# Potential Vorticity in the Boundary Layer

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KITP
Program on Planetary Boundary Layers
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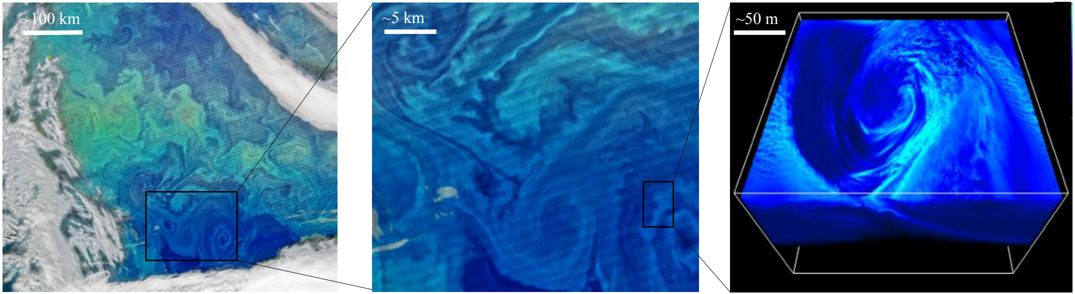
# How does the submesoscale interact with other scales?



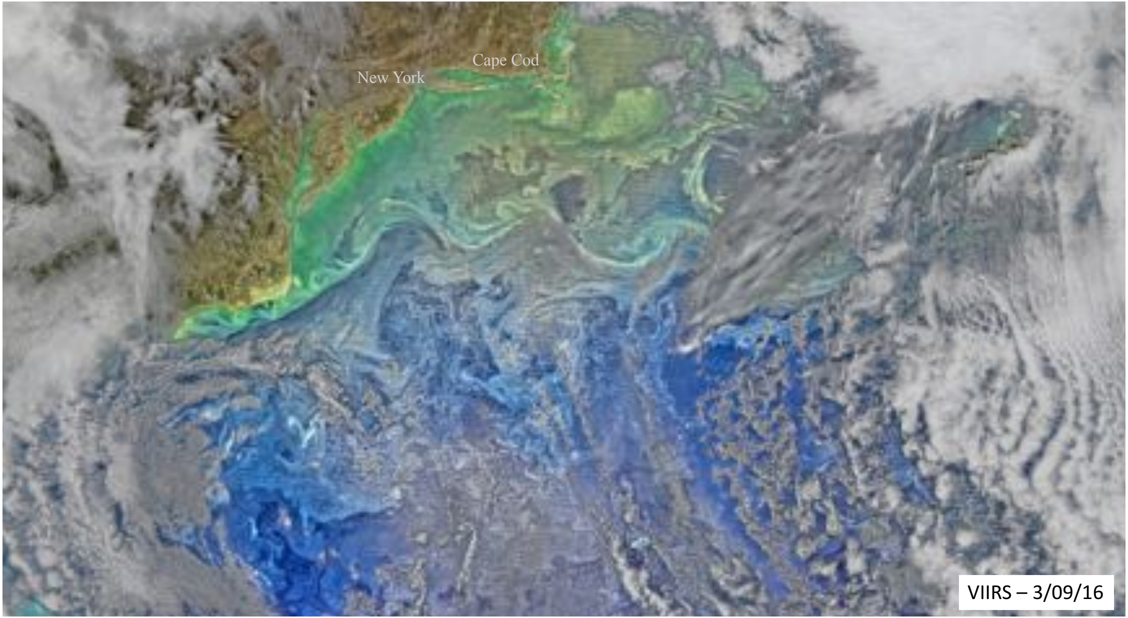
VIIRS-9/15/16

J.R. Taylor

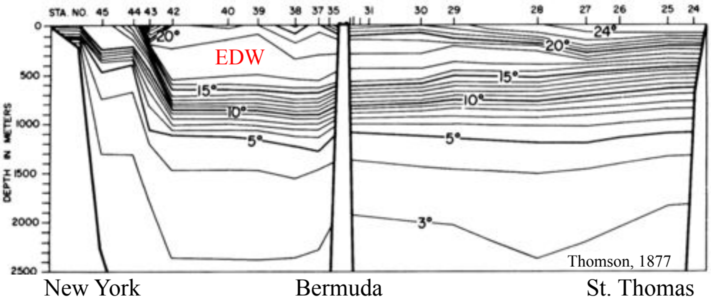
Mesoscale $\longleftrightarrow$ Submesoscale $\longleftrightarrow$ Boundary Layer Turbulence

Submesoscale dynamically characterized by $O(1)$ Rossby and Richardson numbers.

Energetics and potential vorticity provide two frameworks for considering multi-scale interactions.


Cape Cod
New York
VIIRS – 3/09/16

# Gulf Stream region affects entire gyre and climate



Thomson, 1877

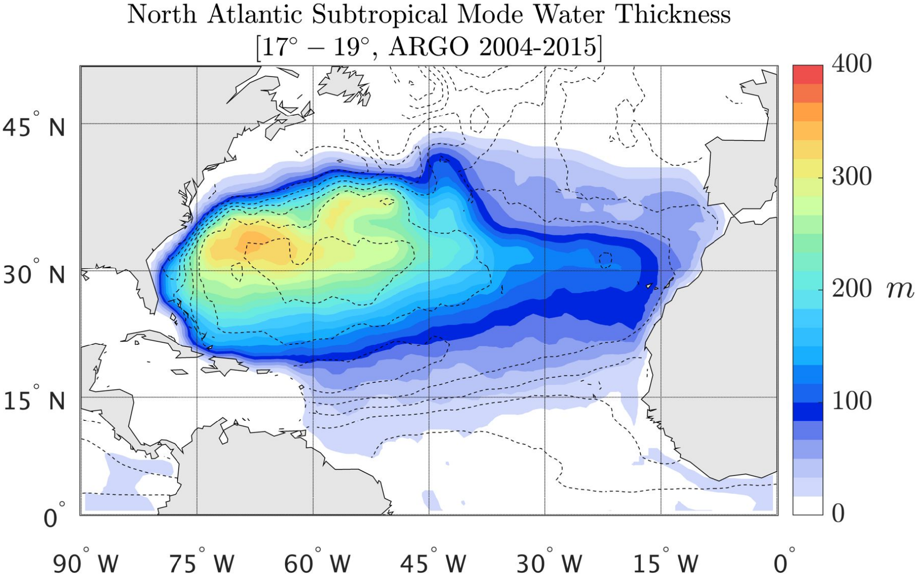
H.M.S. Challenger preparing to Sound, 1872.

Ocean mode waters are thick layers of vertically homogeneous water

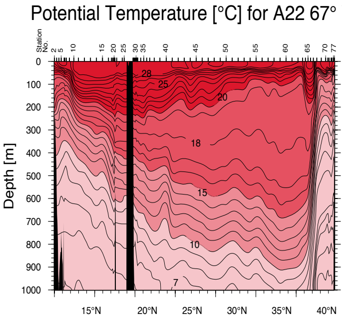
Found in every ocean basin

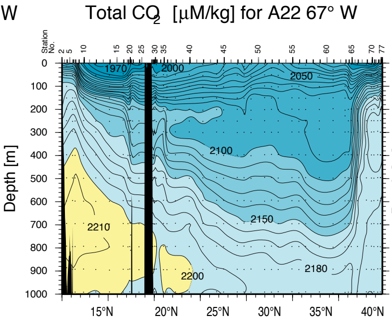
Strong signal evident even in the early observational record!

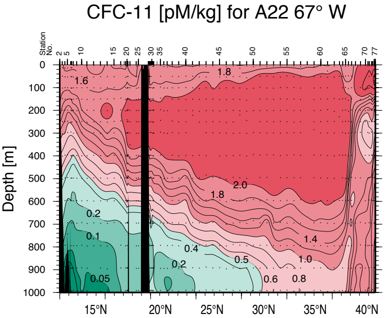
# Mode waters exert strong dynamical control on gyre circulation

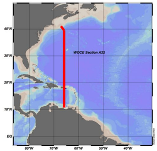


# Mode waters export a history of air-sea interaction to the interior


Potential Temperature [°C] for A22 67° W


Total $CO_2$ [µM/kg] for A22 67° W


CFC-11 [pM/kg] for A22 67° W

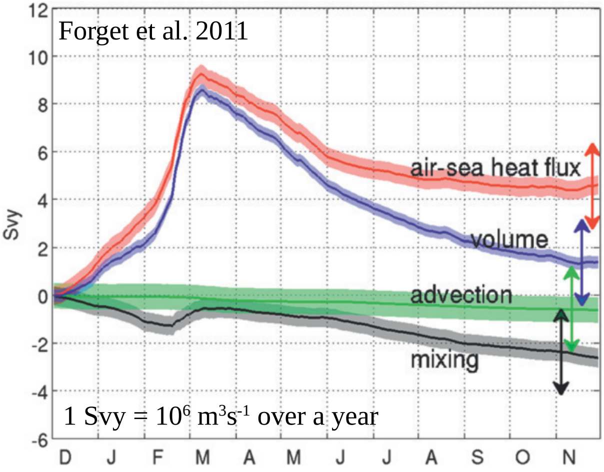


WOCE Atlas V2

18° water is depleted in nitrate and phosphate.

Balance between convective entrainment of nutrient rich water and biological utilization. (Palter et al. 2005)

# Mode water formed through surface buoyancy loss

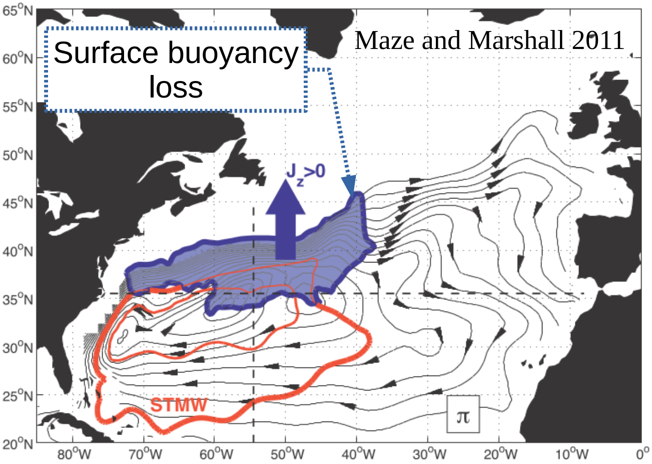


Mode water formation dominated by air-sea heat flux.

Subducted mode water only $\sim 1/3$ of volume implied by air-sea fluxes (Kelly and Dong 2013).

Interior mixing, or export, required to close budget.

# Mechanisms of mode water 'destruction' not clear

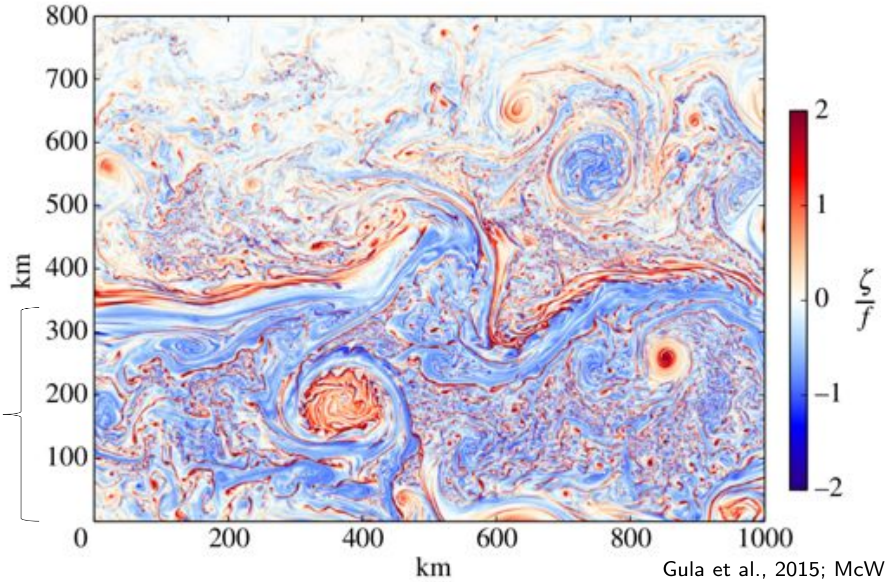


Observationally a very challenging problem, large spatial scales, long timescales.

Climate models exhibit drift in mode water properties.(Carman and McClean 2011)

Most analysis are *very* coarsely resolved horizontally.

# Mode water formation regions are sites of active submesoscale turbulence



18° water formation region

Gula et al., 2015; McWilliams, 2016

# How do surface submesoscale dynamics affect the flux of PV?

## Potential Vorticity Basics

- PV flux framework and the role of turbulence in the boundary layer

## Submesoscale ⟷ Turbulence

- How do interactions between submesoscale processes and boundary layer turbulence modify the flux of PV at the surface?

## Application to mode water PV budget

- What role does the submesoscale play in the seasonal PV budget of the 18° water?

# Potential Vorticity, a dynamical tracer

## Unstratified fluid

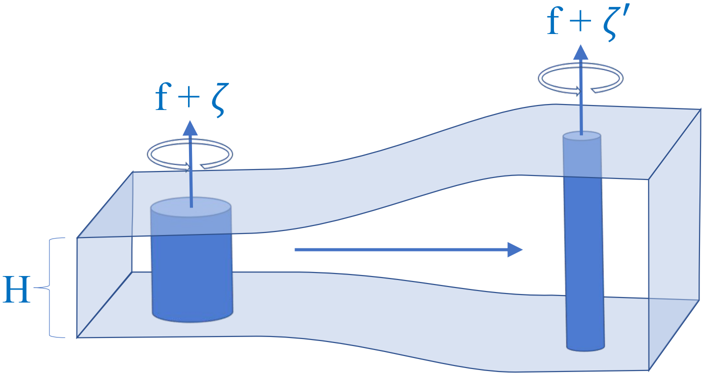
Barotropic PV,

$$q_{BT} = \frac{f + \zeta}{H}$$

## PV is conserved

Outside boundary layers,

$$\frac{Dq_{BT}}{Dt} = 0$$



Coriolis frequency
$f = 2\Omega \sin\theta$

Relative vorticity:
$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

# Potential Vorticity, a dynamical tracer

## Stratified fluid

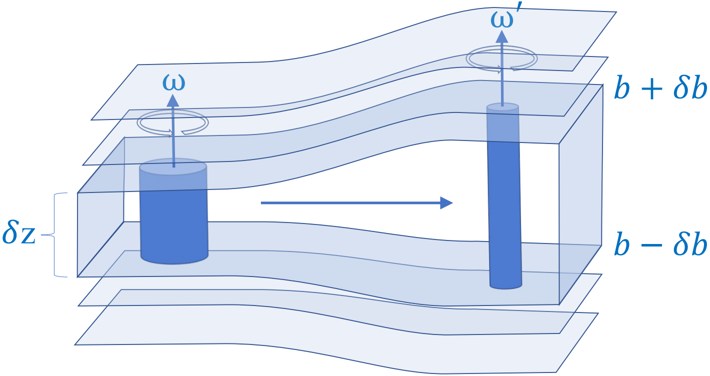
Ertel PV,

$$q \quad = \quad \boldsymbol{\omega} \cdot \nabla b$$

All terms matter at the submesoscale,

$$q \quad \approx \quad fN^2(1 + Ro - Ri^{-1})$$

where $Ro = \zeta_g/f$ and $Ri = N^2/(\partial u_g/\partial z)^2$.



Buoyancy:
$$b = -\frac{g\rho}{\rho_o}$$

Absolute vorticity:
$$\boldsymbol{\omega} = f\hat{\boldsymbol{k}} + \nabla \times \boldsymbol{u}$$

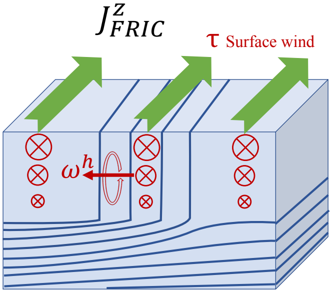
# Non-conservative boundary layer processes flux PV

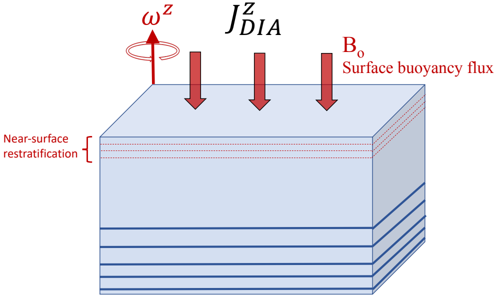
## Flux form of the PV equation

$$\frac{Dq}{Dt} = -\nabla \cdot \left( \underbrace{\nabla b \times \mathbf{F}}_{J_{FRIC}} - \underbrace{\omega \mathcal{D}}_{J_{DIA}} \right)$$

Friction: $\mathbf{F} = \frac{D\mathbf{u}}{Dt} + f\hat{k} \times \mathbf{u} + \frac{1}{\rho}\nabla p$

Diabatic: $\mathcal{D} = \frac{Db}{Dt}$





# Total PV in an isopycnal layer only changes through boundary fluxes!

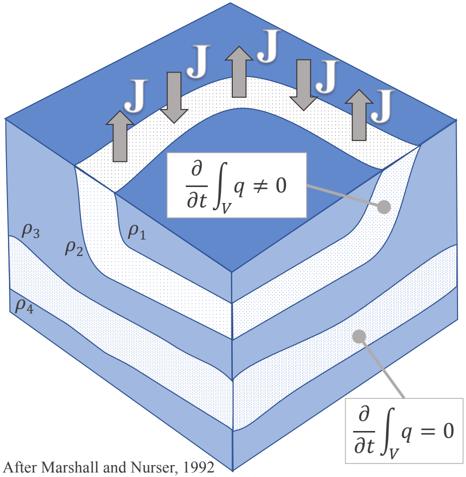
## Isopycnals are ‘impermeable’ to PV

$$\frac{\partial}{\partial t}\int_V q = -\int_A \left(J^z_{FRIC} + J^z_{DIA}\right)$$

(Haynes and McIntyre 1987)

Holds even in the presence of interior mixing.

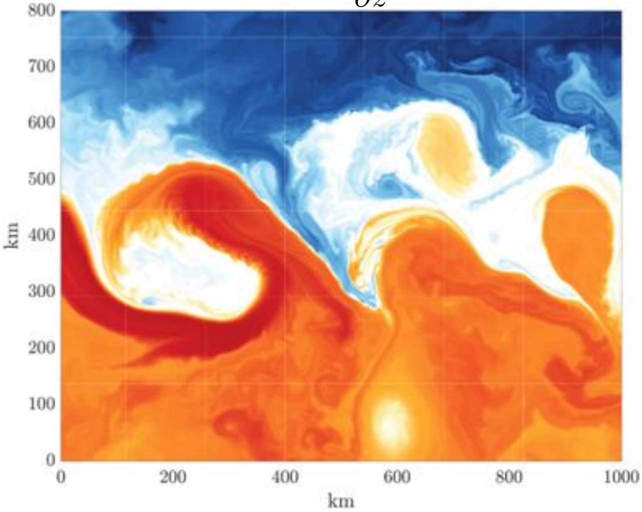
Boundary processes set the interior PV.



After Marshall and Nurser, 1992

# Understanding PV flux at the submesoscale

$$q \approx f\frac{\partial b}{\partial z}$$



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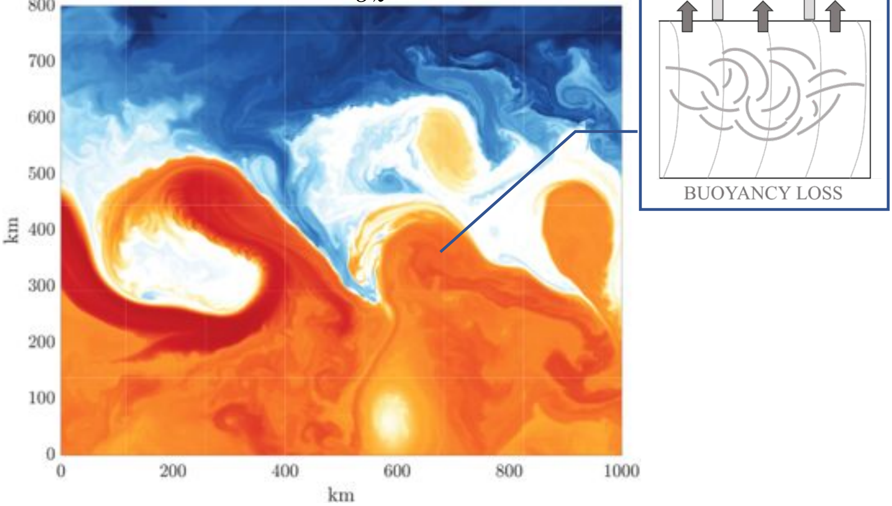
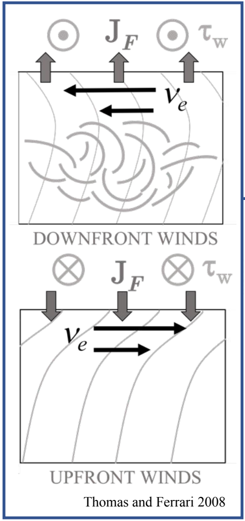
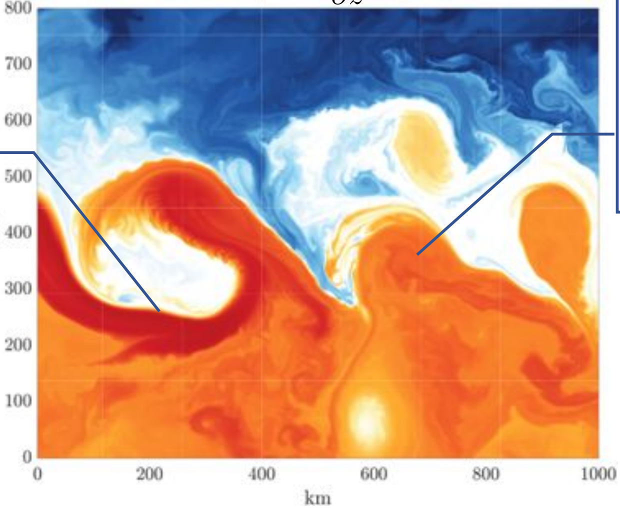


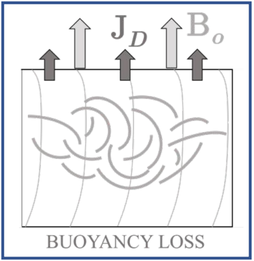
# Understanding PV flux at the submesoscale



Thomas and Ferrari 2008

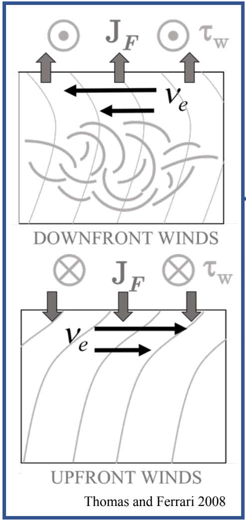
$$q \approx f\frac{\partial b}{\partial z}$$



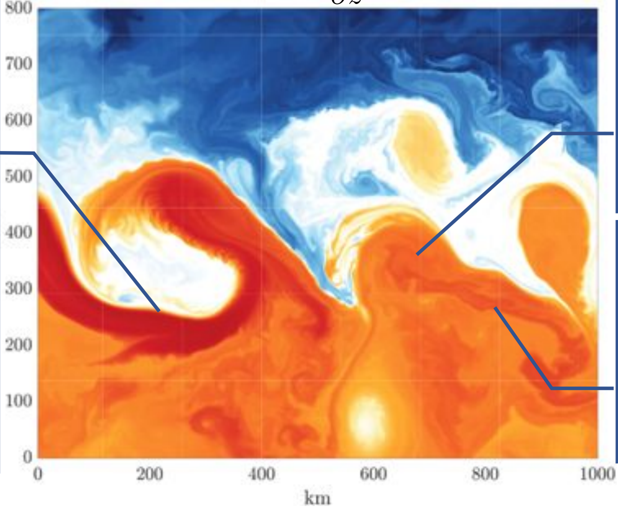


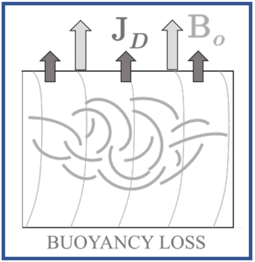
# Understanding PV flux at the submesoscale

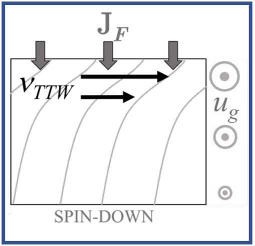
$$q \approx f \frac{\partial b}{\partial z}$$



Thomas and Ferrari 2008

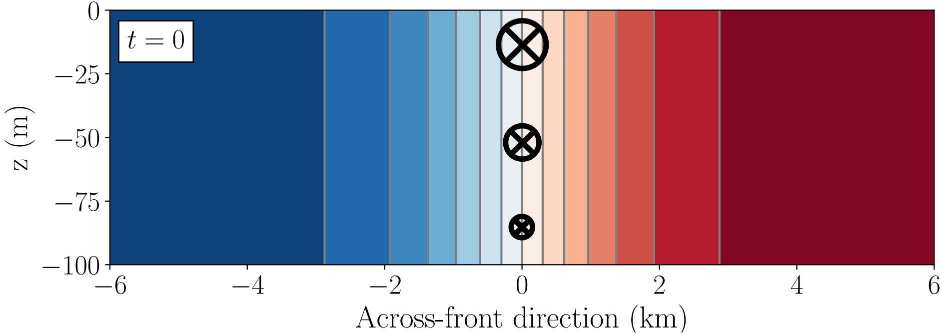






# Turbulent Thermal Wind

$$f\hat{k} \times \mathbf{u_g} = -\frac{1}{\rho}\nabla p$$



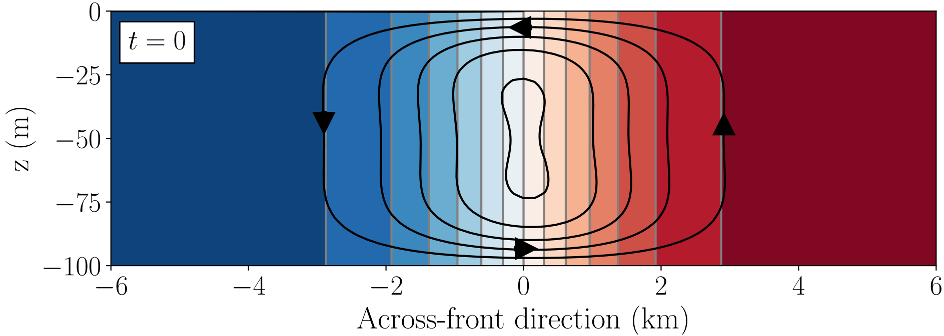
Buoyancy (colorscale)

Classic flow decomposition is in terms of Ekman and thermal wind components.

McWilliams et al. 2015, Wenegrat and McPhaden JPO 2016

# Turbulent Thermal Wind

$$f\hat{k} \times \mathbf{u_{TTW}} = \frac{\partial}{\partial z}\left(\nu \frac{\partial \mathbf{u_g}}{\partial z} + \nu \frac{\partial \mathbf{u_{TTW}}}{\partial z}\right)$$



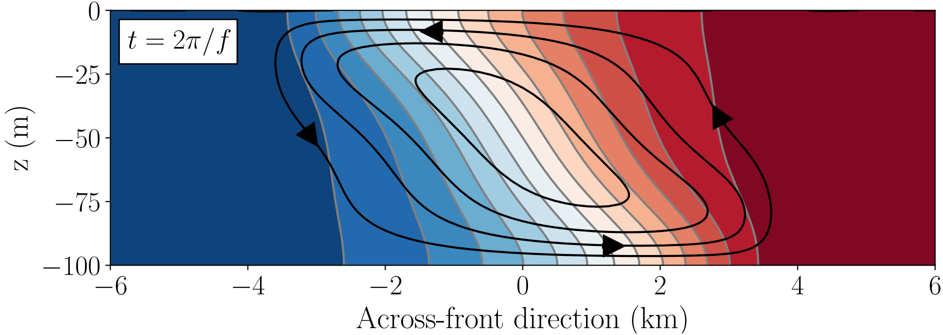
TTW streamfunction (black), Buoyancy (colorscale)

Turbulence breaks the thermal wind balance, couples thermal wind and Ekman.

McWilliams et al. 2015, Wenegrat and McPhaden JPO 2016

# Turbulent thermal wind

$$J^z_{TTW} \approx f u_{TTW} \frac{\partial b}{\partial x}$$



TTW streamfunction (black), Buoyancy (colorscale)

Secondary circulation is restratifying, acts as a PV source at the surface.

Wenegrat et al. JPO *in review*

# Surface flux of PV can be connected to the TKE budget

The non-advective PV flux can be approximated as,

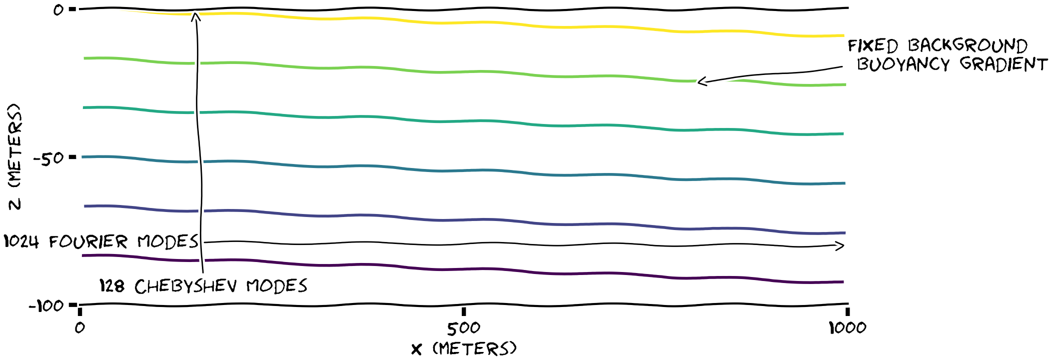
$$J^z \approx f \frac{\partial}{\partial z} \left[ \langle w' b' \rangle - \frac{\langle v' w' \rangle}{f} \frac{\partial b}{\partial x} \right]$$

## PV flux depends on the structure of turbulent fluxes in the boundary layer

$$J^z(0) \approx -\frac{f}{h} \left[ \overbrace{-B_o}^{\text{Surface buoyancy flux}} + \underbrace{\langle w' b' \rangle|_{z=-h}}_{Buoyancy\ production} \overbrace{- \frac{\tau^y}{\rho f} \frac{\partial b}{\partial x}}^{Ekman\ buoyancy\ flux} \underbrace{- \frac{\langle v' w' \rangle}{f} \frac{\partial b}{\partial x}|_{z=-h}}_{Geostrophic\ Shear\ Production} \right]$$

cf. Taylor and Ferrari 2010

# Submesoscale modification of momentum fluxes

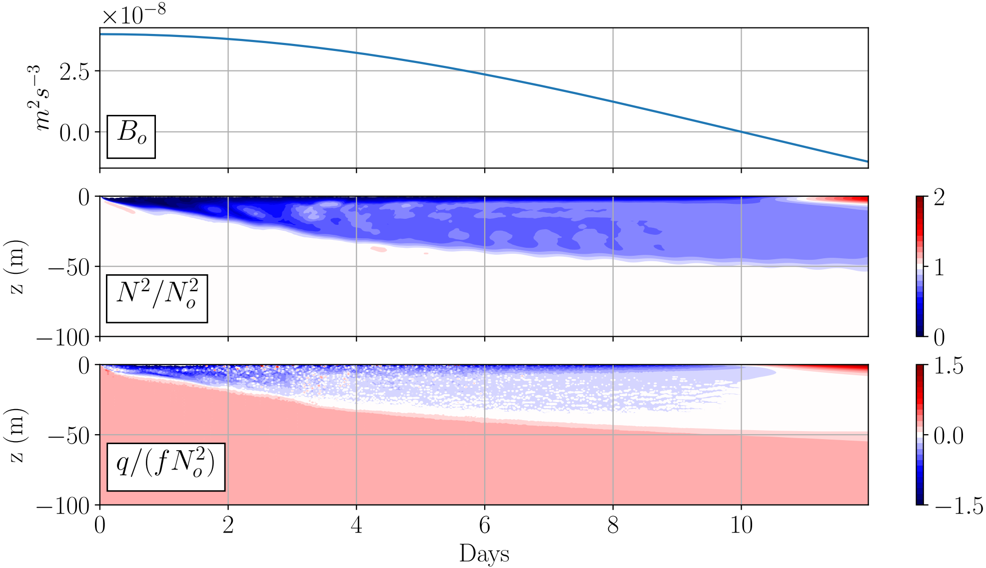


- Solutions found using Dedalus
- Horizontally periodic 2D domain
- Surface buoyancy flux (no winds)

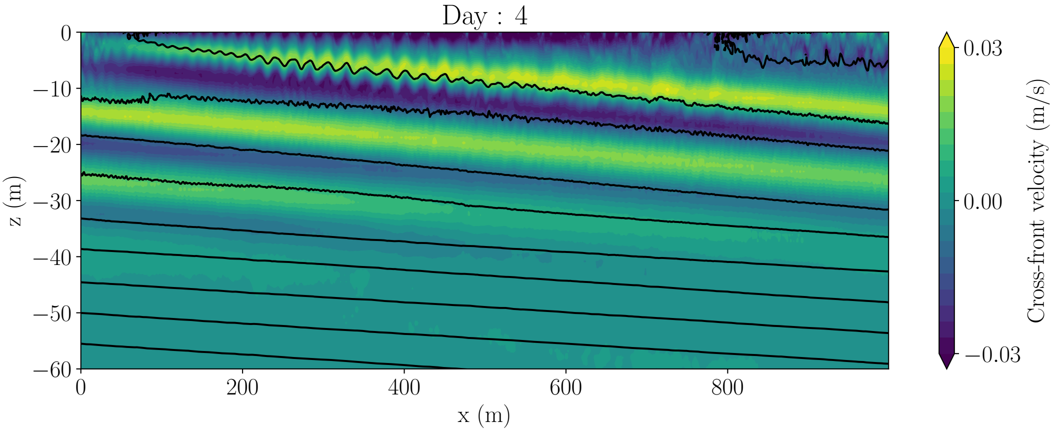
- $\nu = \kappa = 10^{-4}\ m^2 s^{-1}$
- $\partial b/\partial x = 8.48 \times 10^{-7}\ s^{-2}$
- Initial $Ri = 1.25$

cf. Taylor and Ferrari 2010

# 2D front with time-varying surface buoyancy flux

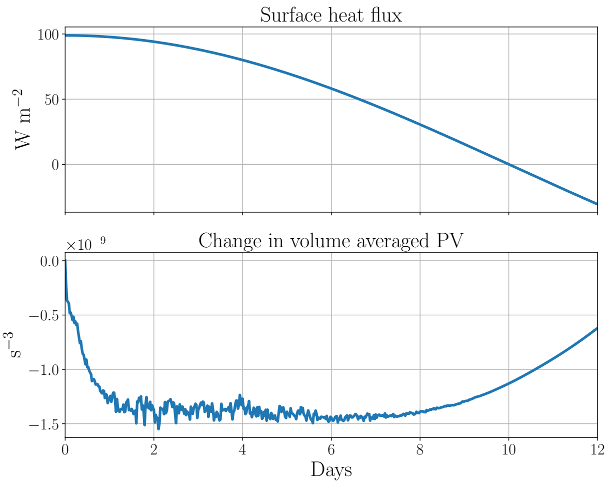


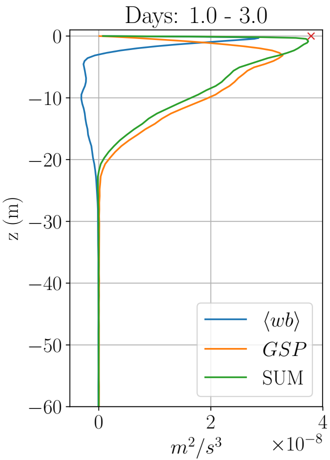
# Flow becomes unstable to symmetric instability



- Fast growing instability in flows where $fq < 0$ and $f(f+\zeta) > 0$ (Hoskins 1974)
- Energy source is geostrophic shear production $(GSP = -\langle v'w' \rangle V_z^g)$ (Thomas et al. 2013)

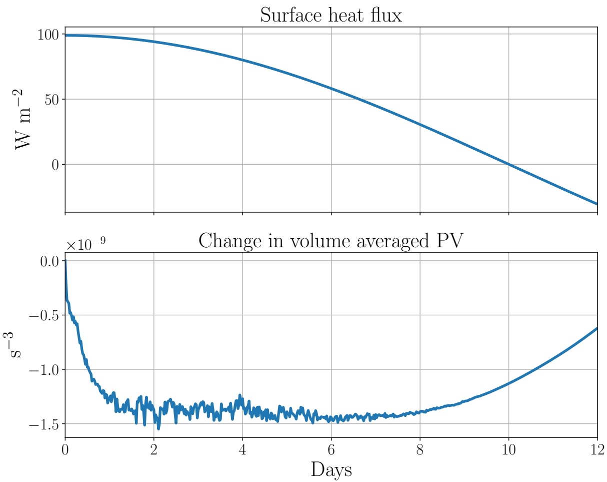
# TKE production terms are modified by the front

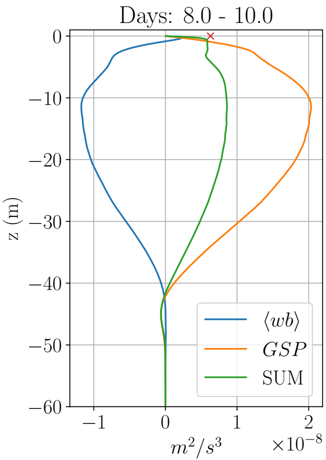




- Forced-SI regime develops after $\sim$ 1 day (Taylor and Ferrari 2010, Thomas et al. 2013).

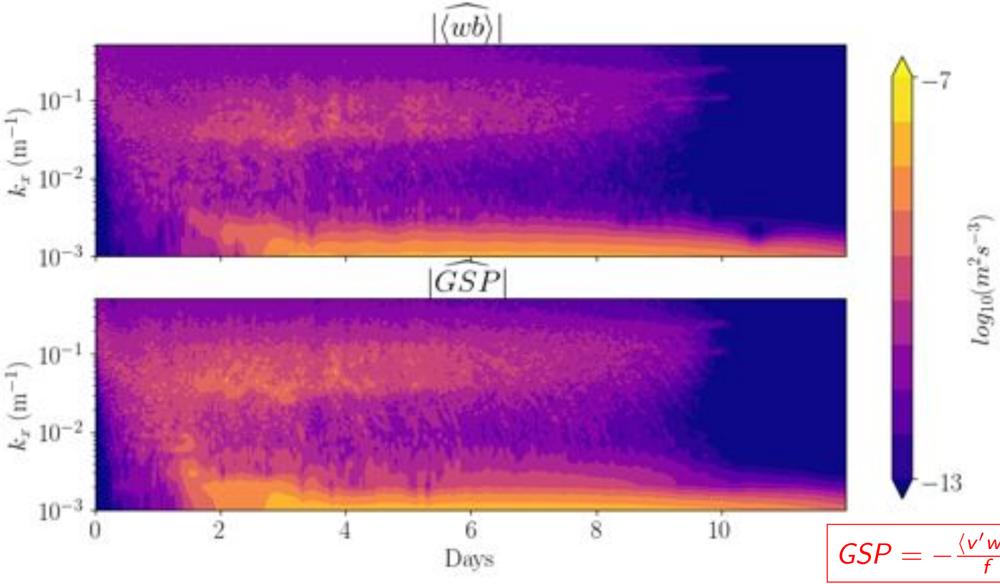
# PV begins to increase before change in sign of surface buoyancy flux





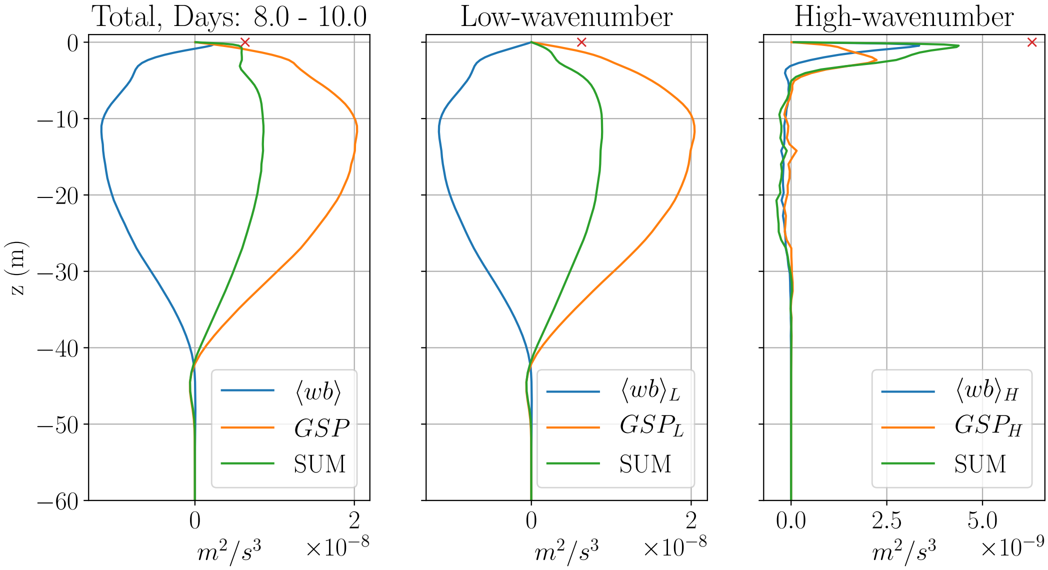
- GSP enhancement through increase in $|\langle v'w' \rangle|$.

# Enhanced fluxes originate from both turbulent and SI scales

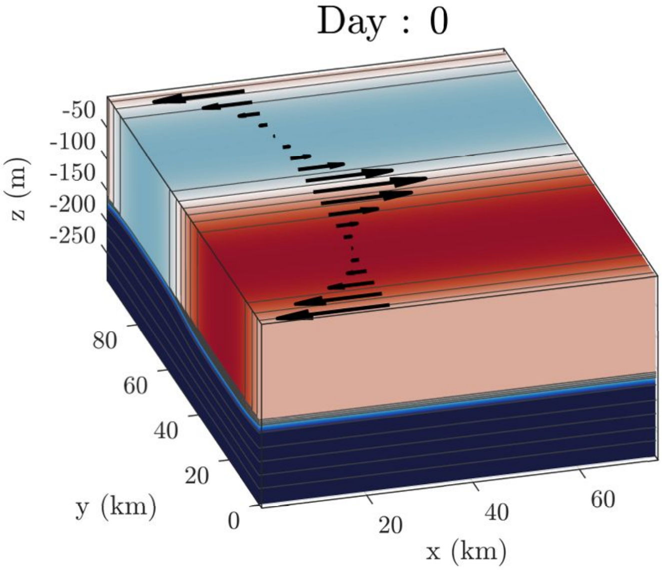


$$GSP = -\frac{\langle v'w' \rangle}{f}\frac{\partial b}{\partial x}$$

# SI momentum fluxes drive PV flux/restratification



# 3D simulations - What is the role of baroclinic instability?



MITgcm - hydrostatic

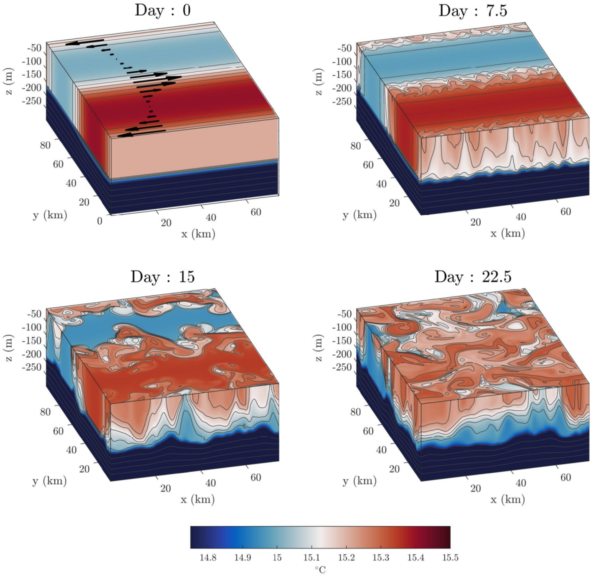
Doubly periodic domain

KPP turbulence closure

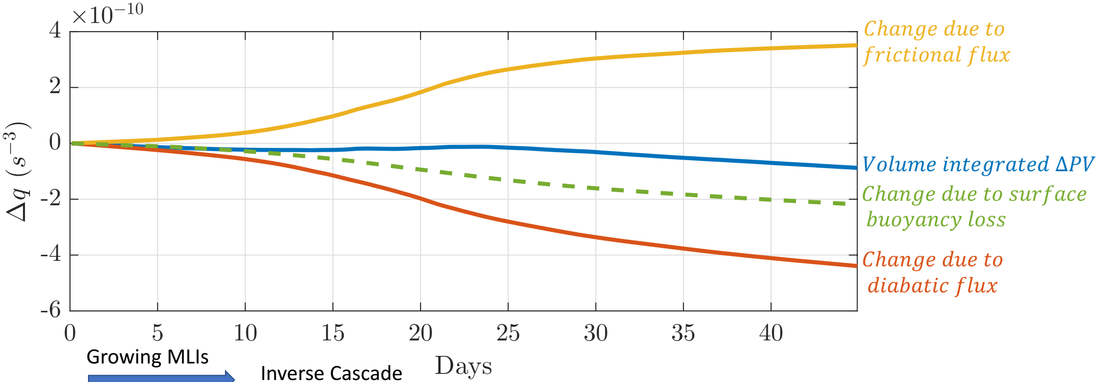
No surface wind stress

Initial buoyancy gradient and heat flux are varied between runs.

# Example frontal spindown



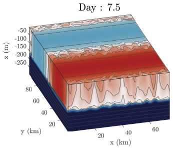
# Frictional injection of PV at sharp fronts can offset buoyancy loss

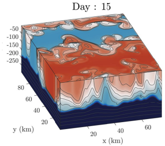


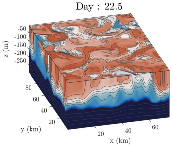
Growing MLIs

Inverse Cascade

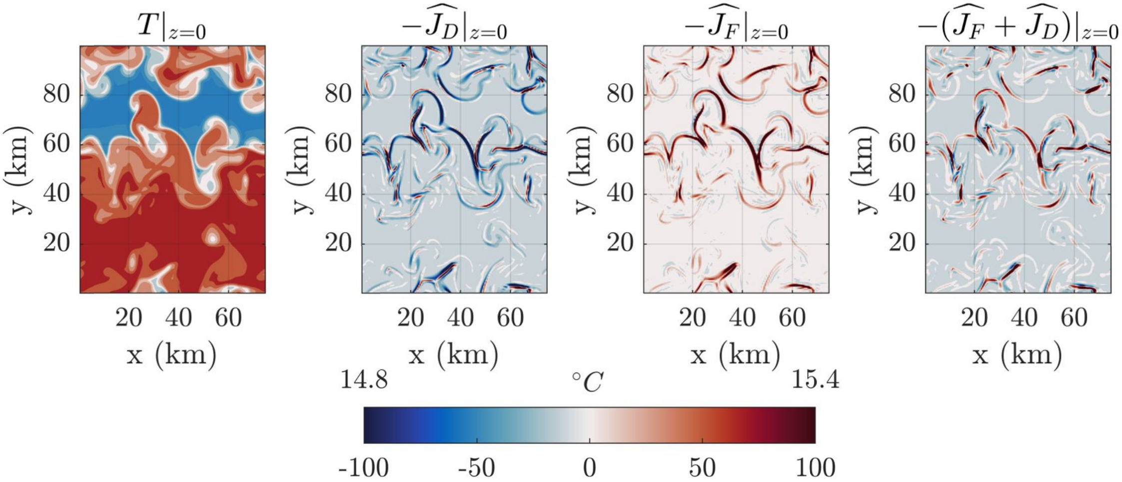
Spin-down







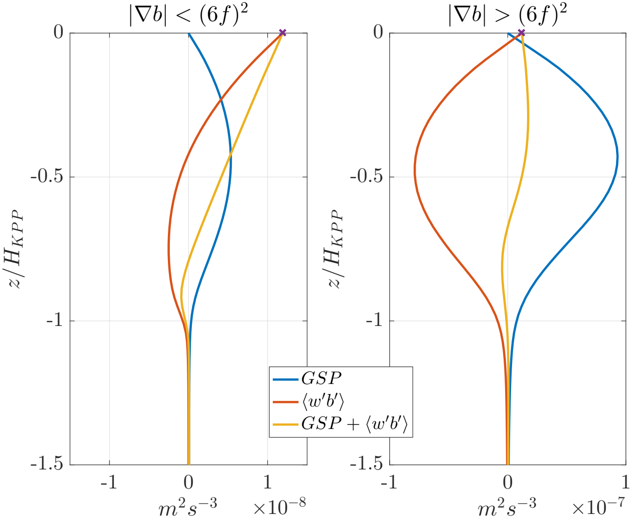
# Frontogenesis enhances rate of PV flux



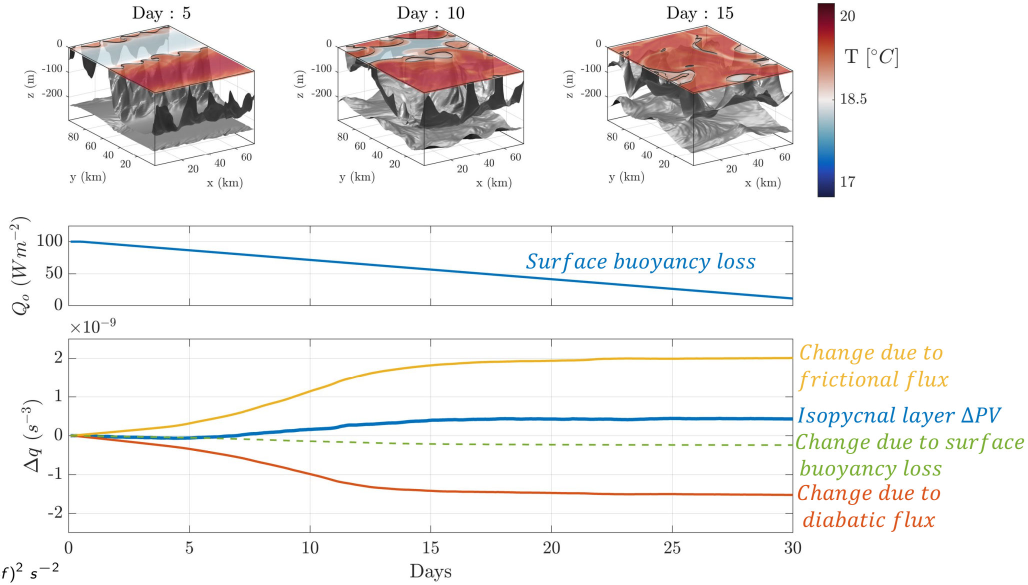
Normalized by surface buoyancy PV flux

# Strong fronts enhance GSP, lead to PV injection

$$J^z \approx f \frac{\partial}{\partial z}\left[\langle w'b'\rangle - \frac{\langle v'w'\rangle}{f}\frac{\partial b}{\partial x}\right]$$



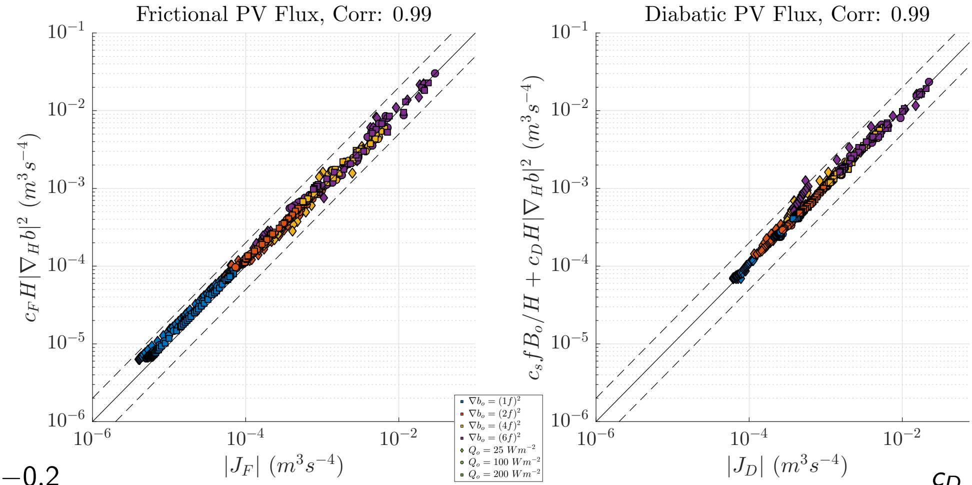
# Isopycnal layers can gain PV, even during surface buoyancy loss



$\nabla b_o = (4f)^2\ s^{-2}$

# TTW PV fluxes can be accurately approximated with a simple scaling

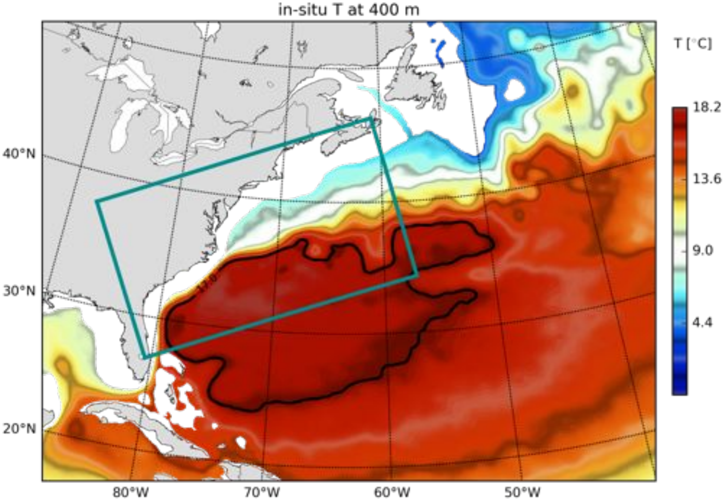
$$J^z_{TTW} \approx -0.05H|\nabla b|^2$$



$c_F = -0.2$

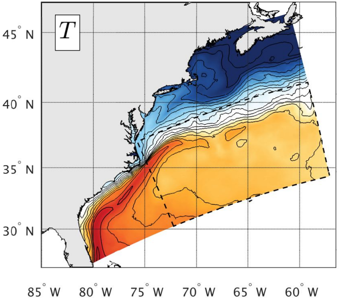
$c_D = 0.15$

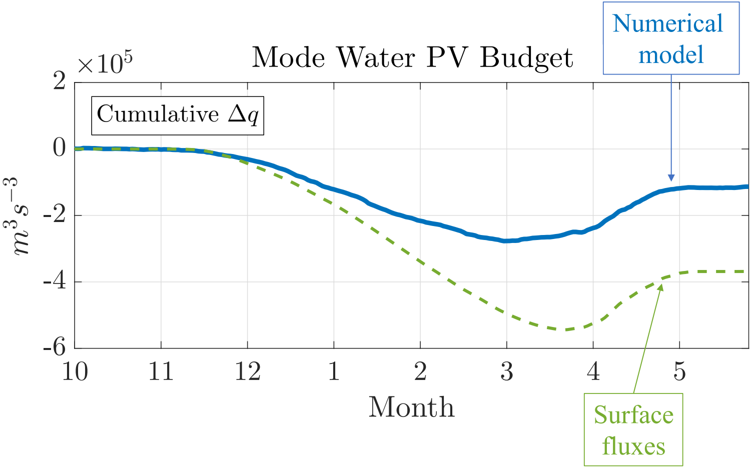
# Evaluation in a realistic model of the North Atlantic



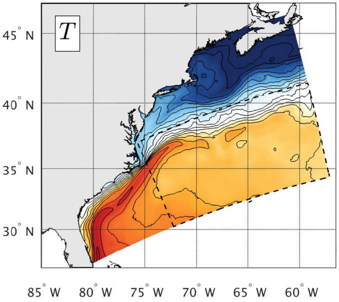
- 1.5 km resolution (nested simulation)
- Regional Ocean Modeling System (ROMS)
- Forced by climatological winds, surface fluxes
- Described in Gula et al. 2015 JPO, McWilliams QJRMS 2016.

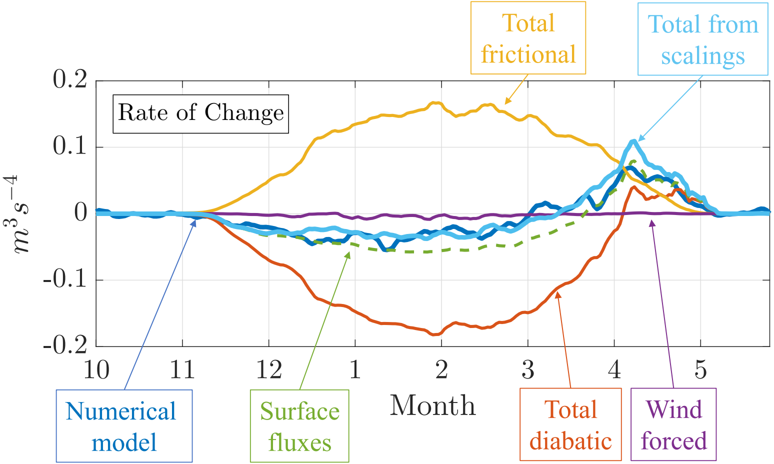
# Air-sea PV flux overestimates mode water formation in high res model





# Submesoscale effects greatly modify the mode water PV budget

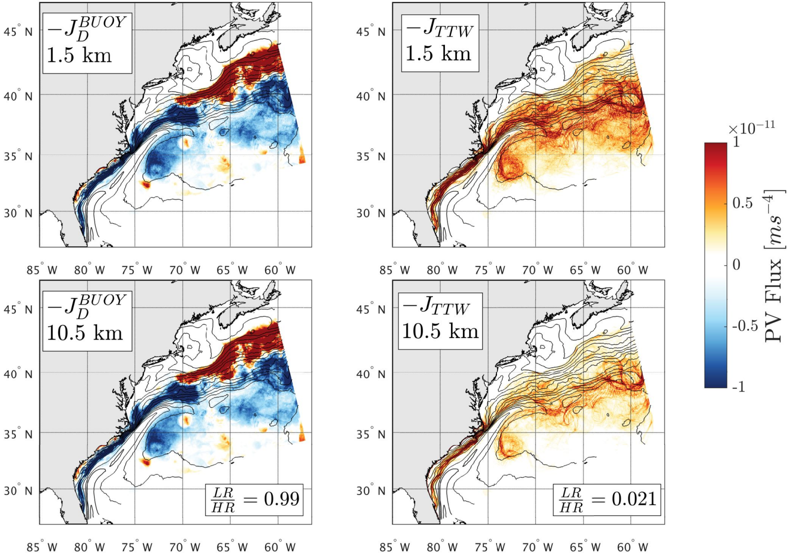




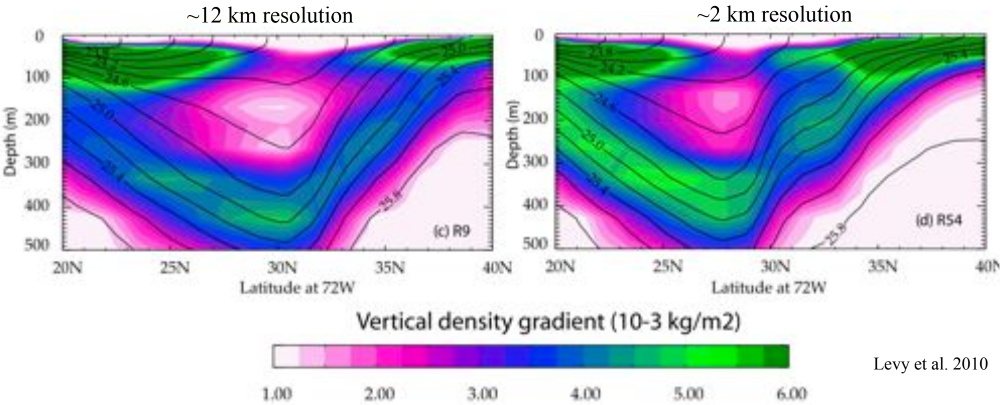
Wintertime mode water formation rate reduced by $\sim$ **50%**

Formation season shortened by $\sim$ **3** weeks.

# Missing physics in climate models, a future parameterization challenge



# From the submesoscale to the gyre scale



Levy et al. 2010

Reduced meridional heat transport with resolved submesoscale (Levy et al. 2010)

Boundary layer turbulence ↔ submesoscales ↔ gyre circulation

# Summary

## Potential Vorticity Basics

- PV provides a framework for linking dynamical processes across scales.

## Submesoscale ⟷ Turbulence

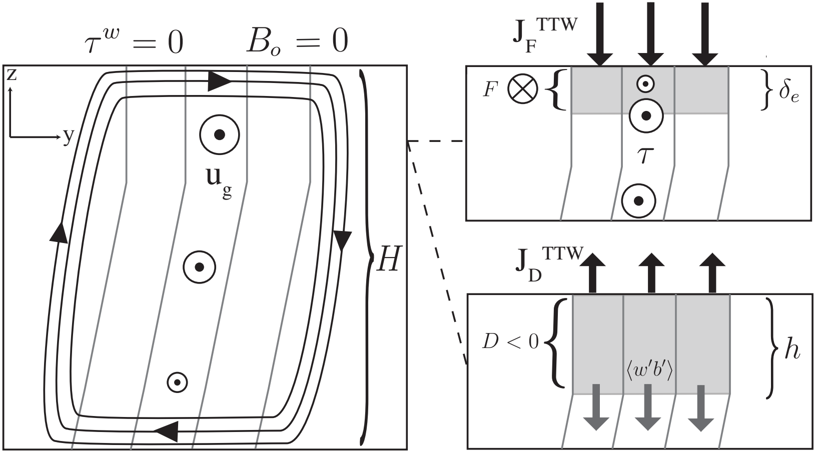
- Symmetric instability PV flux through enhanced vertical momentum fluxes.
- Baroclinic instability PV flux through frontogenesis.
- PV injection at the submesoscale can outpace effect of air-sea fluxes.

## Application to mode water PV budget

- Submesoscales modify the rate and timing of mode water formation/destruction.
- Contributes at leading order to the 18° water seasonal PV budget.

# Extra Slides

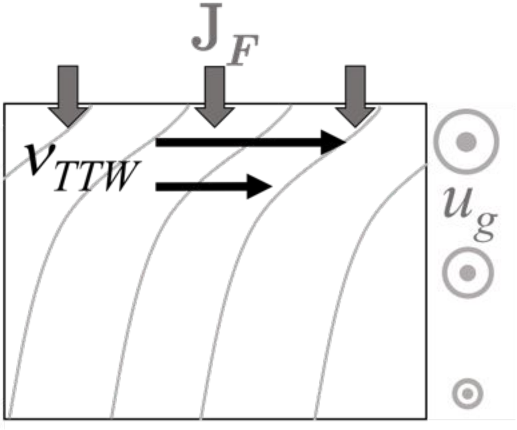
# TTW Secondary Circulation



$$f\hat{k}\times\mathbf{u}_e = \frac{\partial}{\partial z}\left[\nu\left(\frac{\partial \mathbf{u}_e}{\partial z}+\frac{\partial \mathbf{u}_g}{\partial z}\right)\right]$$

$$\rho\nu\left(\frac{\partial \mathbf{u}_e}{\partial z}+\frac{\partial \mathbf{u}_g}{\partial z}\right)=\boldsymbol{\tau}^w, \quad \text{at z} = 0,$$

# Geostrophic stress is a source of PV at the submesoscale



Wenegrat et al. *in review.*

Frictional J-vector:

$$J_F = \nabla b \times \mathbf{F}$$

For subintertial flows, $fu_e \approx F$. Ekman balance then gives,

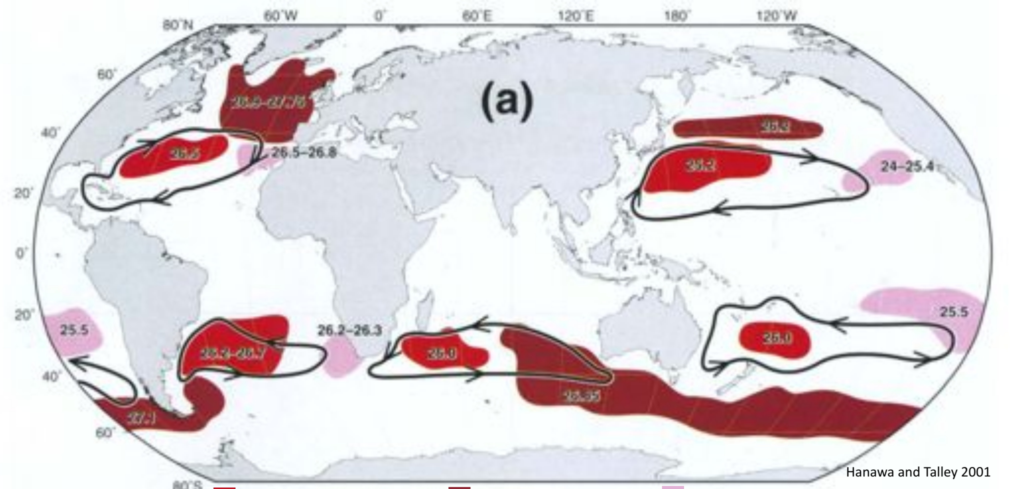
$$F \sim \frac{\tau_g}{\rho h_e} + \frac{\tau_w}{\rho h_e}$$

$$\tau_g \sim \frac{\rho u^* H}{f} |\nabla b|$$

Geostrophic stress term always *injects* PV,

$$\boxed{J_{TTW} \propto -H|\nabla_h b|^2}$$

# Mode waters are of global importance



Hanawa and Talley 2001

🟥 Subtropical mode waters 🟥 Subpolar mode waters 🟪 Eastern Subtropical mode waters

Implications for gyre circulations, understanding climate variability, and ocean ventilation.