Correlation effects in mesoscopic systems:
From the Kondo effect in quantum dots to the 0.7 anomaly in quantum point contacts

Yigal Meir

Program on Realistic theories of strongly correlated systems

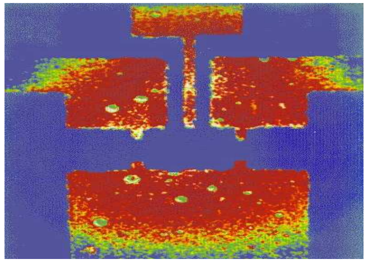
Outline

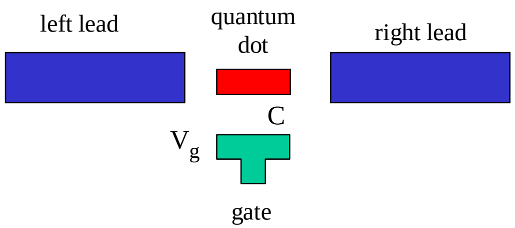
The Kondo effect in Quantum dots
- transport through quantum dots
- the Kondo effect in metals
- Kondo in quantum dots

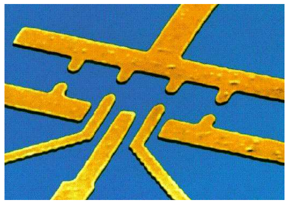
Quantum point contacts:
- conductance quantization
- the 0.7 anomaly
- DFT motivation of the model
- relation to transport through QDs
- model vs. experiments
- predictions and ramifications

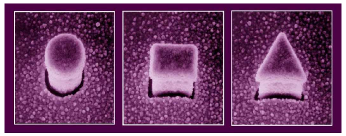
Luttinger Liquid
- nanotubes
- quantum wires
- quantum dots in the FQHE regime
- quantum dot at zero magnetic fields

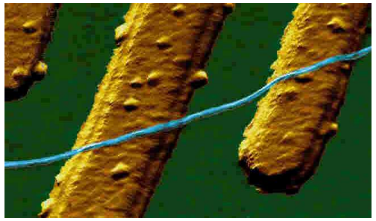
Quantum Dots





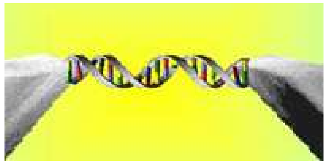


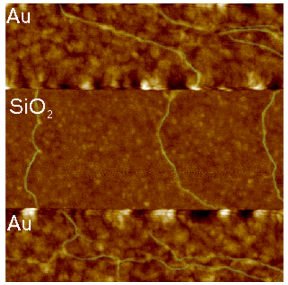




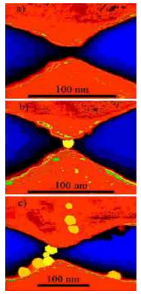
nanotube

DNA molecule

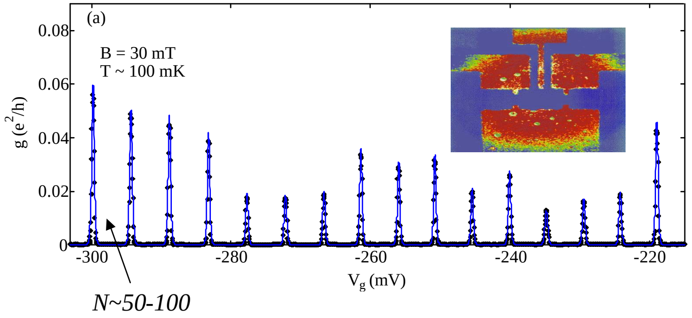




single grain



Tunneling through a quantum dot: Coulomb blockade



Semi-classically:

$$E_N = U\frac{N(N-1)}{2} - NeV_g$$

$$E_{N+1} - E_N = UN - eV_g$$

"Well, once again, here we are."

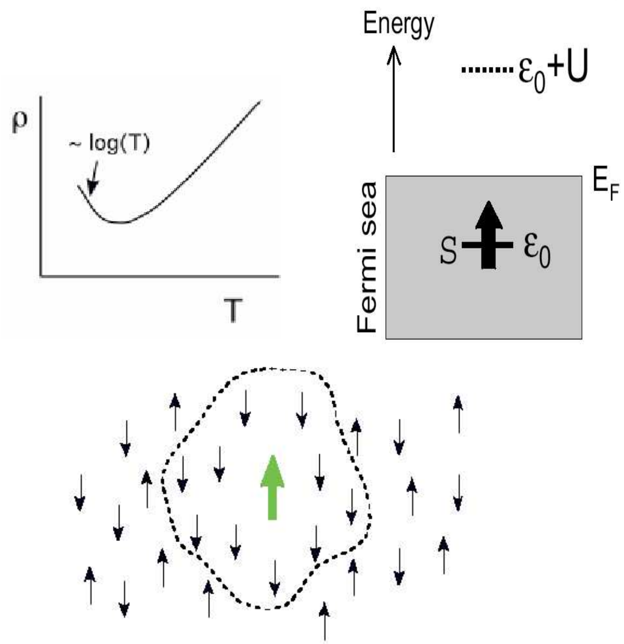
Quantum mechanical model

$$H = \sum_{\substack{k\in L,R\\ \sigma}} \varepsilon_k c^{+}_{k\sigma} c_{k\sigma} + \sum_{n\sigma} \varepsilon_n d^{+}_{n\sigma} d_{n\sigma} + U \sum_{n\sigma\neq n'\sigma'} \hat{n}_{n\sigma}\hat{n}_{n'\sigma'} + \sum_{\substack{k\in L,R\\ \sigma}} V_{nk\sigma}(d^{+}_{n\sigma} c_{k\sigma} + h.c)$$

described quantitatively by an Anderson model. Kondo effects ?

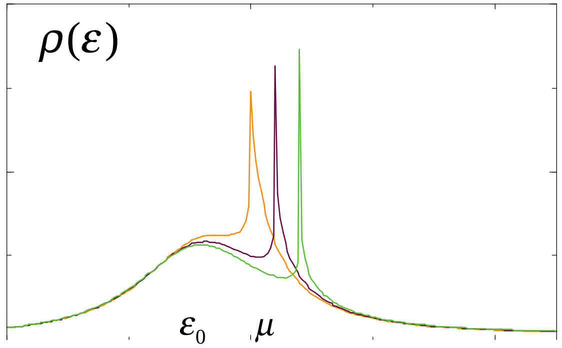
Ng and Lee (1988)
Glazman and Raikh (1988)

Kondo Effect in Metals



A peak in the density of states at the Fermi energy

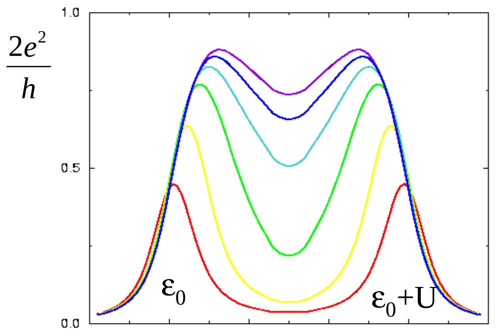
Density of states:



$$I = \frac{e}{\hbar} \int d\varepsilon \left[f_L(\varepsilon) - f_R(\varepsilon) \right] \Gamma(\varepsilon)\, \rho(\varepsilon)$$

$$T_K \propto e^{-\pi(\mu - \varepsilon_0)/\Gamma}$$

conductance



chemical potential

$\Gamma \gg T \gg T_K$:

$$G=\frac{2e^2}{h}\left[(1-n)\frac{\Gamma^2}{(\mu-\varepsilon_0)^2+\Gamma^2}+n\frac{\Gamma^2}{(\mu-\varepsilon_0-U)^2+\Gamma^2}\right]$$

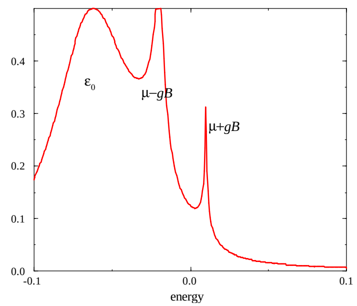
T=0:

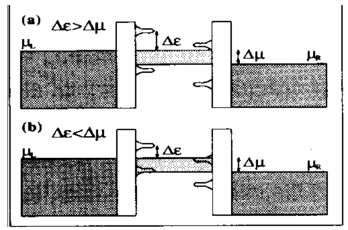
$$G=\frac{2e^2}{h}\sin^2\left(\pi\langle n_\sigma\rangle\right)\rightarrow\frac{2e^2}{h}$$

finite bias and magnetic fields

Meir, Wingreen and Lee (1993,1994)

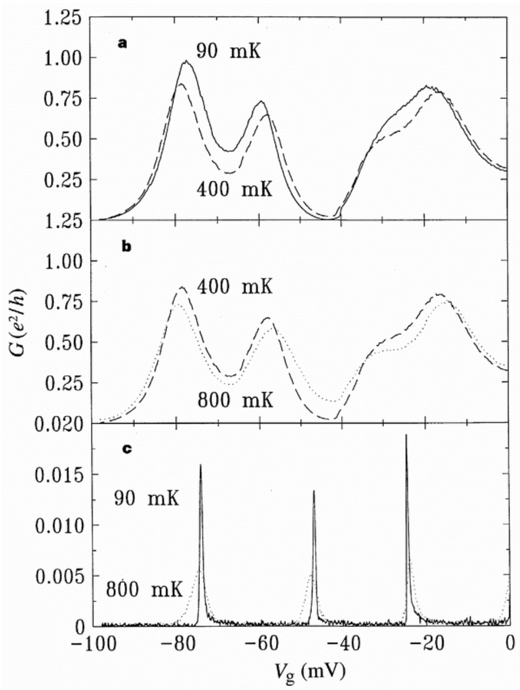
density of states

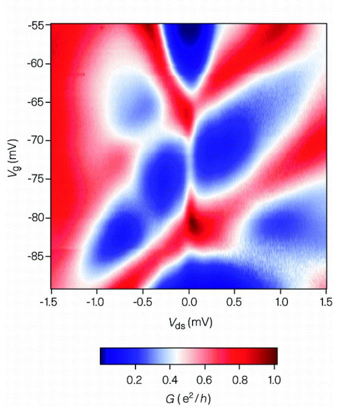


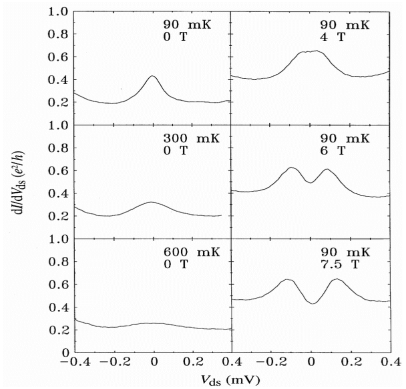


Zero-bias anomaly that splits in finite magnetic fields by *2gB*

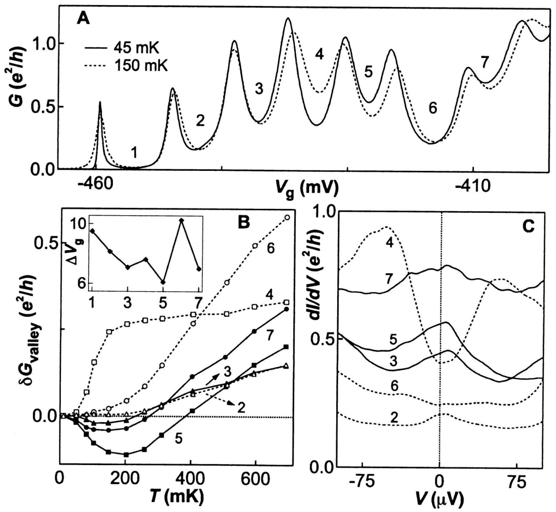
Goldhaber-Gordon, Kastner (1998)

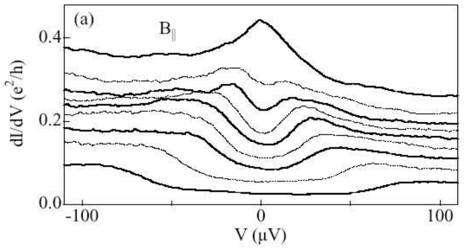






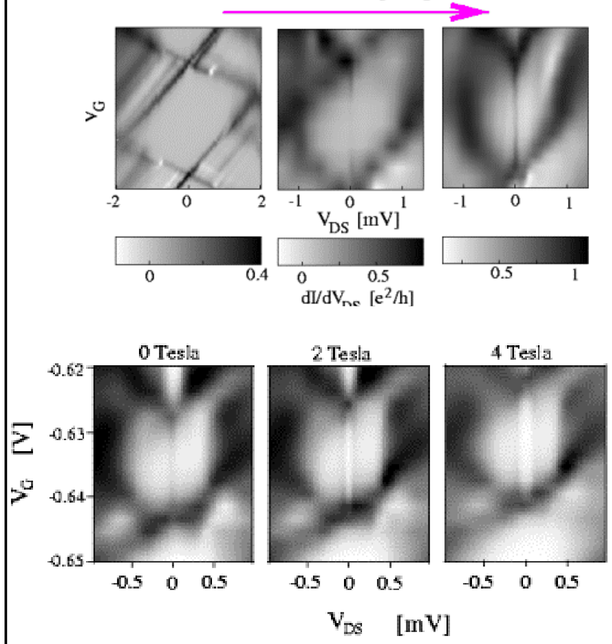
Cronenwett et al. (1998)





Schmidt et al. (1999)

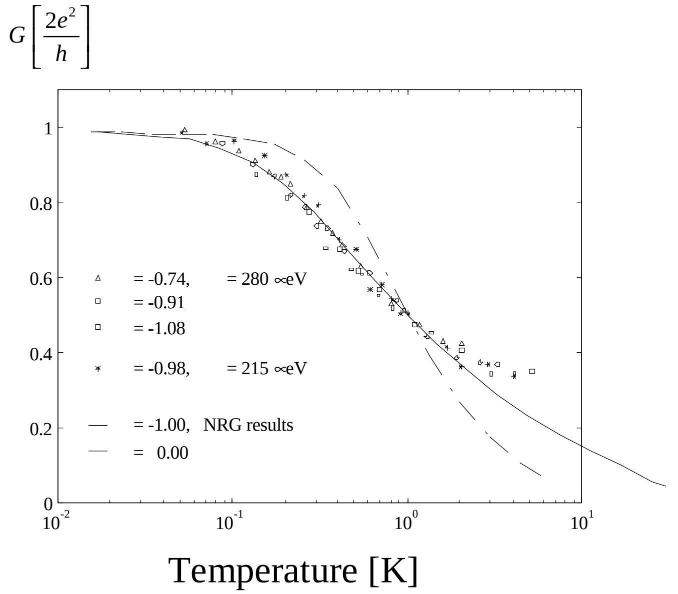
Zero-Bias Anomaly with Increasing the
Tunnel Coupling to the Leads



Quantum dots vs magnetic impurities

- fully tunable, but parameters unknown
- the full crossover between the Kondo limit, the mixed valence regime and the non-Kondo limit (comparison to NRG)
- Kondo effect out of equilibrium
- the unitarity limit
- the enhancement of the Kondo effect by a magnetic field
- the phase of the transmission coefficient
- Kondo effect in the quantum Hall regime
- absence of even-odd parity
- magnetic field induced Kondo effect (singlet-triplet degeneracy)
- Kondo effect due to excited states
- two-impurities ?
- multi-channel Kondo effect ?
- noise in the Kondo regime
- the Kondo effect under external irradiation
- how long does it take ?

Kondo scaling



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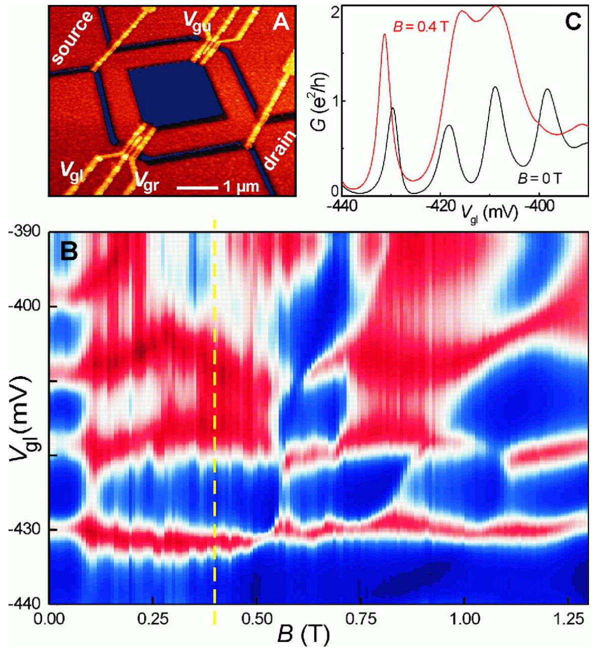
Unitarity limit in quantum dots (Delft)



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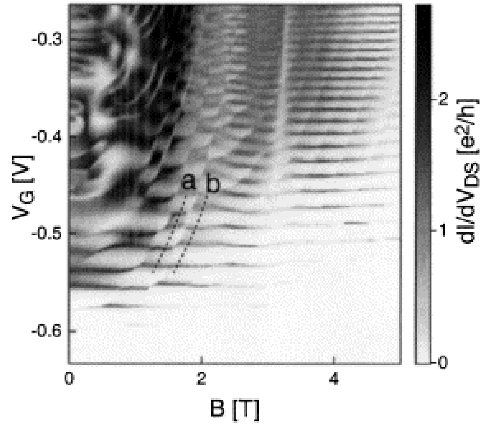
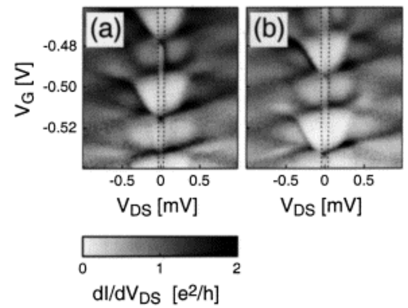
Kondo effect in the quantum Hall regime (Stuttgart)





Quantum dots vs magnetic impurities

- fully tunable, but parameters unknown
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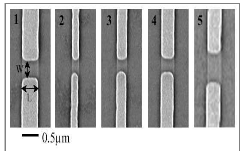
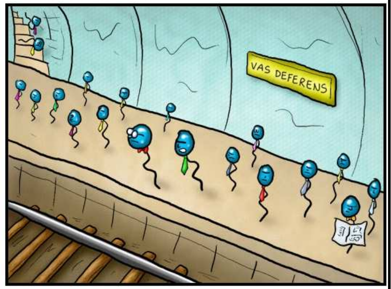
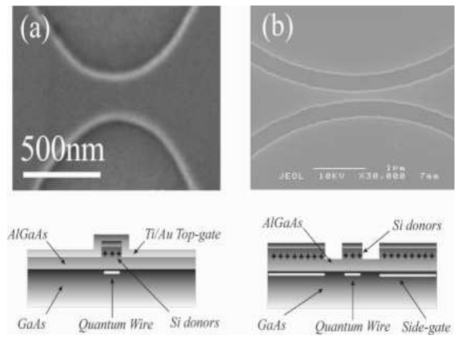
Quantum point contacts and quasi-1d systems



size \ QPC	1	2	3	4	5
W (μm)	0.4	0.3	0.3	0.3	0.7
L (μm)	0.5	0.2	0.3	0.4	0.4

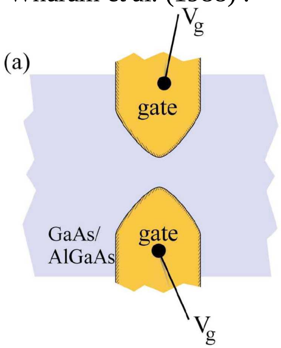


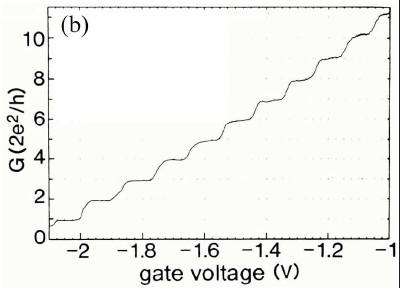
"I heard the tube's tied."



Conductance quantization

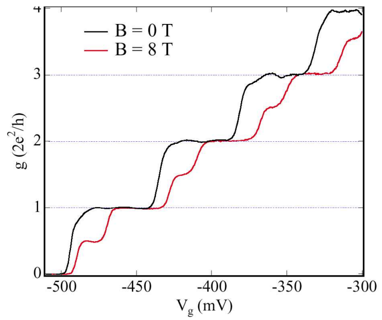
Van wees et al. (1988)
Wharam et al. (1988) :



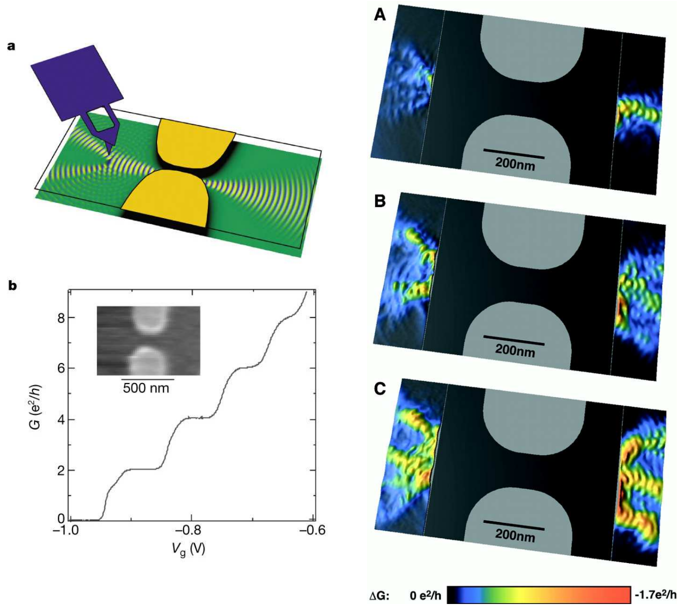


Landauer formula:

$$G = \frac{e^2}{h} \sum_{n,\sigma} T_{n\sigma}$$



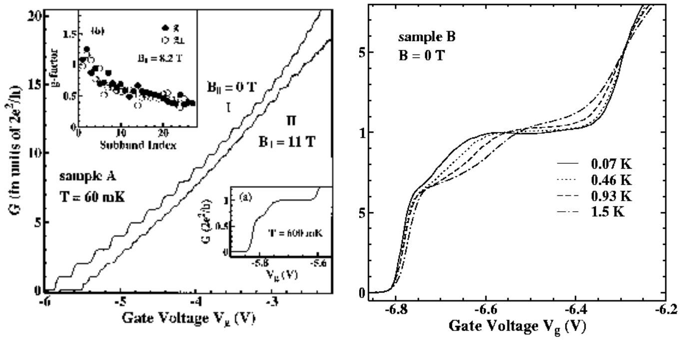
Seeing the different modes
(Topinka et al. 2000/1)

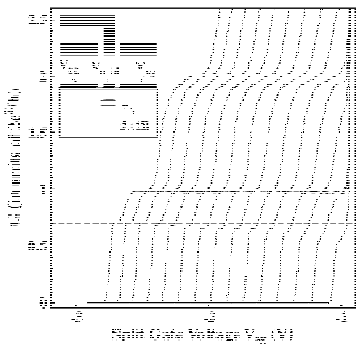


The 0.7 anomaly

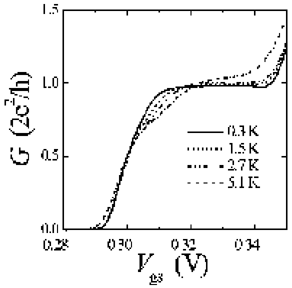
With Ned Wingreen (NECI)
Kenji Hirose (NECI Japan)

Thomas et al. (1996,1998,2000)

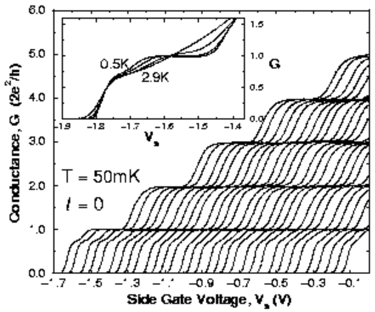




Kristensen et al. (2000)

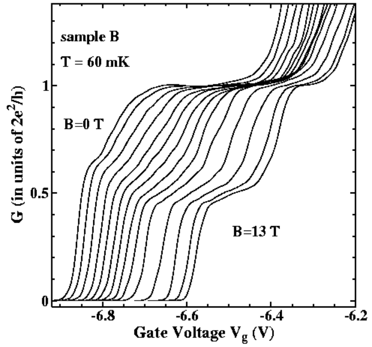


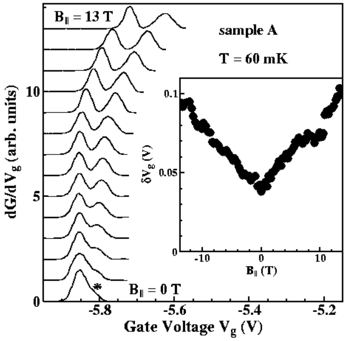
Reilly et al. (2000)

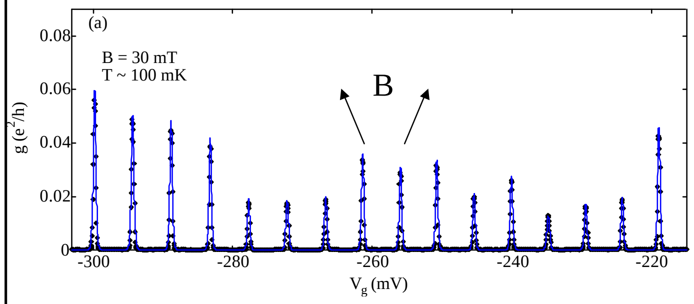


magnetic field dependence

Thomas et al. (1996)



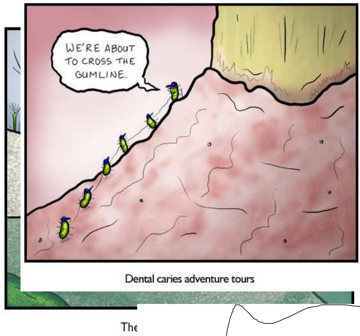


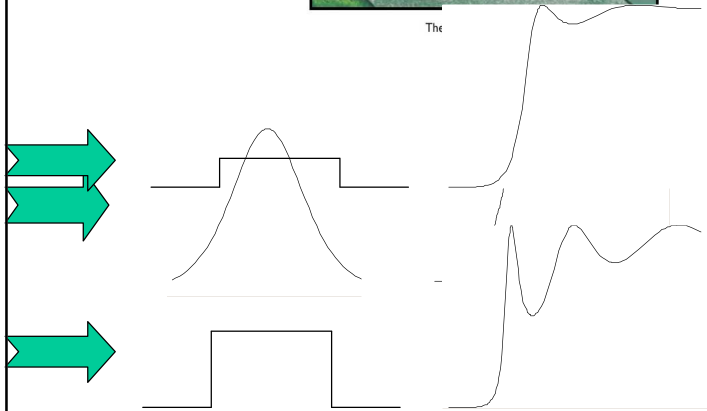


Transmission through a barrier:

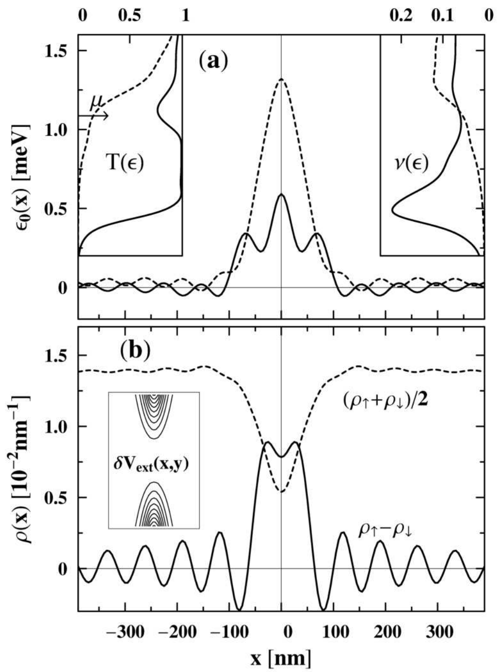
potential step

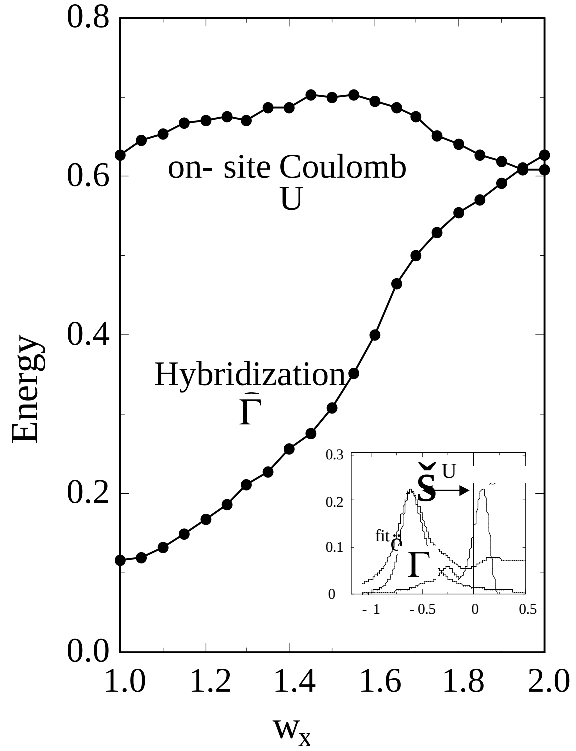
smooth barrier:





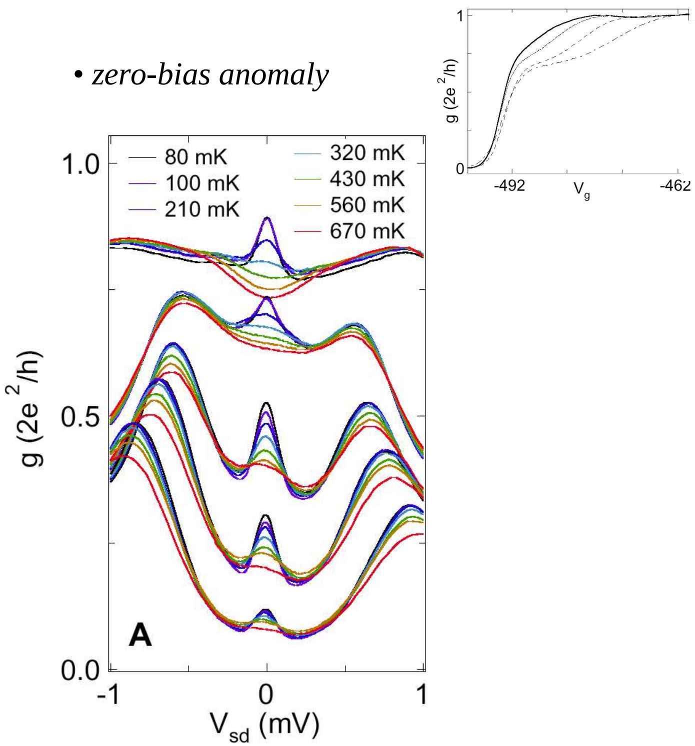
Spin-density functional theory of a QPC: Formation of local moment



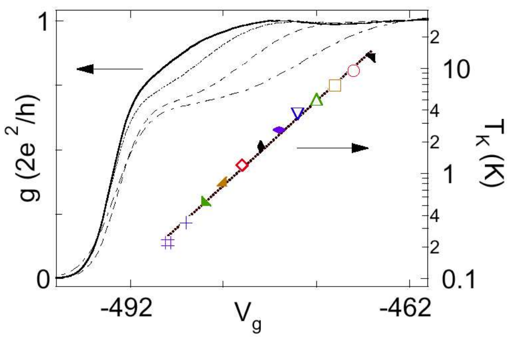


Is the 0.7 anomaly related to the Kondo effect ?

• *zero-bias anomaly*

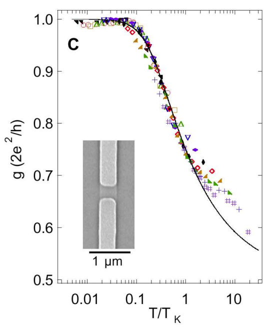


• *temperature dependence*

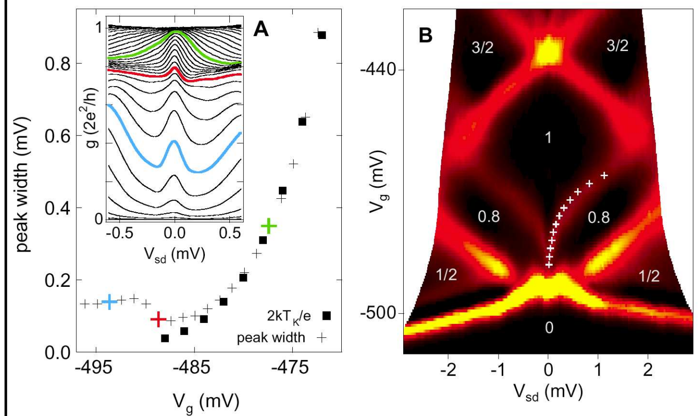


$$g = e^2/h\ [f(T/T_K) + 1]$$

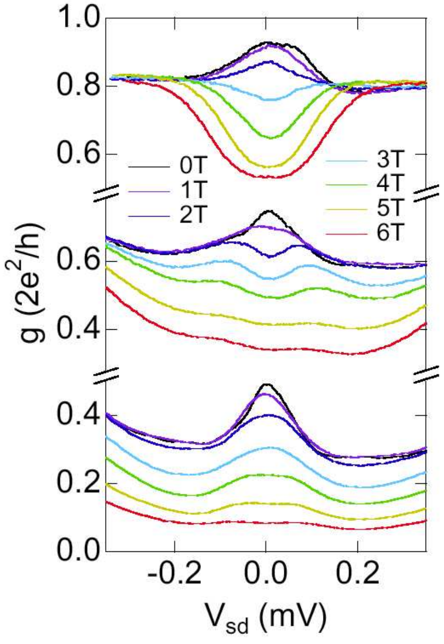
$$f(T/T_K) \sim [1 + (2^{1/S} - 1)\ (T/T_K)^2]^{-S}$$



• *peak width vs. Kondo temperature*



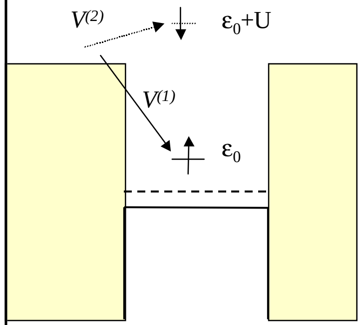
- *splitting of the zero bias anomaly with magnetic field*

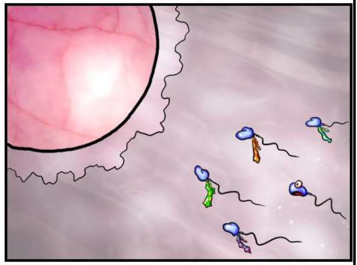


What is different in QPCs compared to QDs ?

- Large background conductance at high temperatures , G ~ 0.5 $(2e^2/h)$
- Energy dependent transparency
- Zero bias anomaly for small conductances

The model





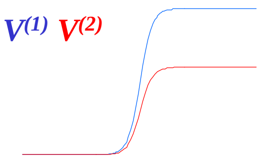
"Hey! Was I supposed to wear a tie?"

single-impurity Anderson model

$$H = \sum_{\substack{k\in L,R \\ \sigma}} \varepsilon_k c^{+}_{k\sigma} c_{k\sigma} + \sum_{\sigma} \varepsilon_0 \, d^{+}_{\sigma} d_{\sigma} + U\, \hat{n}_{\uparrow}\hat{n}_{\downarrow}$$

$$+ \sum_{\substack{k\in L,R \\ \sigma}} V^{(1)}_{k\sigma} (1-\hat{n}_{\bar{\sigma}})(d^{+}_{\sigma} c_{k\sigma} + h.c)$$

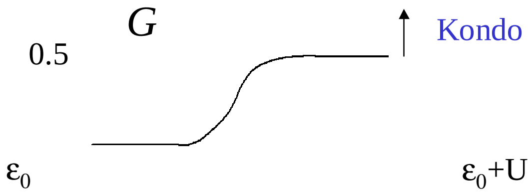
$$+ \sum_{\substack{k\in L,R \\ \sigma}} V^{(2)}_{k\sigma}\, \hat{n}_{\bar{\sigma}} (d^{+}_{\sigma} c_{k\sigma} + h.c)$$



without Kondo correlations

$$G = \frac{2e^2}{h}\left[(1-n)\frac{\Gamma_1^2}{(\mu-\varepsilon_0)^2+\Gamma_1^2} + n\,\frac{\Gamma_2^2}{(\mu-\varepsilon_0-U)^2+\Gamma_2^2}\right]$$

The background conductance is dominated by $0\leftrightarrow 1$ fluctuations (Γ_1)



The Kondo effect dominated by $1\leftrightarrow 2$ fluctuations (Γ_2)

Coexistence of Kondo and mixed-valence regimes

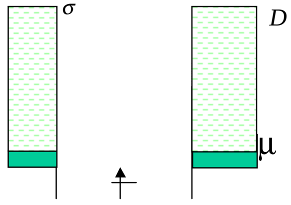
Actual Calculation:

(1) S-W transformation into a Kondo model

$$H_K = \sum_{\substack{k,k'\in L,R \\ \sigma\sigma'}} J_{kk'} C^{+}_{k\sigma} \sigma_{\sigma\sigma'} C_{k'\sigma'} \cdot \vec{S} + \sum_{\substack{k,k'\in L,R \\ \sigma}} W_{kk'} C^{+}_{k\sigma} C_{k'\sigma}$$

$$J_{kk'} \sim \frac{V_k^{(1)} V_{k'}^{(1)}}{\mu - \varepsilon_0} + \frac{V_k^{(2)} V_{k'}^{(2)}}{\varepsilon_0 + U - \mu}$$

$$W_{kk'} \sim \frac{V_k^{(1)} V_{k'}^{(1)}}{\mu - \varepsilon_0} - \frac{V_k^{(2)} V_{k'}^{(2)}}{\varepsilon_0 + U - \mu}$$

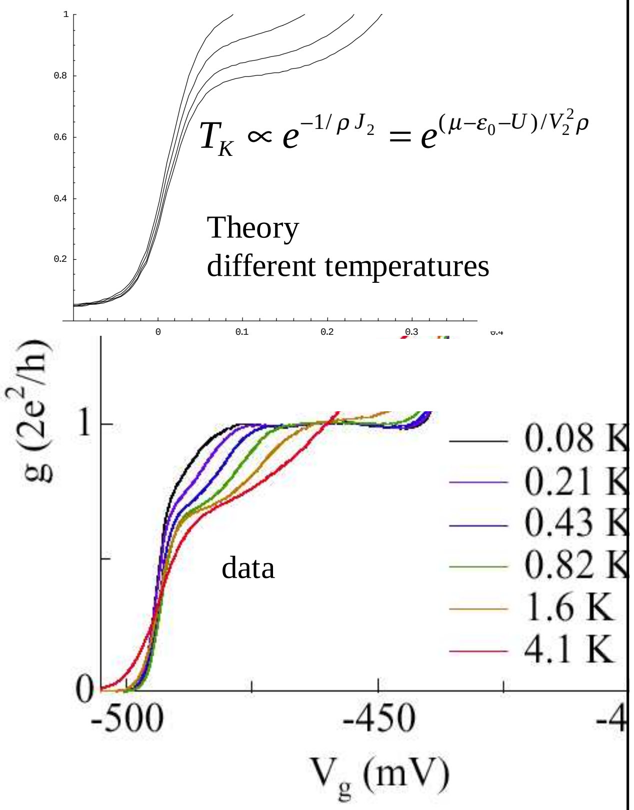


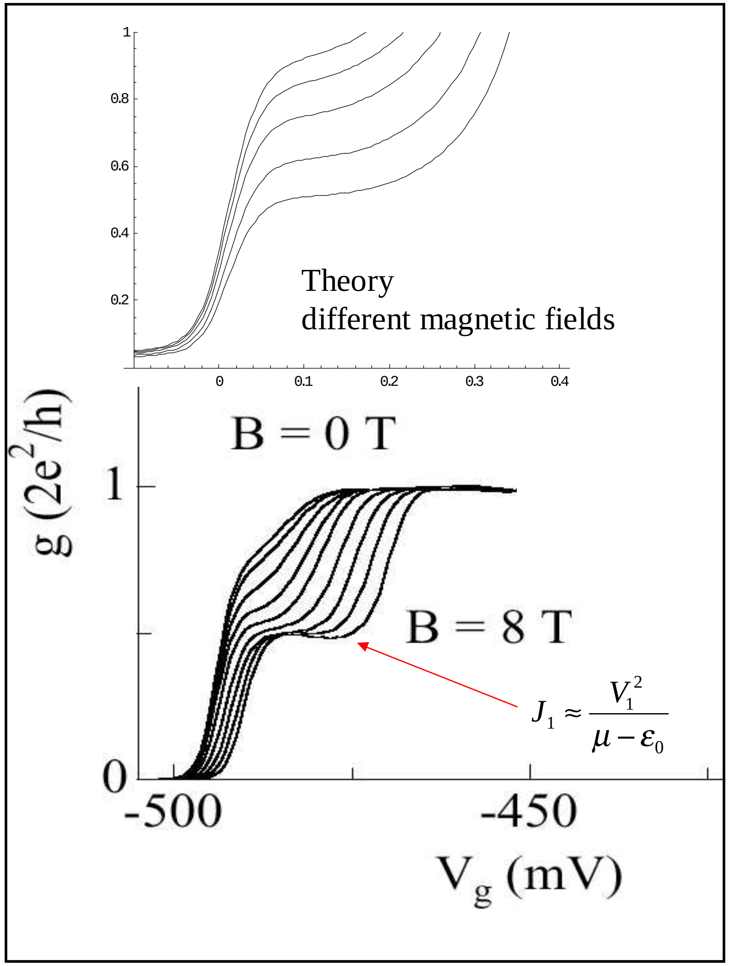
(2) high-T perturbation theory $J_2 \log \frac{D}{T}$

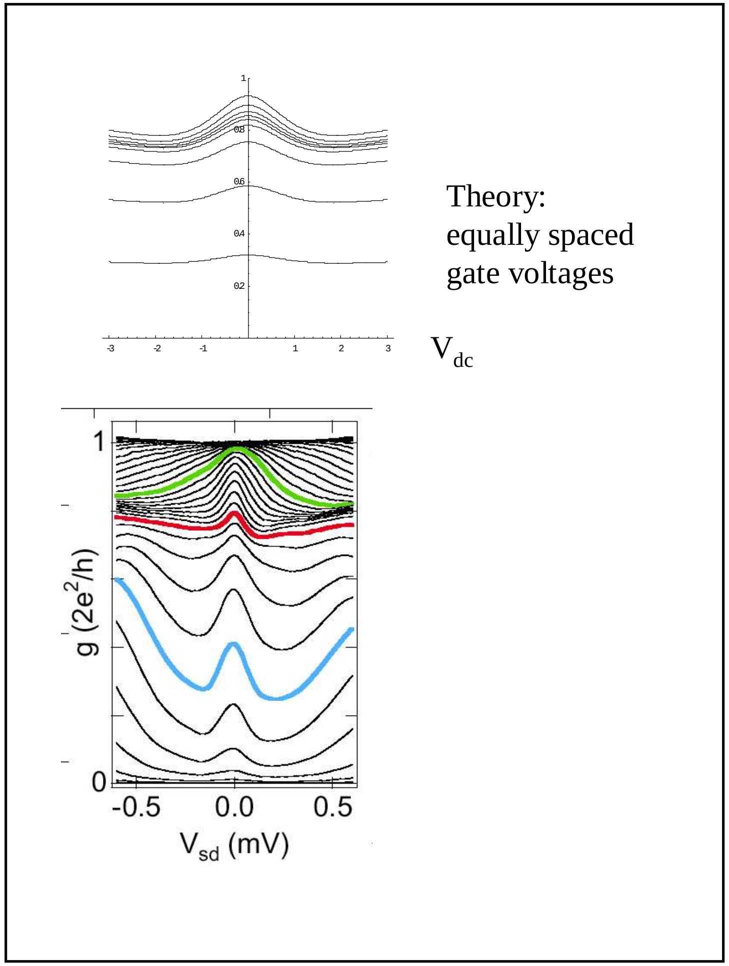
3rd order in J and W,

finite magnetic field and finite voltage

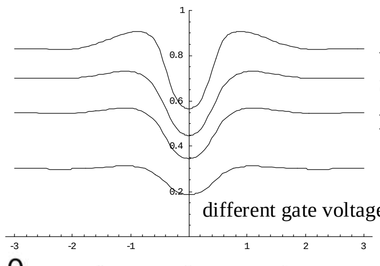
Appelbaum (1967)

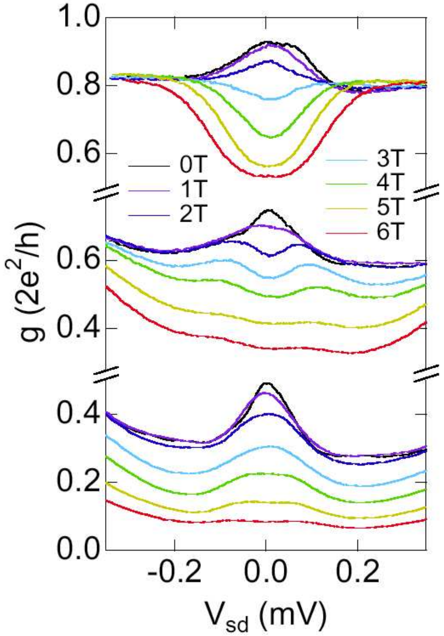



Theory
different magnetic fields
B = 0 T
B = 8 T
g (2e^2/h)
V_g (mV)
-500
-450
$J_1 \approx \frac{V_1^2}{\mu - \varepsilon_0}$

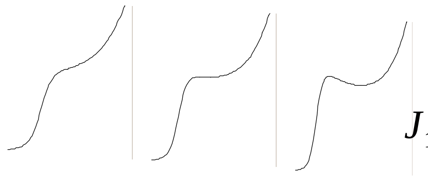

Theory:
equally spaced
gate voltages
V_dc
g (2e^2/h)
V_sd (mV)

Theory:
zero bias anomaly splits in a finite magnetic field





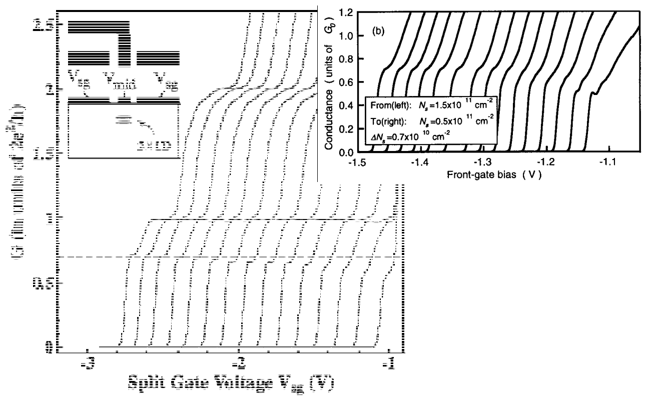
changing the depth of the level (theory):



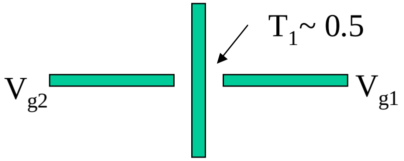
$$J_1 \propto \frac{V_1^2}{\mu - \varepsilon_0}$$

changing back-gate voltage (data):

Thomas et al. *Nuttinck et al.*

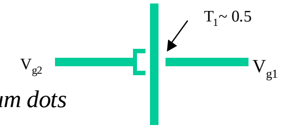


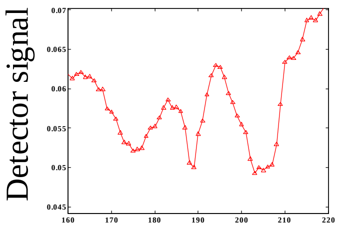
observing the bound state:

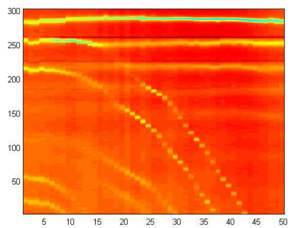


keep V_{g1} constant such that T_1~ 0.5.
scan V_{g2} (around T_2~ 0) – when the bound state gets occupied, T_2 should jump.

Sprinzak et al.:
Probing bound states in quantum dots

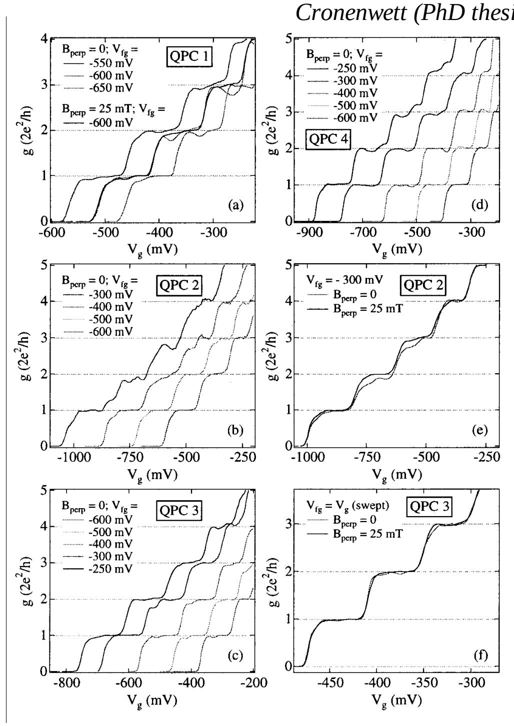




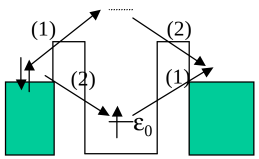


Where are the resonances ?

Cronenwett (PhD thesis, 2002)

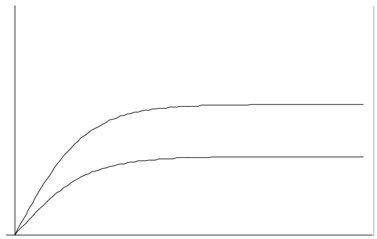


Spin - current



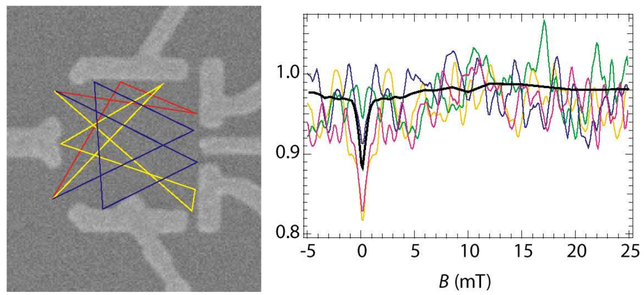
$G_{\uparrow,\downarrow} \propto J_{1,2}^2$

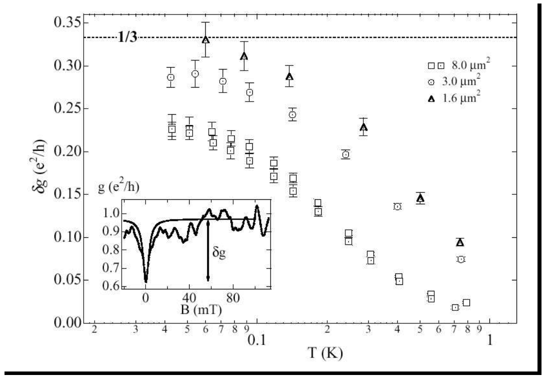
$$G_\sigma \equiv G_\uparrow - G_\downarrow = \frac{2e^2}{h}\rho_L\rho_R\pi^2\langle M\rangle(J_1^2 - J_2^2)$$

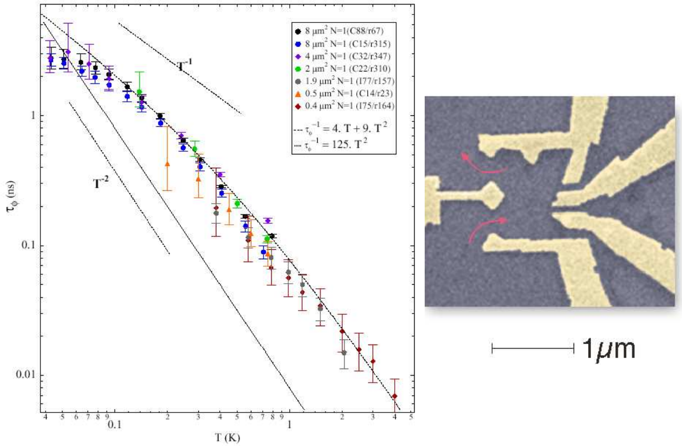


speculation – relation to dephasing in semiconductors ?

Huibers et al.



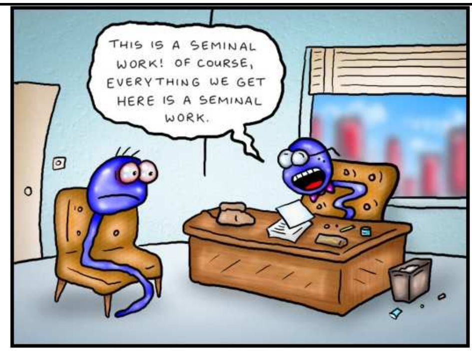




can the finite dephasing time be due to "magnetic impurities" at the QPCs ?

can be easily checked experimentally !

Conclusions:



Sperm publishers

- A Kondo-like model was introduced to explain the 0.7 anomaly in QPCs.
- Semi-quantitative agreement between high-T expansion and experiments. Better agreement with one-loop RG calculations.
- More accurate approaches ? (e-o-m, NCA, NRG)
- Prediction: spin current
- Prediction: observation of a bound states ?
- Relation to dephasing ?

Luttinger liquid:
In one dimension, the phase space for quasi-particle decay does not get smaller at low energies.
→ Elementary excitations are density waves (bosons).

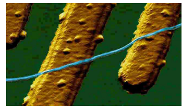
Tunneling of an electron into a Luttinger liquid is reduced (orthogonality catastrophe), and vanishes at zero energy:

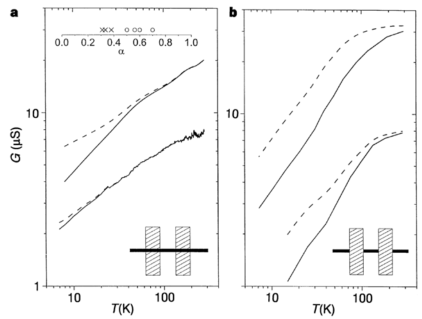
$$T(\varepsilon) \propto \varepsilon^{\alpha-1}$$

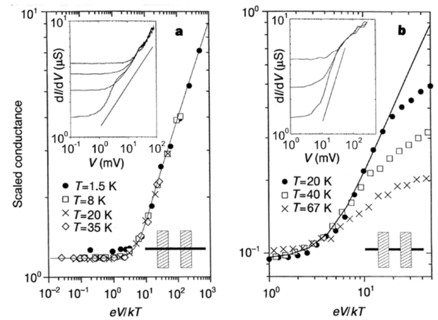
α=1 for non-interacting electrons

Similar power-law behavior is expected for tunneling through an impurity embedded in a LL (Kane and Fisher)

Observations of LL behavior
• tunneling in nanotubes – M. Bockrath et al. (1999)

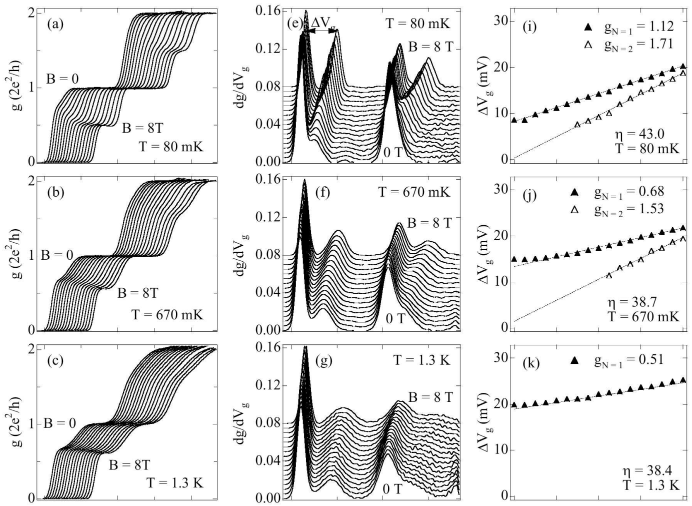


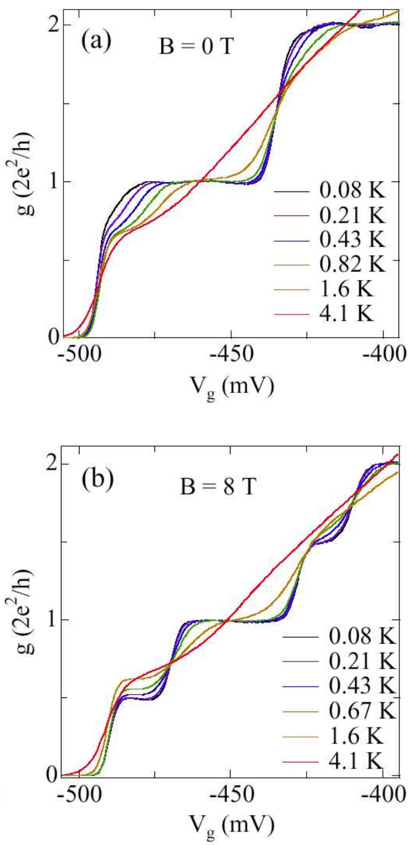




• tunneling in a clean 1d systems (cleaved edge overgrowth) – O. Auslaender et al. (1999)

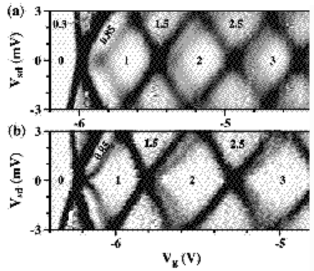
Cronenwett et al. (2002)



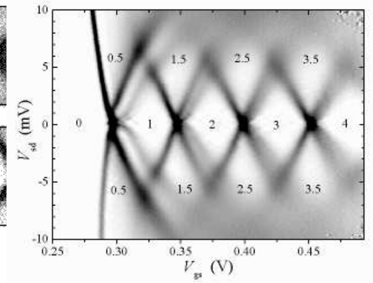


Nonlinear response

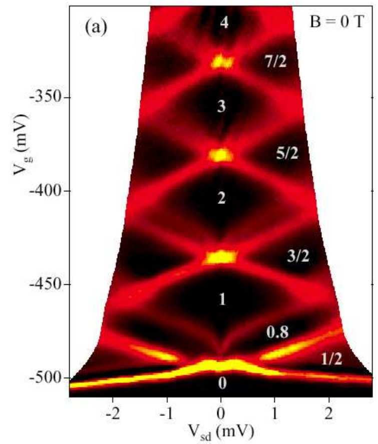
Thomas et al.



Kristensen et al.

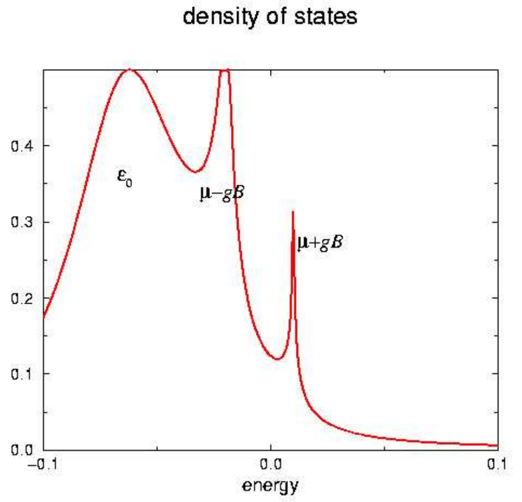


Cronenwett et al.



Nonlinear response and finite magnetic fields

Meir, Wingreen and Lee (1993,1994)



Zero-bias anomaly that splits in finite magnetic fields by $2gB$