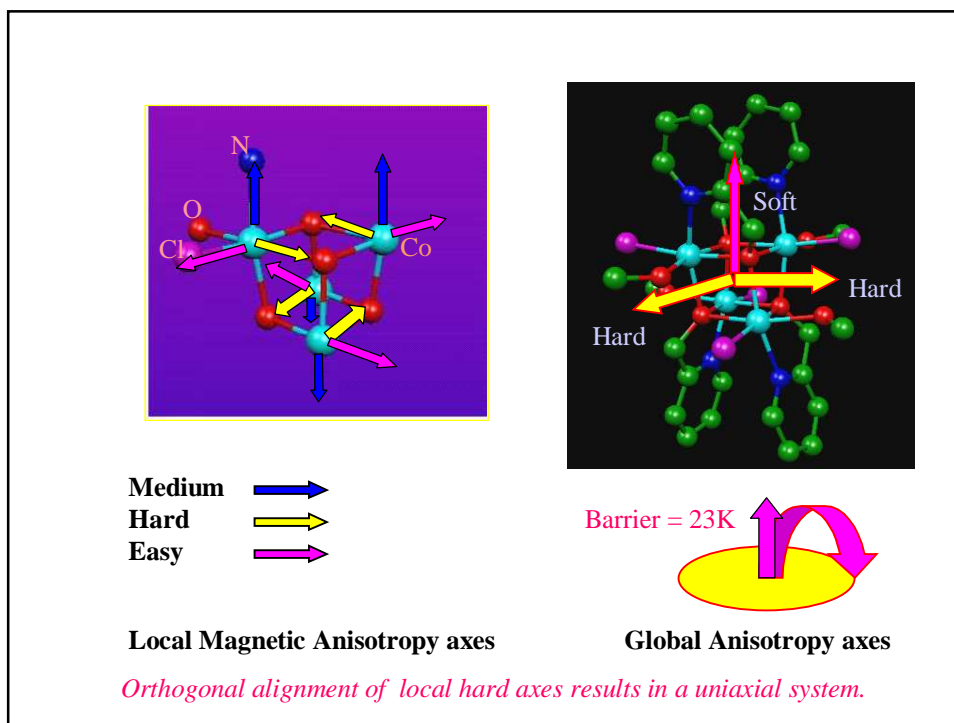
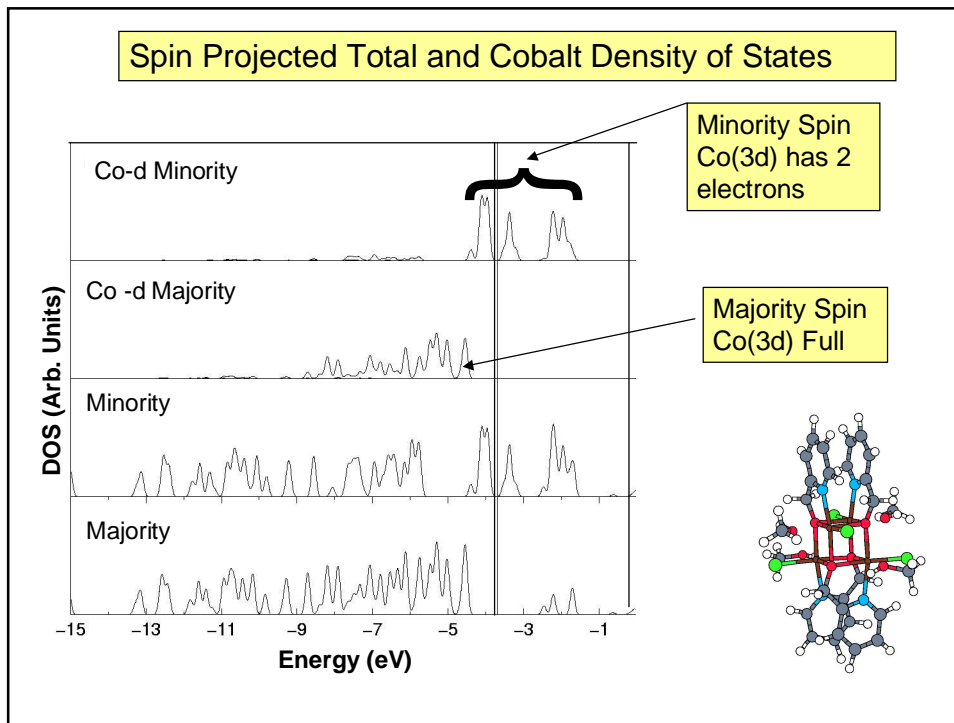
Uniaxial alignment

Global easy axis along Z



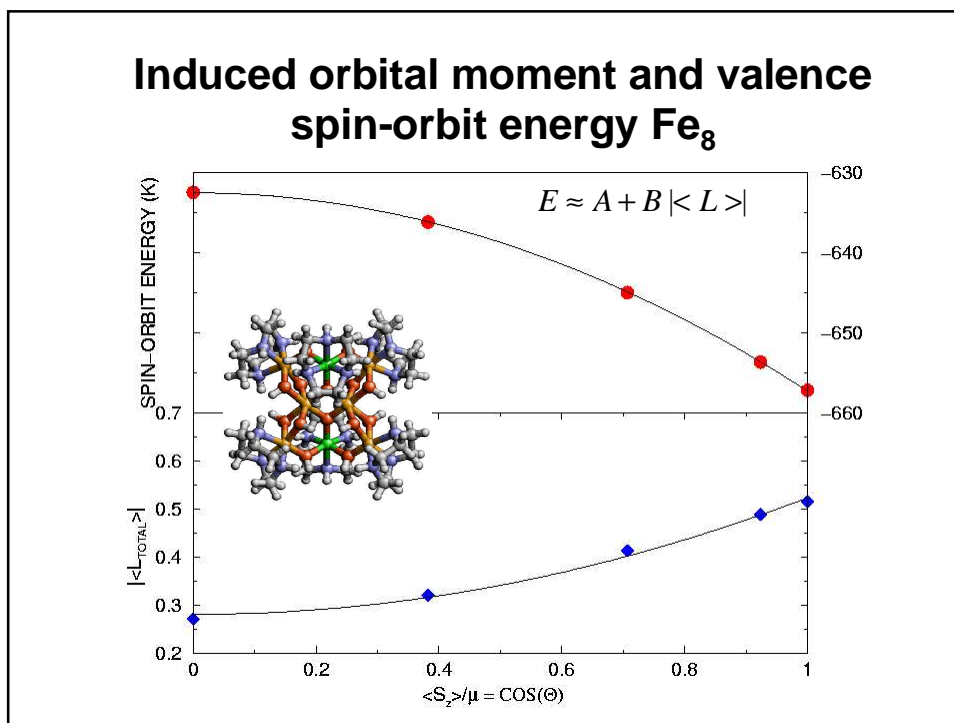
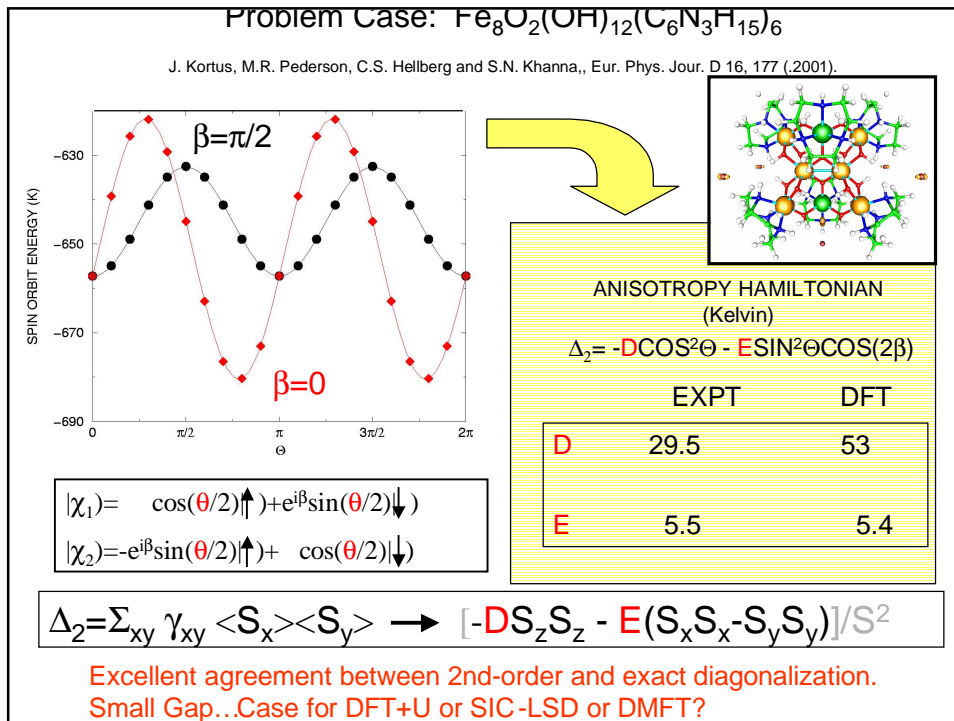
Lowest-energy staggered structure

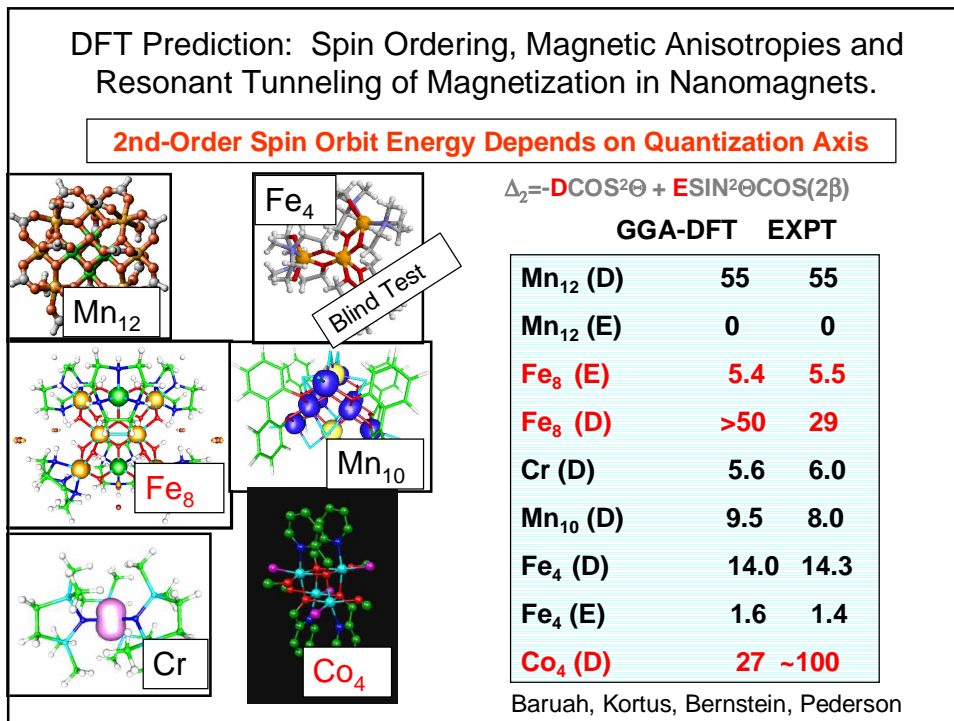
Molecular Magnets within DFT: Tunnel Splittings and Magnetic Anisotropies





# Molecular Magnets within DFT: Tunnel Splittings and Magnetic Anisotropies





## Vibrational Contribution to Magnetic Anisotropies.

- Spin Orbit Interaction Depends on Electric Fields and Kohn-Sham Orbitals
- Electric Fields and Kohn-Sham Orbitals depend on Atomic Positions/ Vibrational Displacements
- Zero Point Energy of a Vibrational Mode Changes as a function of Spin Projection due to **spin-orbit-vibron coupling**.
- Lowest-Order effect is  $1/[\text{Speed of Light}]^8$

4th-Order Anisotropy (responsible for tunnel splittings)

Higher order terms in  $L \cdot S$ :

- exact electronic (non-self-consistent) total energy with  $L \cdot S$
- coupling of spins to vibrations

$$\Delta E_4 = G S_z S_z S_z S_z + H [S_x S_x S_x S_x + S_y S_y S_y S_y] =$$

$A_1(4)[S^2(S_z^2 - S^2/3)] + A_2(4)[3S^4 + 35S_z^4 + 30S^2 S_z^2] +$	}	cubic harmonics
$B_1(4)[S_x^4 + S_y^4 - 6S_x^2 S_y^2] + \dots$		

Can have different angular dependence and different scaling with  $1/[\text{speed of light}]$

**SPIN-ORBIT MEDIATED SPIN VIBRON INTERACTION**

$$[P^2 + \omega^2 Q^2]/2 + \gamma_{zz} S_z^2 + Q \sum_{ab} (d\gamma_{ab}/dQ) S_a S_b$$

$$E = \omega/2 + \gamma_{zz} M^2 - (A + B M^2)^2 / (2\omega^2)$$

$S(S+1)[d/dQ(\gamma_{xx} + \gamma_{yy})]/2$

$d/dQ[\gamma_{zz} - (\gamma_{xx} + \gamma_{yy})/2]$

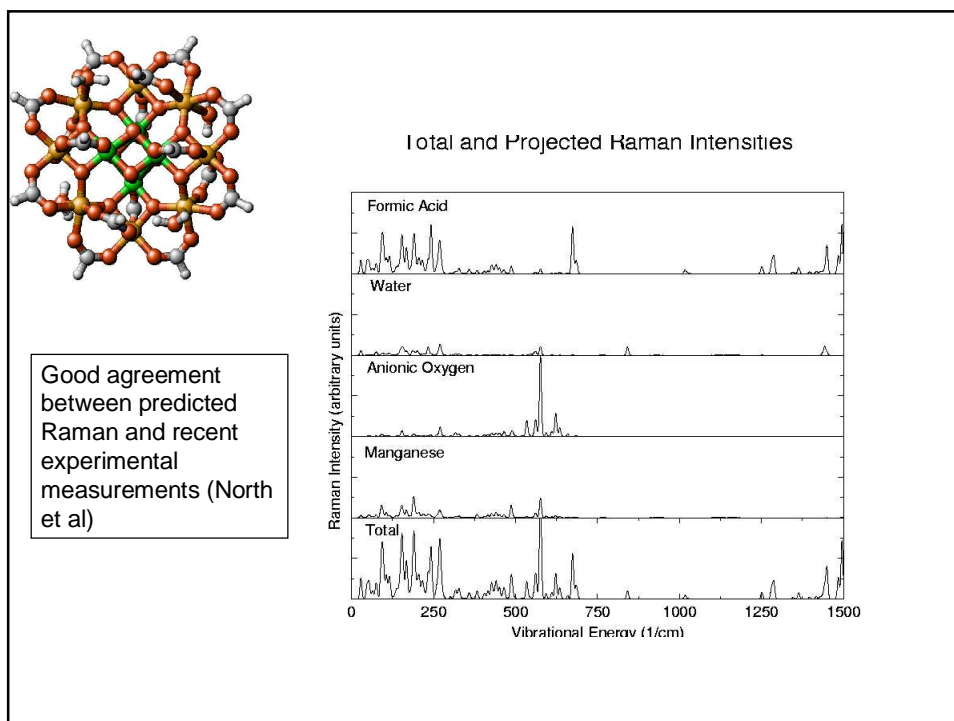
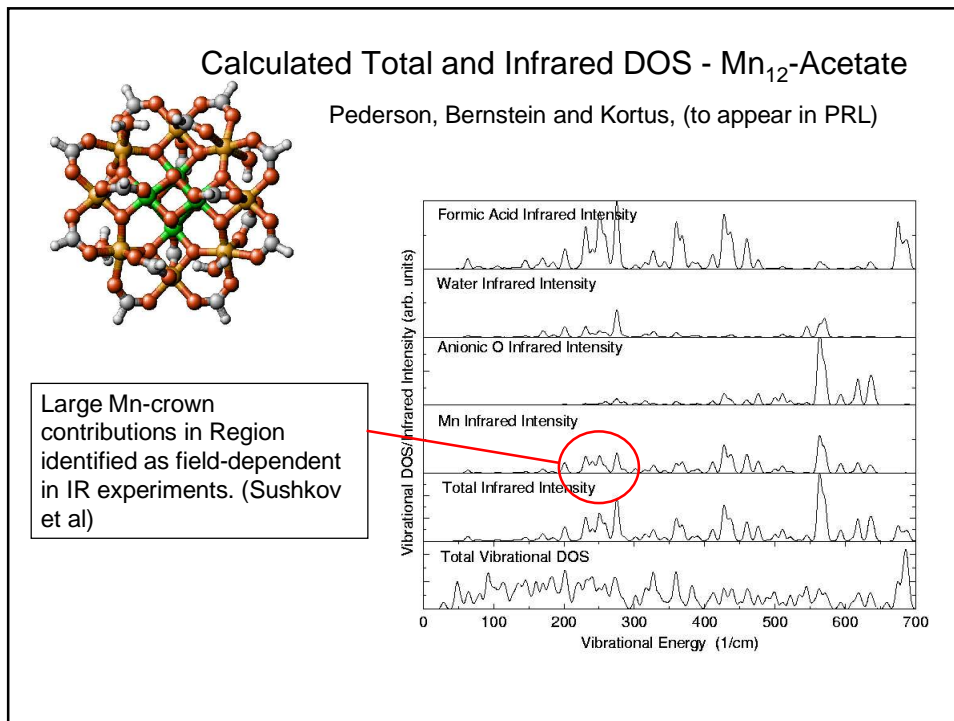
Compute total energy, forces,  $\gamma_{ab}$  for all atomic displacements

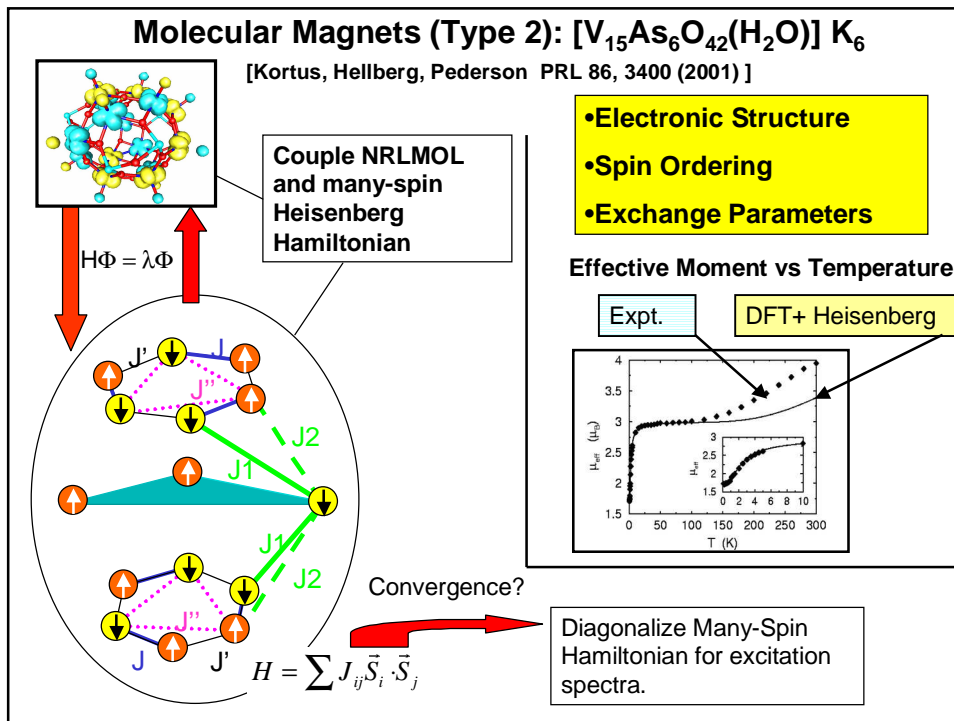
Extract: vibrations (IR, Raman) from dynamical matrix

vibration-spin coupling from  $d/dQ(\gamma_{ab})$

Difficult calculations (shortcuts): must compare to experiment

# Molecular Magnets within DFT: Tunnel Splittings and Magnetic Anisotropies





## Conclusions

- MAE : barrier to spin flips, controlled by spin-orbit coupling
- Accurate calculation of 2nd order MAE using DFT
- Vibrational spectra (IR, Raman) good agreement with exper.
- 4th order MAE
  - electronic: wrong sign, magnitude
  - vibrational coupling: preliminary results good.
- Open questions:
  - Importance of this effect in other nanomagnets
  - Source of tunnel splittings:
    - other transverse terms
    - symmetry breaking terms at 2nd order. energy