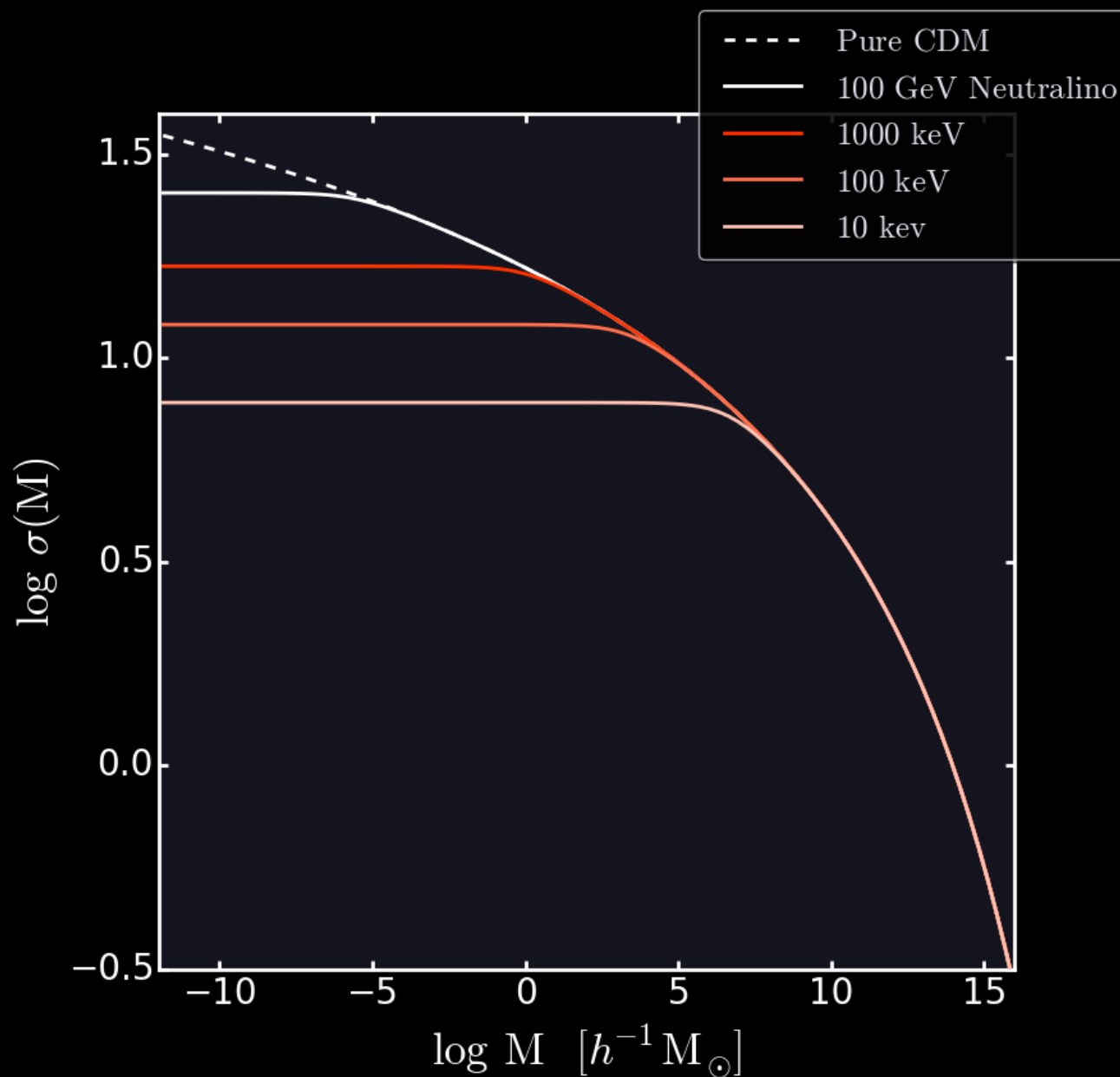
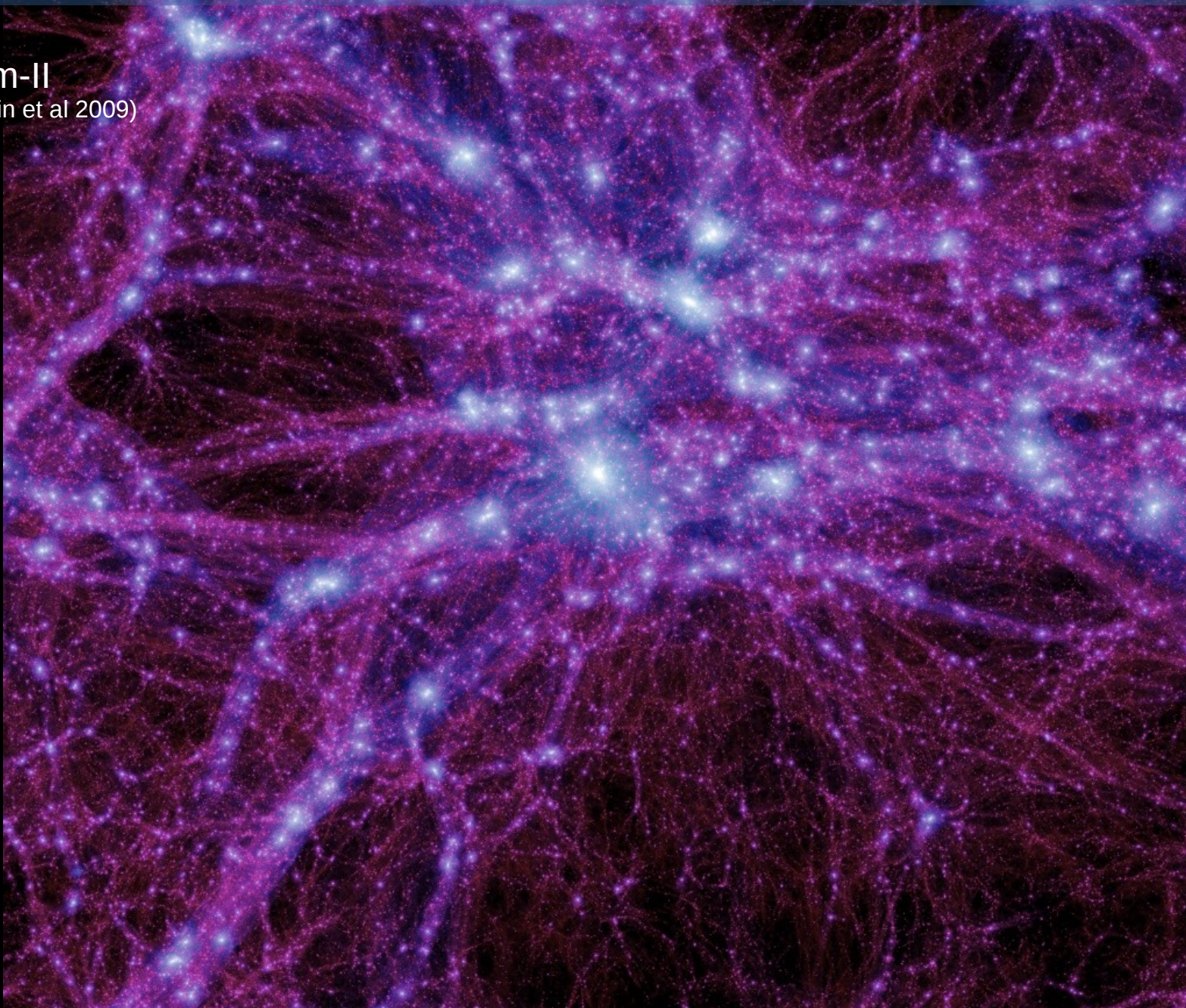


# Cold, Warm and Neutralino DM halos



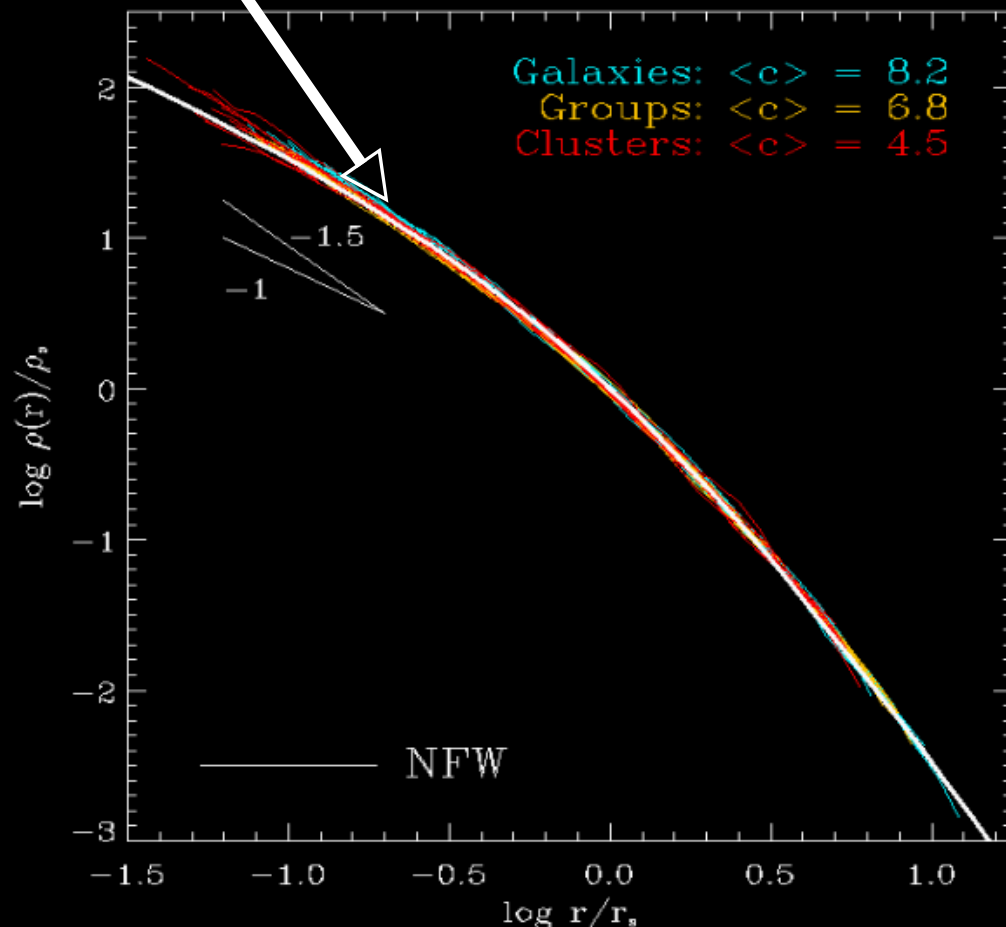
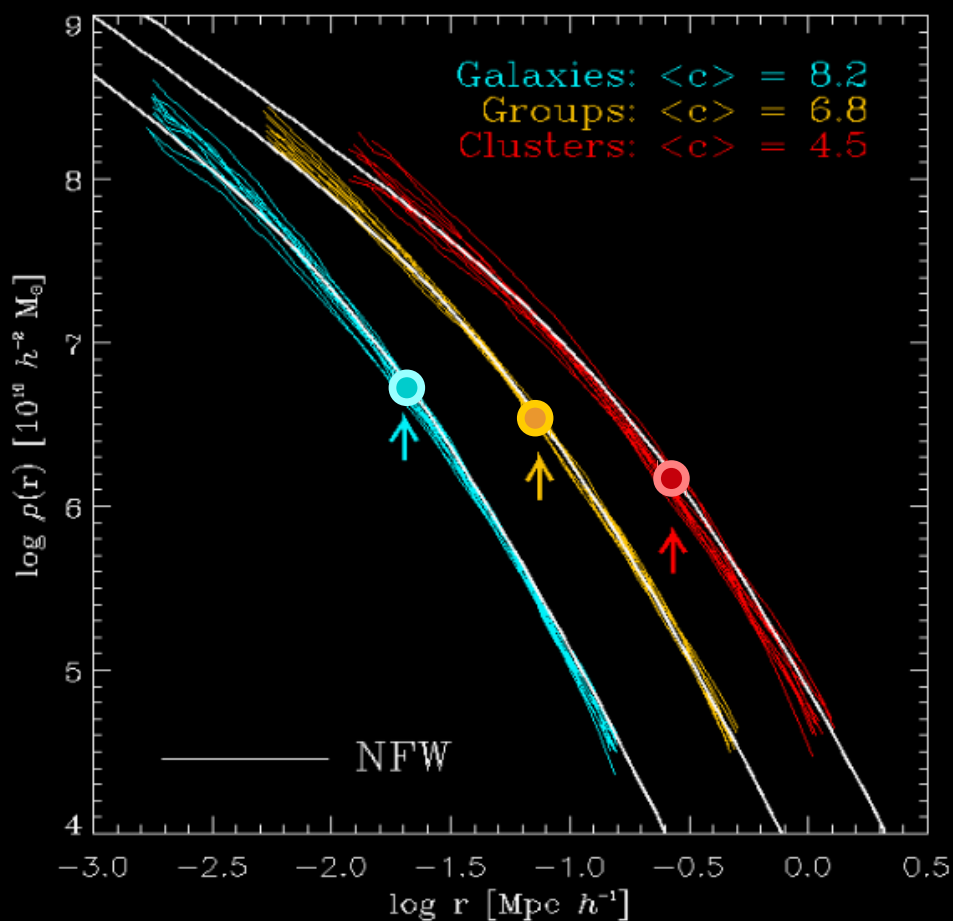
# Cold DM halos

Millennium-II  
(Boylan-Kolchin et al 2009)



# Start with CDM: Navarro, Frenk & White “NFW” profile

**Universal**  
 “NFW profile” :  $\rho(r) = \frac{\rho_{\text{crit}} \delta_c}{\frac{r}{r_s} \left(1 + \frac{r}{r_s}\right)^2}$       concentration:  $C = \frac{r_{200}}{r_s}$



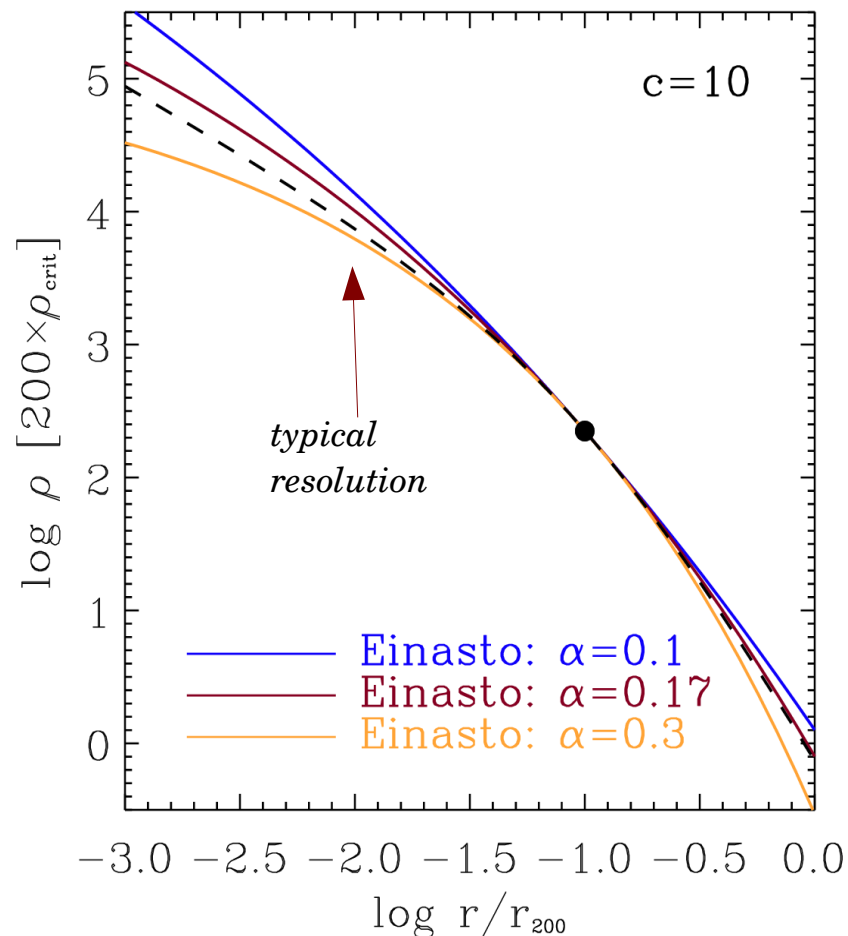
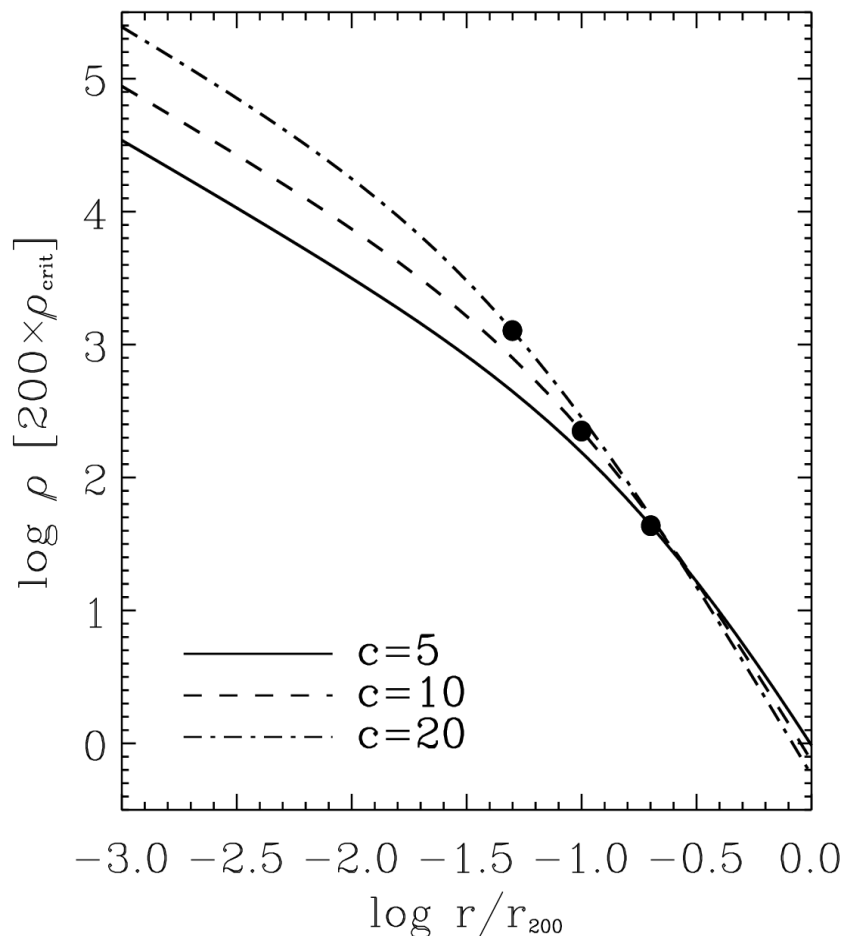
# Start with CDM: Navarro, Frenk & White “NFW” profile

$$\rho(r) = \frac{\rho_0}{cx(1+cx)^2}$$

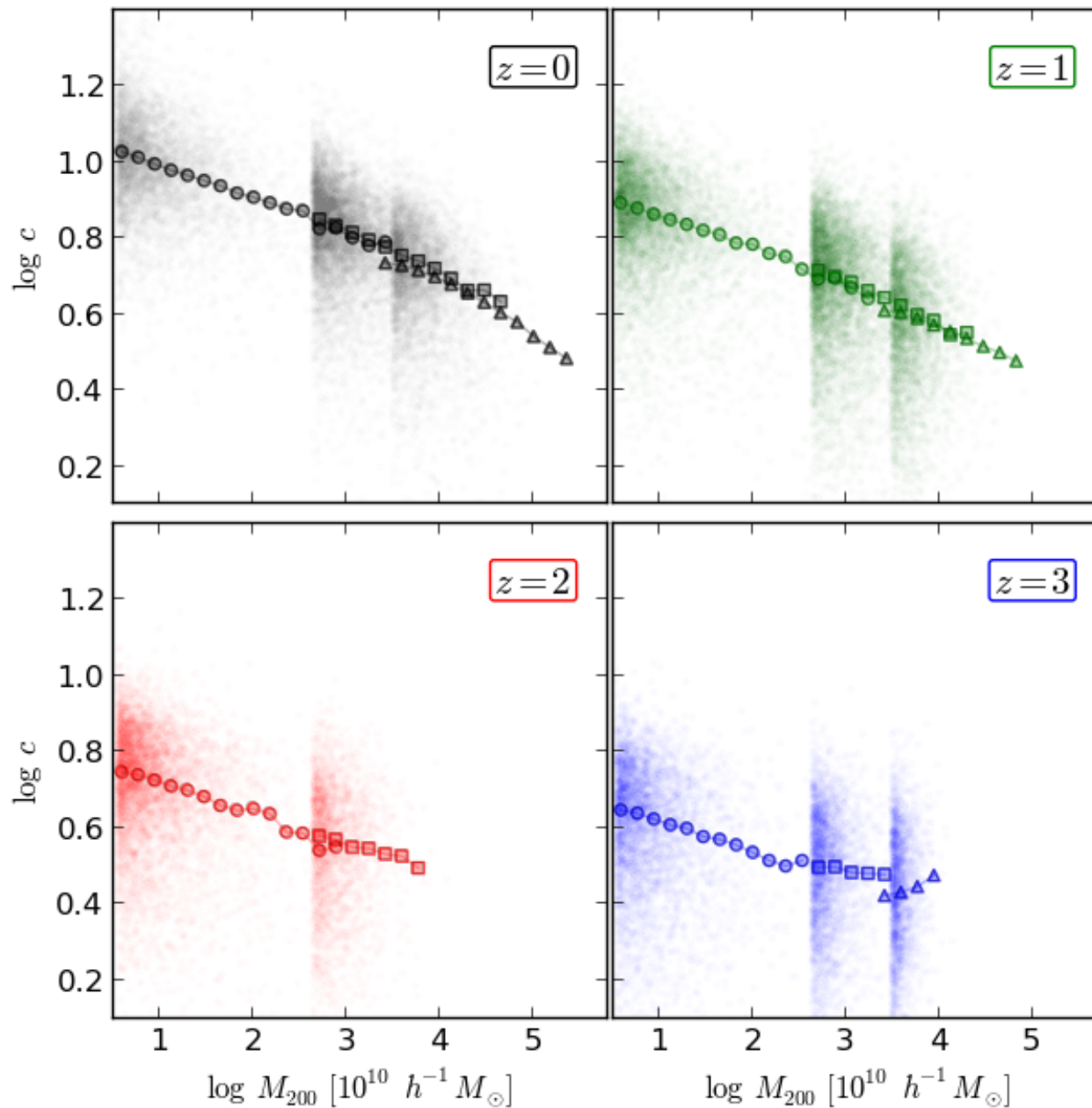
“concentration”

$$\frac{d \ln \rho}{d \ln r} \propto -2(cx)^\alpha$$

“Shape”

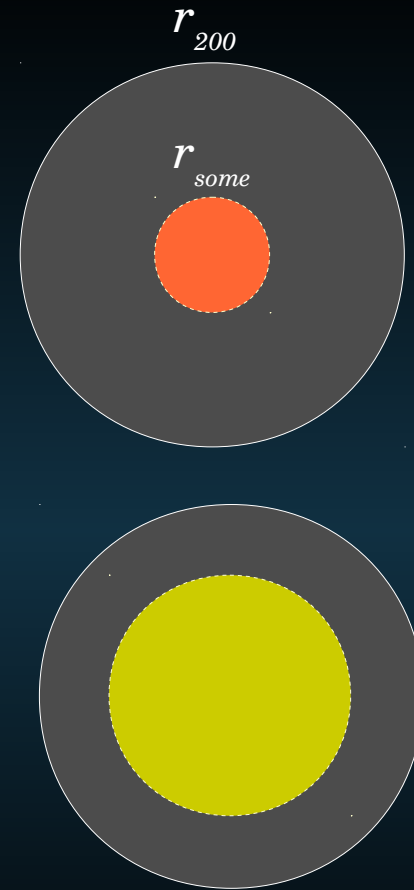
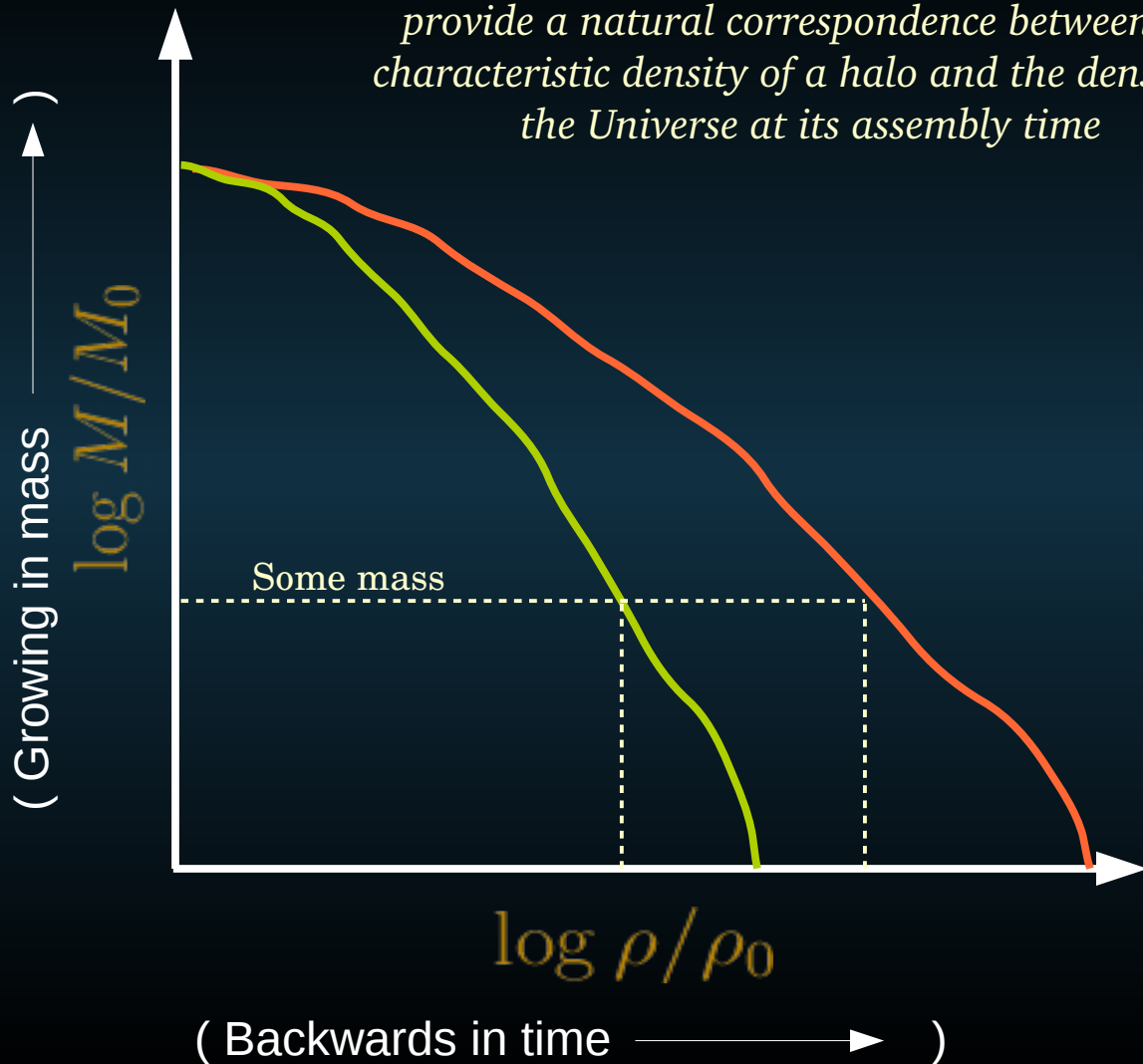


# CDM halos: The concentration-mass-redshift relation



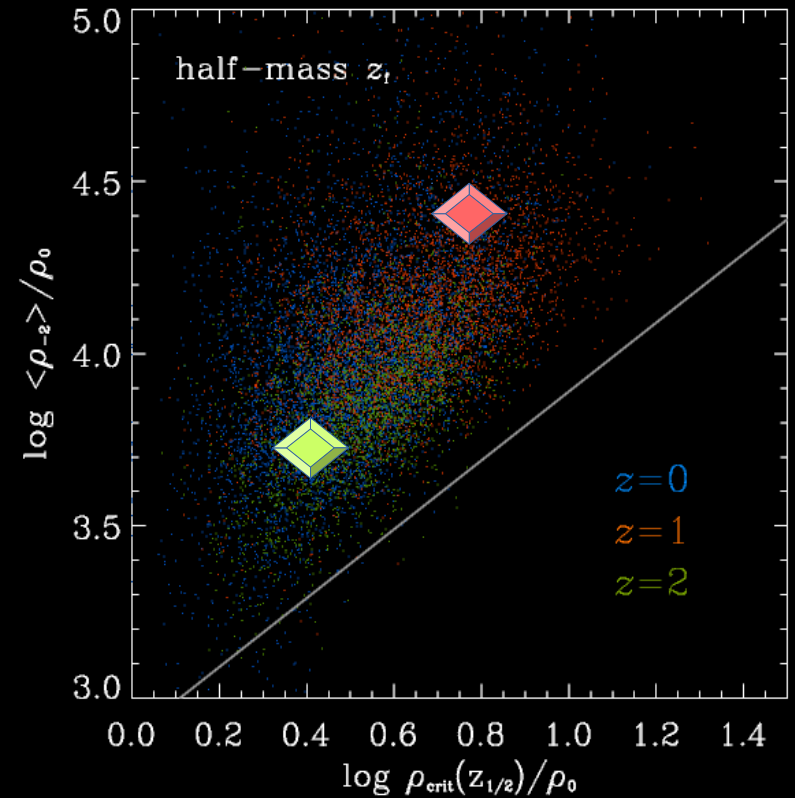
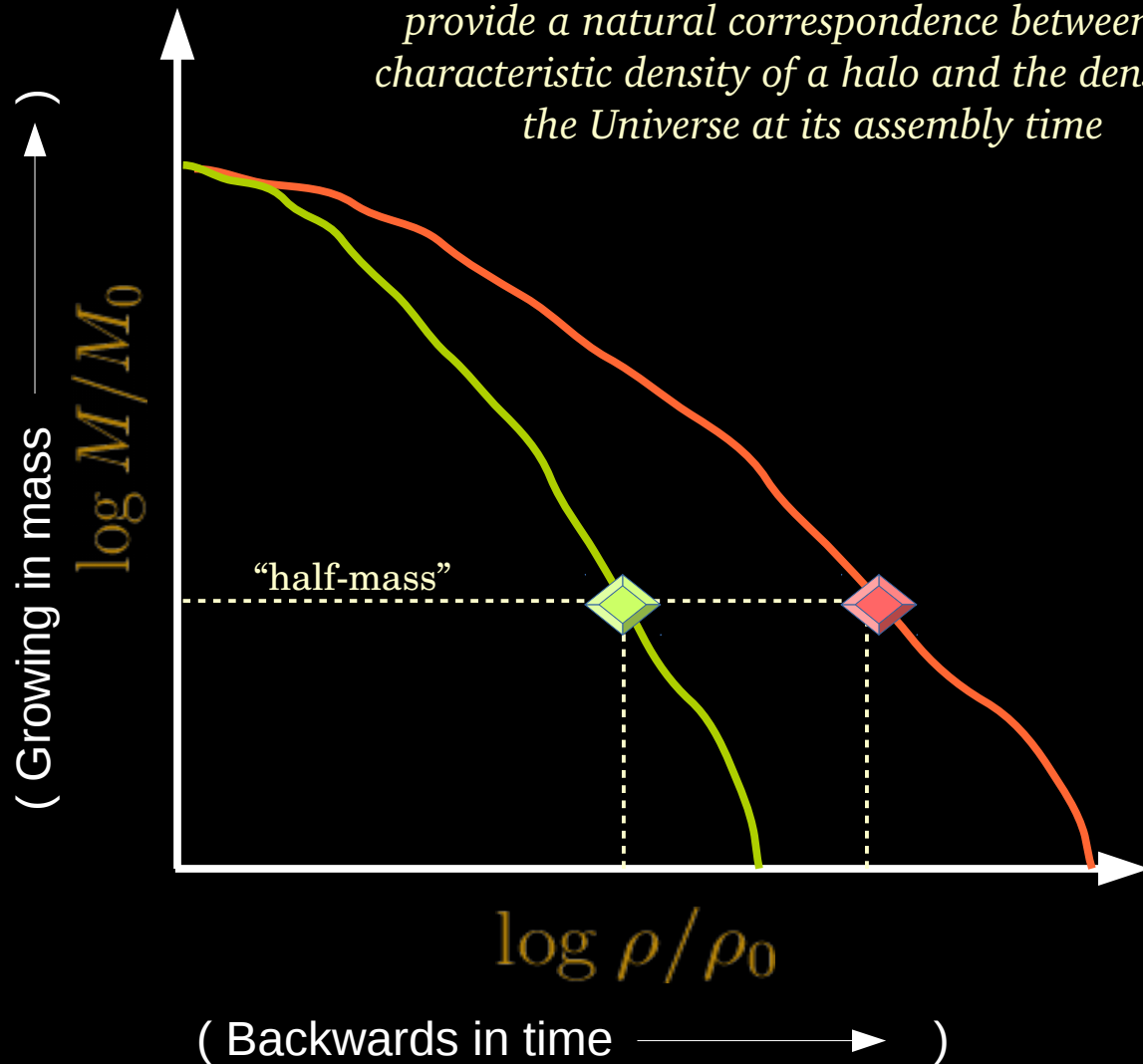
# Mass Profiles and Accretion Histories

*An ideal definition of “formation time” would provide a natural correspondence between the characteristic density of a halo and the density of the Universe at its assembly time*



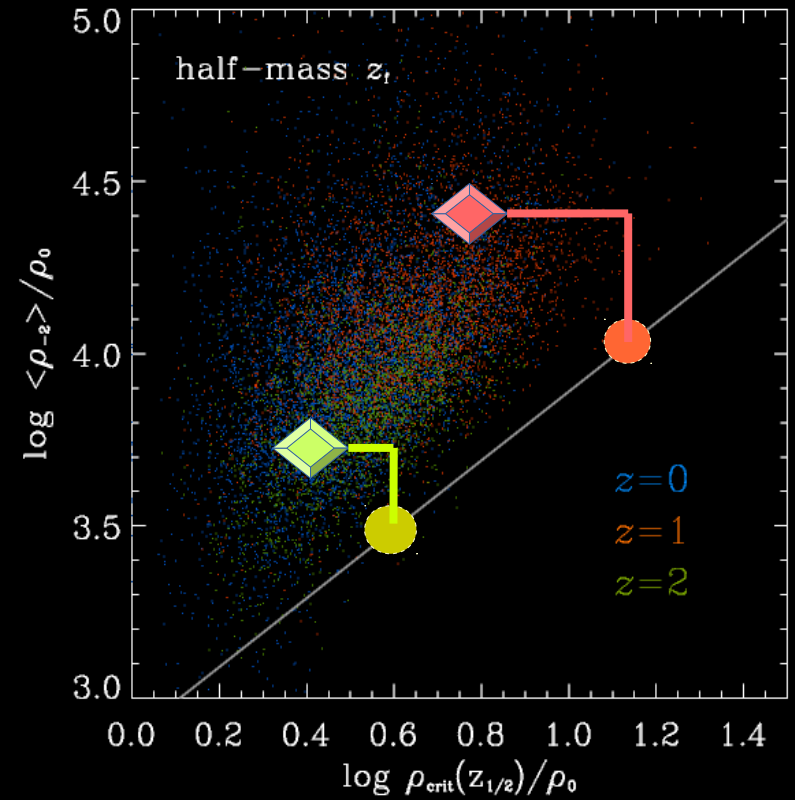
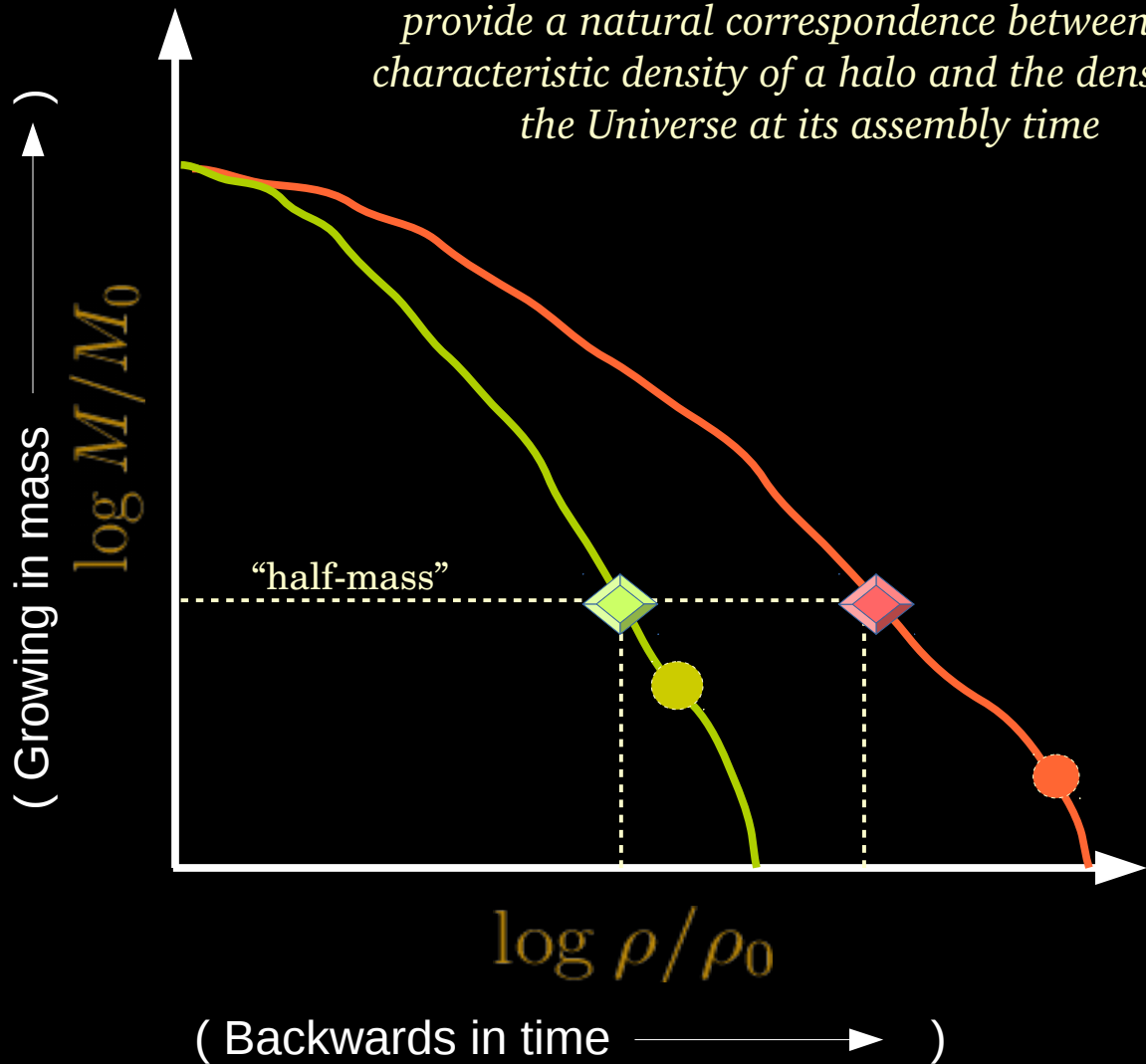
# Mass Profiles and Accretion Histories

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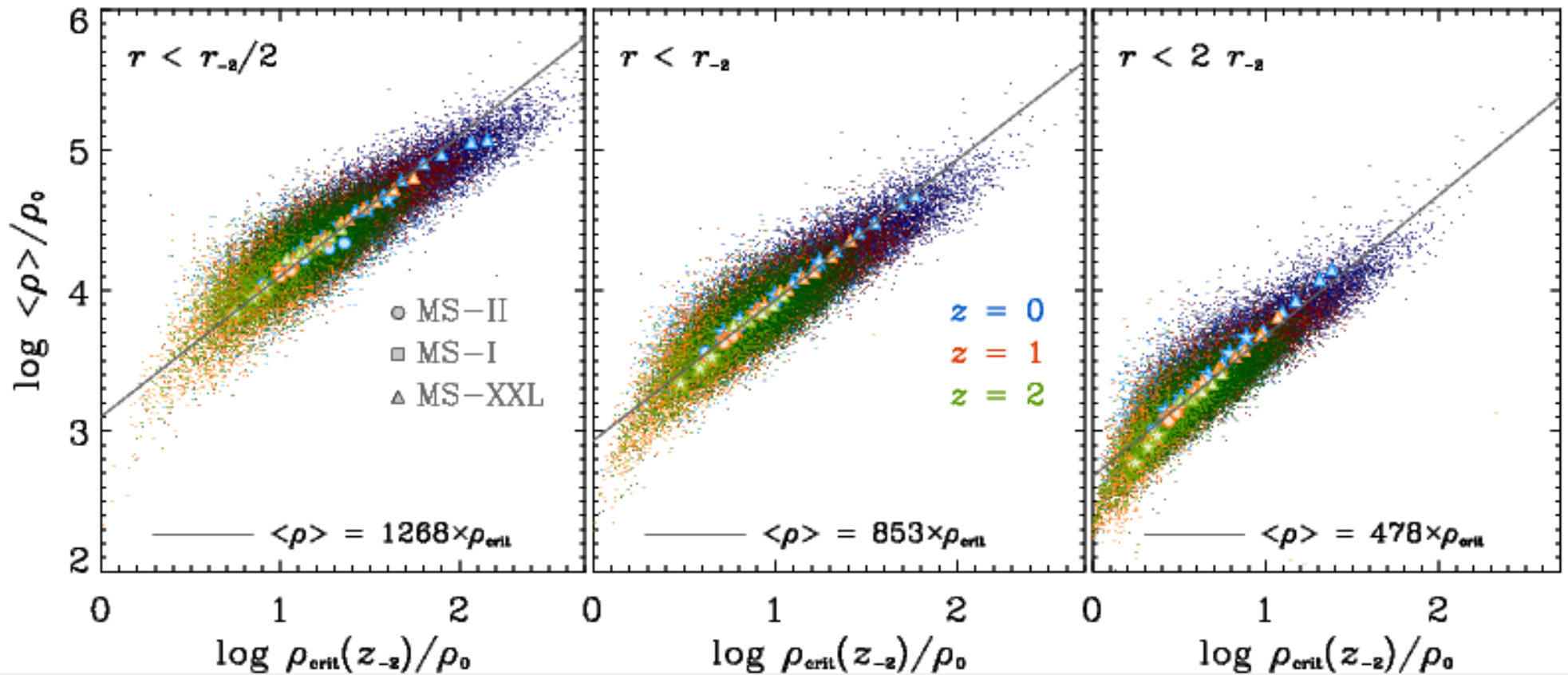
# Mass Profiles and Accretion Histories

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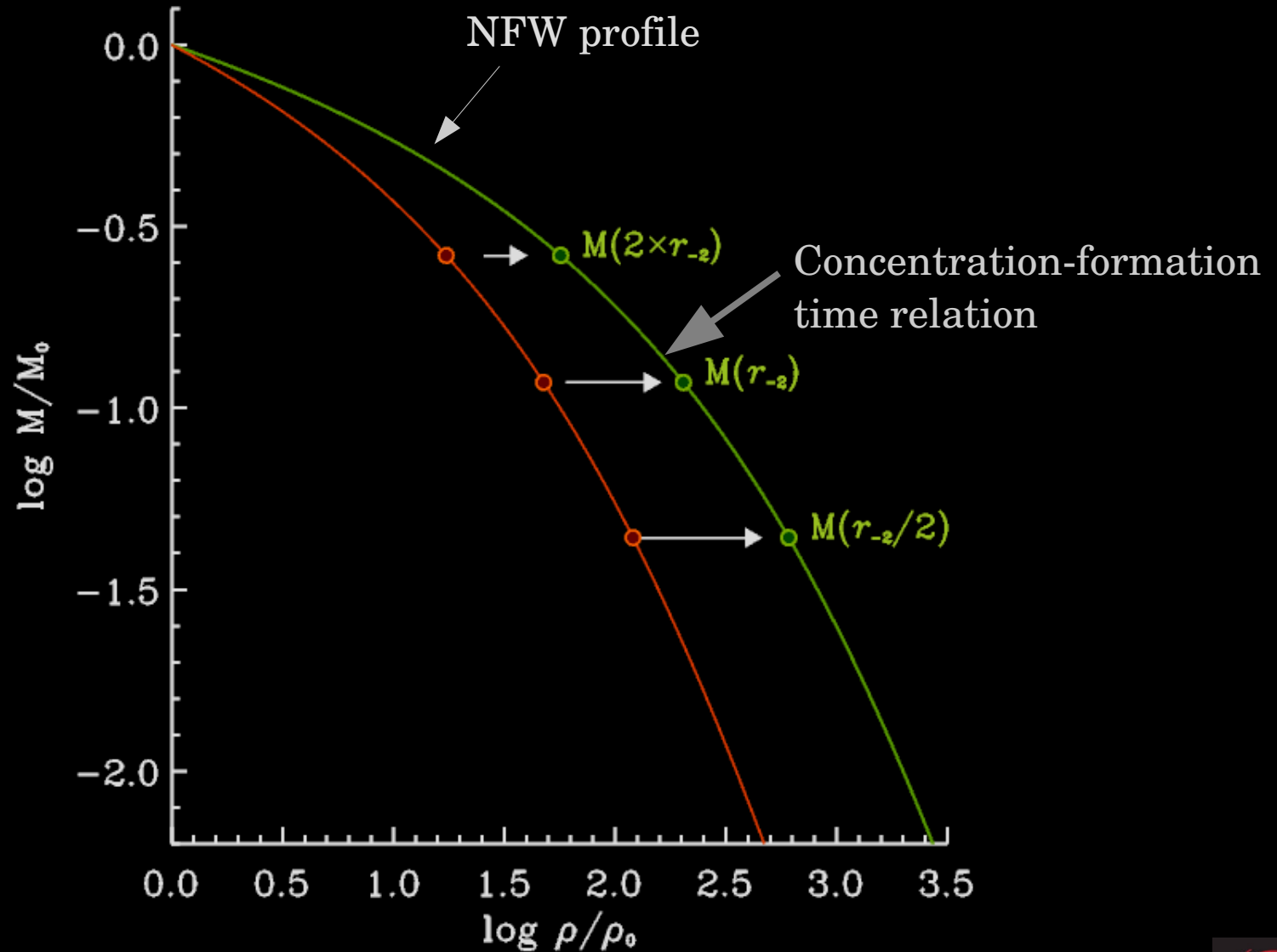


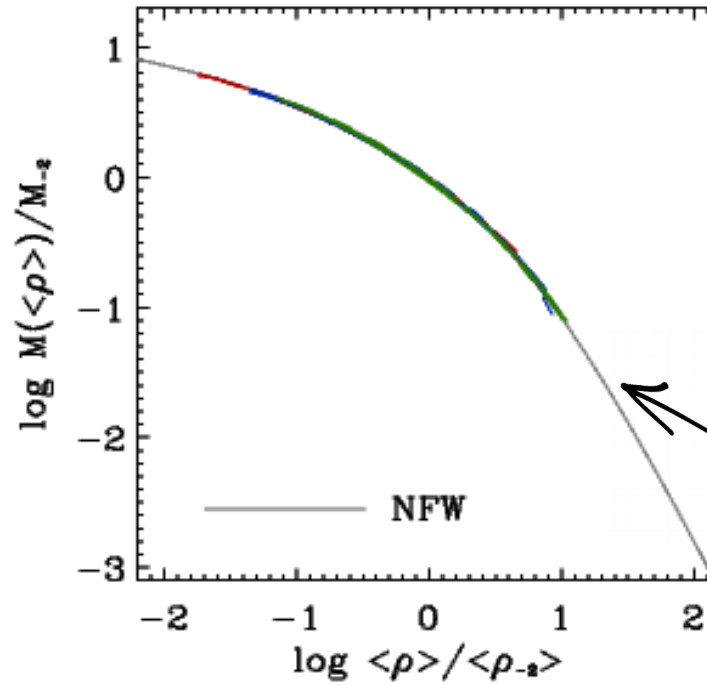
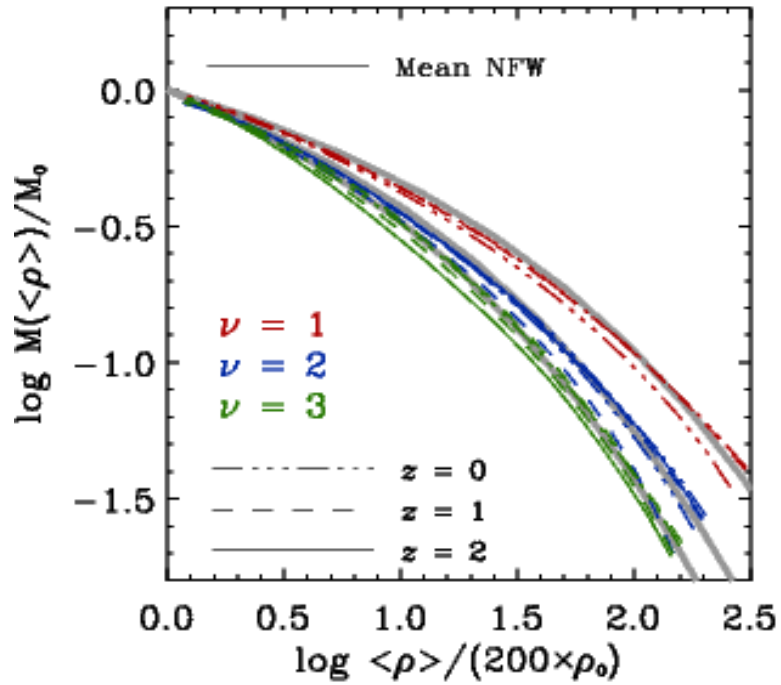


# CDM: concentration-formation time relation

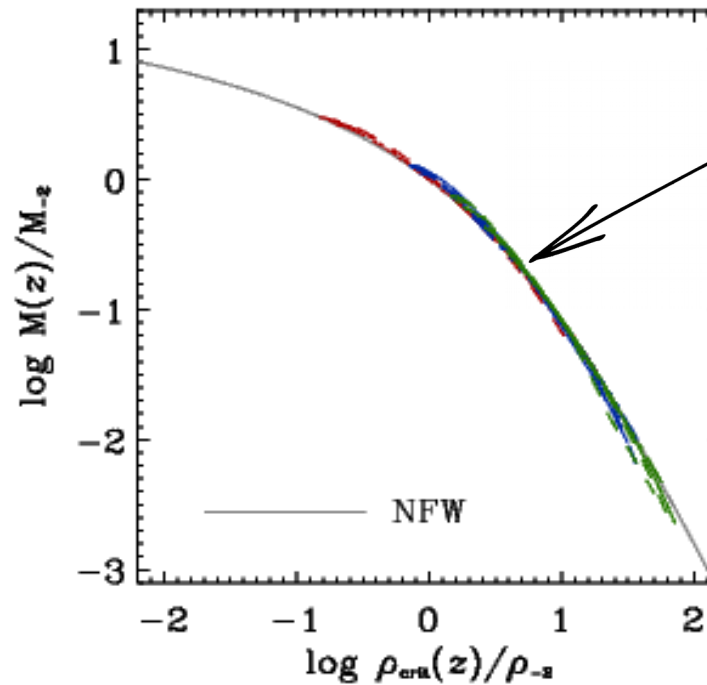
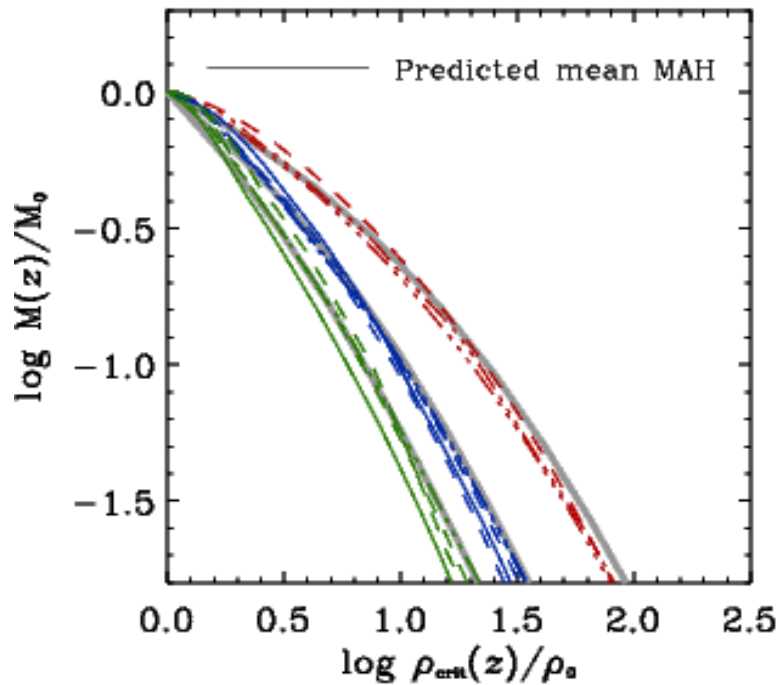


# Accretion Histories can be mapped to mass profiles...

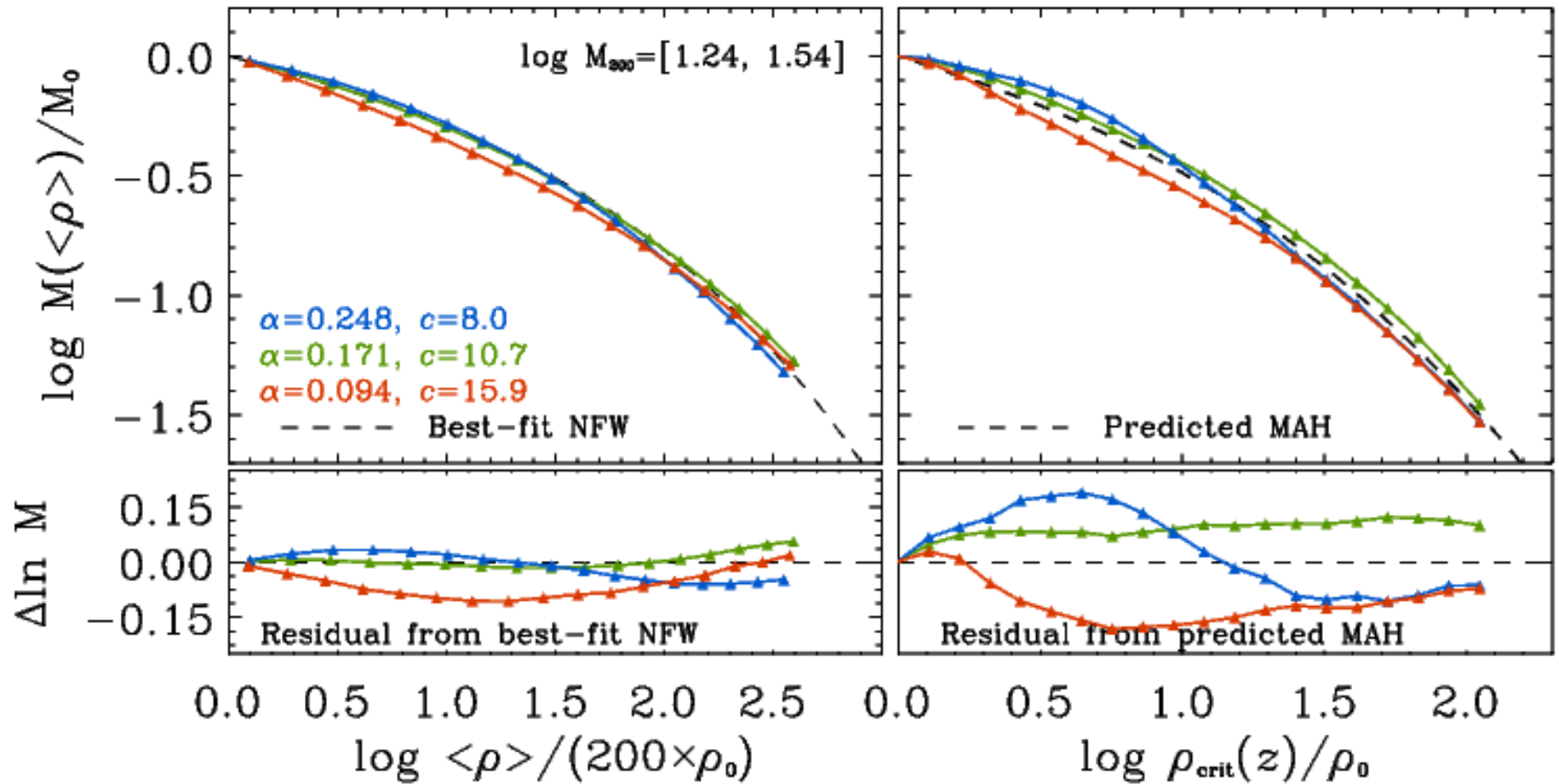




*Universal  
NFW profile*

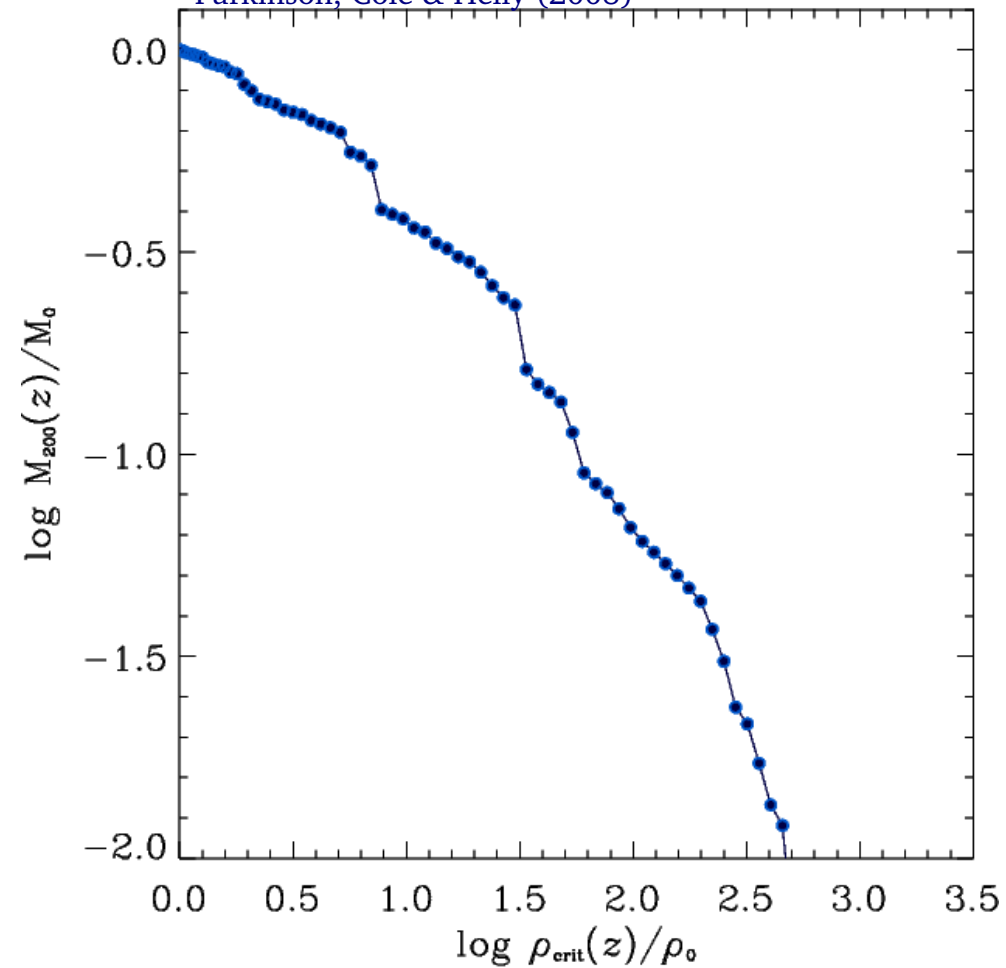


# Departure from universal NFW...



# Predicting mass profiles in practice

Parkinson, Cole & Helly (2008)



Extended Press-Schechter formalism (Bond *et al* 1991, Bower 1991) predicts the conditional probability

$$P(M_1, z_1 | M_0, z_0) dM_1$$

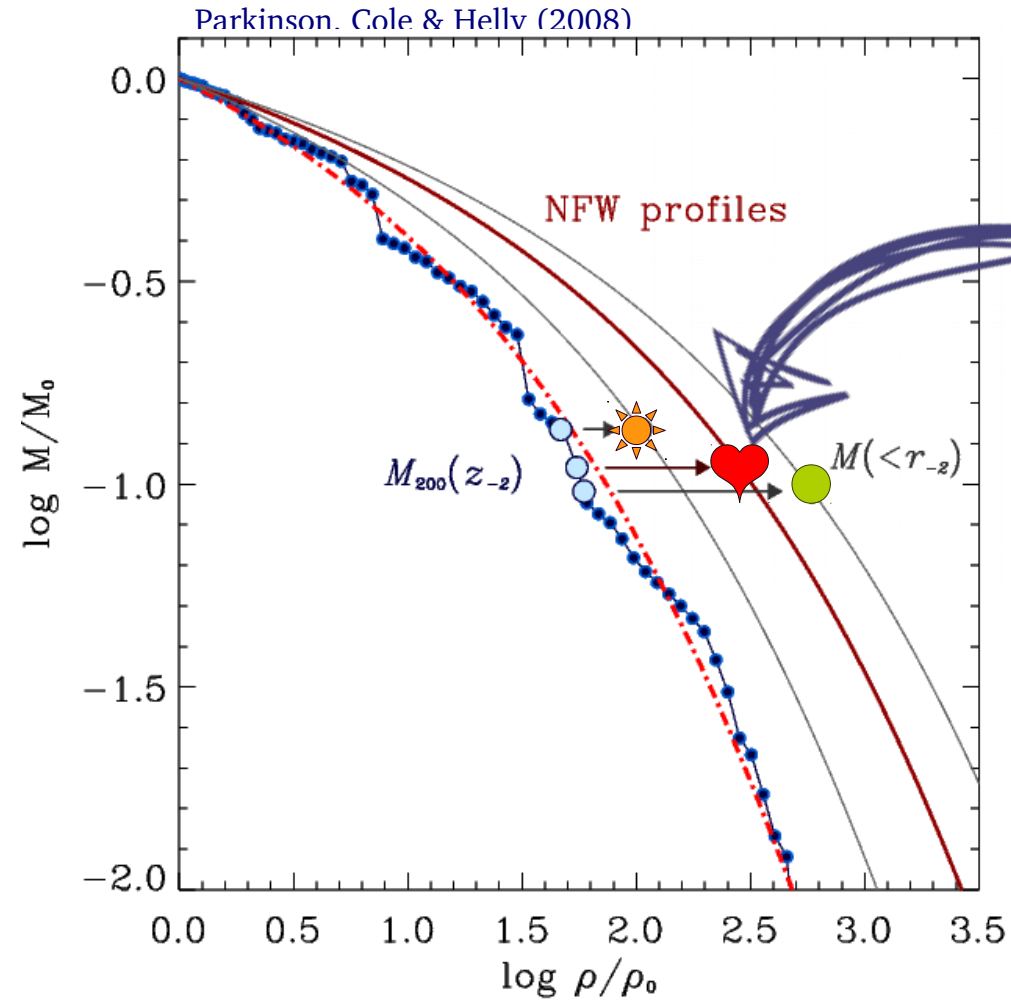
that a halo of mass  $M_0$  at  $z_0$  had a progenitor at  $z_1 > z_0$  in the range

$$[M_1, M_1 + dM_1]$$

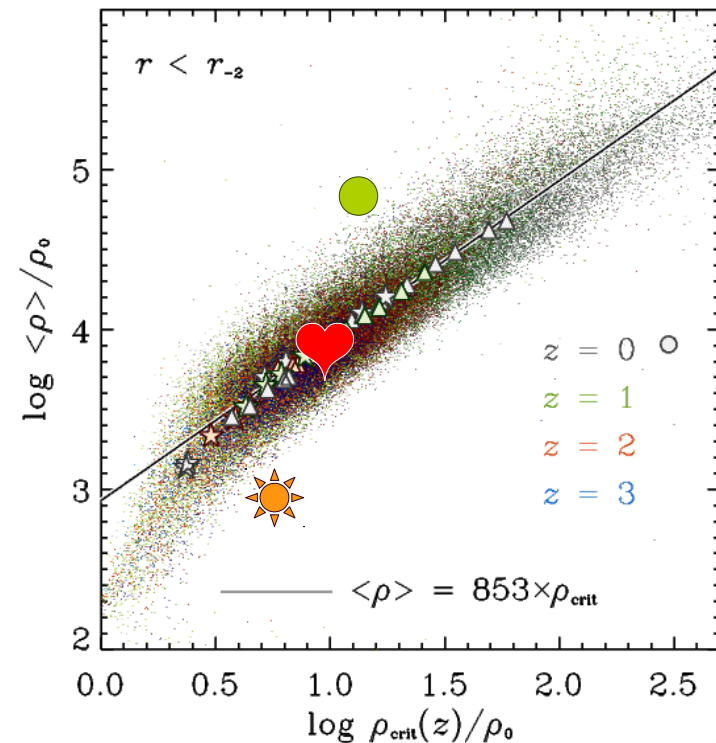
The “progenitor mass function” is the mass-weighted conditional probability

$$n(M_1, z_1 | M_0, z_0) dM_1 = \frac{M_0}{M_1} P(M_1, z_1 | M_0, z_0) dM_1$$

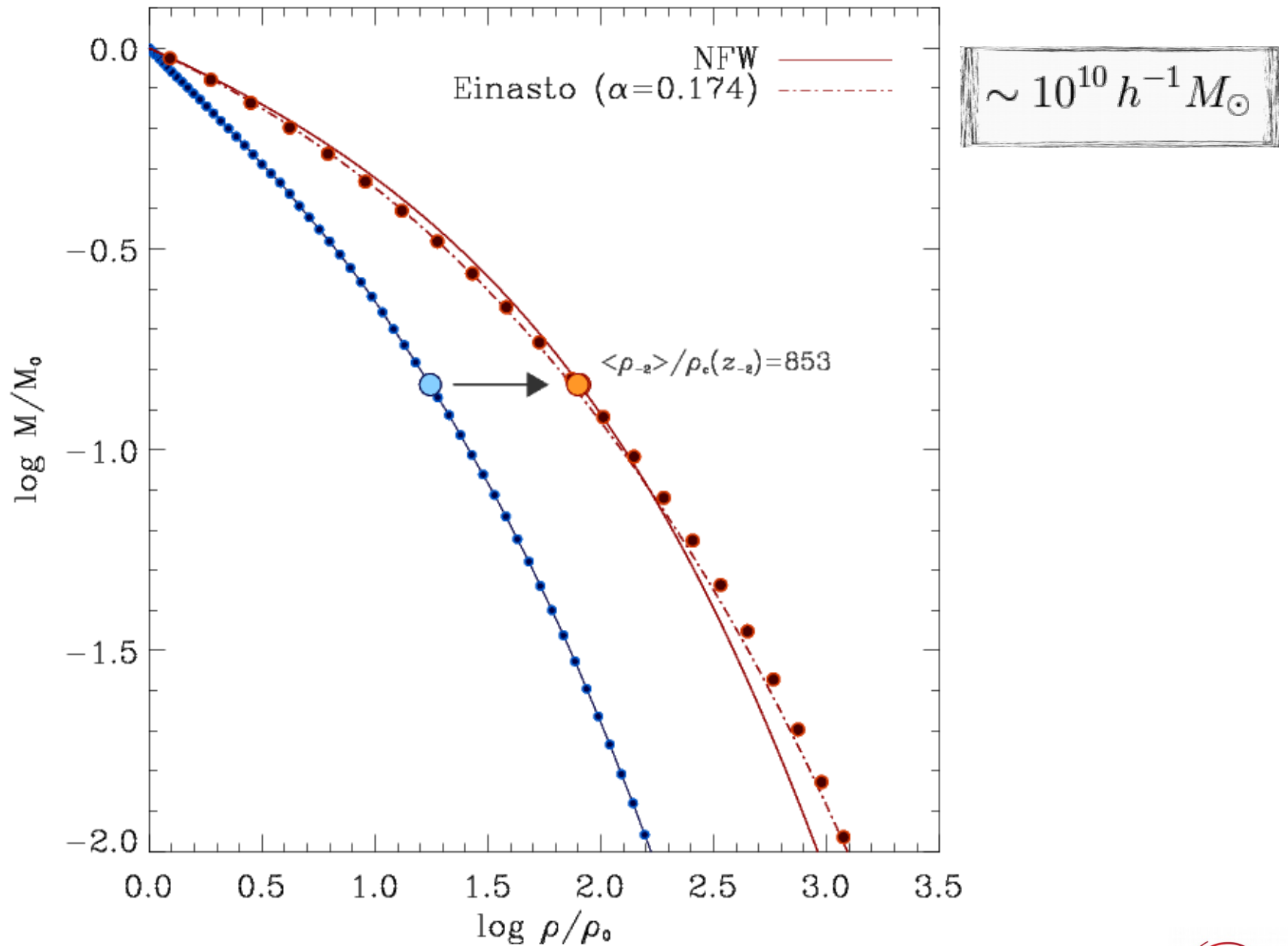
# Predicting mass profiles in practice



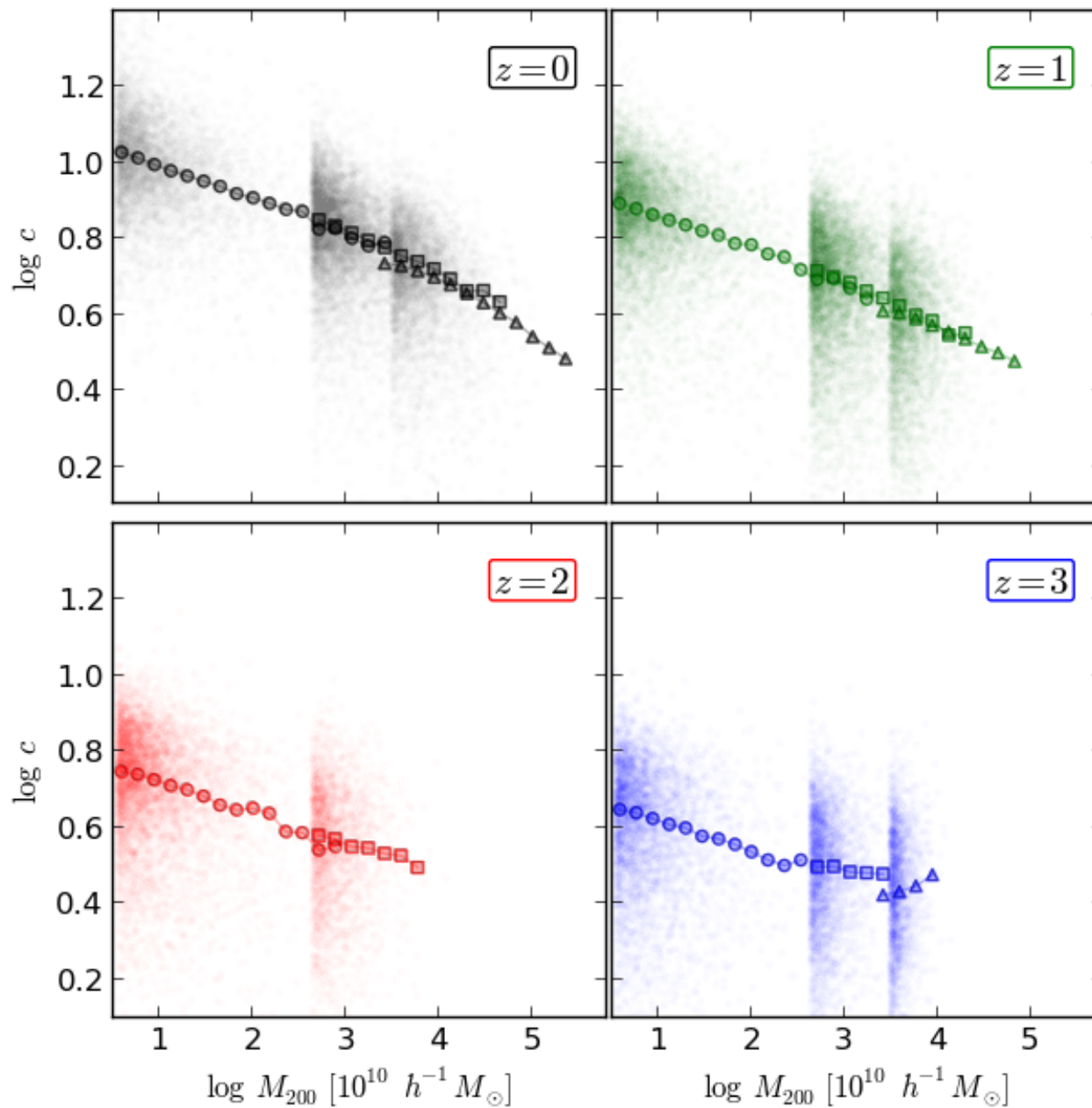
$$\langle \rho(r_{-2}) \rangle = 853 \times \rho_{\text{crit}}(z_{-2})$$



# Predicting mass profiles in practice

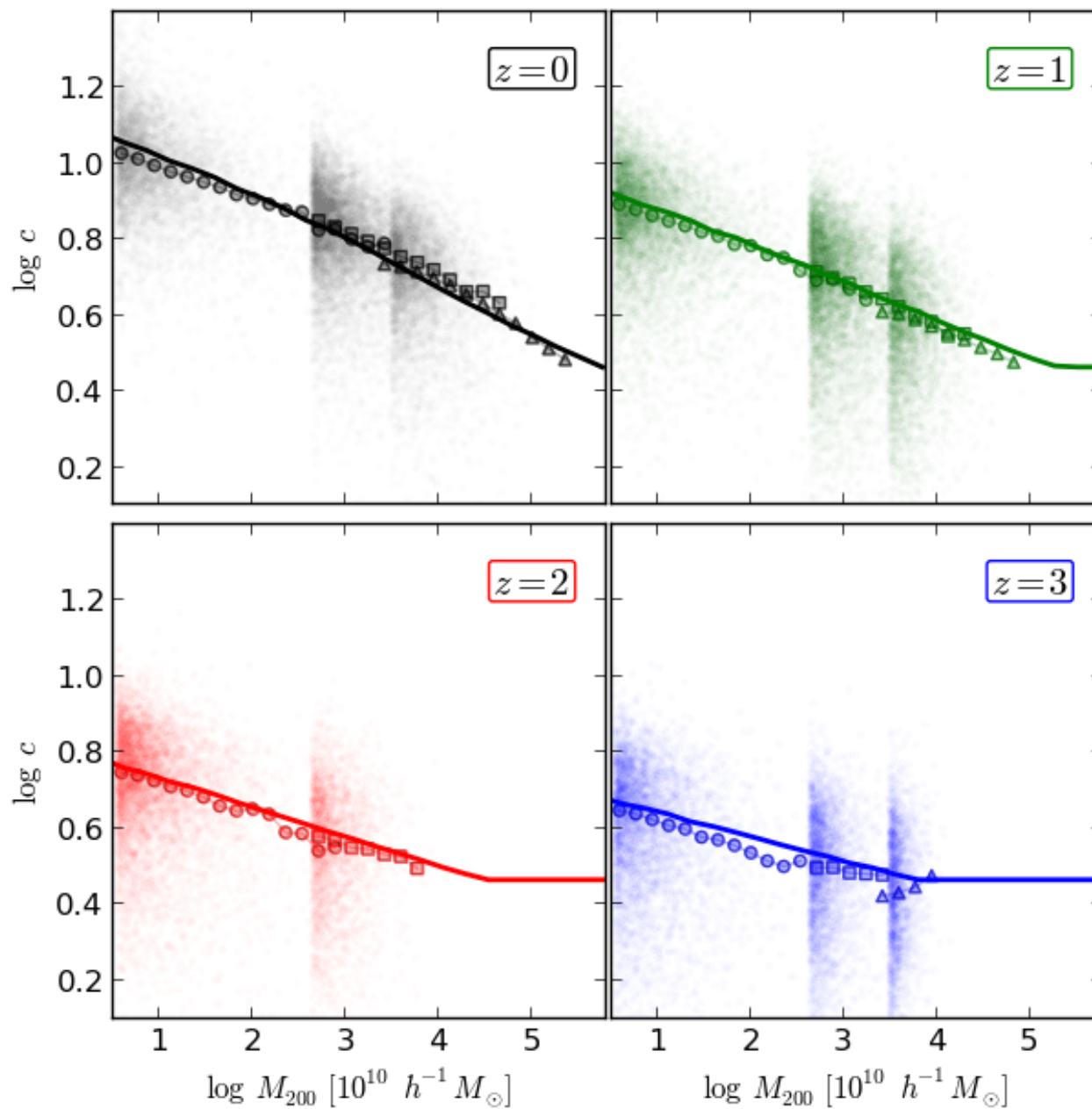


# A simple model for the $c(M,z)$ relation





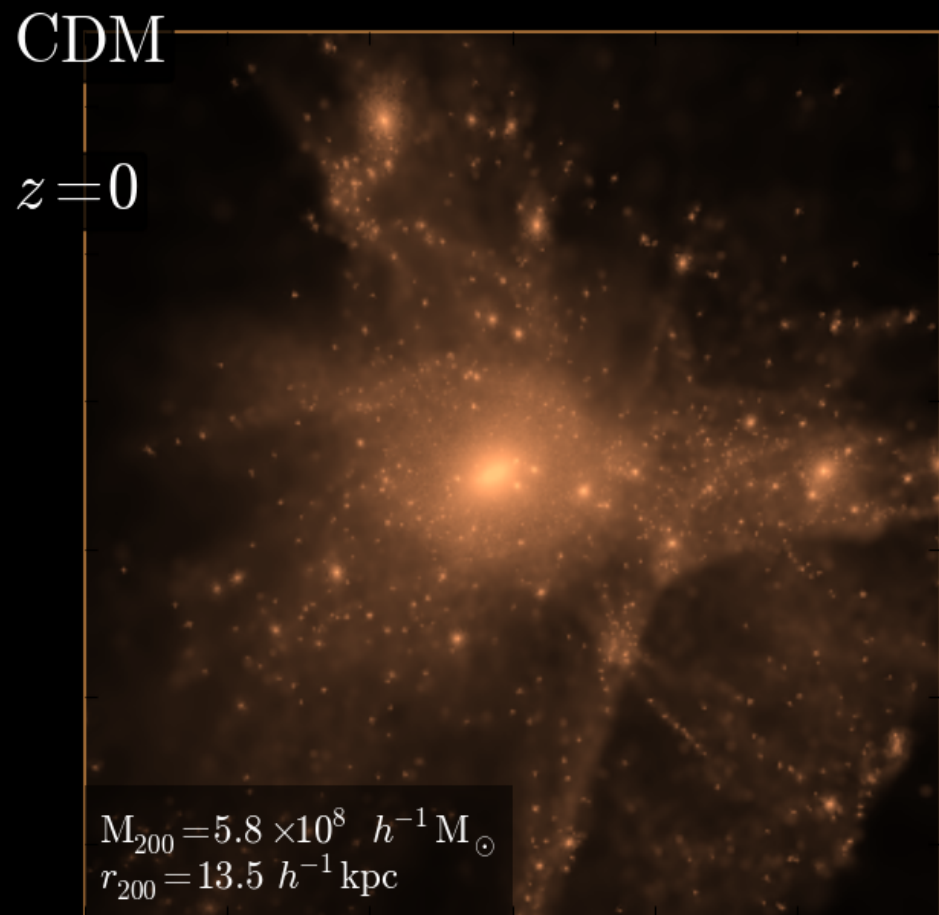
# A simple model for the $c(M,z)$ relation



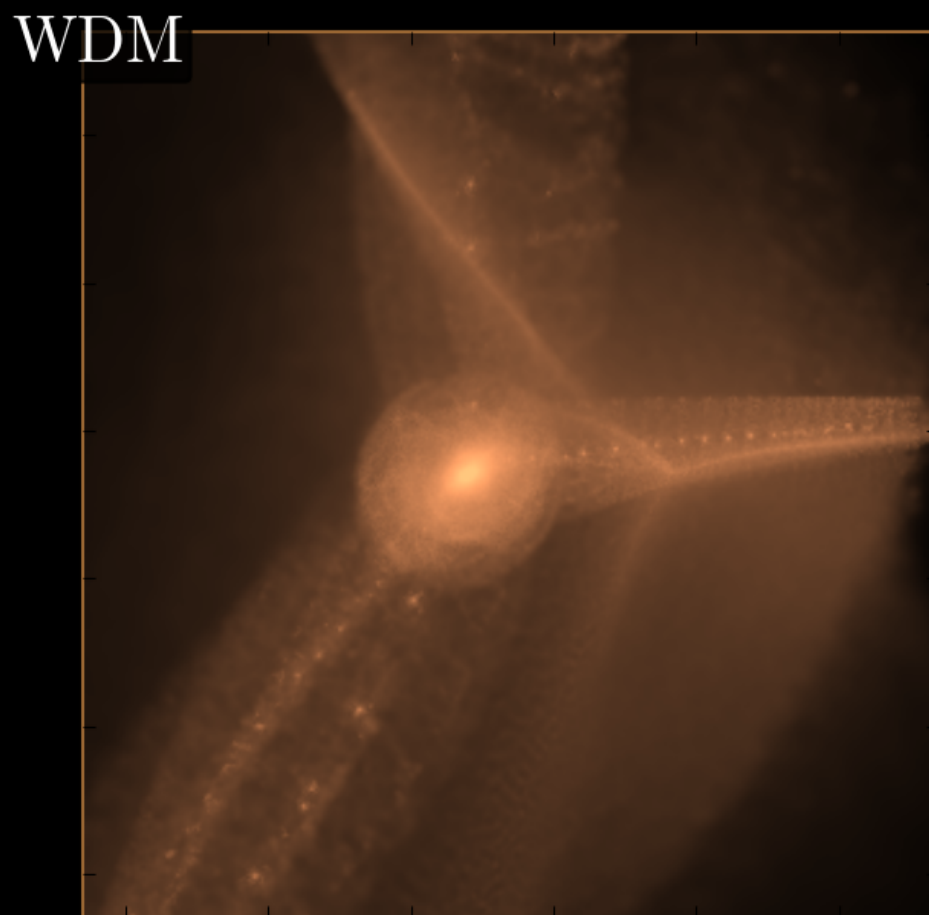
## CDM summary...

- CDM mass profiles and mass accretion histories are self-similar:
- Both follow an NFW profile and are linked by a simple scaling law:
$$\langle \rho(< r_{-2}) \rangle \approx 853 \times \rho_{\text{crit}}(z_{-2})$$
- This allows halo concentrations to be predicted from MAHs alone
- Mass accretion histories that depart from the self-similar form give rise to mass profiles that depart from NFW in a correlated way giving rise to a third structural parameter

# What about Warm DM...



Clumpy

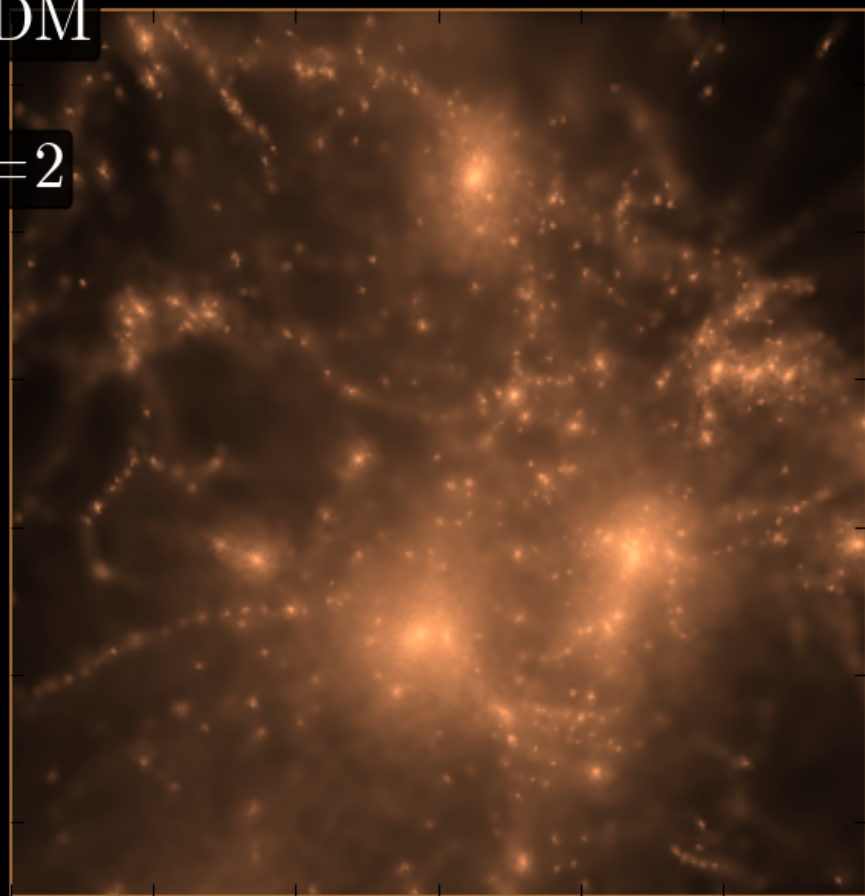


Smooth

# What about Warm DM...

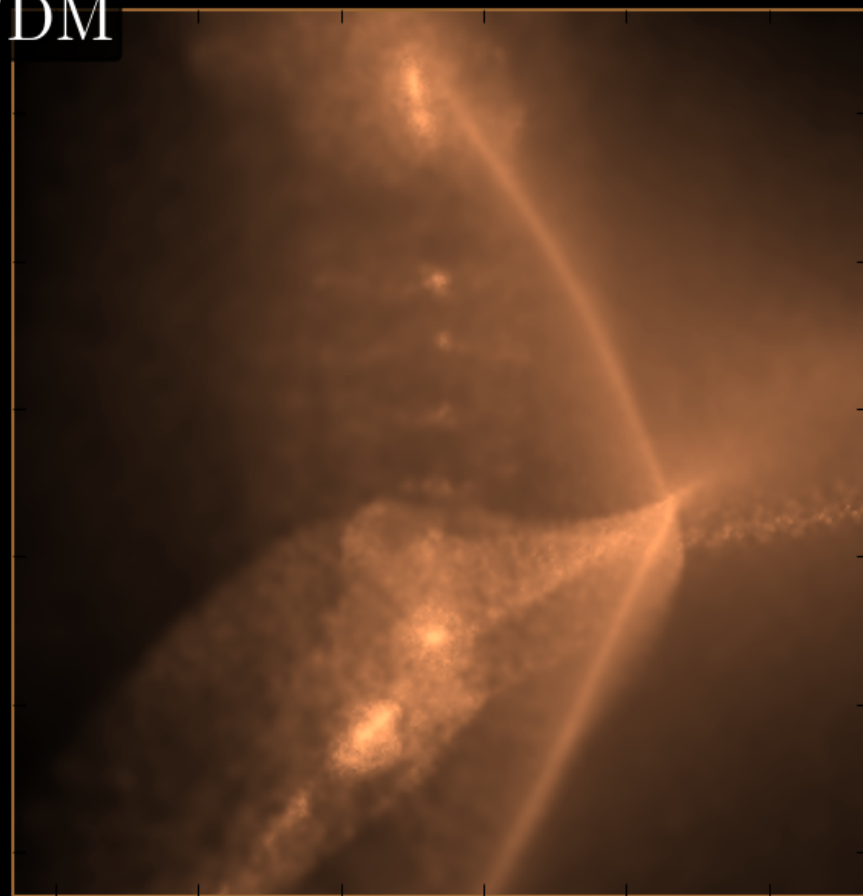
CDM

$z=2$



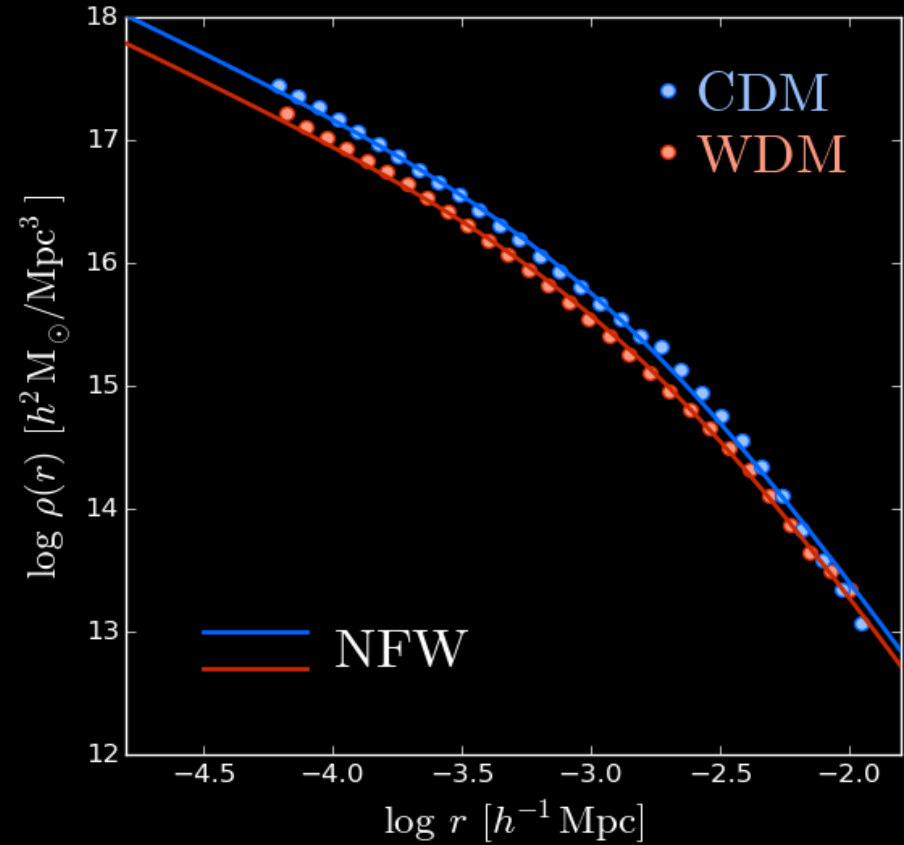
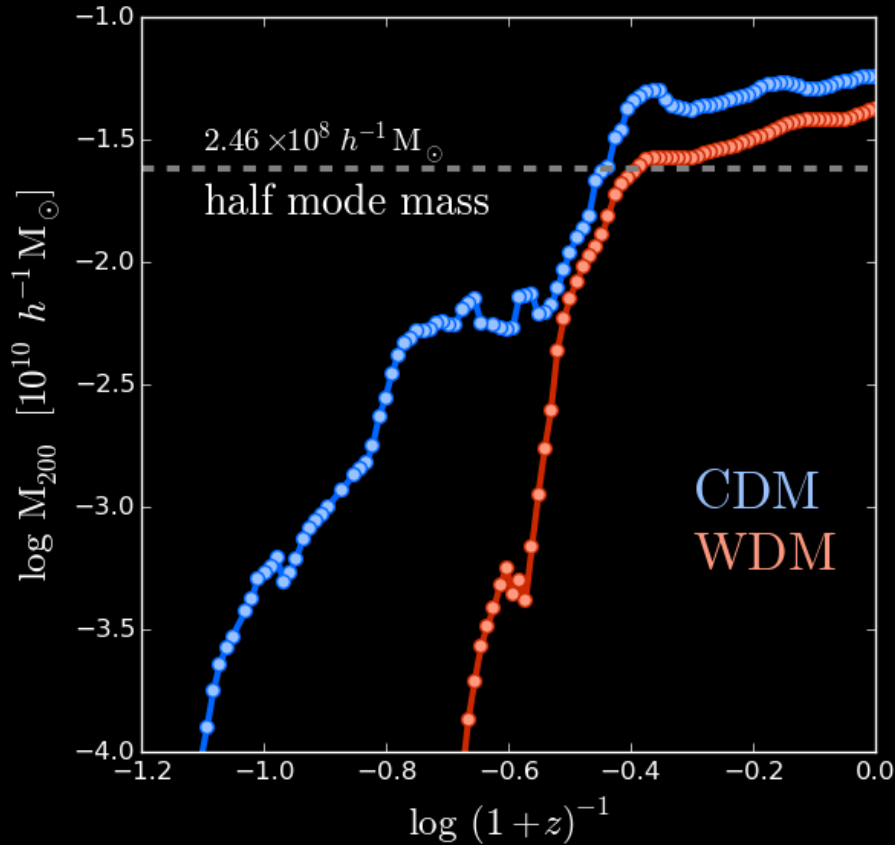
Clumpy

WDM

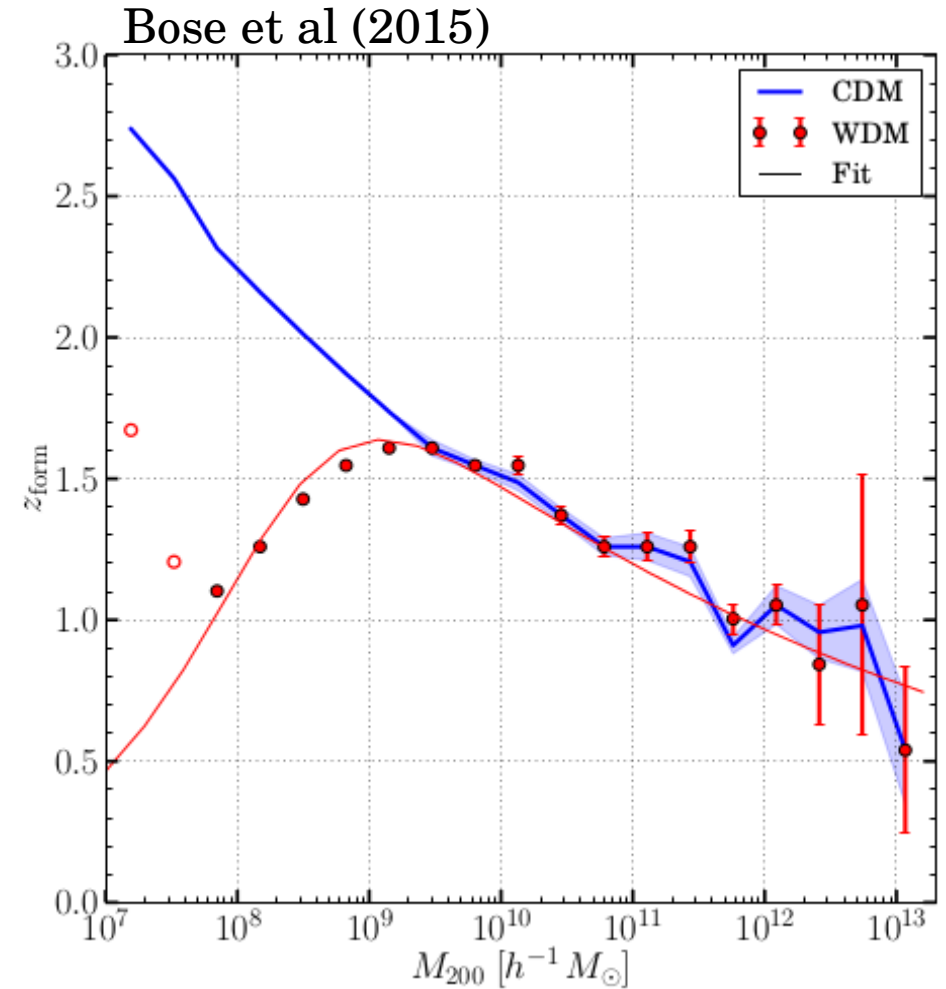
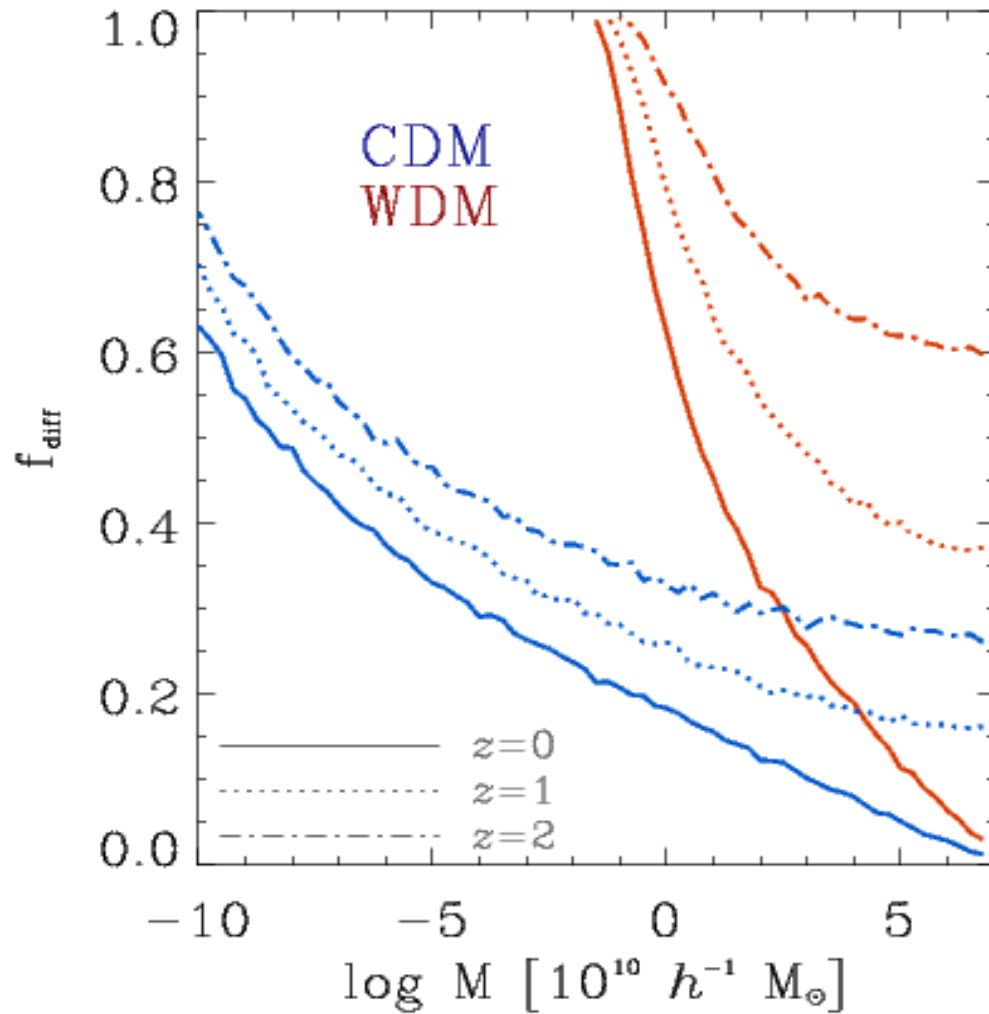


Smooth

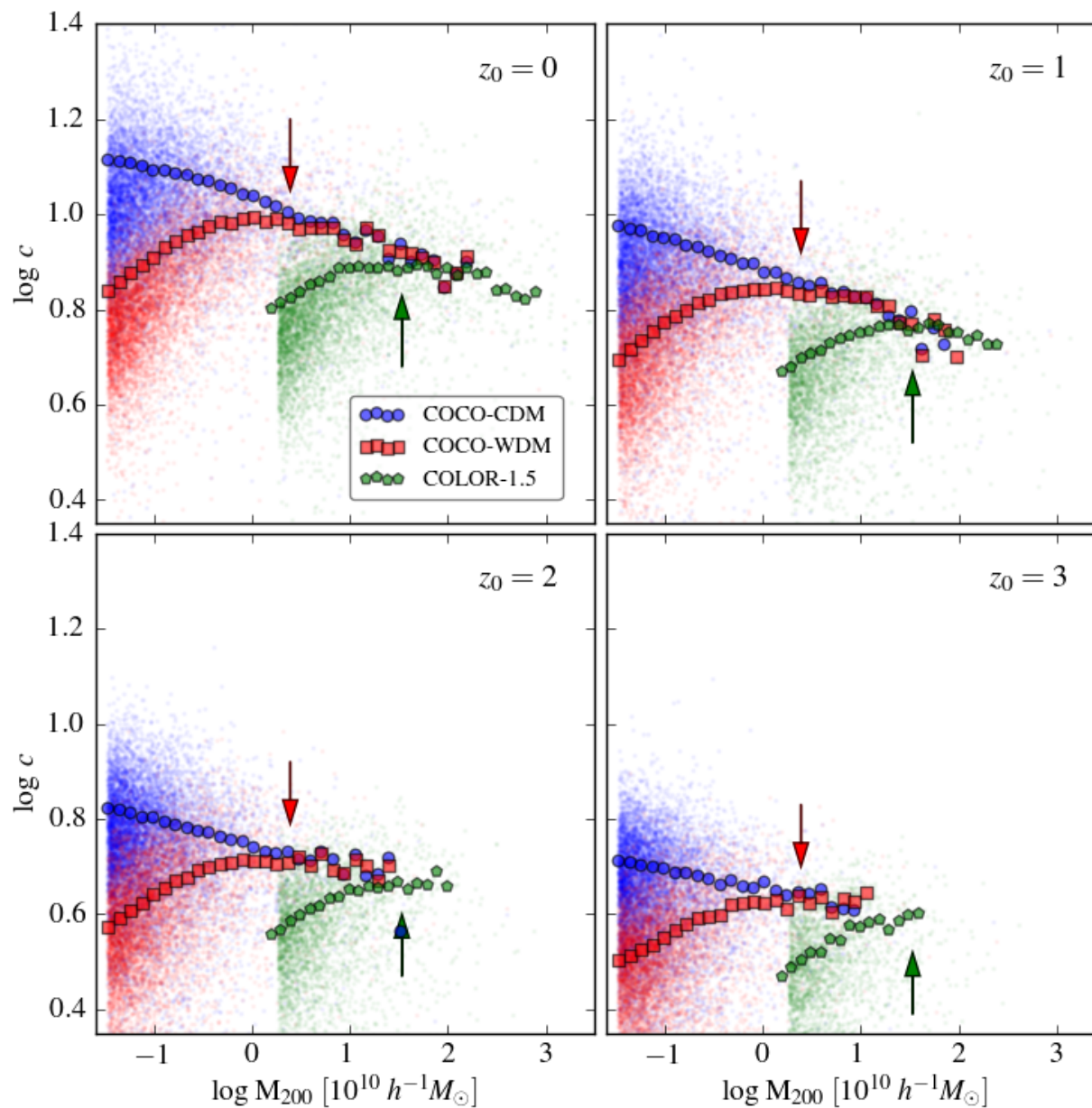
# What about Warm DM...



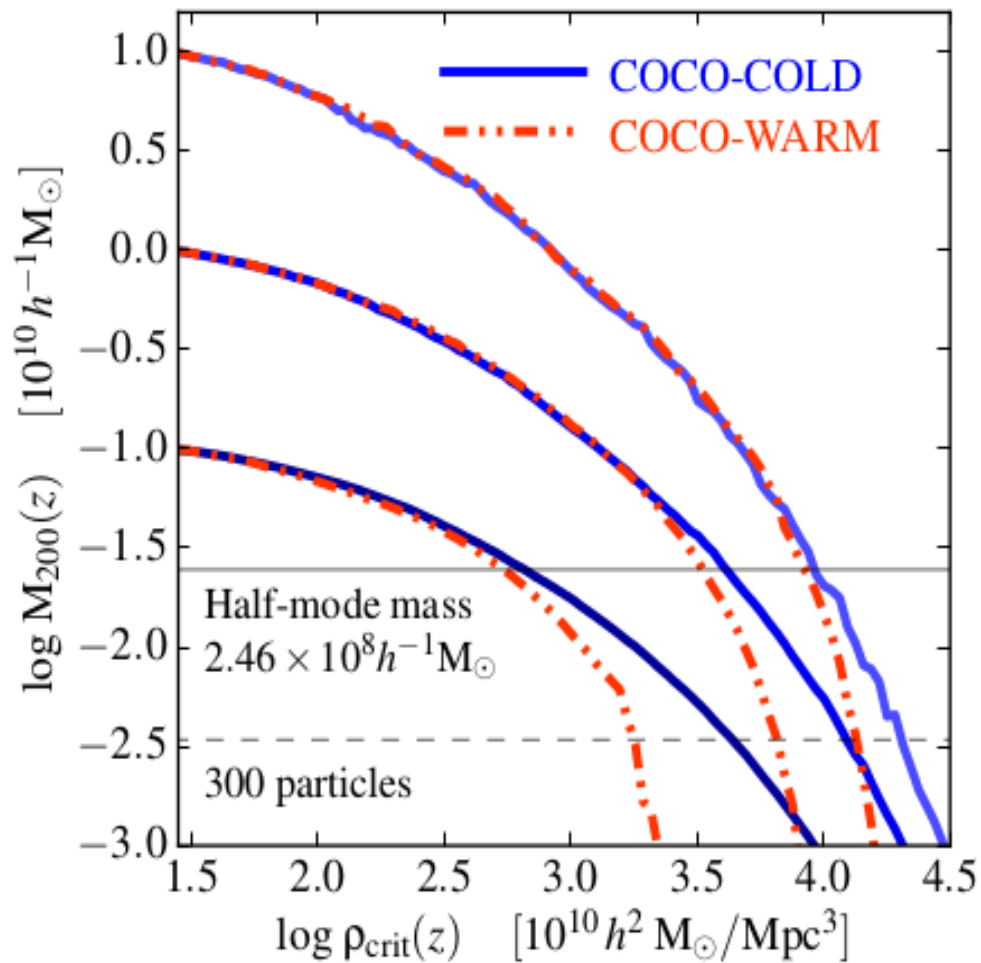
# Diffuse versus clumpy accretion



# WDM halos: the $c(M,z)$ relation

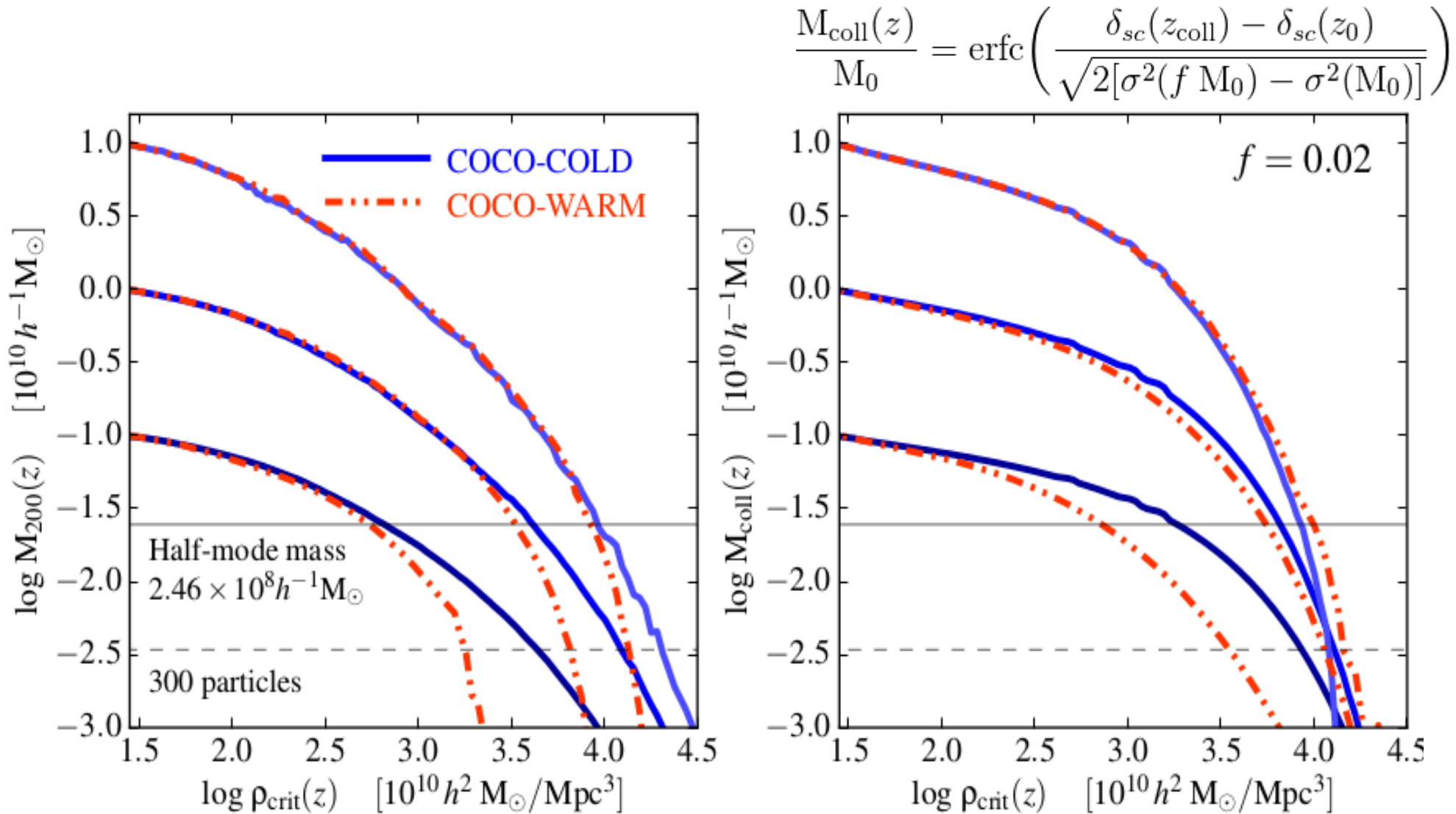


# Mass accretion histories of WDM halos

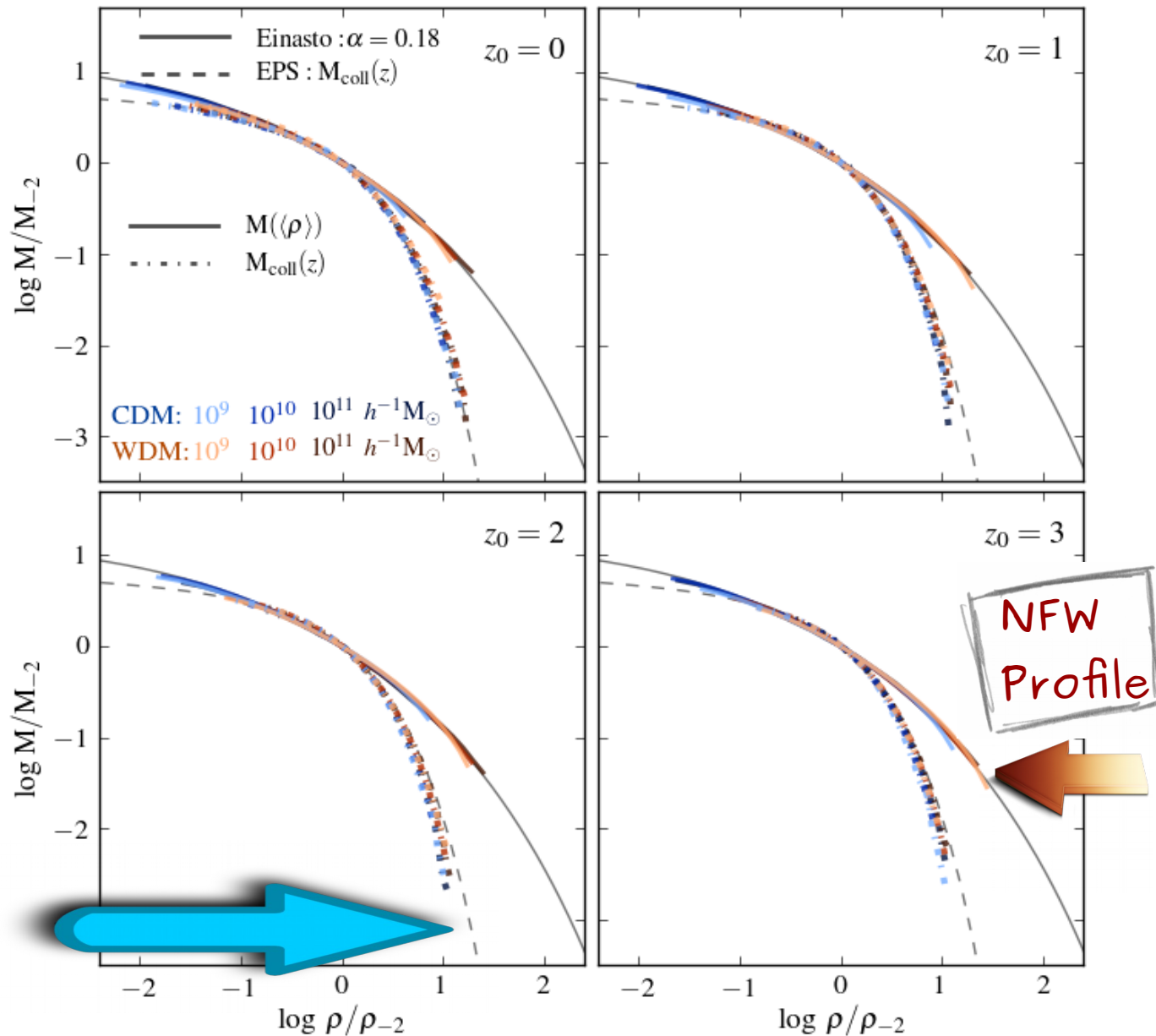




# Mass accretion histories of WDM halos

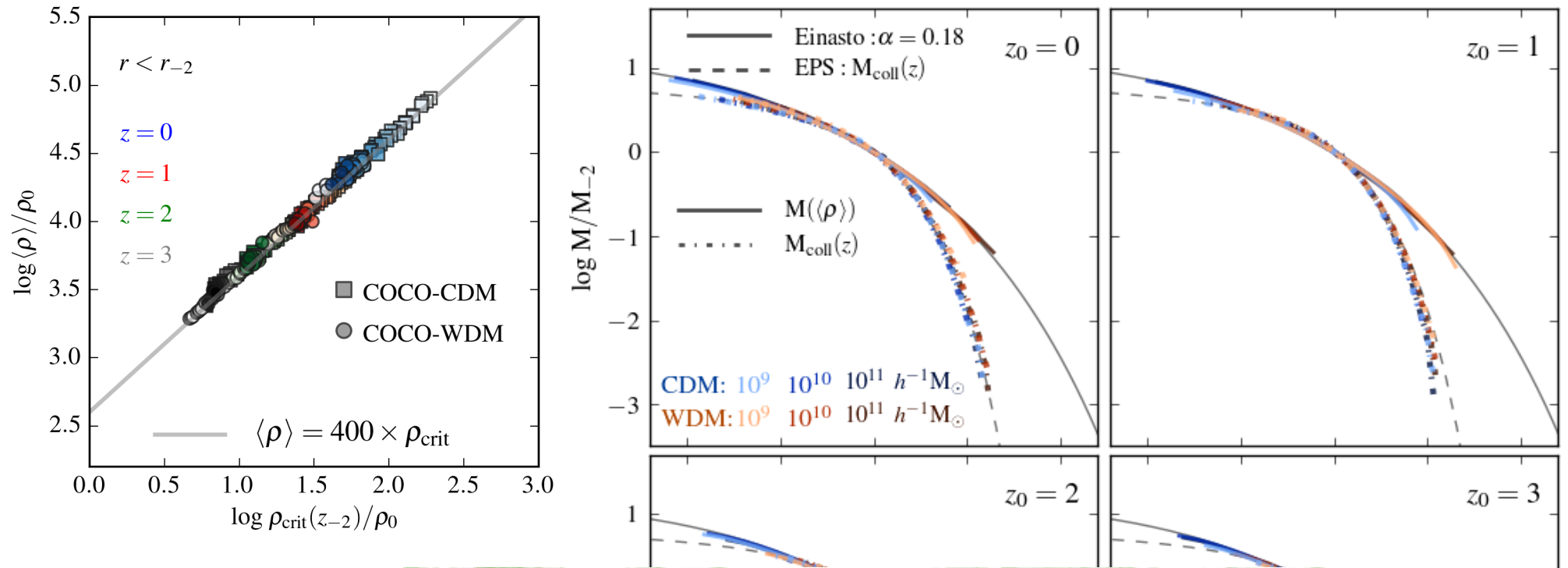


# Mass accretion histories of CDM & WDM halos



$$\frac{M_{\text{coll}}(z)}{M_0} = \text{erfc} \left( \frac{\delta_{sc}(z_{\text{coll}}) - \delta_{sc}(z_0)}{\sqrt{2[\sigma^2(f M_0) - \sigma^2(M_0)]}} \right)$$

# Mass accretion histories of CDM & WDM halos



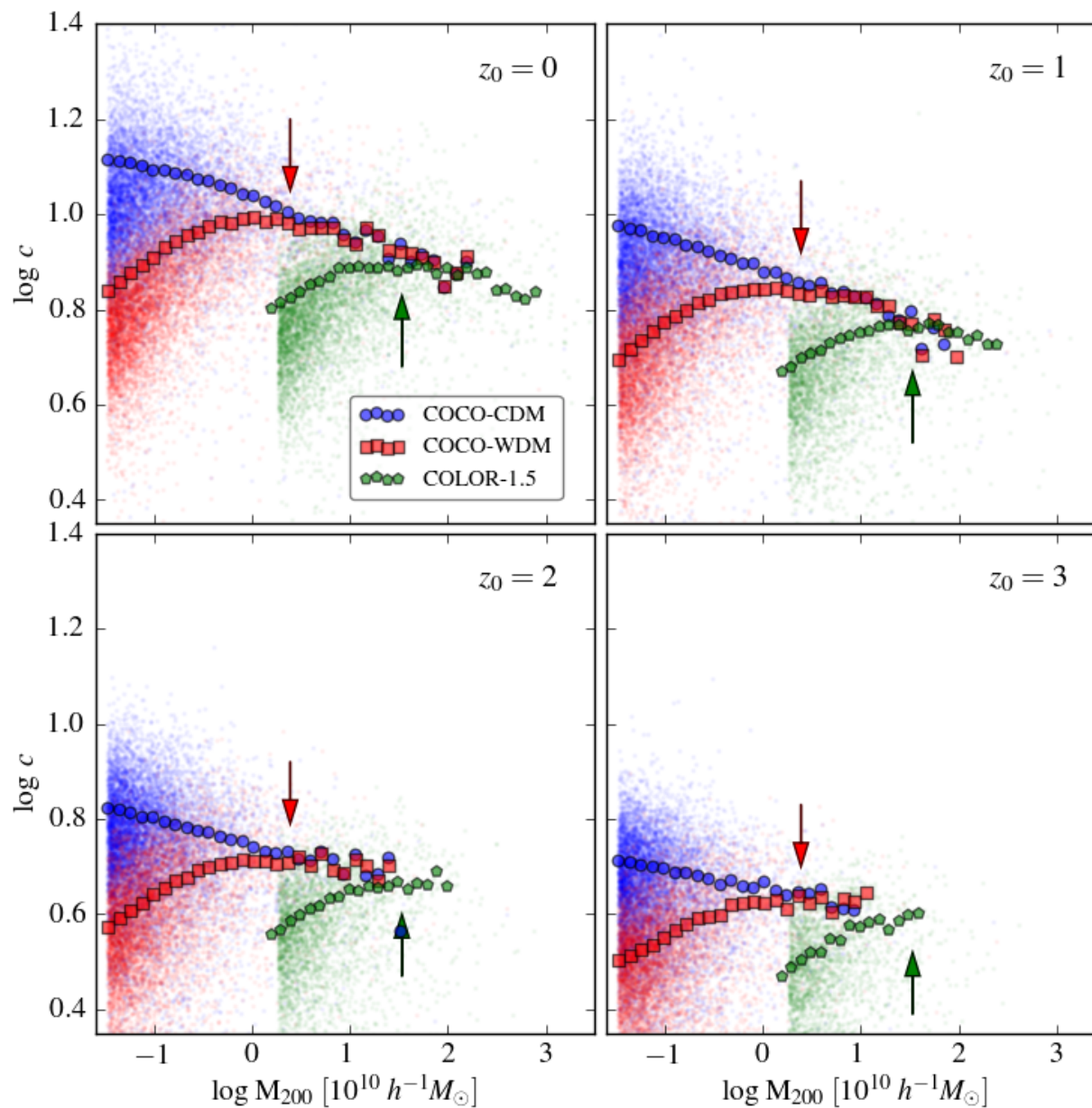
## Analytic model

$$\langle \rho_{-2} \rangle = A \times \rho_{\text{crit}}(z_{-2})$$

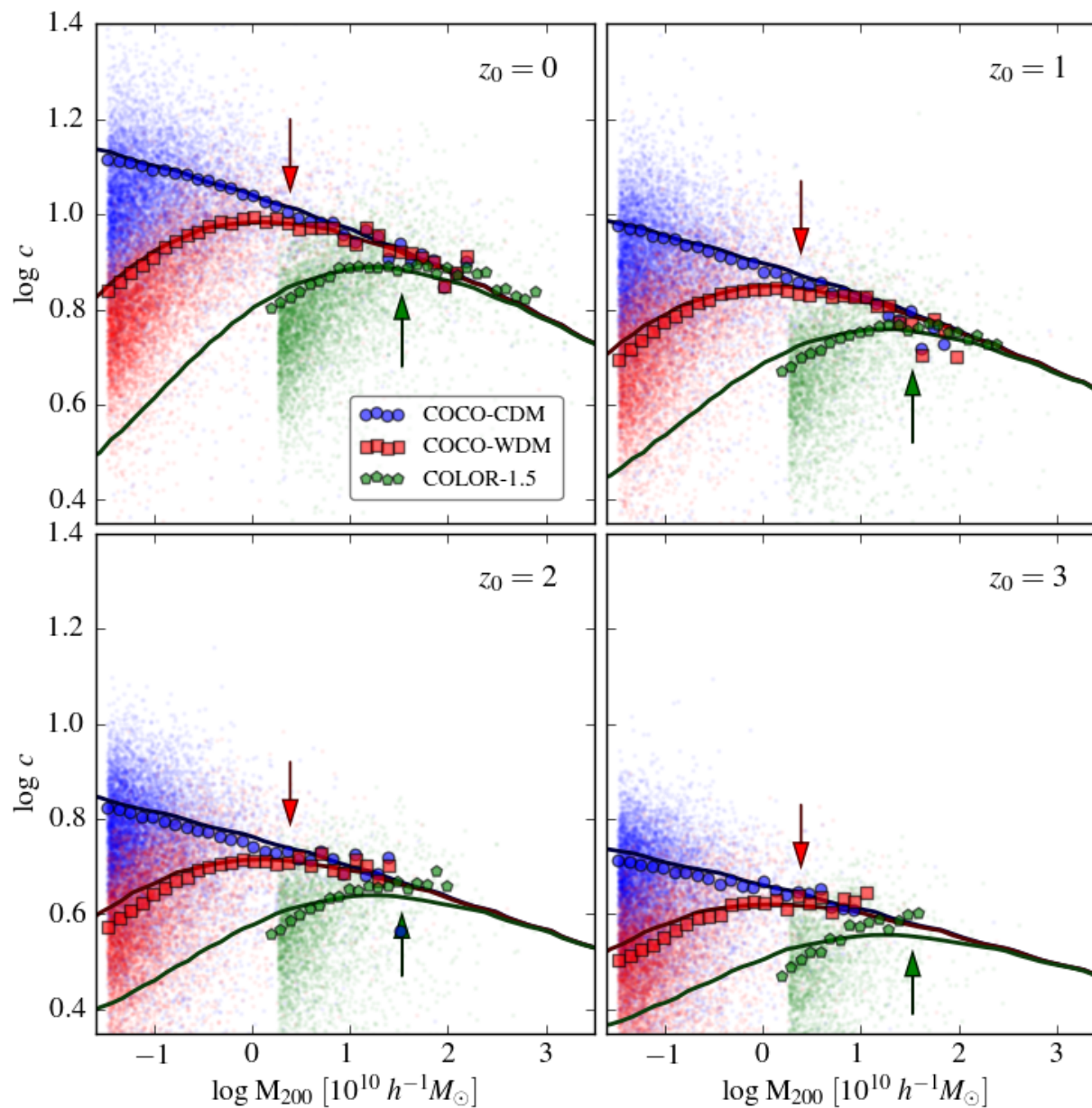
&

$$\frac{M_{-2}}{M_0} = \text{erfc} \left( \frac{\delta_{\text{sc}}(z_{-2}) - \delta_{\text{sc}}(z_0)}{\sqrt{2[\sigma^2(f M_0) - \sigma^2(M_0)]}} \right)$$

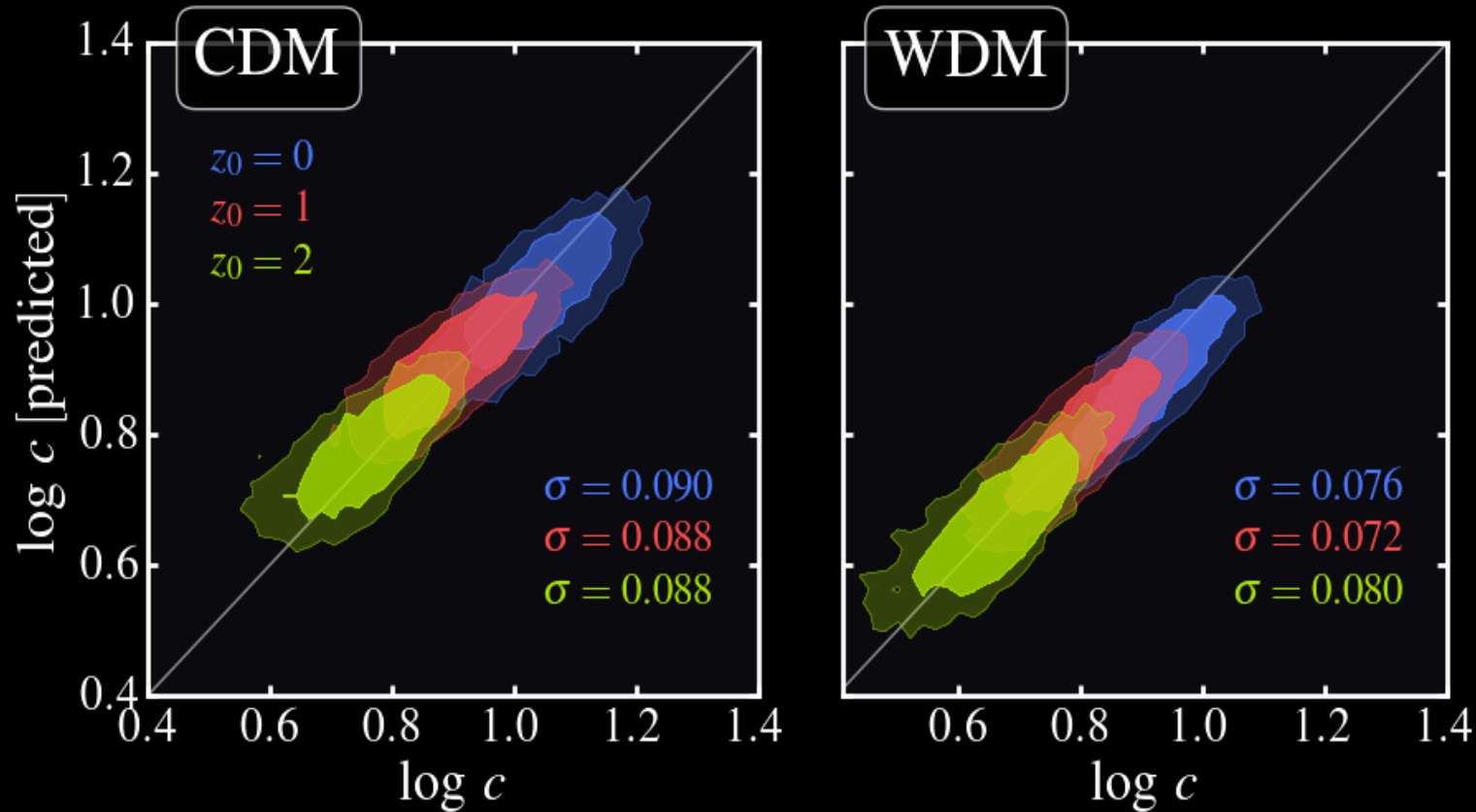
# WDM halos: the $c(M,z)$ relation



# WDM halos: the $c(M,z)$ relation



# Works for individual halos...

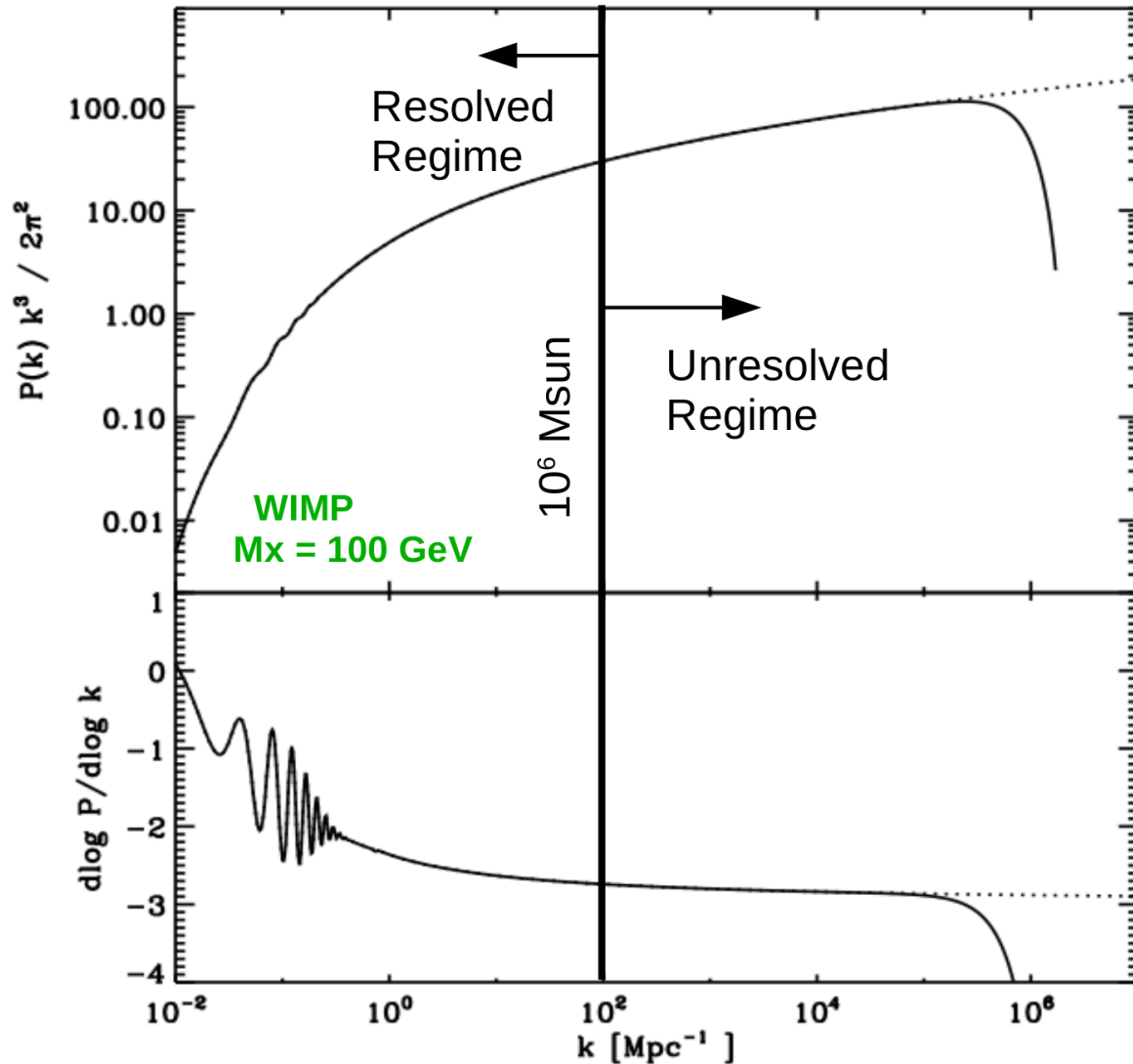


Individual halo concentrations predicted within  $\sim 20\%$

# Structure formation for neutralino DM

A simulation of the full DM hierarchy might require  $10^{21}$  particles

14 orders of magnitude in mass



Slides from Raul Angulo  
(Based on Angulo, Hahn,  
Ludlow & Bonoli, 2017)

Collapse times scale like...

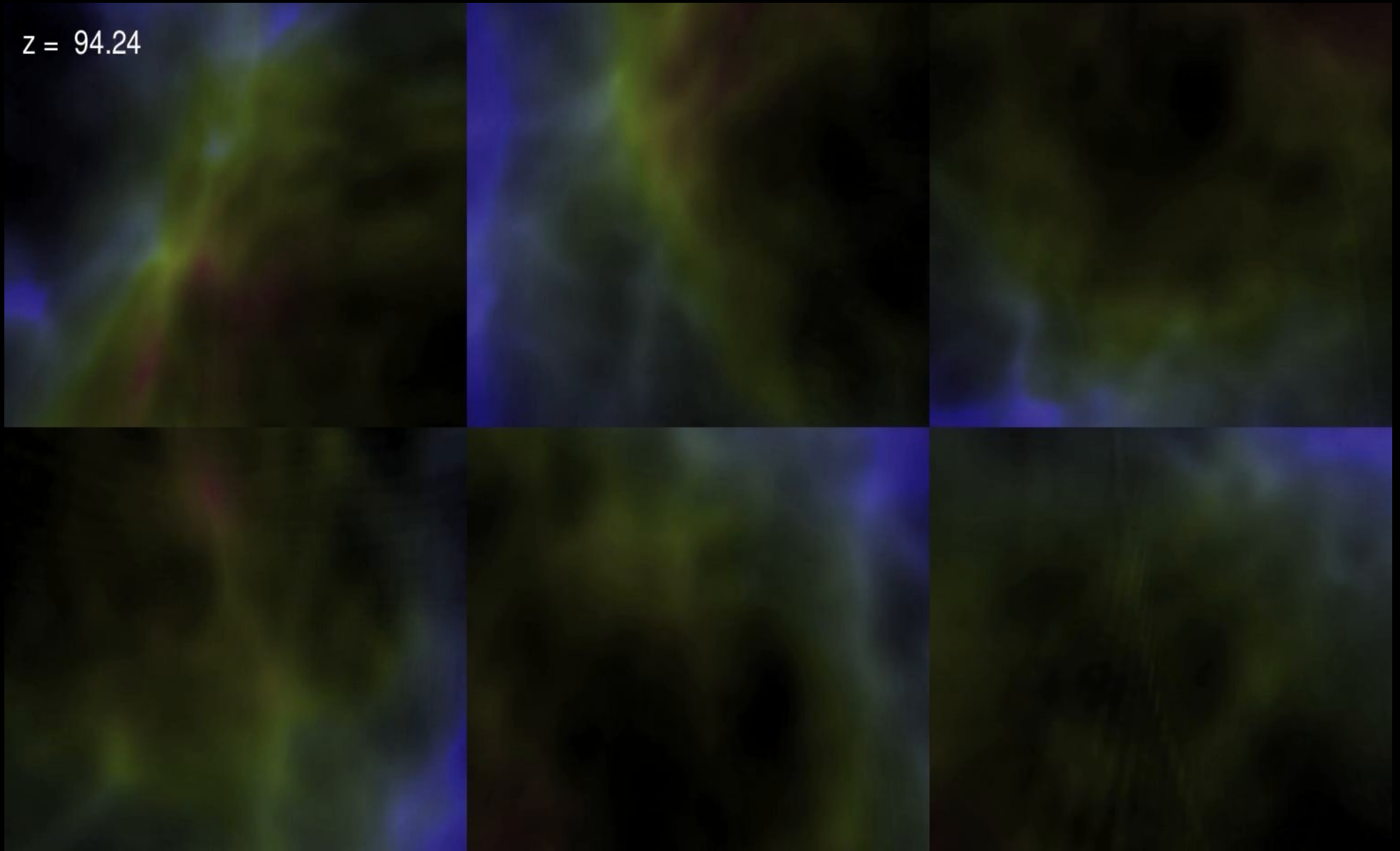
$$\sigma^2 \propto \int k^2 P(k) dk$$

$$= \int k^{2-n} dk \propto k^{3-n}$$

Lowest masses in CDM have  $n \sim 3$ ...

⇒ all structure collapses at once

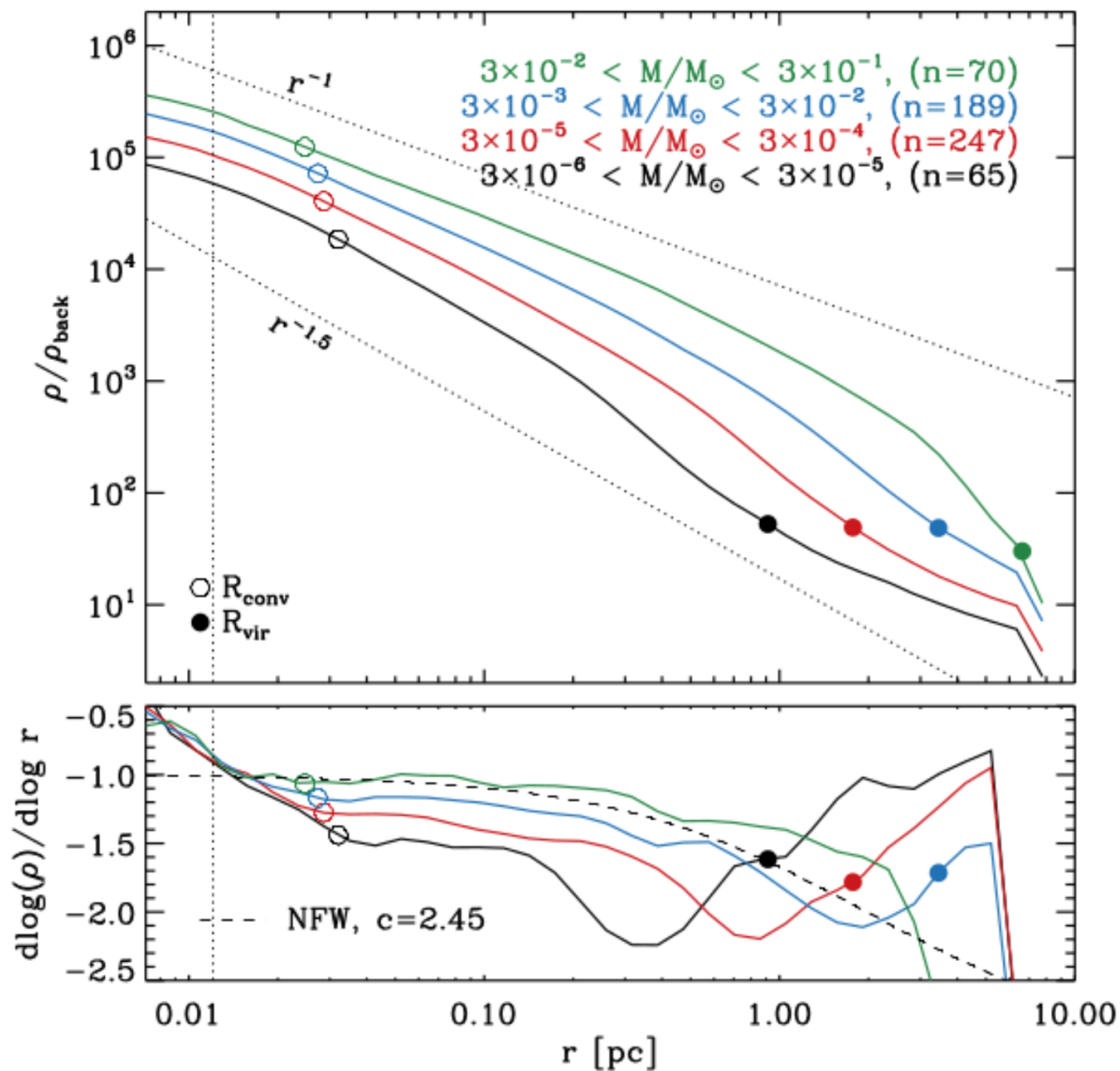
$z = 94.24$





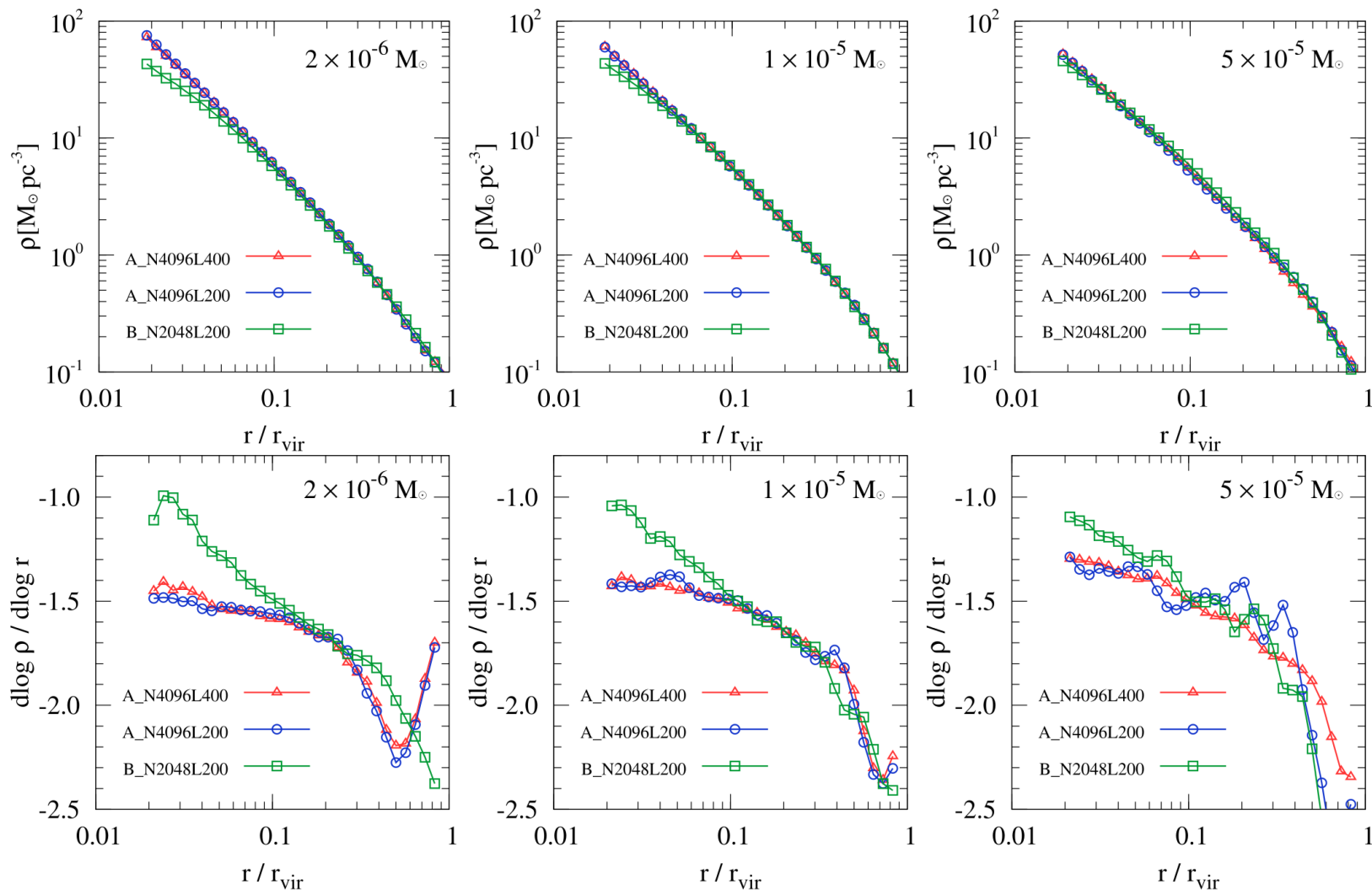
# Structure formation at the free streaming mass

The density profile of microhalos is steeper than a NFW



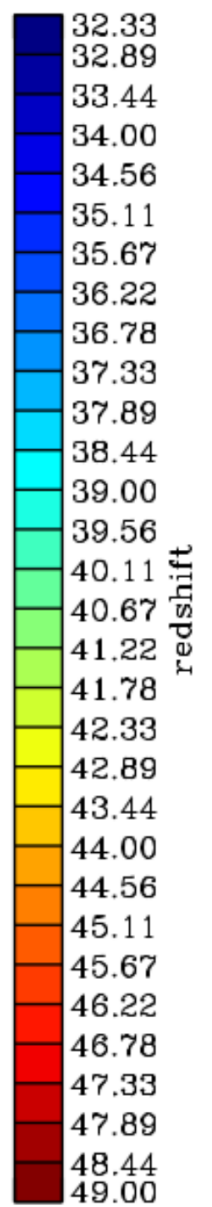
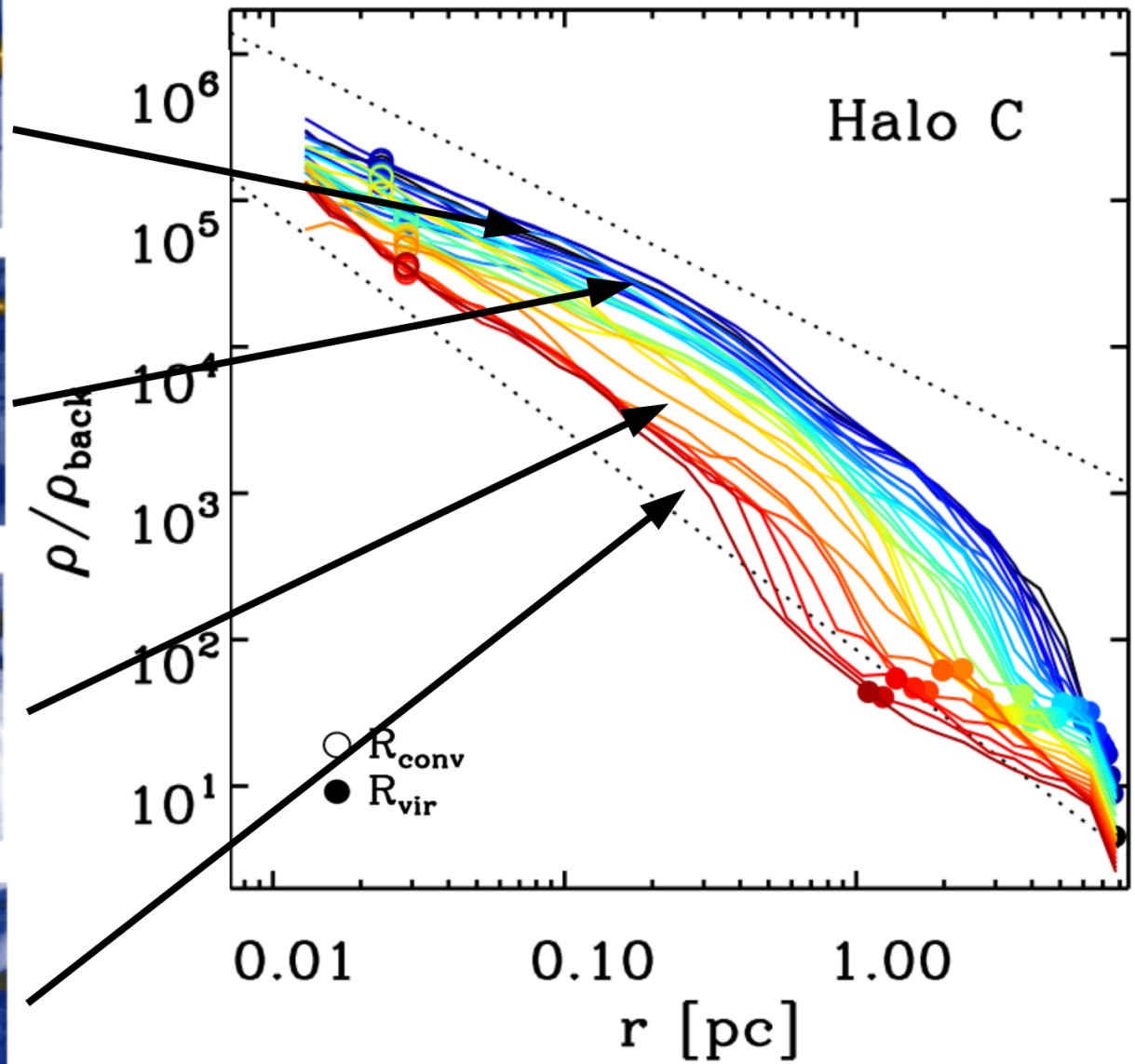
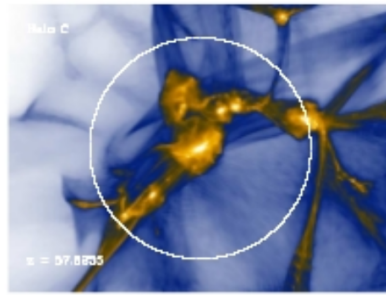
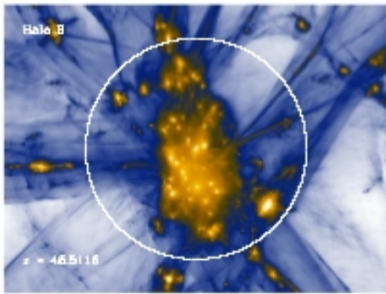
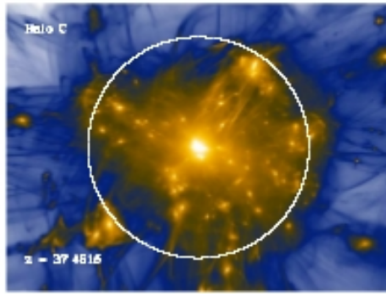
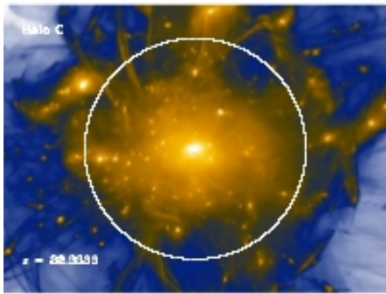
# Structure formation at the free streaming mass

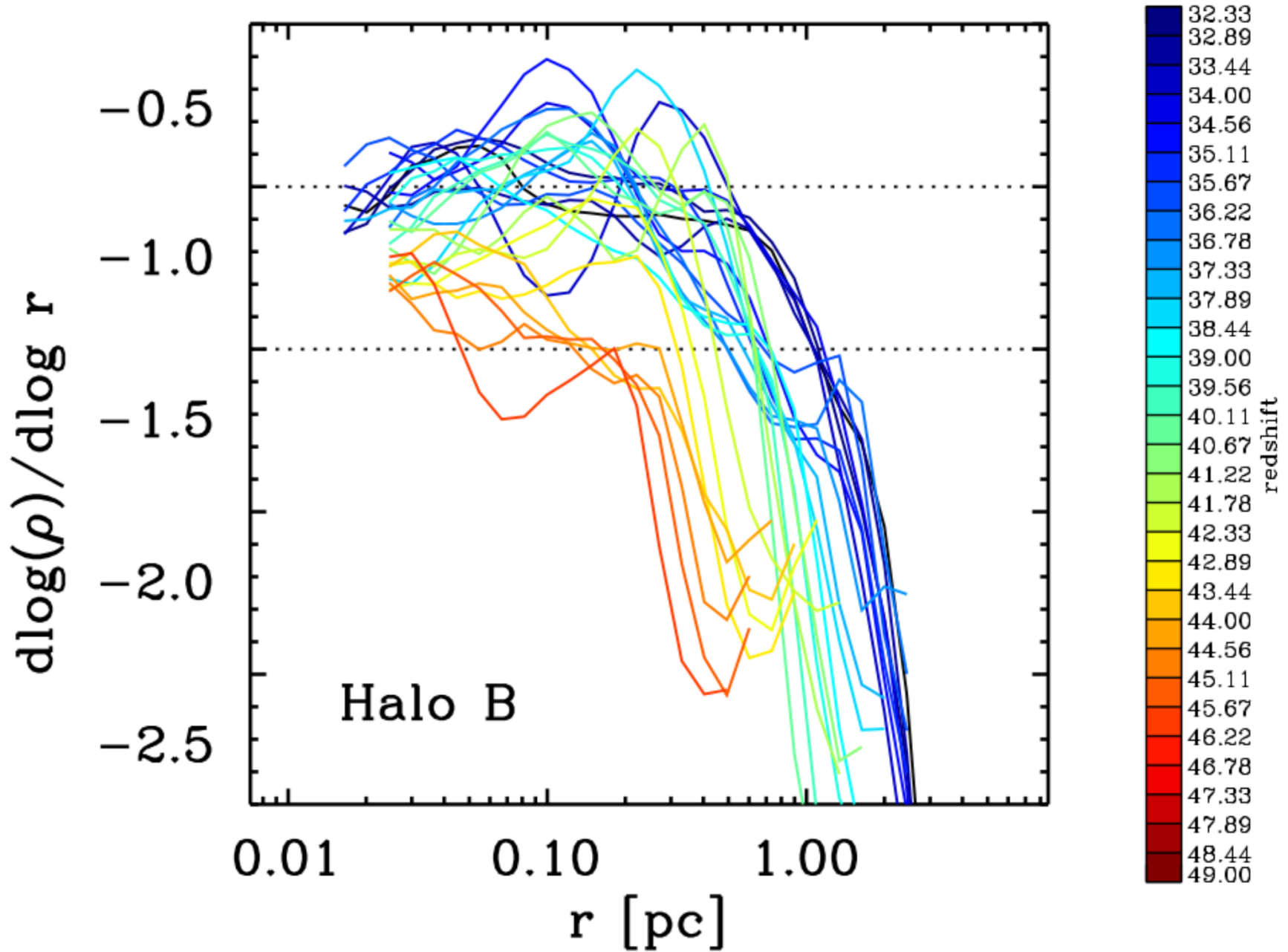
## The density profile of microhalos is steeper than a NFW



Colors: **No cut-off in PS**  
**With cut-off**  
**With cot-off**

Ishiyama 2014





# Controlled simulations of equal-mass mergers

With an explicit calculation of the exact phase-space distribution

## Initial density profiles

$$\rho = \rho_s x^{-\gamma} \begin{cases} 1 & \text{for } 0 \leq x \leq 1 \\ x^\kappa \exp[-\kappa(x-1)] & \text{for } x \geq 1 \end{cases}$$

$$x \equiv r/r_s = r/R_{vir}$$

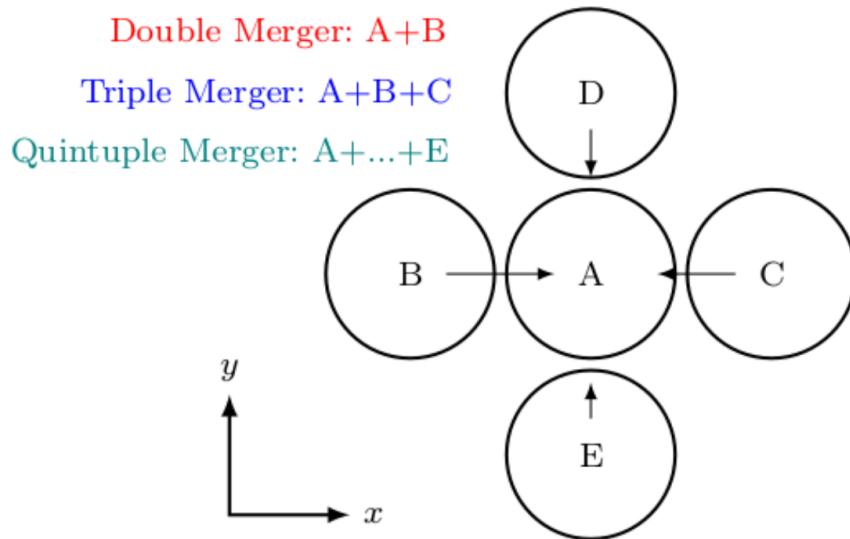


Figure 10.

- Isotropic velocity profile
- One million particles
- Softening = 0.05% of  $R_{vir}$
- Evolution for 15 Gyrs

# Controlled simulations of equal-mass mergers

Multiple mergers change the density profile (and mass is not conserved)

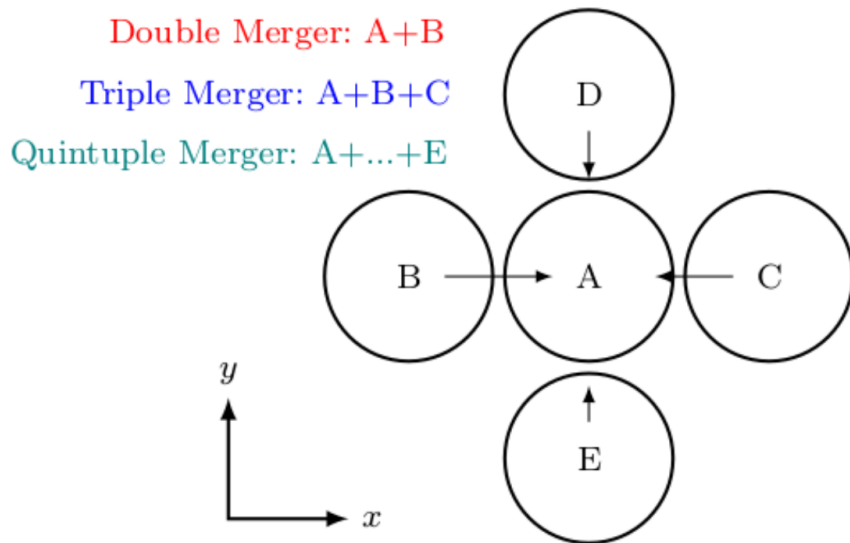
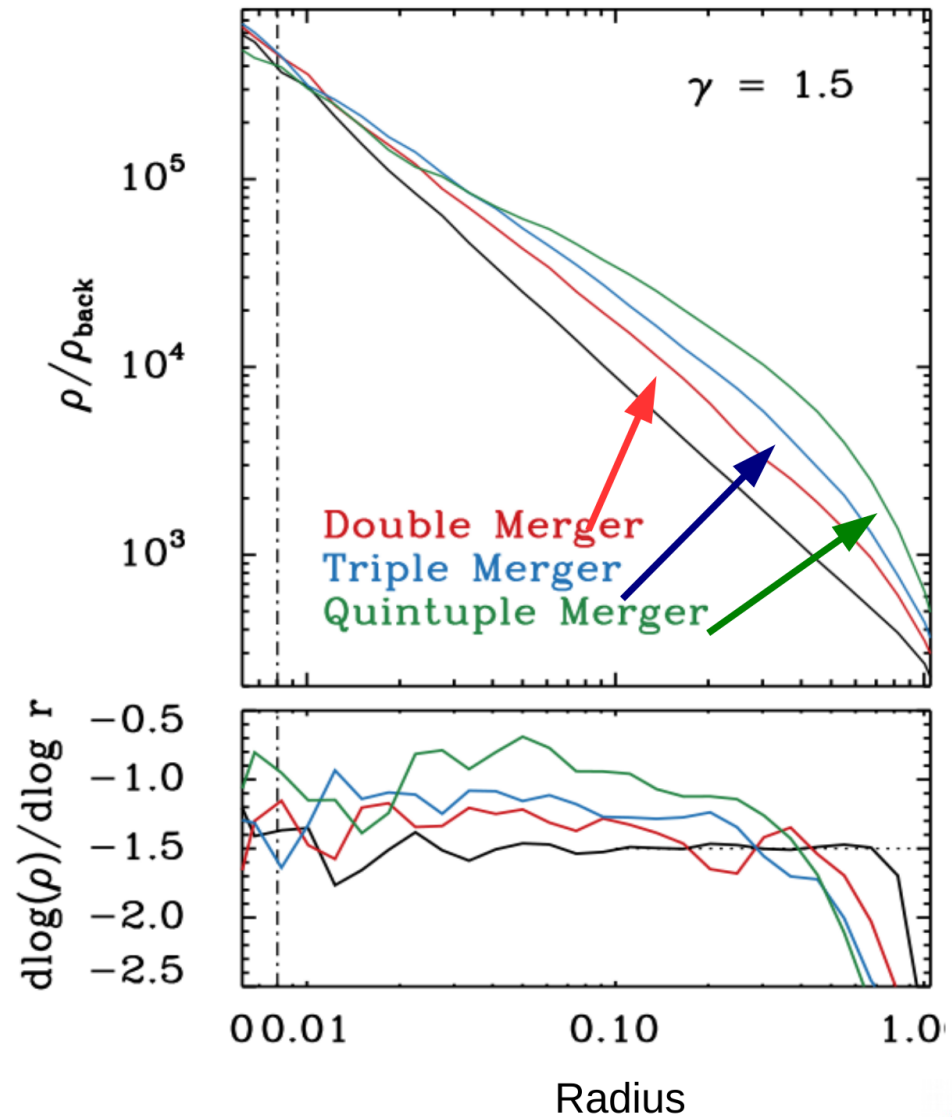


Figure 10.



# End result depends on the initial profile

