Self Sustained Activity and Failure in a Small World Network of Excitable Neurons

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Small World Network

Local connectivity $W_{ij}=1$ for $j=1,2,\ldots,k$

+ a fraction $p$ of unidirectional couplings $W_{ij}=1$ (an average number $pN$ of shortcuts)

(a) $k=3$

(b)

Shortcuts provide a means to transmit information efficiently around the network.

Average shortest distance $L$ between two nodes scales as $\ln N$.
**Integrate-and-Fire Neurons**

The differential equation describing the membrane potential $V_i$ of neuron $i$ is:

$$\tau_m \frac{dV_i}{dt} = -V_i + I_{ext} + g_{syn} \sum_j \sum_n \omega_{ij} \delta(t-t_j^{(n)}, T_D)$$

- $\tau_m$ is the membrane time constant.
- $I_{ext}$ is the external input current.
- $g_{syn}$ is the synaptic conductance.
- $\omega_{ij}$ is the synaptic weight from neuron $j$ to neuron $i$.
- $T_D$ is the delay time.
- $t_j^{(n)}$ is the time the neuron $j$ fires.

**Fast Waves: $T_D = 1.0$**

- In the absence of mutual inhibition ($p = 0$), the system leads to extinction.
- With mutual inhibition ($p = 0.05$), the system exhibits persistent activity.
- With higher inhibition ($p = 0.10$), the system fails to sustain activity.

- $I_{ext} < V_{thr}$, $\Rightarrow$ excitatory neuron.
Persistent Activity

As $p$ increases: faster establishment of oscillations, increasing oscillation amplitude, increasing recruitment of neurons into collective oscillation

But: increased probability of failure!
Failure to reinject activity due to the dynamics of neuronal recovery. After the emission of a spike, \( V(t=t_{\text{spike}})=0 \). Time is needed for the neuron to recover from the reset potential to a new potential such that \( V + g_{\text{syn}} \geq V_{\text{thr}} = 1 \). A single synaptic input will be able to reinject activity and elicit a spike only if the elapsed time exceeds:

\[
T_R^{(1)} = T_m \ln \left[ \frac{I_{\text{ext}}}{I_{\text{ext}} + g_{\text{syn}} - 1} \right]
\]

For \( T_m = 10 \), \( I_{\text{ext}} = 0.85 \), \( g_{\text{syn}} = 0.20 \),

\[ T_R^{(1)} = 28.3 \]
**Recurrence Time**

What is the time needed for activity to spread across the whole network, $T_A(p)$?

$$T_A(p) = T_d L_A(p)$$

Where $L_A(p)$ is the largest distance across the network. This distance has been computed for bidirectional shortcuts using a mean-field approach (Newman, Moore, and Watts, PRL 14, 3201 (2000)).

The mean-field theory can be extended to uni-directional short cuts to obtain:

$$\left(1 + \frac{4}{pN}\right)^{1/2} \tanh\left(\left(1 + \frac{4}{pN}\right)^{1/2} \frac{pT_A(p)}{2T_d}\right) = 1$$

Then, $T_A(p) = T_R''$ determines a critical concentration $p_{cr}(N)$ for failure to sustain activity by reinjection.

**Failure Transition**

A well defined transition to failure at $N=\infty$. Note that $p_{cr}(N) \sim \ln N$.

Only for sufficiently fast waves, small $T_d$
Slow Waves

As $T_D$ increases, more complex network dynamics. Reentrant phenomena: a nonmonotonic probability of failure with increasing $p$.

$T_D = 1.6$, $p = 0.8$

Regular activity, collective oscillations

Failure after a few cycles
Slow Waves: Persistence

Irregular firing patterns, long transients

Quasi Quiescent Epochs

Neurons that receive $n$ excitatory inputs during one cycle of network activity can have short recovery times:

$$T_{R}^{(n)} = T_{m} \ln \left[ \frac{I_{ext}}{I_{ext} + ng_{syn}} - 1 \right]$$

These neurons carry network activity across silent epochs of neurons that need $T_{R}^{(n)}$.
**Failure Times**

Cumulative distribution at \( p = 1.0 \)

\[ 1.5 \leq \tau_{D} \leq 1.7 \]

Fits a stretched exponential:

\[ F(t) = f_{\infty}(\tau_{D}) - C e^{-\alpha t^\beta} \]

For \( \tau_{D} = 1.65 \), runs up to \( t = 300000 \) provide

\[ 0.97 \leq f_{\infty}(\tau_{D} = 1.65) \leq 1.0 \]

**Failure Rates**

Failure rates \( F(t, \tau_{D}) \) at fixed \( t \) exhibit complex structure as function of \( \tau_{D} \), suggestive of "resonances"
Bistability

The network is bistable between an "on" state of persistent, self-sustained activity, and an "off" quiet state. The simultaneous activation of a small number of neurons suffices to switch between them.

Summary

- A small-world network of integrate-and-fire neurons can sustain persistent activity.
- A low density of shortcuts provides a mechanism for reinjection of activity.
- Propagating pulses of activity are sustained by branching and reinjection.
- Network bistability provides a neural substrate for short-term "working" memory.

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