September 2010, KITP

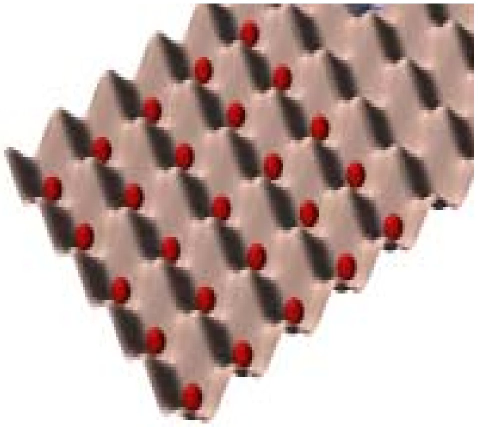
Transport Theory of Lattice Bosons

Assa Auerbach, Technion, Israel

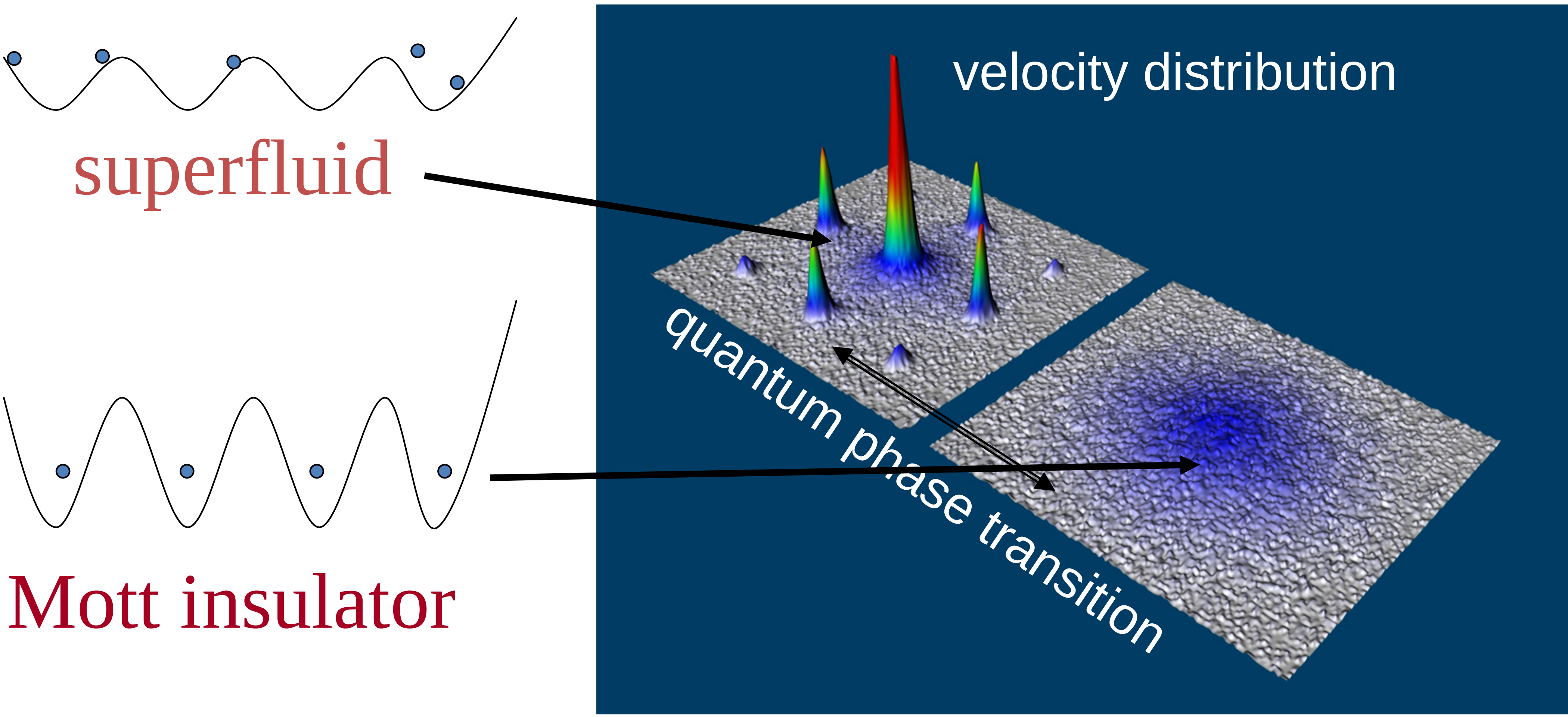
1. Hard core bosons = quantum magnetism
2. Difference between low density versus half filling.
3. Quantum vortices: Studies of the Gauged Torus.
4. Hall conductance and V-spin.
5. Temperature dependent dynamical conductivity and resistivity.

Netanel H. Lindner and AA, ***Phys. Rev. B81, 054512, (2010).***
Netanel H. Lindner AA and Daniel P. Arovas, ***Phys. Rev. Lett. 101, 070403 (2009)***
+ Phys. Rev.B (in press)**;arXiv:1005.4929**

Boson on an Optical Lattice



I. Bloch



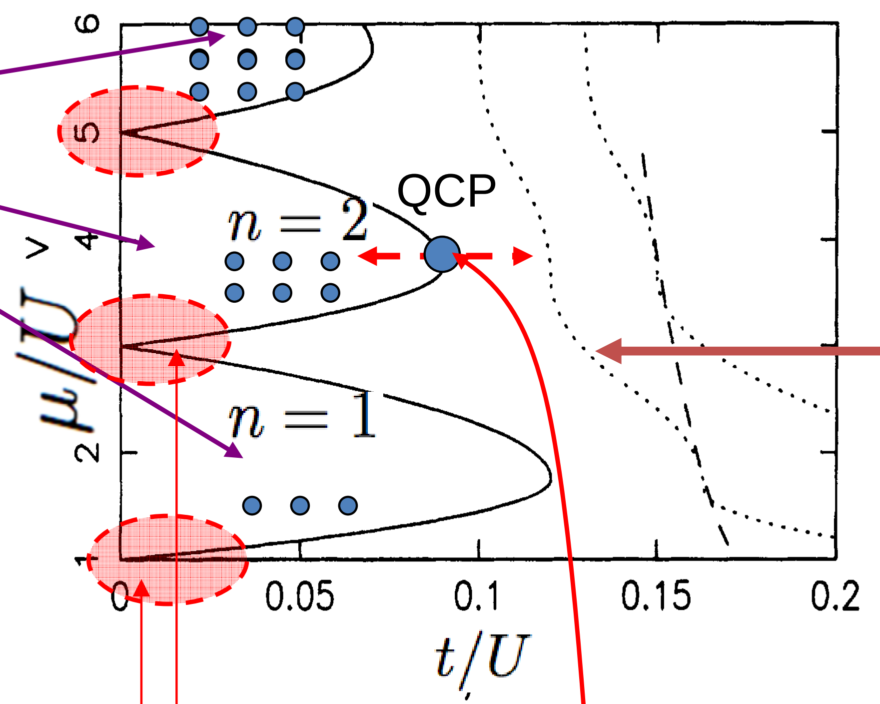
Bose Hubbard Model

Single (s-wave) band

$$\mathcal{H} = -t \sum_{ij} a_i^\dagger a_j + U \sum n_i{}^2 - \mu \sum_i n_i$$

Mott insulators

incompressible

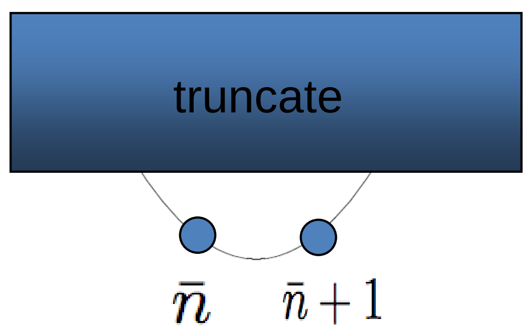


Zimanyi et. al (1994)

superfluid

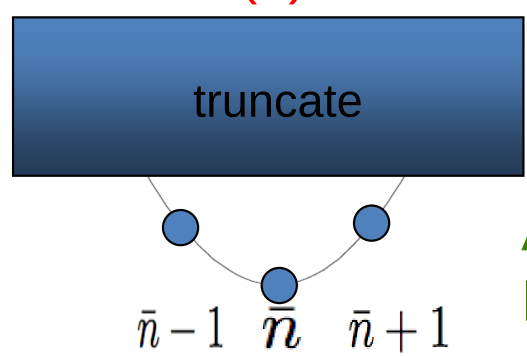
phase order

S=1/2 , XXZ model



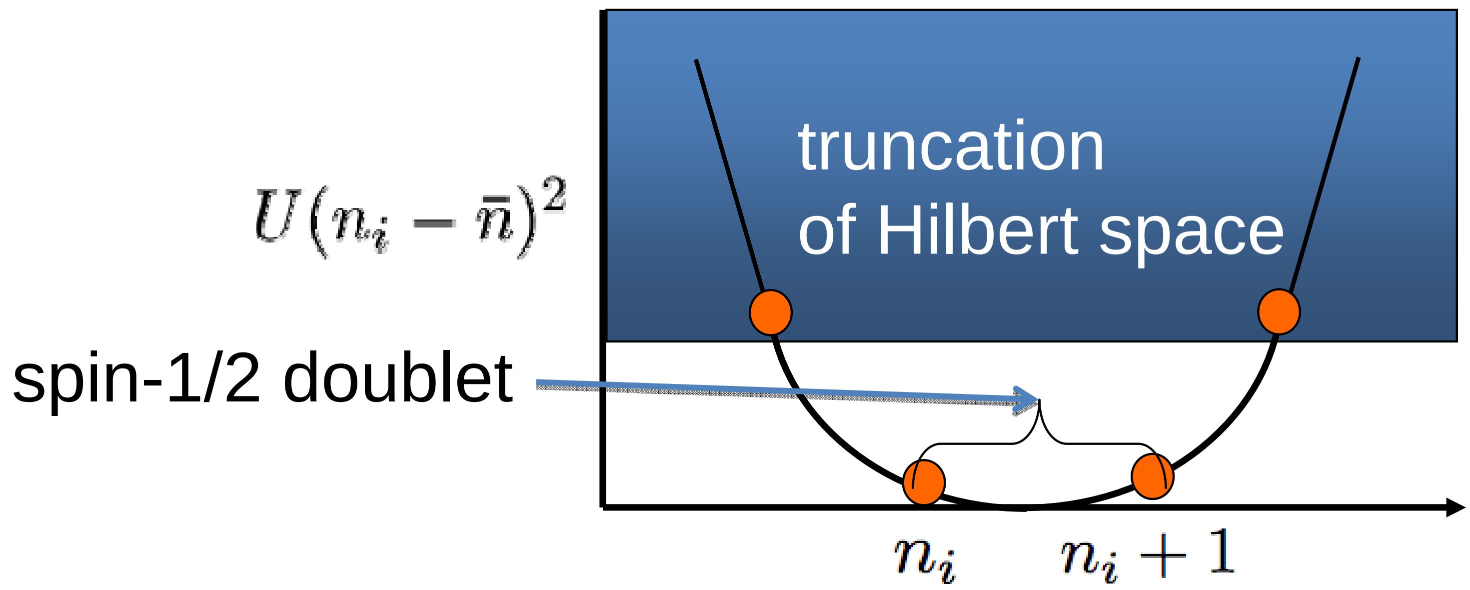
Altman AA, PRL 81, (1998)

S=1, O(2) relativistic GP



Altman AA, PRL 89, (2002)

Hard Core Bosons = spin half representation



$$\tilde{a}_i^\dagger = \mathcal{P}\, a_i^\dagger\, \mathcal{P} = S_i^+$$

$$\tilde{a}_i = \mathcal{P}\, a_i\, \mathcal{P} = S_i^-$$

$$n_i = \tilde{a}_i^\dagger \tilde{a}_i = S_i^z + \frac{1}{2}.$$

HCB are different than free bosons $[\tilde{a}_i\,,\,\tilde{a}_j^\dagger] = (1 - 2n_i)\,\delta_{ij}$

especially around half filling (n=1/2)

Effective Hamiltonian

1. Gauged Bose-Hubbard model

$$\mathcal{H}_U = -2J \sum_{\langle ij \rangle} \left(e^{iqA_{ij}} a_i^\dagger a_j + a^{-iqA_{ij}} a_j^\dagger a_i \right)$$

$$+4V \sum_{\langle ij \rangle} \left(n_i - \tfrac{1}{2}\right)\left(n_j - \tfrac{1}{2}\right) - \mu \sum_i n_i + \tfrac{1}{2}U \sum_i n_i (n_i - 1)$$

2. large U/J → Gauged Spin ½ XXZ model

$$\mathcal{H} = -2J \sum_{\langle ij \rangle} \left(e^{iqA_{ij}} S_i^+ S_j^- + e^{-iqA_{ij}} S_i^- S_j^+ \right)$$

$$+4V \sum_{\langle i,j \rangle} S_i^z S_j^z - \mu \sum_i \left(S_i^z + \tfrac{1}{2}\right).$$

$$N_b = \sum_i \langle S_i^z \rangle + \frac{1}{2} \qquad \langle b_i^\dagger \rangle = \langle S_i^x \rangle + i \langle S_i^y \rangle$$

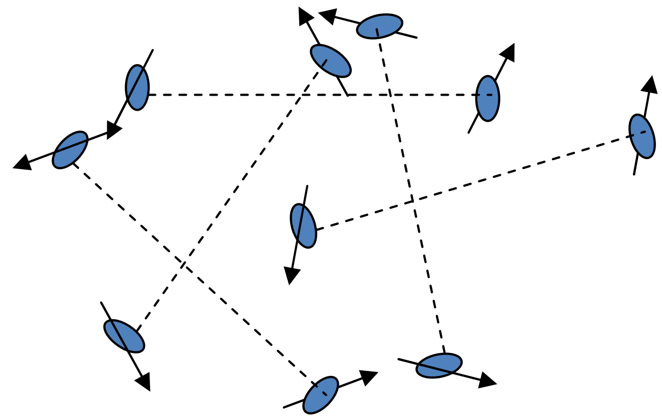
BCS vs HCB Superconductors

Cooper pair operator $$b^\dagger_{\mathbf{x}} = \sum_{\mathbf{r}} f(\mathbf{r}) c^\dagger_{\mathbf{x}-\mathbf{r}/2\uparrow} c^\dagger_{\mathbf{x}+\mathbf{r}/2\downarrow}$$

$$f(\mathbf{r}) = a e^{-r/\xi_{BCS}}$$

BCS regime = large coherence length

$$k_F \xi_{BCS} \sim \frac{\epsilon_F}{2\pi\Delta} >> 1$$

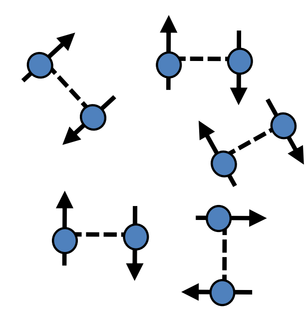


Limit of short coherence length:

$$k_F \xi \leq 1 \qquad b^\dagger_{\mathbf{x}} = c^\dagger_{\mathbf{x}\uparrow} c^\dagger_{\mathbf{x}\downarrow}$$

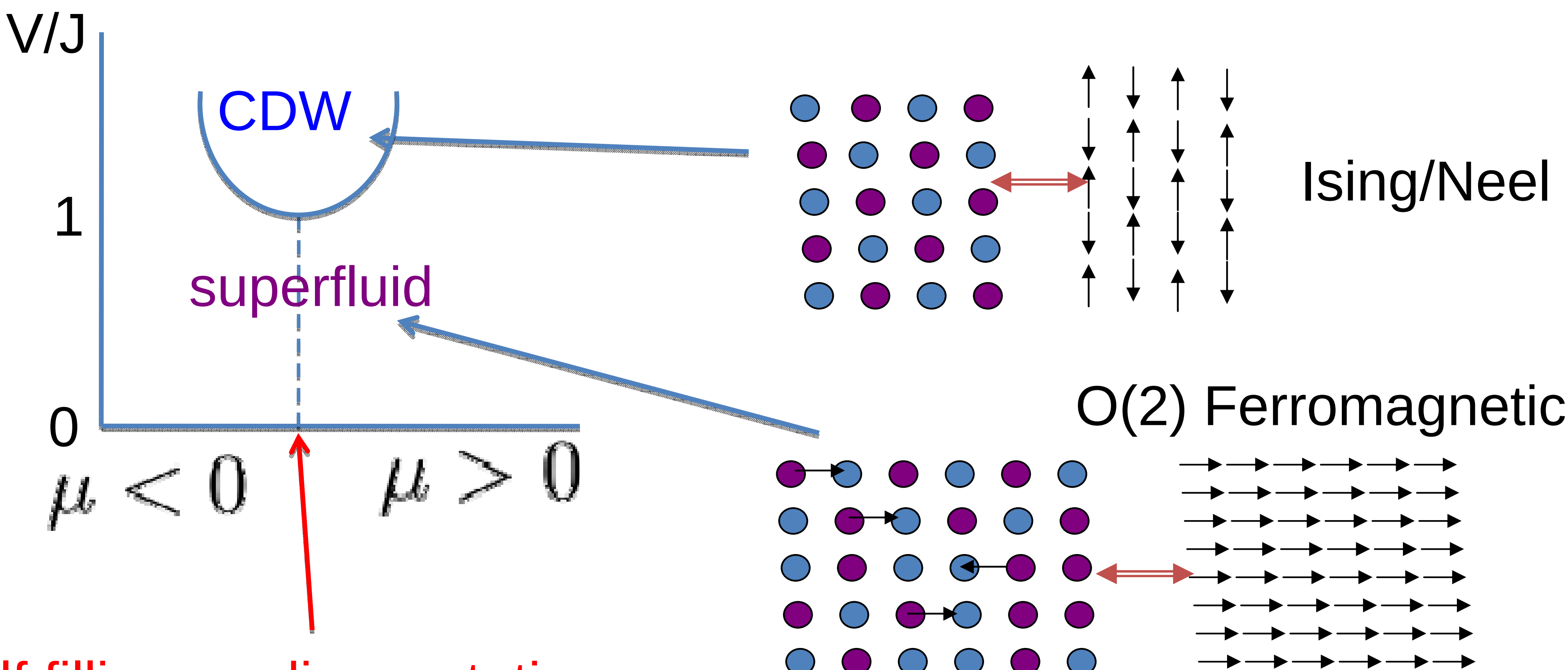
→ hard core bosons

$$(b^\dagger_{\mathbf{x}})^2 = 0 \qquad \left[b_{\mathbf{x}}, b^\dagger_{\mathbf{x}'} \right] = (1 - 2n_{\mathbf{x}}) \delta(\mathbf{x} - \mathbf{x}')$$



Ground states of Hard Core Bosons

Mean field theory $|\Psi^{var}\rangle = \prod_i |\vec{S}(x_i)\rangle$



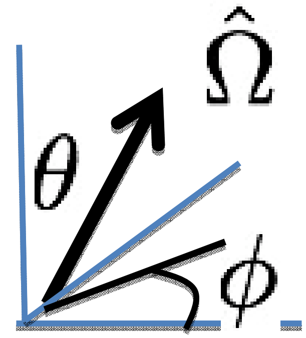
half filling: ordinary statics,

- interesting dynamics!

Semiclassical theory

Spin coherent states path integral

$$Z = \int \mathcal{D}\widehat{\Omega}(\tau) \exp\left(\int_0^\beta d\tau \left(iK - H^{cl} \right) \right)$$



$$H^{\rm cl}[\widehat{\Omega}, A] = -J \sum_{\langle i,j \rangle} \sin\theta_i \sin\theta_j \cos\left(\phi_i - \phi_j + qA_{ij}\right) + V \sum_{\langle i,j \rangle} \cos\theta_i \cos\theta_j - \frac{\mu}{2} \sum_i \cos\theta_i .$$

Berry phases: $K[\widehat{\Omega}, \dot{\widehat{\Omega}}] \equiv \frac{1}{2} \sum_i (1 - \cos\theta_i)\, \dot{\phi}_i$

1. Classical (mean field) theory: $\delta H^{\rm cl}[\theta_i, \phi_i] \Big|_{\theta^{cl}, \phi^{cl}} = 0$

2. Semiclassical dynamics:

$$\dot{\phi}_i = \frac{\partial H^{\rm cl}}{\partial \cos\theta_i} \qquad \dot{\cos\theta_i} = -\frac{\partial H^{\rm cl}}{\partial \phi_i}$$

Low density limit

$$n << 1, \qquad \cos(\theta_i) \simeq -1$$

$$\frac{1}{2}\sin(\theta_i)e^{i\phi_i} \;\rightarrow\; \sqrt{n}e^{i\phi} = \psi(x_i)$$

→ Gross Pitaevskii theory

$$Z_{\rm GP} \tilde{=} \int \mathcal{D}\psi^* \mathcal{D}\psi \, \exp\left(- S_{\rm GP}[\psi^*, \psi, \boldsymbol{A}] + \ldots \right)$$

$$S_{\rm GP} = \int d^2x \int dt \left[\psi^*(\partial_t - \mu)\psi \quad + \frac{1}{2m^*} \left| (-i\nabla - q\boldsymbol{A})\psi \right|^2 + \tfrac{1}{2} g |\psi|^4 \right]$$

Approximate Gaililean symmetry (lattice is unimportant)

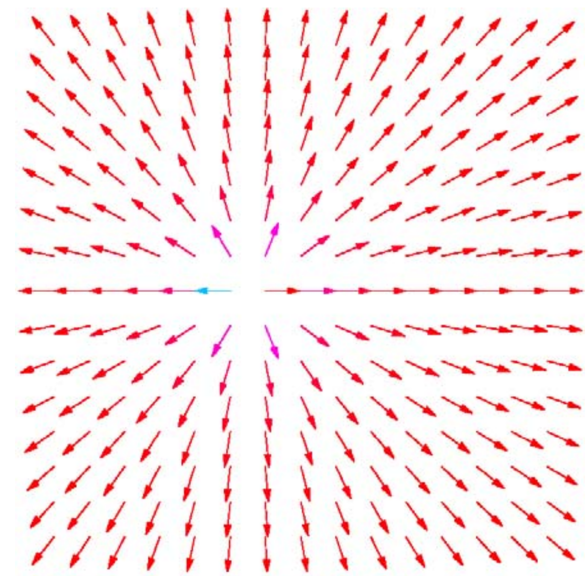
A Vortex in Gross Pitaevskii theory

Pitaevskii, Stringari, BEC (Oxford, 2003)

$$\left(K(\boldsymbol{A}) + V - \mu + g\left|\widetilde{\varphi}(\boldsymbol{x})\right|^2\right)\widetilde{\varphi}(\boldsymbol{x}) = 0$$

$$\lim_{|x|\to\infty} |\tilde{\varphi}| \to \sqrt{n_0}$$

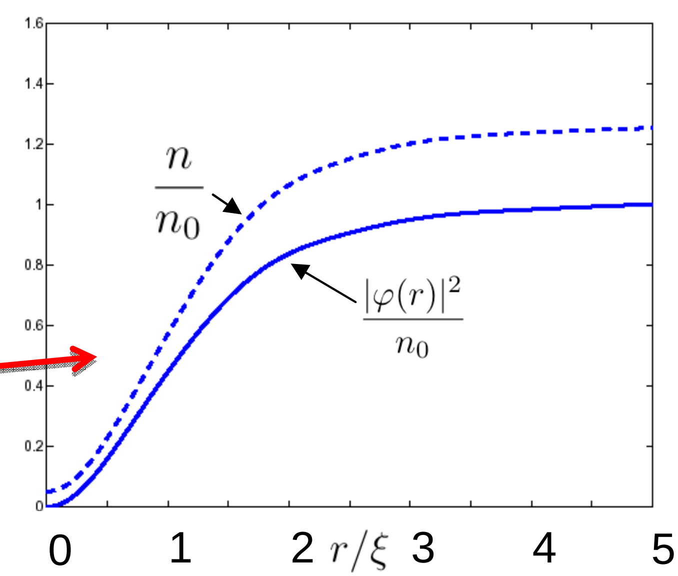
coherence lengthscale $\xi = \frac{\hbar}{\sqrt{g n_0 m}}$



In a uniform magnetic field:

Vortex (approx.) solution: $\widetilde{\varphi} \approx \frac{\sqrt{n_0}\, r\, e^{i\phi}}{\sqrt{r^2 + \xi^2}}$

large core depletion



Dynamics of a Gross-Pitaevskii Vortex

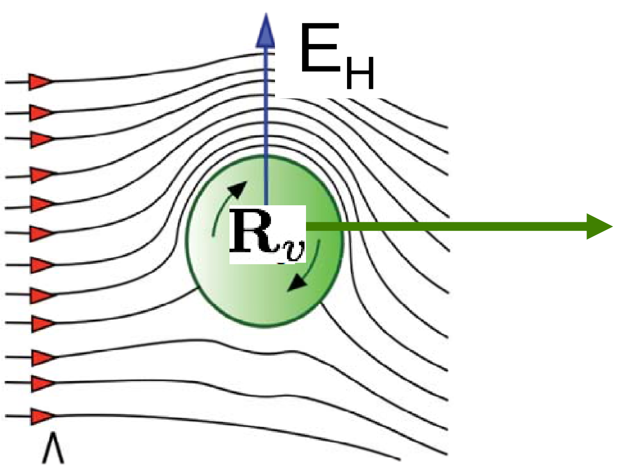
$$\dot{\mathbf{R}} = \hat{\mathbf{z}} \times \vec{\nabla} H[\mathbf{R}]$$

Without *tunneling:*

1. Vortices move on equipotential contours

2. No energy dissipation

→ Vortices 'Go with the Flow' → trivial transport



$$\mathbf{V}_v = \mathbf{v}_s$$

$$\rho_{xx} = 0$$

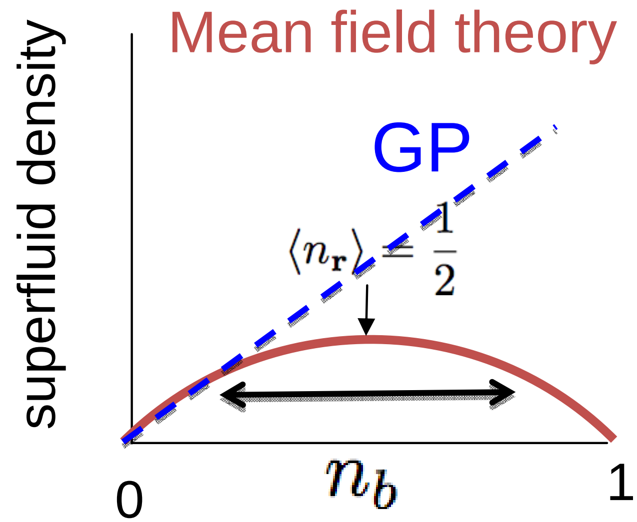
$$\rho_{xy} = \frac{B}{nq}$$

classical Hall number

Emergent Charge Conjugation Symmetry

$$\rho_s^{cl} = 4Jn(1-n)$$

$$T_c$$



Charge conjugation in XXZ model:

$$n \to (1-n)$$

$$C \equiv \exp\left(i\pi \sum_{\mathbf{r}} S^x_{\mathbf{r}}\right)$$

$$S_i^z \to -S_i^z$$
$$S_i^+ \to S_i^-$$

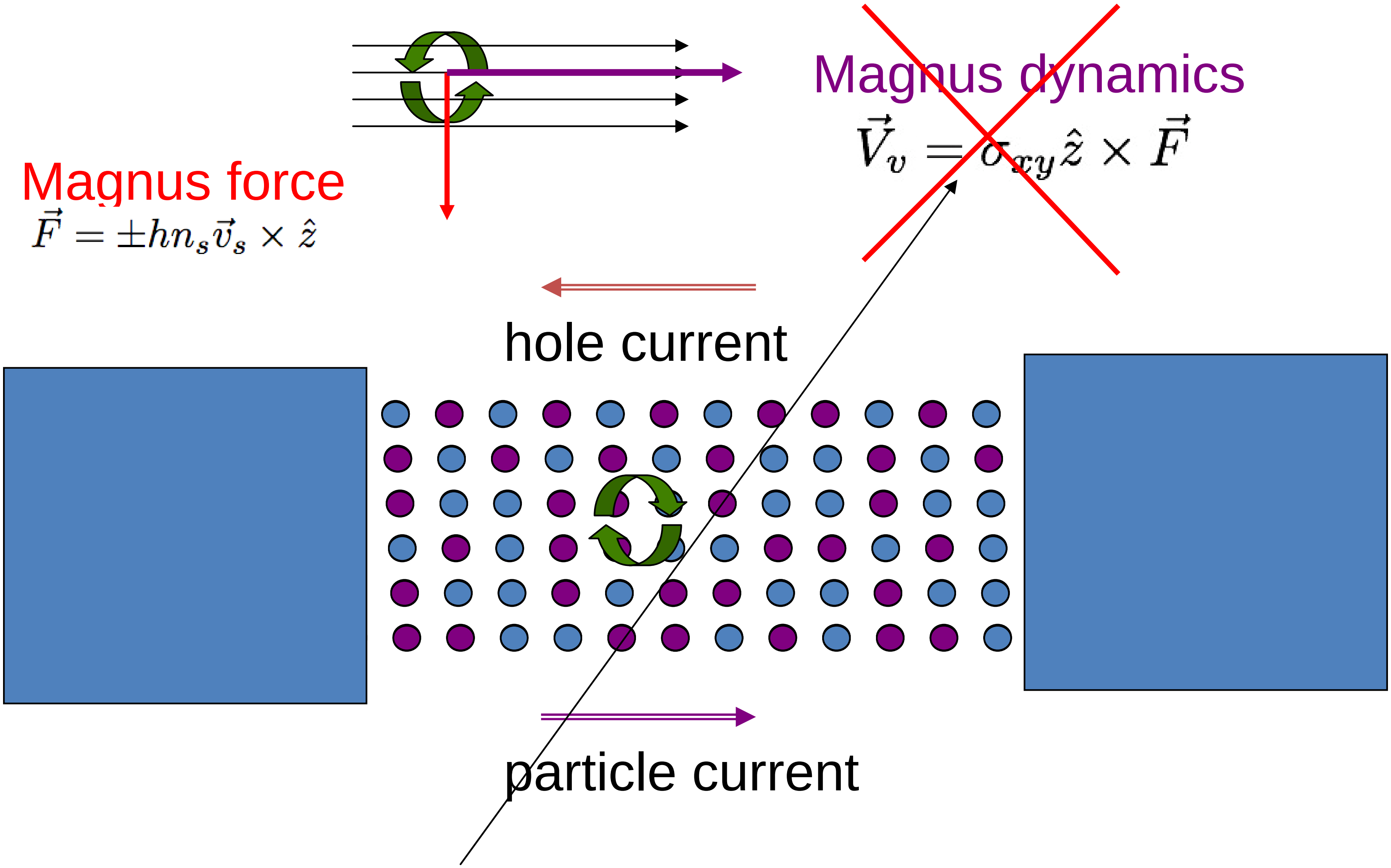
$$C^\dagger \mathcal{H}[\mathbf{A}, n_b] C = \mathcal{H}[-\mathbf{A}, 1-n_b]$$

1. In Hard core limit
2. On ALL lattices

symmetry of transport coefficients:

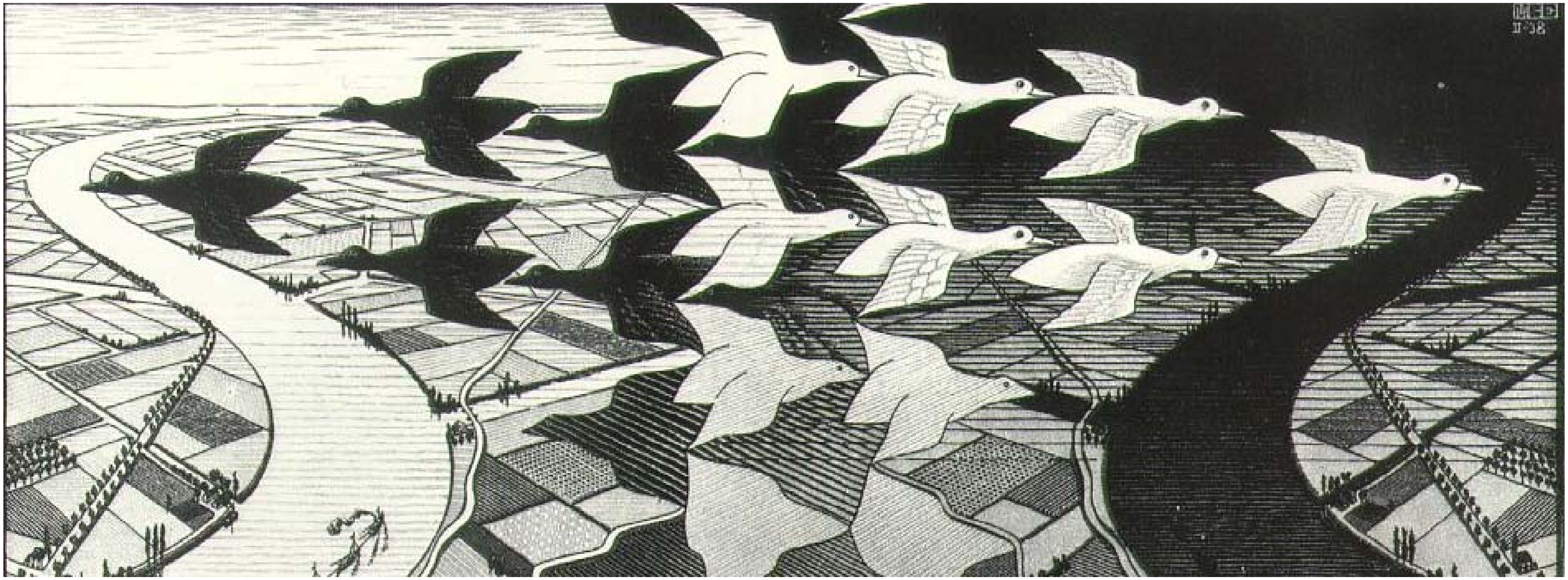
$$\sigma_{xx}(n) = \sigma_{xx}(1-n)$$
$$\sigma_{xy}(n) = -\sigma_{xy}(1-n)$$

No Magnus Field at Half Filling



Magnus dynamics turned off!
No Hall conductivity

Escher, Day and Night, 1938



Half filling – Quantum Antiferromagnet

sublattice rotation $S^z \to (-1)^i S_i^z$

$$\mathcal{H}^{xxz} = \sum_{\langle ij \rangle} J\mathbf{S}_i \cdot \mathbf{S}_j - J^z S_i^z S_j^z$$

easy-plane S=1/2 antiferromagnet

Haldane continuum representation:

Neel field — canting field

$$\hat{\Omega}_i = \eta_i \, \hat{n}(x_i) \sqrt{1 - \left(L(x_i)/S\right)^2} + L(x_i)/S$$

Non Linear Sigma Model (easy plane)

$$\mathcal{L}_{\mathrm{E}} = \tfrac{1}{2}\chi_{\perp} \left|\dot{n}_{\perp}\right|^2 + \tfrac{1}{2}\chi_z \, \dot{n}_z^2 \qquad (24)$$
$$+ \tfrac{1}{2}\rho_{\mathrm{s}} \left|(\boldsymbol{\nabla} - iqA)\, n_{\perp}\right|^2 + \tfrac{1}{2}\rho_{\mathrm{s}}^z \left(\boldsymbol{\nabla} n_z\right)^2 + m_z^2 \, n_z^2 ,$$

+ Berry phases

relativistic (2^{nd} order) dynamics

Superfluid phase

order parameter field $\Psi(x) = n_x(x) + i n_y(x)$

Relativistic Gross-Pitaevskii = O(2) field theory (Higgs)

$$S_{RGP} = \int d^2x \int d\tau \tfrac{1}{2}|\dot{\Psi}|^2 + \frac{\rho_s}{2\Delta^2}|\nabla\Psi|^2 - \frac{m}{8\Delta^2}\left(|\Psi|^2 - \Delta^2\right)^2$$

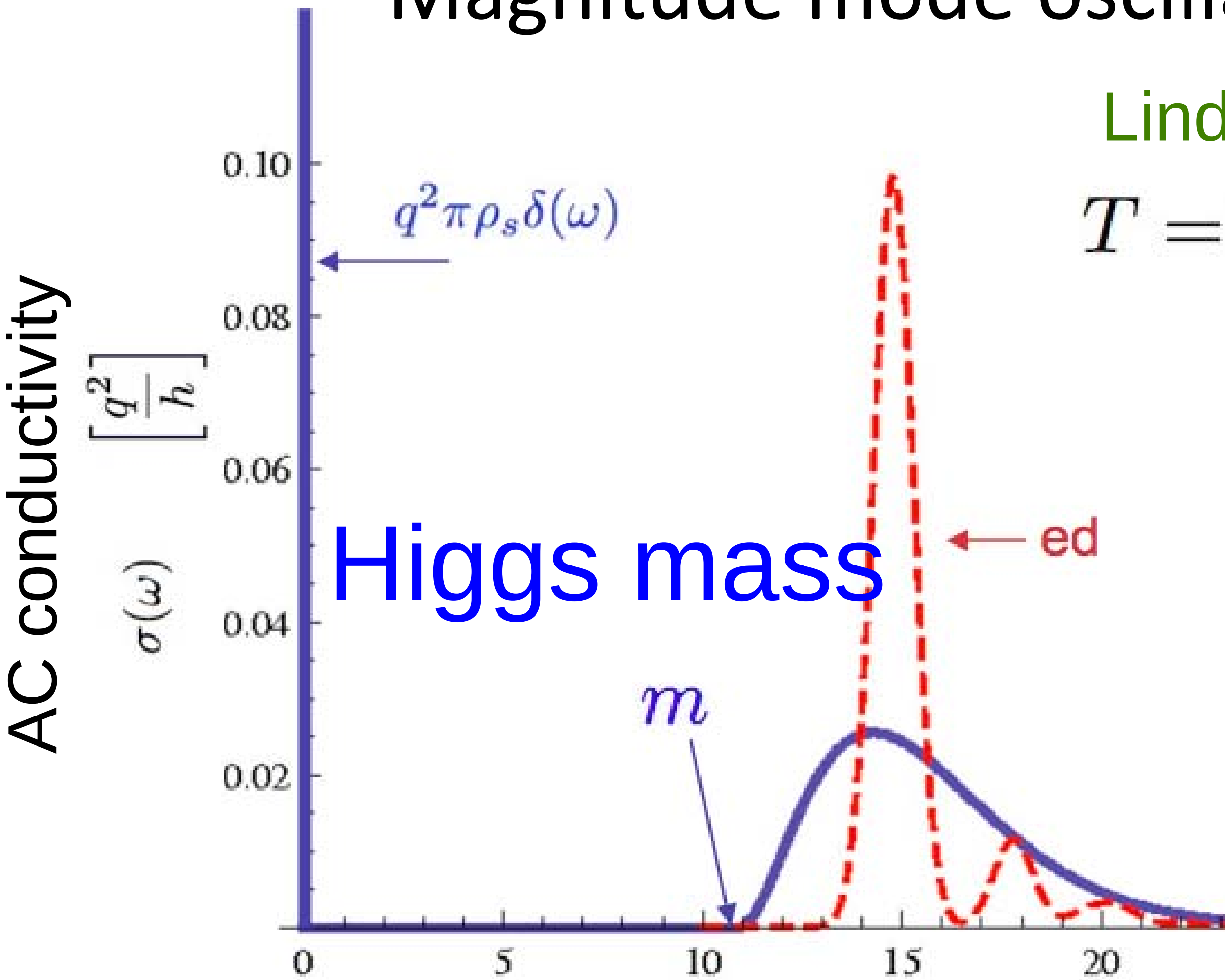
$$+iS \int d\tau \sum_i (-1)^i \dot{\phi}$$

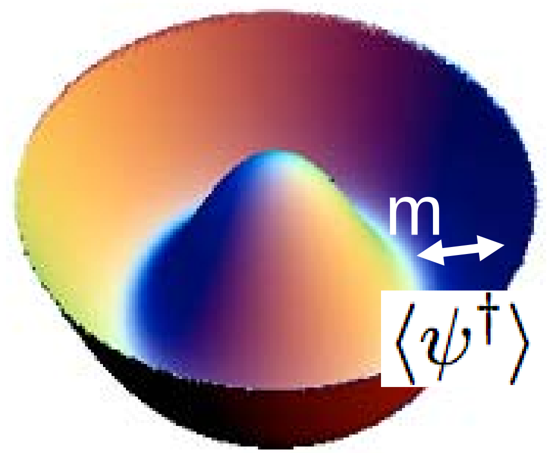
Berry phases

1. Irrelevant for static corelations in superfluid phase.
2. Relevant for quantum disordered phases, (Haldane, Read, Sachdev)
3. Relevant for vortex dynamics, degeneracies(Lindner AA Arovas).

Magnitude mode oscillations

Lindner, AA, PRB (2010)





Analogues: (w Daniel Podolsky)

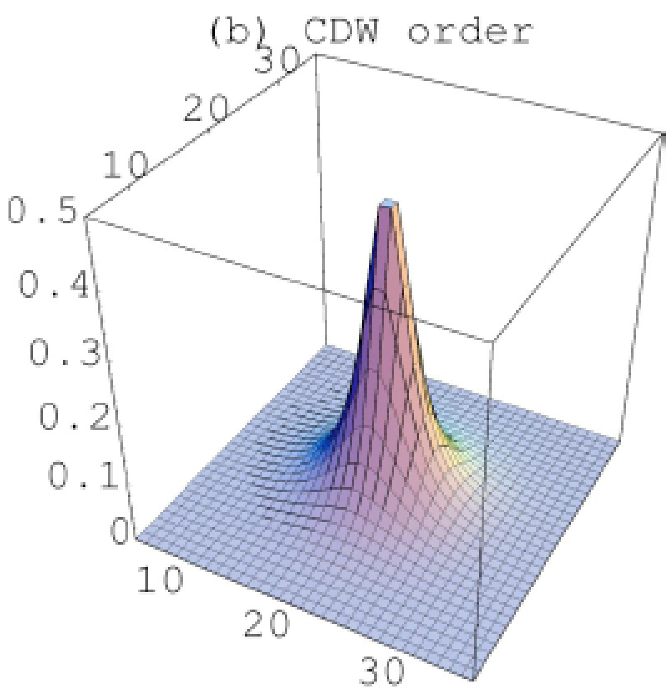
Oscillating coherence near Mott phase of optical lattices

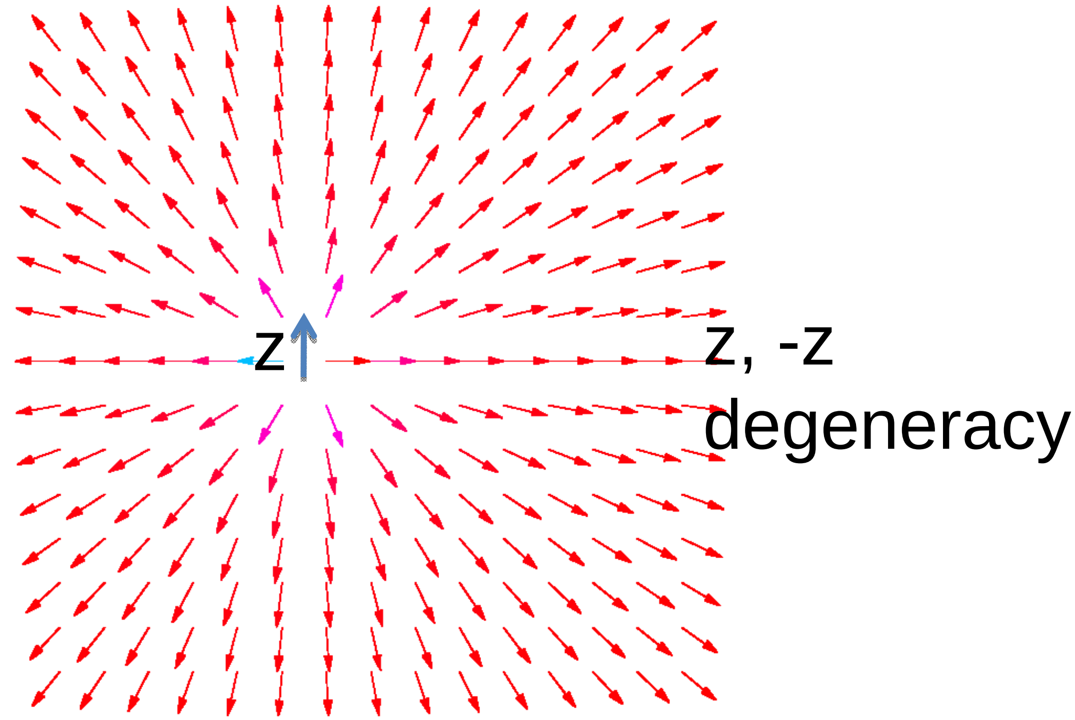
Magnitude mode in 1-D CDW's

2-magnon Raman peaks in O(3) antiferromagnets

Vortex profile at half filling (classical)

Vortex = meron (half skyrmion)





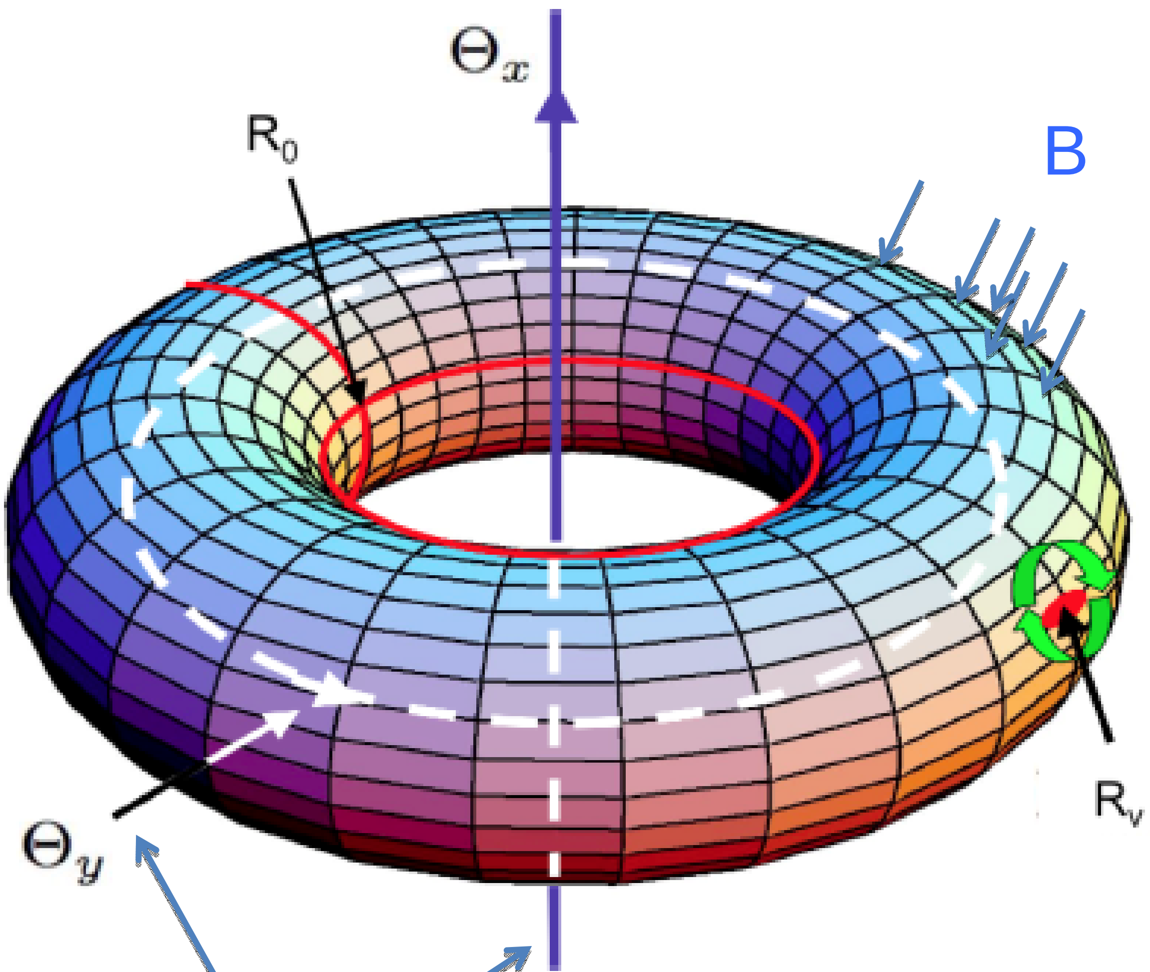
exponentially localized core

staggered charge density wave – no net charge

CDW in vortex matter: Lannert, Fisher, Senthil, PRB (01)
Tesanovic, PRL 93 (2004), C. Wu et. al, Phys Rev A 69 (2004)
Balents, Bartosch, Burkov, Sachdev, and Sengupta, PRB 71, (2005).

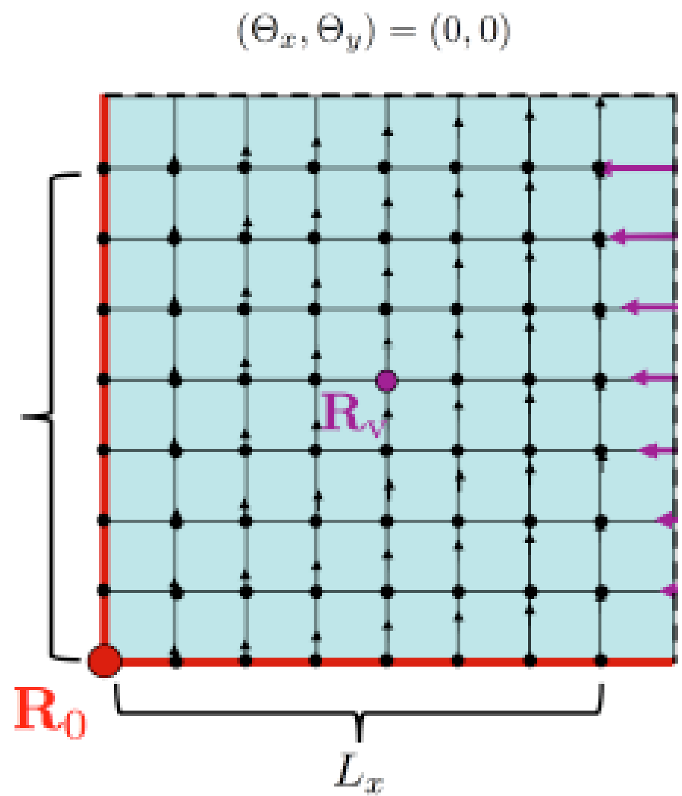
Study of Quantum Vortices: the Gauged Torus

$$\mathcal{H} = -2J \sum \left(e^{iqA_{ij}} S_i^+ S_j^- + e^{-iqA_{ij}} S_i^- S_j^+ \right)$$



Aharonov-Bohm fluxes

gauge field



Lindner AA Arovas, ***PRL (2009)***
+ Phys. Rev.B (in press);
arXiv:1005.4929

Hall conductance

Avron and Seiler, PRL (85)

Ground state Chern class = Hall conductance (finite system)

$$\sigma_{xy}(N) = \frac{q^2}{h\pi} \int_0^{2\pi} d\Theta_x \int_0^{2\pi} d\Theta_y \, \mathrm{Im} \left\langle \frac{\partial \Psi_0}{\partial \Theta_x} \middle| \frac{\partial \Psi_0}{\partial \Theta_y} \right\rangle$$ = integer

Aharonov Bohm fluxes

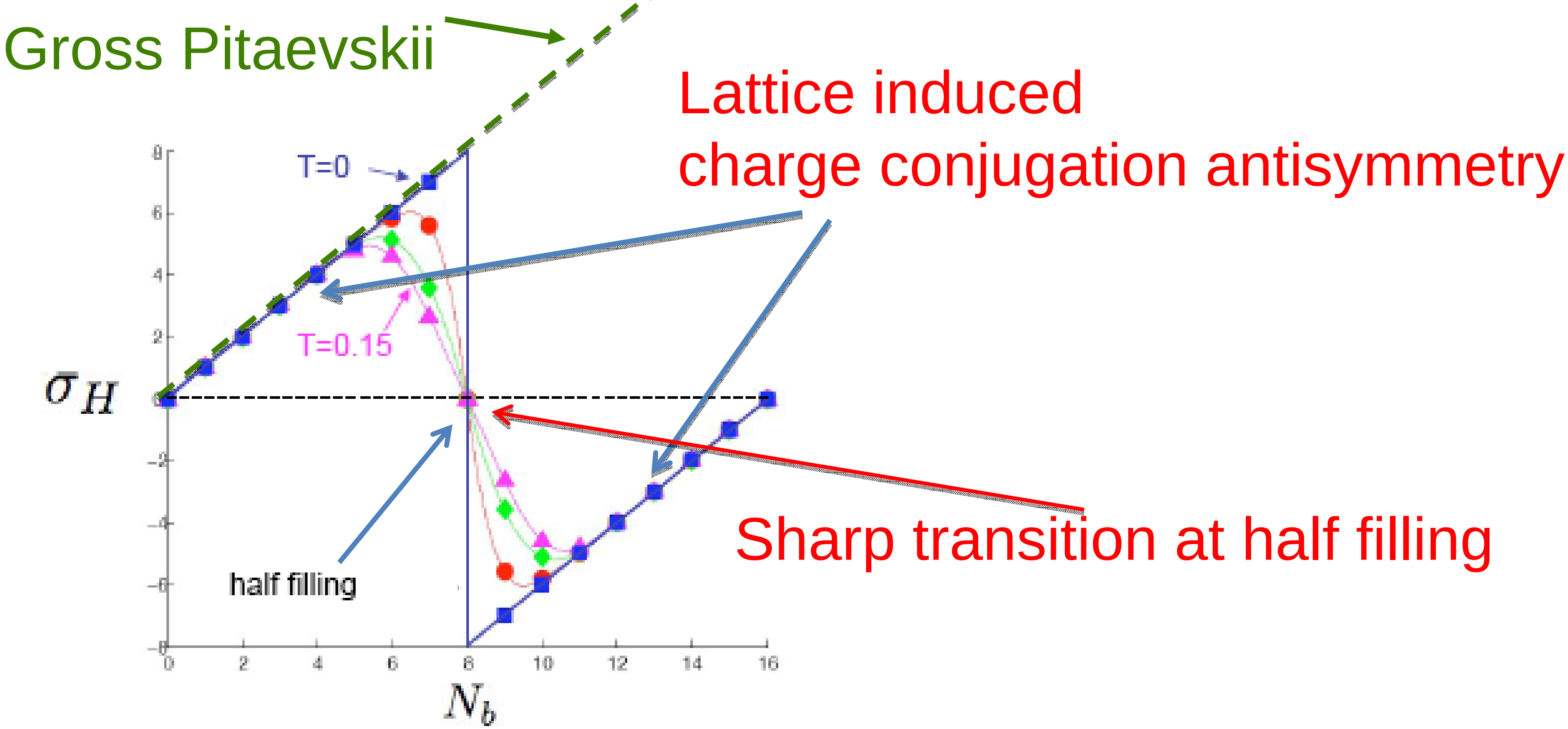
“adiabatic curvature”

Hall Conductance of Hard Core Bosons

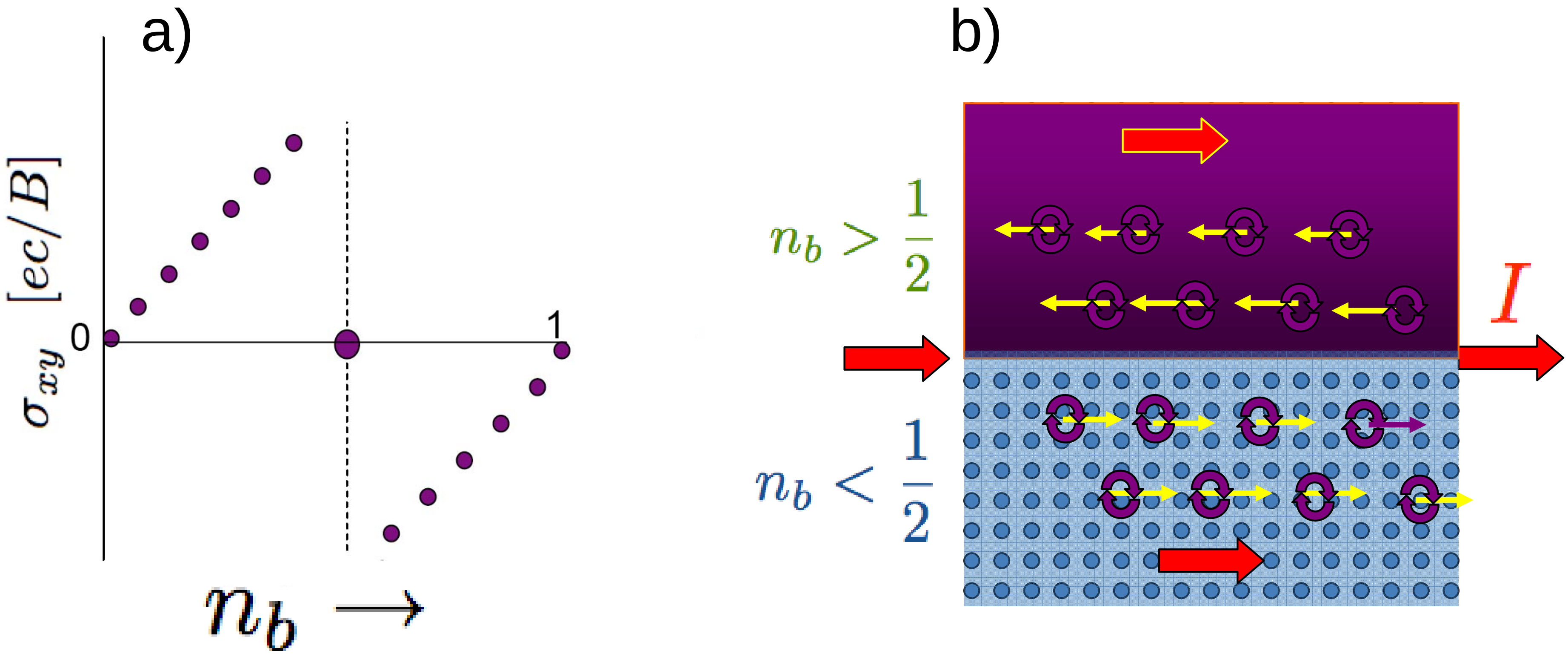
Thermally averaged Chern numbers

$$\sigma_{\mathrm{H}}(n_b, T) = \frac{1}{4\pi} \sum_{n=0}^{\infty} \int_0^{2\pi} \int_0^{2\pi} d^2\Theta \; \frac{e^{-E_n/T}}{Z} \; \mathrm{Im} \left\langle \frac{\partial \psi_n}{\partial \Theta_x} \middle| \frac{\partial \psi_n}{\partial \Theta_y} \right\rangle$$

$$\sigma_{xy}^{cont} = \frac{N_b q}{BA}$$

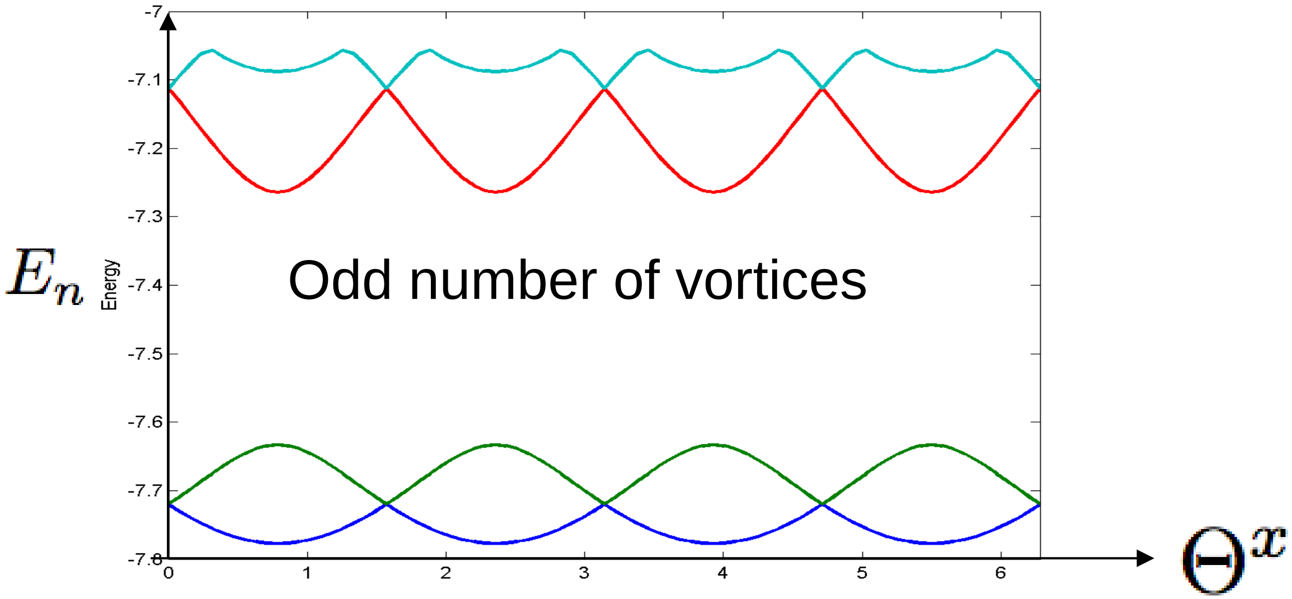


Drift direction reversal: proposed cold atoms experiment



Quantum Degeneracies in Vortex states

Exact spectrum of the gauged torus at half filling



Theorem:

Doublet degeneracies, of all eigenstates, occur when the vorticity center is situated precisely on any lattice site.

Proof:

We construct a non commuting algebra of symmetries

The Pi operators

$$\Pi^x = P^x[\mathbf{R}_v] \cdot C \cdot U^x[\mathbf{A}]$$
$$\Pi^y = P^y[\mathbf{R}_v] \cdot C \cdot U^y[\mathbf{A}]$$

1. Reflection about $\mathbf{R}_v$

2. Charge conjugation $C = e^{i\pi \sum_{\mathbf{r}} S^x_{\mathbf{r}}}$

3. Gauge transformation $U^\alpha = \exp\left(i \sum_{\boldsymbol{r}} \chi^\alpha(\boldsymbol{r})\, S^z_{\boldsymbol{R}}\right)$

$$\chi^\alpha(\boldsymbol{r}) = \int_{\boldsymbol{R}_0}^{\boldsymbol{r}} d\boldsymbol{r}' \cdot \left(\boldsymbol{A}(\boldsymbol{r}') + \tilde{\boldsymbol{A}}^\alpha(\boldsymbol{r}')\right)$$

4. Compute Commutation $\Pi^y_{\mathrm{V}} \Pi^x_{\mathrm{V}} = \exp\left(i \sum_{\boldsymbol{r}} (\chi^y - \chi^x(P^y_{\mathrm{V}}[\boldsymbol{r}])) S^z_{\boldsymbol{r}}\right) P^y_{\mathrm{V}} P^x_{\mathrm{V}},$

$$\Pi^x_{\mathrm{V}} \Pi^y_{\mathrm{V}} = \exp\left(i \sum_{\boldsymbol{r}} (\chi^x - \chi^y(P^x_{\mathrm{V}}[\boldsymbol{r}])) S^z_{\boldsymbol{r}}\right) P^y_{\mathrm{V}} P^x_{\mathrm{V}}$$
$$= e^{-i\Upsilon} \Pi^y_{\mathrm{V}} \Pi^x_{\mathrm{V}},$$

-1 for odd flux

The v-spin algebra

$$\Pi^x = P^x[\mathbf{R}_v] \cdot C \cdot U^x[\mathbf{A}]$$
$$\Pi^y = P^y[\mathbf{R}_v] \cdot C \cdot U^y[\mathbf{A}]$$

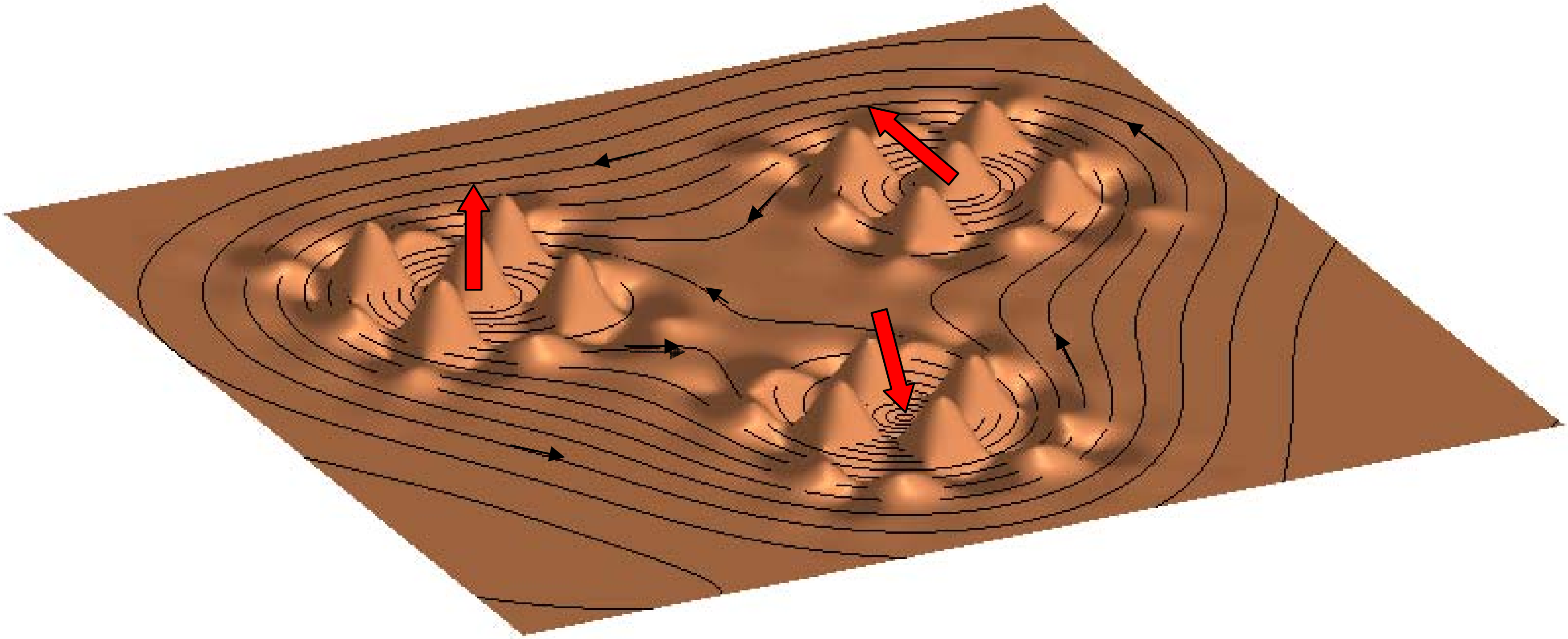
Point group symmetries $\left[\mathcal{H}[\Theta], \Pi^x[\mathbf{R}_v]\right] = \left[\mathcal{H}[\Theta], \Pi^y[\mathbf{R}_v]\right] = 0$

For odd vorticity $\Pi^x \Pi^y = (-1)^{N_\phi} \Pi^y \Pi^x \equiv i\Pi^z$

=> All states are doubly degerenate

$$\Pi^y \Pi^x |E_n, \pi^x\rangle\rangle = -\Pi^x \Pi^y |E_n, \pi^x\rangle \Rightarrow \Pi^y |E_n, \pi^x\rangle = |E_n, -\pi^x\rangle$$

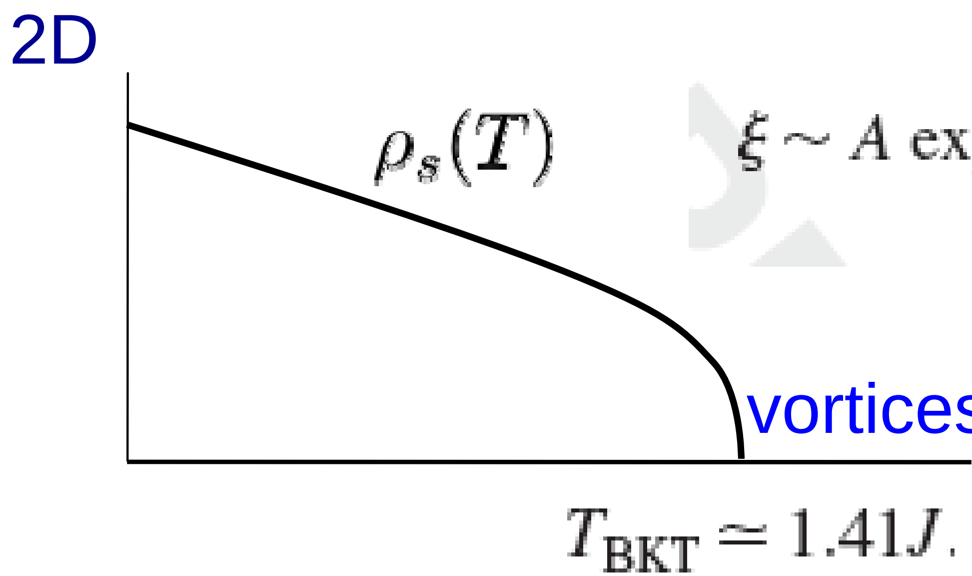
Illustration of 3 vortices with v-spin



Implications of v-spins:

1. v-spin order → supersolid in the vortex lattice
2. Low temperature entropy of v-spins

Conductivity above BKT transition

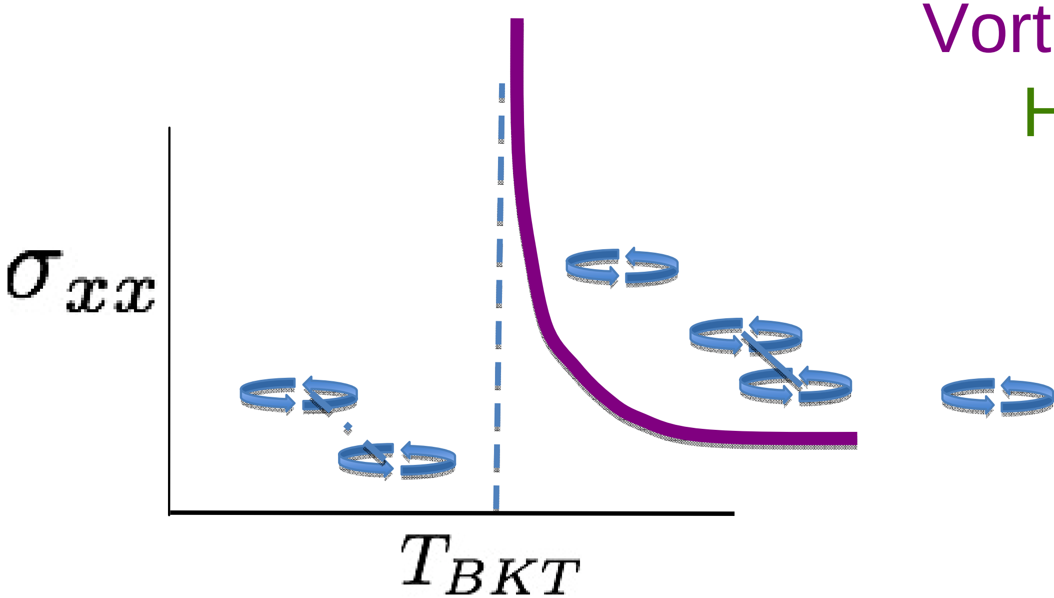


$$\xi \sim A \exp\left(\frac{B}{\sqrt{(T - T_{\mathrm{BKT}})/T_{\mathrm{BKT}}}} \right)$$

Vortex plasma conductivity

Halperin- Nelson

$$\sigma_{xx}^{HN} = \frac{q^2 T}{h D_v} \xi^2(T) + \sigma_N(T)$$



$$\sigma_N(T), D_v = ?$$

PHYSICAL REVIEW B 81, 054512 (2010)

Conductivity of hard core bosons: A paradigm of a bad metal

Netanel H. Lindner and Assa Auerbach

HCB Current Operator $J_x = \frac{4qJ}{\sqrt{N}} \sum_{\mathbf{r}} \left(S^x_{\mathbf{r}} S^y_{\mathbf{r}+\hat{\mathbf{x}}} - S^y_{\mathbf{r}} S^x_{\mathbf{r}+\hat{\mathbf{x}}} \right)$

Real Conductivity:

current fluctuations function

superfluid stiffness

$$\sigma(\beta,\omega) = q^2 \pi \rho_s(\beta) \delta(\omega) + \frac{\tanh(\beta\omega/2)}{\omega} G''(\beta,\omega)$$

$$G''(\beta,\omega) = \frac{1}{2} \int_{-\infty}^{\infty} dt e^{-i\omega t} \langle \{ J_x(t), J_x(0) \} \rangle_\beta .$$

Current fluctuations function

$$G''(\beta,\omega) = -\frac{1}{Z}\mathrm{Im}\mathrm{Tr}\left(e^{-\beta H}\left\{J_x, \frac{1}{\omega - \mathcal{L} + i\epsilon}J_x\right\}\right),$$

Liouvillian hyper-operator $\mathcal{L} = [\mathcal{H}, \cdot]$

Moments expansion:

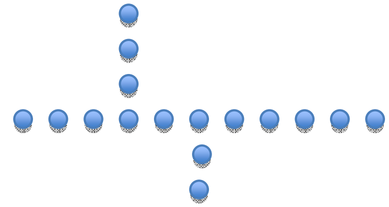
$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi}\omega^k G''(\beta,\omega) = \langle\{J_x, \mathcal{L}^k J_x\}\rangle_\beta \equiv \mu_k(\beta)$$

Static correlators:
amenable to high T expansion

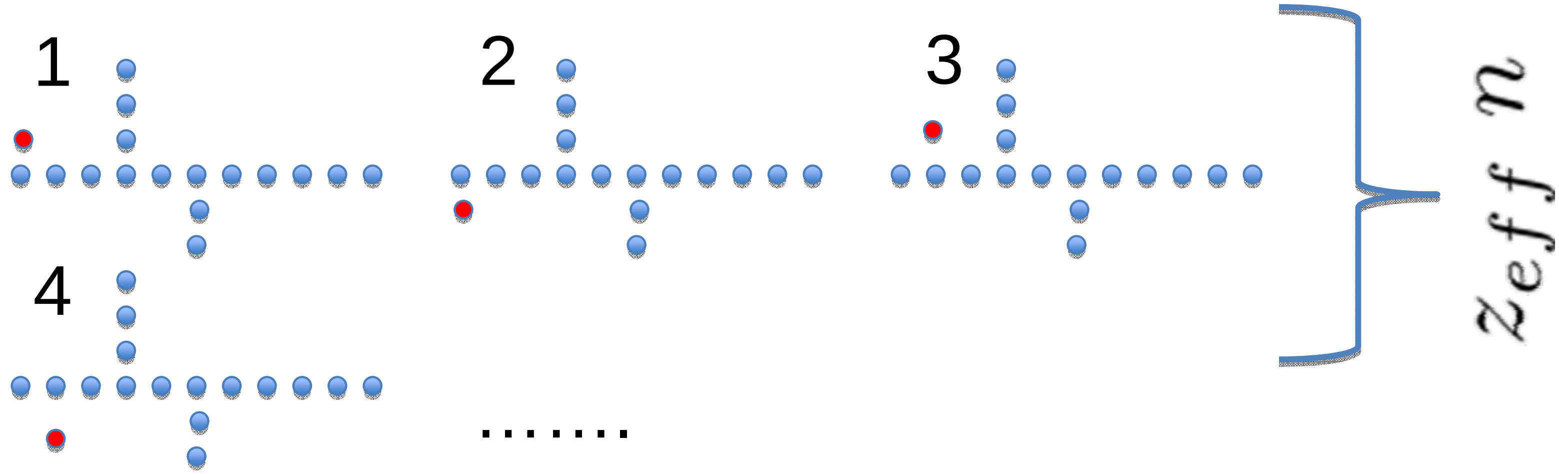
We can invert a finite set of moments
if we know the high order asymptotics!

"Gaussian Termination" – high coordination

$$\hat{O}_n = \sum c^{\alpha_1,\alpha_2,\ldots,\alpha_n}_{i_1,i_2,\ldots,i_n} S^{\alpha_1}_{i_1} S^{\alpha_2}_{i_2}, \ldots, S^{\alpha_n}_{i_n}$$



$$\mathcal{L}\hat{O}_n =$$



Linear recurrents→Gaussian dissipation

$$\mathrm{Im}\ \langle 0| \left(\omega + i\epsilon - \mathcal{L}\right)^{-1} |0\rangle =$$

$$\mathrm{Im} \begin{pmatrix} \omega + i\epsilon & -\Omega & 0 & 0 & \ldots \\ \Omega & \omega + i\epsilon & -\sqrt{2}\Omega & 0 & \\ 0 & -\sqrt{2}\Omega & \omega + i\epsilon & -\sqrt{3}\Omega & \\ 0 & 0 & -\sqrt{3}\Omega & \omega + i\epsilon & \end{pmatrix}^{-1}_{11} \propto e^{-\omega^2/\Omega^2}$$

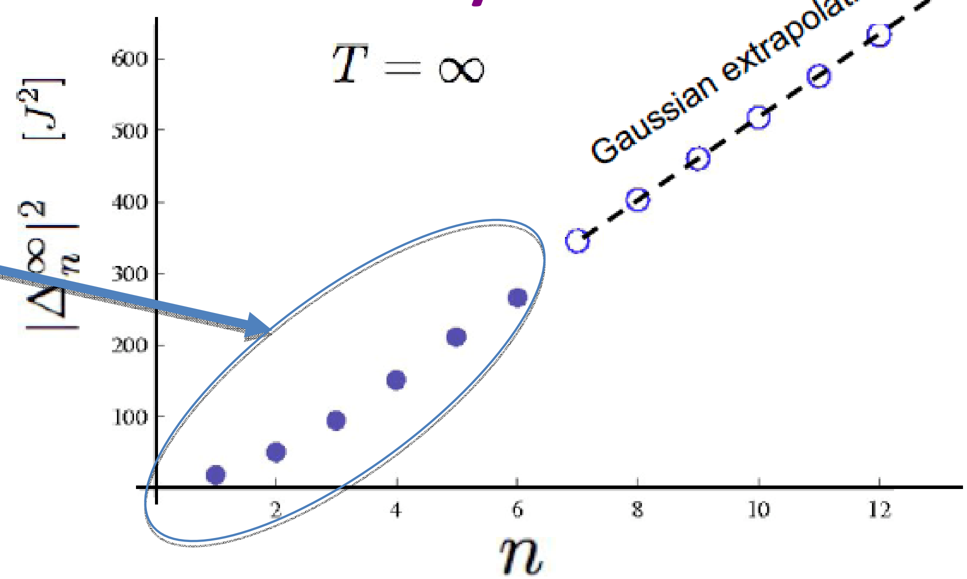
$\sqrt{n}\Omega$

$$G(t) \sim e^{-\Omega^2 t^2}$$

Different from Boltzmann transport

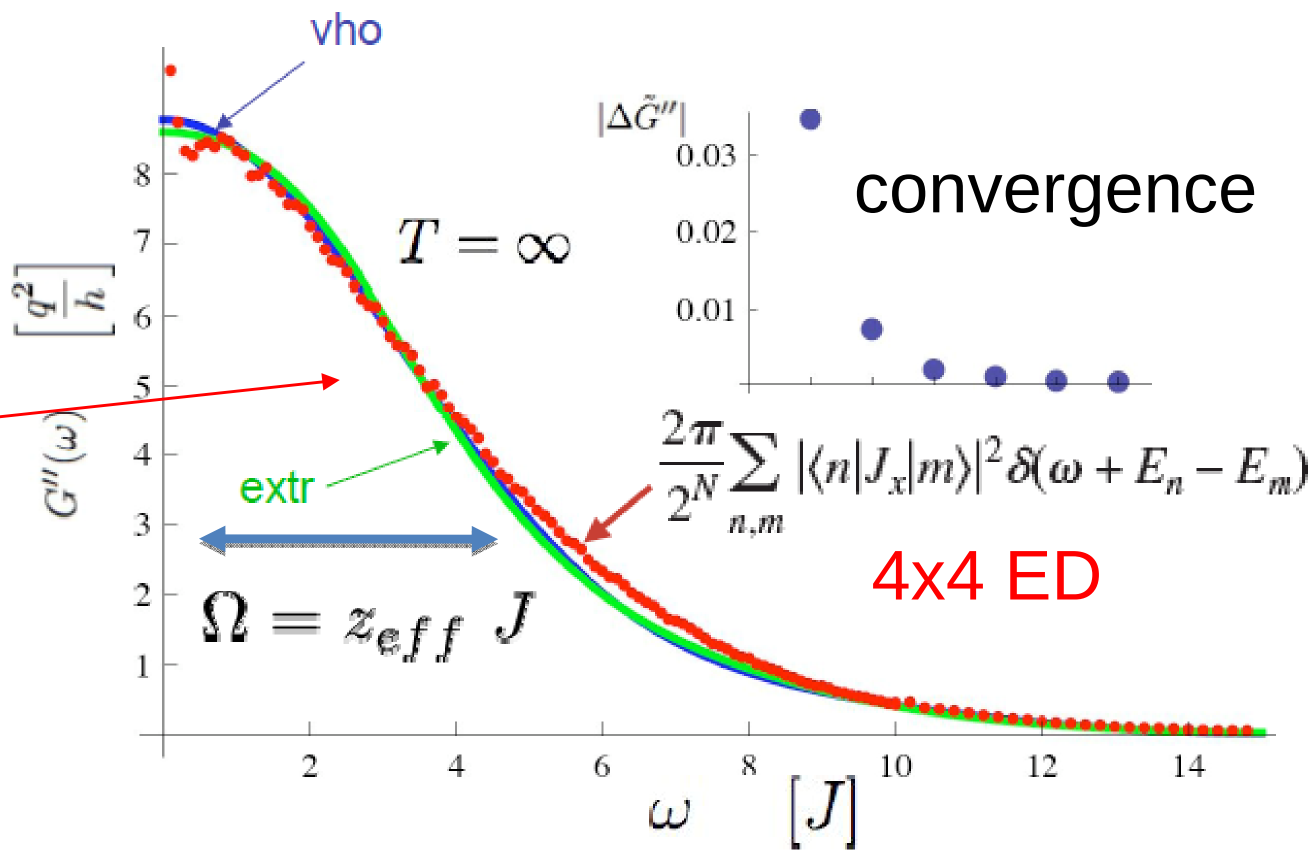
Dynamical conductivity

Computed recurrents



$$\sigma_{\beta \to 0} = \frac{\beta}{2} G''_\infty(\omega)$$

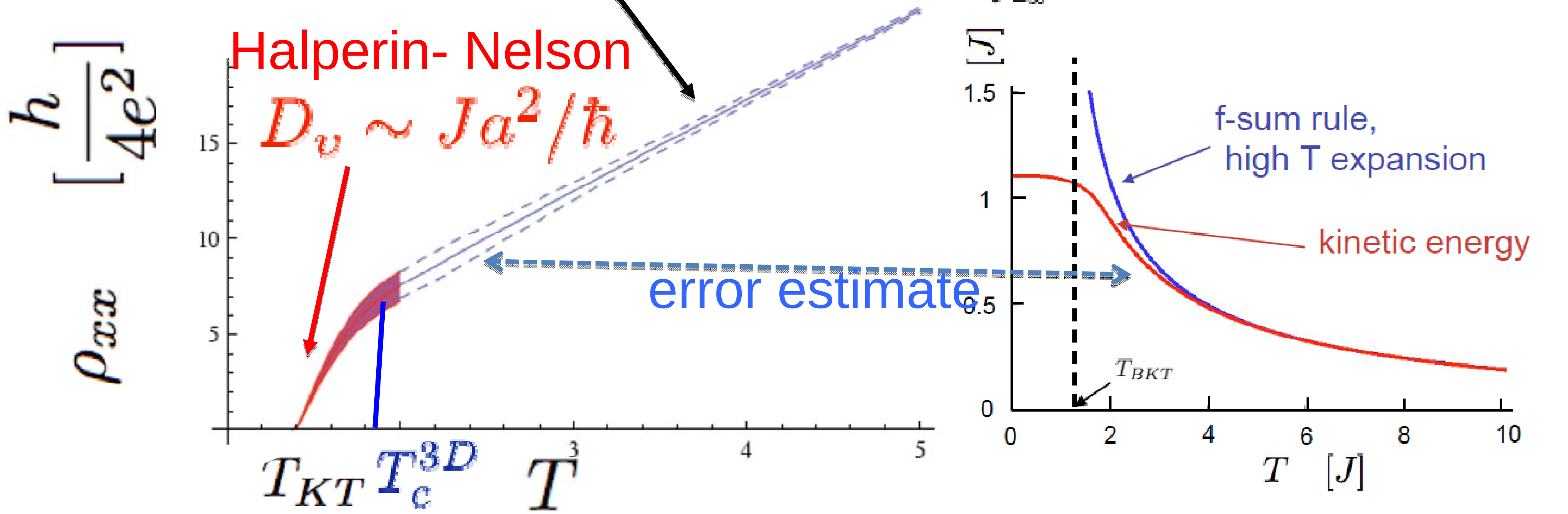
Gaussian decay



High temperature resistivity

$$R(T) = 0.23 R_Q \frac{T}{J}\{1 - 2.9(J/T)^2 + \mathcal{O}[(J/T)^4]\}$$

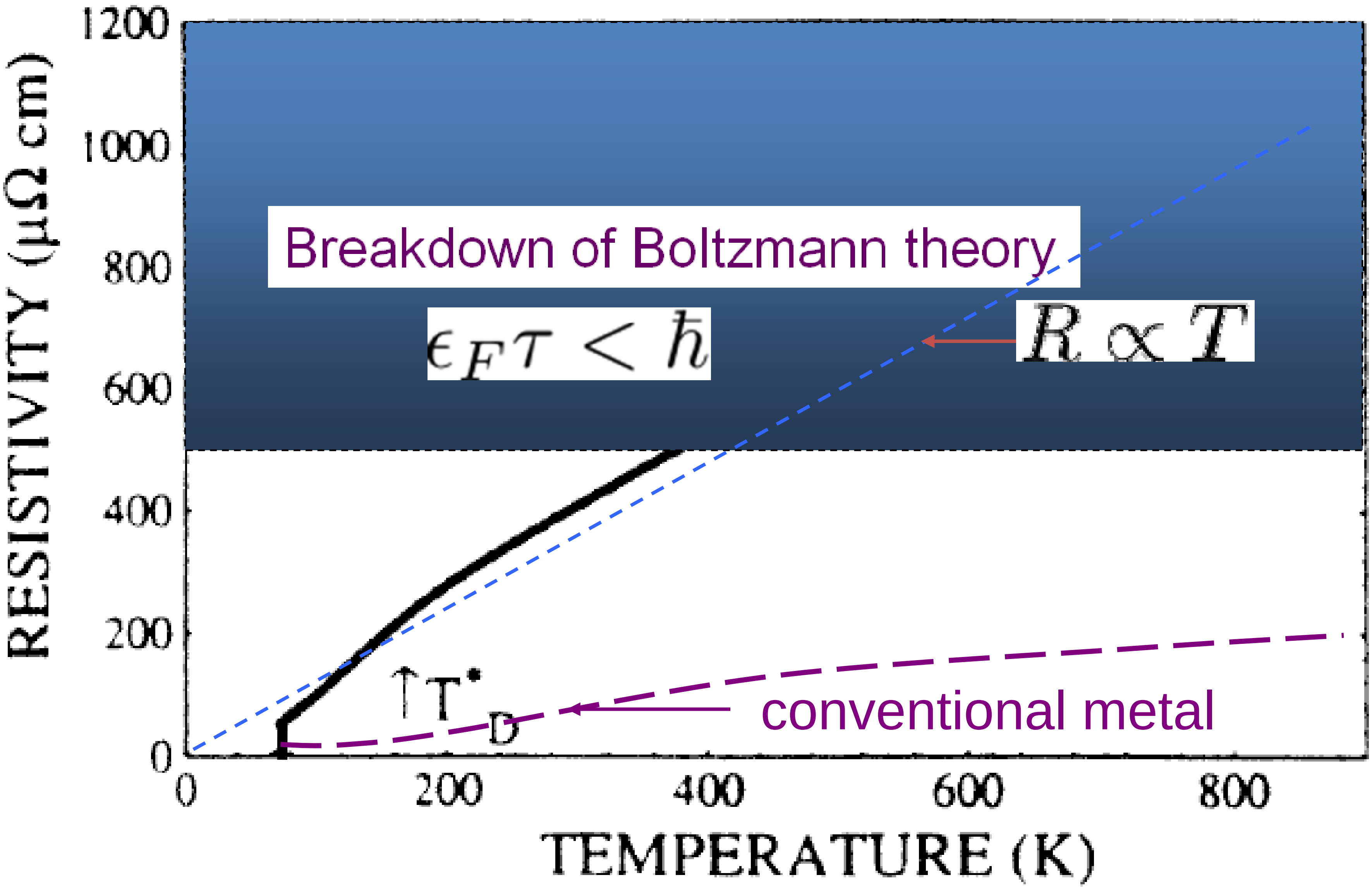
$$\int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\tanh(\beta\omega/2)}{\omega} G''(\beta,\omega) = \langle -q^2 K_x \rangle_\beta$$



"Bad Metal":

linear increase, no resitivity saturation

`Bad Metal'



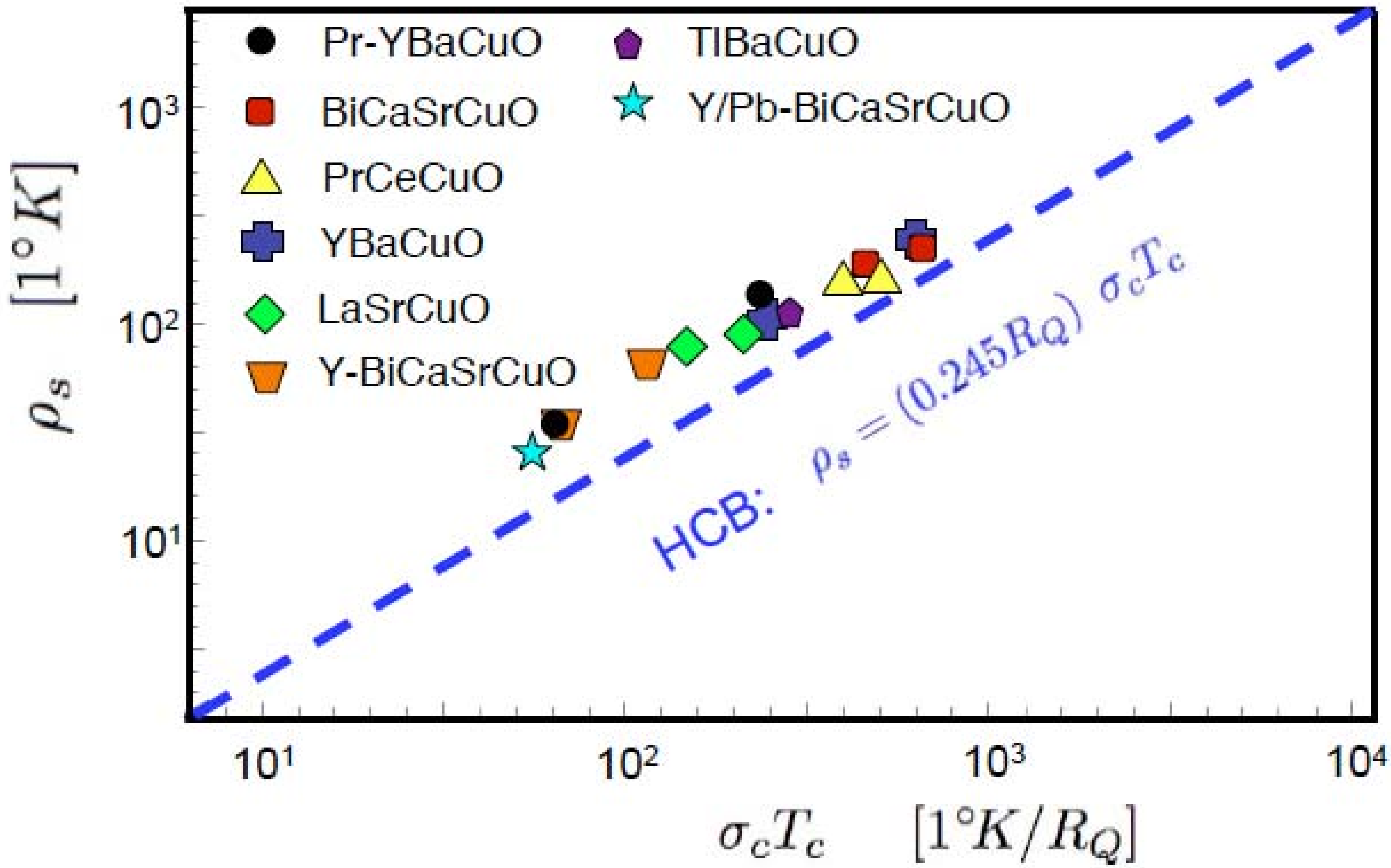
Emery & Kivelson `Bad Metal' behavior

"Homes law" of HCB

$$\rho_s(0) = 0.245\frac{R_Q}{R_c}T_c.$$

$R_Q = h/q^2$ is the boson quantum of resistance $= 6453\ \Omega$

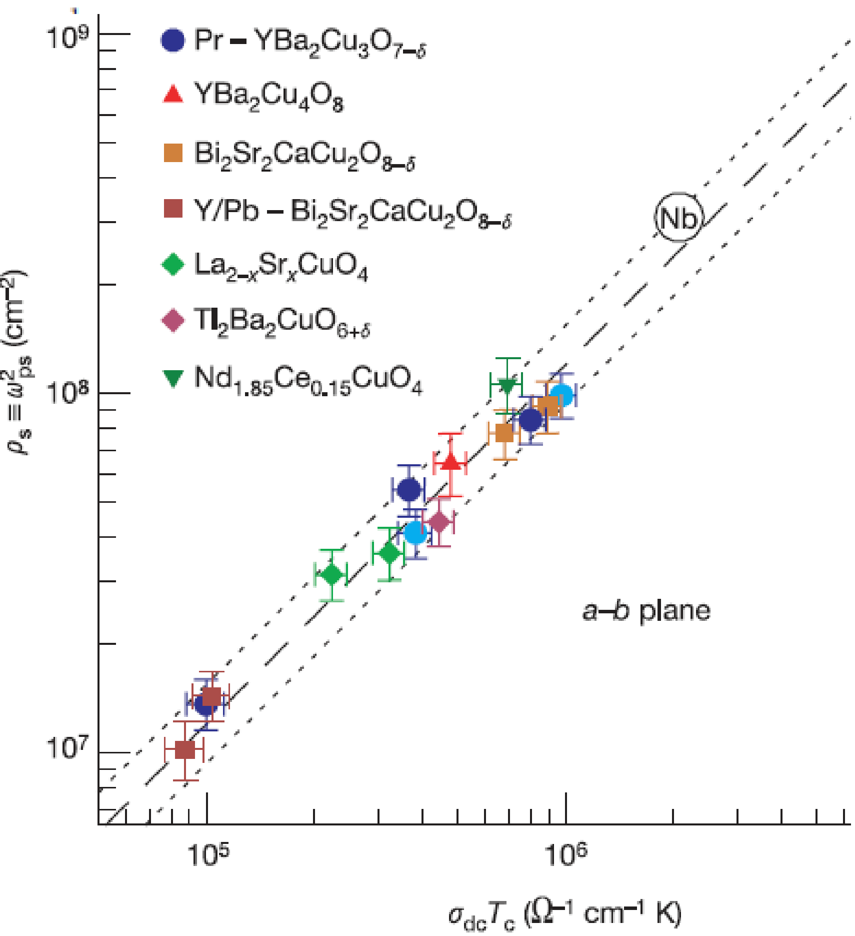
Data: Homes et. al.

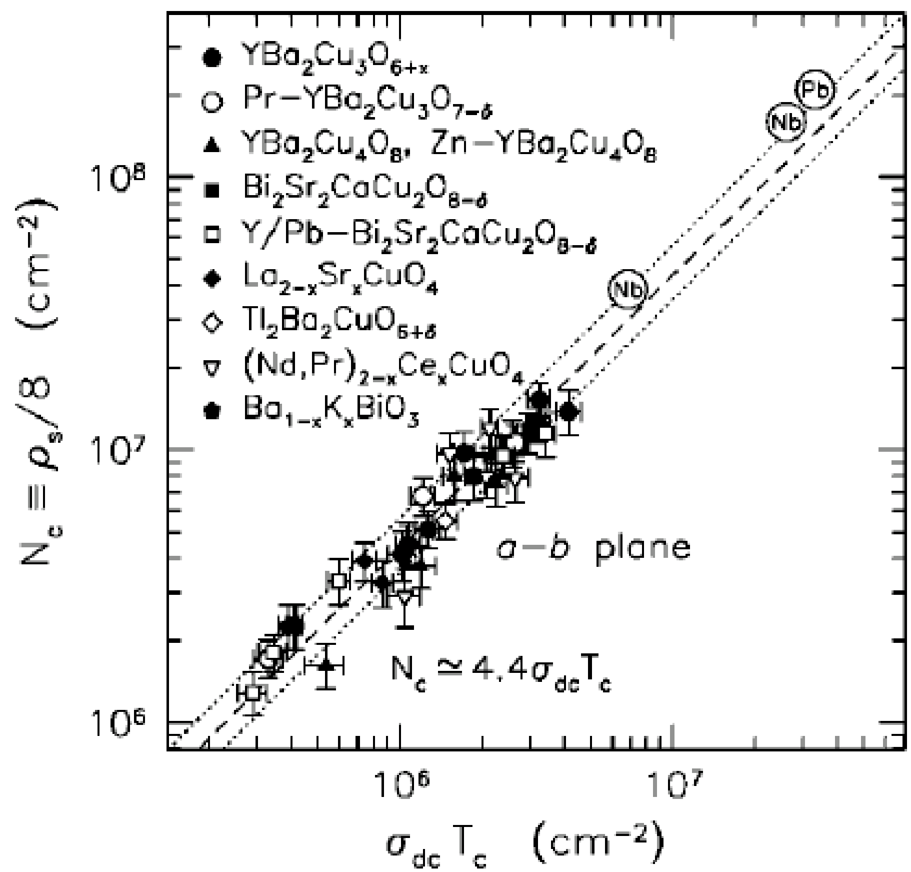


"Homes Law"

A universal scaling relation in high-temperature superconductors

C. C. Homes[1], S. V. Dordevic[1], M. Strongin[1], D. A. Bonn[2], Ruixing Liang[2], W. N. Hardy[2], Seiki Komiya[3], Yoichi Ando[3], G. Yu[4], N. Kaneko[5*], X. Zhao[5], M. Greven[5,6], D. N. Basov[7] & T. Timusk[8]





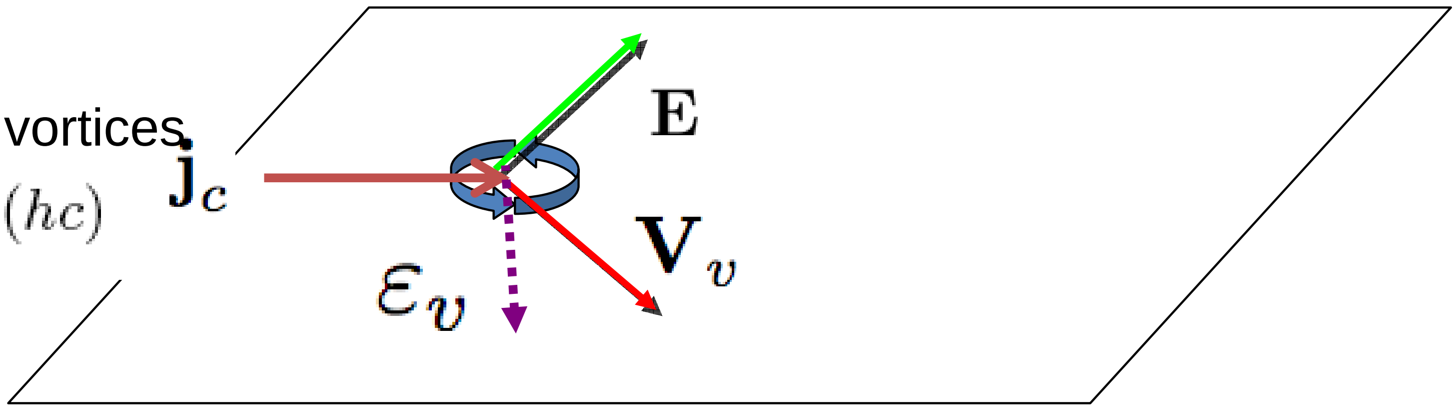
Summary

1. Lattice effects on hard core bosons and their vortices are crucial around half filling.
2. Hall effect reverses sign at half filling, where Vortices acquire spin-half ("V-spin") degeneracies.
3. The Higgs amplitude mode should be observable at integer and half fillings.
4. At high temperatures, HCB and free vortices strongly scatter. The liquid exhibits non-Boltzmann "bad metal" resistivity.
5. HCB may capture some unconventional transport properties of strongly fluctuating superconductors…

Vortex-Charge Duality

density of induced vortices

$$n_v = Bq/(hc)$$



Vortex driving (Magnus) field $\varepsilon_v = \frac{h}{q} j_c \times \hat{z}$

Vortex Induced EMF $\quad E = -c^{-1} \mathbf{V}_v \times B = -\frac{h}{q} \mathcal{J}_\mathbf{v} \times \hat{z}$

Vortex Transport Equation $\quad \mathcal{J}_v^\alpha = \sum_\beta \sigma_v^{\alpha\beta} \varepsilon_v^\beta$

Boson resistivity

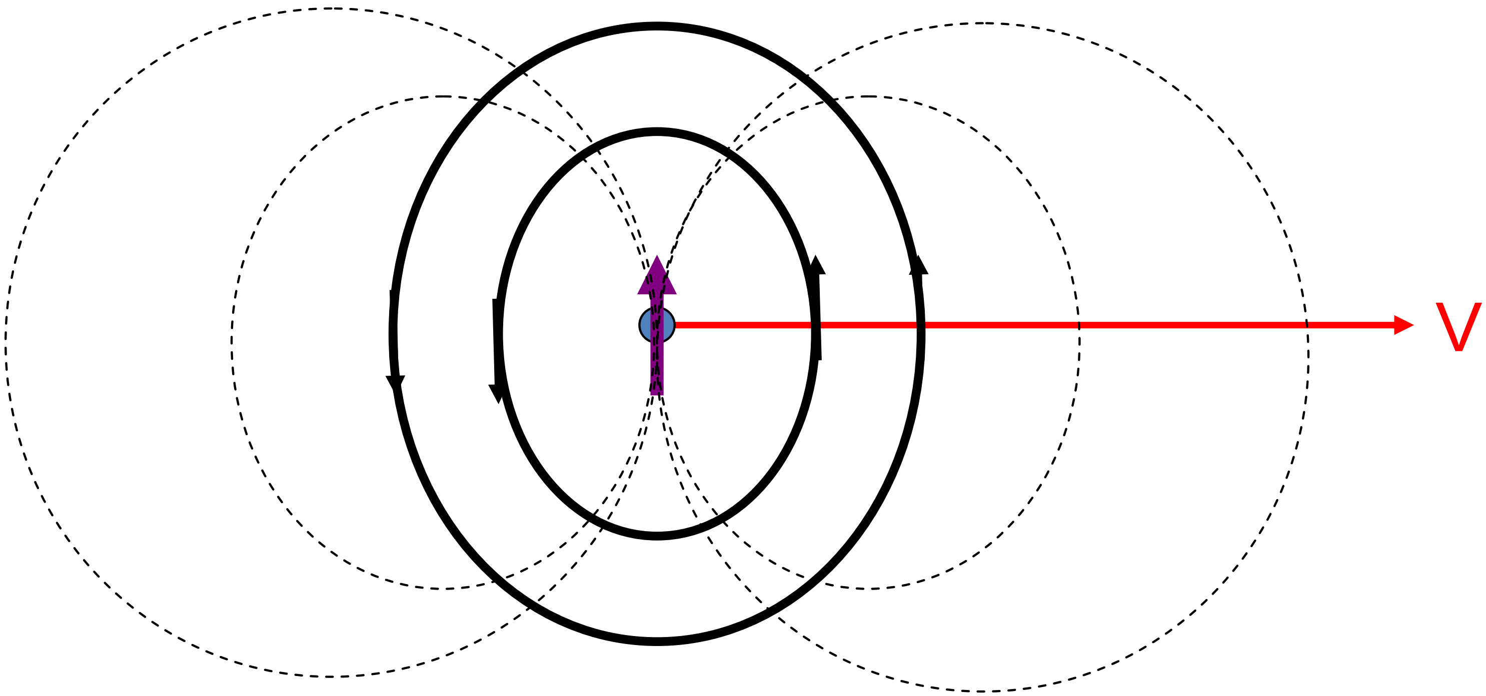
$$\rho^{xx} = \left(\frac{h}{q}\right)^2 \sigma_v^{yy}, \quad (\& \quad y \to x),$$

$$\rho^{xy} = -\left(\frac{h}{q}\right)^2 \sigma_v^{yx}.$$

Vortex conductivity

Vortex Motion induced EMF

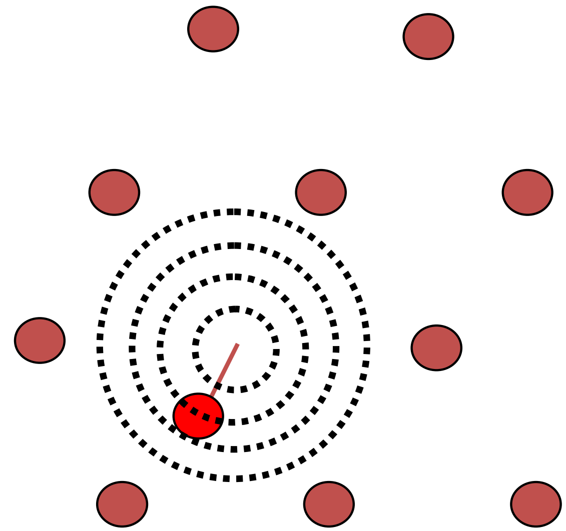
vorticity current $\mathcal{J}_v(\mathbf{x}) = \sum_i k_i \mathbf{V}_i \delta(\mathbf{x} - \mathbf{X}(t))$



Multiple moving
vortices create a stress field $\Sigma \equiv m\dot{\mathbf{v}}(\mathbf{x}) = \hbar \vec{\nabla} \dot{\phi} = h\hat{\mathbf{z}} \times \mathcal{J}_v$

=Hydrodynamic `Lorentz' field $\mathbf{E} = -\frac{\mathbf{V}}{c} \times \mathbf{B}$

Quantum Melting



Multivortex hamiltonian = Bose coloumb liquid

$$\mathcal{H}^{\rm mv} = \sum_{i,s=\uparrow\downarrow} \frac{\mathbf{p}_i^2}{2M_{\rm v}} + \frac{\pi t}{4} \sum_{i \neq j} \log(|\mathbf{r}_i - \mathbf{r}_j|) - \frac{n_{\rm v} \pi^2 t}{4} \sum_i |\mathbf{r}_i|^2 + \mathcal{H}^{\rm ret}(\omega).$$

Magro and Ceperley: Wigner solid melts at $r_s = 12$

$$r_s^{-2} = \pi n_{\rm v} a_0^2. \qquad a_0 = \left(\frac{\hbar^2}{\pi t M_{\rm v}}\right)^{1/2}$$

Therefore, the vortex lattice should quantum melt at

$$n_{\rm v}^{\rm cr} \leq \left(6.5 - 7.9 \frac{V}{t}\right) \times 10^{-3} \text{ vortices per site.}$$

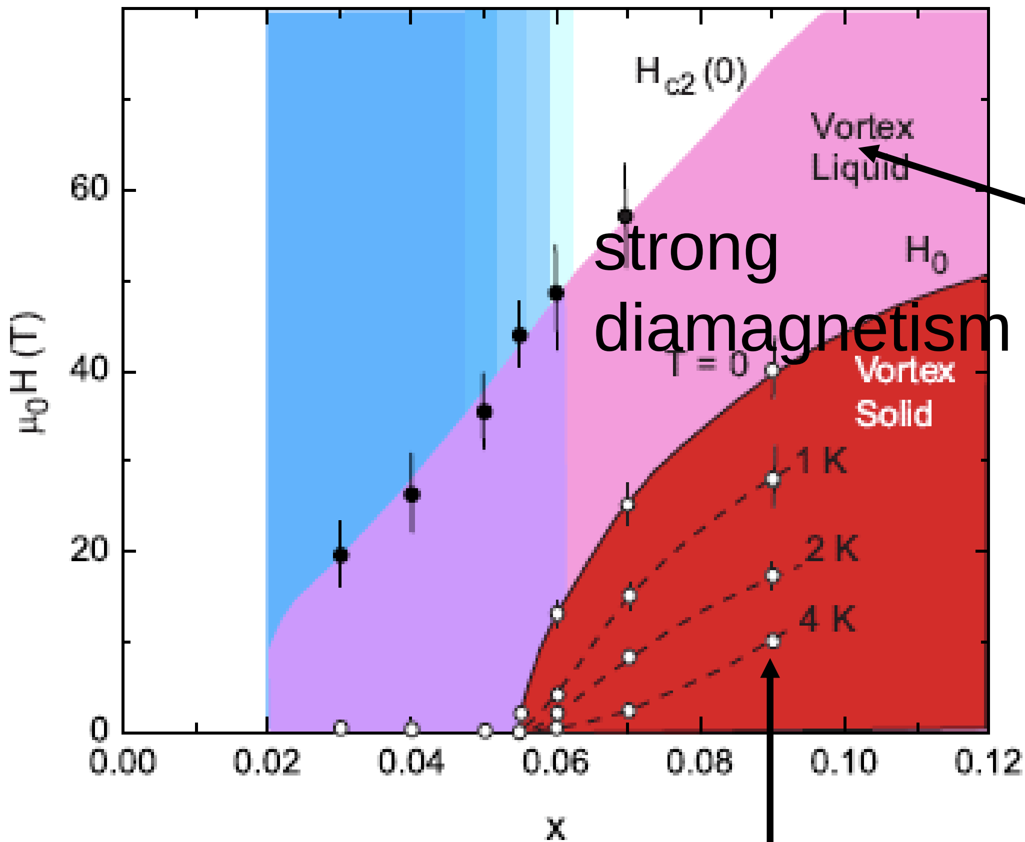
Quantum Vortex liquid: not Bose condensed!

Quantum melting in cuprates

Low temperature vortex liquid in $La_{2-x}Sr_xCuO_4$

Nature Physics 3, 311-314 (2007)

Lu Li[1], J. G. Checkelsky[1], Seiki Komiya[2], Yoichi Ando[2], and N. P. Ong[1*]

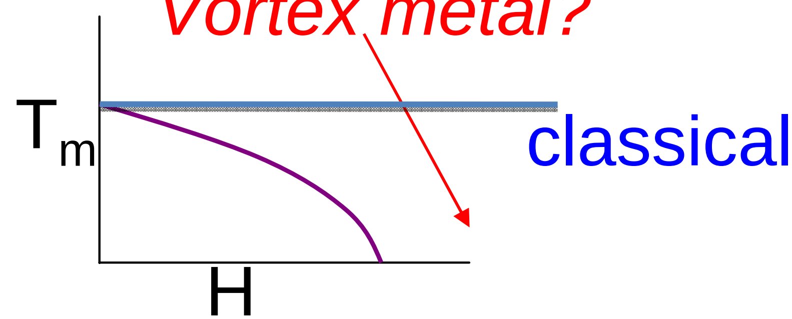


strong diamagnetism

A quantum phase:

Vortex condensate?

Vortex metal?



T_m decreases with field. (Non classical)