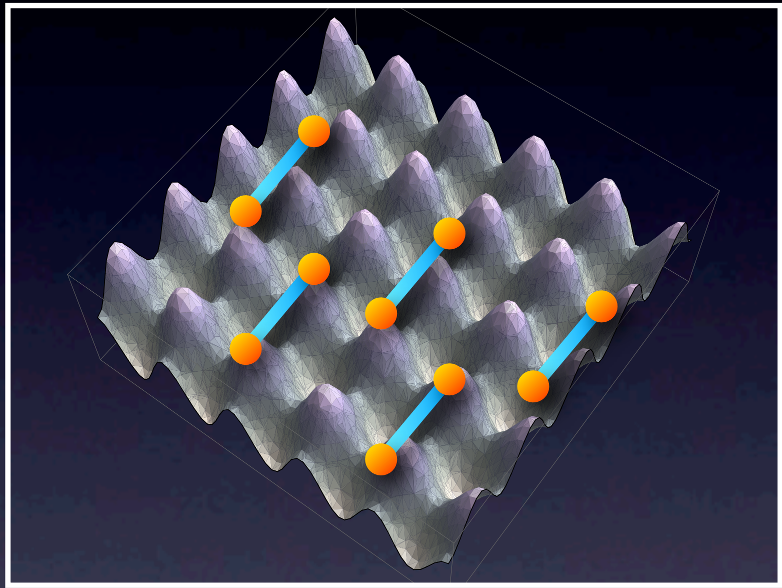
Witnessing order parameters and dynamics in strongly correlated systems



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Quantum Optics & Cold Gases & Condensed matter & Quantum Information & High Energy Physics

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B. Malomed (Israel)
G. Birkl (Germany)

Outline of the Talk

A- Quantum state transfer in spin chains

- Ultracold atoms as quantum simulators
- Dynamics: entanglement transfer in many body systems
- Quantum state transfer in spin 1 chains

B- Detection of strongly correlated systems

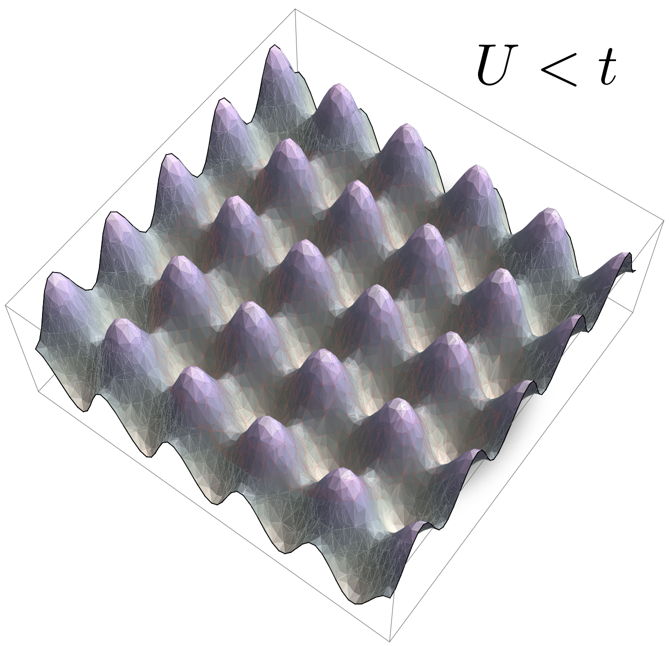
- Detection / Discrimination ?
- Measuring Order Parameters of Non Trivial Phases
- Probing and Manipulating Non-equilibrium Dynamics?

K. Ekert, O. Romero-Isart & Sanpera PRA (2007)
K. Ekert, O. Romero-Isart, M. Rodgriguez, M. Lewenstein, E. Polzik and A. Sanpera Nat, Phys, (2008)
T. Roscilde, M. Rodriguez. O.Romero-Isart, M. Lewenstein, E. Polzik, A. Sanpera NPJ (2009)
G. de Chiara, O. Romero-Isart, A. Sanpera (submitted 2010)
O. Romero-Isart, M. Rizzi, C. Muschik, T. Roscilde, M. Lewenstein and A. Sanpera (in preparation)

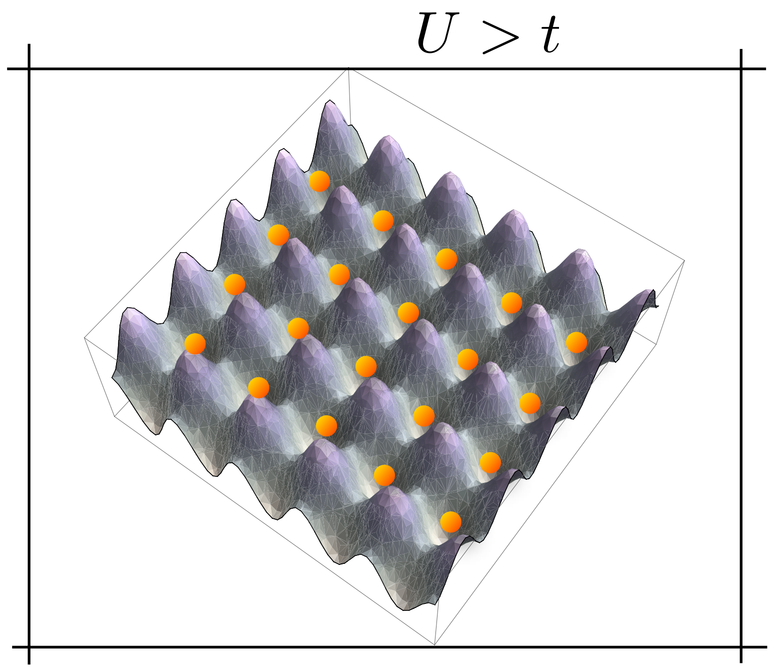
Motivation: Ultracold gases & Quantum phase transitions

- Paradigmatic example: The Bose-Hubbard Mott-Superfluid transition

$$H = -t \sum_{\langle ij \rangle} \left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i \right) + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \sum_i \epsilon_i \hat{n}_i$$



Superfluid



Mott-Insulator

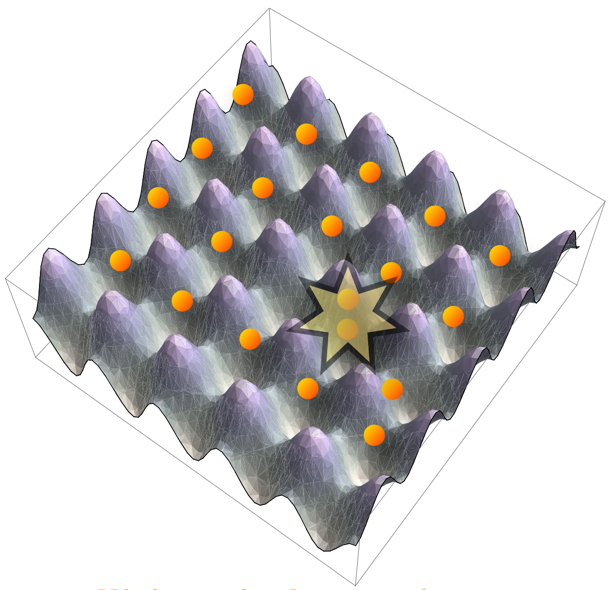
M. P.A. Fisher et al, Phys. Rev. B **40**, 546 (1989)   D. Jaksch et al, Phys. Rev. Lett. **81**, 3108 (1998)

M. Greiner, *et al.* Nature **415**, 39 (2002)

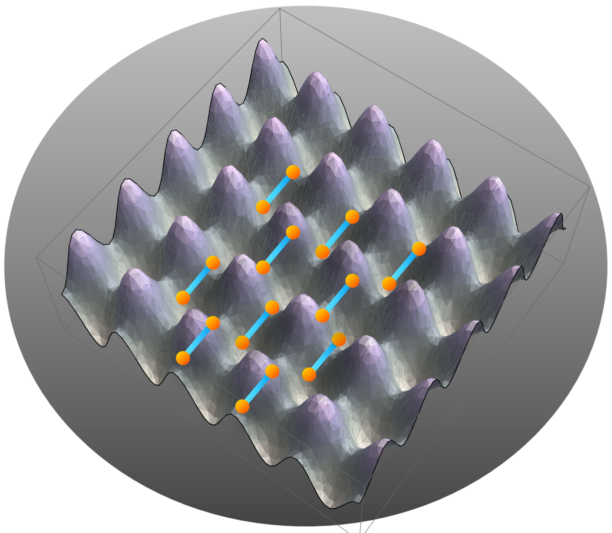
Motivation: Ultracold gases & Quantum Simulators

- Exchange Interactions: Quantum Information
- Super-exchange Interactions: Quantum Magnetism

$$H_{\text{eff}} = \pm \frac{t^2}{U} \sum_{\langle i,j \rangle} \vec{S}_i \vec{S}_j$$



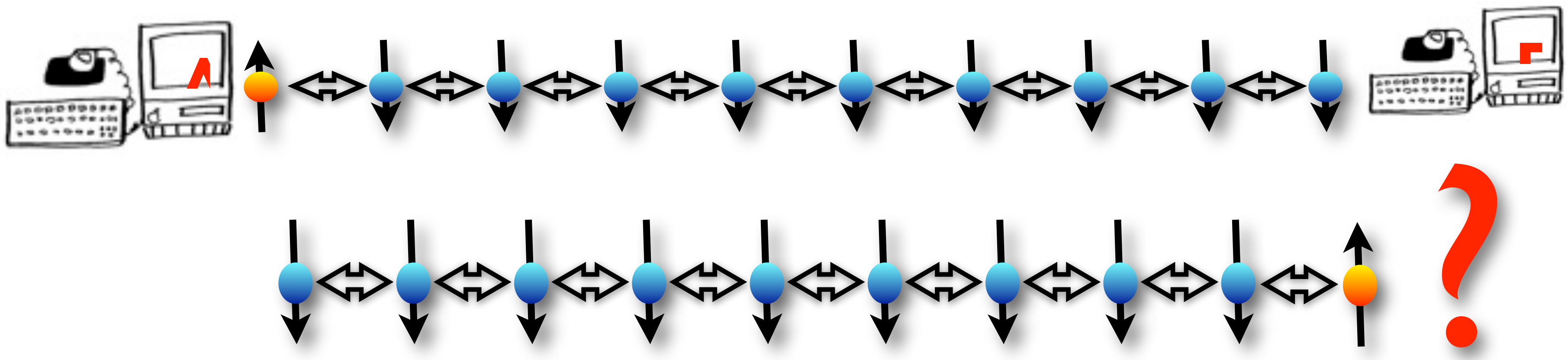
collisions induce spin-exchange interaction: gates



Virtual tunneling induces spin-exchange interaction

L.-M. Duan, E. Demler, and M. D. Lukin Phys. Rev. Lett. **91**, 090402 (2003). J. J. Garcia-Ripoll and J. I. Cirac, New J. Phys. **5**, 76 (2003). S. Trotzky, *et al.* Science **319**, 295 (2008), Bloch et al. (2008)

Quantum Communication via spin-1/2 Hamiltonians

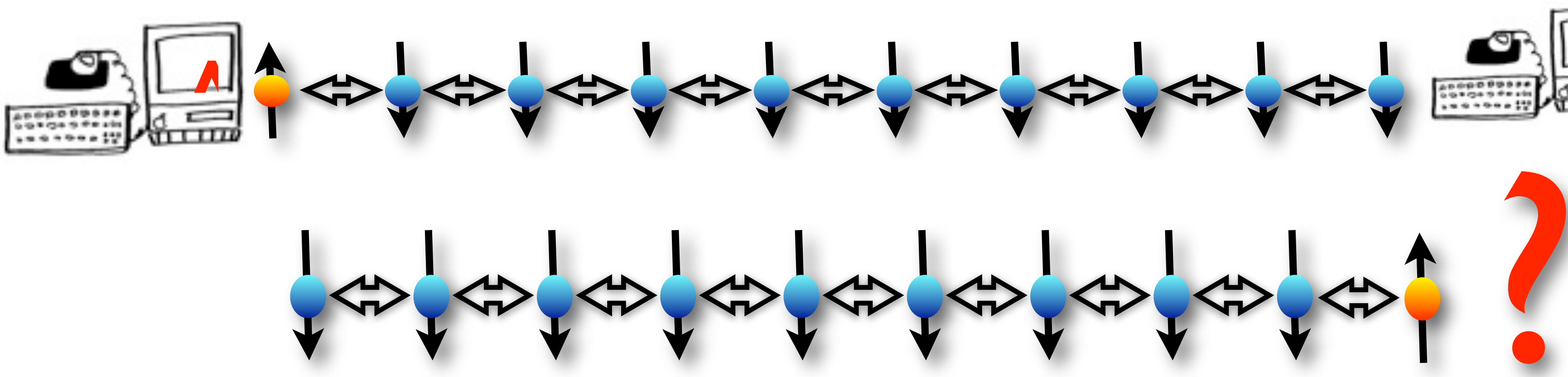


$$H_{\text{eff}} = \pm \frac{t^2}{U} \sum_{\langle i,j \rangle} \vec{S}_i \vec{S}_j$$

Quantum Benchmark

“Quantum Communication through an unmodulated Spin Chain”, S. Bose, PRL **91**, 207901 (2003)

Quantum Communication via spin-1/2 Hamiltonians



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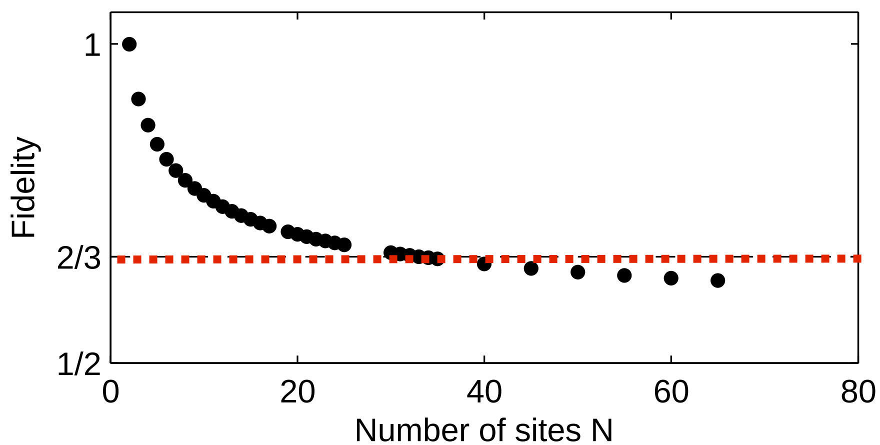


Quantum Benchmark

"Quantum Communication through an unmodulated Spin Chain", S. Bose, PRL **91**, 207901 (2003)

A- Transfer of entanglement in spin Hamiltonians.

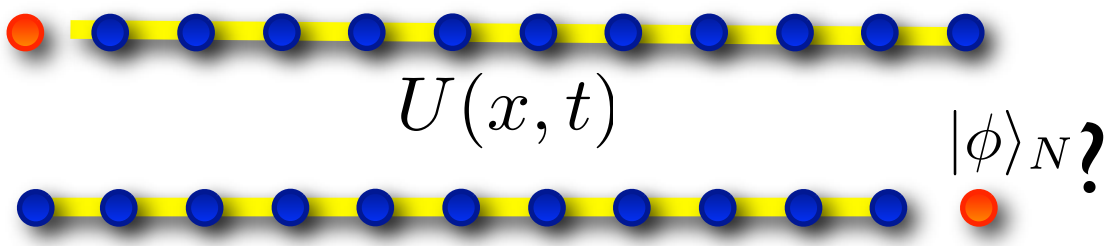
Can we use dyanamics & transfer of entanglement as a resource?

Can the quality of the entanglement transfer characterize the many-body system?

Quantum Communication

- Spin chains as a quantum channels: $\Lambda(\rho) = \rho'$ $\quad F = \langle \rho | \rho' \rangle$

$$|\psi_{\rm in}\rangle_{1,N} = |\phi\rangle_1 \otimes |{\rm in}\rangle_{2,N}$$



$$\Lambda[|\phi\rangle\langle\phi|] = {\rm tr}_{1,\ldots,N-1}\left[U(x,t)|\psi_{\rm in}\rangle_{1,N}\langle\psi_{\rm in}|U^\dagger(x,t)\right]$$

d dimension of the spin space

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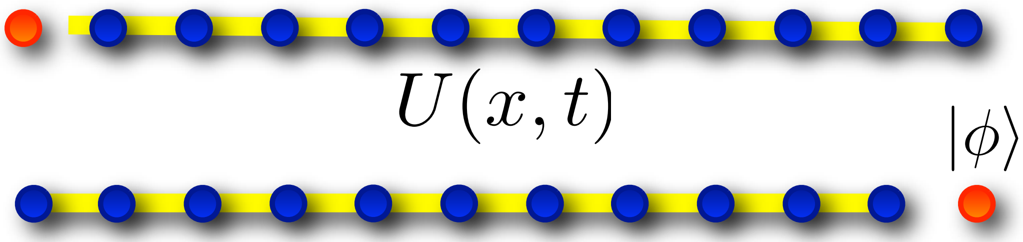


$U(x,t)$

$|\phi\rangle_N$?

$$\Lambda[|\phi\rangle\langle\phi|] = {\rm tr}_{1,\ldots,N-1}\left[U(x,t)|\psi_{\rm in}\rangle_{1,N}\langle\psi_{\rm in}|U^\dagger(x,t)\right]$$

- Quantum benchmark

d dimension of the spin space

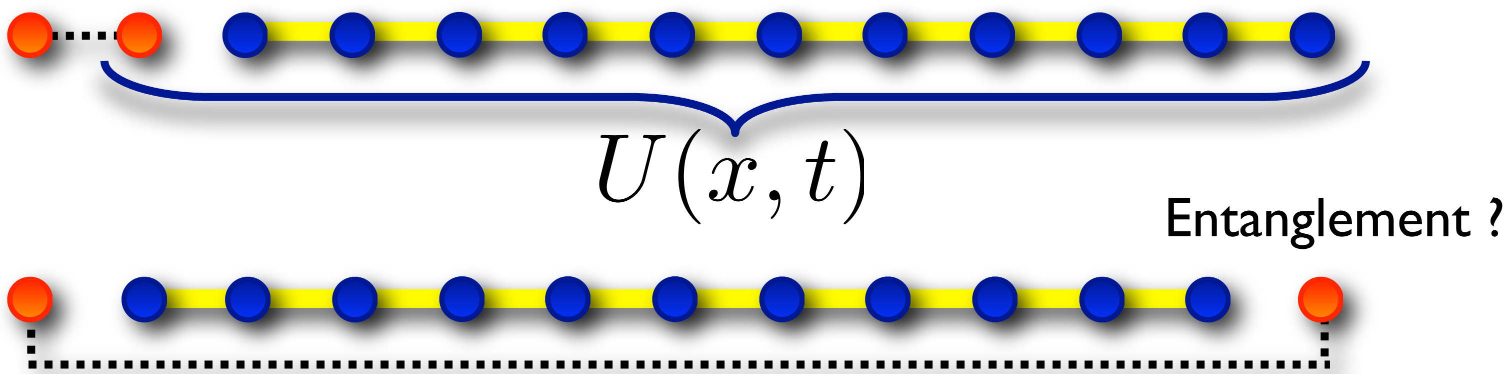
Channel fidelity:

$$F(\Lambda) = \int d\phi \langle\phi|\Lambda\left[|\phi\rangle\langle\phi|\right]|\phi\rangle$$

$$F(\Lambda^{\rm nonclassical}) > \frac{2}{d+1}$$

- Many-body systems as quantum channels: entanglement transfer

$$|\psi_{\rm in}\rangle_{0,N} = |\psi^+\rangle_{01} \otimes |{\rm in}\rangle_{2,N}$$



$$(\mathbb{I} \otimes \Lambda)\left[|\psi^+\rangle\langle\psi^+|\right] = \rho_\Lambda$$

- Quantum benchmark

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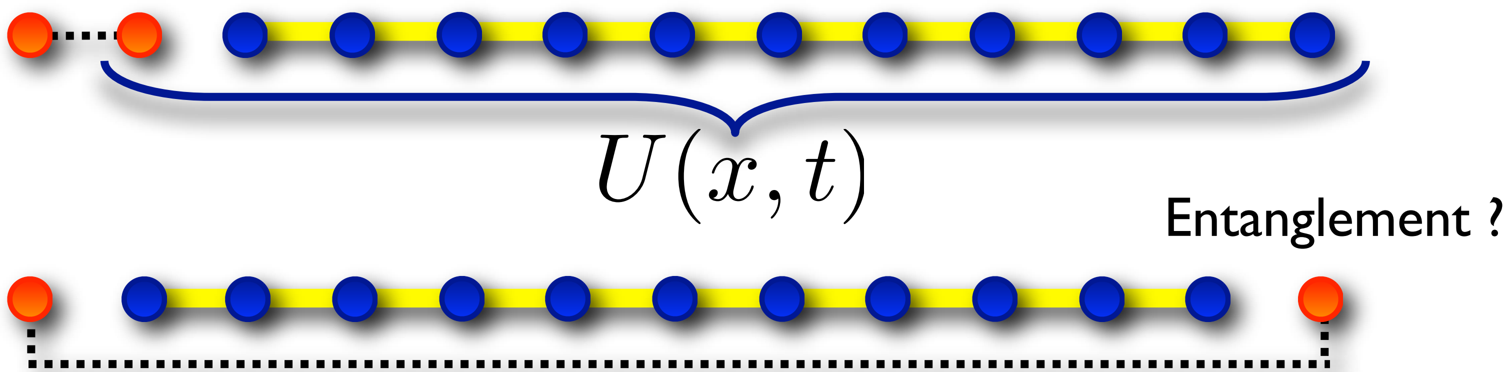


$$(\mathbb{I} \otimes \Lambda)\left[|\psi^+\rangle\langle\psi^+|\right] = \rho_\Lambda$$

- Quantum benchmark

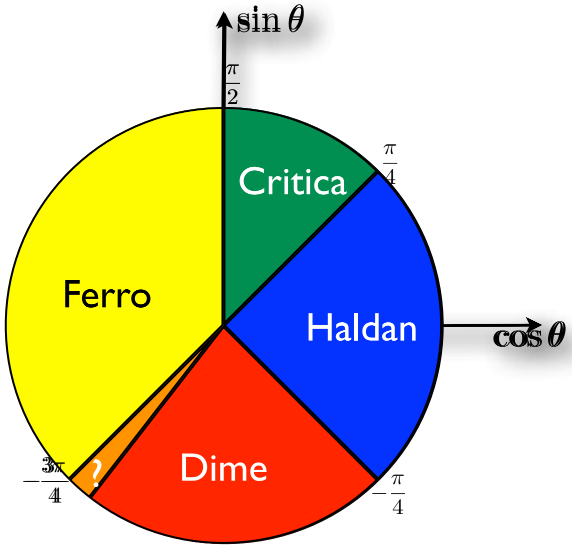
Entanglement measure (Log. negativity):

$$\text{LN}(\rho_\Lambda) = \log_2 ||\rho_\Lambda^\Gamma||_1$$

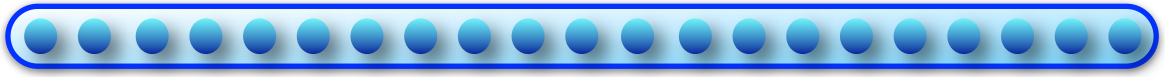
$$\text{LN}(\Lambda^{\text{nonclassical}}) \equiv \text{LN}(\rho_\Lambda) \neq 0$$

Quantum Communication in spin-1 chains: Bilinear-Biquadratic Hamiltonian

- Restricted to nearest-neighbours the most general isotropic Hamiltonian



$$\hat{H}(\theta) = J \sum_{<ij>} \left[\cos\theta(\vec{S}_i\vec{S}_j) + \sin\theta(\vec{S}_i\vec{S}_j)^2\right]$$



Quantum Communication in spin-1 chains: Bilinear-Biquadratic Hamiltonian

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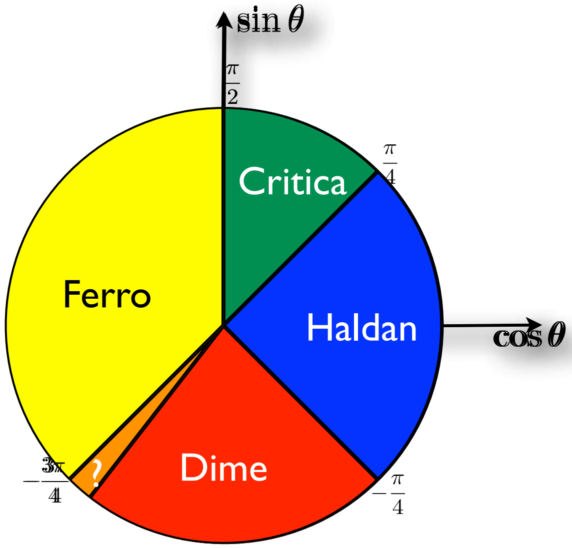

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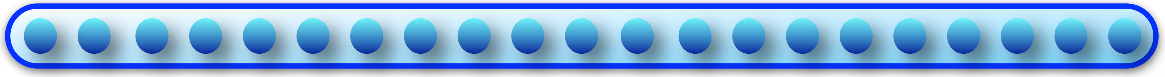
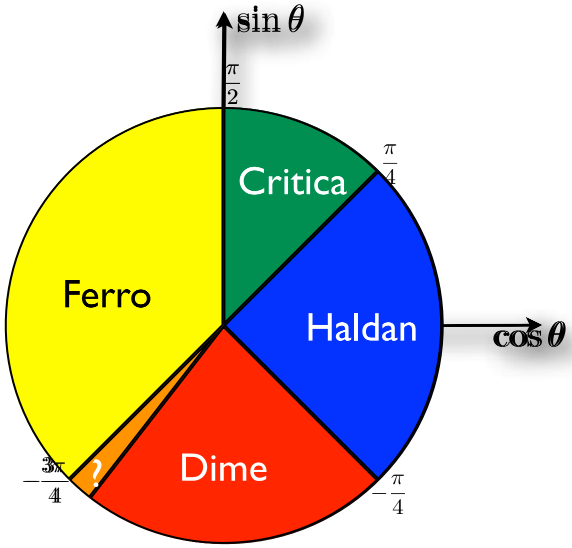


- Emblematic strongly-correlated quantum many-body system in 1D

Quantum Communication in spin-1 chains: Bilinear-Biquadratic Hamiltonian

- Restricted to nearest-neighbours the most general isotropic Hamiltonian



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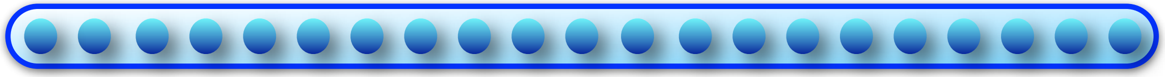

- Emblematic strongly-correlated quantum many-body system in 1D

- The whole phase-diagram is realizable loading spin-1 atoms in an optical lattice and realizing that H_(AF)=-H_F

A. Imambekov, M. Lukin, and E. Demler, PRA **68**, 063602 (2003);

Garcia-Ripoll, M.A. Martin Delgado and J.I. Cirac, PRL (2004)

Bilinear-Biquadratic Hamiltonian

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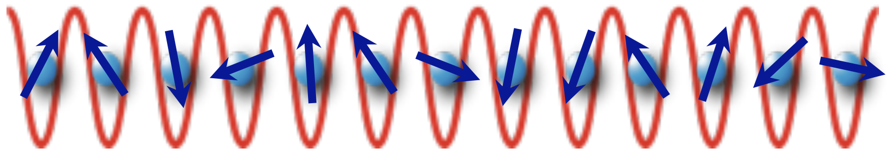
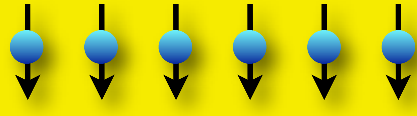
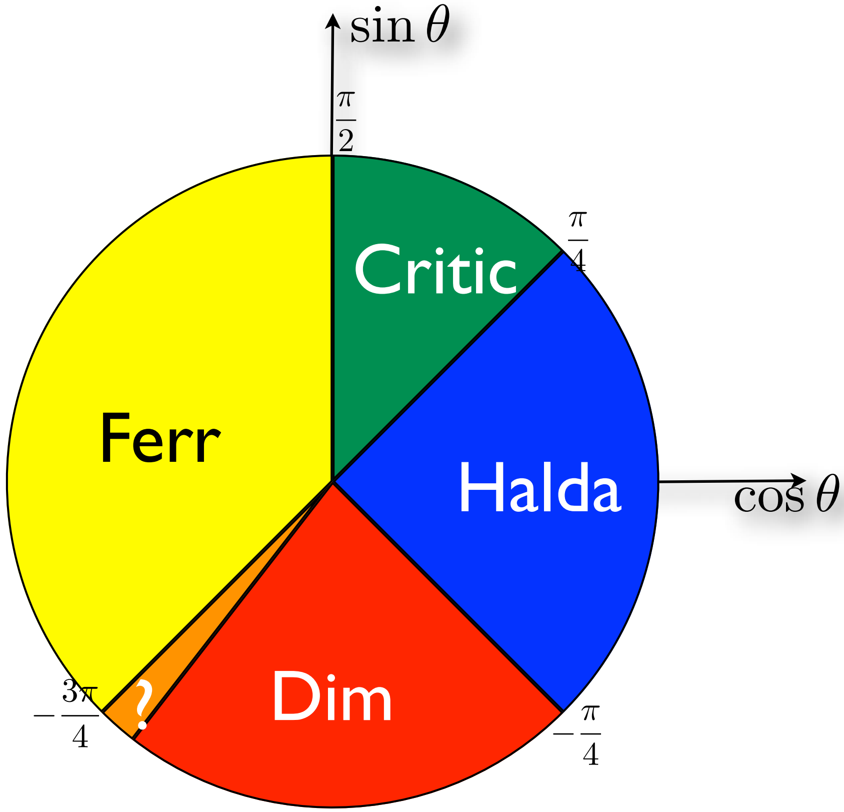


- Ferromagnetic





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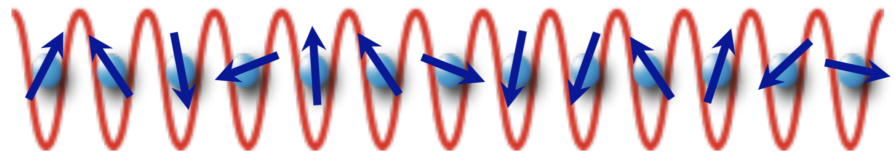

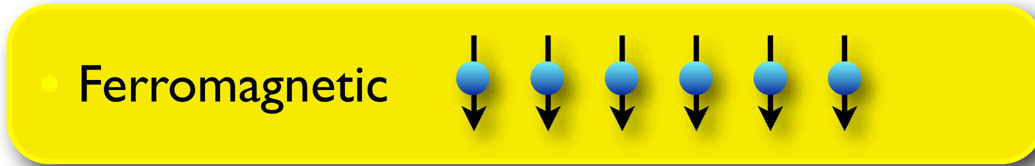

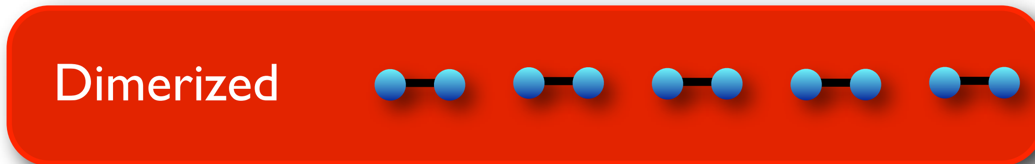

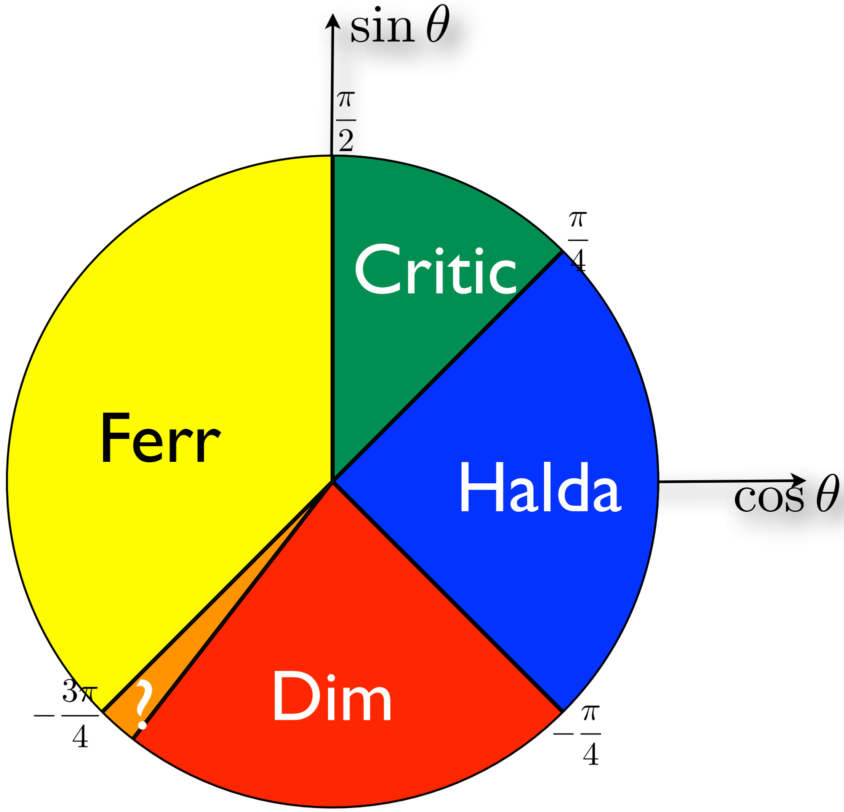

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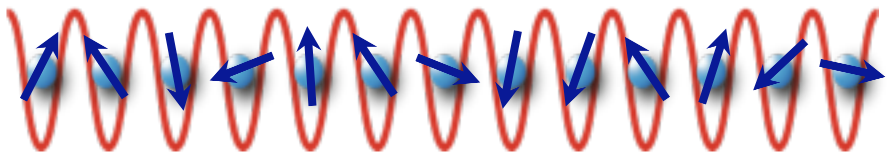

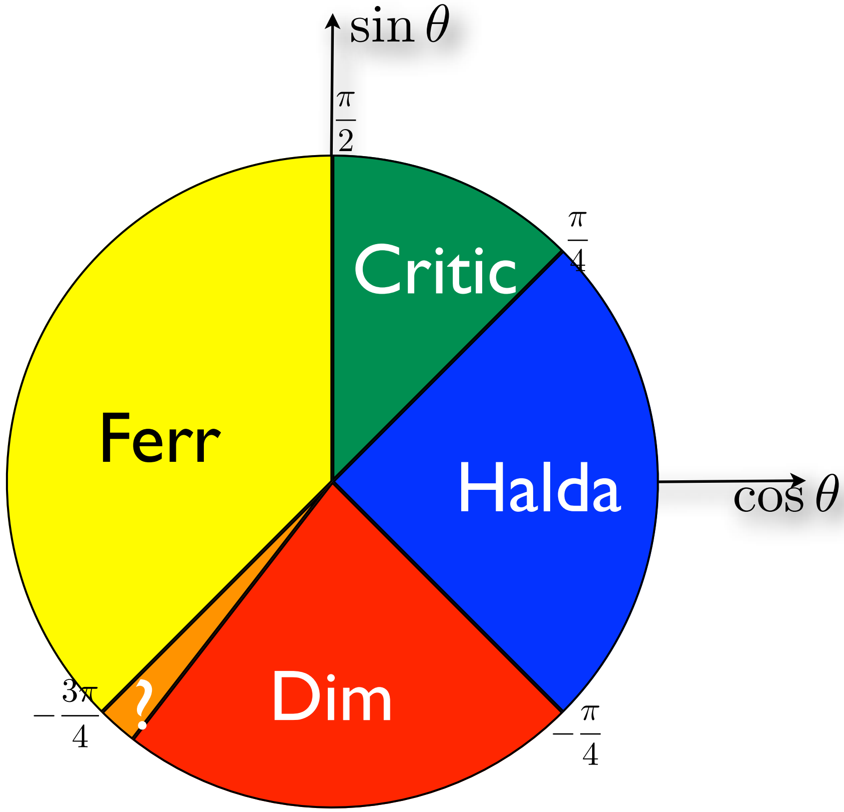

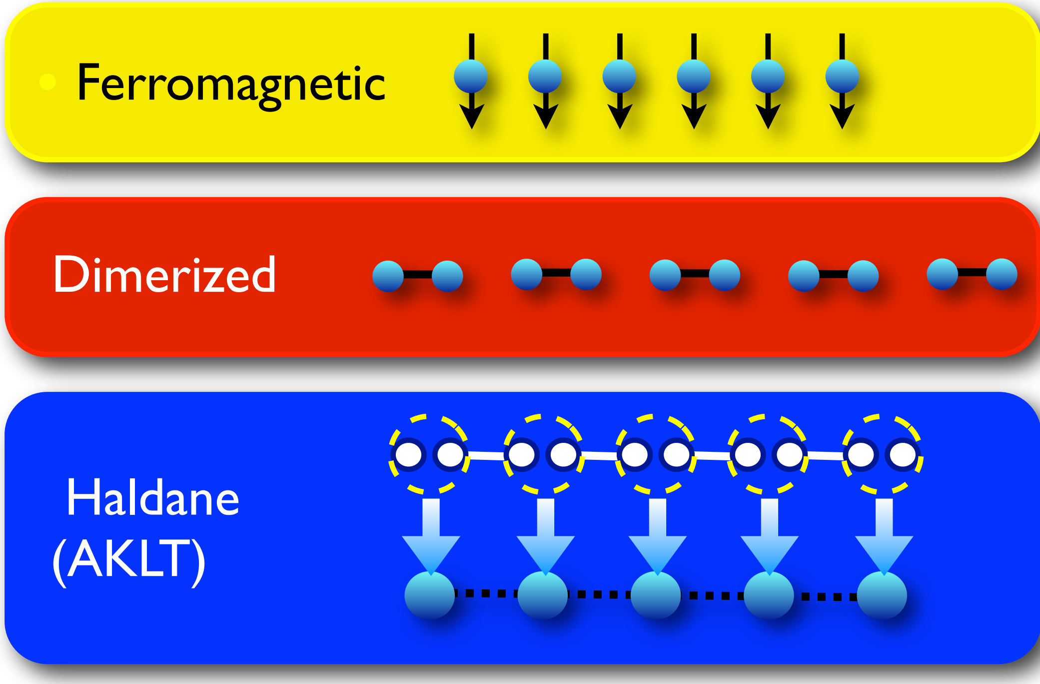

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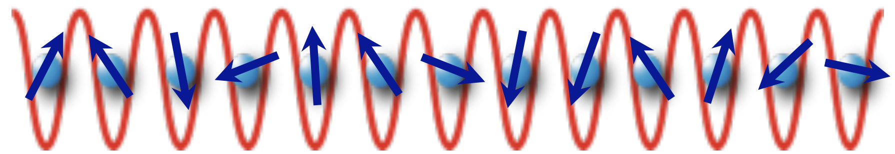

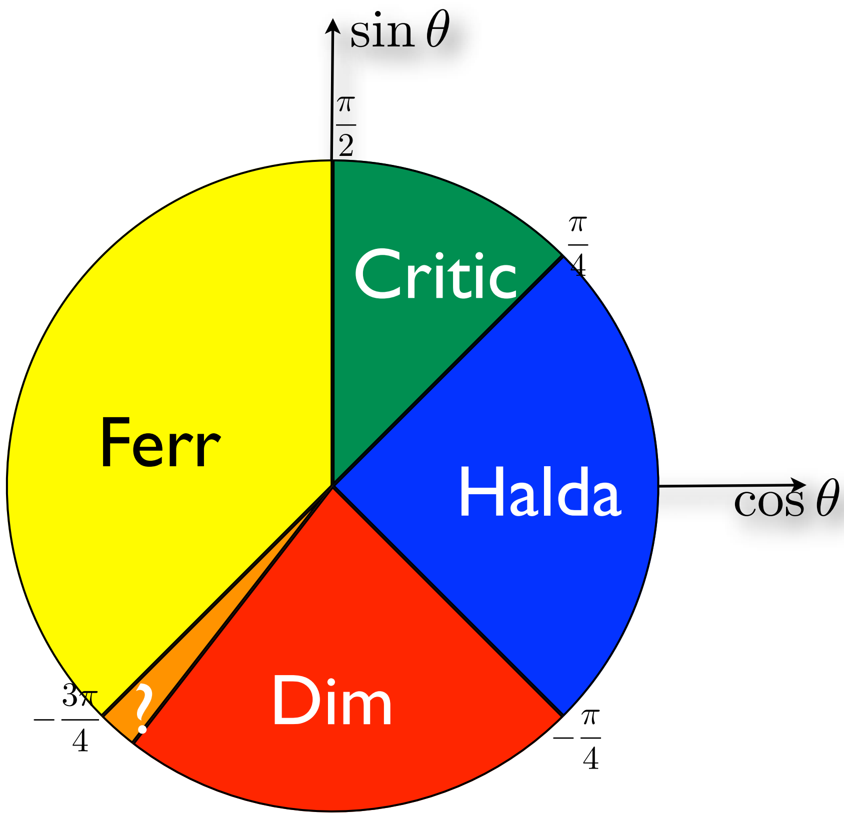

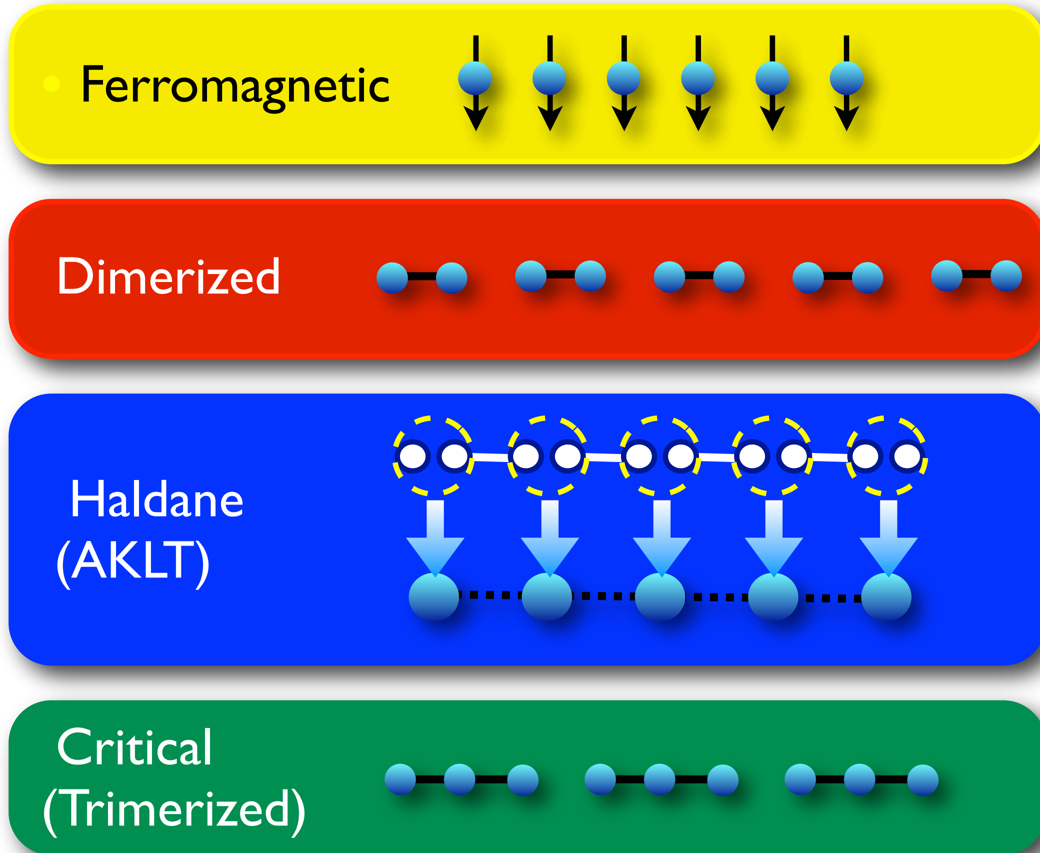

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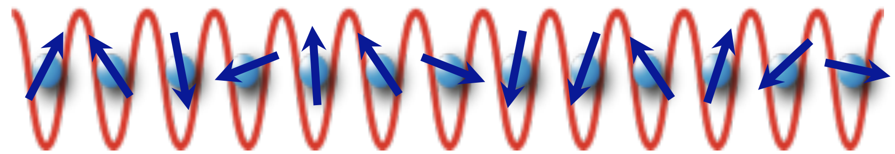
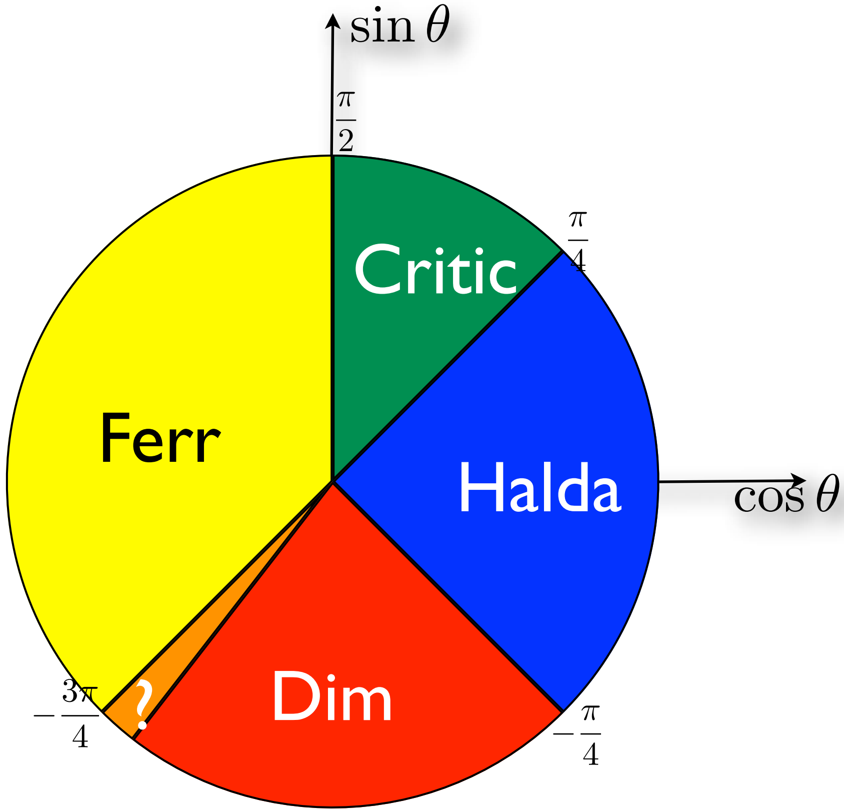
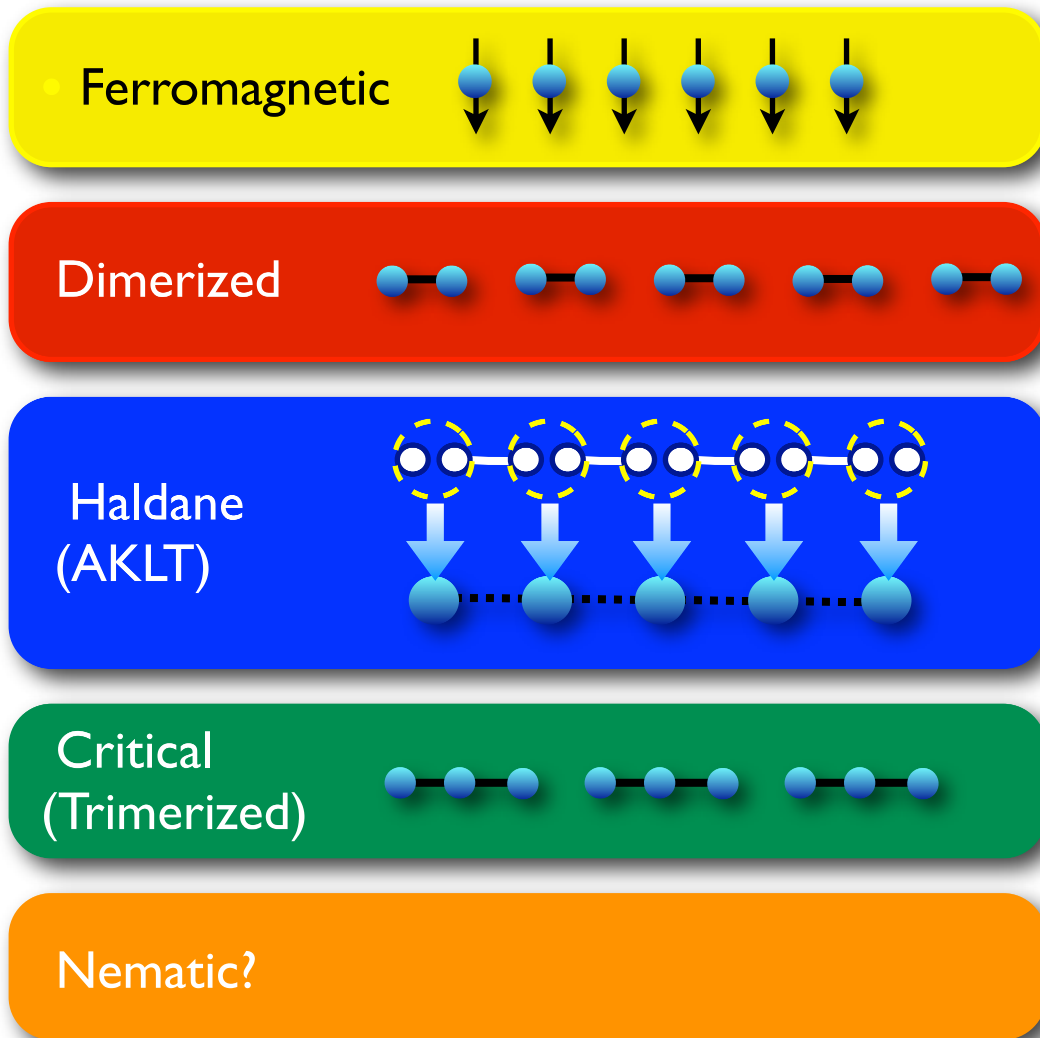
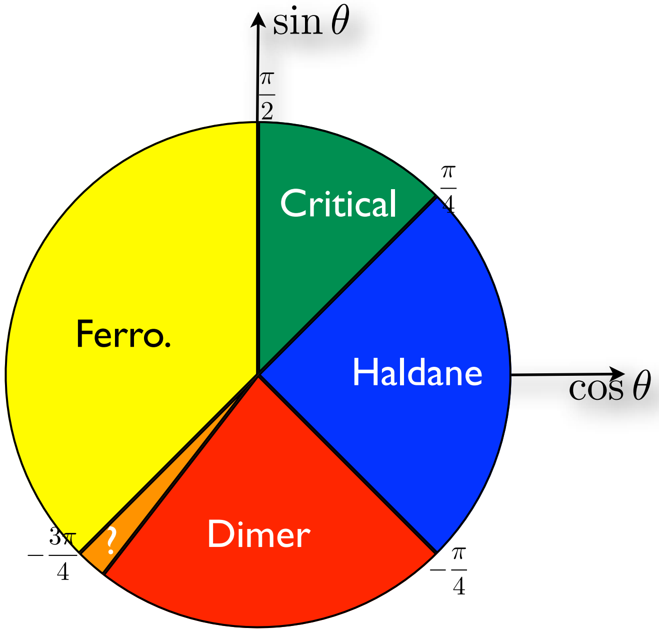
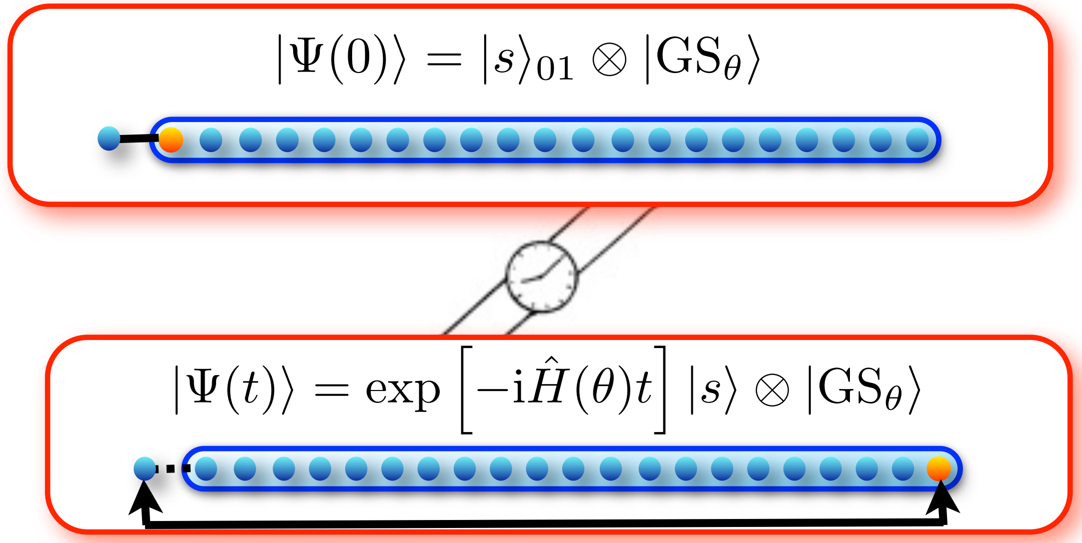






Quantum Transfer versus Magnetic Ordering





$$|\Psi(0)\rangle = |s\rangle_{01} \otimes |\mathrm{GS}_\theta\rangle$$

$$|\Psi(t)\rangle = \exp\left[-\mathrm{i}\hat{H}(\theta)t\right] |s\rangle \otimes |\mathrm{GS}_\theta\rangle$$

Log negativity ~entanglement $\mathrm{LN}(\theta,t) = ||\rho_{0N}^{\Gamma}(\theta,t)||_1$

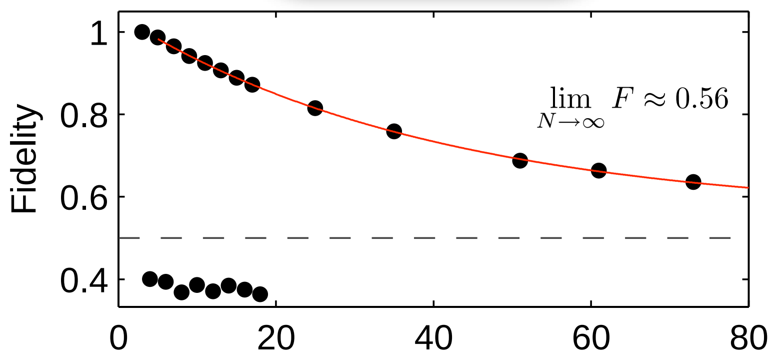
Fidelity: (Haar Measure) $F(\theta,t) =< \langle\phi|\rho_N(\theta,t)|\phi\rangle >$

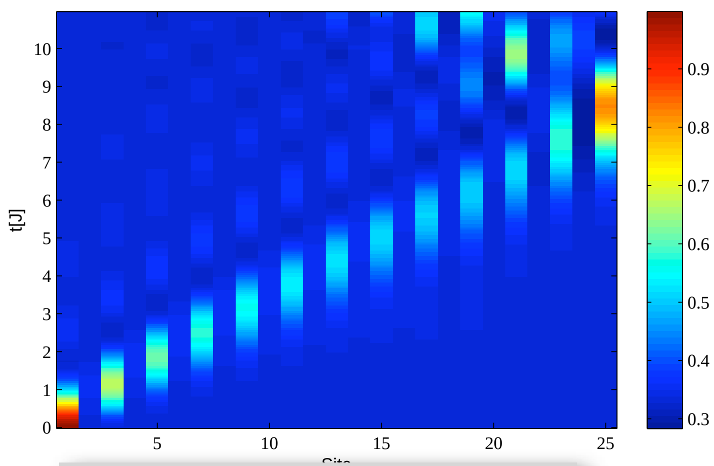
Calculate the ground state of the system (MPS or DMRG) for the whole phase diagram add one excitation and propagate on time (t-DMRG).

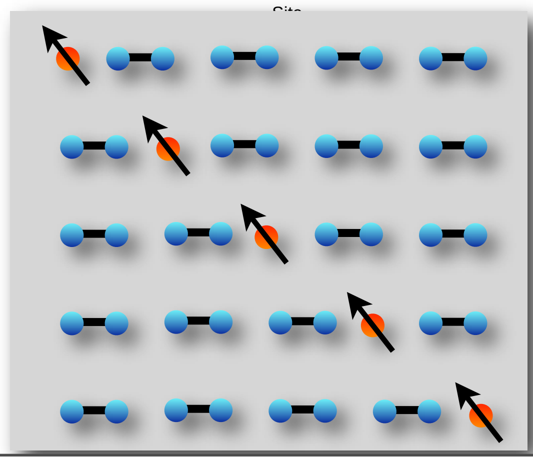
Probing the properties of the ground state + low energy excitations at once + quantifying spin-spin correlations

Quantum Transfer versus Magnetic Ordering

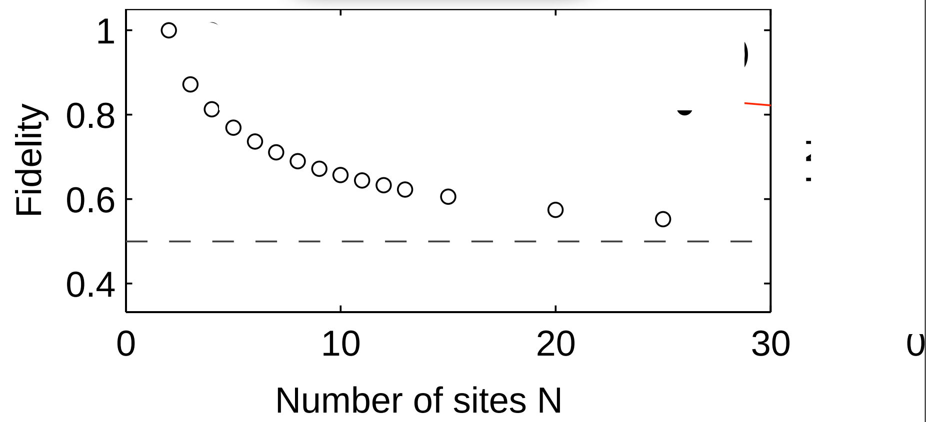
Dimerized

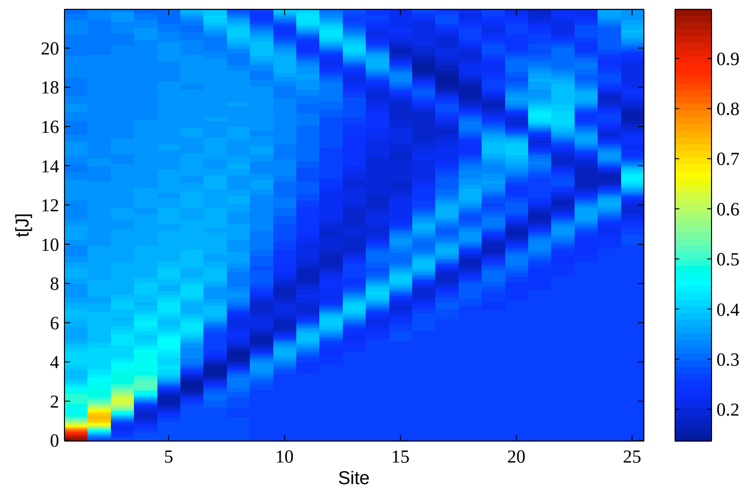






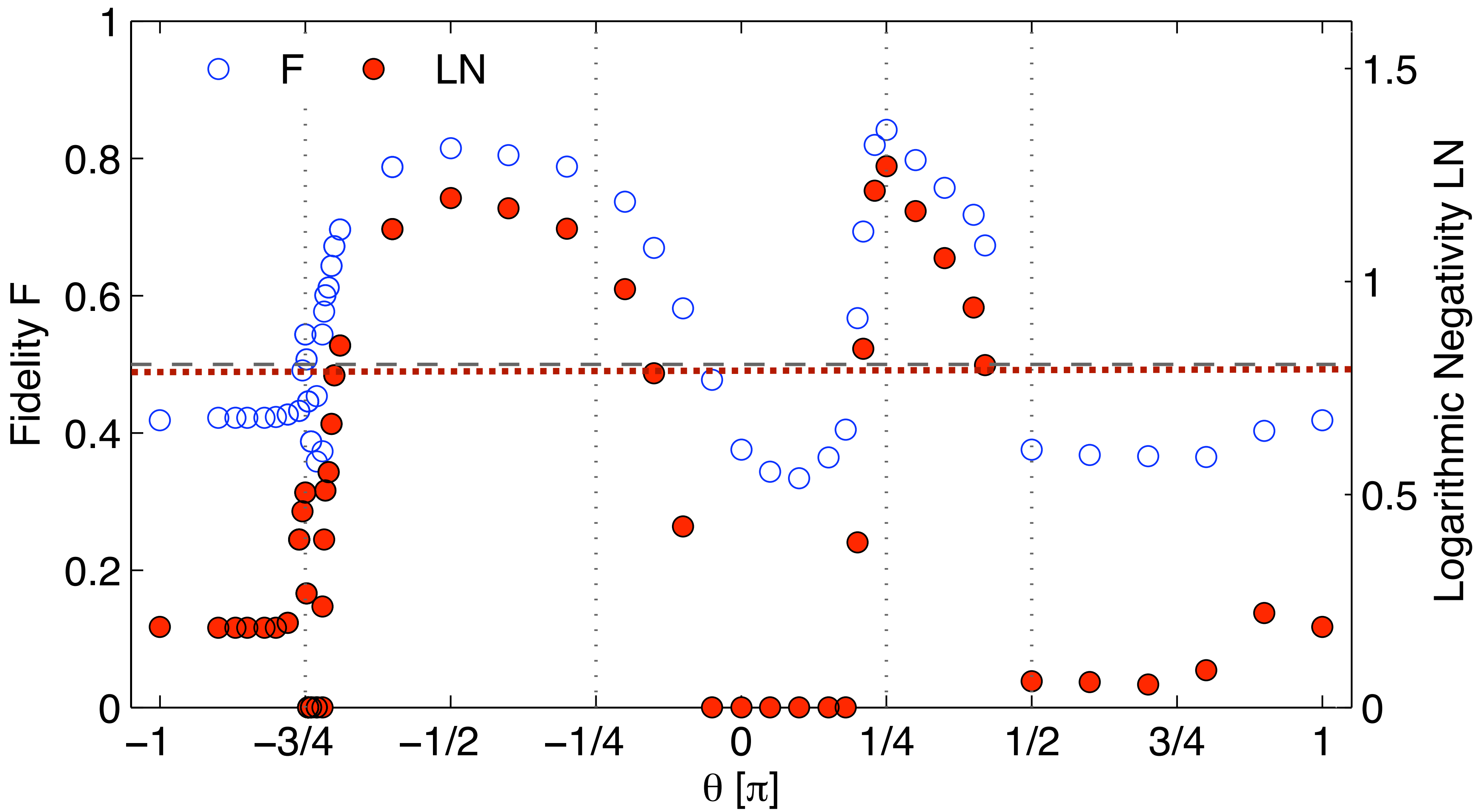
Ferromag.





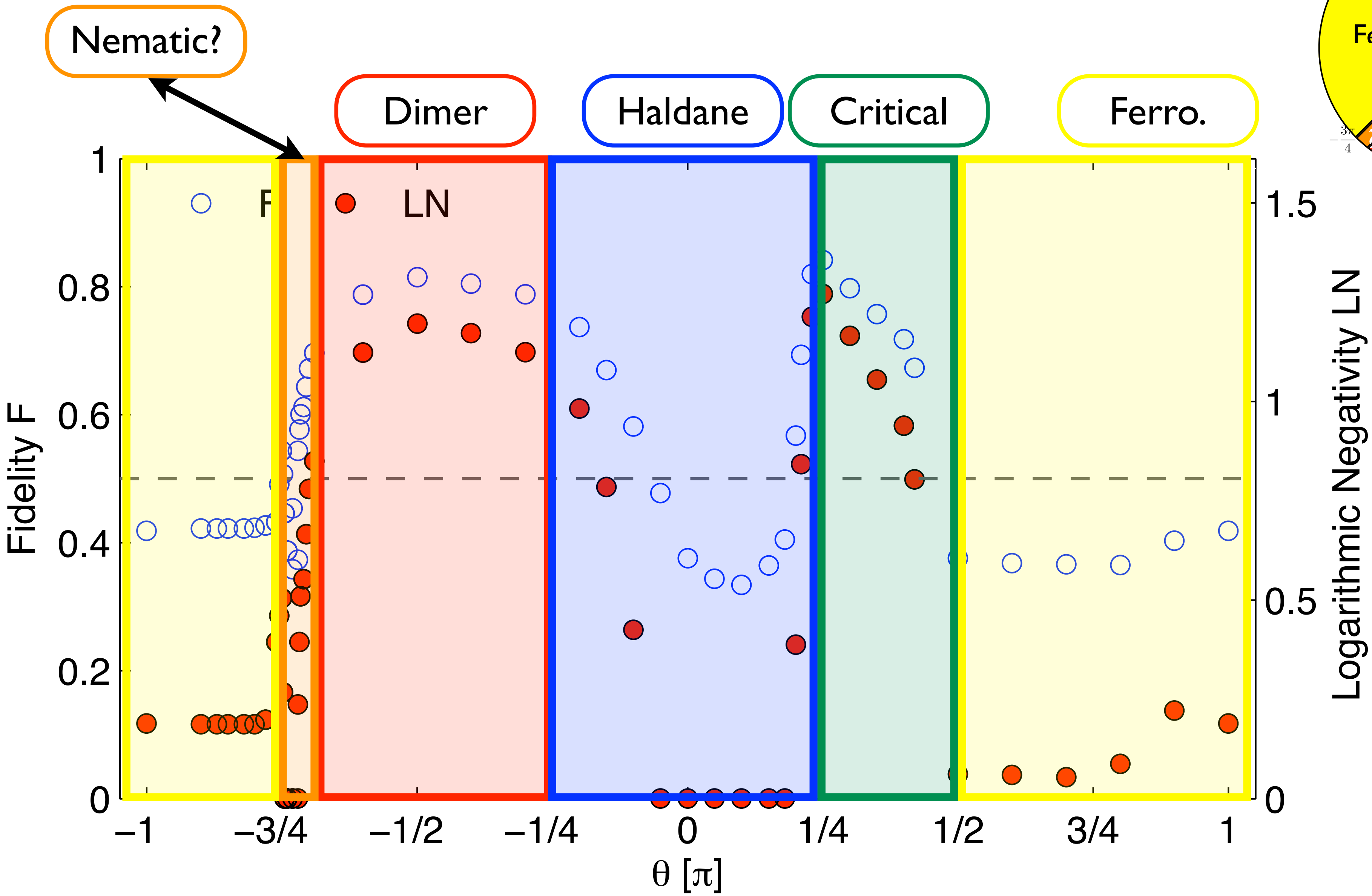
ORI, K. Eckert, and A. Sanpera (2007)

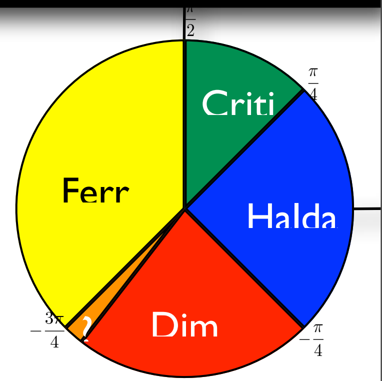
Transfer Fidelity vs Magnetic Order



ORI, K. Eckert, and A. Sanpera, PRA(R) in press, quant-ph/0610210

Transfer Fidelity vs Magnetic Order





ORI, K. Eckert, and A. Sanpera PRA(2007)

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B- Detection of strongly correlated systems

- Detection / Discrimination ?
- Measuring Order Parameters of Non Trivial Phases
- Probing and Manipulating Non-equilibrium Dynamics?

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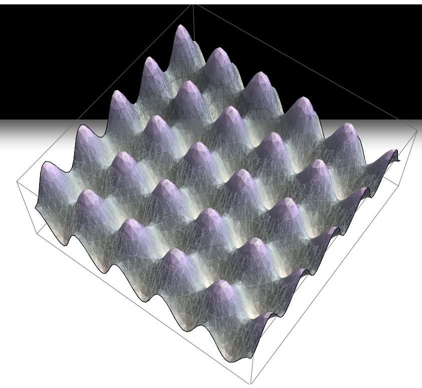
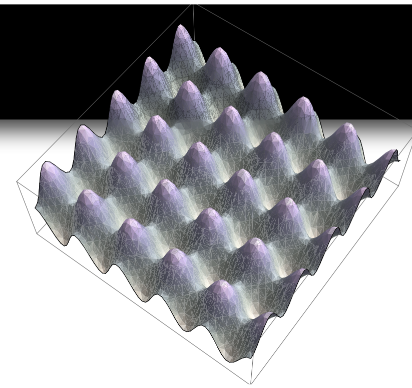
G. de Chiara, O. Romero-Isart, A. Sanpera (submitted 2010)
O. Romero-Isart, M. Rizzi, C. Muschik, T. Roscilde, M. Lewenstein and A. Sanpera (in preparation)



B- Detection of strongly correlated systems

- Detection / discrimination ?

$\langle A \rangle \neq 0$ $\longleftrightarrow$ Broken Symmetry



B- Detection of strongly correlated systems

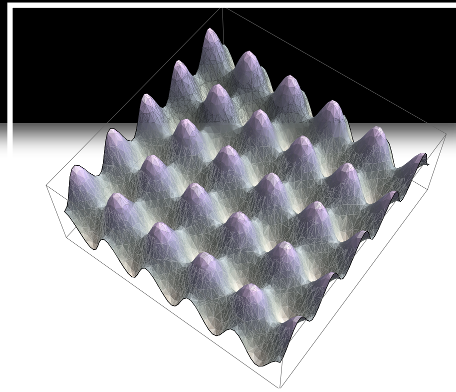
- Detection / discrimination ?

➡ 1. Many body systems- large # states: full tomography is not possilbe

➡ 2. Often the ground state of the system is not even analytically known: state estimation / state discrimination is not possible

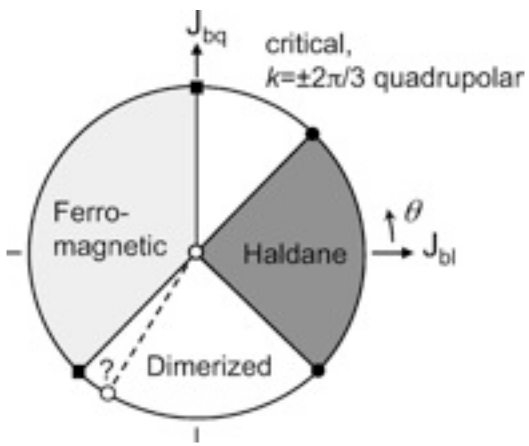
➡3. A faithful determinatin of the state LANDAU theory of phase transition and looks for an order parameter (not valid if the state is topological)

$$\langle A \rangle \neq 0 \quad \longleftrightarrow \quad \text{Broken Symmetry}$$



Detection of strongly correlated systems

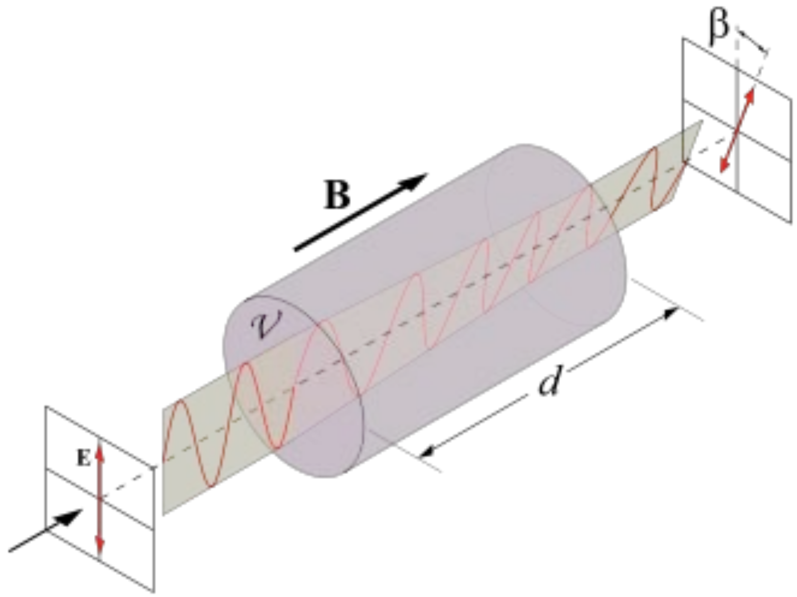
- Non trivial phases
- Spatial resolution ←┈┈┈→ high order correlation
- Works for fermions and/or bosons
- Quantum non demolishing
- Allows for time dependent correlations



Quantum Polarization Spectroscopy !

Faraday interaction with spatial resolution: QPS

Quantum Polarization Spectroscopy is based on the Faraday interaction between atoms and light.



Light beam x-polarized: Stokes operators (Schwinger representation)

$$\hat{S}_1 = \frac{1}{2}\left(\hat{n}_x - \hat{n}_y\right)$$

$$\hat{S}_2 = \frac{1}{2}\left(\hat{n}_{+45^o} - \hat{n}_{-45^o}\right)$$

$$\hat{S}_3 = \frac{1}{2}\left(\hat{n}_+ - \hat{n}_-\right)$$

$$\hat{S}_1 = S_1 = \frac{N_{\rm ph}}{2}$$

$$\left[\hat{S}_2, \hat{S}_3\right] = \frac{N_{\rm ph}}{2}\mathrm{i}$$

Atom-Light interaces: Quantum non-demolition detection

Light beam x-polarized: Stokes operators (Schwinger representation)

$$\hat{S}_1 = \frac{1}{2}(\hat{n}_x - \hat{n}_y)$$

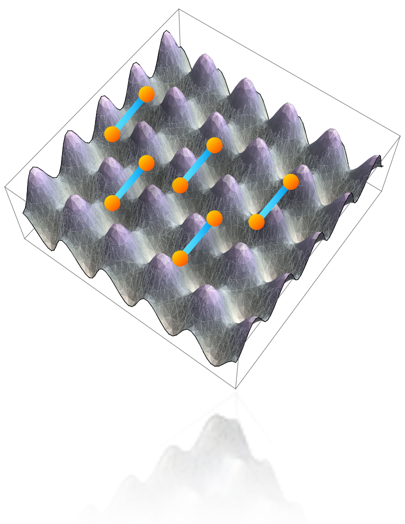
$$\hat{S}_2 = \frac{1}{2}(\hat{n}_{+45^o} - \hat{n}_{-45^o})$$

$$\hat{S}_3 = \frac{1}{2}(\hat{n}_+ - \hat{n}_-)$$

$$\hat{S}_1 = S_1 = \frac{N_{\mathrm{ph}}}{2}$$

$$\left[\hat{S}_2, \hat{S}_3\right] = \frac{N_{\mathrm{ph}}}{2}\mathrm{i}$$

Atoms: collective pseudo-spin



$$\hat{\mathbf{J}} = (\hat{J}_x, \hat{J}_y, \hat{J}_z) = \sum_{n=1}^{N_{at}} \mathbf{j}_n$$

$$\hat{H}_{in} \propto \hat{S}_3 \hat{J}_z$$

Quantum non-demolition detection: QPS

Faraday Interaction: Off-resonant dipole interaction between the light and atoms of the sample maps light polarization into spin polarization and viceversa

$$\hat{H}_{in} \propto \hat{S}_3 \hat{J}_z \longrightarrow \hat{S}_2^{\,out} = \hat{S}_2^{\,in} + \kappa_P \hat{J}_z$$

1.- If the spin Hamiltonian $[H_s, J_z] = 0 \Longrightarrow$ The measure is Non Demolishing

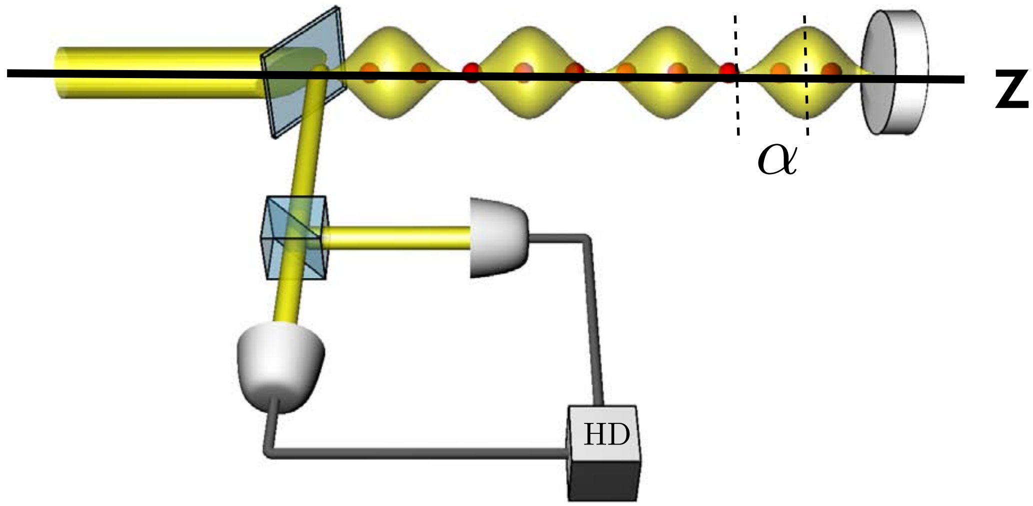
2. The mapping is quantum: fluctuations of matter into fluctuations of light (and viceversa)

$$\langle(\Delta \hat{S}_2^{\text{out}})^2\rangle = \langle(\Delta \hat{S}_2^{\text{in}})^2\rangle + \kappa_P^2 \langle(\Delta \hat{J}_z)^2\rangle$$

Photon shot noise   Spin Correlations

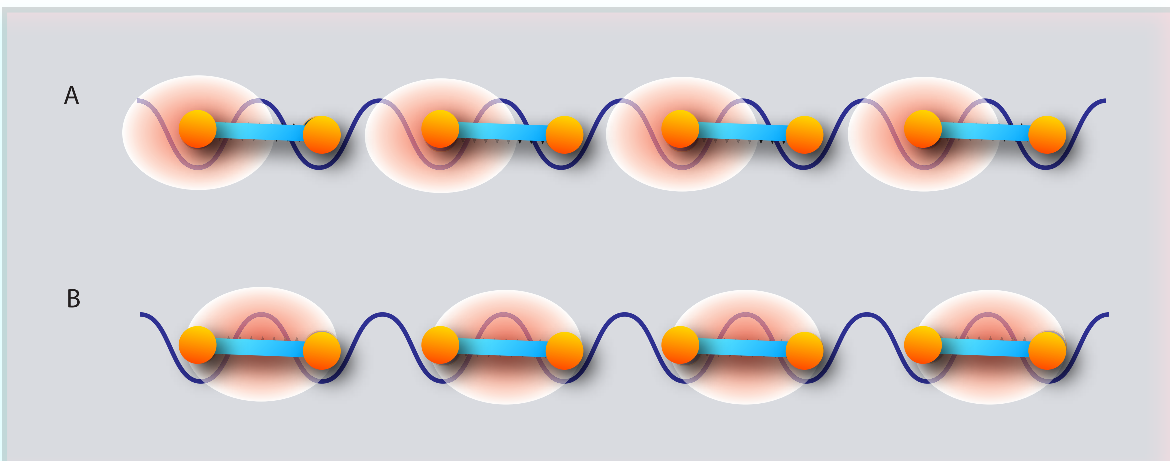
QND detection with spatial resolution in free space (no CAVITY)

Light beam x-polarized in a standing wave configuration



$$\langle \hat{J}_z^{eff} \rangle = \sum_{i=1}^{N} \cos^2[k_p(z_i - \alpha)] \langle \hat{j}_z^i \rangle$$

$$\langle (\Delta \hat{J}_z^{eff})^2 \rangle = \sum_{i,j=1}^{N} \cos^2[k_p(z_i - \alpha)] \cos^2[k_p(z_i - \alpha)] (\langle \hat{j}_z^i \hat{j}_z^j \rangle - \langle \hat{j}_z^i \rangle \langle \hat{j}_z^j \rangle)$$

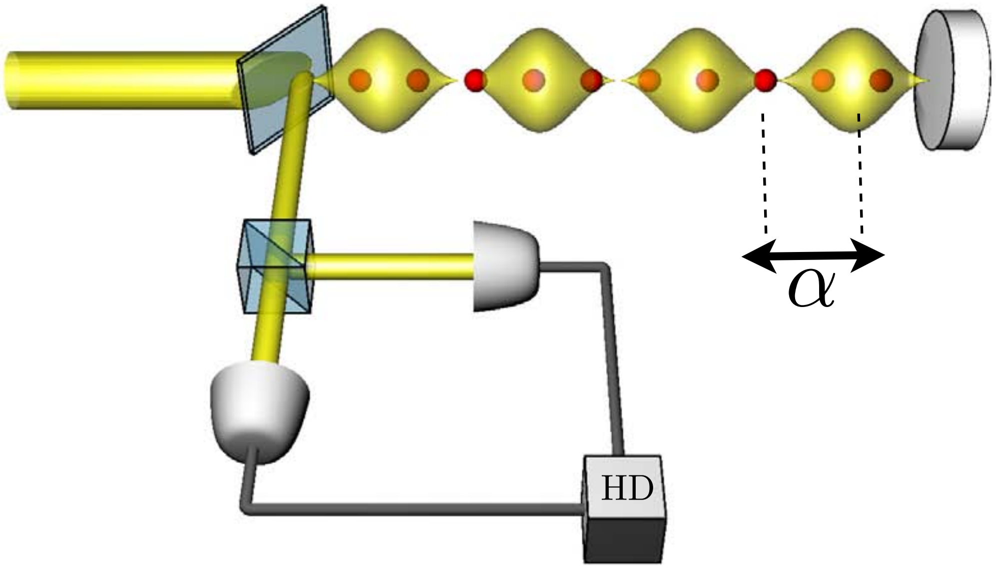


$$\langle \hat{J}_z^{\text{eff}} \rangle = 0$$
$$\langle (\Delta \hat{J}_z^{\text{eff}})^2 \rangle \neq 0$$

$$\langle \hat{J}_z^{\text{eff}} \rangle = 0$$
$$\langle (\Delta \hat{J}_z^{\text{eff}})^2 \rangle = 0$$

“Quantum non demolition detection of strongly correlated systems”, Nature Physics **4**, 50-54 (2008)

QND of strongly correlated systems



$$c_n = 2\cos^2[k_P d(n-\alpha)]$$

$$\varepsilon(k_P,\alpha) \equiv (\Delta J_z^{eff})^2 = \frac{1}{L}\sum_{nm} c_m c_n(\langle S_{zm}S_{zn}\rangle - \langle S_{zm}\rangle\langle S_{zn}\rangle)$$

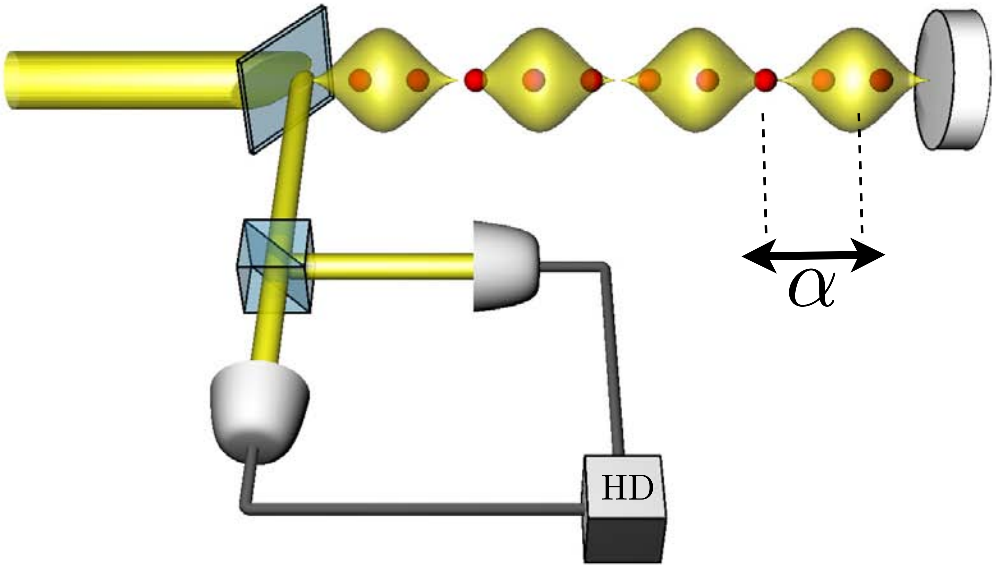
Structure factor

$$S(q) = 1/L\sum_{mn}\exp[iqd(m-n)]\langle S_{zm}S_{zn}\rangle$$

$$\bar{\varepsilon}(k_P) \equiv \int d\alpha\varepsilon(k_P,\alpha) = \frac{1}{2}S(2k_P).$$

T. Roscilde et al. NPJ(09)

QND of strongly correlated systems



$$c_n = 2\cos^2[k_P d(n-\alpha)]$$

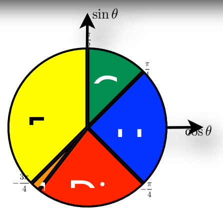
Spatial Resolution permits to detect spin correlations

Examples:

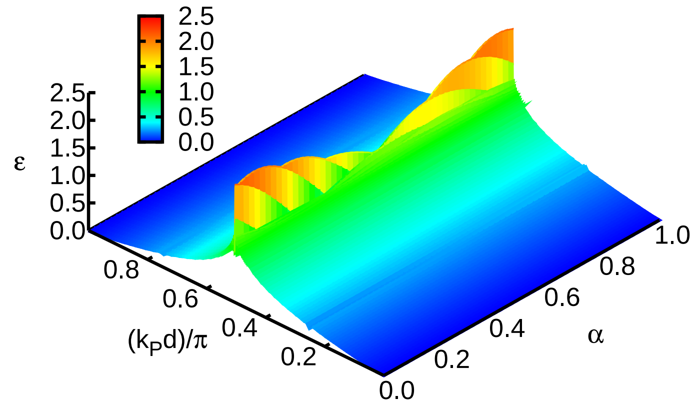
1- Non Magnetic phases in Bilinear Biquadratic spin 1chain
2- FFLO phase in Imbalance Fermi Systems

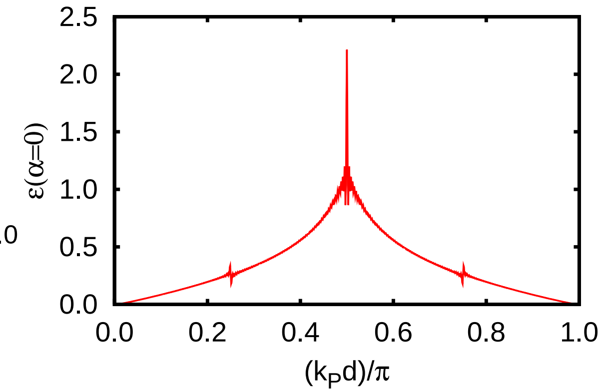
T. Roscilde et al. NPJ(09)

Example 1: Antiferromagnetic phases in the bilinear biquadratic spin 1

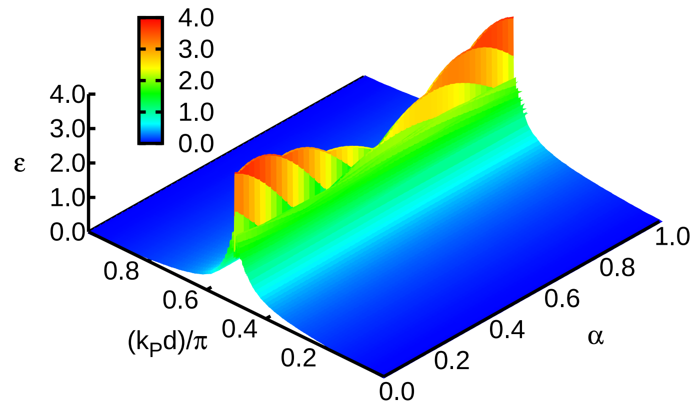


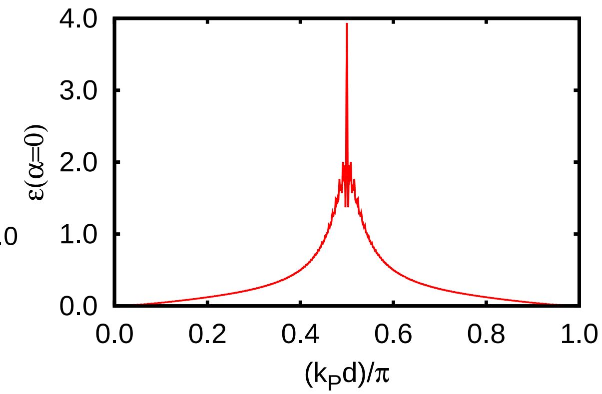
Dimer



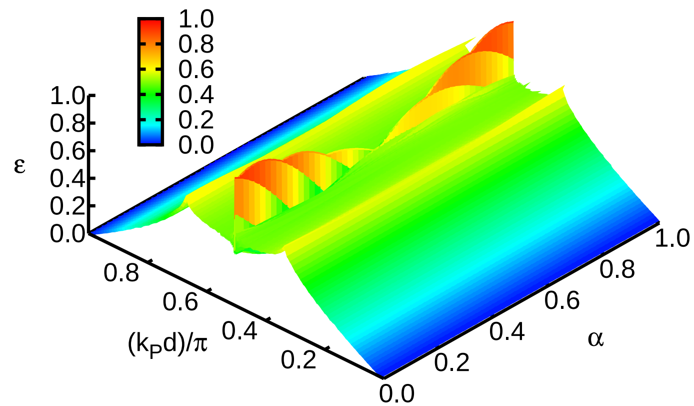


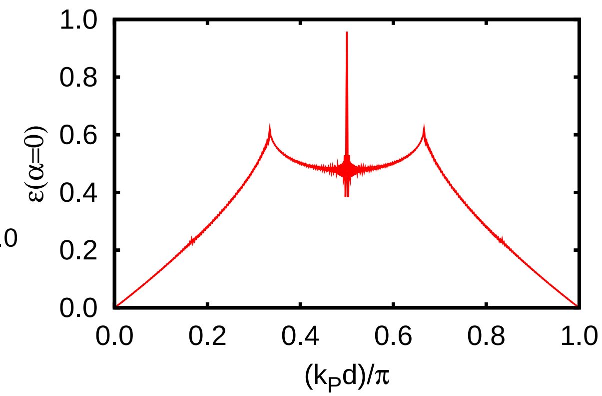
Haldane



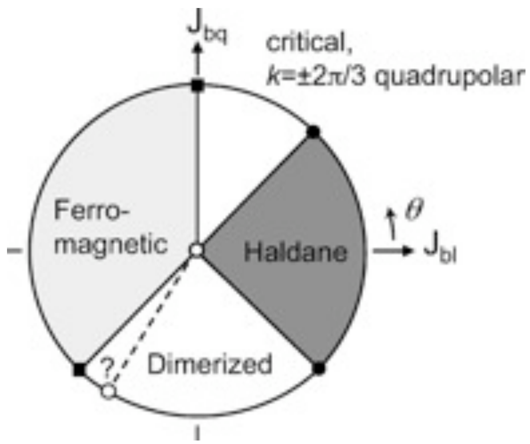


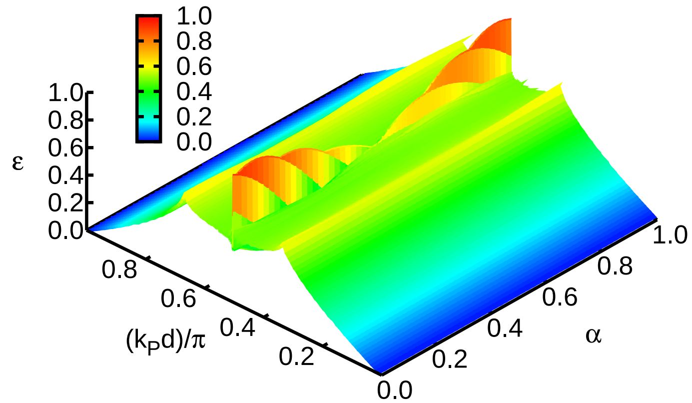
Critical

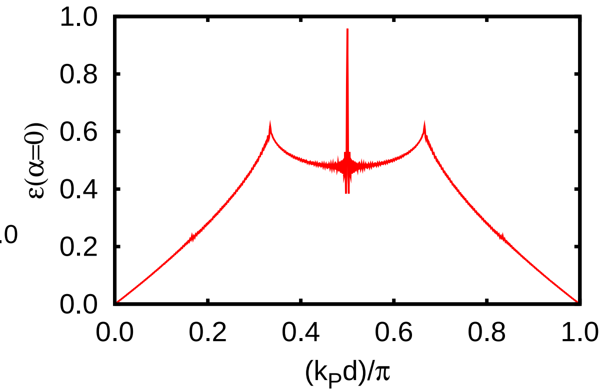




Detection quantum phases in the spin 1-Chain





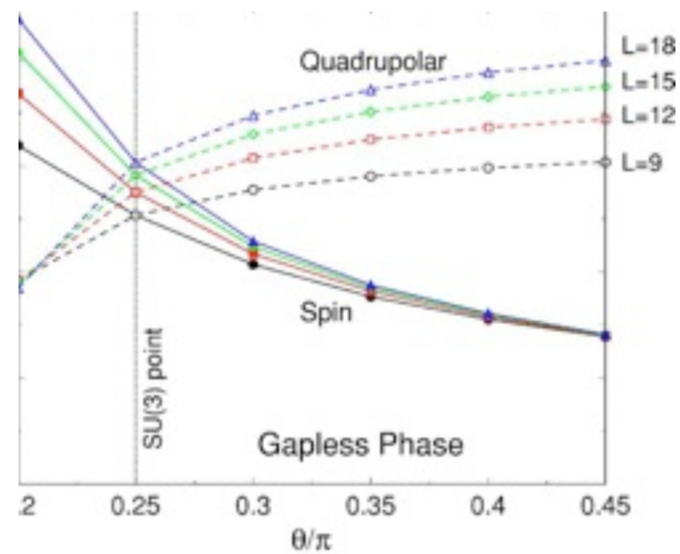


The enigmatic (critical) 3 period- phase

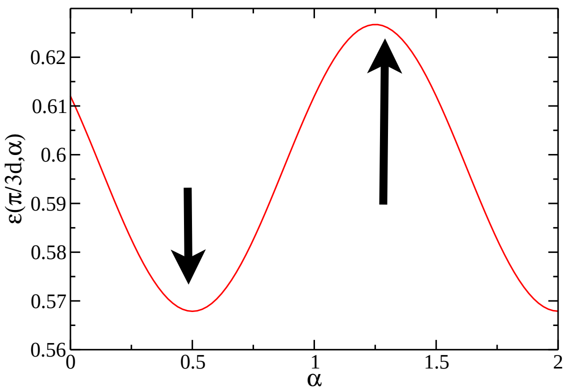
A. Lauchli, G. Schmidt, and S. Trebs PRB (2006)

"We clarify the dominant correlation function in the enigmatic 3 period phase which is spin quadrupolar"

$$\langle S^2(k) S^2(-k) \rangle$$



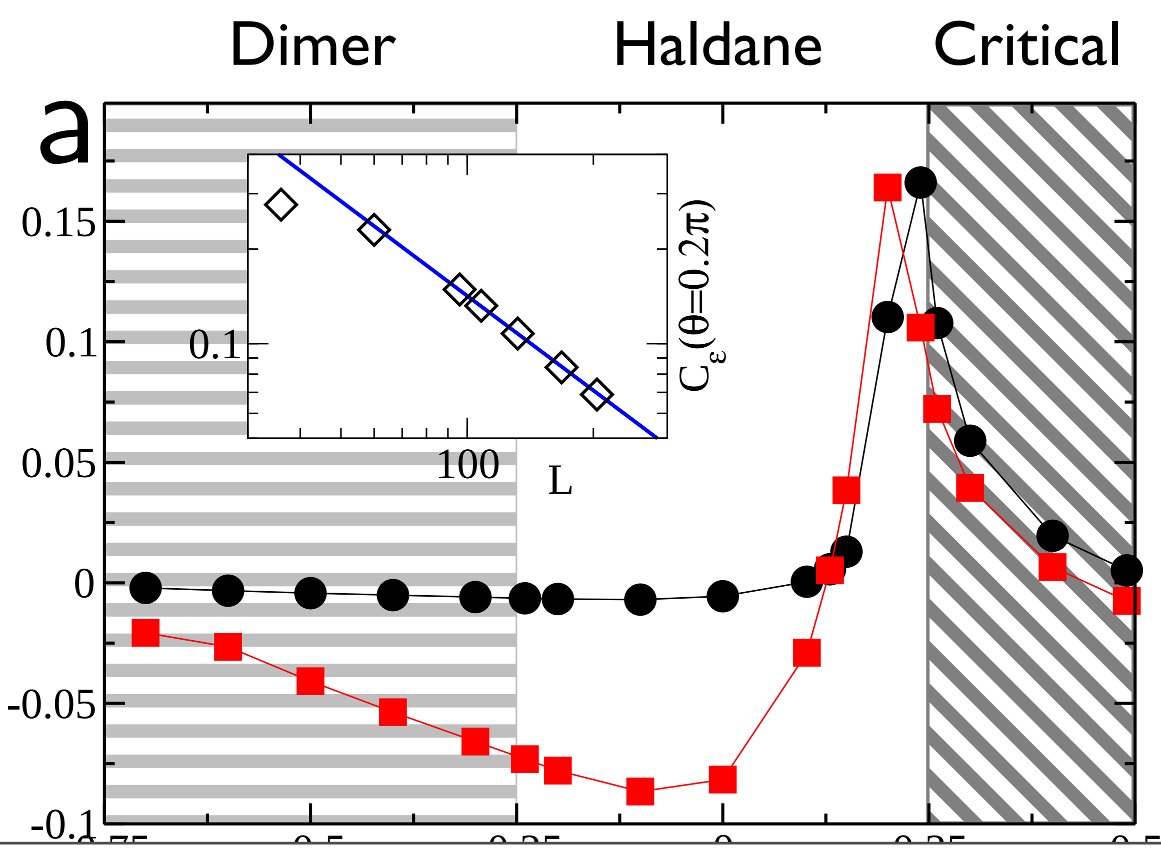
Detection quantum phases in the spin 1-Chain



$$C_\varepsilon = \Delta\varepsilon(k_P, \alpha_1, \alpha_2) \equiv \varepsilon(k_P, \alpha_1) - \varepsilon(k_P, \alpha_2)$$

$$C_\varepsilon = \varepsilon(2\pi/3d, 5/4) - \varepsilon(2\pi/3d, 1/2)$$

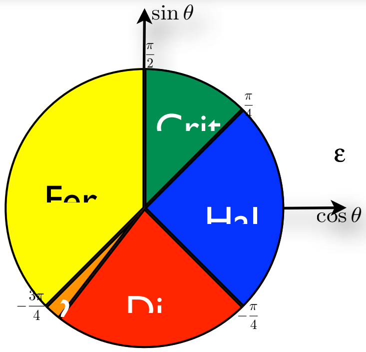
$$\bullet\ \mathcal{C}_\varepsilon = \frac{1}{L}\sum_{mn} \cos\left[\frac{2\pi}{3}(m+n) + \frac{\pi}{3}\right]\langle S^z_m S^z_n \rangle$$

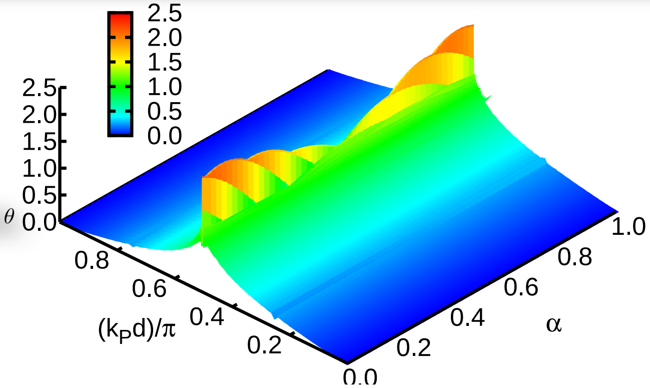


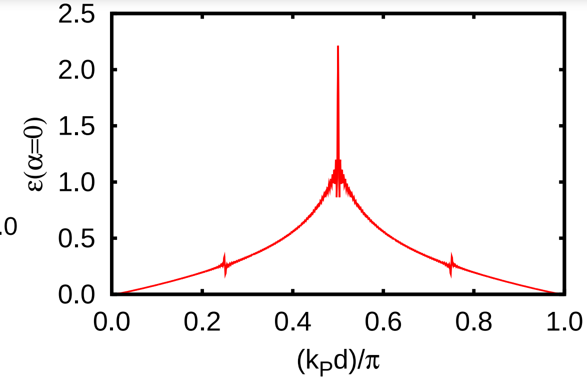
Structure Factor

$$[S(2\pi/3d) - 1]$$

Detection quantum phases in the spin 1- Chaim



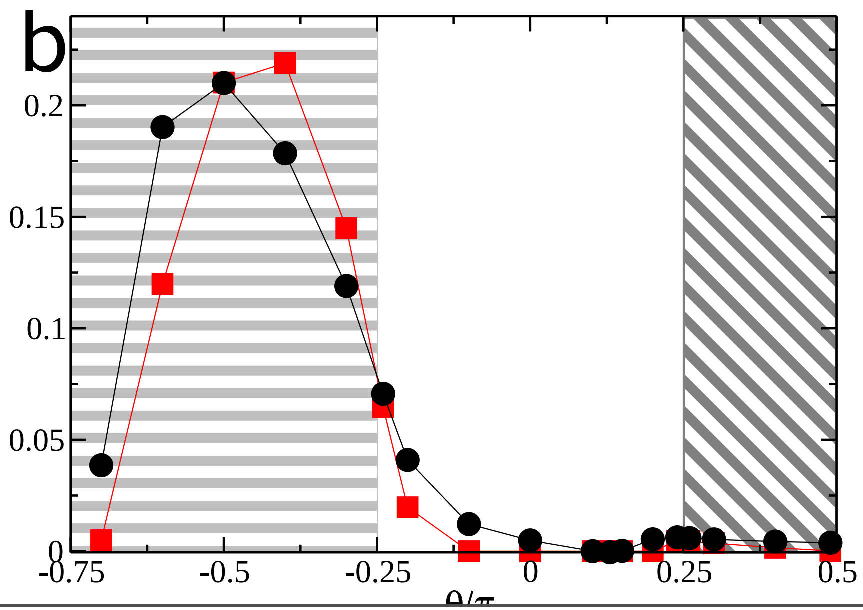




Dimerized Phase

$$\bullet\ \mathcal{D}_\varepsilon \equiv \Delta\varepsilon\left(\frac{\pi}{4d},\frac{1}{2},\frac{3}{2}\right) = -\frac{1}{L}\sum_{mn}\sin\left[\frac{\pi}{2}(m+n)\right]\langle S_m^z S_n^z\rangle$$

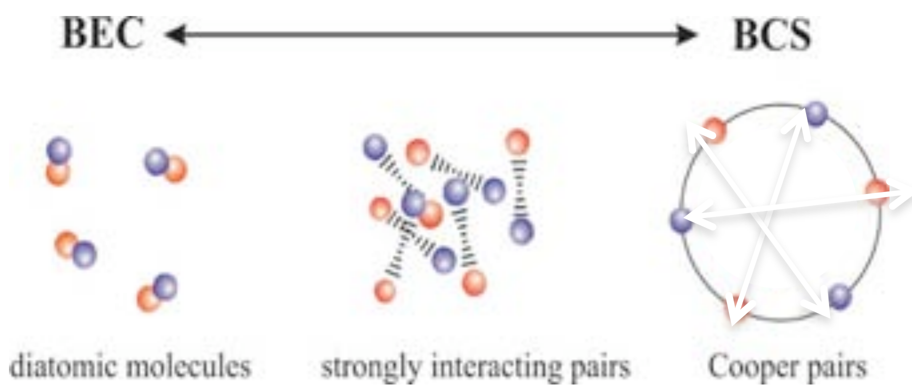
Dimer Haldane Critical

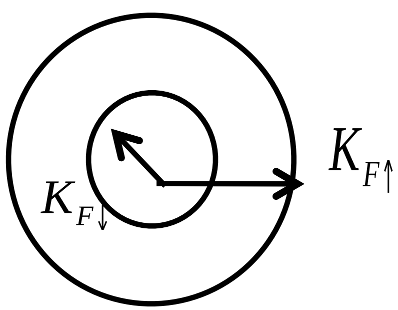


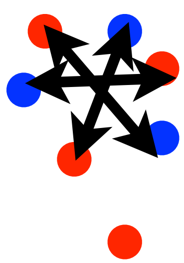
Dimer order parameter

$$\blacksquare\ D = \sum_i (\langle H_{i,i+1}\rangle - \langle H_{i+1,i+2}\rangle)$$

Example 2: Spin-Spin correlations in Imbalance Fermi systems

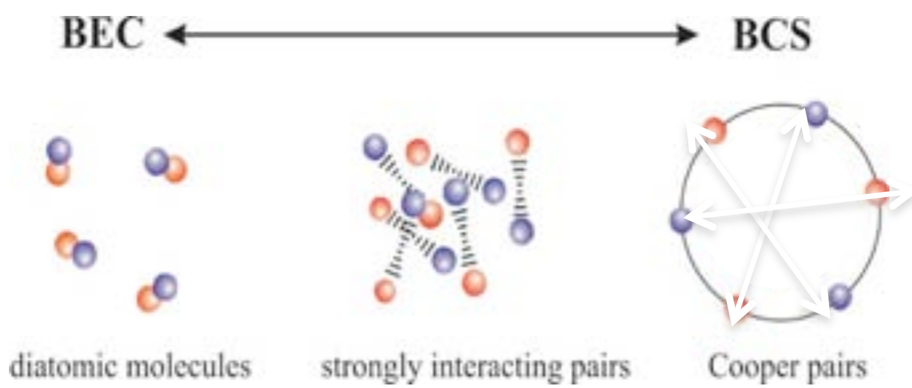




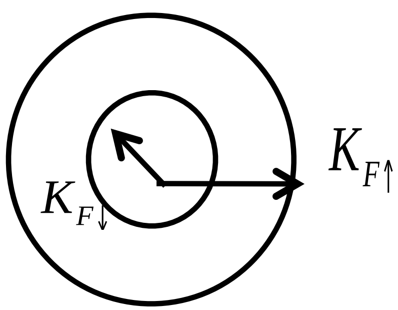


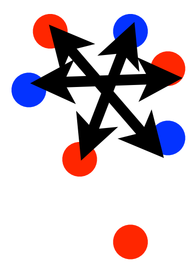
F. & F Phys. Rev 135 (64)
L & O JEPT 20 (64)

Example 2: Spin-Spin correlations in Imbalance Fermi systems



Problem: **Imbalance Fermi mixture :** 2 alternative pictures





F. & F Phys. Rev 135 (64)
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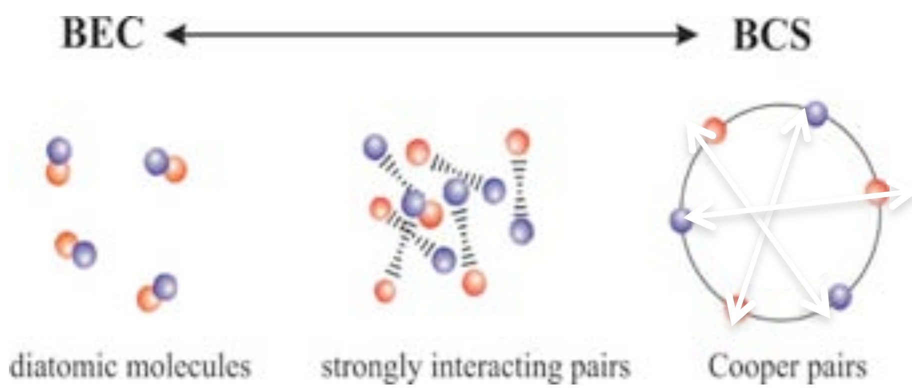
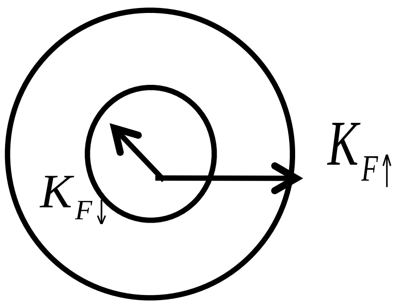
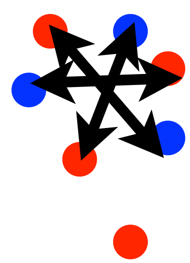
Example 2: Spin-Spin correlations in Imbalance Fermi systems



Problem: **Imbalance Fermi mixture :** 2 alternative pictures

(i) FFLO phase: Cooper pairs with finite momentum $Q = | K_{F\uparrow} - K_{F\downarrow} |$





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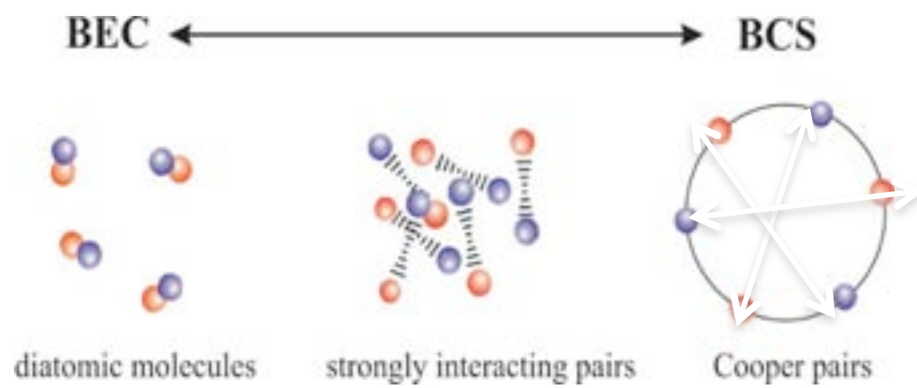
Example 2: Spin-Spin correlations in Imbalance Fermi systems



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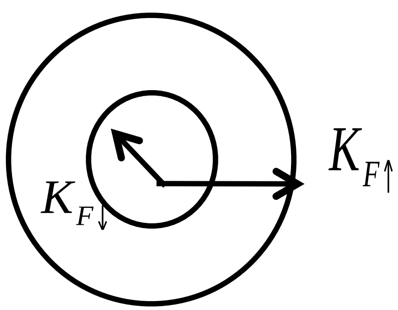

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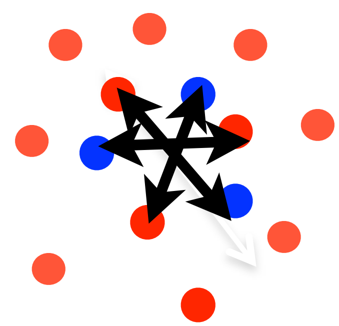
(ii) Breached Pairing: superfluid + normal translational invariant



Spin-Spin correlations in Imbalanced 1D Fermi systems

Imbalance Fermi mixture FFLO phase

As a consequence of the imbalance (unequal populations) fermions with atractive interactions will PAIR with a non-zero center of mass momentum $Q = | K_{F\uparrow} - K_{F\downarrow} |$ leading to spatial inhomogenities in the pairing and in the spin correlations.

$$H = -t \sum_{i,\sigma} \hat{c}^{+}_{i\sigma} \hat{c}_{i+1\sigma} + hc + U \sum_{i} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

$$Q = | K_{F\uparrow} - K_{F\downarrow} |$$

Spin-Spin correlations in Imbalanced 1D Fermi systems

Imbalance Fermi mixture FFLO phase

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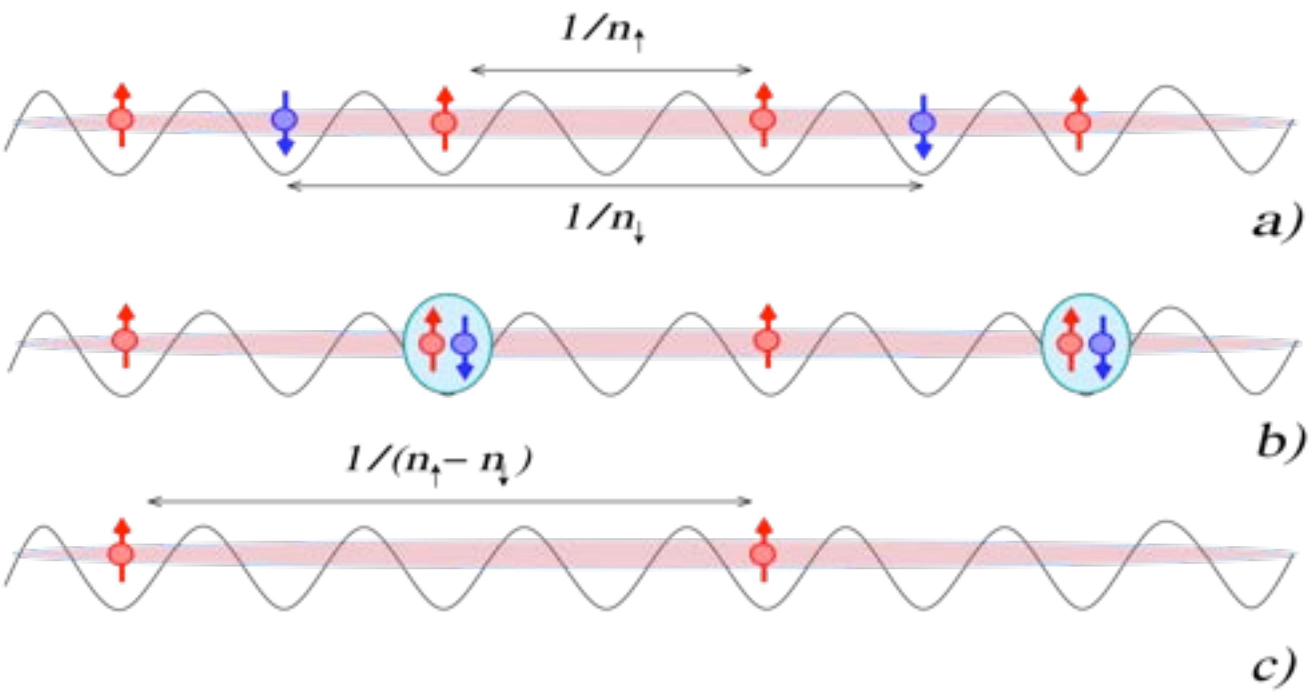
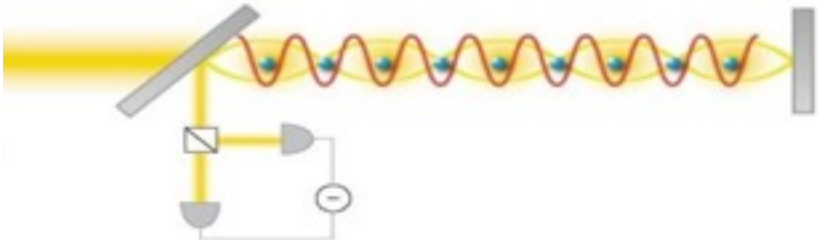
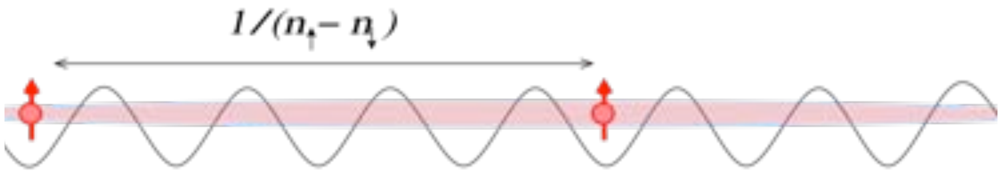


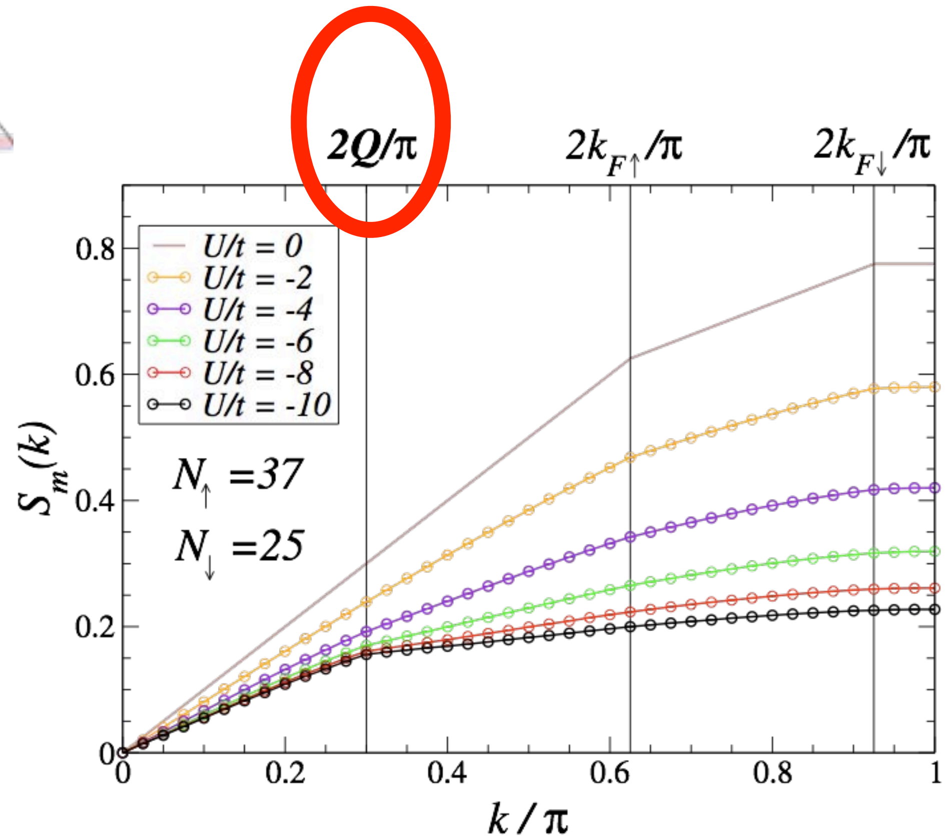
$Q = | K_{F\uparrow} - K_{F\downarrow} |$

Examples of QND: Spin-Spin correlations in Imbalance Fermi systems



$$2Q/\pi = (n_\uparrow - n_\downarrow)$$



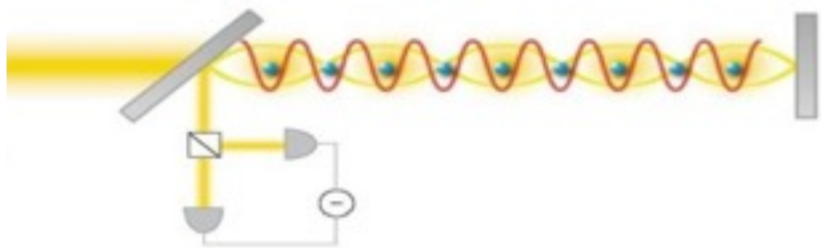


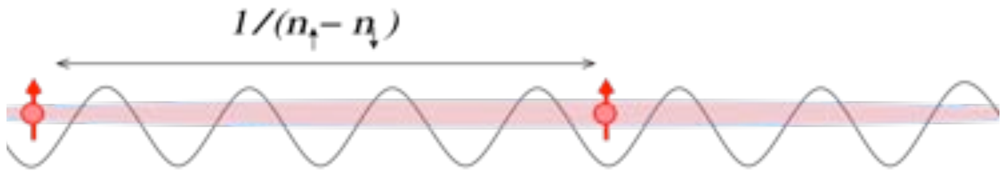
$$(\Delta S_y^{out}(\vec{k}))^2 = \frac{1}{2} + \kappa S_m(\vec{k})$$

T. Roscilde et al. NJP 2009

FFLO signature

Examples of QND: Spin-Spin correlations in Imbalance Fermi systems



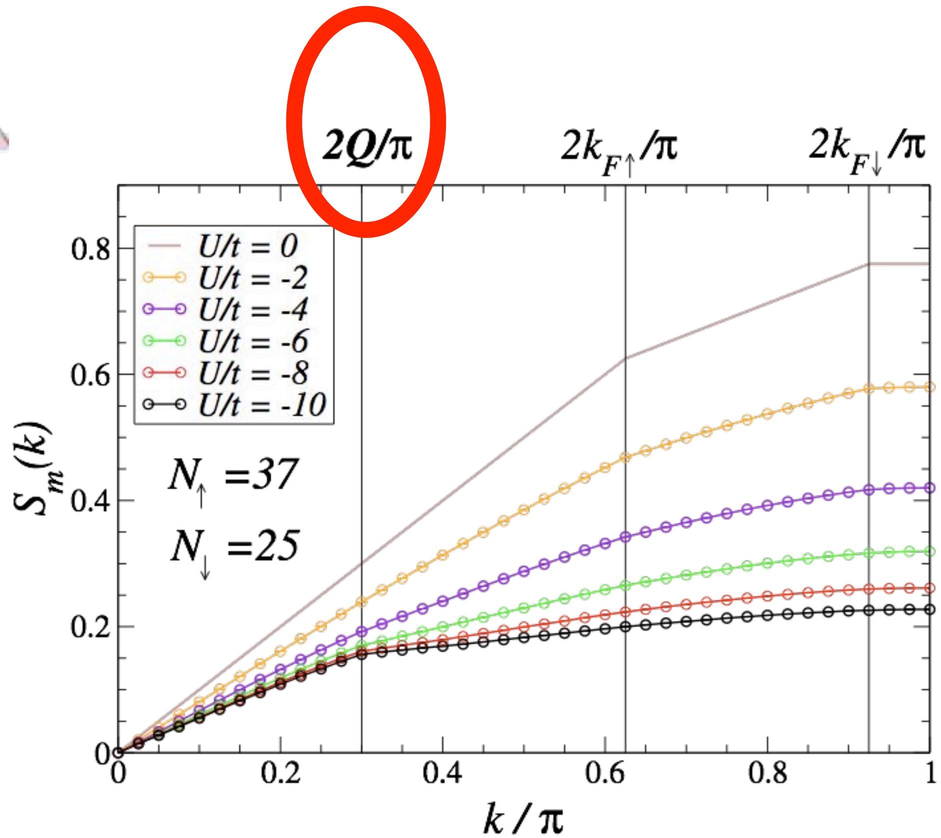


$$2Q/\pi = (n_\uparrow - n_\downarrow)$$

QND Faraday interface

$$\left\langle \left(\Delta S_y^{out}\right)^2 \right\rangle = \frac{1}{2} + \frac{\kappa^2}{2N} \left\langle \left(J_z - \langle J_z \rangle\right)^2 \right\rangle$$

$$(\Delta S_y^{out}(\vec{k}))^2 = \frac{1}{2} + \kappa S_m(\vec{k})$$



T. Roscilde et al. NJP 2009

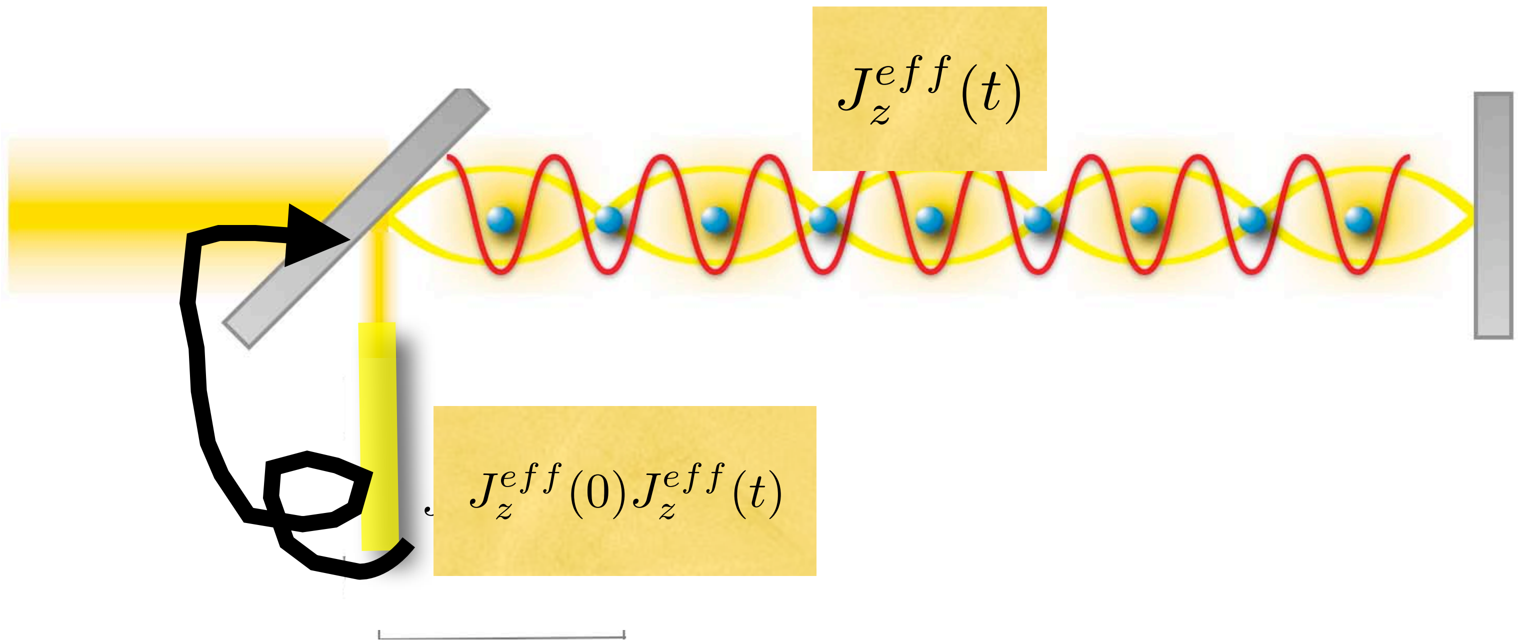
FFLO signature

Example 3: Time dependent correlations: learning the low excitation spectrum

Can we use the QPS scheme not only to detect the quantum phase but also **dynamical correlations?**

$$\langle \hat{J}(0)\hat{J}(t)\rangle$$

QUANTUM MEMORY SPECTROSCOPY



Time dependent correlations: learning the low excitation spectrum

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$$\langle \hat{J}(0)\hat{J}(t)\rangle$$

Can we use the QPS scheme not only to detect the quantum phase but also **dynamical correlations?**

$$\langle \hat{J}(0)\hat{J}(t)\rangle$$

$$[\hat{H}_s, \hat{A}] \neq 0$$

$$\begin{aligned} C_A(t) &= \langle \hat{A}(0)\hat{A}(t)\rangle \\ &= \sum_n e^{+i(E_0-E_n)t/\hbar} |\langle 0|\hat{A}|n\rangle|^2 \\ &= \sum_n e^{+i(E_0-E_n)t/\hbar} A_n^2 \end{aligned}$$

FOURIER the signal

$$\begin{aligned} C_A(\omega) &= \sum_n \int dt\, e^{i\omega t} C_A(t) \\ &= \sum_n A_n^2 \delta(\omega - (E_n - E_0)) \end{aligned}$$

Time dependent correlations: learning the low excitation spectrum

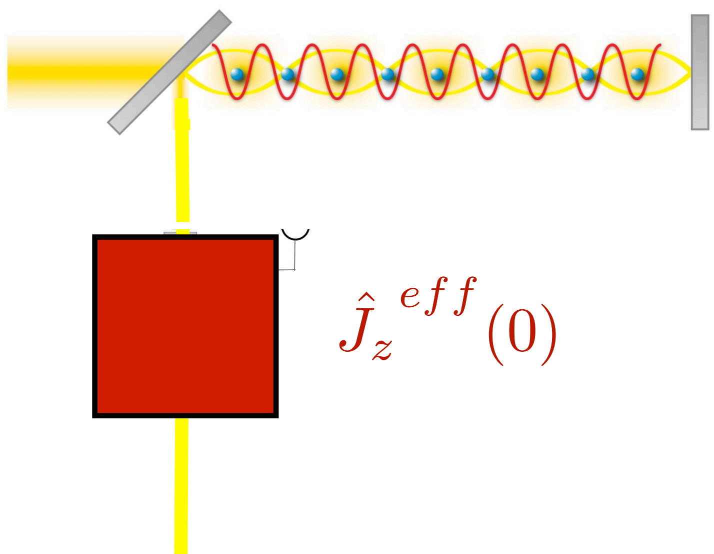
Protocol:

1. Time dependent correlation $\longleftrightarrow$ $C_J(t) = \langle \hat{J}_z^{eff}(0)\hat{J}_z^{eff}(t)\rangle$

2. Quantum Non Demolition + spatial resolution $\longleftrightarrow$ $[\hat{H}_s, \hat{J}_z^{eff}] \neq 0$

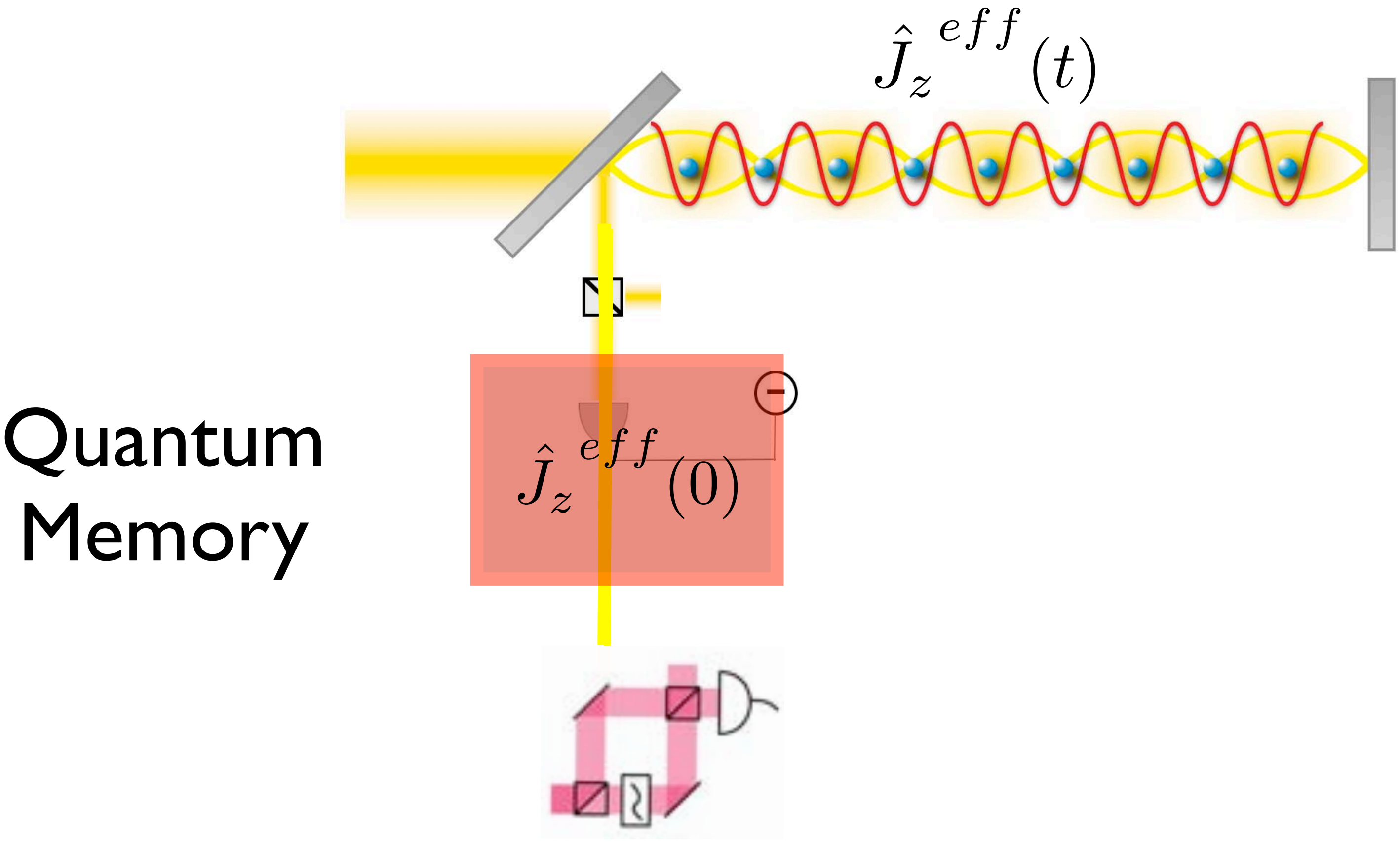
3. Map system correlations into a Quantum Memory via light interface

Quantum Memory



Time dependent correlations: learning the low excitation spectrum

4. Second beam + QND measurement at t=T: map system correlations into the Quantum Memory via light interface, measure the outgoing light



$$\langle(\Delta X_2^{out})^2\rangle = \frac{1}{2} + \kappa_1\kappa_1\kappa_3\kappa_4\langle\{J_z^{ef}(0), J_z^{ef}(t)\}\rangle$$

TOY MODEL: A lattice of plaquettes of spin 1/2

$$H = J \sum_{i=1}^{4} \vec{S}_i \vec{S}_{i+1} = \frac{1}{2}(\vec{S}_T^2 - \vec{S}_{13}^2 - \vec{S}_{24}^2)$$

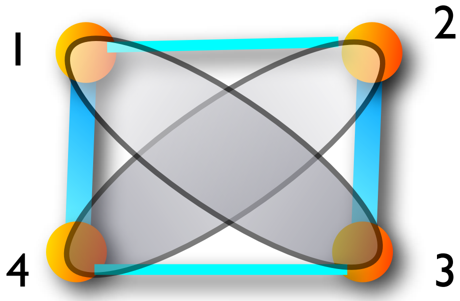
$$(1/2) \otimes (1/2) \otimes (1/2) \otimes (1/2) = 2 \oplus 1 \oplus 0$$

Spectrum

$$\begin{aligned} E_0 &= -2 \\ E_1 &= -1 \\ E_2 &= 0 \\ E_3 &= 1 \end{aligned}$$

Time dependent correlations: learning the low excitation spectrum

TOY MODEL



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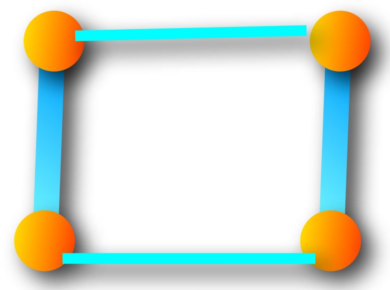

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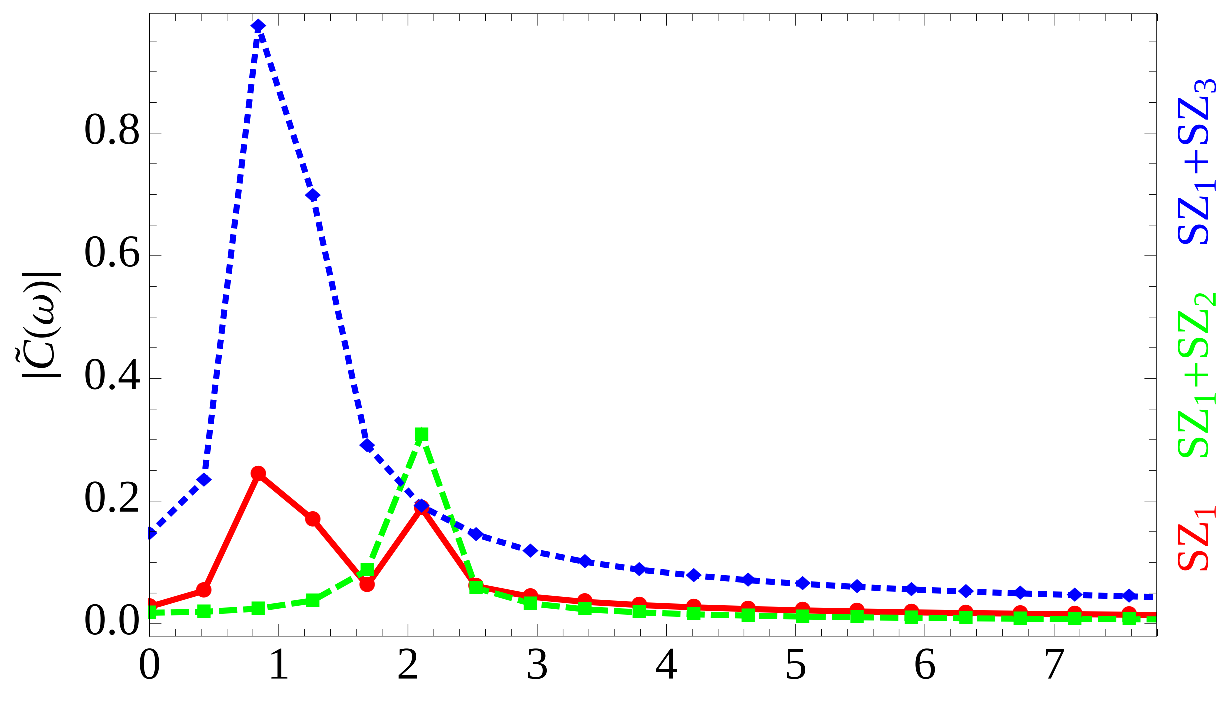

Time dependent correlations: learning the low excitation spectrum

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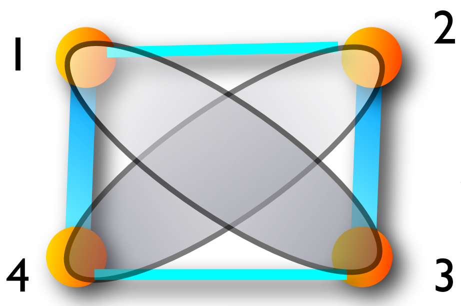

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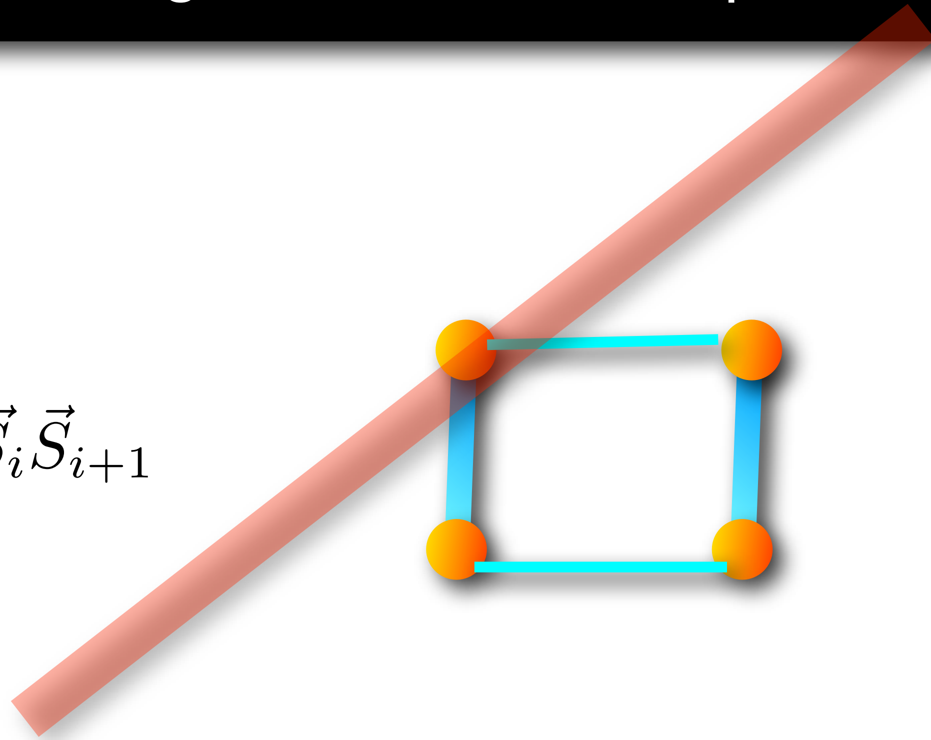

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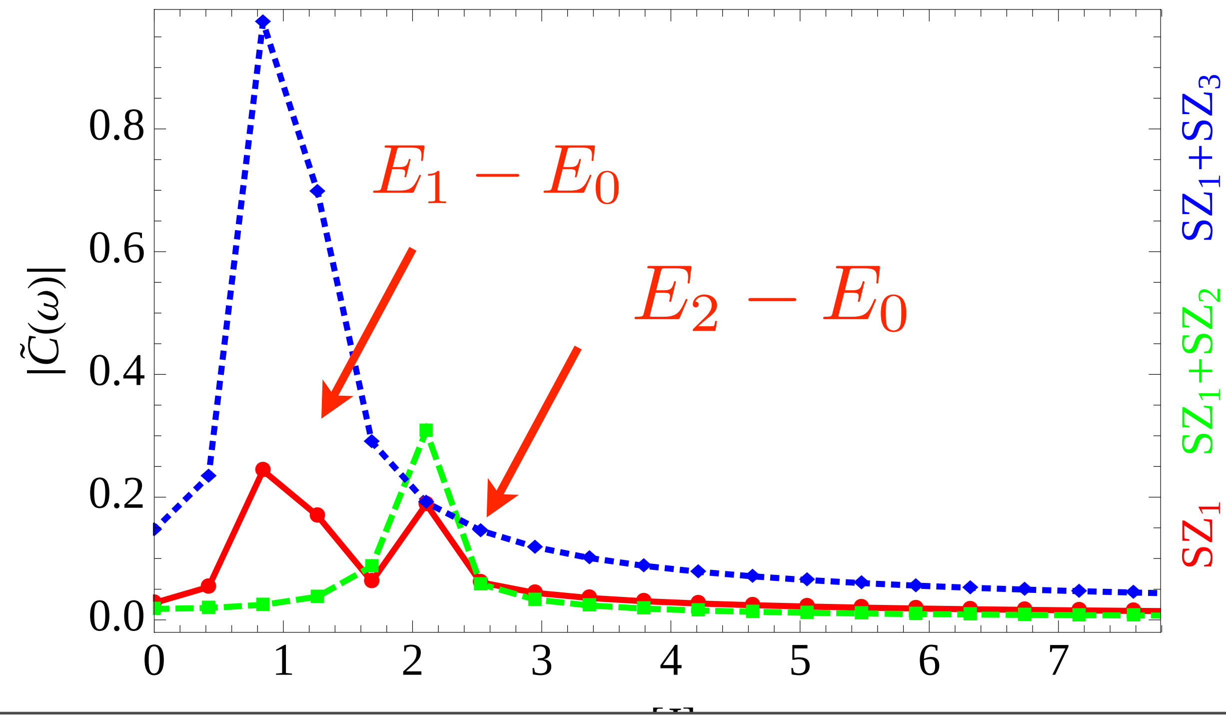

Time dependent correlations: learning the low excitation spectrum

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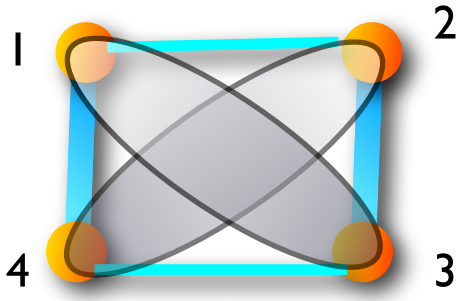

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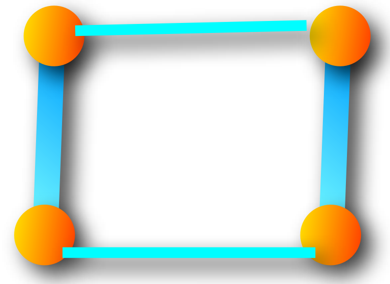

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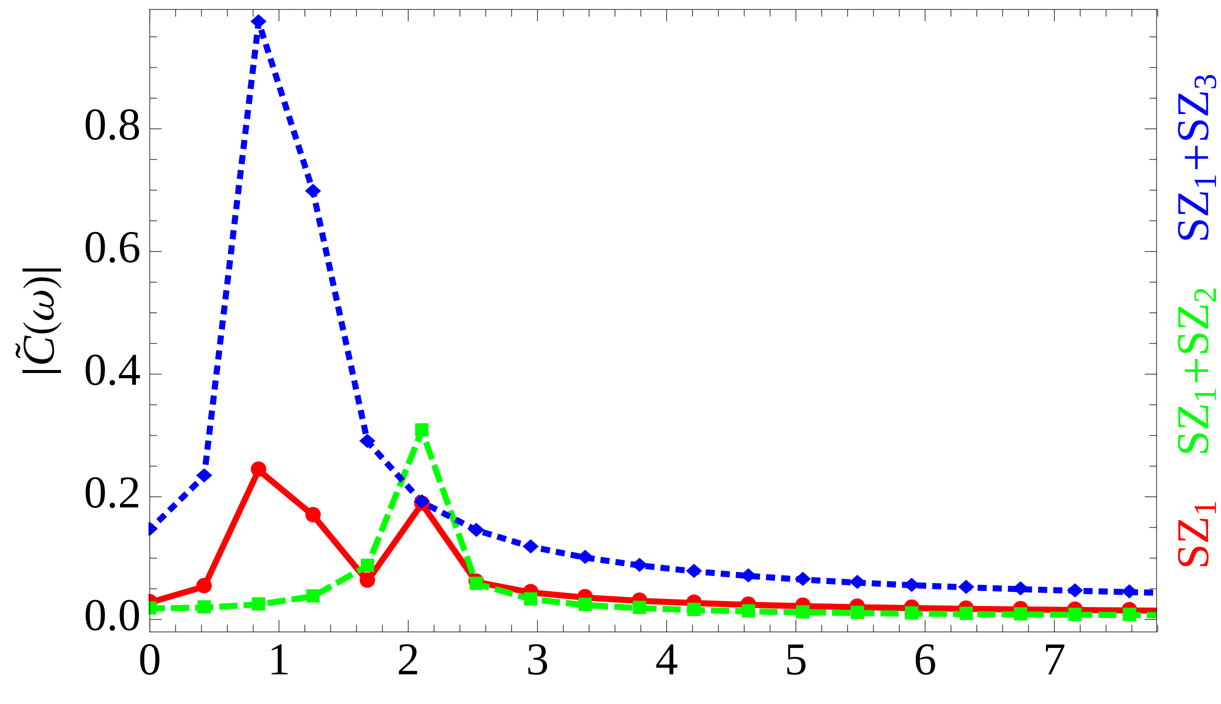

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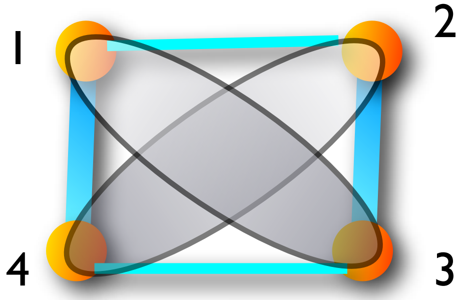

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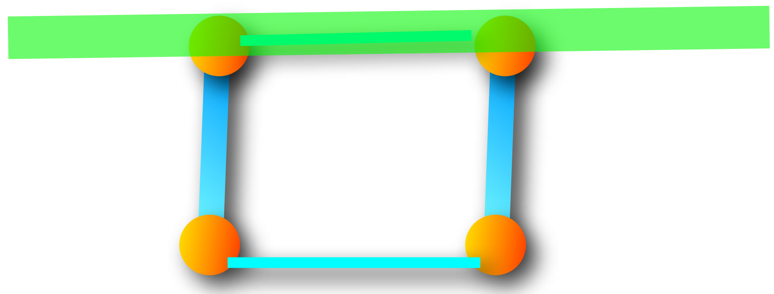

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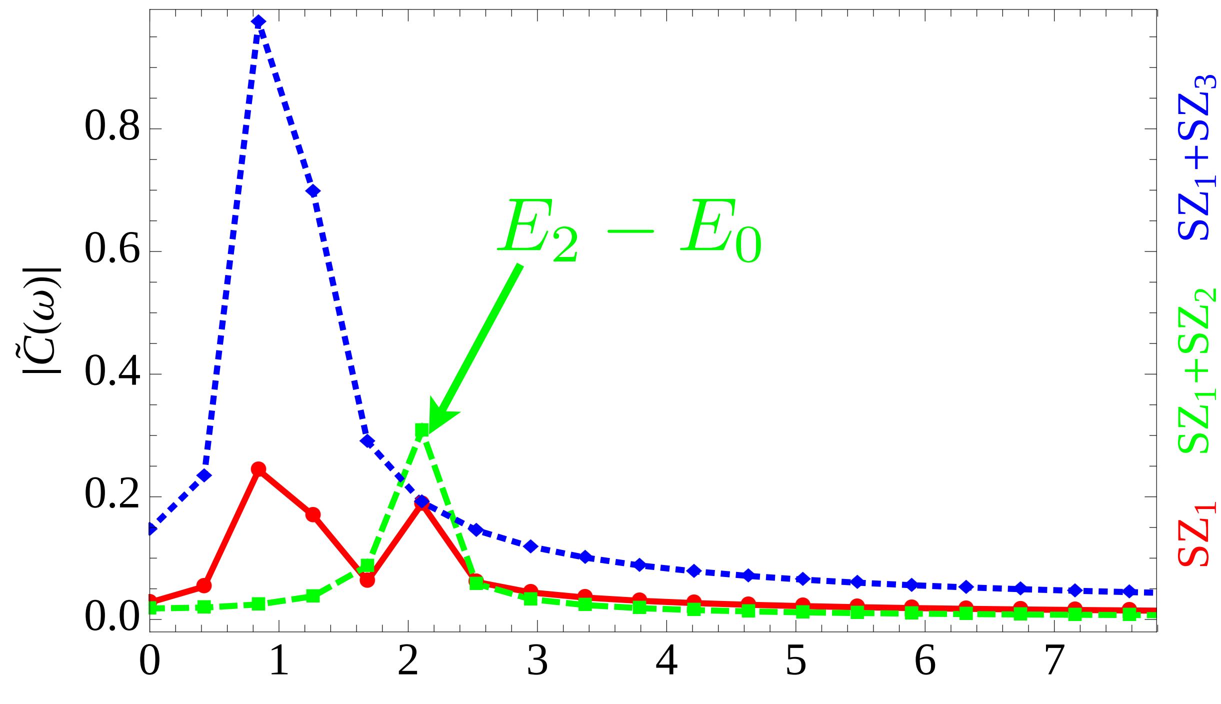
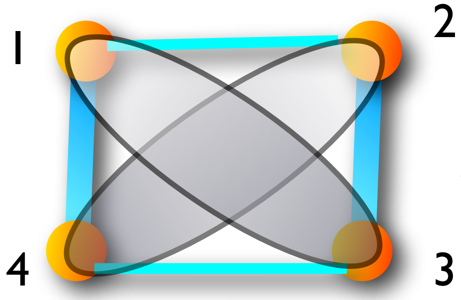


Time dependent correlations: learning the low excitation spectrum

TOY MODEL



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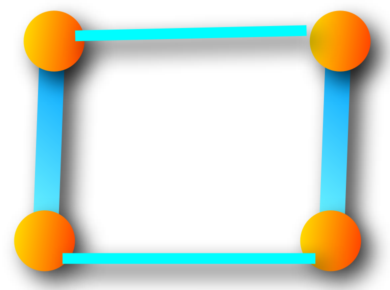

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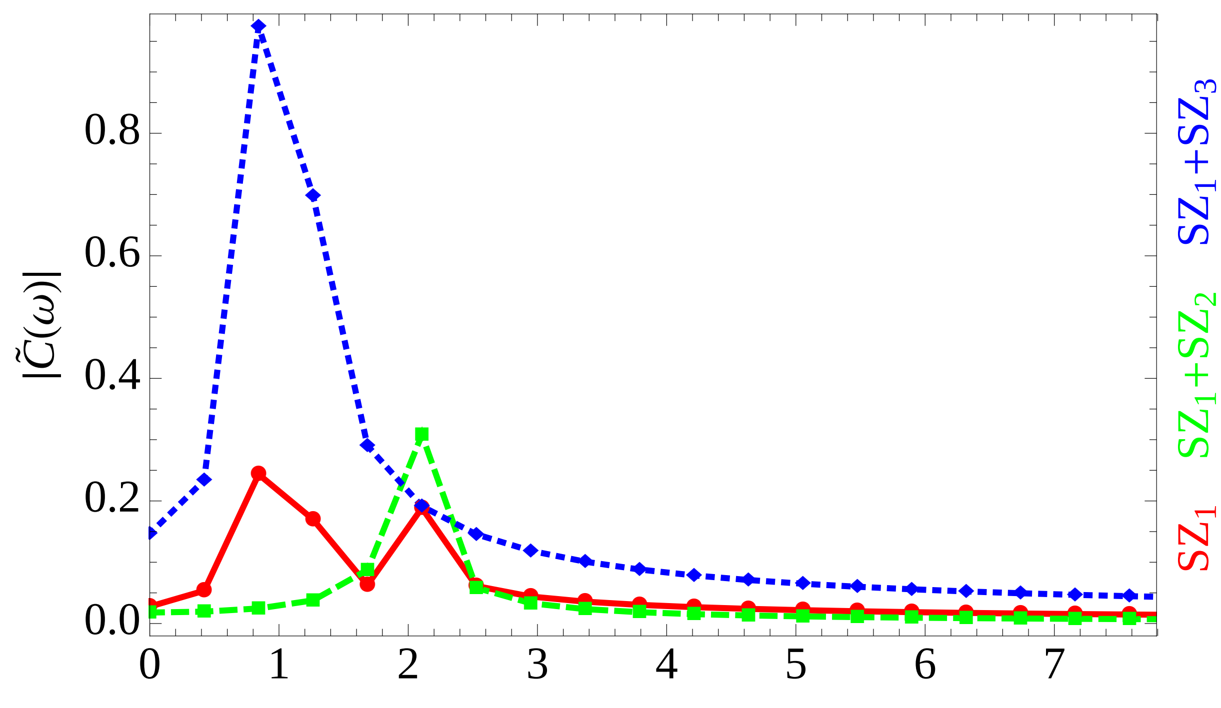

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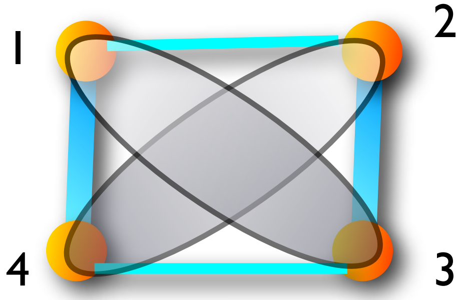

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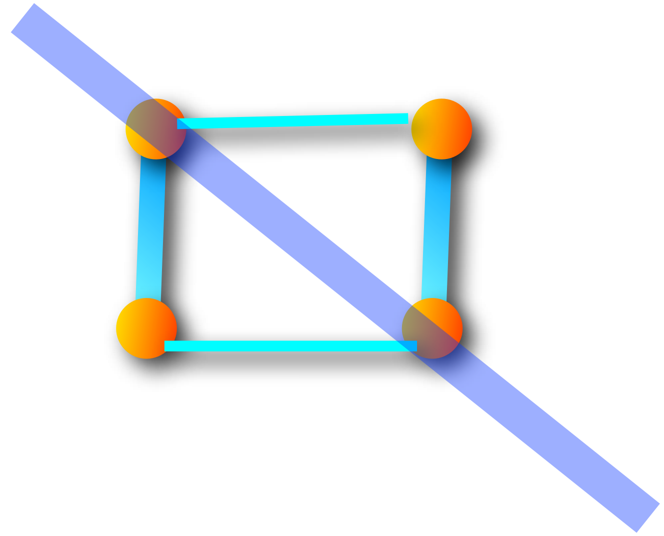

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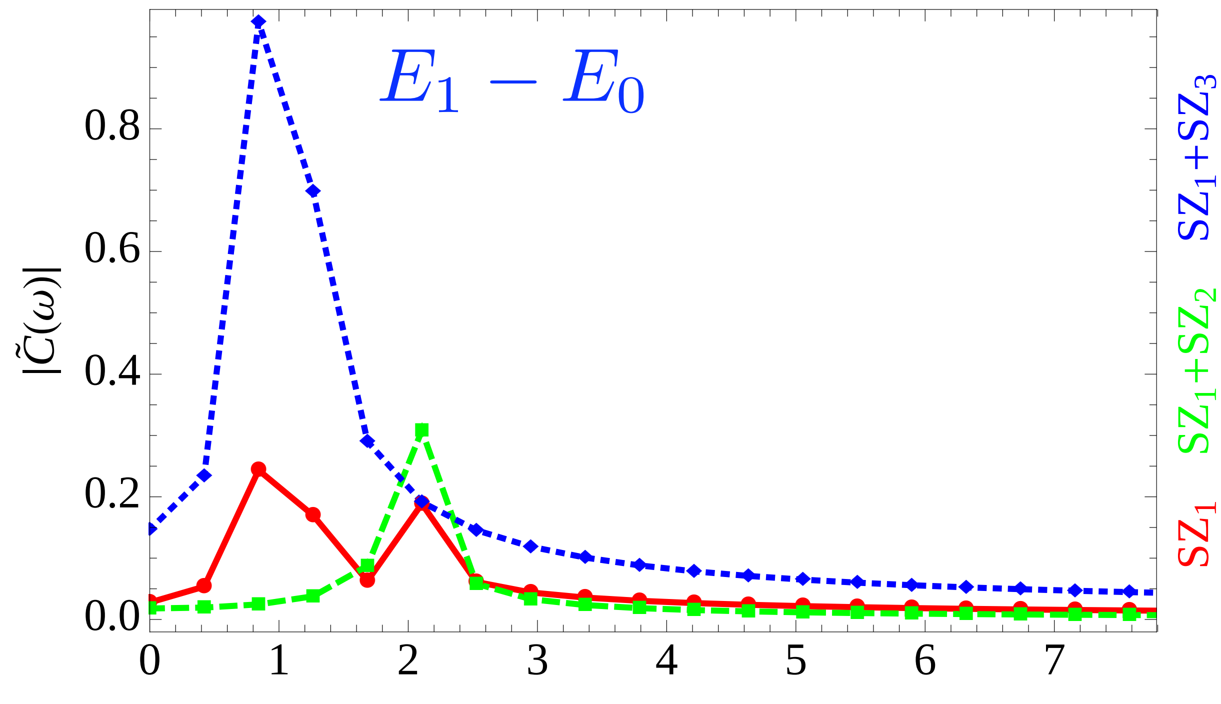

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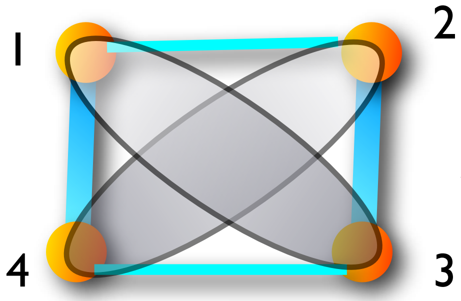

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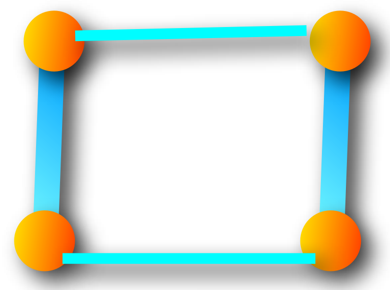

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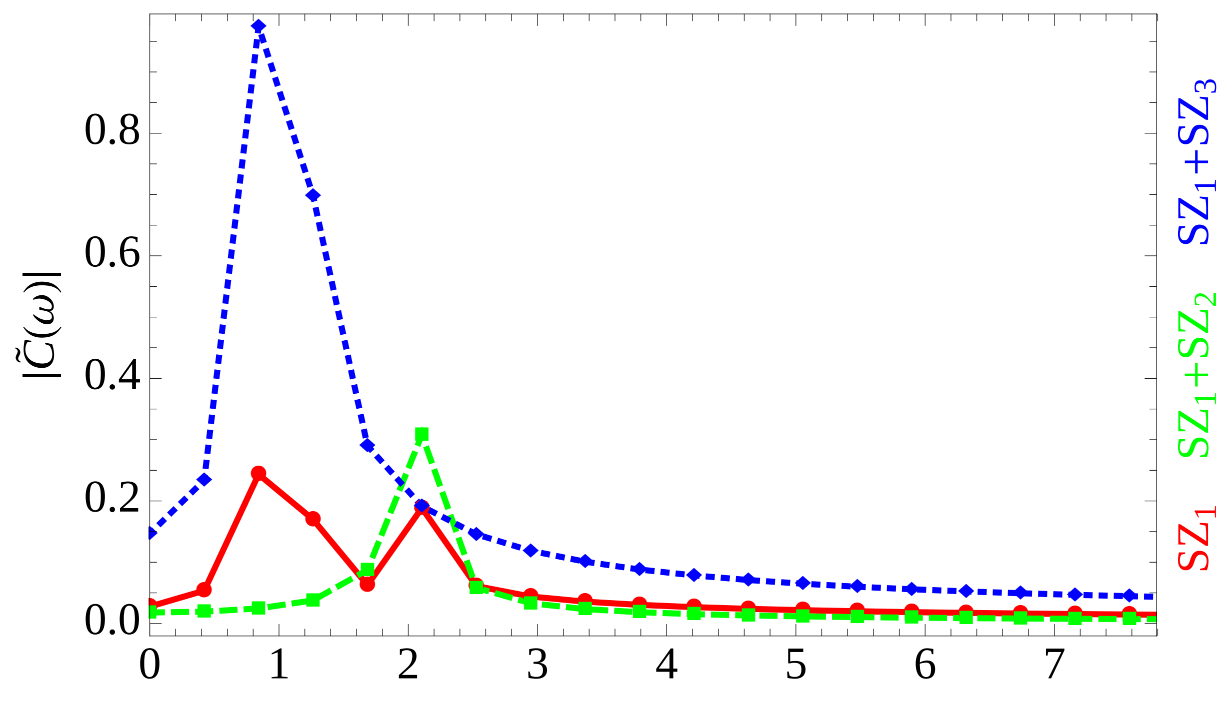
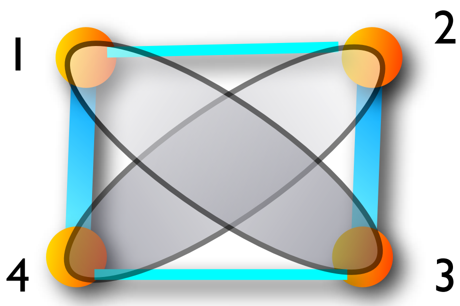


Time dependent correlations: learning the low excitation spectrum

TOY MODEL



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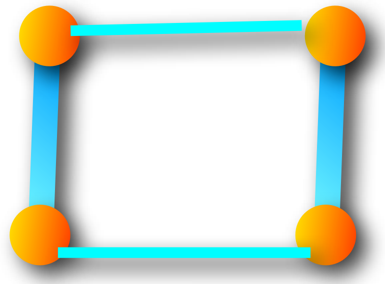

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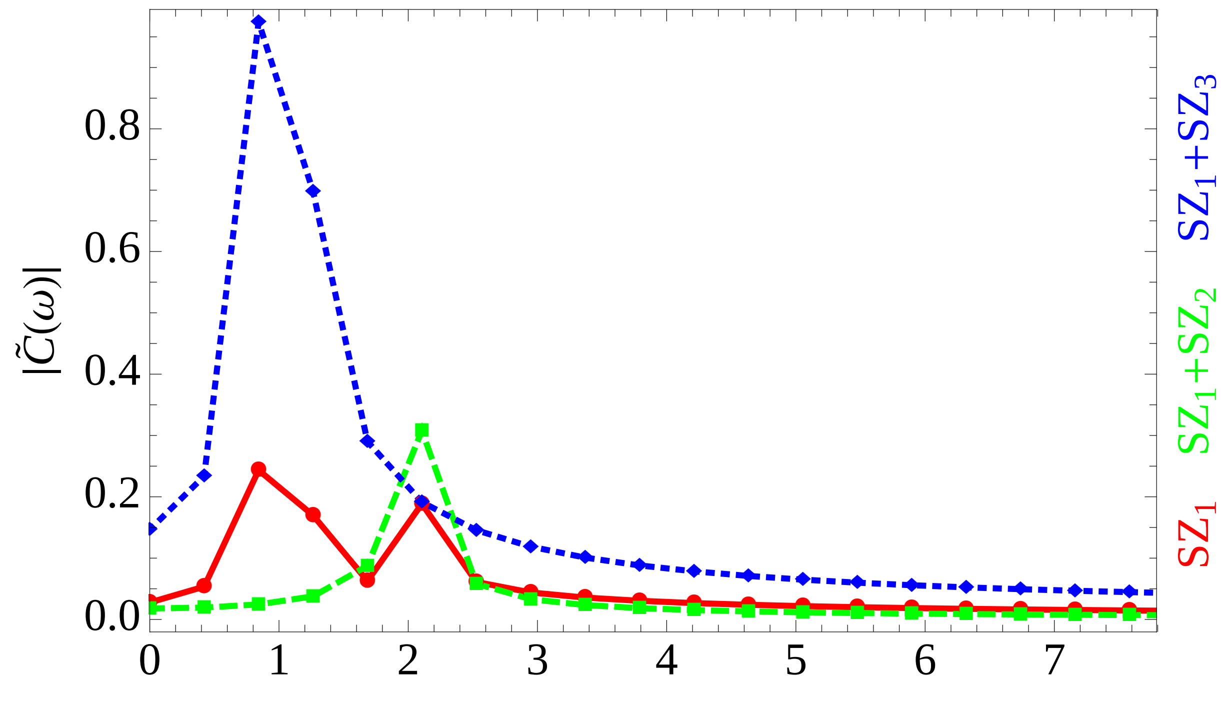


It works !

Summary

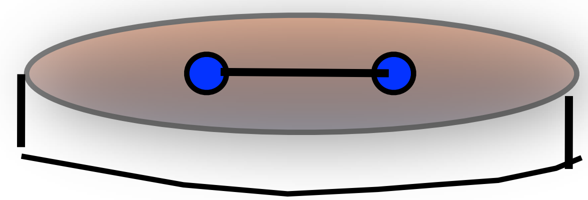
Dynamics: Entanglement transfer in many body quantum systems can be used as a resource

Quantum Faraday Detection can be used to detect in and out of equilibrium many body systems

p-wave superfluidity in the lattice

➡Topological order versus confining geometry:

- ✴ pairing ordering in 2D



Read & Green, PRB 61, 10267(2000)

p-wave superfluidity in the lattice

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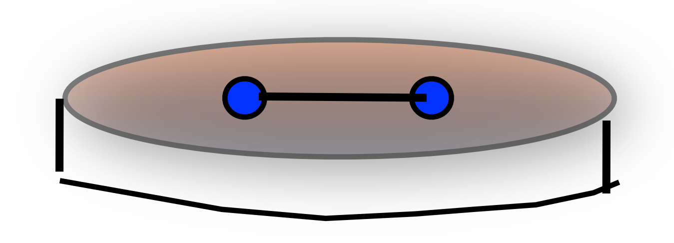

$l = 0$

s-wave pairing

BEC-BCS crossover

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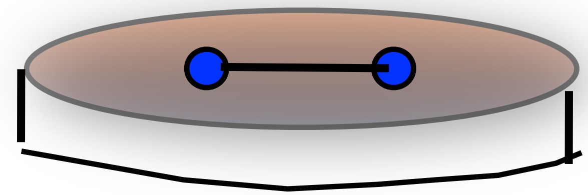

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s-wave pairing — BEC-BCS crossover

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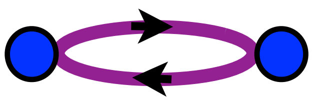

d-wave pairing — high Tc-superconductivity

Read & Green, PRB 61, 10267(2000)

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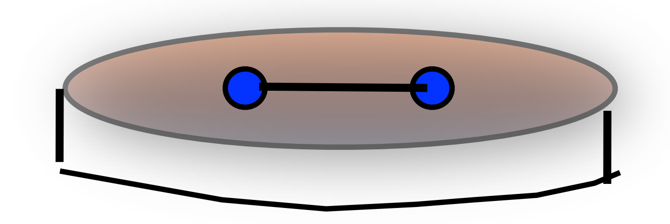
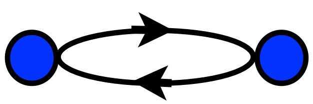


$l = 0$

s-wave pairing — BEC-BCS crossover

$l = 1$



p-wave pairing

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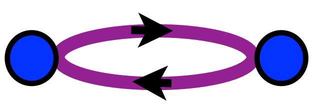

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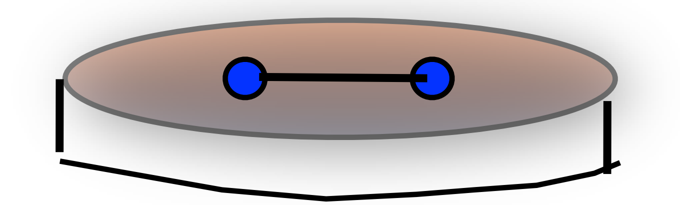

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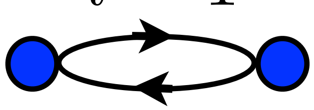
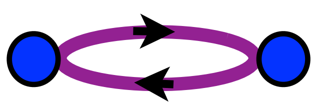


p-wave pairing

- ✦BEC-BCS phase transtion
- ✦p-wave superfluid= Pfaffian state
- ✦zero energy Majorana fermions in the presence of vortex or edges
- ✦non abelian anyons

$l = 2$

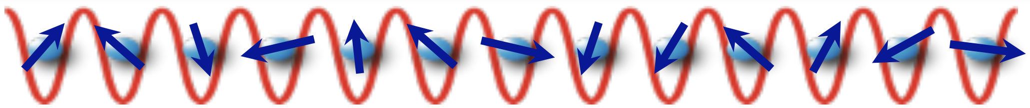


d-wave pairing

high Tc-superconductivity

Read & Green, PRB 61, 10267(2000)

From Bose-Hubbard to the Bilinear-Biquadratic Hamiltonian



Bose-Hubbard Model

$$H = -t \sum_{<i,j>,\sigma} (a^\dagger_{i,\sigma} a_{j,\sigma} + h.c) + \frac{U}{2} \sum_i n_i(n_i - 1) + \frac{U_2}{2} \sum_i (\vec{S}_i^2 - 2n_i) - \mu \sum_i n_i$$

Spin 1 operators $\vec{S}_i = \sum_{\sigma,\sigma'} a^\dagger_{i,\sigma} T_{\sigma,\sigma'} a_{j,\sigma'}$

2n order PT

$$H = H_0 - t \sum_{<i,j>,\sigma} (a^\dagger_{i,\sigma} a_{j,\sigma} + h.c) \rightarrow H_{eff} = \sum_{<i,j>} H_{ij} = -J_1 \sum_{<i,j>} (\vec{S}_i \vec{S}_j) - J_2 \sum_{<i,j>} (\vec{S}_i \vec{S}_j)^2$$

$$\hat{H}(\theta) = J \sum_{<ij>} \left[\cos\theta (\vec{S}_i \vec{S}_j) + \sin\theta (\vec{S}_i \vec{S}_j)^2 \right]$$

$$\cos(\theta) = \frac{-J_1}{\sqrt{J_1^2 + J_2^2}}$$

$$\sin(\theta) = \frac{-J_2}{\sqrt{J_1^2 + J_2^2}}$$

Time dependent correlations: learning the low excitation spectrum

- PROTOCOL   Time dependent correlations
- Atom-light interface with standing light configuration
- reading the SCS at t=0 with a first pulse of light
- mapping such state of light into an atomic quantum memory and discard the light
- reading the SCS at t=t with a second pulse of light and read the Quantum Memory and then measure light