"CDM?" program - KITP - 4/13/18

# Cosmology of flavor-mixed dark matter 

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## $\Lambda \mathrm{CDM}$ at small scales

- core / cusp problem



## $\Lambda \mathrm{CDM}$ at small scales

- substructure problem (missing satellites)


CDM

obs.


## $\Lambda \mathrm{CDM}$ at small scales

- too-big-to-fail problem



## Cosmic Web: Small Scale Structure (SSS) - problems

- core / cusp problem
- substructure problem (missing satellites)
- too-big-to-fail problem


CDM

obs.


log halo mass

## WIMP miracle

## Traditional Cold Dark Matter paradigm

```
WIMP miracle
-- mass ~ hundreds GeV - few TeV
-- weak cross-section \(\sim 10^{-37} \mathrm{~cm}^{2}\)
```

$$
\Omega \propto \frac{1}{\langle\sigma v\rangle} \propto\left(\frac{g_{h}^{4}}{m_{h}^{2}}\right)^{-1}
$$

seems to fail
or, at least, many scenarios ruled out
-- direct detection experiments push cross-section
by orders of magnitude to $<10^{-44-45} \mathrm{~cm}^{2}$

## Possible solutions

- Baryonic physics
- NS, BH feedback
- outflows
- star formation
- CR, turbulence
- Dark Matter physics


## Possible solutions

- Baryonic physics
- NS, BH feedback
- outflows
- star formation
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inconclusive (or need too strong feedback)
- large $\sigma$ in dark sector
- multi-flavor *
* naturally, $N$-component flavor-mixed DM (named a la Pontecorvo model of neutrinos)


## 2-component mixed particle

B. Pontecorvo (1957)

Interactions do not care about propagation (mass) eigenstates;

Propagation does not care about interaction (flavor) eigenstates.


## Illustrative model

## Schrödinger equation

$$
i \partial_{t}\binom{m_{h}(x, t)}{m_{l}(x, t)}=\frac{\left[\left(\begin{array}{cc}
-\partial_{x x}^{2} / 2 m_{h} & 0 \\
0 & -\partial_{x x}^{2} / 2 m_{l}-\Delta m
\end{array}\right)+\left(\begin{array}{cc}
m_{h} \phi(x) & 0 \\
0 & m_{l} \phi(x)
\end{array}\right)+\left(\begin{array}{cc}
V_{h h} & V_{h l} \\
V_{l h} & V_{l l}
\end{array}\right)\right]\binom{m_{h}(x, t)}{m_{l}(x, t)}}{H_{\text {free }}} \frac{H_{\text {grav }}}{V}
$$



## No flavor mixing case




## With flavor mixing




## Space-Time diagram



## Quantum evaporation - the "Münchhausen effect"

Particle gradual escape from a gravitational potential (in "elastic" collisions) without changing particle's identity



## "Münchhausen effect"



Baron von Münchhausen lifted himself (and his horse) out of the mud by pulling on his own pigtail.

It is one of the "true" stories from
"The Surprising Adventures of Baron Munchhausen" by R.E. Raspe

## Flavor-mixed $N \mathrm{cDM}$ model (2cDM)

$$
\text { / } N \text {-component flavor-mixed DM with } N=2(2 \mathrm{cDM}) \text { - simplest/ }
$$

## Postulates

(i) Dark Matter - stable N -component mixed particles

Neutralinos<br>Sterile neutrinos<br>Axion+ photon


(ii) DM halos - self-gravitating ensembles of mass eigenstates $|h\rangle,|l\rangle$
(iii) Quantum evaporation of DM mass eigenstates

$$
|h\rangle+|l\rangle \rightarrow|l\rangle+|l\rangle
$$

## 2cDM kinematics

example: $|h\rangle+|l\rangle \rightarrow|l\rangle+|l\rangle$


$$
\left.\begin{array}{c}
p_{h}+p_{l}=0=p_{l}^{\prime}+p_{l}^{\prime} \\
\left(m_{h}^{2} c^{4}+p_{l}^{2} c^{2}\right)^{1 / 2}+\left(m_{l}^{2} c^{4}+p_{l}^{2} c^{2}\right)^{1 / 2}=2\left(m_{l}^{2} c^{4}+{p_{l}^{\prime}}_{l}^{2} c^{2}\right)^{1 / 2} \\
\Delta m / m \ll 1
\end{array}\right] \begin{aligned}
\Delta v=v^{\prime}-v \simeq\left[(\Delta m / m) c^{2}+v^{2}\right]^{1 / 2}-v \\
\simeq \begin{cases}v_{k}, & \text { if } v \ll v_{k} \\
\frac{1}{2} v_{k}^{2} / v, & \text { if } v>v_{k}\end{cases}
\end{aligned}
$$

"kick" velocity: $v_{k}=c\left(\Delta m / m_{l}\right)^{1 / 2}$

## 2cDM kinematics

## example: $|h\rangle+|l\rangle \rightarrow|l\rangle+|l\rangle$

$$
\text { "kick" velocity: } v_{k}=c\left(\Delta m / m_{l}\right)^{1 / 2}
$$

if $v_{\text {kick }} \gg v_{\text {escape. }}$ dwarf halos destroyed


MM, J Phys A 2010; JCAP 2014
if $v_{\text {kick }} \ll v_{\text {escape. }} \quad$ central cusps softened

halo

## Do halos evaporate completely?



$$
\begin{aligned}
& \text { abundance evolution eqns. } \quad \begin{array}{l}
\dot{n}_{h}=-\left(\sigma_{h h} v\right) n_{h}^{2}-\left(\sigma_{h l} v\right) n_{h} n_{l} \\
\dot{n}_{l}=-\left(\sigma_{h l} v\right) n_{h} n_{l}
\end{array} \\
& \text { then } \frac{d n_{h}}{d n_{l}}=\frac{\sigma_{h h} n_{h}}{\sigma_{h l} n_{l}}+1 \\
& \text { solution } \frac{n_{h}(t)}{n_{h, 0}}=\left(\frac{n_{l, 0} / n_{h, 0}}{1-R}\right)\left(\frac{n_{l}(t)}{n_{l, 0}}\right)+\left(1-\frac{n_{l, 0} / n_{h, 0}}{1-R}\right)\left(\frac{n_{l}(t)}{n_{l, 0}}\right)^{R}
\end{aligned}
$$

$$
R=\sigma_{h h} / \sigma_{h l}
$$

asymptotically $\quad n_{h}(\infty) \rightarrow 0, n_{l}(\infty) \rightarrow n_{l, \infty}$

$$
\frac{n_{l, \infty}}{n_{l, 0}}=\left[1-\frac{n_{h, 0}}{n_{l, 0}}(1-R)\right]^{\frac{1}{1-R}}
$$

complete evaporation is possible when

$$
\frac{n_{l, 0}}{n_{h, 0}} \leq 1-\frac{\sigma_{h h}}{\sigma_{h l}}
$$

## N-body simulations

* GADGET, $50 \mathrm{Mpc} / \mathrm{h}$ box, standard $\Lambda$ CDM cosmology
- At each step:
+ Pairs of nearest neighbors are identified
+ Densities of each species are found at each particle location
+ Conversion probabilities are calculated
+ Monte-Carlo module is used for conversions
+ Energy-momentum is manifestly conserved in every interaction
* 2 free parameters: $\sigma(\mathrm{v}) / \mathrm{m}$ [with $\left.\sigma \propto\left(\mathrm{v} / \mathrm{v}_{\mathrm{k}}\right)^{-1}\right]$ and $\Delta \mathrm{m} / \mathrm{m}\left[\right.$ or $\left.\mathrm{v}_{\mathrm{k}}=\mathrm{c}(2 \Delta \mathrm{~m} / \mathrm{m})^{1 / 2}\right]$

$$
\begin{aligned}
P_{s_{i} t_{i} \rightarrow s_{f} t_{f}}=\left(\rho_{t_{i}} / m_{t_{i}}\right) \sigma_{s_{i} t_{i} \rightarrow s_{f} t_{f}}\left|\mathbf{v}_{t_{i}}-\mathbf{v}_{s_{i}}\right| \Delta t \Theta\left(E_{s_{f} t_{f}}\right) \\
\sigma_{s_{i} t_{i} \rightarrow s_{f} t_{f}}=\sigma_{\mathrm{si}}(v)=\sigma\left(v / v_{0}\right)^{-a} \\
a=1
\end{aligned}
$$

## No change on large scales



## No change on large scales



## No change on large scales



## Less substructure on small scales



## Less substructure on small scales



## Velocity function


$\sigma_{*}=0.75 \mathrm{~cm}^{2} / \mathrm{g}$
$\mathrm{v}_{\mathrm{k}}=50 \mathrm{~km} / \mathrm{s}$
$\Delta \mathrm{m} / \mathrm{m}=10^{-8}$

## Density profiles



## Key: cross-sections

cross-sections

$$
\begin{aligned}
\sigma_{\left(s_{i} t_{i}\right) \rightarrow\left(s_{i} t_{i}\right)} & =\frac{\pi}{k_{i}^{2}} \sum_{l=0}^{\infty}(2 l+1)\left|1-S_{\left(s_{i} t_{i}\right)\left(s_{i} t_{i}\right)}^{(l)}\right|^{2} \\
\sigma_{\left(s_{i} t_{i}\right) \rightarrow\left(s_{f} t_{f}\right)} & =\frac{\pi}{k_{i}^{2}} \sum_{l=0}^{\infty}(2 l+1)\left|S_{\left(s_{i} t_{i}\right)\left(s_{f} t_{f}\right)}^{(l)}\right|^{2}
\end{aligned}
$$

parameterize

$$
\sigma_{i \rightarrow f}(v)= \begin{cases}\sigma_{0}\left(v / v_{0}\right)^{a_{s}} & \text { for scattering, } \\ \sigma_{0}\left(p_{f} / p_{i}\right)\left(v / v_{0}\right)^{a_{c}} & \text { for conversion }\end{cases}
$$

natural: $\mathrm{a}_{\mathrm{s}}=\mathrm{a}_{\mathrm{c}}$
examples:

| $a_{s}=a_{c}=0$ | "hard spheres" (s-wave scattering) |
| :--- | :--- |
| $a_{s}=a_{c}=-1$ | annihilation-like |
| $a_{s}=a_{c}=-2$ | maximum conversion probability |
| $a_{s}=-4$ | Rutherford-like |

## Substructure evaporation

assume profile

$$
\rho(r)=\rho_{0}\left(\frac{r}{R}\right)^{-\beta}
$$

hydrostatic balance yields

$$
v_{t h}^{2}=\frac{4 \pi G \rho_{0} R^{\beta}}{\beta(3-\beta)} r^{2-\beta}
$$

mass-loss per radius

$$
\begin{aligned}
& \frac{d \dot{M}}{d r}=4 \pi r^{2} \dot{\rho}=4 \pi r^{2} \dot{\rho}_{0}\left(\frac{r}{R}\right)^{\lambda=1-\frac{5}{2} \beta+a\left(1-\frac{\beta}{2}\right)} \\
& \dot{\rho}=-(n \sigma v) \rho=-\rho^{2}\left(\frac{\sigma}{m}\right) v=\rho_{0}\left(\frac{r}{R}\right)^{-2 \beta} \frac{\sigma_{0}}{m}\left(\frac{v}{v_{0}}\right)^{a} v
\end{aligned}
$$

integrate to yield the total halo mass-loss $\quad \dot{M}=\frac{3-\beta}{\lambda+3} \frac{\sigma_{0} v_{0}}{m}\left[\frac{G}{v_{0} \beta}\left(\frac{4 \pi \rho_{0}}{3-\beta}\right)^{1 / 3}\right]^{a+1}\left(\frac{r_{c}}{R}\right)^{\lambda+3} M^{1+\frac{2}{3}(a+1)}$

## Substructure evaporation

assume profile

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\rho(r)=\rho_{0}\left(\frac{r}{R}\right)^{-\beta}
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\frac{d \dot{M}}{d r}=4 \pi r^{2} \dot{\rho}=4 \pi r^{2} \dot{\rho_{0}}\left(\frac{r}{R}\right)^{\lambda=1-\frac{5}{2} \beta+a\left(1-\frac{\beta}{2}\right)}
$$

$$
\dot{\rho}=-(n \sigma v) \rho=-\rho^{2}\left(\frac{\sigma}{m}\right) v=\rho_{0}\left(\frac{r}{R}\right)^{-2 \beta} \frac{\sigma_{0}}{m}\left(\frac{v}{v_{0}}\right)^{a} v
$$

integrate to yield the total halo mass-loss $\quad \dot{M}=\frac{3-\beta}{\lambda+3} \frac{\sigma_{0} v_{0}}{m}\left[\frac{G}{v_{0} \beta}\left(\frac{4 \pi \rho_{0}}{3-\beta}\right)^{1 / 3}\right]^{a+1}\left(\frac{r_{c}}{R}\right)^{\lambda+3} M^{1+\frac{2}{3}(a+1)}$

> just a constant
approximately constant

$$
\dot{M}=-|A| M^{\xi}
$$

indep. of halo shape (beta)
solution

$$
\rightarrow M_{0}=\left[(1-\xi) A t+M^{1-\xi}\right]^{1 /(1-\xi)}
$$

## Substructure evaporation

mapping of old to new

$$
M_{0}= \begin{cases}\left(M^{-2 / 3}-\frac{2}{3} A t\right)^{-3 / 2}, & a=0 \\ M e^{A t}, & a=-1 \\ \left(M^{2 / 3}+\frac{2}{3} A t\right)^{3 / 2}, & a=-2\end{cases}
$$

New mass function given the old one $f\left(M_{0}\right)$ is $f\left(M_{0}\right)=f\left(M_{0}(M, t)\right) \equiv f(M, t)$
and similarly for the velocity function

Evaporation resolves substructure \& TBTF problems Shape of mass function tells: index $a_{c}$ (conversion) and $\sigma_{0} / \mathrm{m}$

Velocity function


## 2cDM $\sigma(\mathrm{v})$-simulations



## 2cDM-б(v) -- Substructure



## Message 1

## SUBSTRUCTURE



2cDM: Resolves substructure \& TBTF problems.
Position of break tells:
$\Delta \mathrm{m} / \mathrm{m} \Leftrightarrow \mathrm{v}_{\mathrm{k}}$
Shape of mass function tells:
index $\mathrm{a}_{\mathrm{c}}$ (conversion) and $\sigma_{0} / \mathrm{m}$


Wide parameter region allowed:
$\sigma(v) \sim 1 . . .0 .1 \ldots . .0 .01$ - consistent with all constraints
$\Delta \mathrm{m} / \mathrm{m} \sim 10^{-8} \Leftrightarrow \mathrm{v}_{\mathrm{k}} \sim 50-100 \mathrm{~km} / \mathrm{s}$

## Cusp softening

number of interactions per particle

$$
N_{\text {int }}=n \sigma v t_{H}=\rho_{\mathrm{vir}} \frac{\sigma_{0}}{m} t_{\mathrm{H}} v_{0}\left(\frac{V_{\mathrm{vir}}}{v_{0}}\right)^{a+1}\left(\frac{r_{c}}{R_{\mathrm{vir}}}\right)^{a+1-\frac{\beta}{2}(a+3)}
$$



$$
\begin{aligned}
& \rho_{\mathrm{vir}}=\frac{(3-\beta) M_{\mathrm{vir}}}{4 \pi R_{\mathrm{vir}}^{3}} \\
& V_{\mathrm{vir}}^{2}=G M_{\mathrm{vir}} / R_{\mathrm{vir}} \\
& N_{\mathrm{vir}} \equiv \rho_{\mathrm{vir}} \frac{\sigma_{0}}{m} V_{\mathrm{vir}} t_{\mathrm{H}}
\end{aligned}
$$

core radius

$$
\frac{r_{c}}{R_{\mathrm{vir}}} \simeq\left[\left(\frac{\mathrm{a} \text { few }}{N_{\mathrm{vir}}}\right)\left(\frac{V_{\mathrm{vir}}}{v_{0}}\right)^{-a}\right]^{-\xi} \propto \sigma_{0}^{\xi}
$$

Scattering resolves core-cusp problem Core size tells:
$\sigma_{0} / \mathrm{m}$ and index $\mathrm{a}_{\mathrm{s}}$ (scattering)

$$
\xi=\frac{2}{\beta(a+3)-2(a+1)}
$$

## 2cDM-б(v) -- Profiles (MW-like)



## 2cDM-o(v) -- Profiles (Dwarfs)

$$
\begin{aligned}
& \sigma(v)=\left\{\begin{array}{l}
\sigma\left(v / v_{0}\right)^{a_{s}} \\
\sigma\left(v / v_{0}\right)^{a_{c}}
\end{array}\right. \\
& \sigma=0.01,0.1, i_{i} \\
& \text { consistent with } \\
& \text { observations } \\
& \left(\mathrm{a}_{\mathrm{s},} \mathrm{a}_{\mathrm{c}}\right)
\end{aligned}
$$

(Todoroki \& MM, in prep.)


## 2cDM- $\sigma(v)$-- Core relations (obs vs sim)

Red symbols - simulations Gray crosses - MW sSph
(Data:
Strigari et al. 2008;
Burkert 2015 )

## 2cDM- $\sigma(\mathrm{v}$ ) -- Profiles (Clusters)


(Todoroki \& MM, in prep.)

## Message 2

## PROFILES



Wide parameter region allowed:
$\sigma(\mathrm{v})$ ~ 1...0.1...0.01 - consistent with all constraints
$\Delta \mathrm{m} / \mathrm{m} \sim 10^{-8} \Leftrightarrow \mathrm{v}_{\mathrm{k}} \sim 50-100 \mathrm{~km} / \mathrm{s}$

2cDM: Resolves core-cusp problem. Core size tells: $\sigma_{0} / \mathrm{m}$ and index $\mathrm{a}_{\mathrm{s}}$ (scattering)



## Radial distribution of satellites (CDM)



## 2cDM- $\sigma(\mathrm{v})$-- dwarf distribution



## Message 3

## DISTRIBUTION of

 SATELLITES2cDM: Resolves
substructure radial distribution. Shape of function depends on all parameters


Wide parameter region allowed:
$\sigma(\mathrm{v}) \sim 1 . . .0 .1 . . .0 .01$ - consistent with all constraints
$\Delta \mathrm{m} / \mathrm{m} \sim 10^{-8} \Leftrightarrow \mathrm{~V}_{\mathrm{k}} \sim 50-100 \mathrm{~km} / \mathrm{s}$


## Typical constraints

"Bullet cluster" $\sigma / \mathrm{m}<1 \mathrm{~cm}^{2} / \mathrm{g} \quad$ 2cDM cross-sections $\sigma / \mathrm{m} \sim 1 . . .0 .1$, even... 0.01

Stability to decay "mass-eigenstates must decay to leave the lightest only" $\Delta \mathrm{m} / \mathrm{m} \sim 10^{-8}$-- enough room to avoid: no secondaries to decay into (cf $100 \mathrm{keV} / 1 \mathrm{TeV}$ )

## Typical constraints

"Bullet cluster" $\sigma / \mathrm{m}<1 \mathrm{~cm}^{2} / \mathrm{g} \quad 2 \mathrm{CDM}$ cross-sections $\sigma / \mathrm{m} \sim 1 . . .0 .1$, even...0.01

Stability to decay "mass-eigenstates must decay to leave the lightest only"

$$
\begin{aligned}
& \Delta \mathrm{m} / \mathrm{m} \sim 10^{-8} \text {-- enough room to avoid: no } \\
& \text { secondaries to decay into (cf } 100 \mathrm{keV} / 1 \mathrm{TeV} \text { ) }
\end{aligned}
$$

Early universe "catastrophe"
2cDM looks like any multi-species / composite DM -- allows rapid "reactions" $\mathrm{Y} \rightarrow \mathrm{X}$
$\Rightarrow$ abundance of heavy states must be exponentially suppressed


## Typical constraints

if wave packets overlap, particles interact coherently (as flavor states) - no conversions
time-dependent wave packet of a mass eigenstate

$$
\psi_{j}(x, t)=\left[2 \pi\left(\Delta_{0}+\frac{i \hbar t}{2 m_{j} \Delta_{0}}\right)^{2}\right]^{-1 / 4} \exp \left[-\frac{\left(x-x_{0}-v_{j} t\right)^{2}}{4 \Delta_{0}^{2}+2 i \hbar t / m_{j}}+\frac{i}{\hbar}\left(m_{j} v_{j} x-\frac{m_{j} v_{j}^{2}}{2} t\right)\right]
$$

wave packet width $\Delta_{j}^{2}(t)=\Delta_{0}^{2}+\left(\frac{\hbar}{2 m_{j} \Delta_{0}}\right)^{2} t^{2}$
packet spreads as fast as it propagates
interaction amplitude $\sim$ overlap of two mass states

$$
I(t)=\int_{-\infty}^{\infty} A_{h}(x, t) A_{l}(x, t) d x \quad I(\infty) \simeq 1-\left(\frac{\Delta m}{m}\right)^{2} \xi+\mathcal{O}\left(\frac{\Delta m^{3}}{m^{3}}\right)
$$

$$
\xi=\frac{1}{4}+\frac{p^{2} \Delta_{0}^{2}}{2 \hbar^{2}} \sim \frac{1}{4}+\frac{p^{2}}{2(\Delta p)^{2}} \sim \mathcal{O}(1)
$$

Catastrophe isn't a problem for 2 cDM : conversions do not occur before structure formation starts (needed to separate mass states)

$$
\sigma_{\mathrm{conv}}^{\mathrm{fst}} \sim(\Delta m / m)^{4} \sigma_{\mathrm{conv}}
$$

before structure formation

## Caveats

some fine tuning of $\sigma$ to Hubble time
large $\sigma \sim 1 \mathrm{~cm}^{2} / \mathrm{g} \sim 1$ barn $/ \mathrm{GeV}$
high degeneracy $\Delta \mathrm{m} / \mathrm{m} \sim 10^{-8}$


$$
0.001<\sigma / \mathrm{m}<1 \mathrm{~cm}^{2} / \mathrm{g}
$$

too few collisions too many collisions

- uninteresting
- collapse
not very natural in particle physics
not very natural in particle physics possibly needs light mediator
small mass splitting:
Y. Zhang, Phys. Dark Univ. 15 (2017)
K. Schutz, T.R. Slatyer, JCAP 01 (2015) 021
J. Kopp et al. JHEP 12 (2016) 033
M. Baumgart et al. JHEP 0904:014,2009
$\qquad$


## 2cDM predictions

direct detection
"inelastic recoil"

$-\Delta m c^{2}$
indirect detection
" $\gamma$-ray annihilation line triplet"

(MM, PRL 2014)

## Conclusions

## flavor-mixed DM - just works

+ resolves small-scale problems simultaneously across many scales
+ cosmologically interesting ( $\mathrm{v} \sim 100 \mathrm{~km} / \mathrm{s}$ )
+ $\sigma(\mathrm{v}) / \mathrm{m} \sim 1 \ldots 0.1 \ldots . .0 .01$ - consistent with all obs. constraints
+ $\Delta \mathrm{m} / \mathrm{m} \sim 10^{-8}$ - can be naturally stable
+ passes the "early universe catastrophe" challenge
- makes predictions for DM detection experiments: inelastic recoil, gamma triplet further study - "realistic" simulations with star formation, baryons, feedback,...

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