

SU(N) Magnetism with Cold Atoms and Chiral Spin Liquids

Victor Gurarie

collaboration with M. Hermele, A.M. Rey



KITP Santa Barbara, Oct 7 2010

In this talk

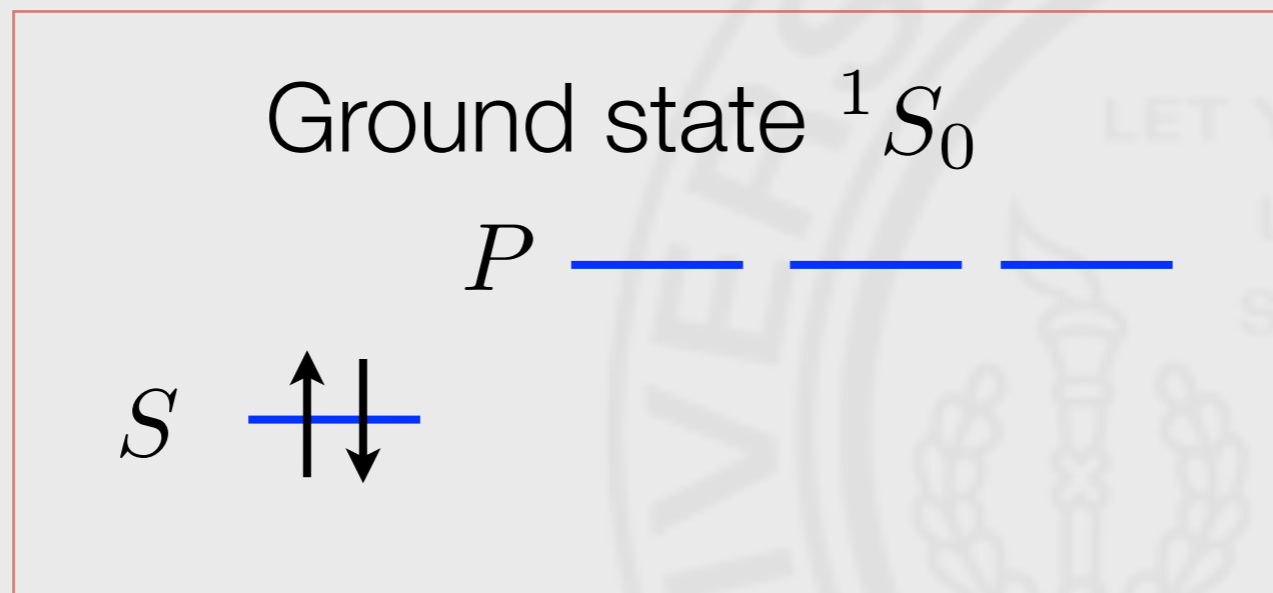
- ▶ Alkaline earth atoms can be thought of as having **SU(N) spins**, generalization of the usual SU(2) spins.
- ▶ **New world of the SU(N) physics** opens up for experimental exploration.
- ▶ “Heisenberg antiferromagnets” of the SU(N) spins can be **Chiral Spin Liquids**, spins counterparts of **quantum Hall effect**, states of matter having excitations with **fractional and non-Abelian statistics**. Those, as is well known, can be used for quantum computation.

$SU(N)$ magnetism



Alkaline earth atoms: $SU(N)$ spins

two electrons in the outer shell

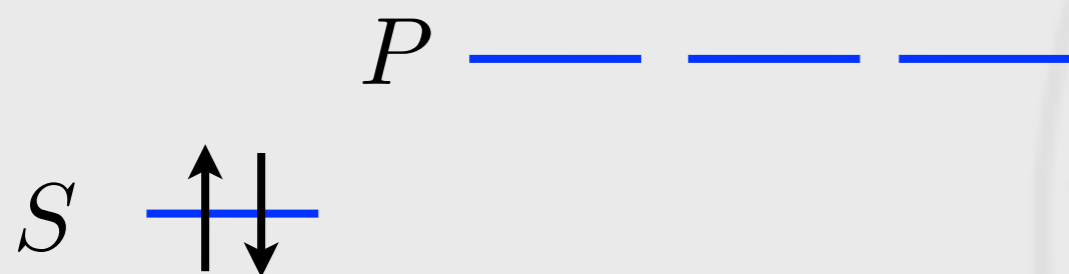


$J=0$, so the nuclear spin is decoupled from the electronic spin.

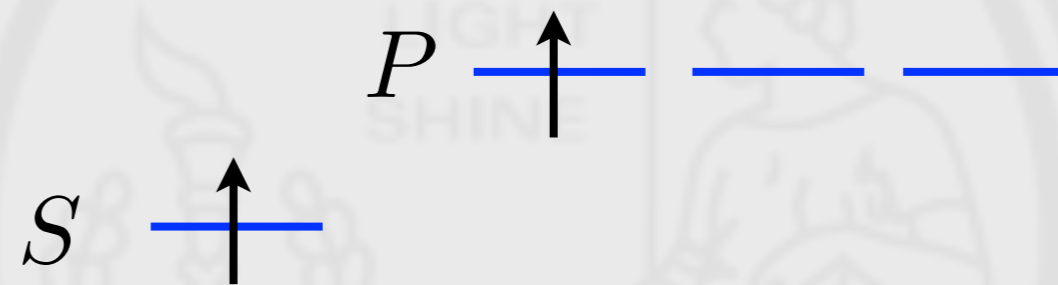
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Ground state 1S_0



Excited state 3P_0



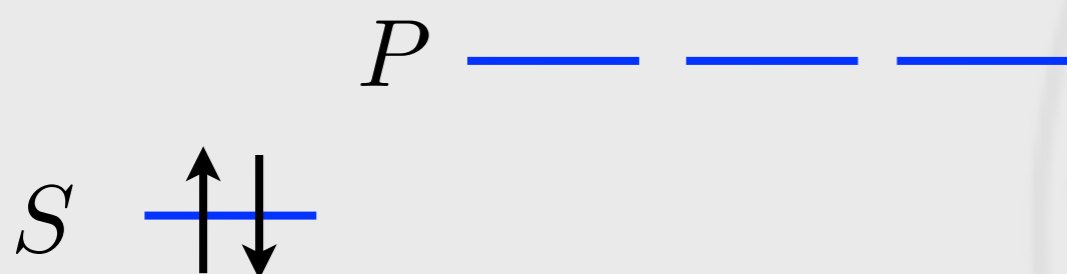
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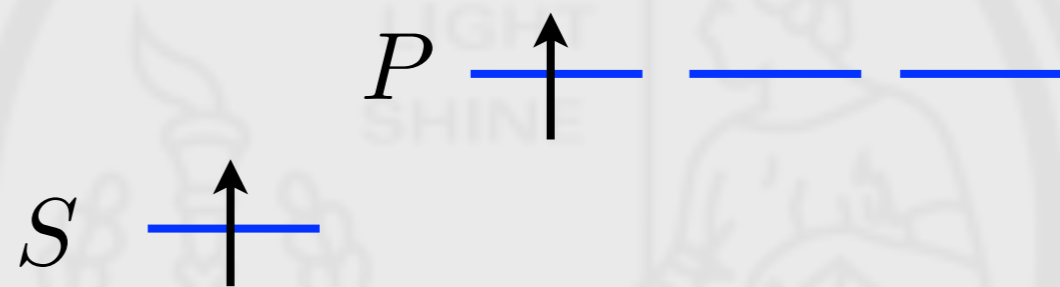
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Not alkali: those
have one electron!

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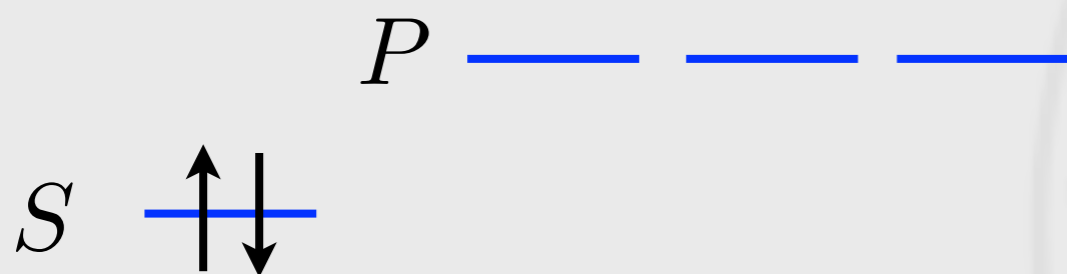


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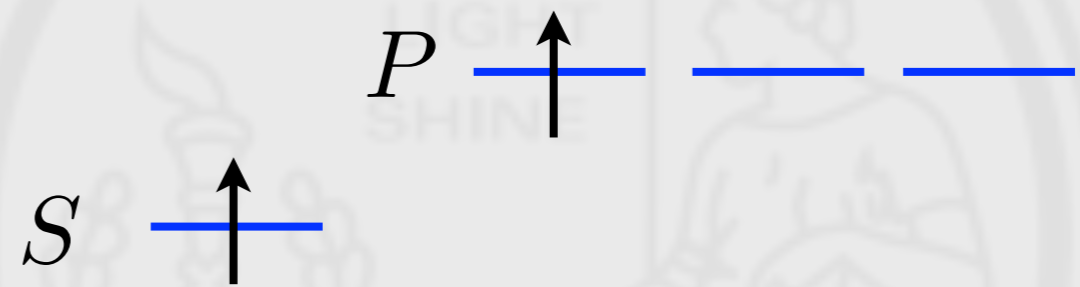
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This (sometimes large) nuclear spin can play the role of the **$SU(N)$ spin**.

For example, ^{87}Sr : $I=9/2$, $N=2I+1=10$.

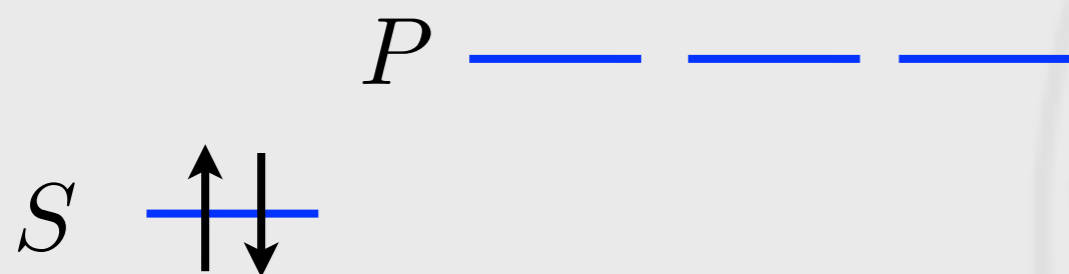
A.V. Gorshkov, M. Hermele, VG, C. Xu, P.S. Julienne,
J. Ye, P. Zoller, E. Demler, M.D. Lukin, A.M. Rey. (2010)

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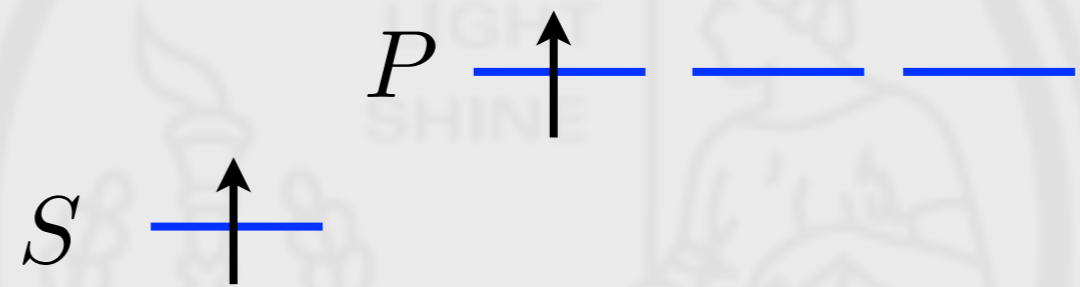
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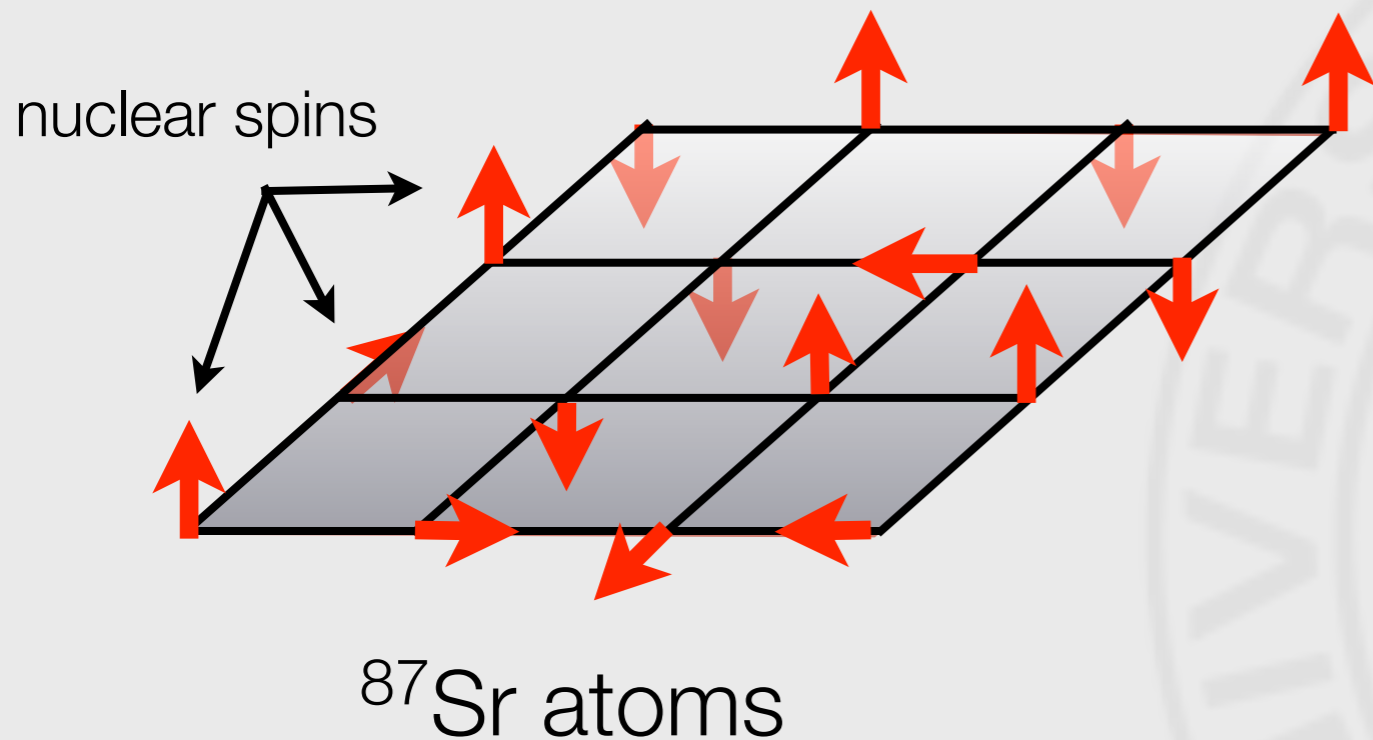
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Interesting twist: only fermionic atoms have $N > 1$
(bosons have even-even nuclei, whose $I=0$)

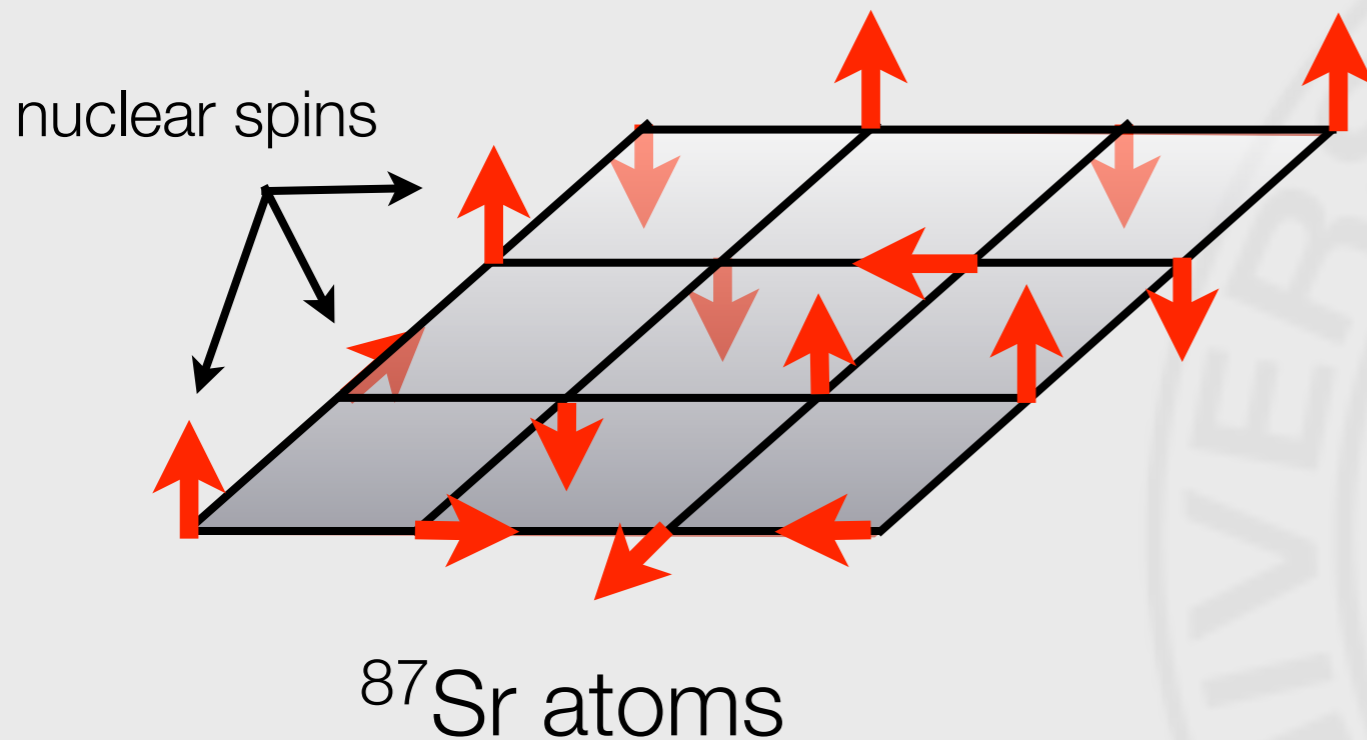
Mott insulators of the alkaline earth atoms

Place these atoms on an optical potential



Mott insulators of the alkaline earth atoms

Place these atoms on an optical potential



$$f_{i,\alpha}^\dagger$$

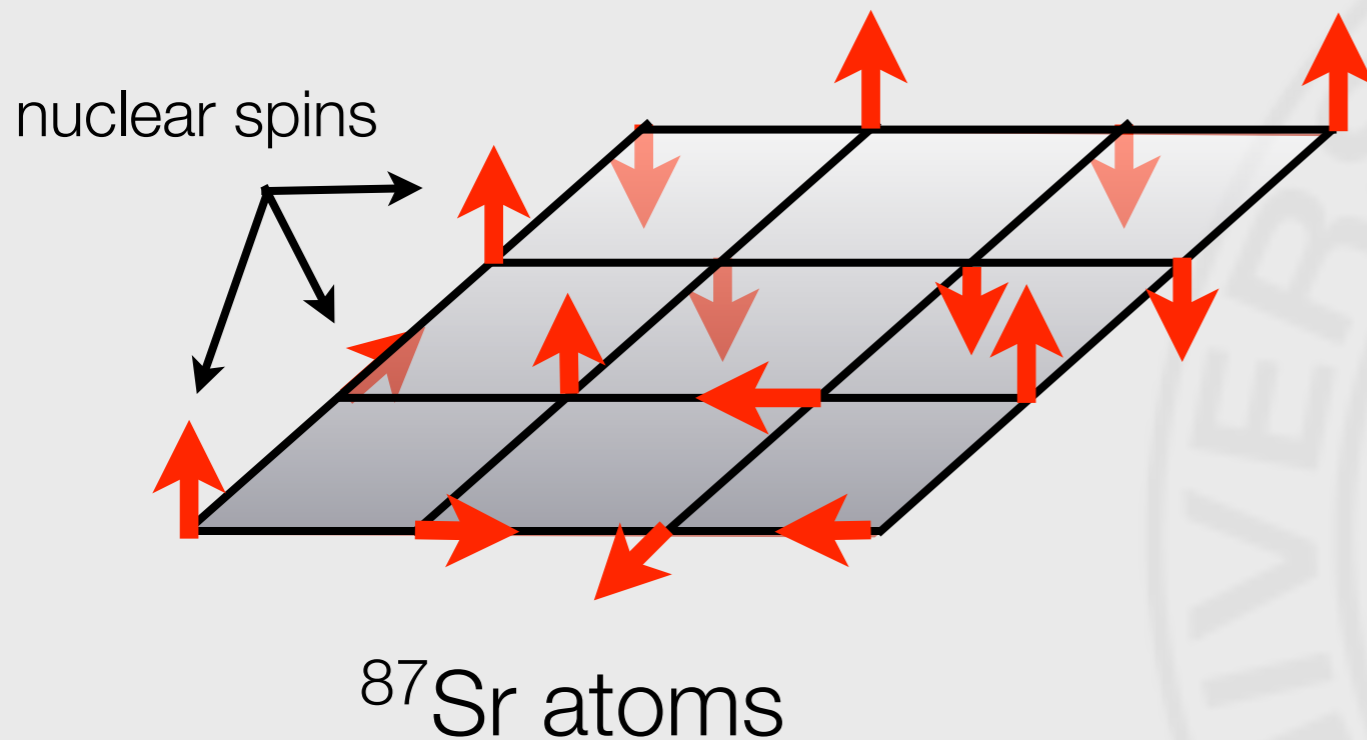
Creates an atom on a site i ,
with nuclear spin α .

SU(N) symmetric Hubbard model

$$\hat{H} = t \sum_{\langle ij \rangle, \alpha=1, \dots, N} \left(\hat{f}_{i,\alpha}^\dagger \hat{f}_{j,\alpha} + h.c. \right) + U \sum_{i, \alpha, \beta} \hat{f}_{i,\alpha}^\dagger \hat{f}_{i,\alpha} \hat{f}_{i,\beta}^\dagger \hat{f}_{i,\beta}$$

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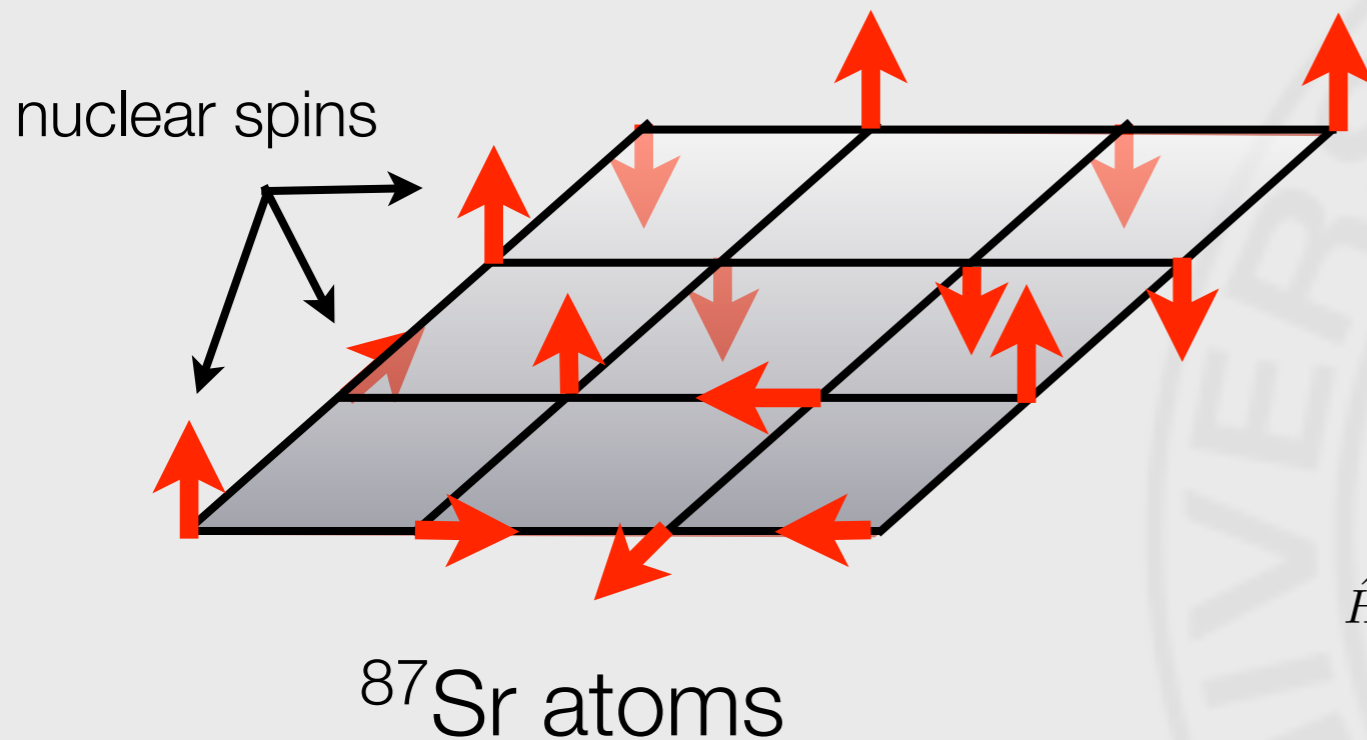
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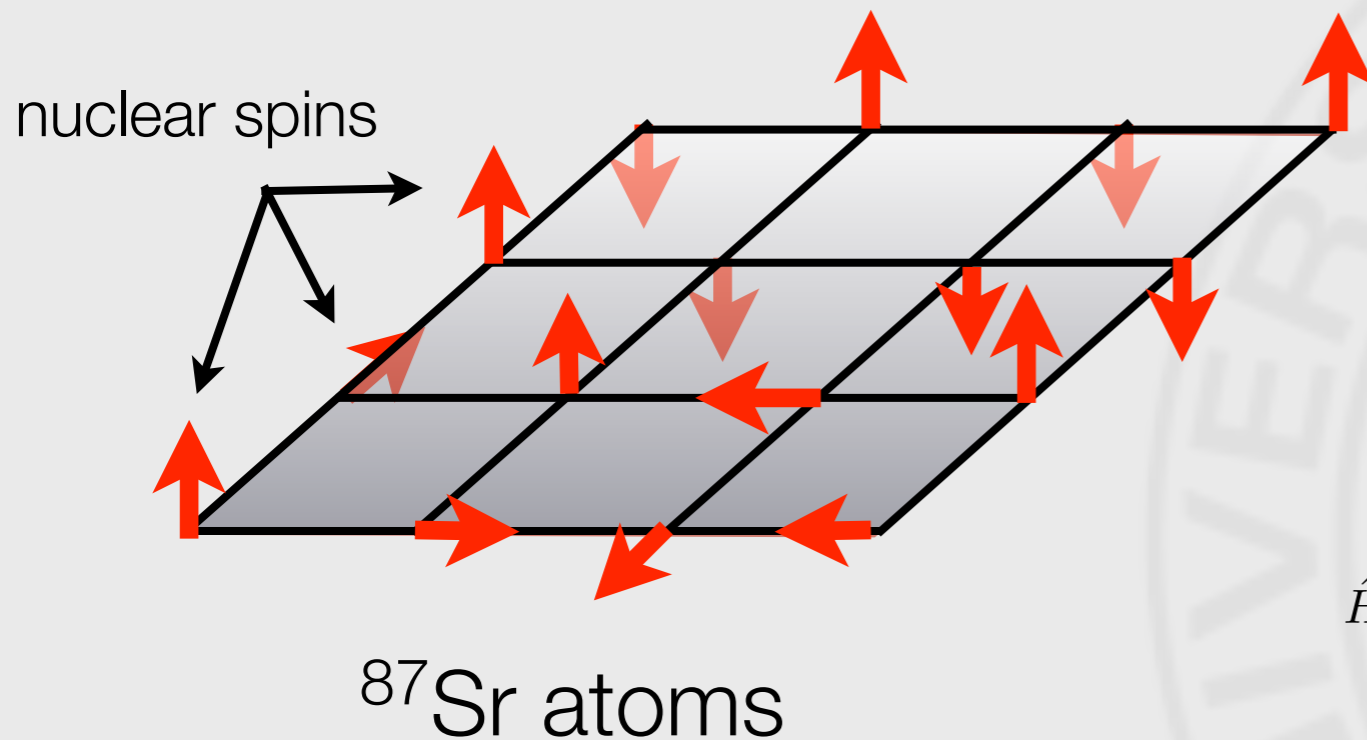
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SU(N) spins

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SU(N) spins

$$\hat{f}_{i,\alpha} \rightarrow \sum_{\beta=1}^N U_{\alpha,\beta} \hat{f}_{i,\beta}$$

SU(N) symmetry

Bottom line: these are SU(N) spin antiferromagnets



Degenerate Fermi Gas of ^{87}Sr

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Department of Physics and Astronomy, Rice University, Houston, Texas, 77251, USA

(Received 5 May 2010; published 13 July 2010)

We report quantum degeneracy in a gas of ultracold fermionic ^{87}Sr atoms. By evaporatively cooling a mixture of spin states in an optical dipole trap for 10.5 s, we obtain samples well into the degenerate regime with $T/T_F = 0.26_{-0.06}^{+0.05}$. The main signature of degeneracy is a change in the momentum distribution as measured by time-of-flight imaging, and we also observe a decrease in evaporation efficiency below $T/T_F \sim 0.5$.

DOI: [10.1103/PhysRevLett.105.030402](https://doi.org/10.1103/PhysRevLett.105.030402)

PACS numbers: 03.75.Ss

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **82**, 011608(R) (2010)

Double-degenerate Bose-Fermi mixture of strontium

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¹*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria*

²*Institut für Experimentalphysik und Zentrum für Quantenphysik, Universität Innsbruck, 6020 Innsbruck, Austria*

(Received 6 June 2010; published 20 July 2010)

We report on the attainment of a spin-polarized Fermi sea of ^{87}Sr in thermal contact with a Bose-Einstein condensate (BEC) of ^{84}Sr . Interisotope collisions thermalize the fermions with the bosons during evaporative cooling. A degeneracy with $T/T_F = 0.30(5)$ is reached for a ^{87}Sr Fermi sea of 2×10^4 atoms together with an almost pure ^{84}Sr BEC of 10^5 atoms.

DOI: [10.1103/PhysRevA.82.011608](https://doi.org/10.1103/PhysRevA.82.011608)

PACS number(s): 67.85.Hj, 37.10.De, 03.75.Ss

Experiments with ^{173}Yb , $\text{SU}(6)$

Realization of $\text{SU}(2) \times \text{SU}(6)$ Fermi System

Shintaro Taie,^{1,*} Yosuke Takasu,¹ Seiji Sugawa,¹ Rekishu Yamazaki,^{1,2}
Takuya Tsujimoto,¹ Ryo Murakami,¹ and Yoshiro Takahashi^{1,2}

¹*Department of Physics, Graduate School of Science, Kyoto University, Japan 606-8502*

²*CREST, JST, 4-1-8 Honcho Kawaguchi, Saitama 332-0012, Japan*

(Dated: May 21, 2010)

We report the realization of a novel degenerate Fermi mixture with an $\text{SU}(2) \times \text{SU}(6)$ symmetry in a cold atomic gas. We successfully cool the mixture of the two fermionic isotopes of ytterbium ^{171}Yb with the nuclear spin $I = 1/2$ and ^{173}Yb with $I = 5/2$ below the Fermi temperature T_F as $0.46T_F$ for ^{171}Yb and $0.54T_F$ for ^{173}Yb . The same scattering lengths for different spin components make this mixture featured with the novel $\text{SU}(2) \times \text{SU}(6)$ symmetry. The nuclear spin components are separately imaged by exploiting an optical Stern-Gerlach effect. In addition, the mixture is loaded into a 3D optical lattice to implement the $\text{SU}(2) \times \text{SU}(6)$ Hubbard model. This mixture will open the door to the study of novel quantum phases such as a spinor Bardeen-Cooper-Schrieffer-like fermionic superfluid.

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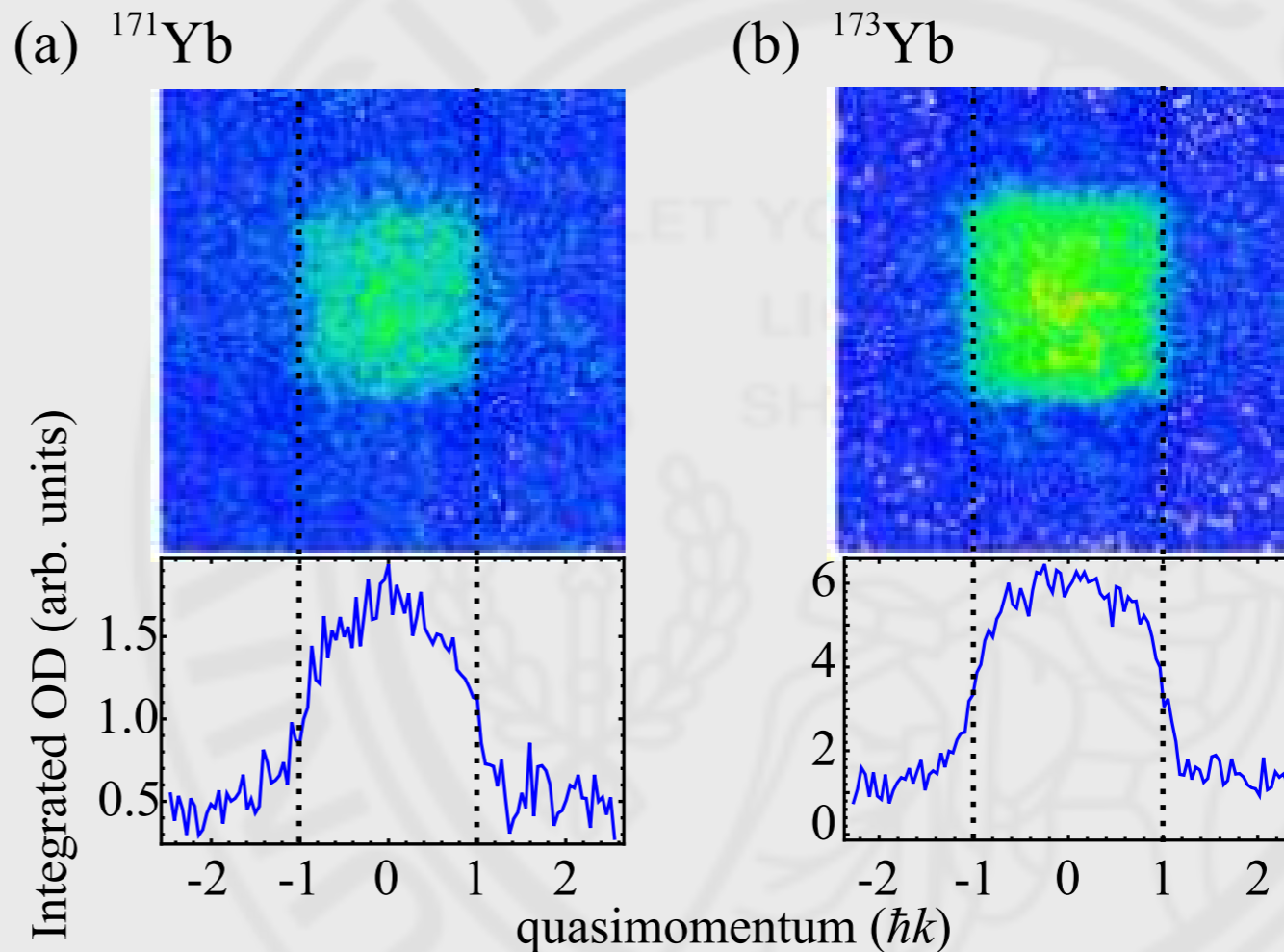


FIG. 5. (Color online) Quasimomentum distribution of (a) ^{171}Yb and (b) ^{173}Yb in the SU(2)×SU(6) two-species mixture in an optical lattice. The density distributions integrated along the vertical direction are also shown below. The atom numbers are 0.4×10^4 for ^{171}Yb and 1.5×10^4 for ^{173}Yb , respectively. The images are taken after linear ramping down of the lattice in 0.5ms, followed by a ballistic expansion of (a) 12ms and (b) 13ms. The dotted lines indicate the domain of the 1st Brillouin zone, which equals twice the recoil momentum $\hbar k$.

New world of $SU(N)$ physics

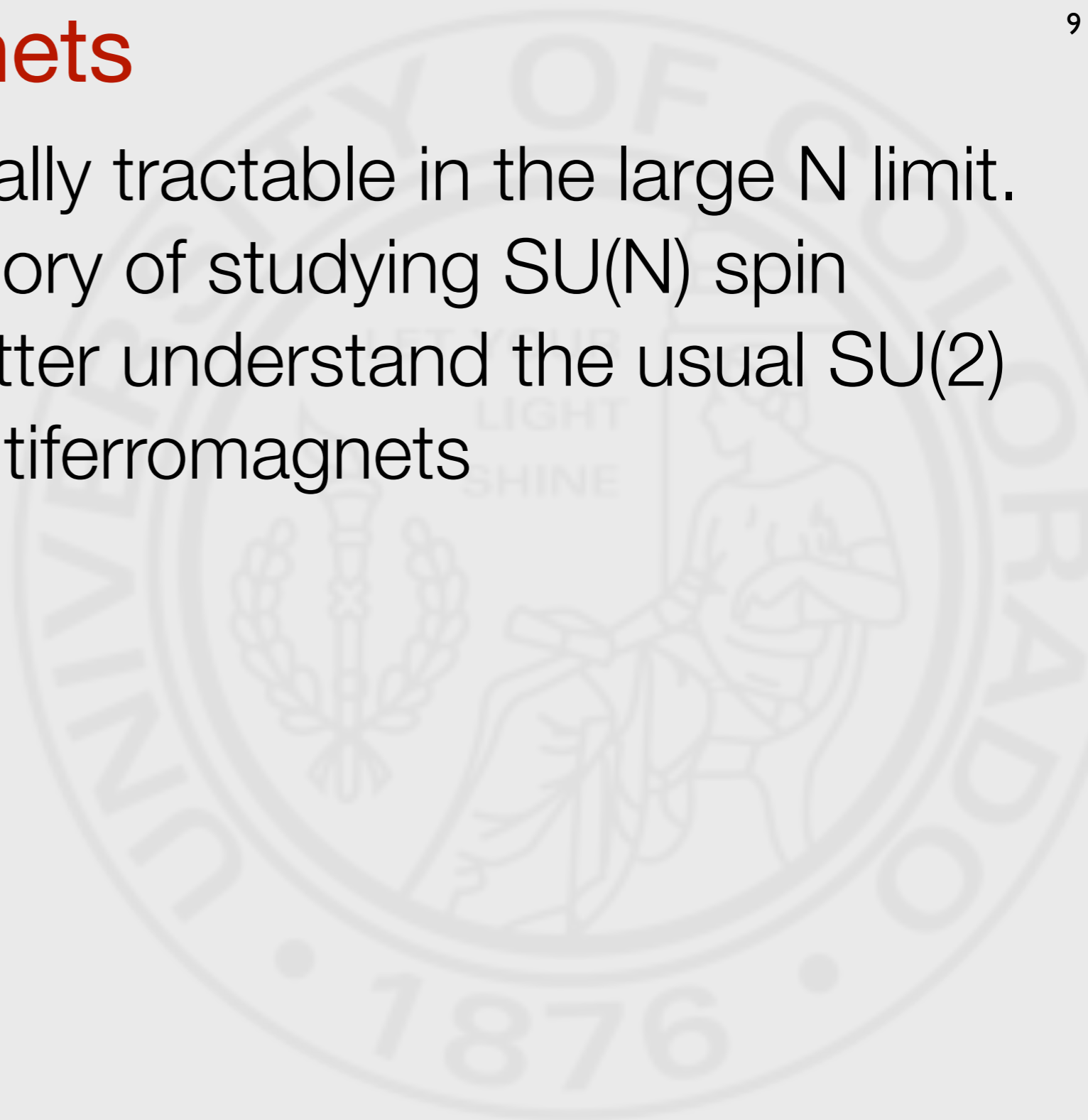
is now accessible to experiment

1. $SU(N)$ Hubbard model
2. $SU(N)$ metal - Mott insulator transitions
3. 1D $SU(N)$ -symmetric integrable models
4. $SU(N)$ -symmetric Luttinger liquid
5. $SU(N)$ (anti)ferromagnetism
6. all sorts of other things

SU(N) antiferromagnets

SU(N) spins are analytically tractable in the large N limit.

There is a long history of studying SU(N) spin antiferromagnets, to better understand the usual SU(2) spin antiferromagnets



SU(N) antiferromagnets

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S. Sachdev



N. Read



I. Affleck



J. B. Marston

Late 1980's

Careful and numerous studies of the SU(N) antiferromagnets. Papers with many 100's of citations.

SU(N) antiferromagnets

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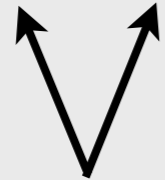
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Does this mean we can just look up the answer in these papers and find out all we need to know about SU(N) antiferromagnets and alkaline earth atoms?

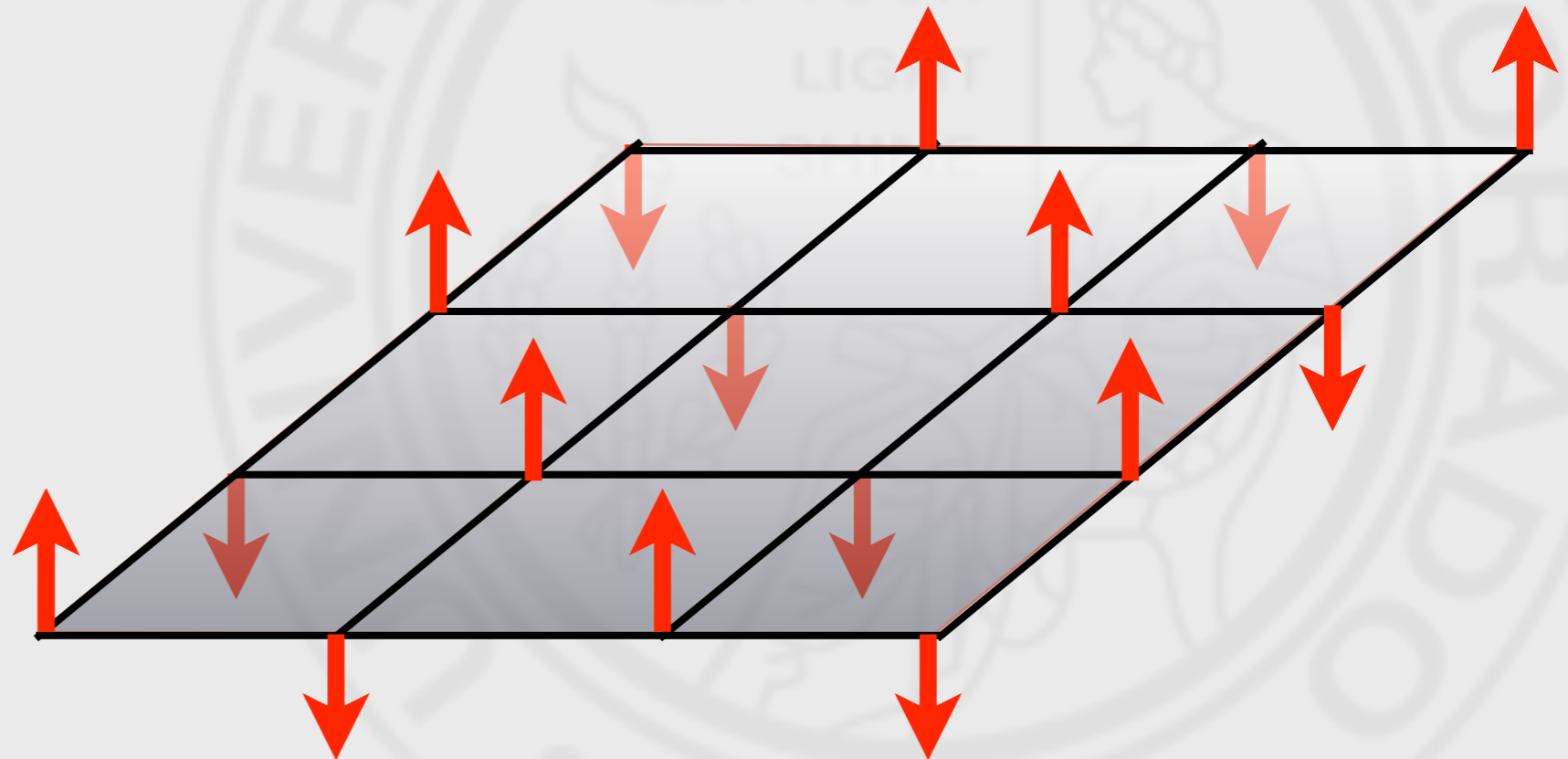
NO!

Standard SU(2) antiferromagnet: Néel state

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



 SU(2) spins



A collection of spin-1/2s on a square lattice in the presence of the antiferromagnetic interactions at $T=0$ forms a Néel state with a long range antiferromagnetic order (this is known numerically and experimentally).

The difference between $SU(2)$ and $SU(N)$

$SU(2)$ spins: two spins- $1/2$
can form a singlet



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$$\phi_{\uparrow}^{(1)} \phi_{\downarrow}^{(2)} - \phi_{\downarrow}^{(1)} \phi_{\uparrow}^{(2)}$$

singlet



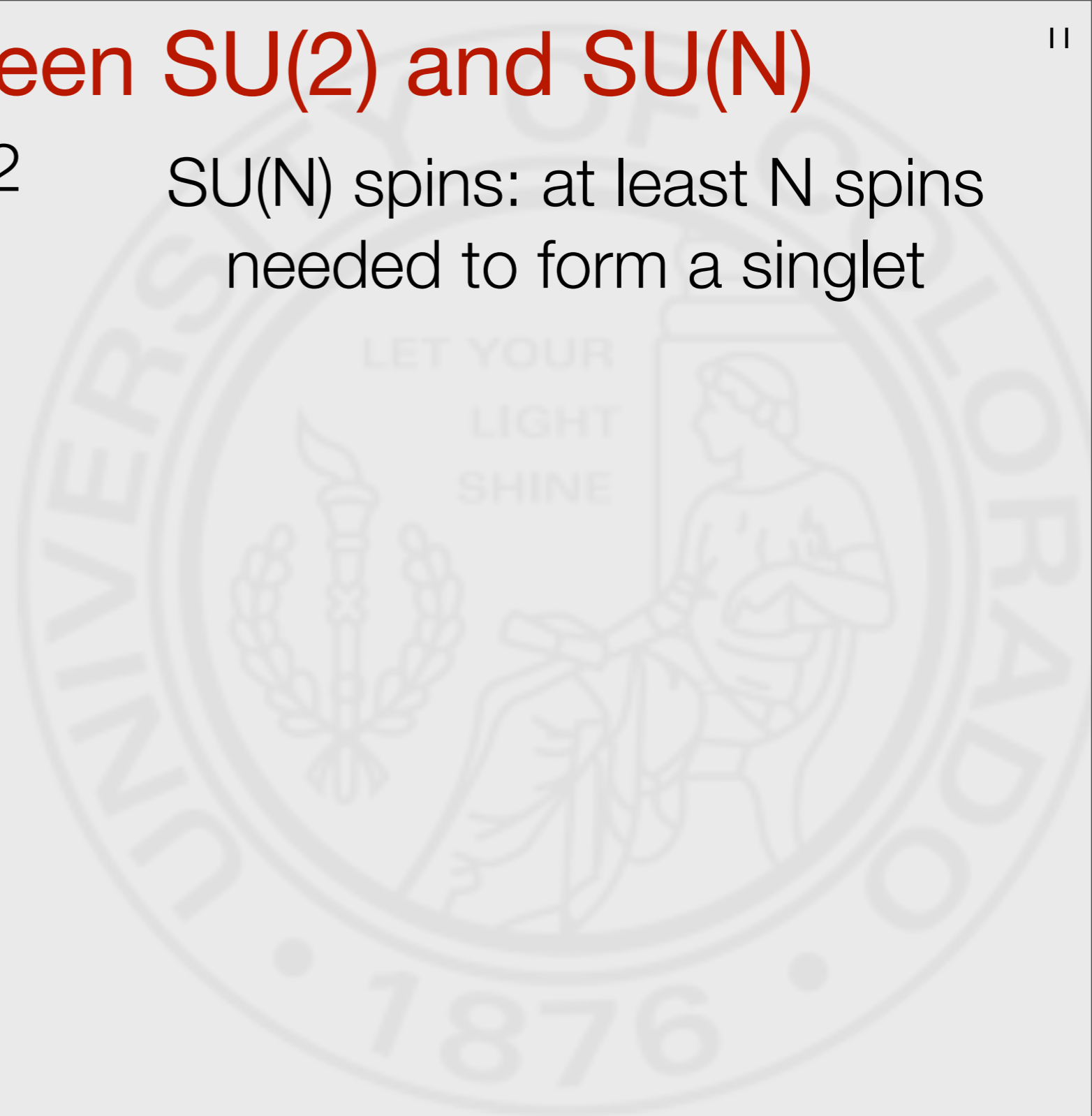
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$\alpha, \beta = 1, 2, \dots, N$

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2 spins:

$$\phi_{\alpha\beta}^{(1,2)} = \phi_{\alpha}^{(1)} \phi_{\beta}^{(2)} - \phi_{\beta}^{(1)} \phi_{\alpha}^{(2)}$$

This is not a singlet, but an antisymmetric N by N tensor, with $N(N-1)/2$ components.

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$$\phi_{\alpha\beta\gamma}^{(1,2,3)}$$

3 spins:

antisymmetric rank 3 tensor:
 $N(N-1)(N-2)/3!$ components

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$$\phi_{\alpha_1 \alpha_2 \dots \alpha_N}^{(1,2,\dots,N)}$$

N spins:
finally, scalar!

Prior studies of the $SU(N)$ magnets

All prior studies were designed so that one was able to form singlets from nearby spins



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Method: Place m and $N-m$ spins on even and odd sublattices respectively

1. Read & Sachdev, 1989: $m=1$
2. Marston & Affleck, 1988: $m=N/2$

site with m spins:

$$\hat{f}_{i,\alpha_1}^\dagger \hat{f}_{i,\alpha_2}^\dagger \cdots \hat{f}_{i,\alpha_m}^\dagger |0\rangle$$

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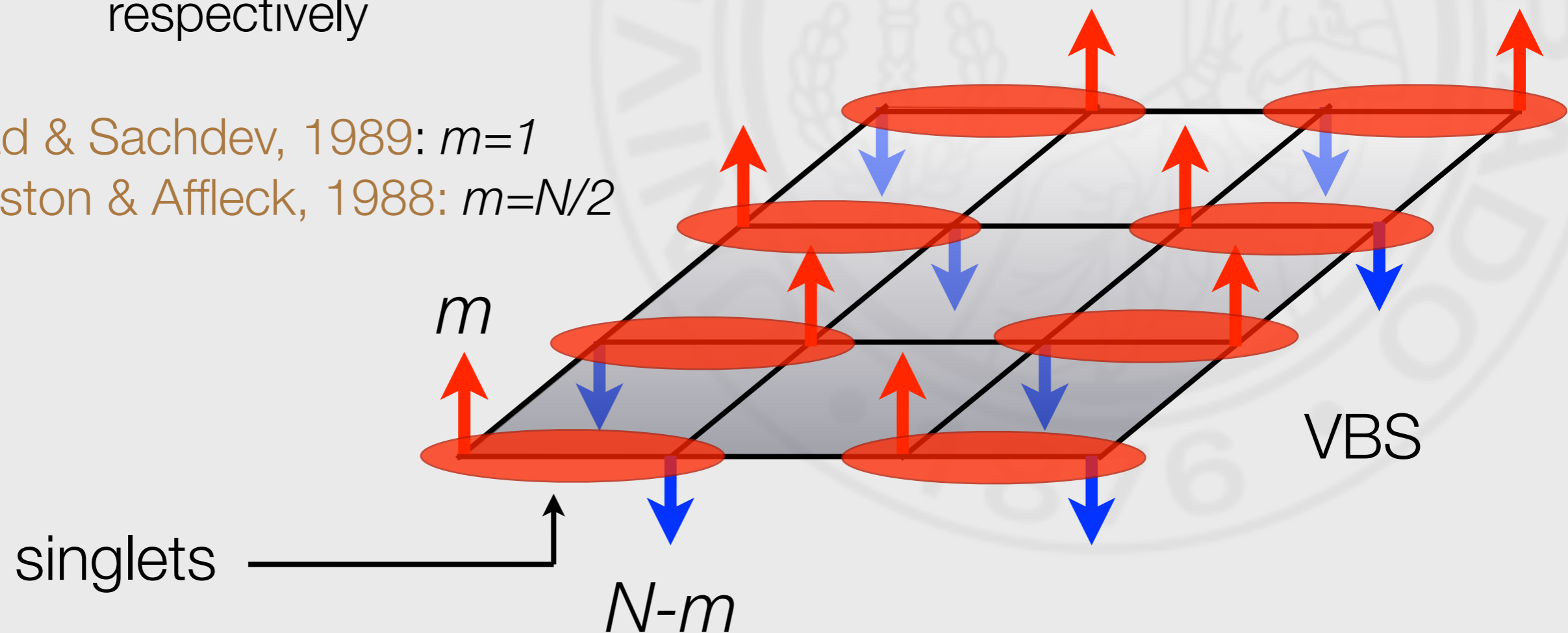
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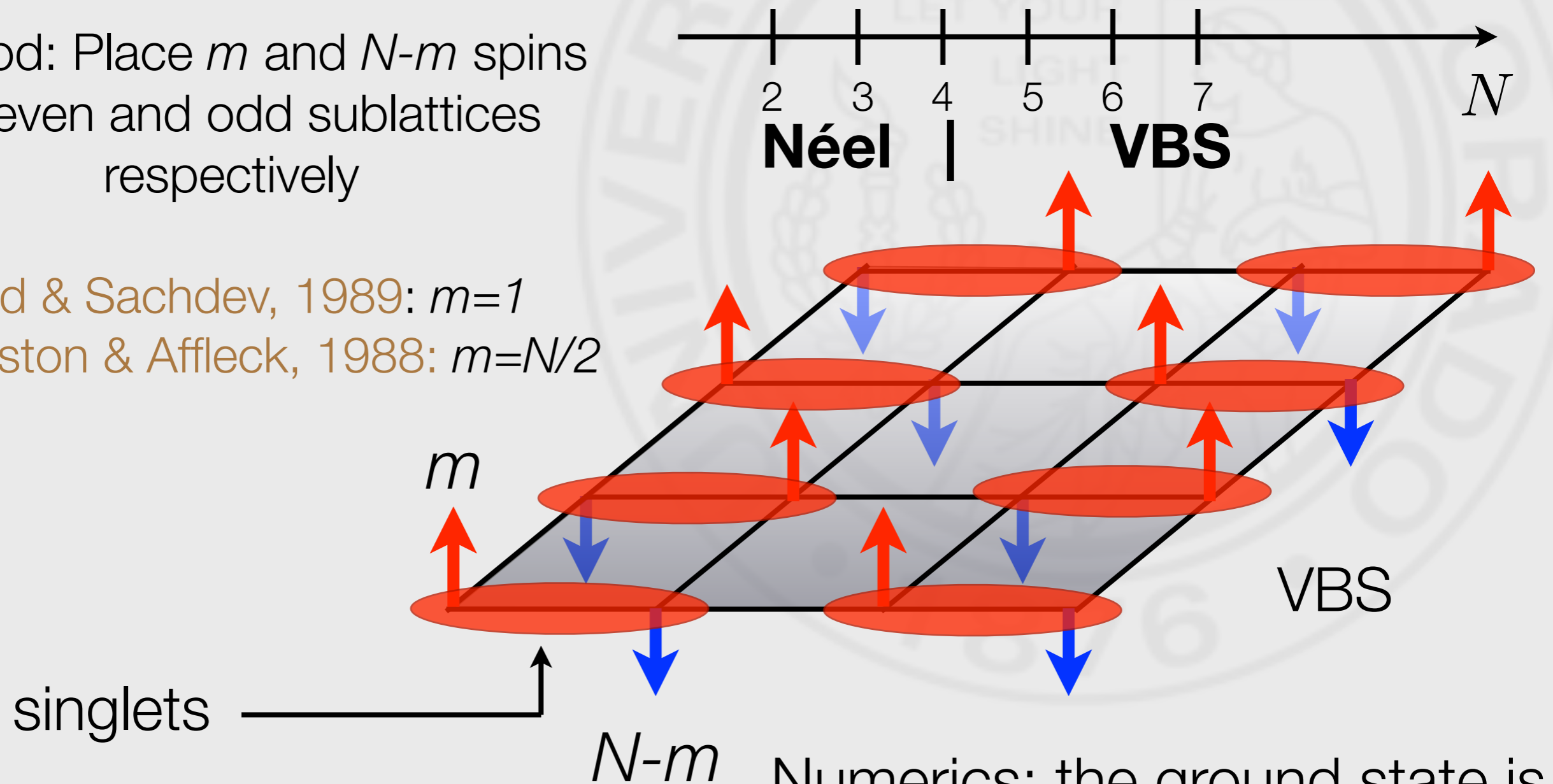
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Numerics: the ground state is

Néel if $N < 4$

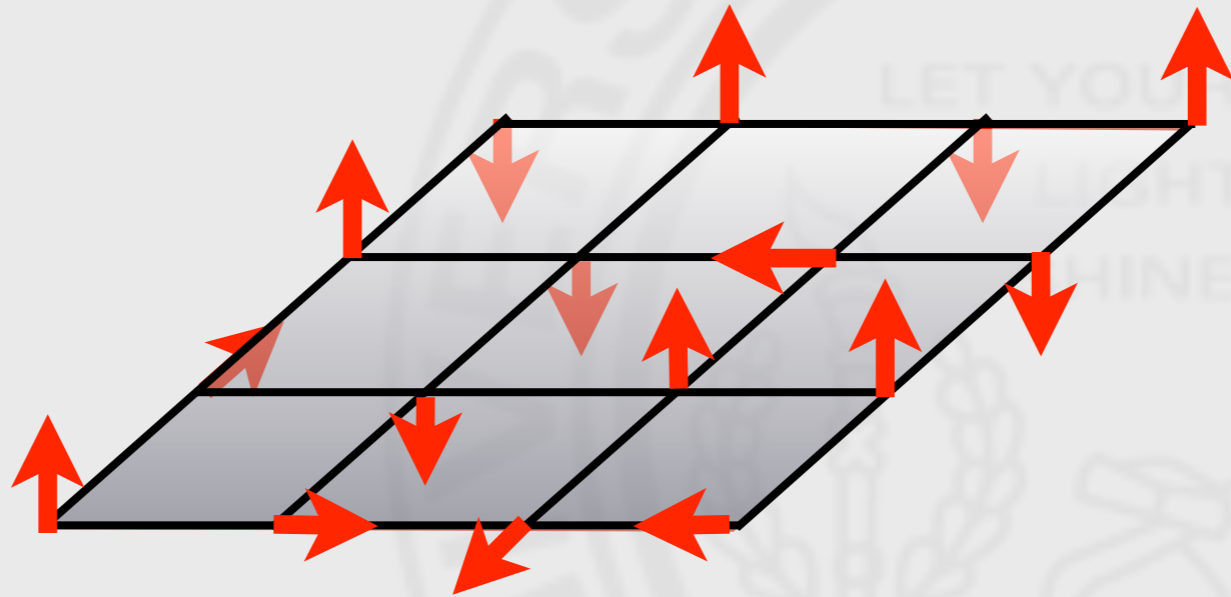
VBS if $N > 4$

Analytics at large N : The ground state is **V**alence **B**ond **S**olid (VBS)

All of this is not relevant for us: we can place one,
at most two atoms ($SU(N)$ spins) on each site

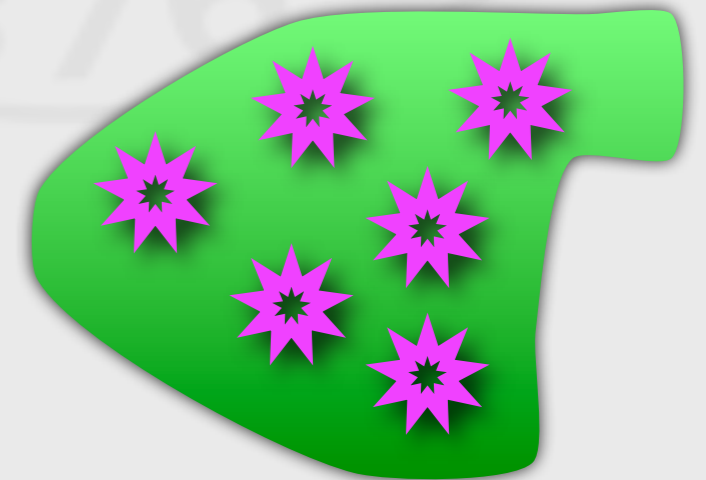
Thus this is a new yet unexplored problem

One atom (or two) per site: experimentally realizable $SU(N)$



at least N (or $N/2$) sites are required to form a singlet

D. Arovas (2008) calls these N -simplexes.

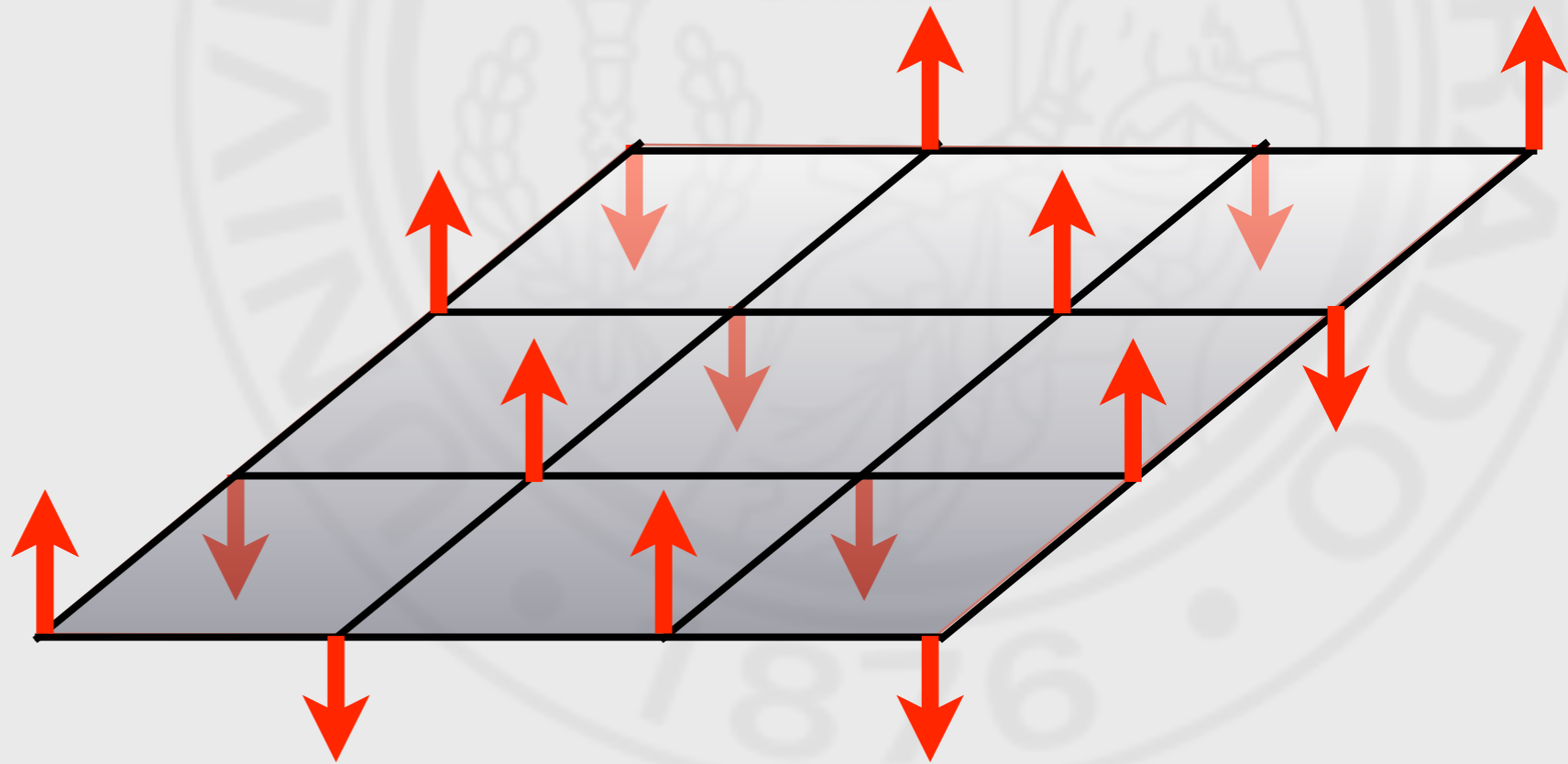


The background features a large, faint watermark of the University of Colorado seal. The seal is circular and contains the text "UNIVERSITY OF COLORADO" around the top edge and "1876" at the bottom. In the center, it depicts a figure holding a torch and a book, with the motto "LET YOUR LIGHT SHINE" written above the figure.

Quasiclassical analysis

Quasiclassical version: $SU(2)$

We'd like to imagine spins as arrows \vec{S}



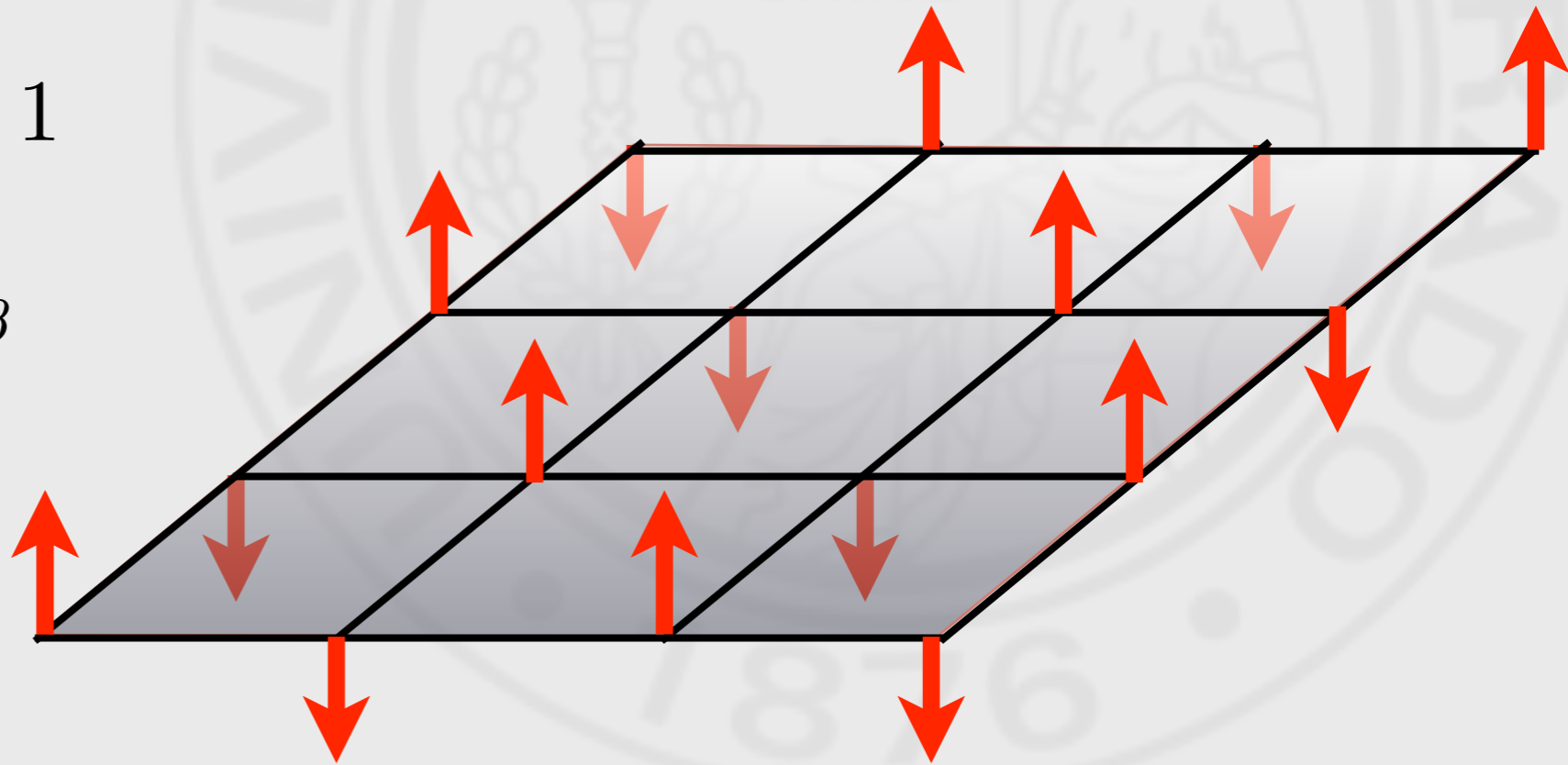
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SU(2): each spin is a complex unit vector

$$z_\alpha, \alpha = 1, 2 \quad z^* \cdot z = 1$$

$$S^a = \sum_{\alpha, \beta=1}^2 z_\alpha^* \sigma_{\alpha, \beta}^a z_\beta$$



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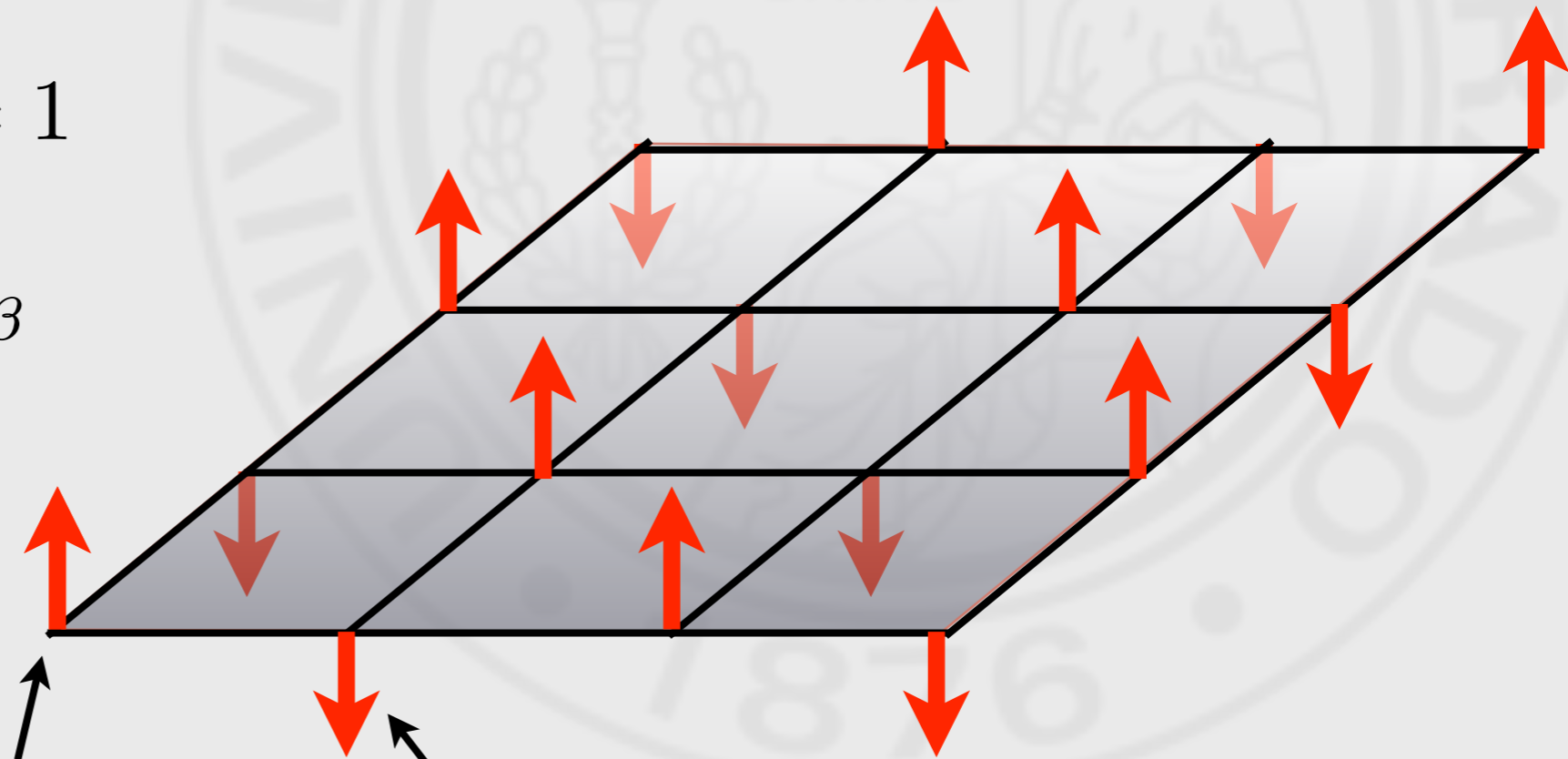
$$S^a = \sum_{\alpha, \beta=1}^2 z_\alpha^* \sigma_{\alpha, \beta}^a z_\beta$$

$$H \sim J \sum_{\langle ij \rangle} |z_i^* \cdot z_j|^2$$

$$z = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$z = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Antiferromagnetism

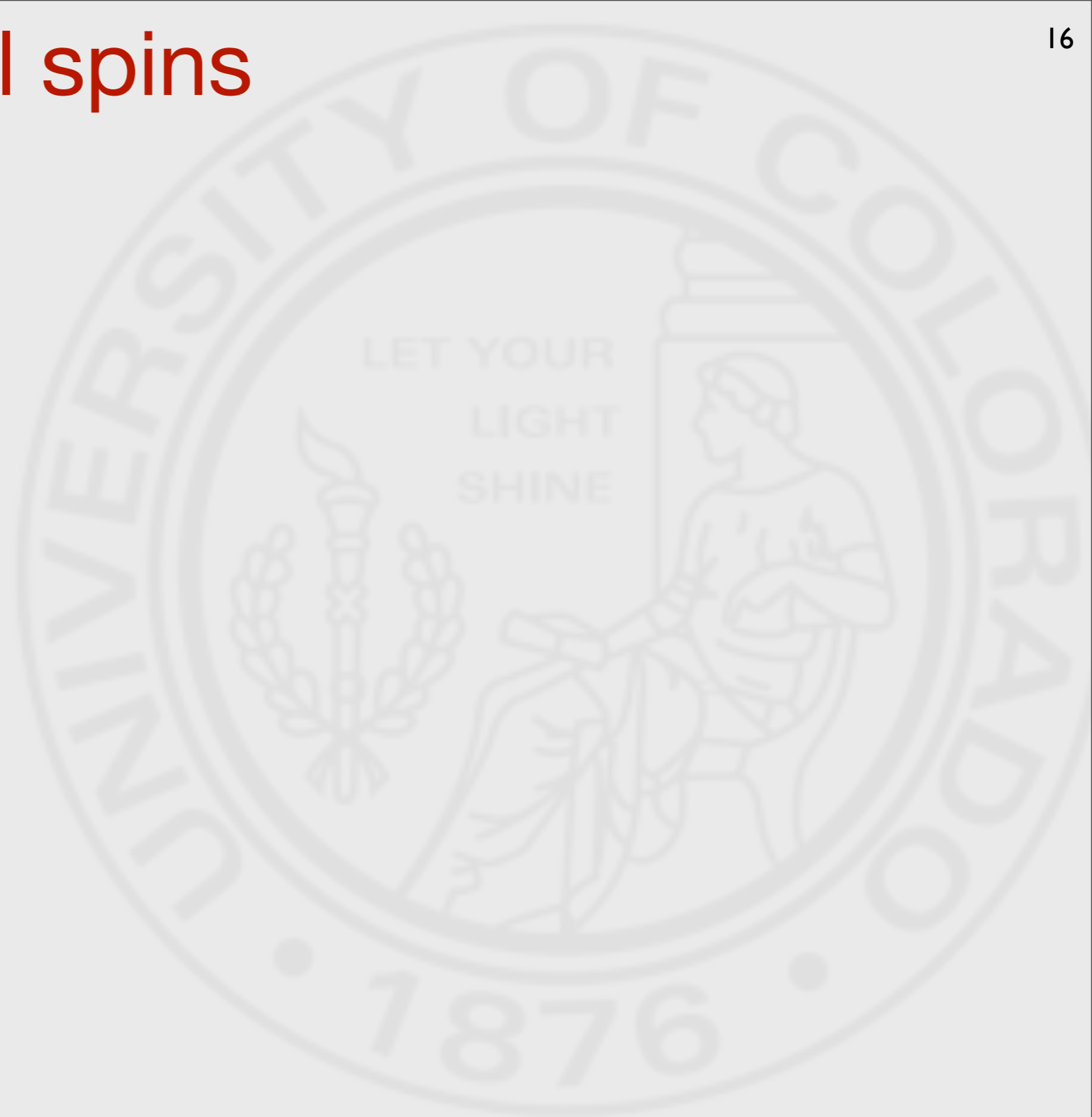


SU(N) quasiclassical spins

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“Orthomagnetism”

(spins are trying to all be orthogonal)

Frustration: large number of classical ground states

SU(N) quasiclassical spins

SU(N): each spin is a complex vector

$$z_\alpha, \alpha = 1, 2, \dots, N$$

“Orthomagnetism”

(spins are trying to all be orthogonal)

Frustration: large number of classical ground states

$$H \sim J \sum_{\langle ij \rangle} |z_i^* \cdot z_j|^2$$

Counting classical ground states (Moessner & Chalker):

$$z : 2N-2 \text{ real degrees of freedom} \quad D = 2N_s(N - 1)$$

$$z_i^* \cdot z_j = 0 \quad 2 \text{ constraints per bond} \quad C \leq 4N_s$$

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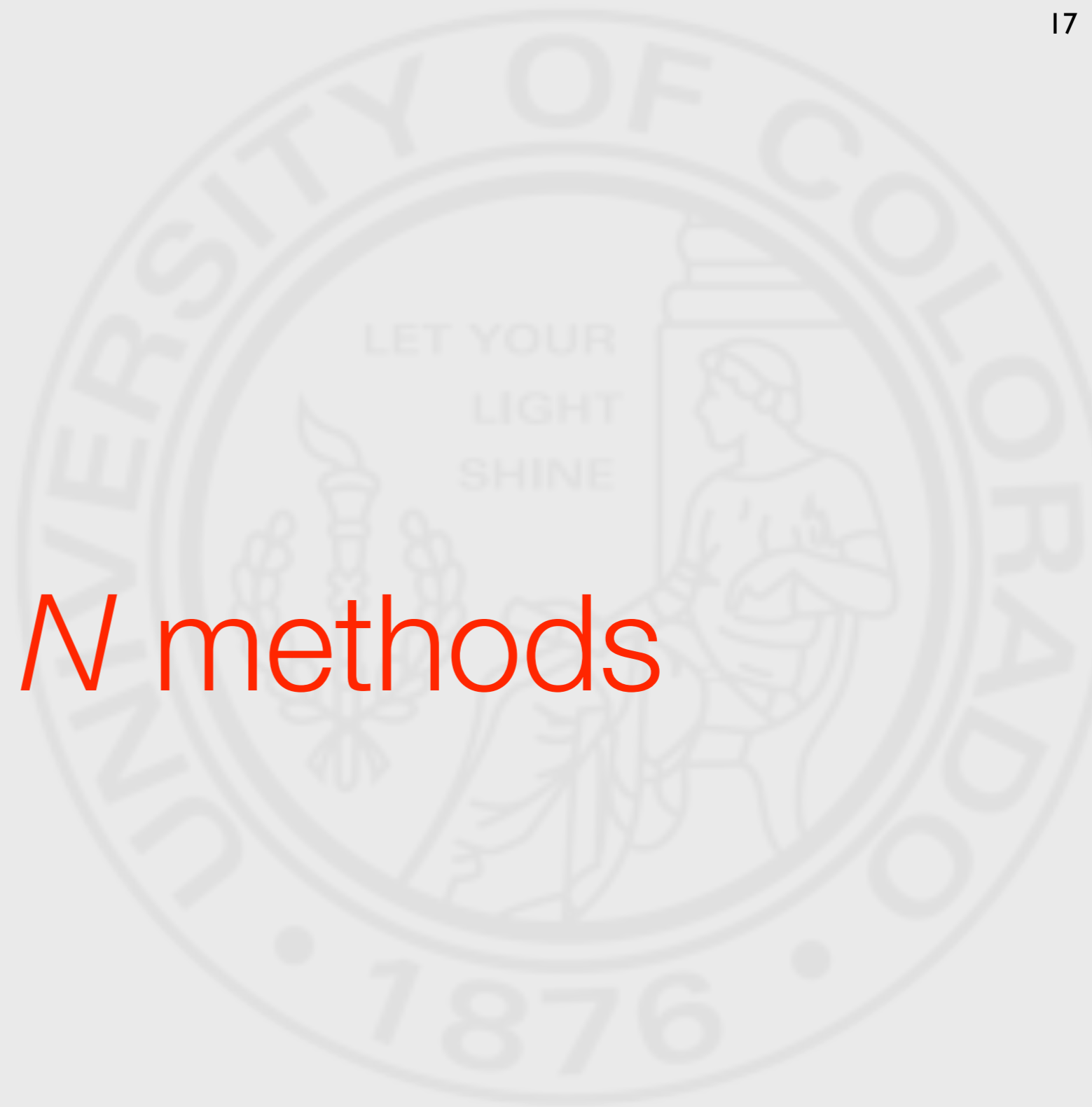
$$\text{ground state degeneracy} \quad G = D - C \geq 2N_s(N - 3)$$

extensive classical ground state if $N > 3$

SU(N) quasiclassical spins

This implies that the standard methods to treat quantum antiferromagnets (sigma models) do not apply. One also expects that magnetic order is unlikely.

Large N methods



Large N methods

Taking N to infinity is problematic: the number of spins required to form a singlet goes to infinity too.

Proposal: Let us put $m = \frac{N}{k}$ atoms on each site.

The number of sites required to form a singlet is now k and is N independent. Then take N to infinity.

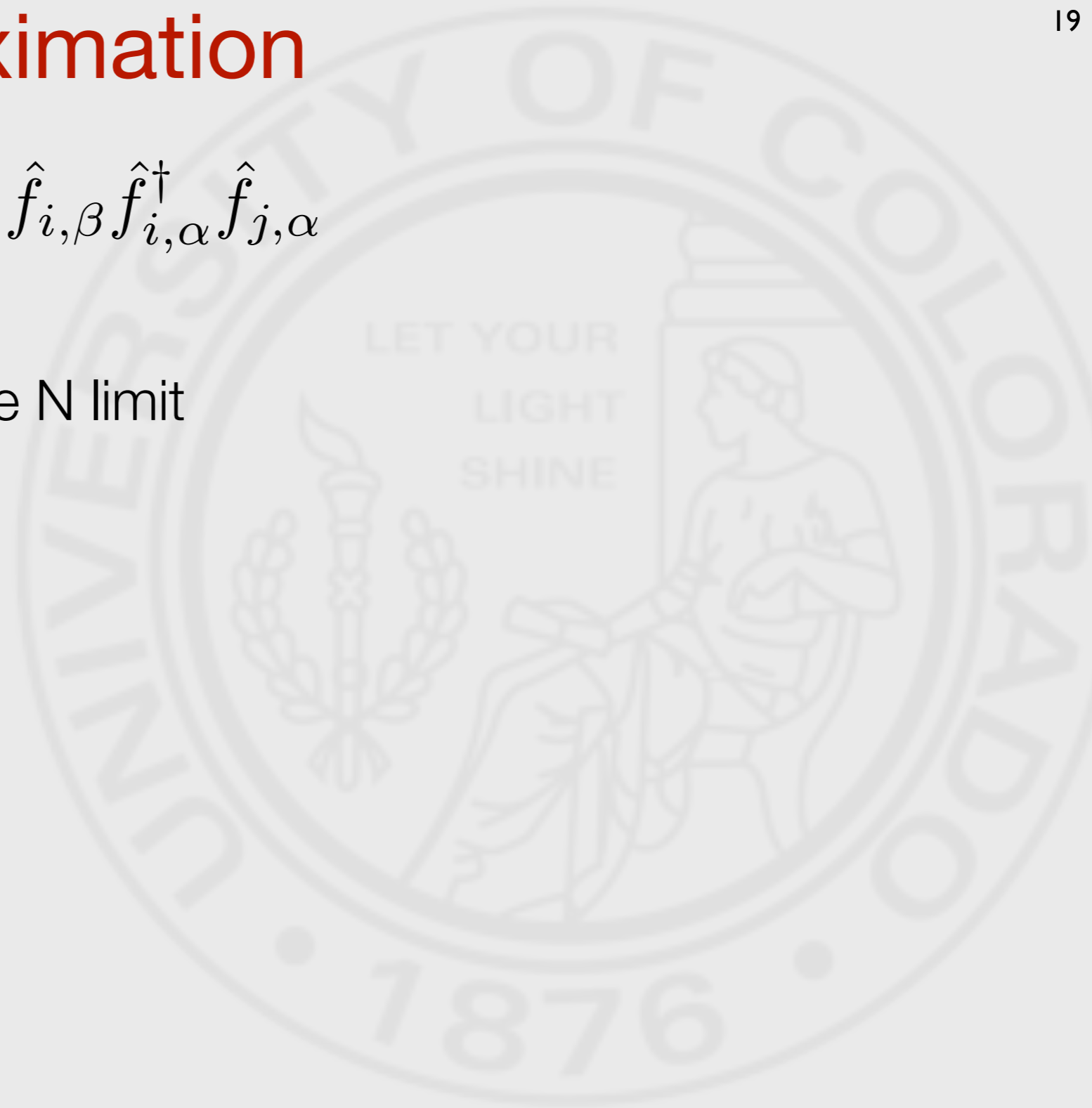
Carrying out this procedure results in the chiral spin liquid ground state if $k > 4$.

M. Hermele, VG, A.-M. Rey (2009)

Saddle point approximation

$$H = -\frac{J}{N} \sum_{\langle ij \rangle, \alpha, \beta=1, \dots, N} \hat{f}_{j, \beta}^\dagger \hat{f}_{i, \beta} \hat{f}_{i, \alpha}^\dagger \hat{f}_{j, \alpha}$$

└ convenient for large N limit



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Hubbard-Stratonovich hopping

$$S = \int d\tau \left[\sum_i \left\{ \sum_\alpha \bar{f}_{i, \alpha} \partial_\tau f_{i\alpha} + i\lambda_i \left(\sum_\alpha \bar{f}_{i, \alpha} f_{i, \alpha} - \frac{N}{k} \right) \right\} + \sum_{\langle ij \rangle} \left\{ \chi_{ij} \sum_\alpha \bar{f}_{i\alpha} f_{j\alpha} + \text{H.c.} + N \frac{|\chi_{ij}|^2}{J} \right\} \right]$$

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Saddle point equations (exact in the large N limit)

$$\frac{1}{k} = \left\langle \hat{f}_{i, \alpha}^\dagger \hat{f}_{i, \alpha} \right\rangle \quad \chi_{ij} = -J \left\langle \hat{f}_{j, \alpha}^\dagger \hat{f}_{i, \alpha} \right\rangle$$

Fermions try to arrange their hopping dynamically to minimize their energy

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$$\chi_{ij} = \chi_{ij}^{(0)} e^{iA_{ij}}$$

saddle point

fluctuations

$S_{\text{eff}}[A_{ij}]$

Fermions try to arrange their hopping dynamically to minimize their energy

Minimization procedure

One needs to find hoppings which minimize the ground state energy of

$$\hat{H}_{\text{MFT}} = N \sum_{ij} \frac{|\chi_{ij}|^2}{J} + \sum_{ij} \sum_{\alpha=1}^N \left(\chi_{ij} \hat{f}_{i,\alpha}^\dagger \hat{f}_{j,\alpha} + \text{h. c.} \right)$$

while having exactly N/k particles per site (on the average).

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Theorem (due to M. Hermele, generalizing the earlier bound by D. Rokhsar)

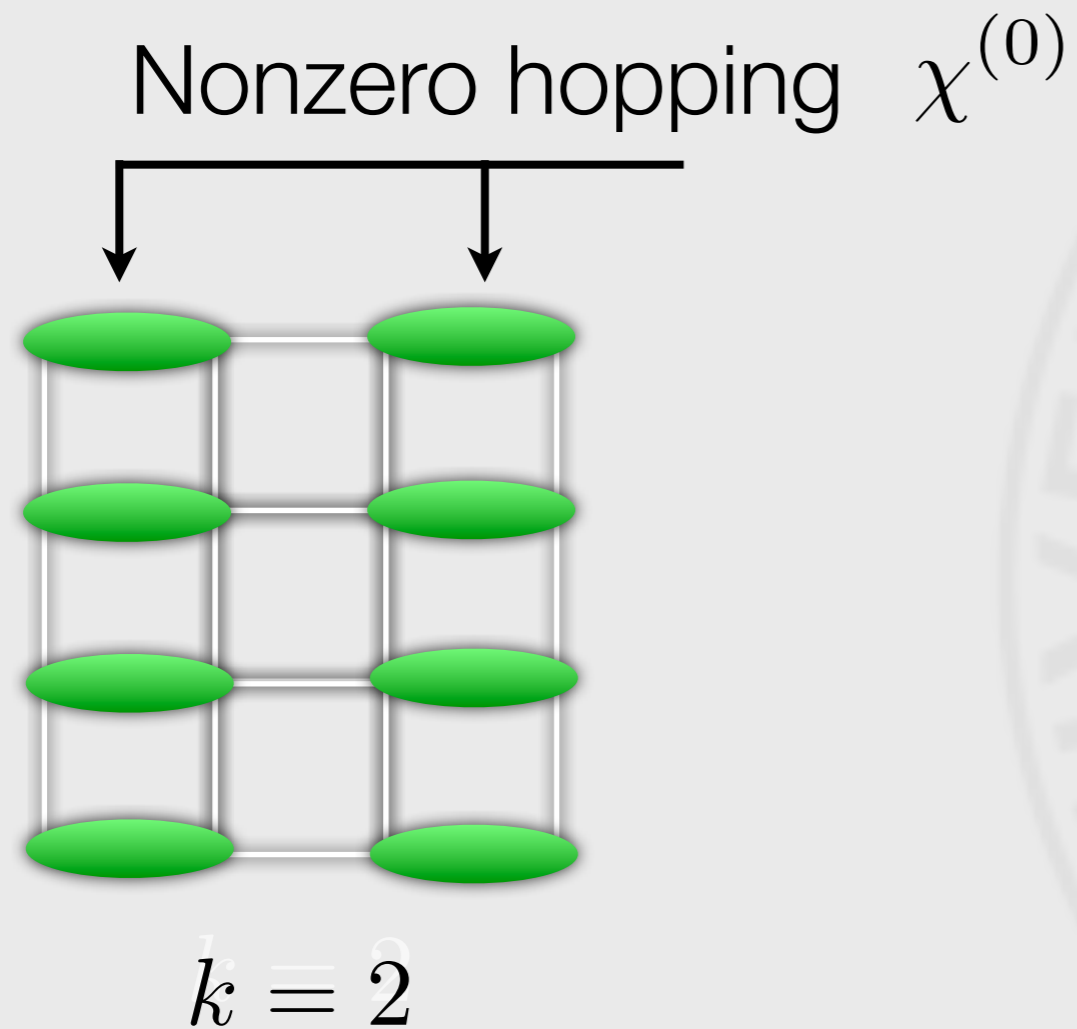
$$\frac{E_{\text{MFT}}}{N_s} \geq -\frac{N}{4k} J$$

number of lattice sites

Saddle points for $k=2, 3, 4$



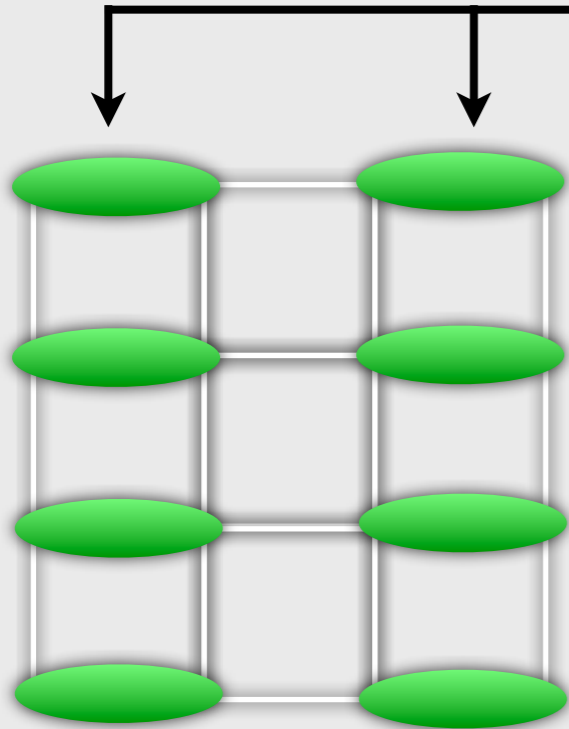
Saddle points for $k=2, 3, 4$



Spins pair up to form
2-spin singlets
This state (VBS) was
found by Affleck &
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Nonzero hopping $\chi^{(0)}$



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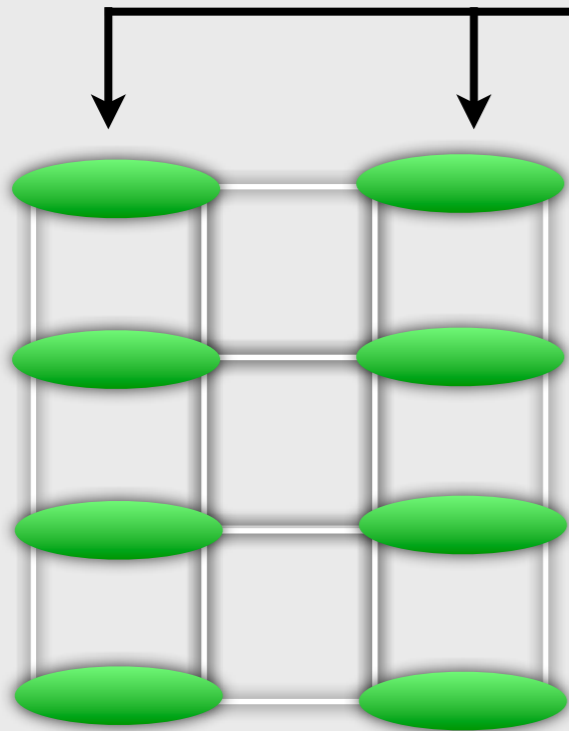
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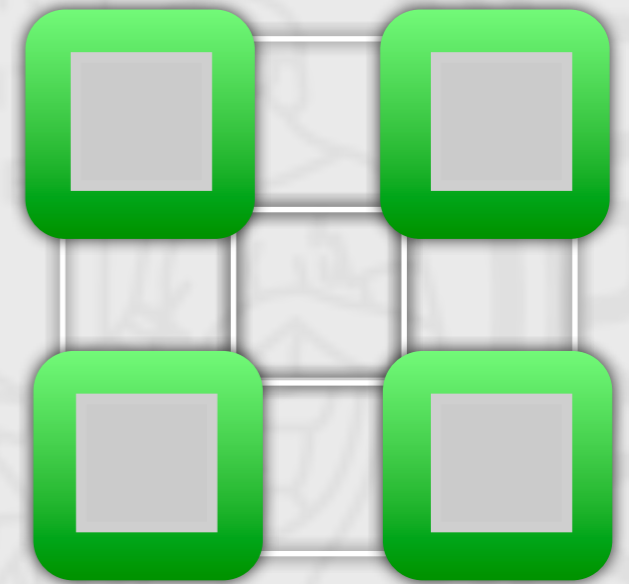
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$$k = 3$$

Spins form 6-spin
singlets
(new result)



$$k = 4$$

Spins form 4-spin
singlets
(new result)

Saddle points for $k > 4$

To minimize their energy, fermions attempt to organize hoppings so that they completely fill a band

The filling fraction for fermions with one of N spin components is

$$\nu = \frac{m}{N} = \frac{N}{kN} = \frac{1}{k}$$

Fermions would like to form a closed band with N_s/k states

Our result: the best way to do that is by arranging a “magnetic flux” of $2\pi/k$ per plaquette and fill the lowest Landau level.

$2\pi/k$	$2\pi/k$
$2\pi/k$	$2\pi/k$

M. Hermele, VG, A.-M. Rey (2009)

The background features a large, faint watermark of the University of Colorado seal. The seal is circular and contains the text "UNIVERSITY OF COLORADO" around the top and "1876" at the bottom. In the center, it says "LET YOUR LIGHT SHINE" above an illustration of a Native American figure holding a bow and arrow, with a torch to the left.

Chiral Spin Liquid

Chiral Spin Liquid (CSL)



Wen, Wilczek & Zee

Kalmeyer & Laughlin

Proposed in 1989

A state of magnets

without any magnetic order (spin liquid),
but breaking parity and time reversal invariance (chiral).

Has to be described by a Chern-Simons theory.

(Local) Hamiltonians whose ground state would be CSL
were unknown until now

Chiral spin liquid ($k > 4$)

$$H = \sum_{\langle ij \rangle, \alpha} \chi_{ij}^{(0)} \left(\hat{f}_{i,\alpha}^\dagger \hat{f}_{j,\alpha} + \text{H.c.} \right)$$

magnetic field with $1/k$ flux through plaquette

like quantum Hall effect

$$\chi_{ij} = \chi_{ij}^{(0)} e^{iA_{ij}}$$

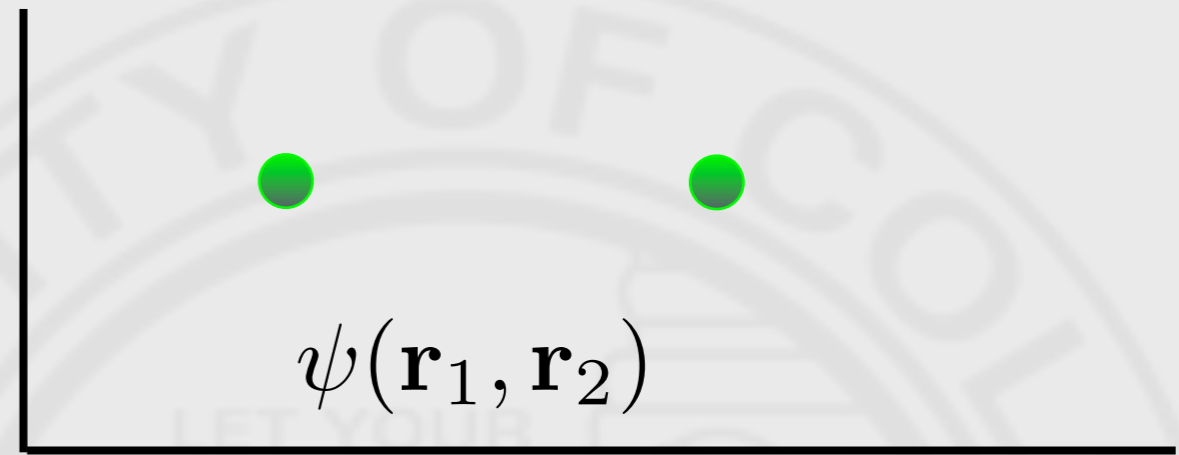
$$S_{\text{eff}} = \frac{N}{4\pi} \int d^2x dt \epsilon_{\mu\nu\rho} A_\mu \partial_\nu A_\rho \quad \text{Level } N \text{ Chern-Simons theory}$$

Fermions acquire fractional statistics with the angle θ :

$$\theta = \pi + \frac{\pi}{N} \quad N = k, 2k, 3k, \dots$$

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$$\psi(\mathbf{r}_2, \mathbf{r}_1) = e^{i\left(\pi + \frac{\pi}{N}\right)} \psi(\mathbf{r}_1, \mathbf{r}_2)$$

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Level N Chern-Simons theory

Fermions acquire fractional statistics with the angle θ :

$$\theta = \pi + \frac{\pi}{N} \quad N = k, 2k, 3k, \dots$$

Who's the carrier of the statistics?

$$\hat{S}_\beta^\alpha(i) = \hat{f}_{i,\alpha}^\dagger \hat{f}_{i,\beta} \quad \text{The spin itself is not fractional}$$

The fermions are fractional, but with each site containing exactly one atom, the fermionic atoms don't have any dynamics, fractional or otherwise

Let's create "holes" - empty atomless sites on the lattice

$$\hat{f}_{i,\alpha} = \hat{b}_i^\dagger \hat{c}_{i,\alpha}$$

↑ atom
 ↑ "holon" - boson carrying atom number
 ↓ "spinon" - fermion carrying spin

Well known construction from high T_c theories - only here justified by large N .

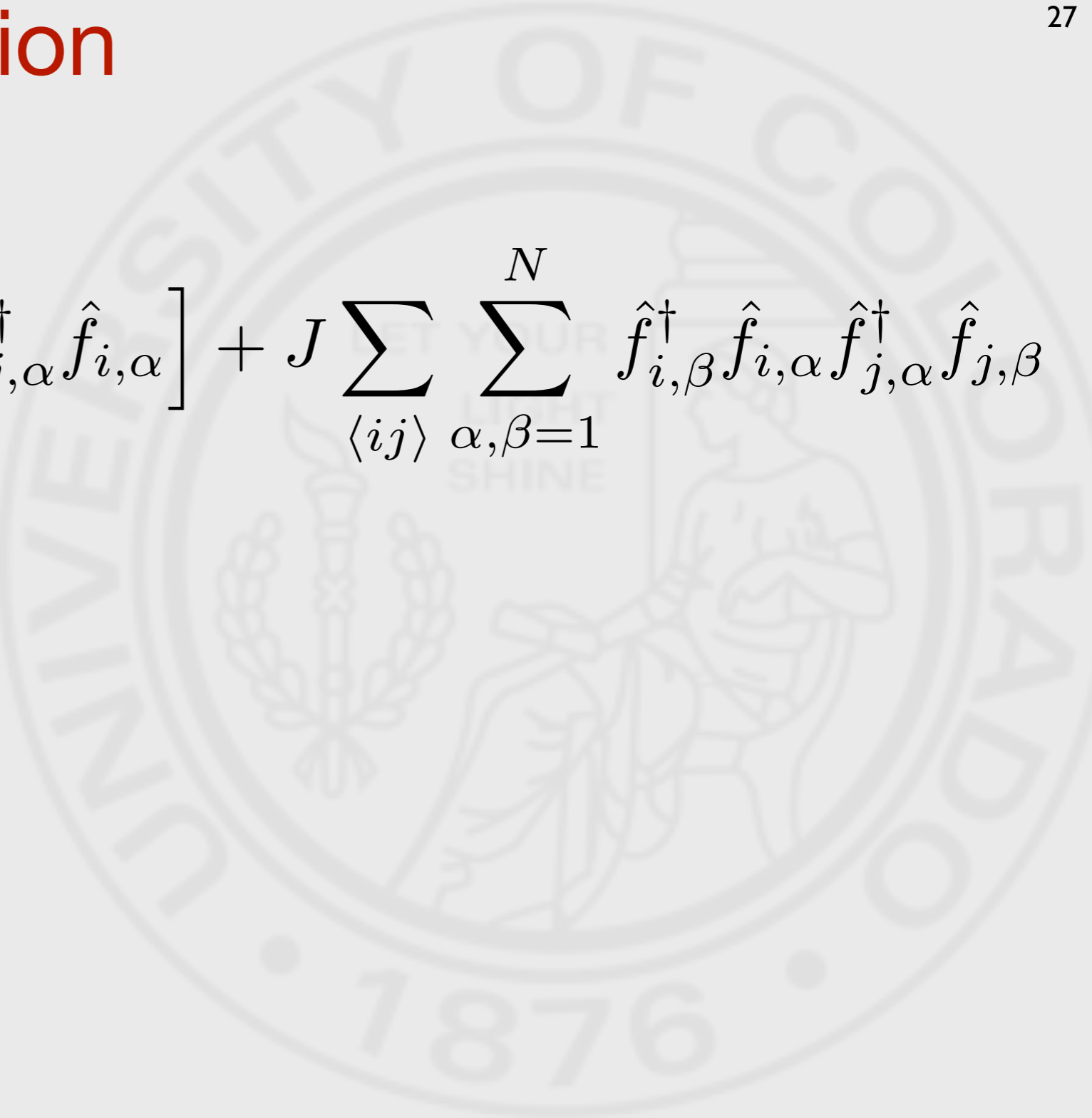
Both spinons and holons are fractional, but only holons respond to an external potential

P.A. Lee, N. Nagaosa (1992)

Spin-charge separation

Start with a t-J model

$$\hat{H} = t \sum_{\langle ij \rangle} \sum_{\alpha=1}^N \left[\hat{f}_{i,\alpha}^\dagger \hat{f}_{j,\alpha} + \hat{f}_{j,\alpha}^\dagger \hat{f}_{i,\alpha} \right] + J \sum_{\langle ij \rangle} \sum_{\alpha,\beta=1}^N \hat{f}_{i,\beta}^\dagger \hat{f}_{i,\alpha} \hat{f}_{j,\alpha}^\dagger \hat{f}_{j,\beta}$$



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$$\sum_{\alpha=1}^N \hat{c}_{i,\alpha}^\dagger \hat{c}_{i,\alpha} + \hat{b}_j^\dagger \hat{b}_j = 1$$

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New effective Hamiltonian

$$\hat{H}_{\text{eff}} = -\frac{J}{N} \sum_{\langle ij \rangle} \left[\left(\sum_{\alpha=1}^N \hat{c}_{i,\alpha}^\dagger \hat{c}_{j,\alpha} + \frac{t}{J} \hat{b}_i^\dagger \hat{b}_j \right) \left(\sum_{\beta=1}^N \hat{c}_{j,\beta}^\dagger \hat{c}_{i,\beta} + \frac{t}{J} \hat{b}_j^\dagger \hat{b}_i \right) - \frac{t^2}{J^2} \hat{b}_i^\dagger \hat{b}_i \hat{b}_j^\dagger \hat{b}_j \right]$$

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Introduce the Hubbard-Stratonovich field, integrate out the fermions (spinons) to find the effective holon Hamiltonian

$$\hat{H} = \sum_{\langle ij \rangle} \left[\frac{t}{J} \chi_{ij}^{(0)} e^{ia_{ij}} \hat{b}_i^\dagger \hat{b}_j + \frac{t}{J} \chi_{ij}^{(0)} e^{-ia_{ij}} \hat{b}_i^\dagger \hat{b}_j \right] + \frac{t^2}{JN} \sum_{\langle ij \rangle} \hat{b}_i^\dagger \hat{b}_i \hat{b}_j^\dagger \hat{b}_j$$

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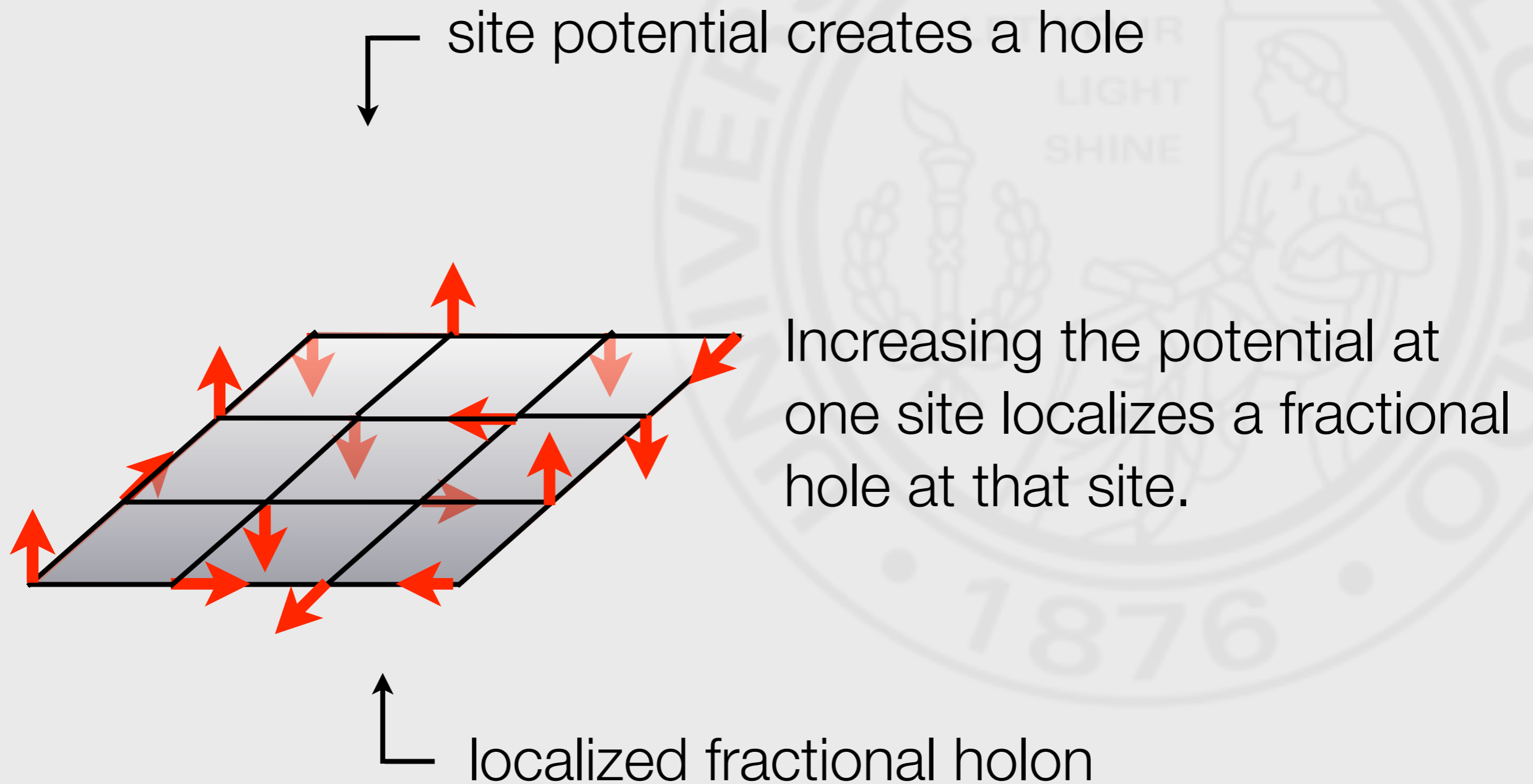
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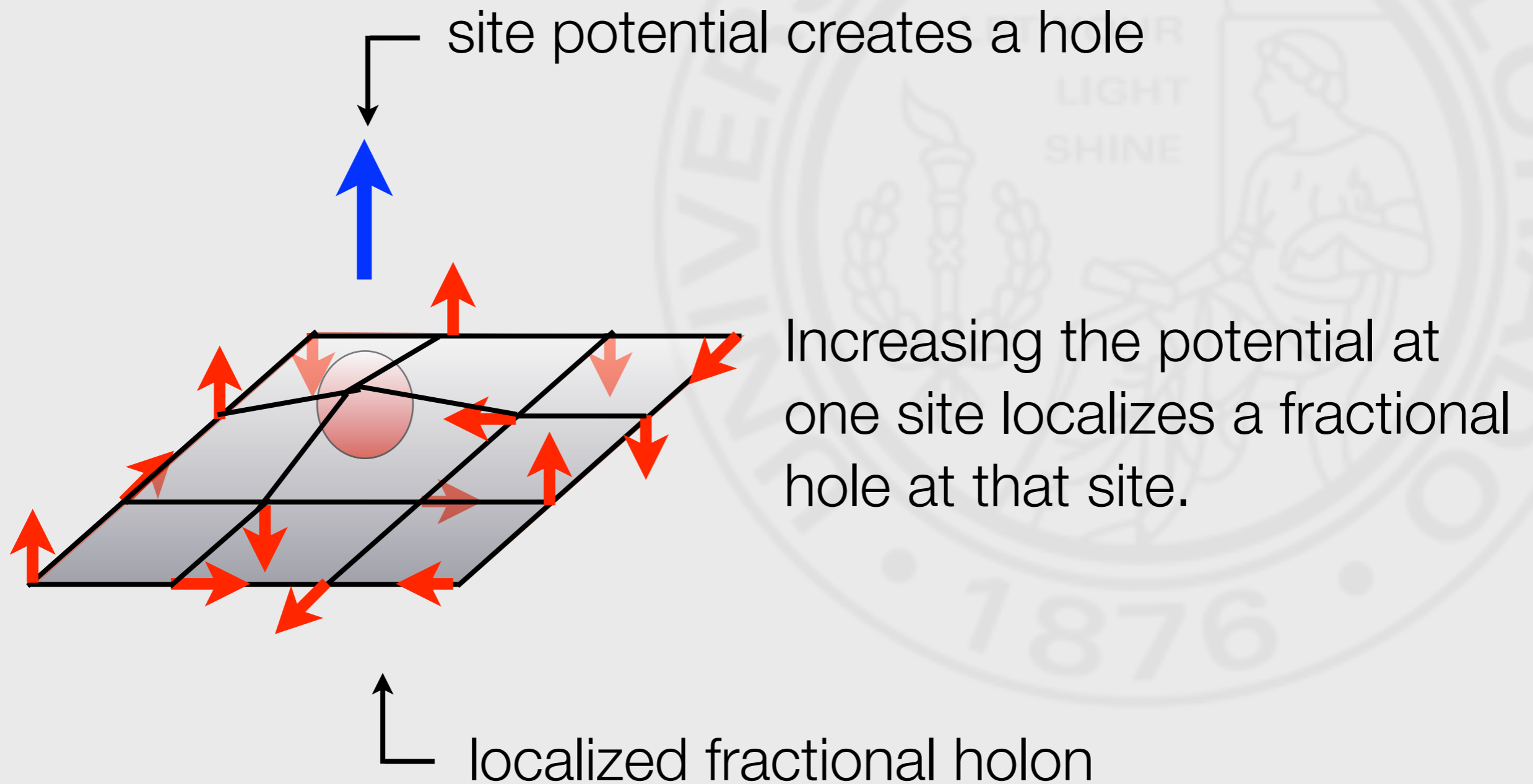
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End result: (weakly) interacting bosons coupled to a Chern-Simons field

Scenario to create fractional excitations



Scenario to create fractional excitations



Non-Abelian chiral spin liquid

Place two species of fermionic atoms on each site, in two distinct electronic states 1S_0 and 3P_0 .

They form either antisymmetric (boring) or symmetric (interesting) state depending on the relative strength of interaction constants (10 author paper)

$a, b = ^1S_0, ^3P_0$ labels species

$$|s\rangle = \frac{1}{\sqrt{2}} \left(\hat{f}_{a,\alpha}^\dagger \hat{f}_{b,\beta}^\dagger + \hat{f}_{a,\beta}^\dagger \hat{f}_{b,\alpha}^\dagger \right) |0\rangle \quad \text{on every site of the lattice}$$

$$H = \sum_{\langle ij \rangle, a, b, \alpha, \beta} \hat{f}_{i,a,\alpha}^\dagger \hat{f}_{i,a,\beta} \hat{f}_{j,b,\beta}^\dagger \hat{f}_{j,b,\alpha}$$

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$$\chi_{ij}^{(0)ab} = \delta_{ab} \chi_{ij}^{(0)} \leftarrow \text{magnetic field}$$

$$S_{CS} = \frac{N}{4\pi} \text{Tr} \int d^2x dt \epsilon_{\mu\nu\rho} \left[A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right]$$

This is non-Abelian Chern-Simons $SU(2)_N$ theory!

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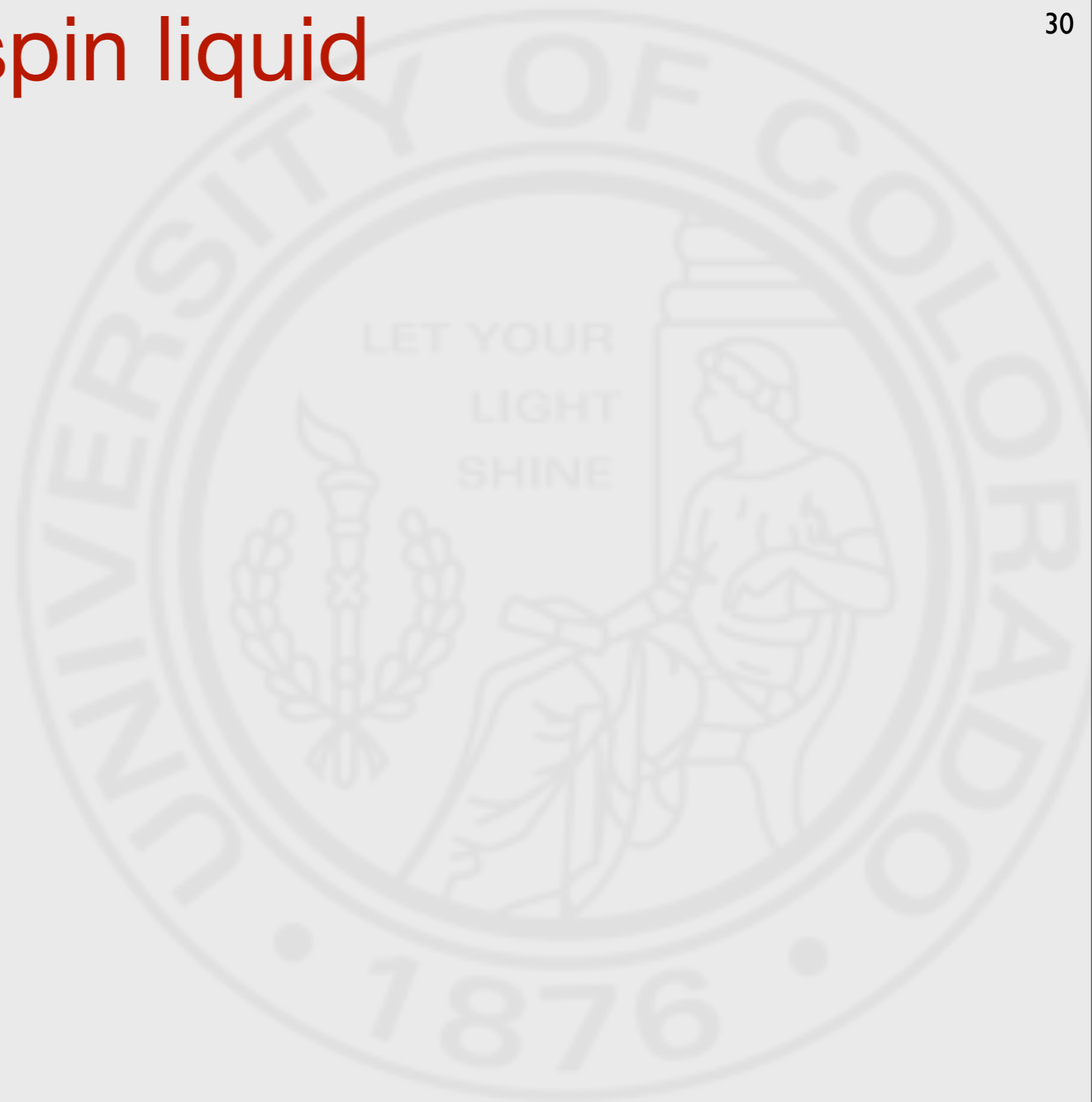
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Topological quantum computing with $SU(2)_{10}$!

Non-Abelian chiral spin liquid

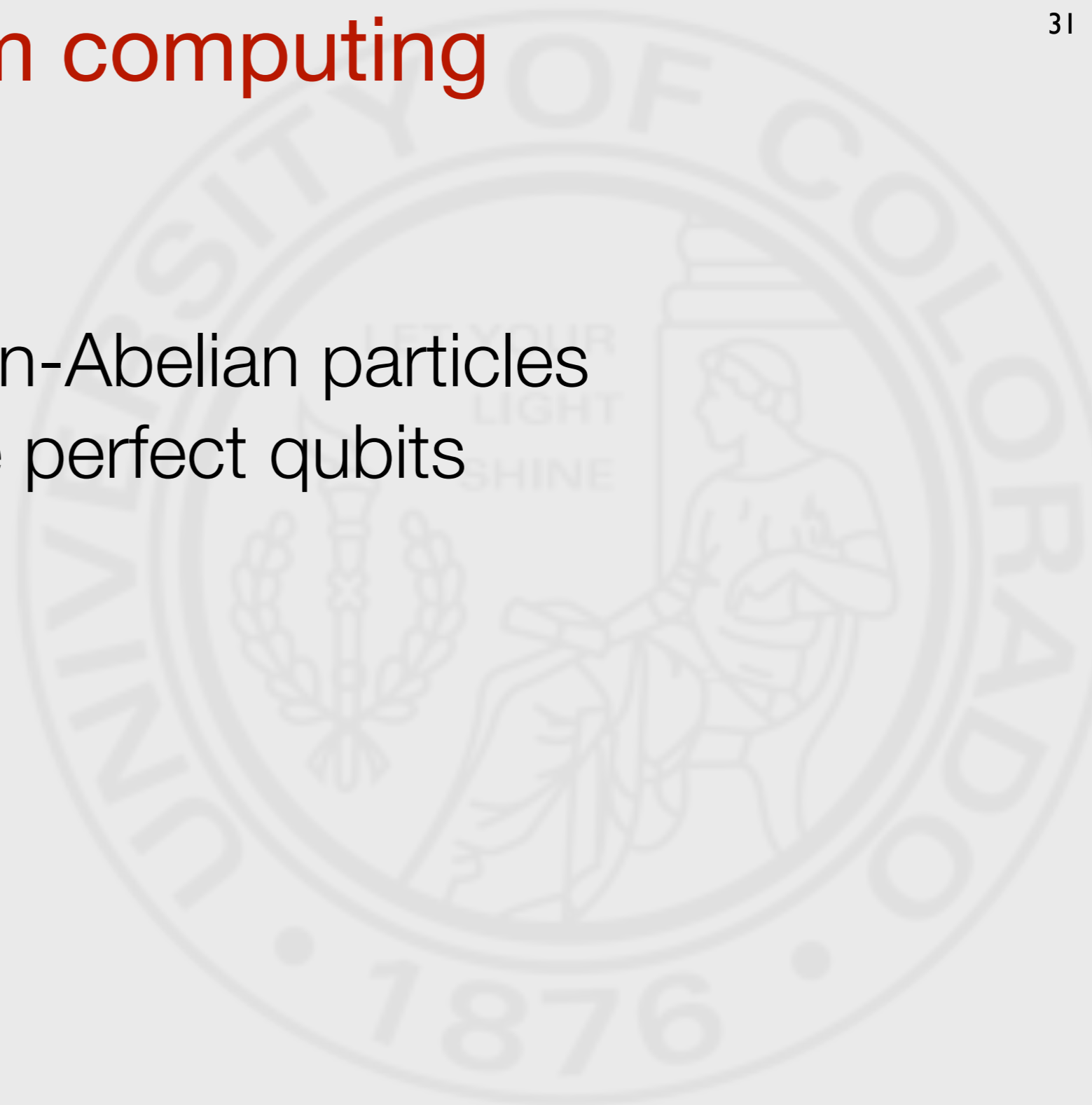


Topological quantum computing



Non-Abelian particles
are perfect qubits

A. Kitaev, 1997

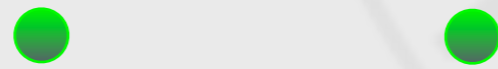


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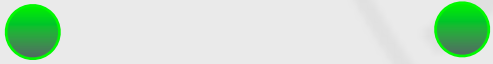
$\psi_\alpha(\mathbf{r}_1, \mathbf{r}_2)$

Topological quantum computing

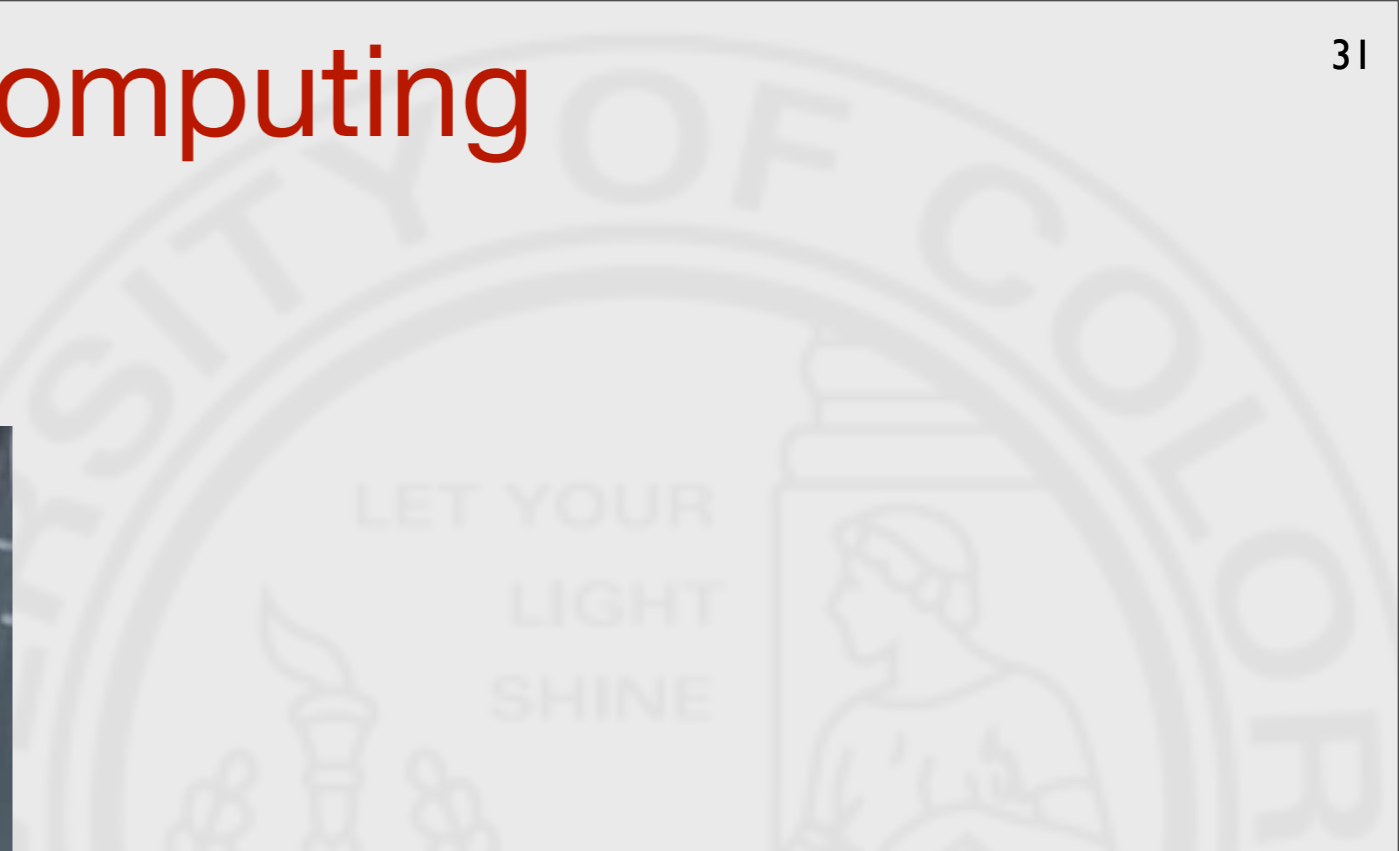


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$$\psi_{\alpha}(\mathbf{r}_2, \mathbf{r}_1) = \sum_{\beta} U_{\alpha, \beta} \psi_{\beta}(\mathbf{r}_1, \mathbf{r}_2)$$

Topological quantum computing



Microsoft | UCSB KITP | UCSB Physics | UCSB Math | CNSI

Station Q

Welcome!

People

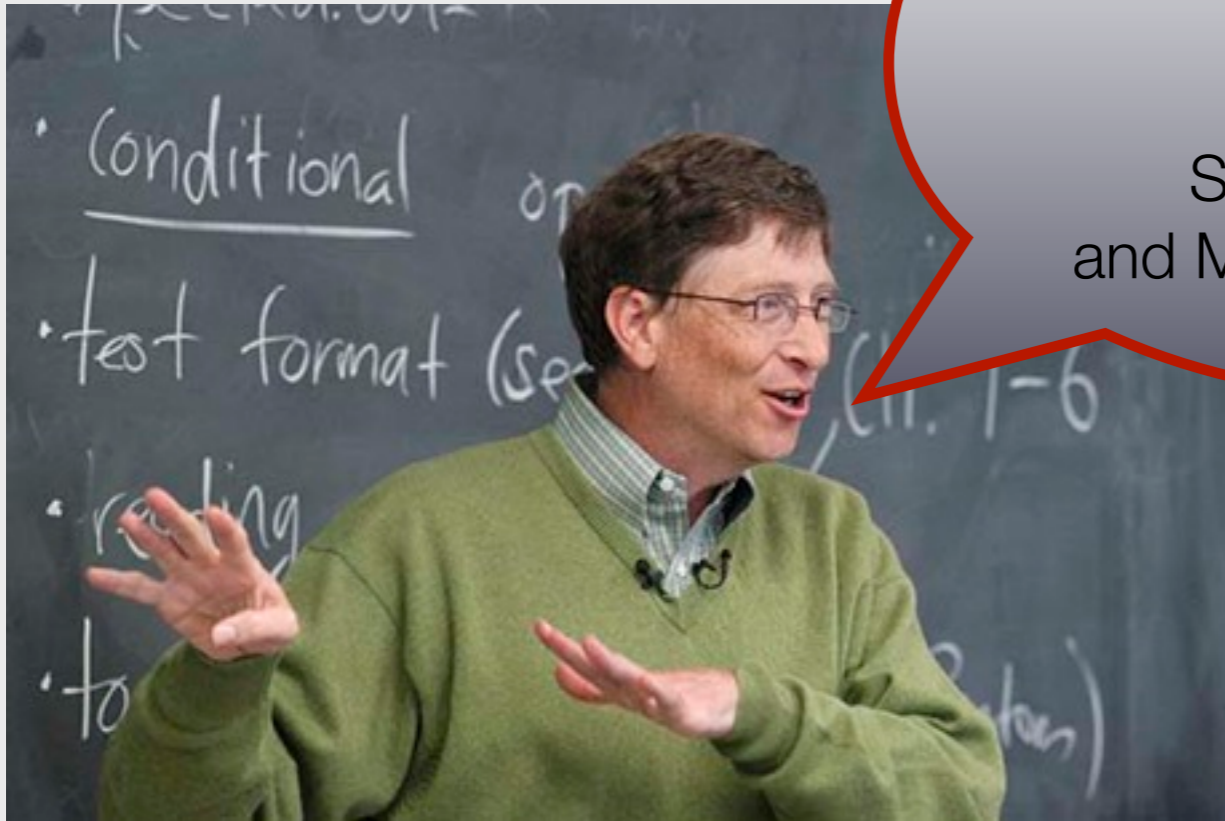
Research

Welcome to Station Q

Station Q is a Microsoft research group working on topological quantum computing. The group combines researchers from math, physics and computer science.

Topological quantum computing

We wish we had an
 $SU(2)_2$
 $SU(2)_2 = \text{Pfaffian}$
 and Majorana fermions...



Microsoft | UCSB

KITP | UCSB Physics | UCSB Math | CNSI

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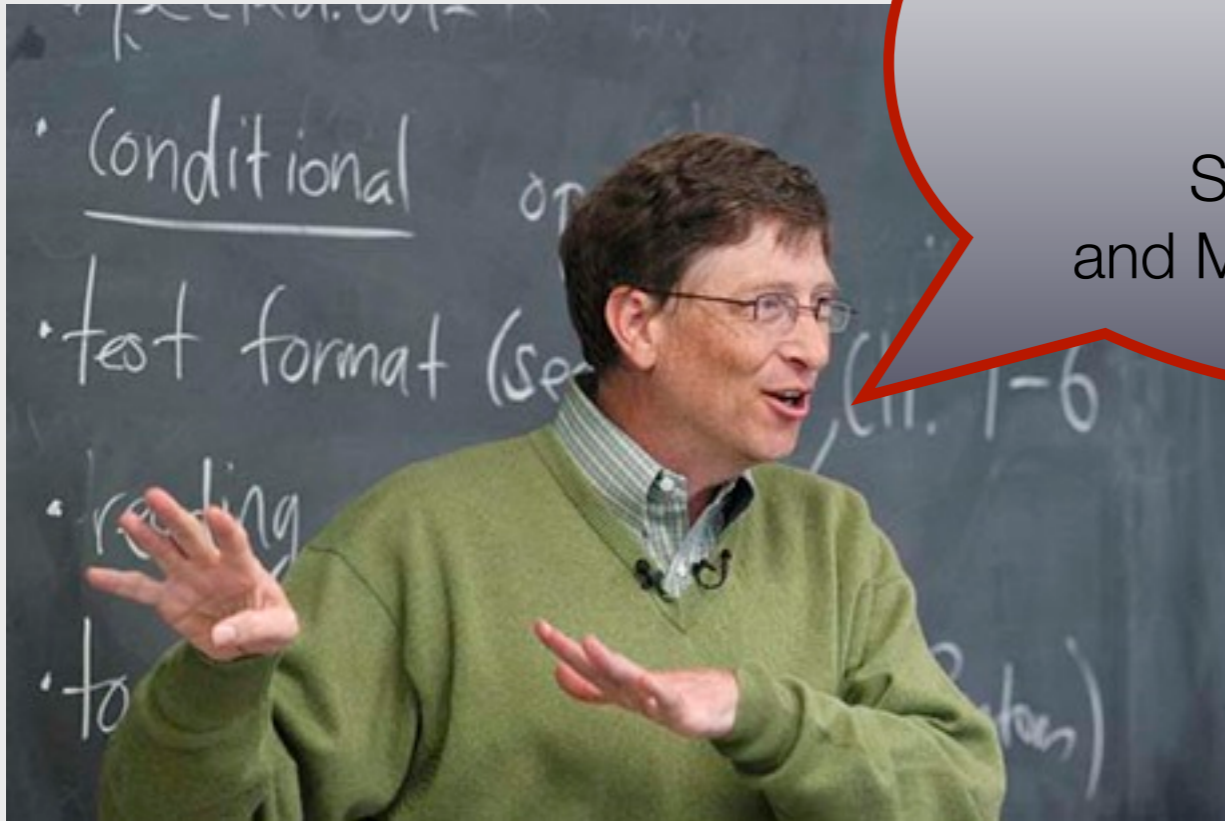
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People

Research

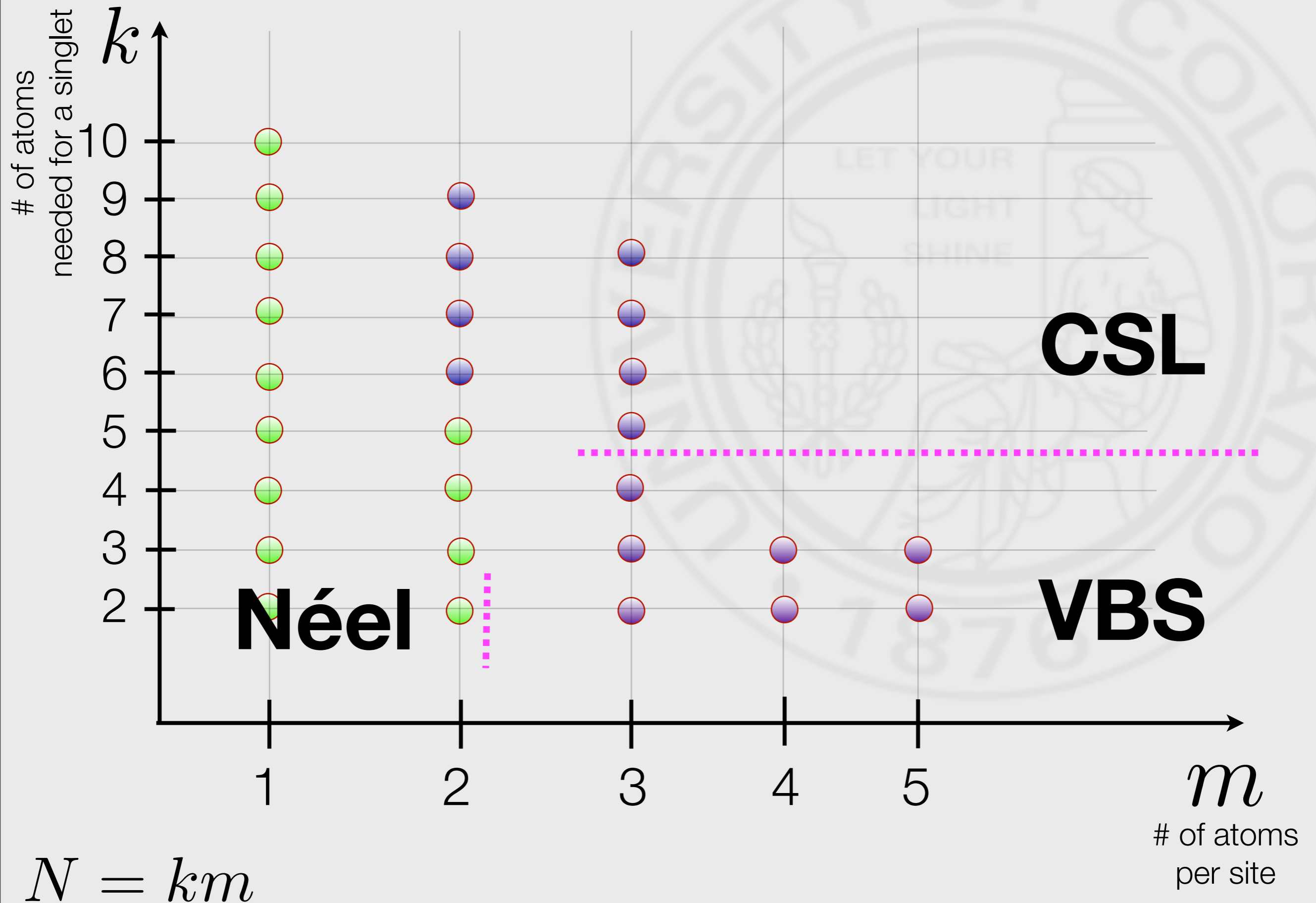
Station Q is a Microsoft research group working on topological quantum computing. The group combines researchers from math, physics and computer science.

But: $SU(2)_2$ not good enough for
 a “universal” quantum computer

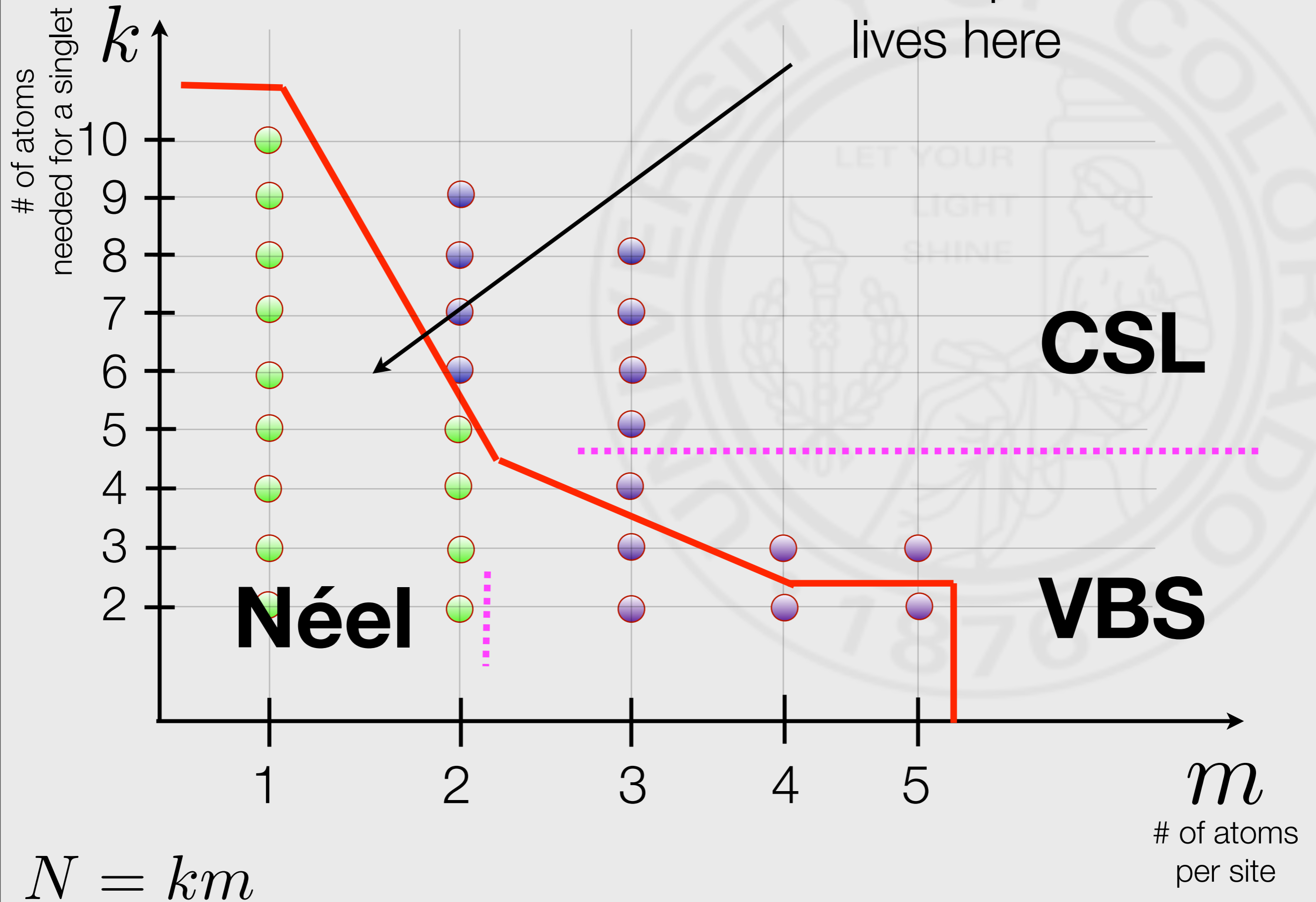
Topological quantum computing

How about $SU(2)_{10}$!

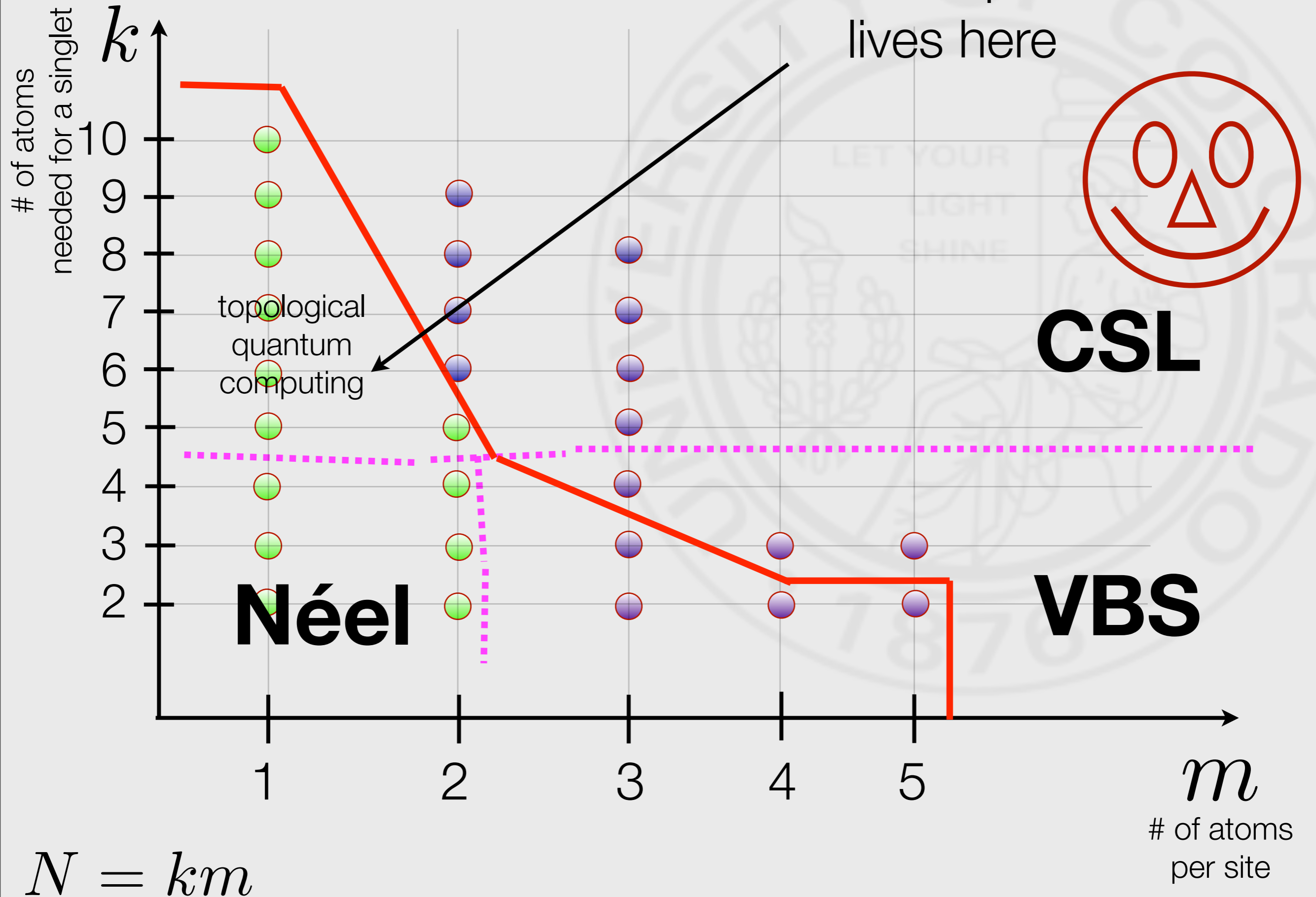
Phase diagram (Abelian)



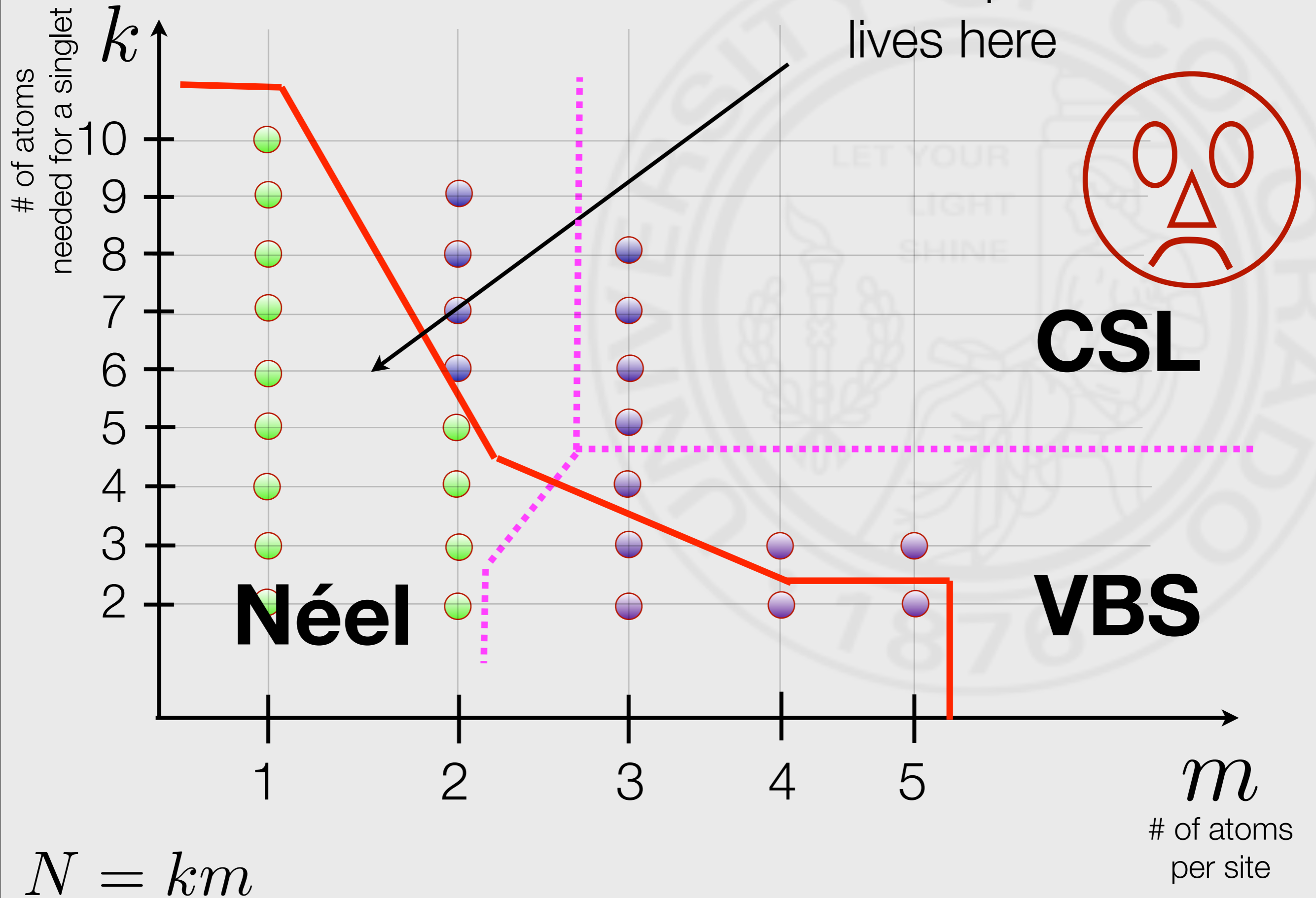
Phase diagram (Abelian) Potential experiment lives here



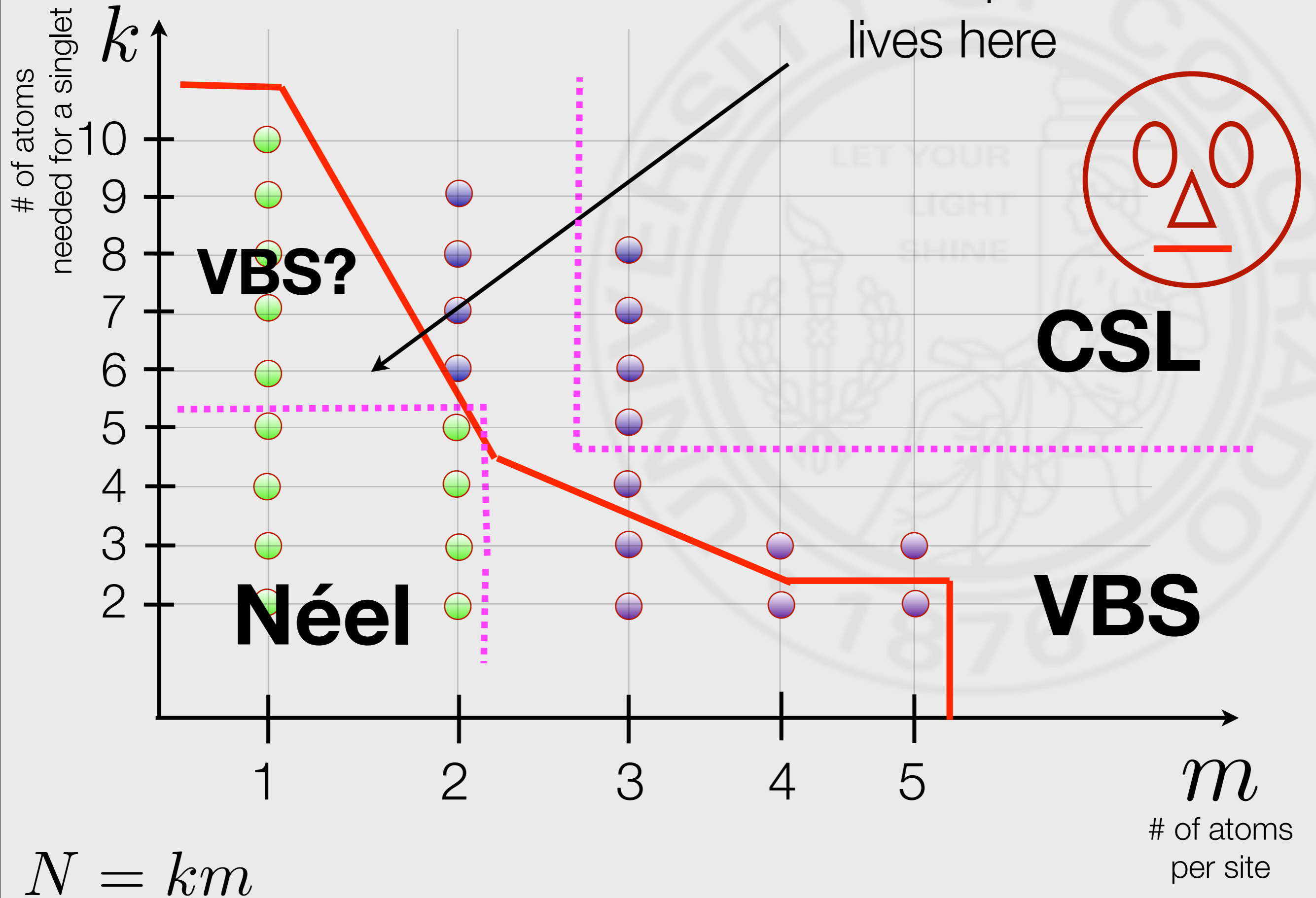
Phase diagram (Abelian) Potential experiment lives here



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Phase diagram (Abelian) Potential experiment lives here



Status of numerics

- Numerical methods are the only way to access the experimentally relevant $m=1$ column
- QMC - sign problem for $k>2$, at least in 2D
- Other 2D numerical methods?

Conclusions

- ▶ $SU(N)$ magnets are a useful theoretical construct due to the existence of the large N techniques
- ▶ $SU(N)$ magnets can have phases going beyond the phases of the $SU(2)$ magnets
- ▶ Nuclear spin of the alkaline earth atoms a perfect realization of the $SU(N)$ spin - so far lacking in condensed matter
- ▶ A version of the $SU(N)$ magnets particularly well suited to realization by the alkaline earth atoms forms chiral spin liquids, a state of matter with fractionalized excitations
- ▶ Possibility of the topological quantum computing with the $SU(N)$ spin magnets??

A large, faint watermark of the University of Colorado seal is centered in the background. The seal is circular and contains the text "UNIVERSITY OF COLORADO" around the top edge and "1876" at the bottom. In the center of the seal, it says "LET YOUR LIGHT SHINE" above a figure holding a torch and a book.

The end