Atmospheric Turbulent Convection

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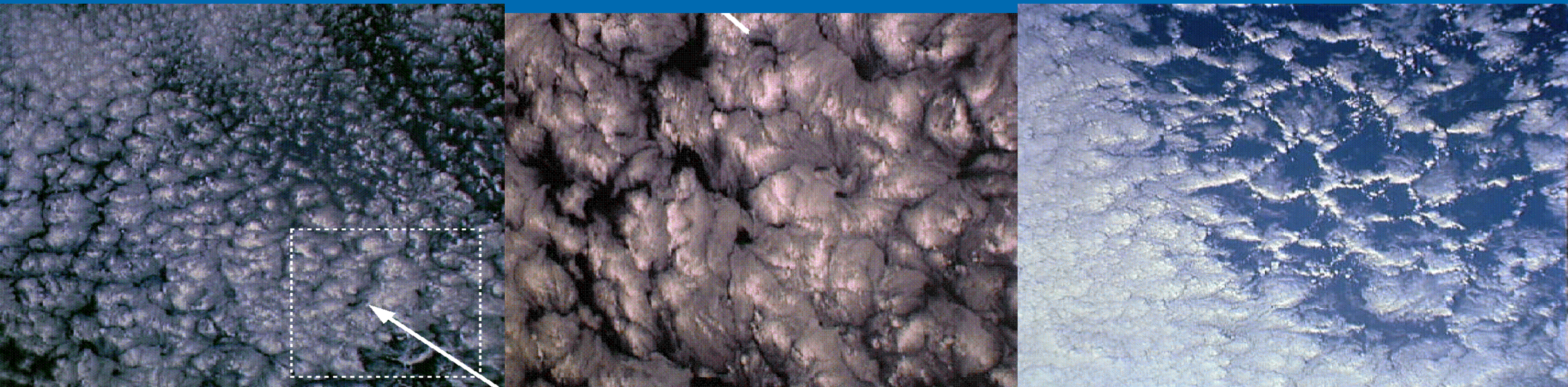
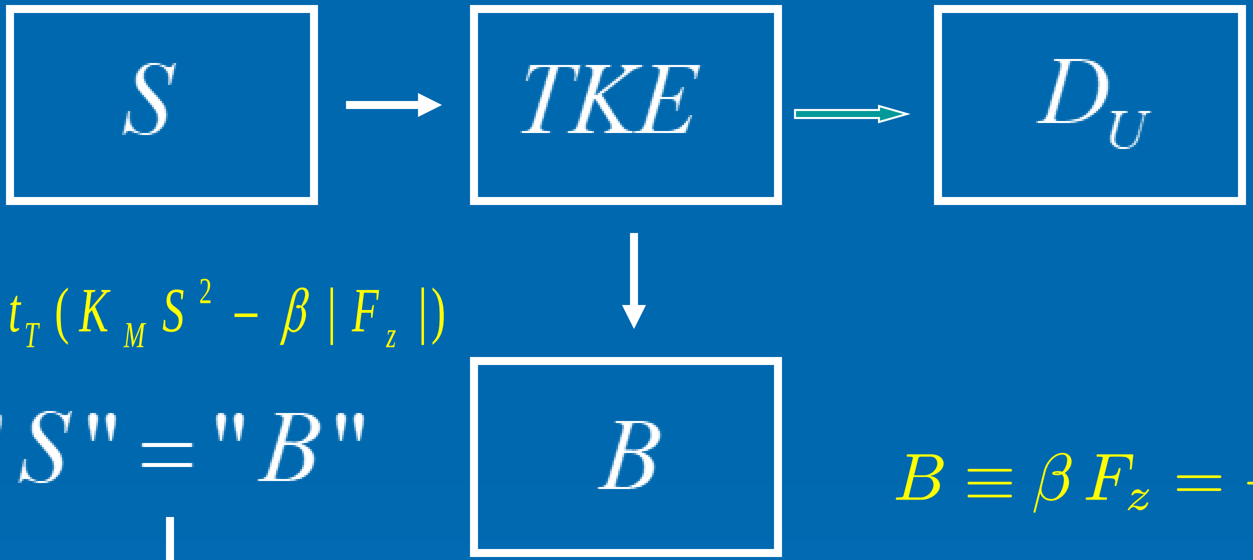


Outline

- Summary: stably-stratified turbulence and EFB
- Mechanism of formation of cloud cells in shear-free convection
- Mechanism of formation of cloud streets in sheared convection
- EFB theory for shear-free convection
- EFB theory for sheared convection
- Convection in Planetary Scales,
  3D Slow Rossby waves and Southern Oscillations

TKE Balance for SBL

$$\frac{\partial E_K}{\partial t} = K_M S^2 - \beta |F_z| - \frac{E_K}{t_T} - T$$



$$E_K \approx t_T (K_M S^2 - \beta |F_z|)$$

$$"S" = "B"$$

$$\mathrm{Ri}_C \approx 0.25$$

$$B \equiv \beta F_z = -(2E_z t_T) \frac{\partial \bar{\Theta}}{\partial z}$$

$$\beta = \frac{g}{T_0}$$

$$\mathrm{Ri} = \frac{N^2}{S^2}$$

$$N^2 = \beta \frac{\partial \overline{\Theta}}{\partial z}$$

$$\Theta = T (P_0 / P)^{1-1/\gamma}$$

Budget Equations for SBL

- Turbulent kinetic energy: $E_K = \frac{1}{2}\left\langle \mathbf{u}^2 \right\rangle$
- Potential temperature fluctuations: $E_\theta = \frac{1}{2}\left\langle \theta^2 \right\rangle$
- Flux of potential temperature : $\mathbf{F} = \left\langle \mathbf{u}\,\theta \right\rangle$

$$\frac{DE_K}{Dt} + \mathrm{div}\,(\mathbf{\Phi}_u) - \Pi - \beta\,F_z = -\,D_K$$

$$\frac{DE_\theta}{Dt} + \mathrm{div}\,(\mathbf{\Phi}_\theta) + \frac{N^2}{\beta} F_z = -\,D_\theta$$

$$\frac{DF_i}{Dt} + \mathrm{div}_j\,(\Phi_{ij}^F) + (\mathbf{F}\cdot\nabla)\overline{U}_i + \frac{N^2}{\beta}\tau_{ij}e_j - 2C_\theta\beta\, e_i\, E_\theta = -\,D_i^F$$

$$D_K = \frac{E_K}{t_T} \qquad D_\theta = \frac{E_\theta}{C_\theta\, t_T} \qquad D_i^F = \frac{F_i}{C_F\, t_T}$$

$$C_\theta\,\beta_i\left\langle \theta^2 \right\rangle = \beta_i\left\langle \theta^2 \right\rangle + \frac{1}{\rho_0}\left\langle \theta\nabla_i\, p \right\rangle$$

$$\Pi = -\,\tau_{ij}\nabla_j\overline{U}_i = K_M\, S^2$$

Budget Equations for SBL

$$\frac{DE_K}{Dt} + \frac{\partial \Phi_K}{\partial z} = \Pi + \beta F_z - D_K$$

$$\frac{DE_P}{Dt} + \frac{\partial \Phi_P}{\partial z} = -\beta F_z - D_P$$

$$E_\theta = \frac{1}{2}\left\langle \theta^2 \right\rangle$$

$$D_K = \frac{E_K}{t_T}$$

$$D_P = \frac{E_P}{C_P\, t_T}$$

$$E_p \equiv \frac{g}{\rho_0}\left\langle \int \rho\, dz \right\rangle = \left(\frac{\beta}{N}\right)^2 E_\theta = \frac{1}{2}\left(\frac{\beta}{N}\right)^2 \left\langle \theta^2 \right\rangle$$

Total Turbulent Energy

$$\frac{DE}{Dt} + \nabla \cdot \mathbf{\Phi} = \Pi - \frac{E}{C_u \, t_T}$$

$$E = E_K + \left(\frac{\beta}{N}\right)^2 E_\theta$$

The turbulent potential energy:

$$E_P = \left(\frac{\beta}{N}\right)^2 E_\theta$$

Production of Turbulent energy:

$$\Pi = -\tau_{ij} \nabla_j \bar{U}_i = K_M S^2$$

$$K_M = 2 C_\tau A_z \, l \sqrt{E_K}$$

Budget Equations for SBL

$$\frac{DE_K}{Dt} + \frac{\partial \Phi_K}{\partial z} = \Pi + \beta F_z - D_K$$

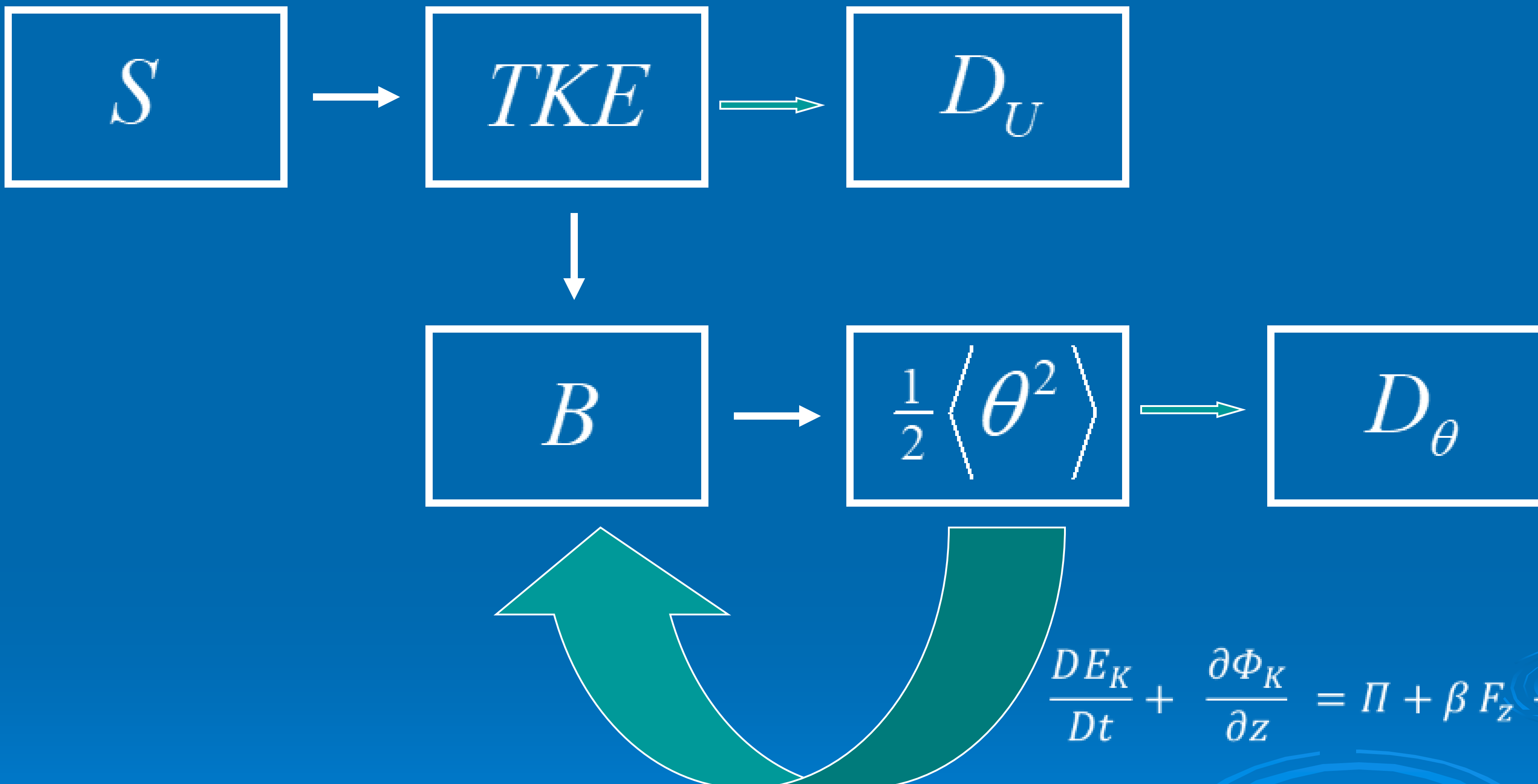
$$\frac{DE_P}{Dt} + \frac{\partial \Phi_P}{\partial z} = -\beta F_z - D_P$$

$$\frac{DF_z}{Dt} + \frac{\partial \Phi_F}{\partial z} = -D_z^F - \langle u_z u_z \rangle \frac{\partial \overline{\Theta}}{\partial z} + 2C_\theta \beta E_\theta$$

$$E_p \equiv (\beta / N)^2 E_\theta = ½(\beta / N)^2 \overline{\theta'^2}$$

$$C_\theta \beta_i \langle \theta^2 \rangle = \beta_i \langle \theta^2 \rangle + \frac{1}{\rho_0} \langle \theta \nabla_i p \rangle$$

No Critical Richardson Number



$$\frac{DE_K}{Dt} + \frac{\partial \Phi_K}{\partial z} = \Pi + \beta F_z - D_K$$

$$\frac{DE_P}{Dt} + \frac{\partial \Phi_P}{\partial z} = -\beta F_z - D_P$$

$$B \equiv \beta F_z = -(2E_z t_T)\frac{\partial \bar{\Theta}}{\partial z} + C_\theta \beta^2 \langle\theta^2\rangle t_T$$

$$C_\theta \beta_i \langle\theta^2\rangle = \beta_i \langle\theta^2\rangle + \frac{1}{\rho_0}\langle\theta \nabla_i p\rangle$$

Atmospheric Turbulent Convection

- **The atmospheric turbulent convection:**
  - the fully organized large-scale flow (the mean flow or mean wind)
  - the small-scale turbulent fluctuations,
  - long-lived large-scale semi-organized (coherent) structures.

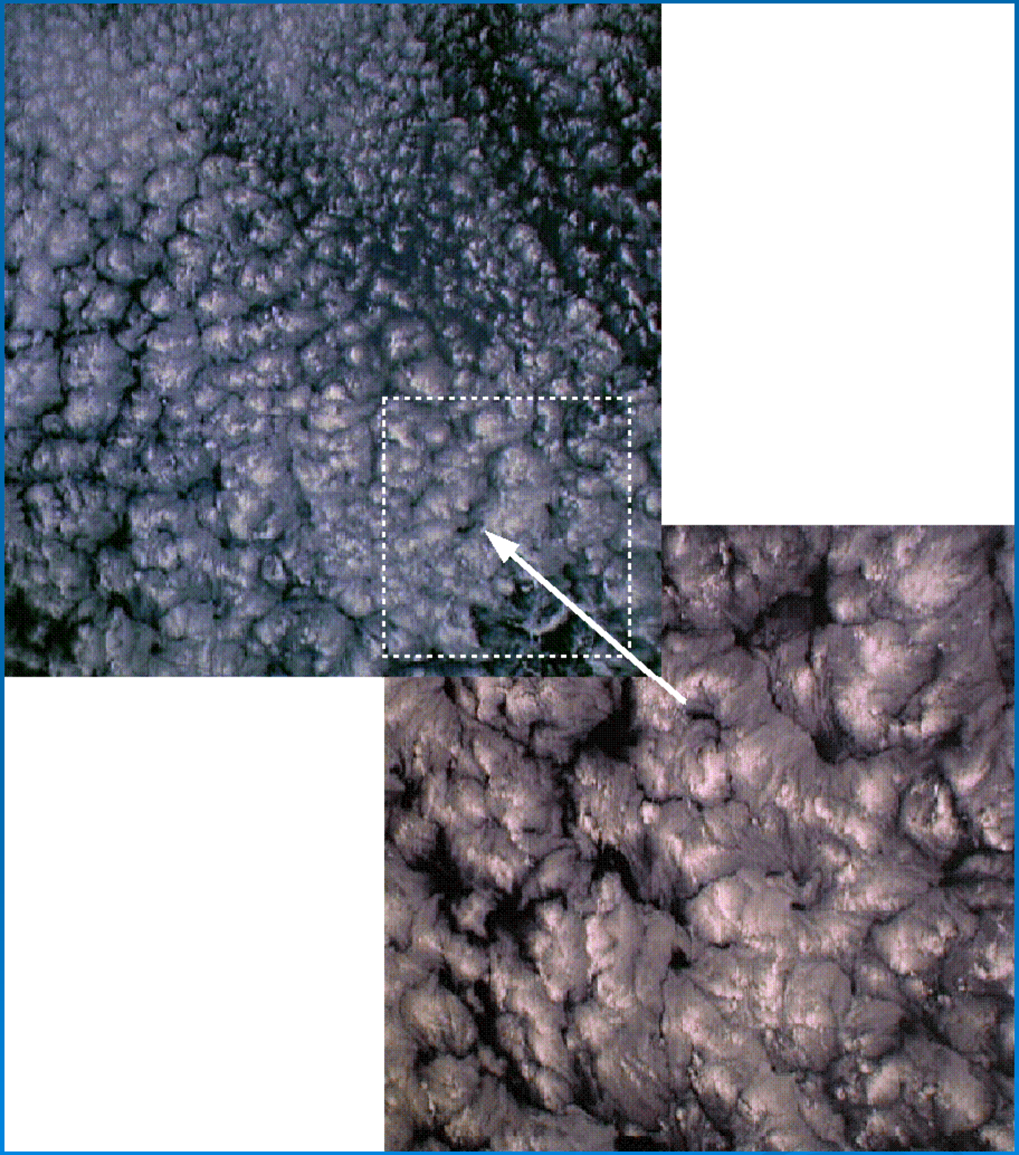
- **Two types of the semi-organized structures:**
  - cloud “streets”
  - cloud cells

- **The life-times and spatial scales of the semi-organized structures are much larger than the turbulent scales.**

Etling, D. and **Brown**, R. A., 1993. *Boundary-Layer Meteorol.*, **65**, 215—248.

Atkinson, B. W. and **Wu Zhang**, J., 1996. *Reviews of Geophysics*, **34**, 403—431.

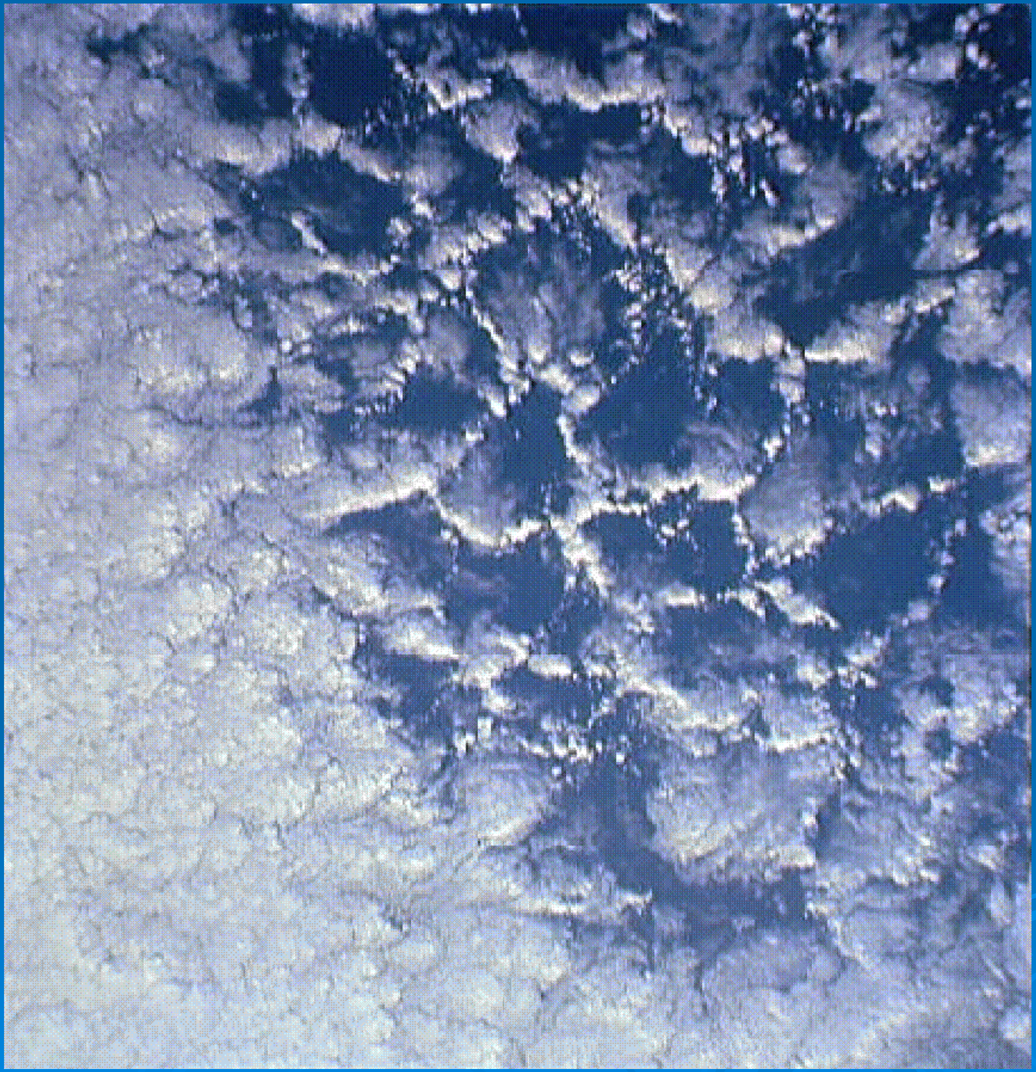
Closed cloud cells over the Atlantic Ocean



$T_{lifetime}$ = Several hours

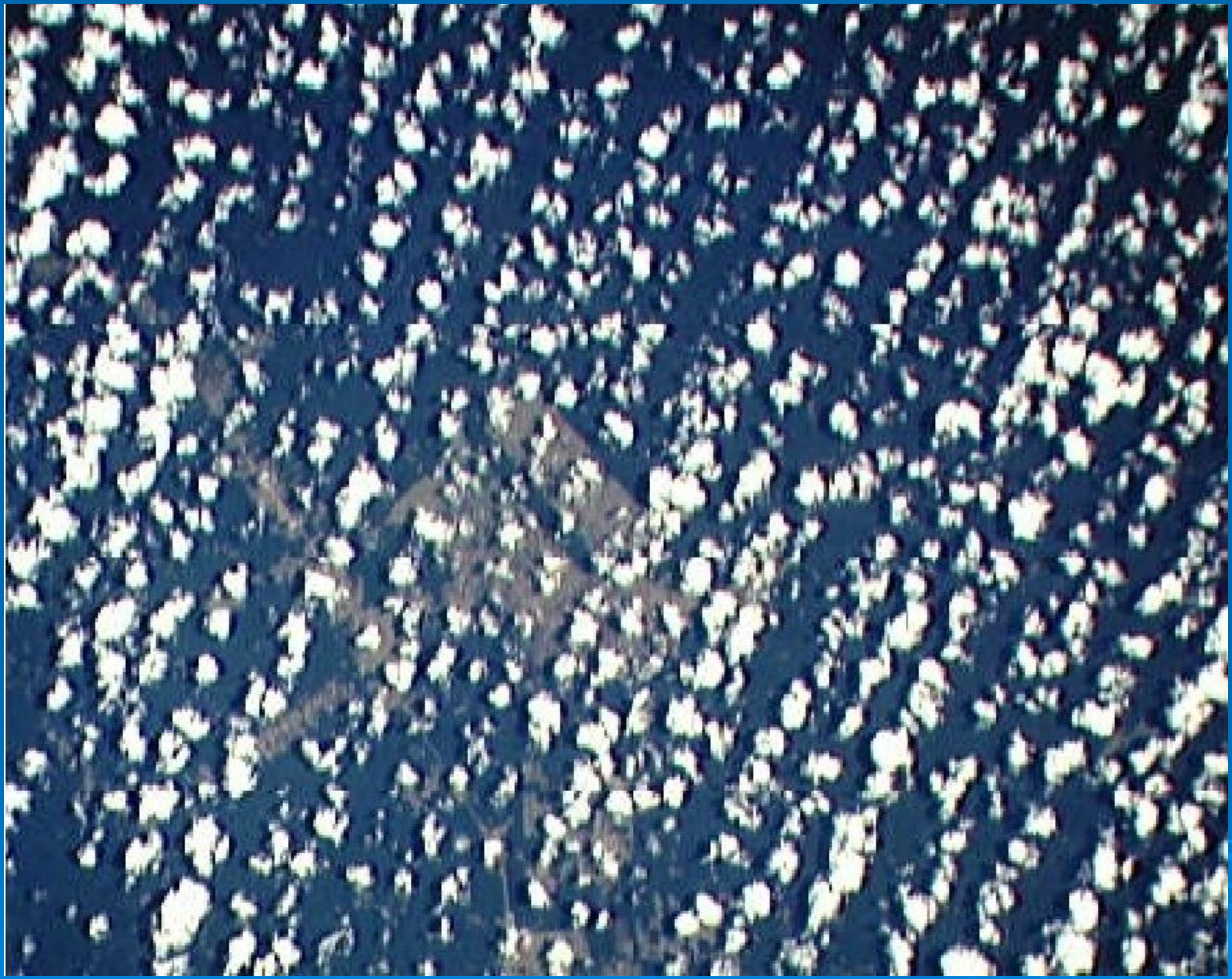
$$L/l_0 = 5 - 20$$

Open cloud cells over the Pacific Ocean



Cloud “streets” over Indian ocean

Cloud "streets" over the Amazon River



Cloud “streets”

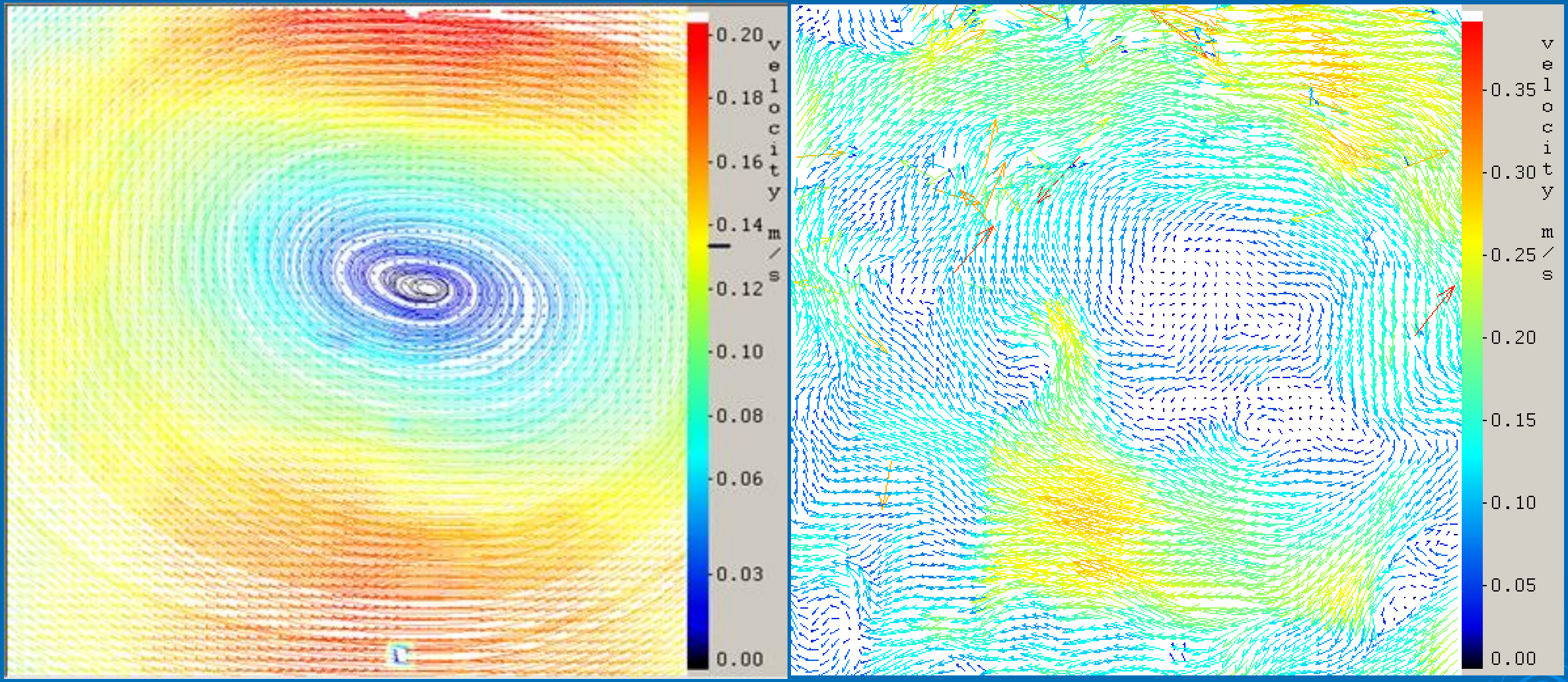
Laboratory Turbulent Convection

- In laboratory turbulent convection (in the Rayleigh-Benard apparatus) coherent organized features of motion, such as large-scale circulation patterns (the "mean wind“) are observed.

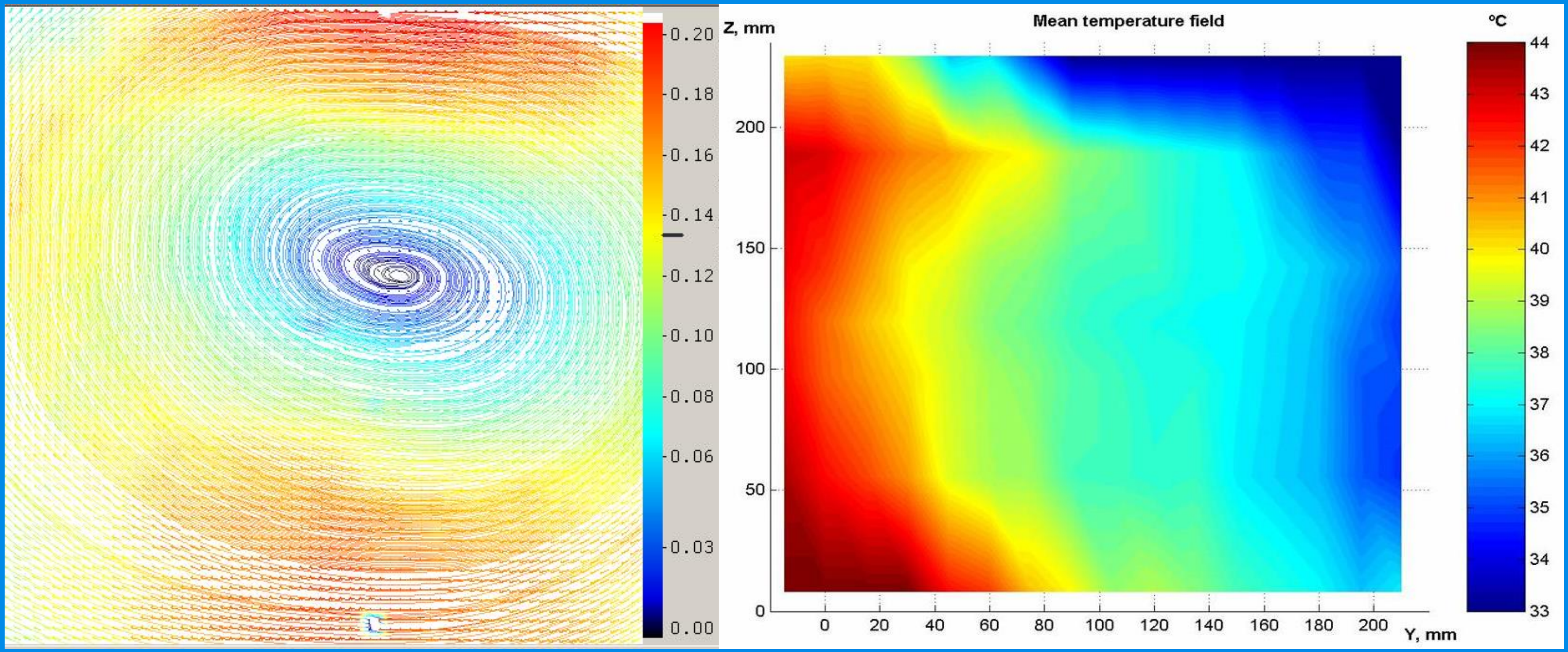
- There are several open questions concerning these flows:

  - How do they arise ?
  - What is the effect of mean wind on turbulent convection?
  - Is it shear-produced turbulence (due to the mean wind) or buoyancy-produced turbulence?

Coherent Structures (Mean Wind) in Laboratory Turbulent Convection

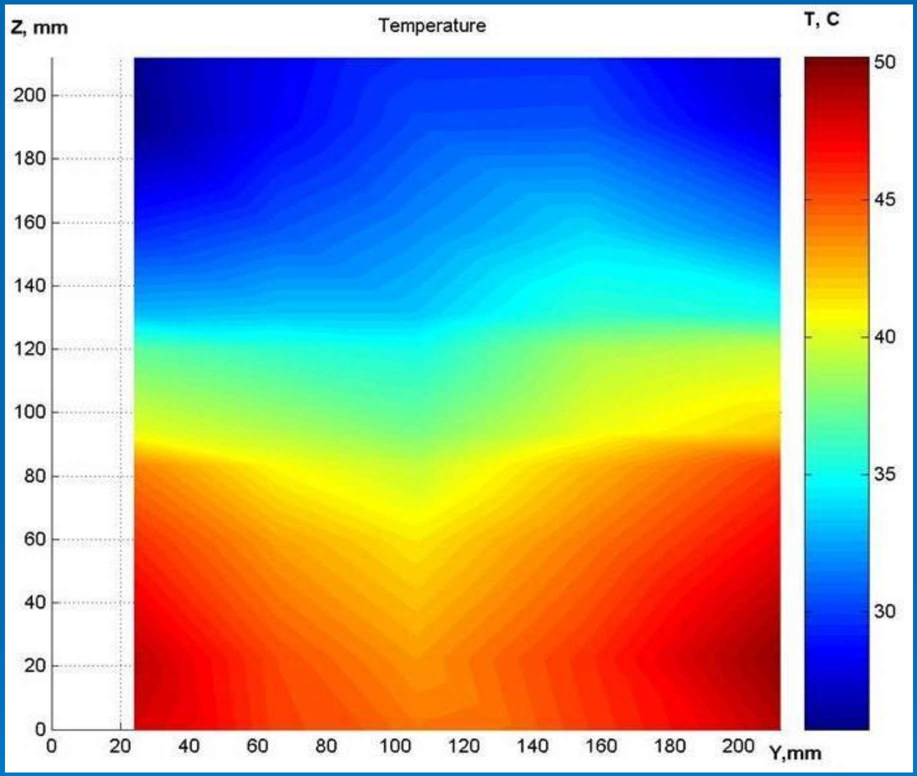


Unforced Convection: A = 1

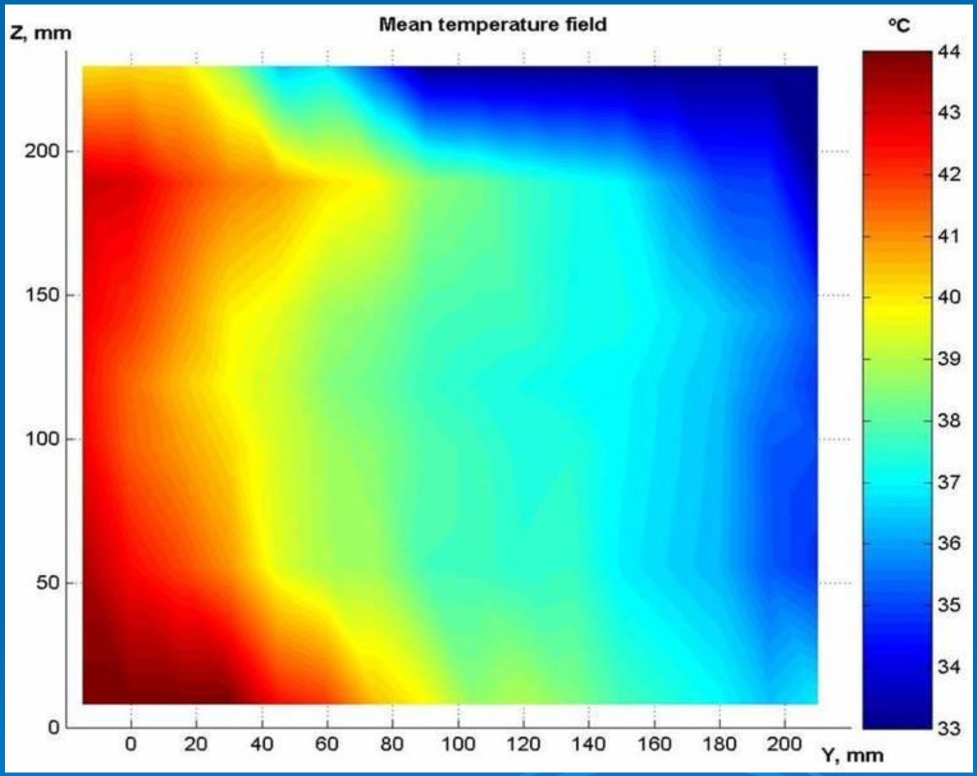


$\overline{U}(y,z)$ $\overline{T}(y,z)$

Temperature Field in Forced and Unforced Turbulent Convection

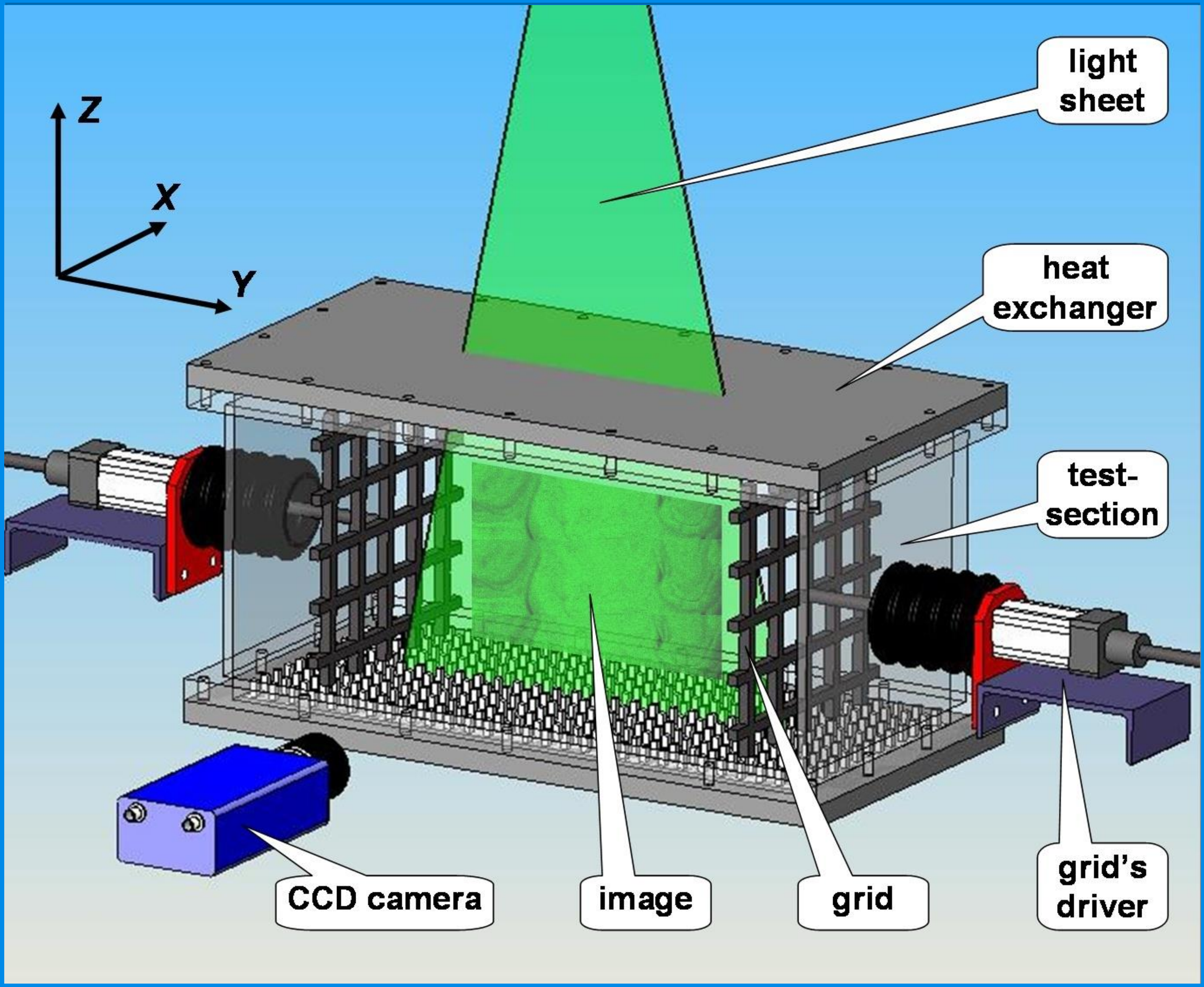


Forced turbulent convection
(two oscillating grids)



Unforced convection

Experimental Set–up for Forced Turbulent Convection



Problems

- The Rayleigh numbers based on the molecular transport coefficients are very large:

$$Ra = \frac{g\,\beta\,\Delta T\,L^3}{\nu\;\kappa} \approx 10^{11} \div 10^{13}$$

  This corresponds to fully developed turbulent convection in atmospheric and laboratory flows.

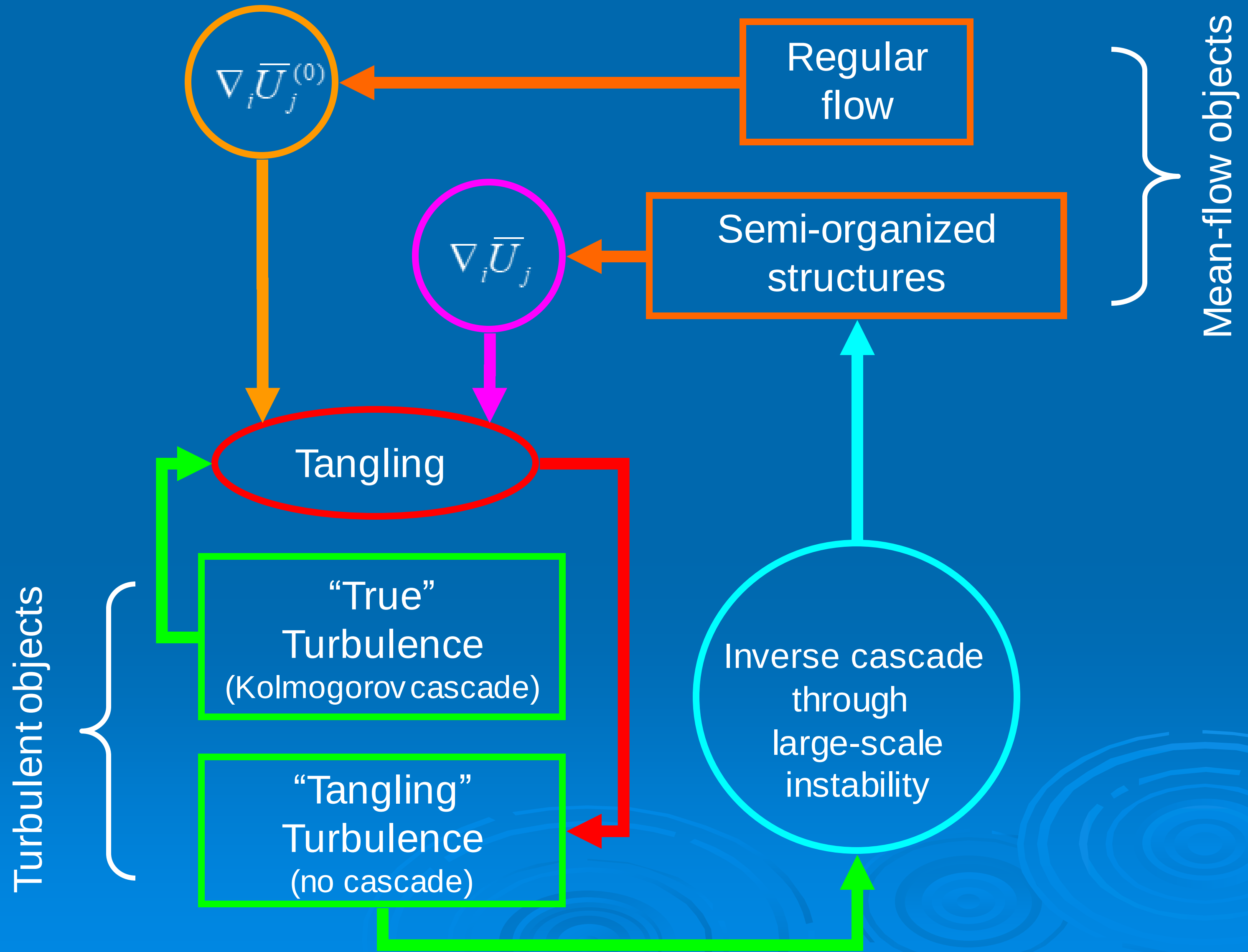
- The effective Rayleigh numbers based on the turbulent transport coefficients (the turbulent viscosity and turbulent diffusivity) are not high.

$$Ra^{eff} = \frac{g\,\beta\,\Delta T\,L^3}{\nu_T\;\kappa_T}$$

  They are less than the critical Rayleigh numbers required for the excitation of large-scale convection.

Hence the emergence of large-scale convective flows (which are observed in the atmospheric and laboratory flows) seems puzzling.

Interaction between mean-flow and turbulent objects



Tangling turbulence in sheared mean flow

Sheared mean flow

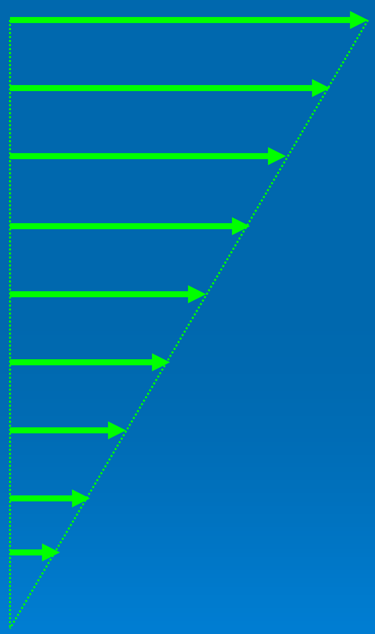
$$\nabla \overline{U}^{(0)} \neq 0$$

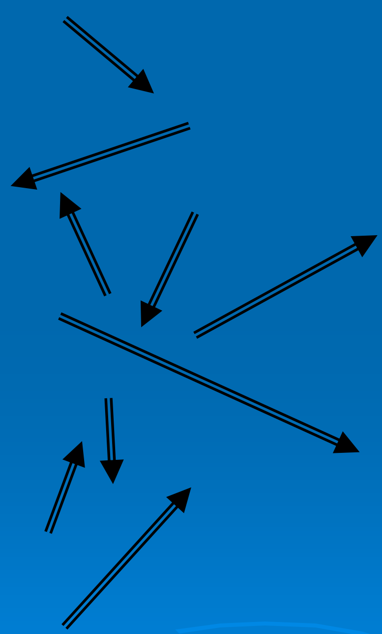
Kolmogorov turbulence

$$\vec{u}$$

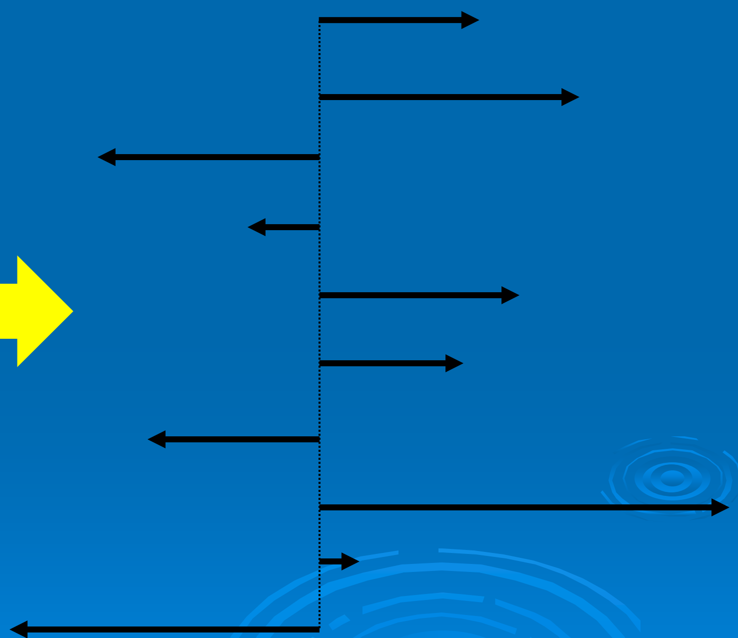
"Tangling" turbulence

$$\delta u \propto (\vec{u} \cdot \vec{\nabla})\, \overline{U}^{(0)}$$



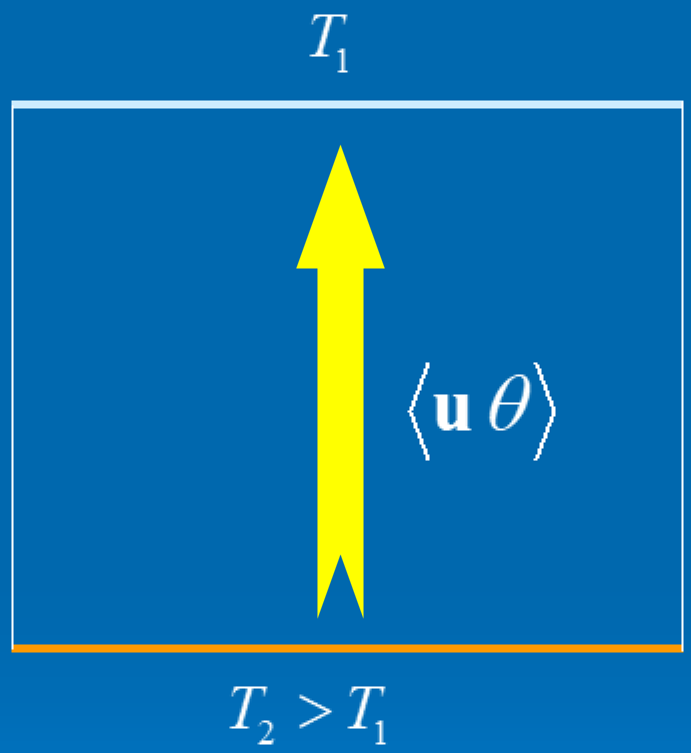


$$k^{-5/3}$$



Lumley (1967) $k^{-7/3}$

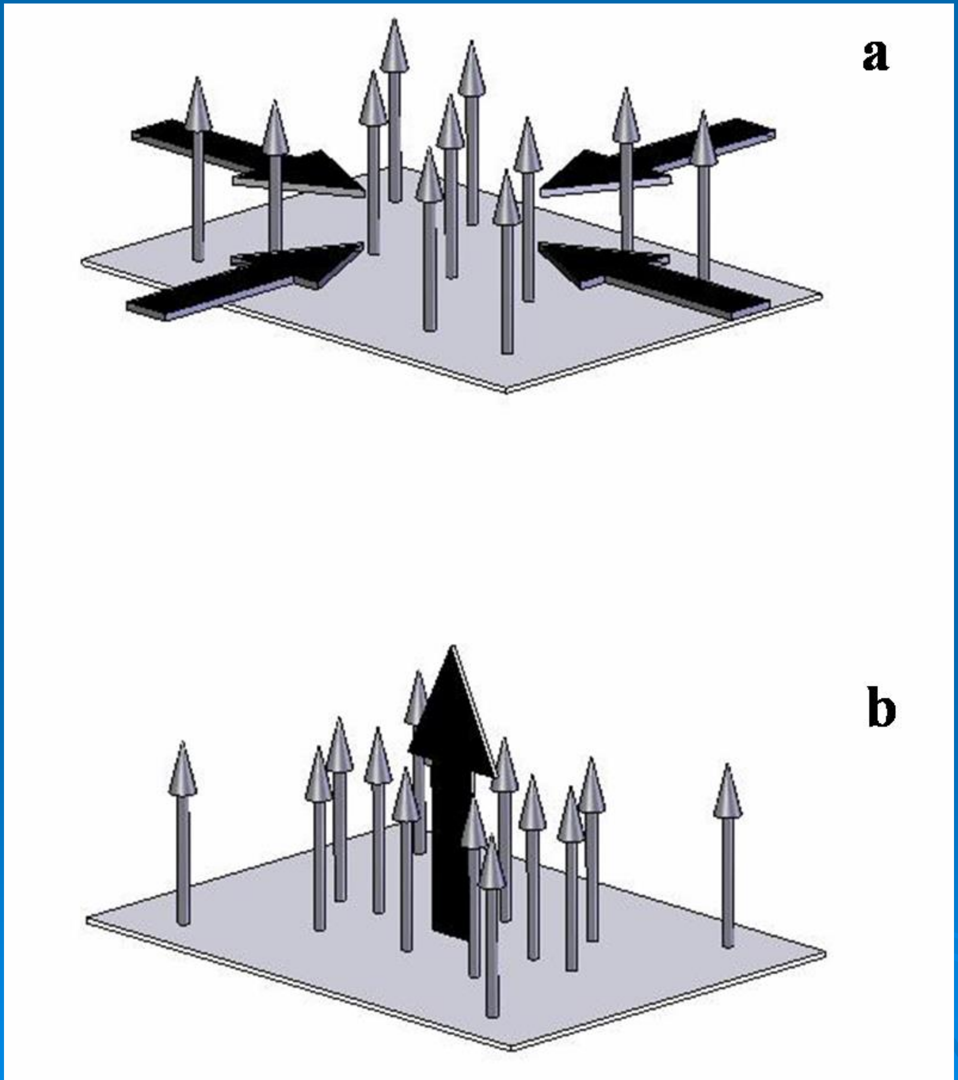
Heat flux



$$\langle \mathbf{u}\,\theta \rangle = -\kappa_T \vec{\nabla} \overline{\Theta}$$

$$\kappa_T \cong \frac{u_0\, l_0}{3}$$

Redistribution of a homogeneous vertical turbulent heat flux by a converging horizontal mean flow



$$\langle \theta \mathbf{u} \rangle = -\kappa_T \vec{\nabla} \overline{\Theta}$$

$$\vec{\nabla} \cdot \overline{\mathbf{U}}_\perp < 0$$

$$\langle \theta \mathbf{u} \rangle \approx -\kappa_T \vec{\nabla} \overline{\Theta} \left[1 - \tau_0 \left(\vec{\nabla} \cdot \overline{\mathbf{U}}_\perp \right) \right]$$

Heat flux

Traditional parameterization

$$\left\langle \theta\, \mathbf{u} \right\rangle = -\kappa_T \vec{\nabla} \overline{\Theta}$$

Modification of the heat flux by tangling turbulence

$$\left\langle \theta\, \mathbf{u} \right\rangle \approx -\kappa_T \vec{\nabla} \overline{\Theta} \left[1 - \tau_0 \left(\vec{\nabla} \cdot \overline{\mathbf{U}}_{\perp} \right) \right]$$

Mean field equations

$$\left(\frac{\partial}{\partial t}+\overline{\mathbf{U}}\cdot\vec{\nabla}\right)\overline{U}_i=-\nabla_i\left(\frac{\overline{P}}{\rho_0}\right)-\nabla_j\left\langle u_i u_j\right\rangle-g_i\overline{\Theta}+\nu\Delta\overline{\mathbf{U}},$$

$$\left(\frac{\partial}{\partial t}+\overline{\mathbf{U}}\cdot\vec{\nabla}\right)\overline{\Theta}=-\nabla_i\left\langle\theta\, u_i\right\rangle+\kappa\Delta\overline{\Theta}$$

$\mathbf{F}\equiv\left\langle\theta\,\mathbf{u}\right\rangle$ is the heat flux

$\left\langle u_i u_j\right\rangle$ are the Reynolds stresses

Method of Derivation

Equations for the correlation functions for:

- **The velocity fluctuations** $(M_{ij}^{(II)})_u \equiv \langle u_i u_j \rangle$
- **The temperature fluctuations** $M_\theta^{(II)} \equiv \langle \theta\, \theta \rangle$
- **The heat flux** $(M_i^{(II)})_\Phi \equiv \langle \theta\, u_i \rangle$

The spectral τ-approximation (the third-order closure procedure)

$$\hat{D}M^{(III)}(\mathbf{k}) - \hat{D}M_K^{(III)}(\mathbf{k}) = -\frac{M^{(II)}(\mathbf{k}) - M_K^{(II)}(\mathbf{k})}{\tau_c(\mathbf{k})}$$

$$\left(\hat{D}M_{ij}^{(III)}\right)_u = -\langle u_j(\mathbf{u}\cdot\nabla)u_i \rangle - \langle u_i(\mathbf{u}\cdot\nabla)u_j \rangle$$

$$\left(\hat{D}M_{ij}^{(III)}\right)_\theta = -2\langle \theta(\mathbf{u}\cdot\nabla)\theta \rangle$$

$$\left(\hat{D}M_i^{(III)}\right)_\Phi = -\langle u_i(\mathbf{u}\cdot\nabla)\theta \rangle - \langle \theta(\mathbf{u}\cdot\nabla)u_i \rangle$$

Heat flux

Traditional parameterization

$$\langle \theta \mathbf{u} \rangle = -\kappa_T \vec{\nabla} \overline{\Theta}$$

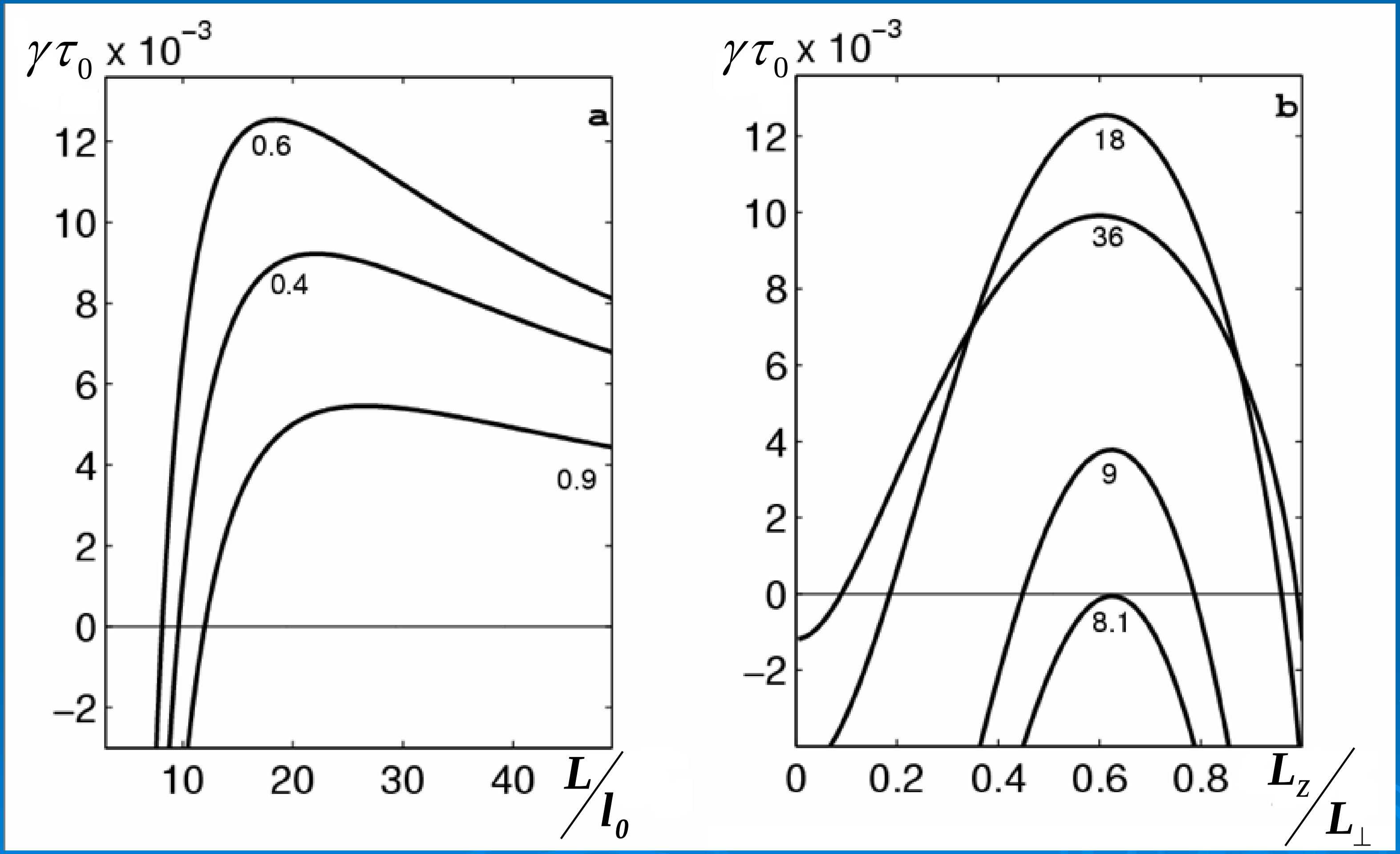
Modification of the heat flux by tangling turbulence

$$\langle \theta \mathbf{u} \rangle = \mathbf{F}^* + \frac{\tau_0}{6}\left[-5\alpha \left(\vec{\nabla} \cdot \bar{\mathbf{U}}_{\perp} \right) \mathbf{F}_z^* + \left(\alpha + \frac{3}{2} \right) \left(\bar{\mathbf{W}} \times \mathbf{F}_z^* \right) + 3 \left(\bar{\mathbf{W}}_z \times \mathbf{F}^* \right) \right]$$

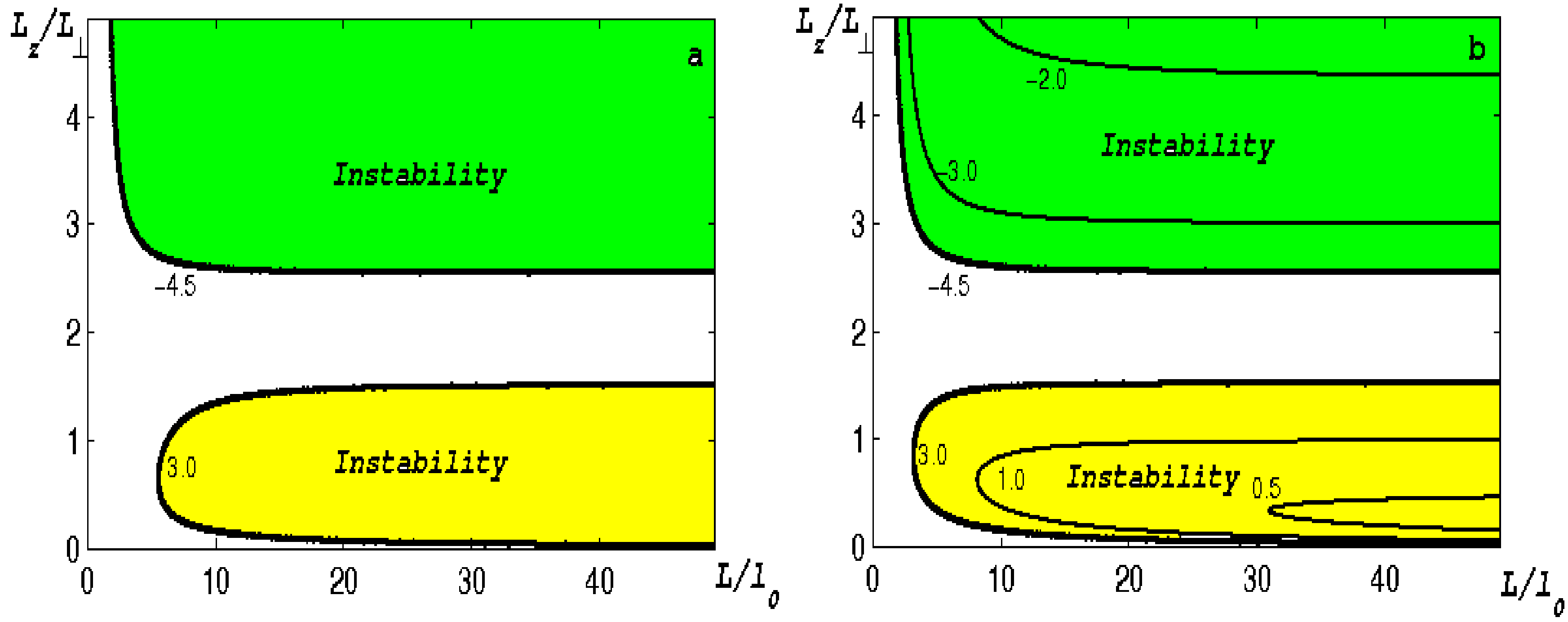
$$\mathbf{F}^* = -\kappa_T \vec{\nabla} \overline{\Theta} - \tau_0 \left(\mathbf{F}_z^* \cdot \vec{\nabla} \right) \bar{\mathbf{U}}^{(0)}(z)$$

$$\overline{\mathbf{W}} = \vec{\nabla} \times \overline{\mathbf{U}}$$

The growth rate of convective wind instability

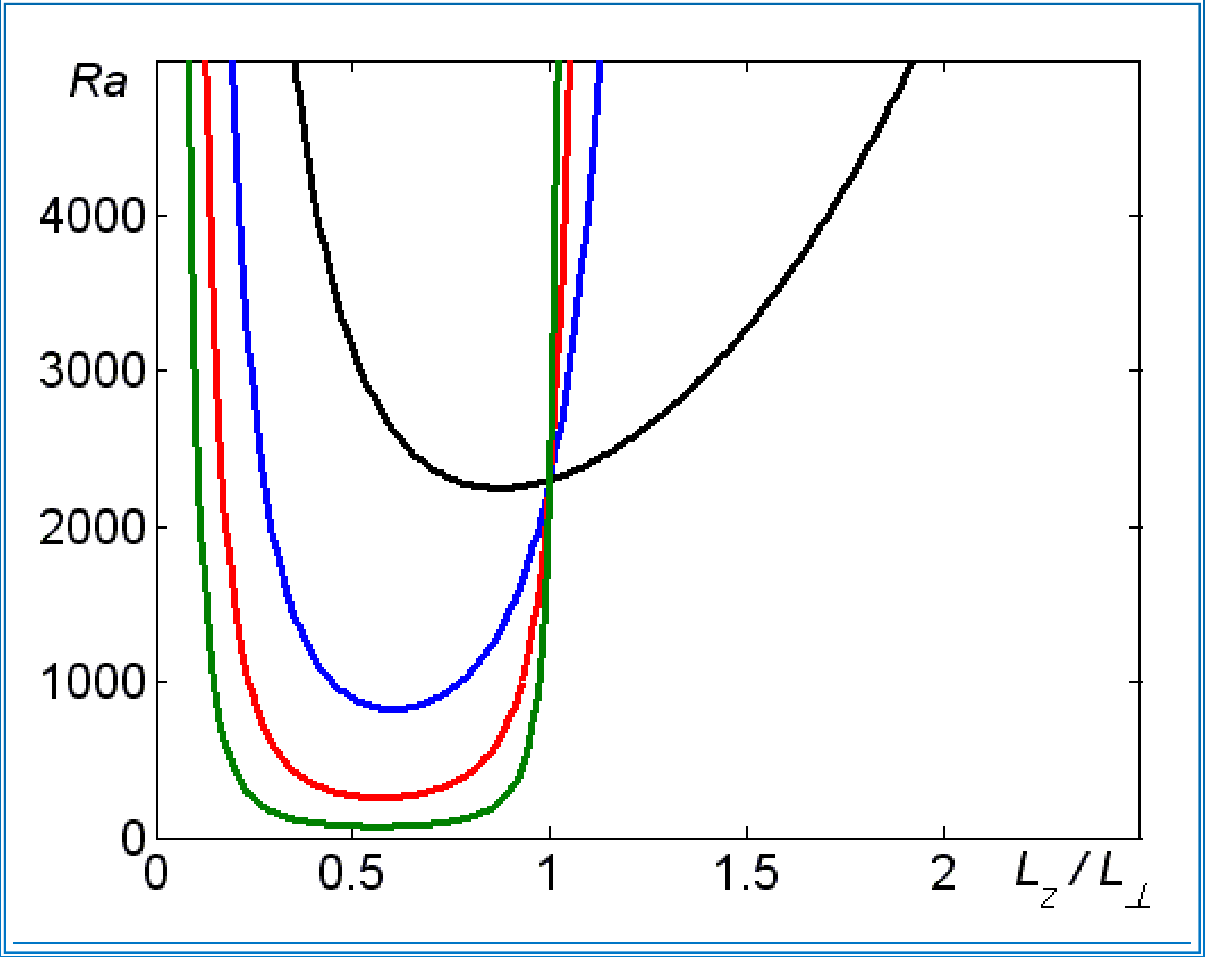


Convective-wind instability



The range of parameters for which the convective-wind instability occurs for different anisotropy of turbulence.

Critical Rayleigh Number



$Ra^{cr} = 2247$

$\mu = 0.7$ $Ra^{cr} = 826$

$\mu = 2$ $Ra^{cr} = 256$

$\mu = 5$ $Ra^{cr} = 72$

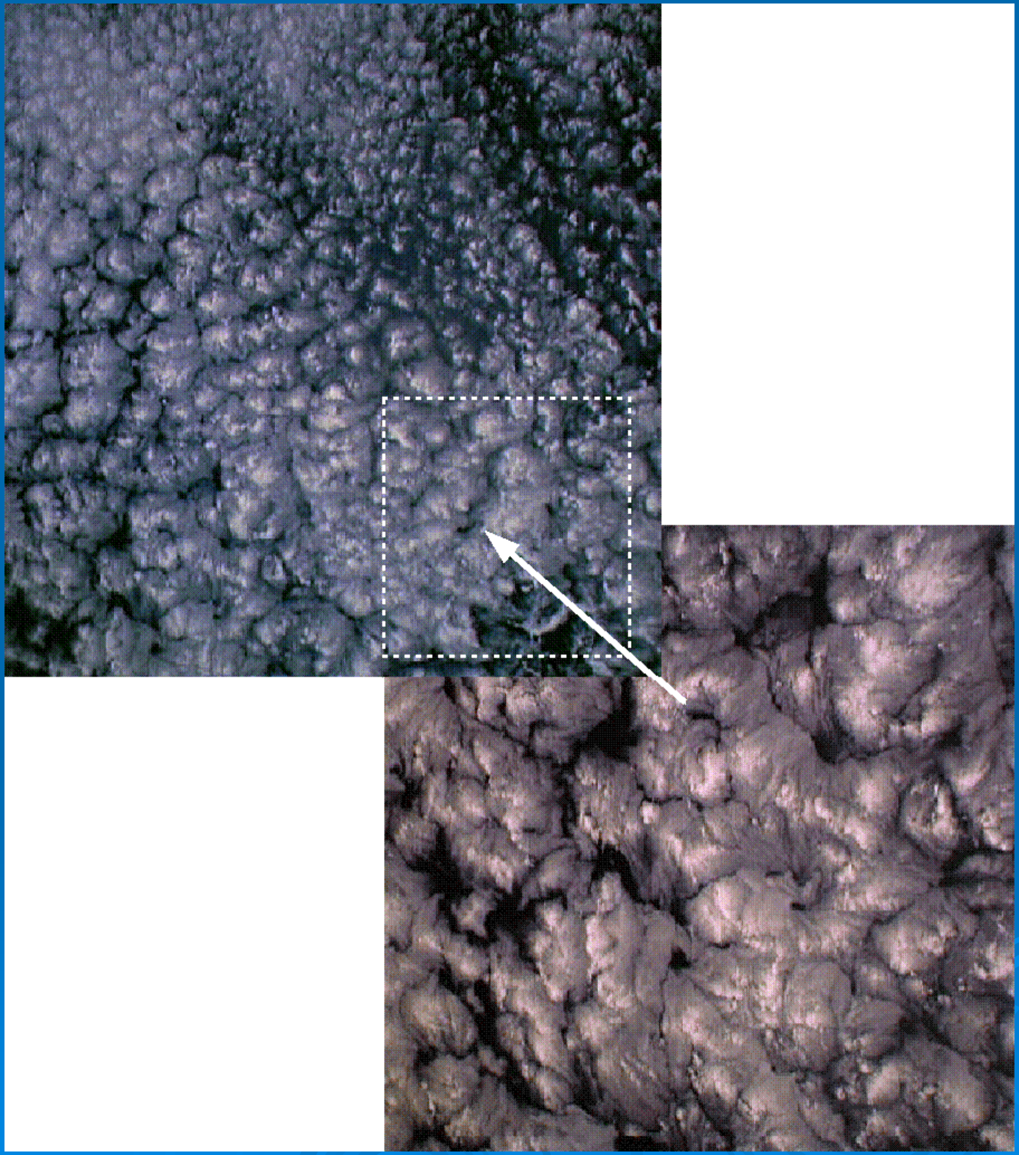
$$\mu = \frac{4 g \tau \langle u_z \, \theta \rangle}{L_z^2 \left| N^2 \right|} \left(\frac{\mathrm{Ra}}{\mathrm{Pr_T}} \right)^{1/3}$$

$$N^2 = -\mathbf{g} \cdot \vec{\nabla} \Theta$$

In laminar convection:

$Ra^{cr} = 657.5$

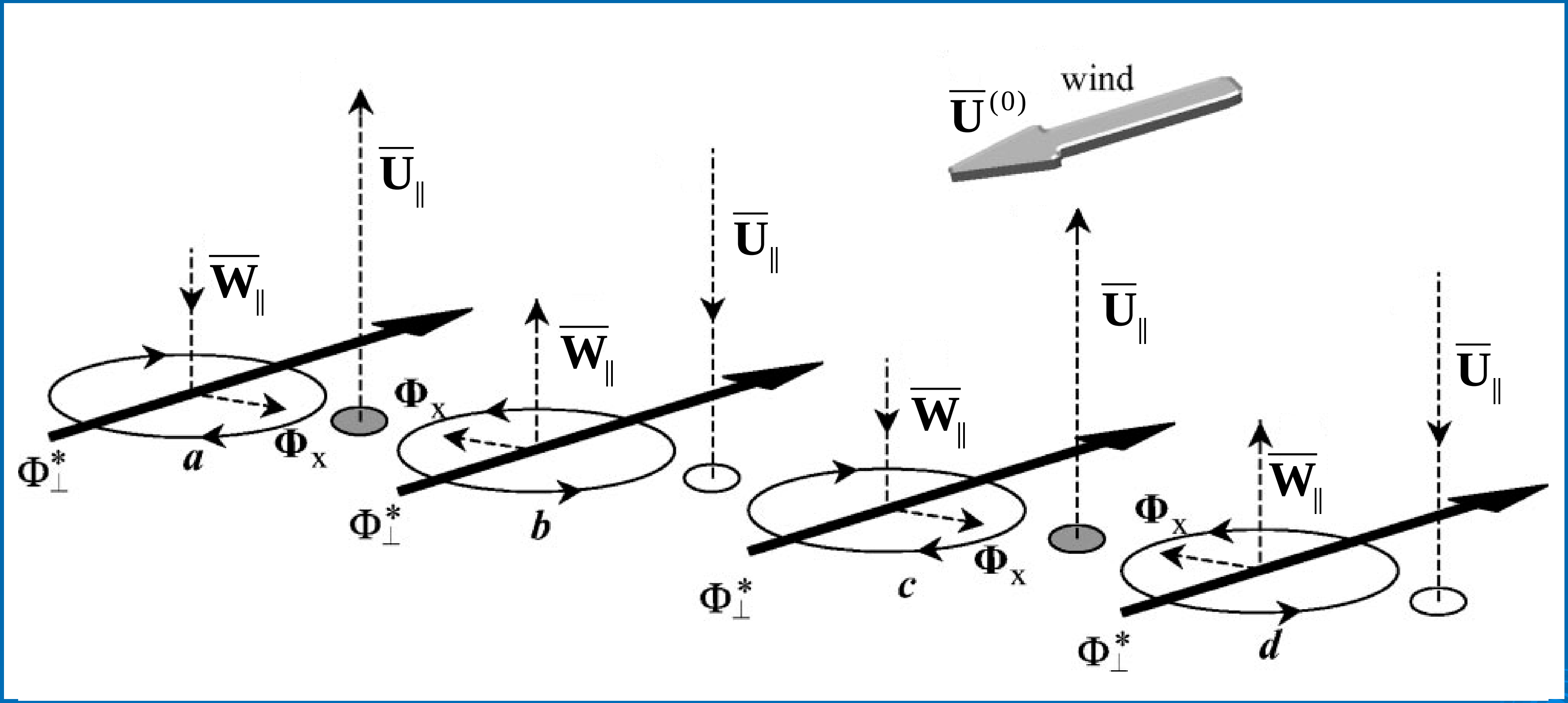
Closed Cloud Cells over the Atlantic Ocean



Cloud cells

	Observations	Theory
$L_z / L_\perp$	$0.05 \div 1$	$0 \div 1$
L / l_0	$5 \div 20$	$5 \div 15$
$T_{lifetime}$	Several hours	$\gamma^{-1} = (25 \div 100)\tau_0$ $= 1 \div 3h$

Mechanism of convective-shear instability

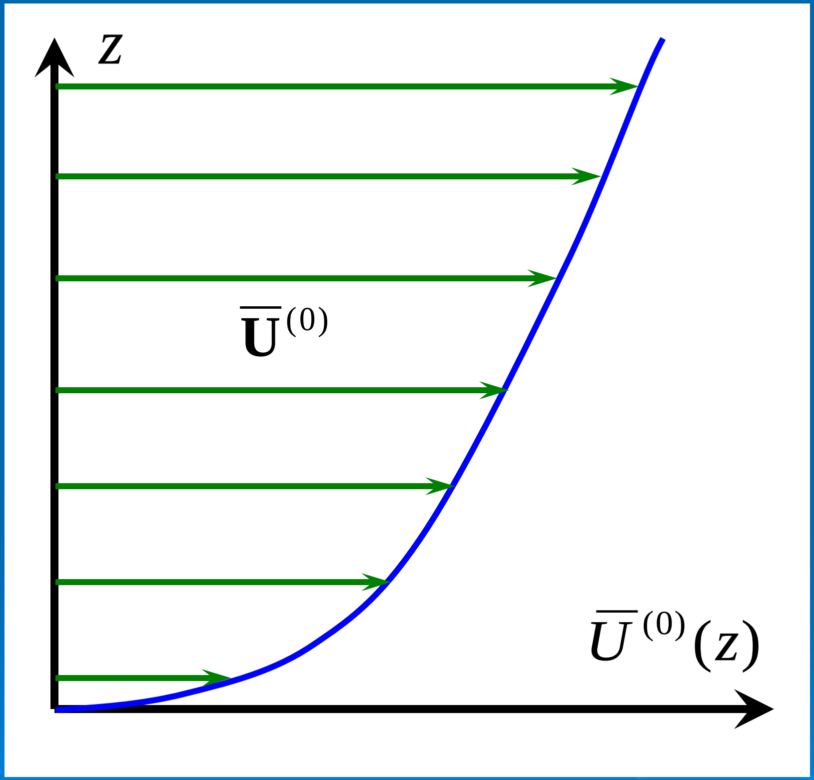


$$\mathbf{\Phi} = \langle \theta\, \mathbf{u} \rangle \propto \tau_0 \left(\bar{\mathbf{W}}_z \times \mathbf{\Phi}^* \right)$$

$$\mathbf{\Phi}^* = \langle \theta\, \mathbf{u} \rangle^* = -\kappa_T \vec{\nabla} \bar{\Theta} - \tau_0 \left(\mathbf{\Phi}_z^* \cdot \vec{\nabla} \right) \bar{\mathbf{U}}^{(0)}(z)$$

Counter wind flux

$$\mathbf{F}^* = \langle \theta \mathbf{u} \rangle^* = -\kappa_T \vec{\nabla} \bar{\Theta} - \tau_0 \left(\mathbf{F}_z^* \cdot \vec{\nabla} \right) \bar{\mathbf{U}}^{(0)}(z)$$



$$\frac{\partial \mathbf{u}}{\partial t} \propto -\left(\mathbf{u} \cdot \vec{\nabla} \right) \overline{\mathbf{U}}^{(0)} + \ldots$$

Tangling fluctuations

$$\delta \mathbf{u} \propto -\tau_0 \left(\mathbf{u} \cdot \vec{\nabla} \right) \overline{\mathbf{U}}^{(0)}$$

$$\langle \theta \, \delta \mathbf{u} \rangle \propto -\tau_0 \left(\mathbf{F}_z^* \cdot \nabla \right) \bar{\mathbf{U}}^{(0)}(z)$$

$$\mathbf{F}_z^* = \langle \theta \mathbf{u}_z \rangle$$

Method of Derivation

Equations for the correlation functions for:

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$$\left(\hat{D}M_{ij}^{(III)}\right)_u = -\langle u_j (\mathbf{u}\cdot\nabla) u_i \rangle - \langle u_i (\mathbf{u}\cdot\nabla) u_j \rangle$$

$$\left(\hat{D}M_{ij}^{(III)}\right)_\theta = -2\langle \theta (\mathbf{u}\cdot\nabla) \theta \rangle$$

$$\left(\hat{D}M_i^{(III)}\right)_\Phi = -\langle u_i (\mathbf{u}\cdot\nabla) \theta \rangle - \langle \theta (\mathbf{u}\cdot\nabla) u_i \rangle$$

Heat flux

Traditional parameterization

$$\langle \theta \mathbf{u} \rangle = -\kappa_T \vec{\nabla} \overline{\Theta}$$

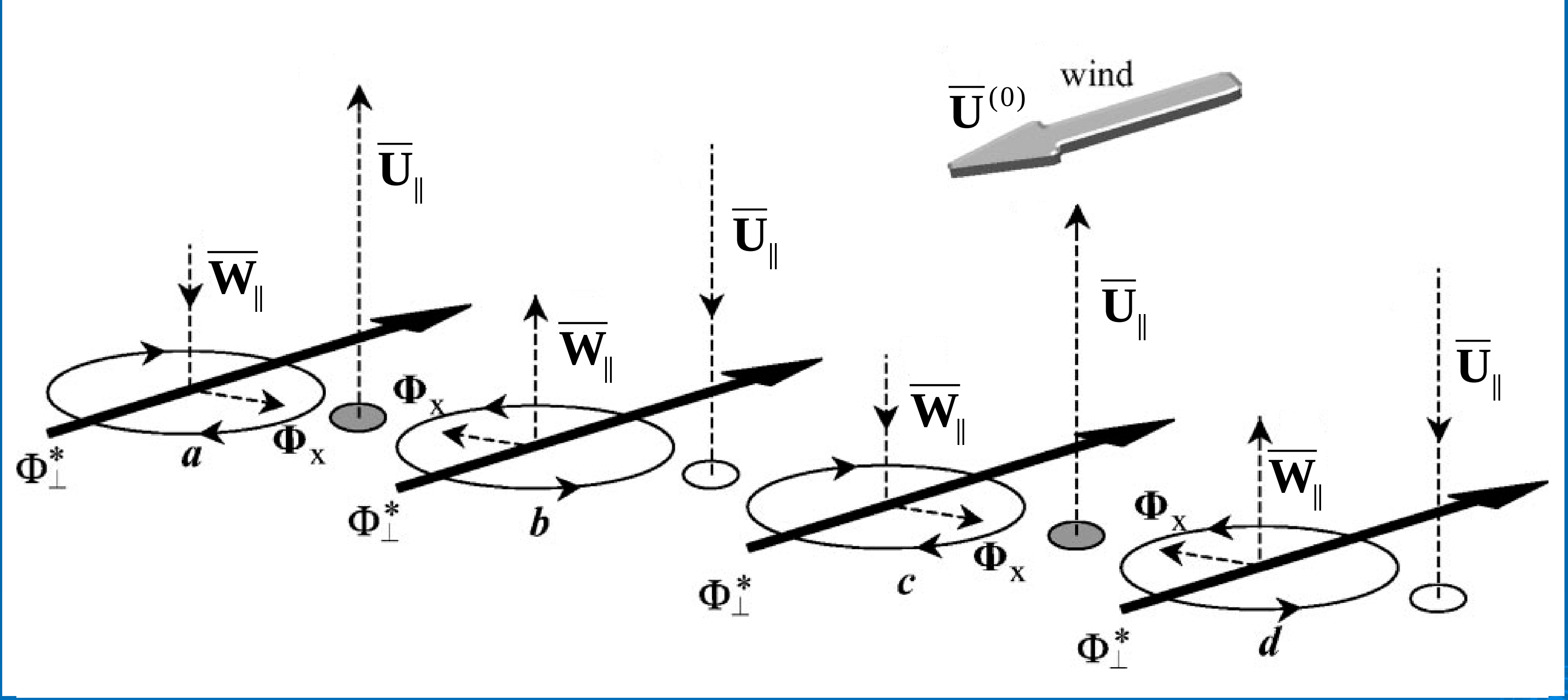
Modification of the heat flux by tangling turbulence

$$\langle \theta \mathbf{u} \rangle = \mathbf{F}^* + \frac{\tau_0}{6}\left[-5\alpha \left(\vec{\nabla} \cdot \bar{\mathbf{U}}_\perp \right) \mathbf{F}_z^* + \left(\alpha + \frac{3}{2} \right) \left(\bar{\mathbf{W}} \times \mathbf{F}_z^* \right) + 3 \left(\bar{\mathbf{W}}_z \times \mathbf{F}^* \right) \right]$$

$$\mathbf{F}^* = -\kappa_T \vec{\nabla} \overline{\Theta} - \tau_0 \left(\mathbf{F}_z^* \cdot \vec{\nabla} \right) \bar{\mathbf{U}}^{(0)}(z)$$

$$\overline{\mathbf{W}} = \vec{\nabla} \times \overline{\mathbf{U}}$$

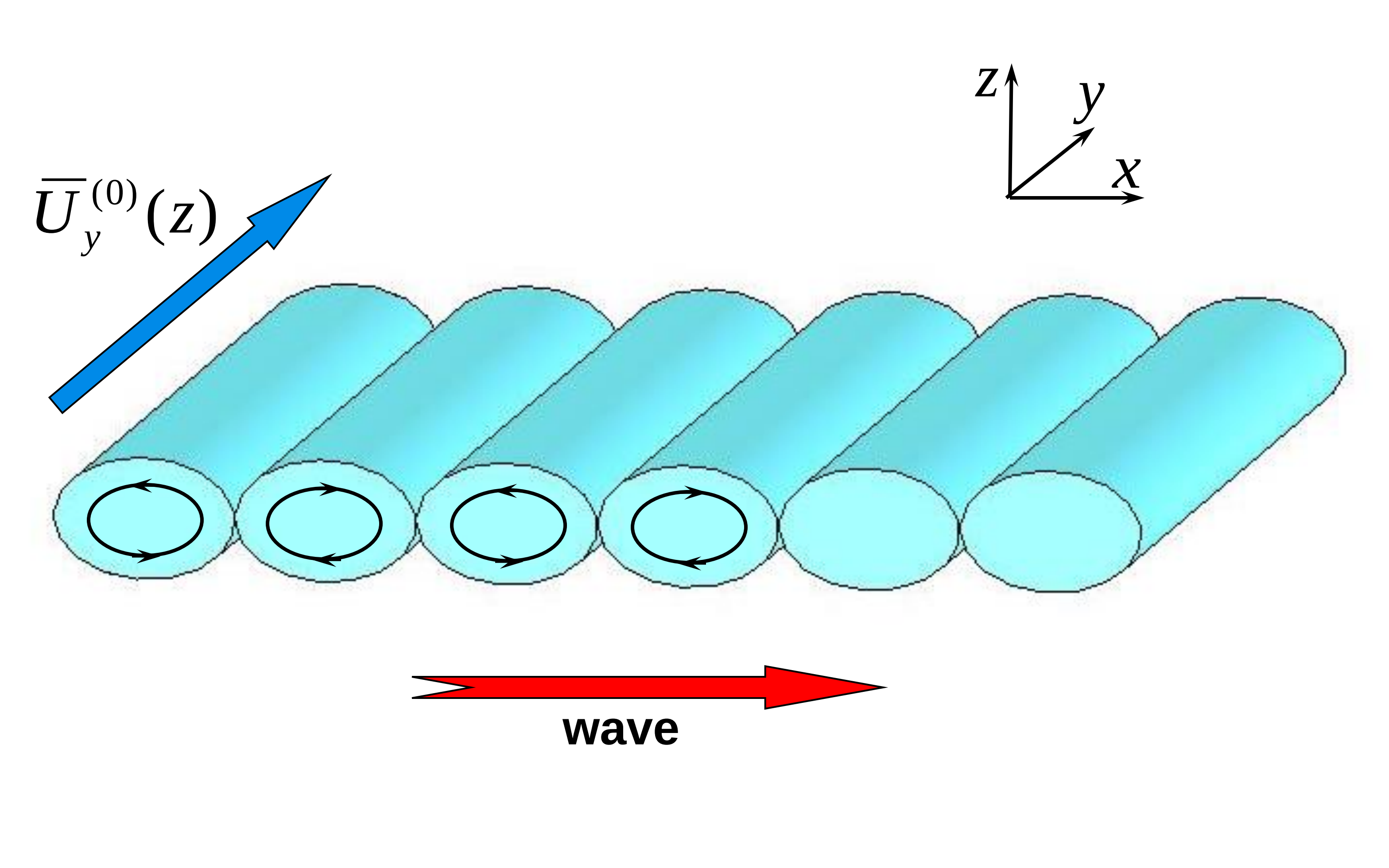
Mechanism of convective-shear instability



$$\mathbf{\Phi} = \langle \theta \mathbf{u} \rangle \propto \tau_0 \left(\bar{\mathbf{W}}_z \times \mathbf{\Phi}^* \right)$$

$$\mathbf{\Phi}^* = \langle \theta \mathbf{u} \rangle^* = -\kappa_T \vec{\nabla} \bar{\Theta} - \tau_0 \left(\mathbf{\Phi}_z^* \cdot \vec{\nabla} \right) \bar{\mathbf{U}}^{(0)}(z)$$

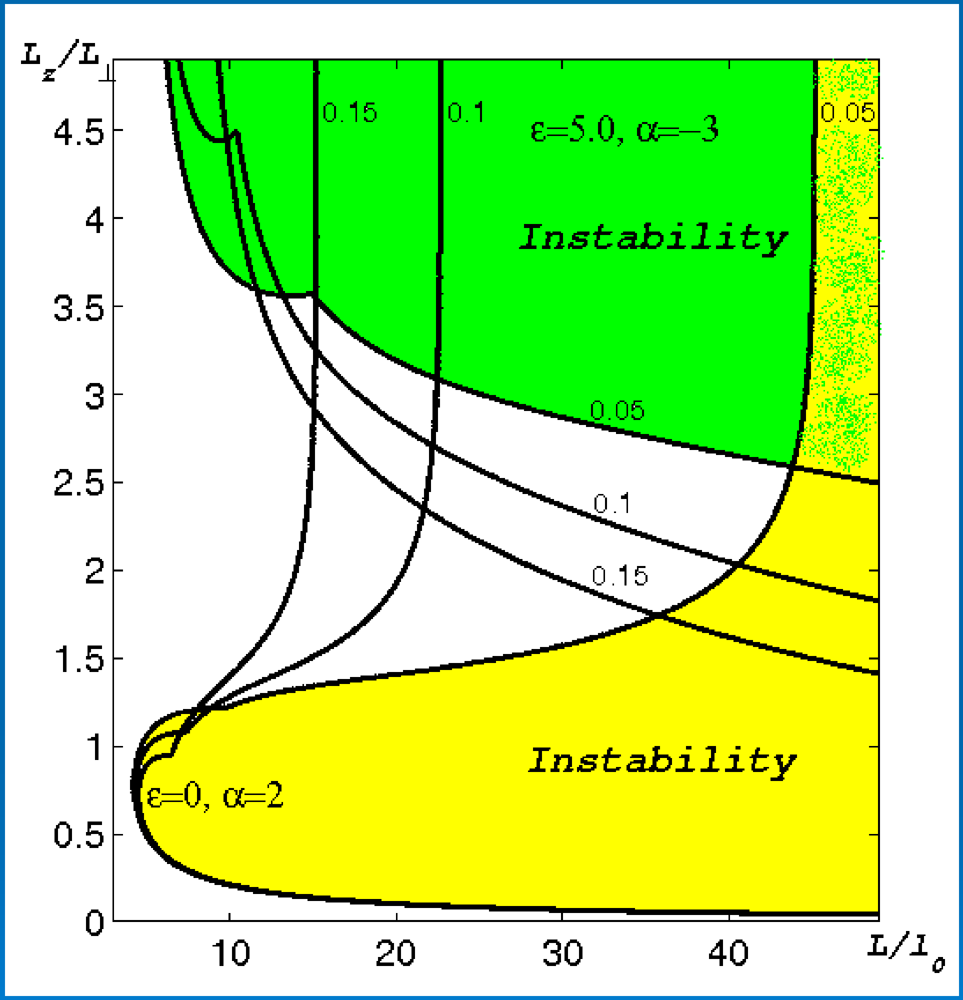
Convective-shear waves



$$\overline{W}_z \propto \cos\left(\omega t - K x\right)$$

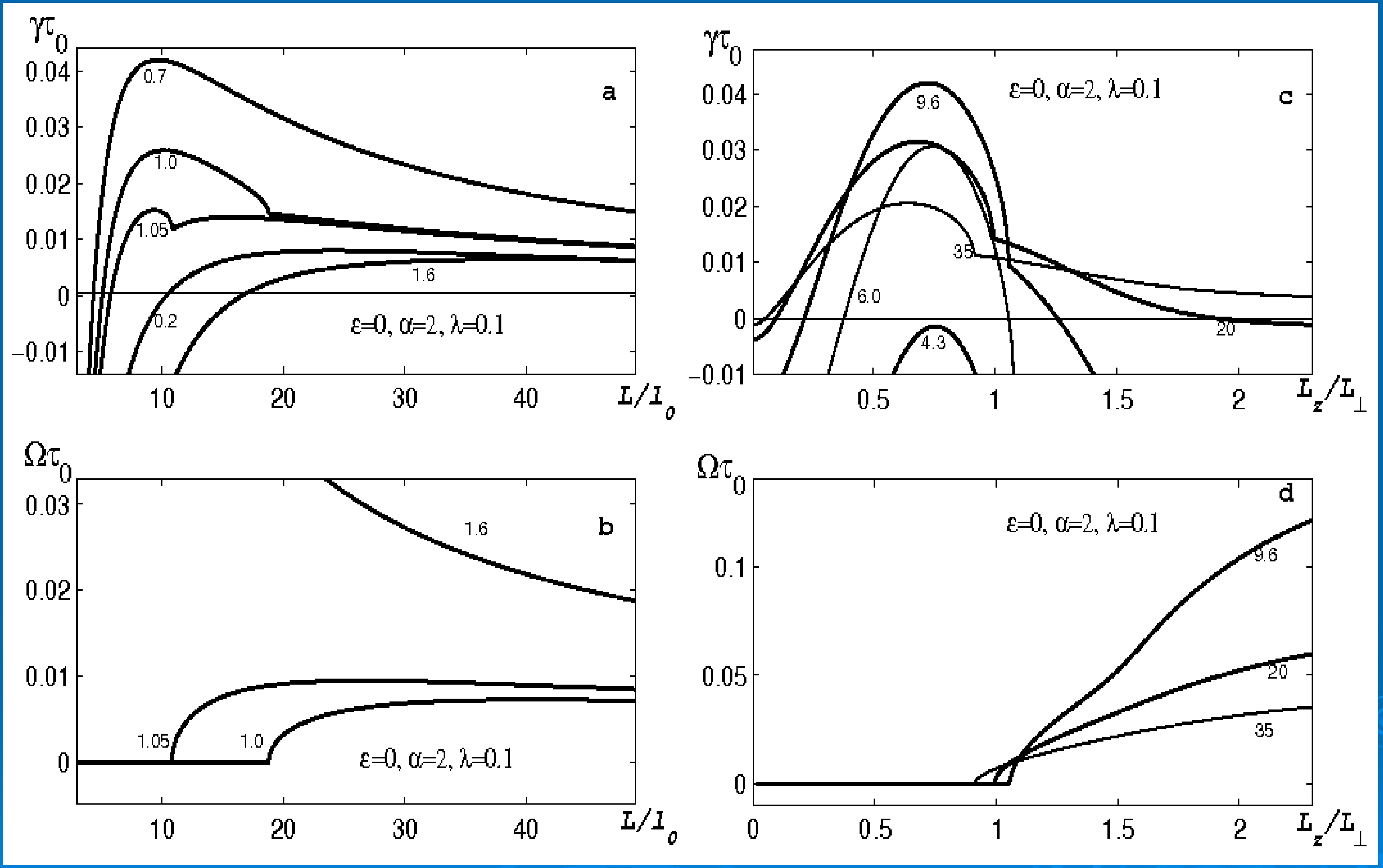
$$\overline{U}_z \propto \cos\left(\omega t - K x - \frac{\pi}{6}\right) \qquad \overline{\Theta} \propto \cos\left(\omega t - K x + \frac{\pi}{6}\right)$$

Convective-shear instability



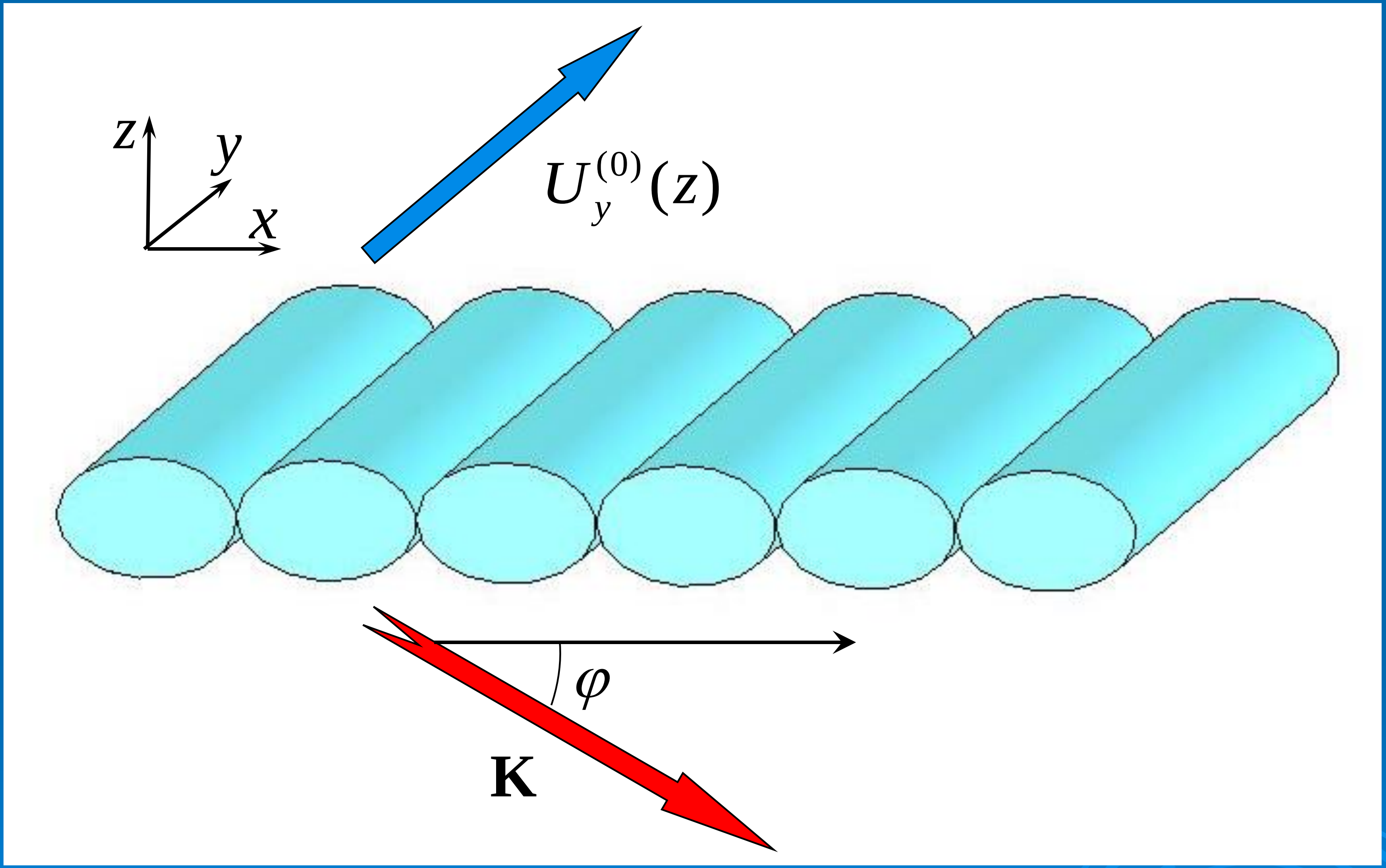
The range of parameters for which the convective-shear instability occurs for different values of shear and anisotropy.

Convective-shear instability

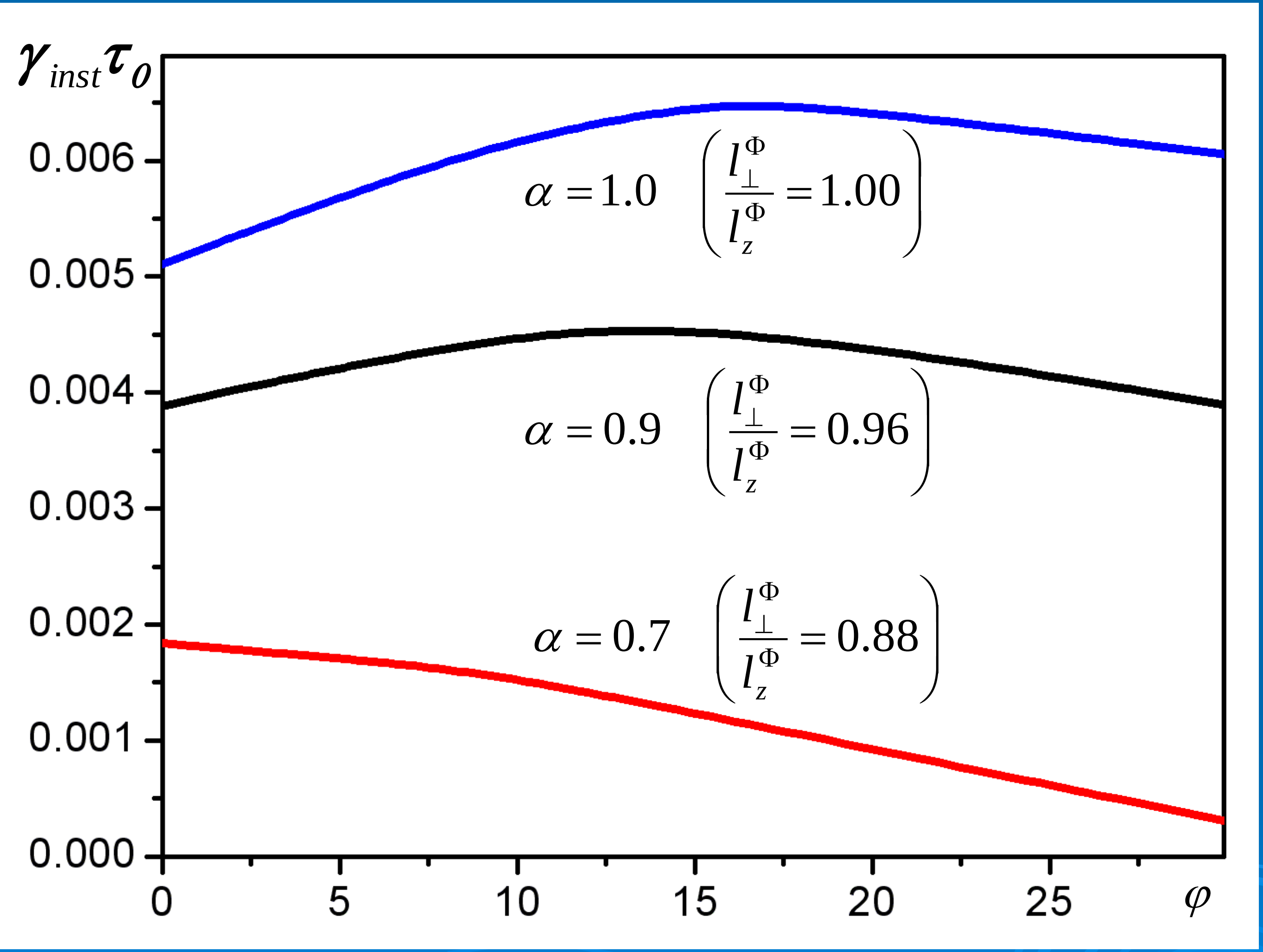


The growth rate of the convective-shear instability and frequencies of the generated convective-shear waves.

Convective-shear instability

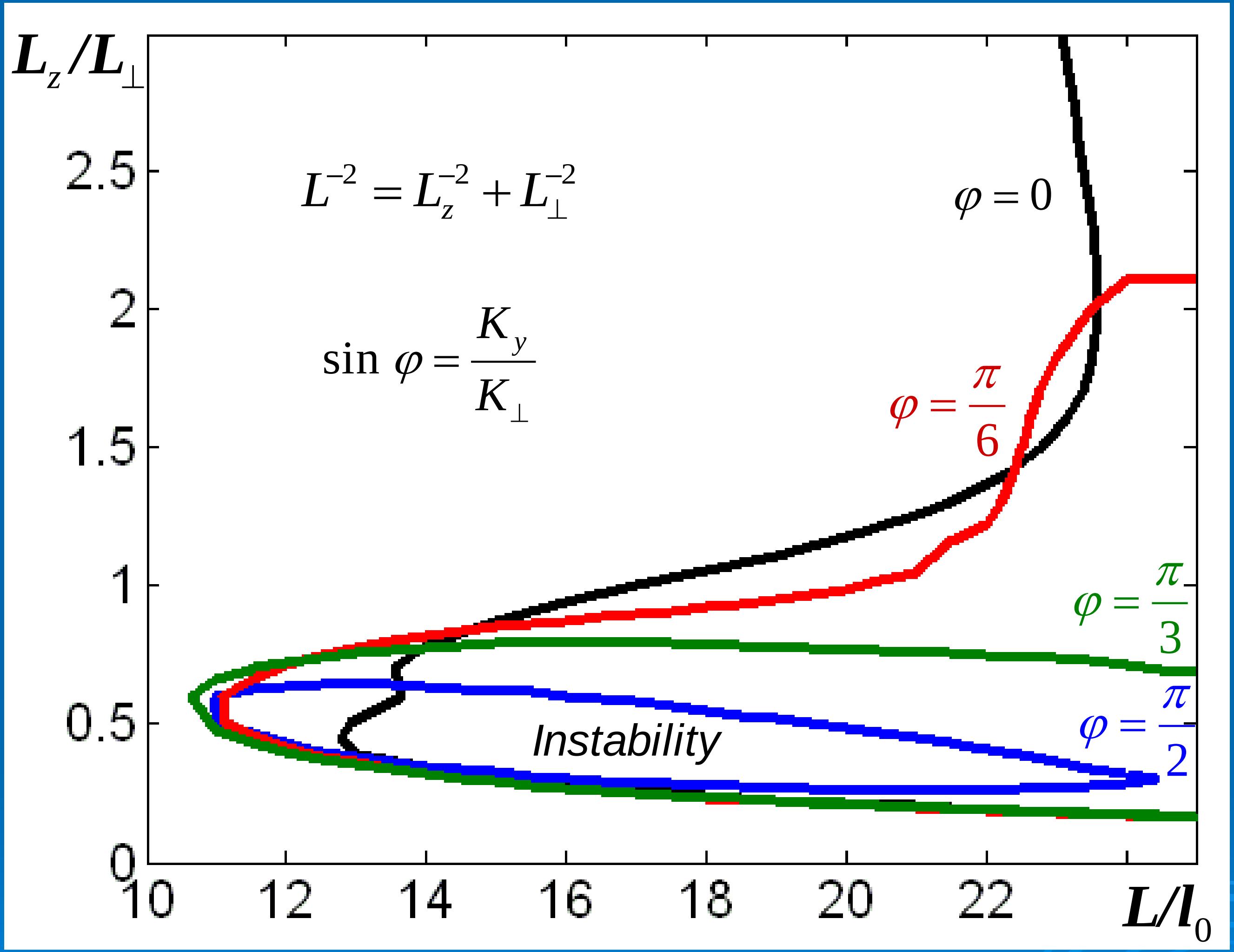


Maximum growth rate



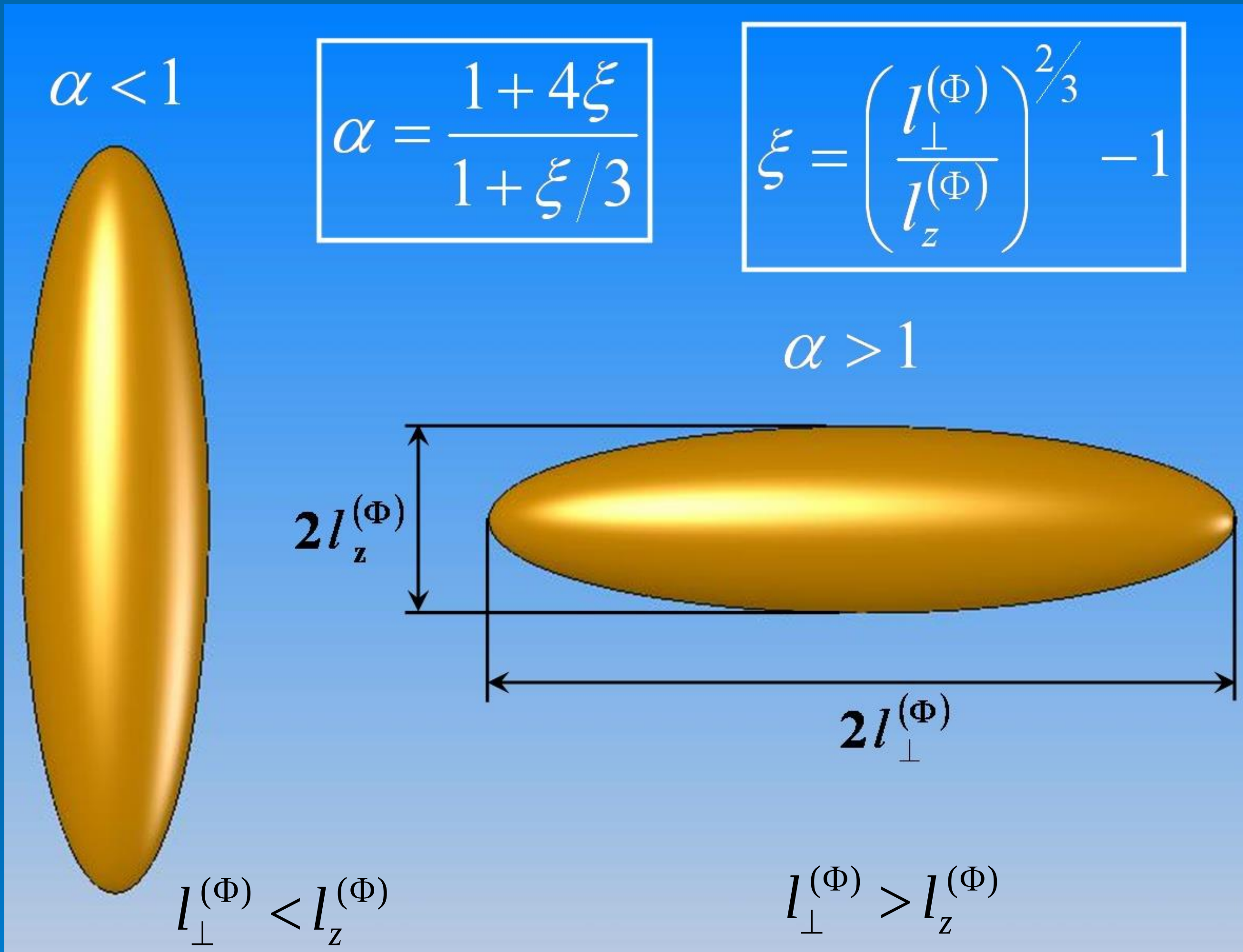
The growth rate of the convective shear instability
for different thermal anisotropy

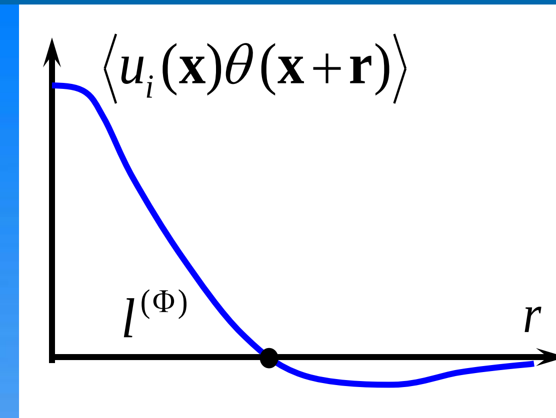
Conditions for the instability



The range of parameters $L_z / L_\perp$ and L / l_0 for which the convective shear instability occurs

Thermal anisotropy

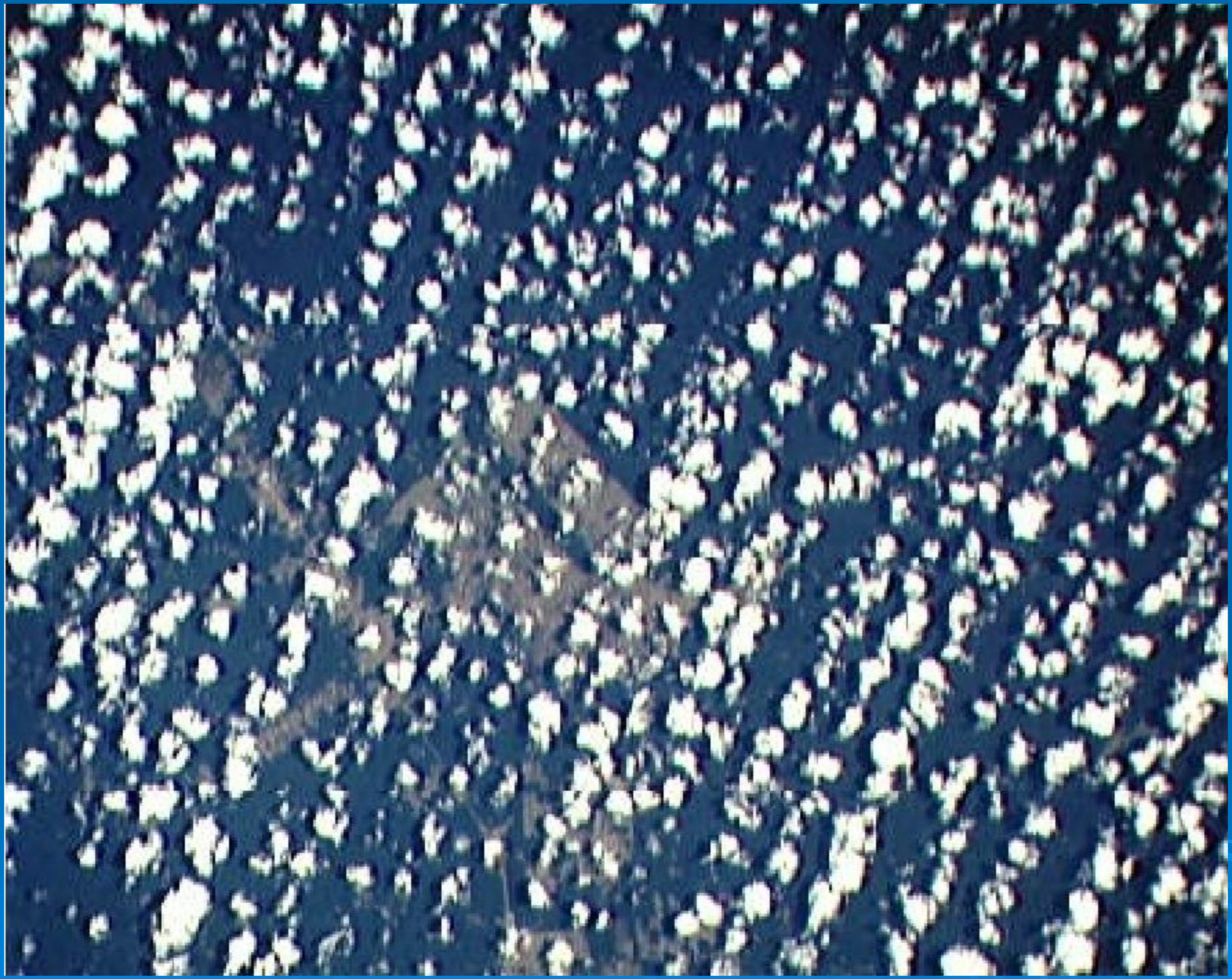




"Column"-like thermal structure

"Pancake"-like thermal structure

Cloud "streets" over the Amazon River



Cloud "streets"

	Observations	Theory
$L_z / L_\perp$	$0.14 \div 1$	$0 \div 1$
L / l_0	$10 \div 100$	$10 \div 100$
$T_{lifetime}$	$1 \div 72\ h$	$\gamma^{-1} = (25 \div 100)\tau_0$ $= 1 \div 3\,h$

The atmospheric convective boundary layer (CBL) consists in three basic parts:

- ***Surface layer*** strongly unstably stratified and dominated by small-scale turbulence of very complex nature including usual 3-D turbulence, generated by mean-flow surface shear and structural shears **(*the lower part of the surface layer*)**, and unusual strongly anisotropic buoyancy-driven turbulence **(the *upper part of the surface layer*)**;

- ***CBL core*** dominated by the structural energy-, momentum- and mass-transport, with only minor contribution from usual 3-D turbulence generated by local structural shears on the background of almost zero vertical gradient of potential temperature (or buoyancy);

- ***Turbulent entrainment layer* at the CBL upper boundary**, characterised by essentially stable stratification with negative (downward) turbulent flux of potential temperature (or buoyancy).

Budget Equations for Shear-Free Convection

$$\frac{DE_K}{Dt} + \nabla \cdot \mathbf{\Phi}_K = -\tau_{ij} \frac{\partial U_i}{\partial x_j} + \beta F_z - \varepsilon_K ,$$

$$\frac{DE_\theta}{Dt} + \nabla \cdot \mathbf{\Phi}_\theta = -(\mathbf{F} \cdot \nabla)\Theta - F_z \frac{\partial \overline{\Theta}}{\partial z} - \varepsilon_\theta ,$$

$$\frac{DF_i}{Dt} + \frac{\partial}{\partial x_j} \Phi_{ij}^{(F)} = \beta \delta_{i3} \langle \theta^2 \rangle - \frac{1}{\rho_0} \langle \theta \nabla_i p \rangle - \tau_{ij} \frac{\partial (\Theta + \overline{\Theta})}{\partial x_j} - F_j \frac{\partial U_i}{\partial x_j} - \varepsilon_i^{(F)} ,$$

$$\tau_{ij} \equiv \langle u_i u_j \rangle = \delta_{ij} \frac{\langle \mathbf{u}^2 \rangle}{3} - K_M (\nabla_i U_j + \nabla_j U_i),$$

$$\varepsilon_K = \frac{E_K}{t_T}, \quad \varepsilon_i^{(F)} = \frac{F_i}{C_F t_T}, \quad \varepsilon_\theta = \frac{E_\theta}{C_P t_T},$$

CBL-Core for Shear-Free Convection

$$\frac{D\boldsymbol{\omega}}{Dt} = K_M \Delta\,\boldsymbol{\omega} - \beta\left(\mathbf{e}\times\nabla\right)\Theta + (\boldsymbol{\omega}\cdot\nabla)\,\mathbf{U}\,,$$

$$\frac{D\Theta}{Dt} = -\nabla\cdot\mathbf{F}\ ,$$

$$\boldsymbol{\omega} = \nabla\times\mathbf{U}$$

$$D/Dt = \partial/\partial t + U_k\partial/\partial x_k\ ,\quad \beta = g/T_0$$

$$\nabla\cdot\mathbf{F} = -t_T\sigma F_z^{*}\left(\mu\Delta_h - \Delta_z\right)U_z - K_H\,\Delta\Theta\,,$$

Solution for Cloud Cells (CBL-core)

$$U_r = -A_* U_{z0} J_1\left(\lambda \frac{r}{R}\right)\cos\left(\frac{\pi z}{L_z}\right),$$

$$U_z = U_{z0} J_0\left(\lambda \frac{r}{R}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\Theta = \Theta_0 J_0\left(\lambda \frac{r}{R}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\boldsymbol{\omega} = \mathbf{e}_\varphi \lambda \frac{U_{z0}}{R}\left(1 + A_*^2\right) J_1\left(\lambda \frac{r}{R}\right)\sin\left(\frac{\pi z}{L_z}\right).$$

$$A_* = \pi R / \lambda L_z$$

$$\frac{U_{z0}}{\Theta_0} = \frac{\beta L_z^2}{\pi^2 K_M} \frac{A_*^2}{\left(1 + A_*^2\right)^2},$$

$$\frac{K_M^2}{\beta F_z t_T R^2 \Pr_T} = \frac{\sigma}{\lambda^2} \frac{A_*^2 - \mu}{\left(1 + A_*^2\right)^3}.$$

Budget Equations for Shear-Free Convection

$$\frac{DE_K}{Dt} + \nabla \cdot \mathbf{\Phi}_K = -\tau_{ij} \frac{\partial U_i}{\partial x_j} + \beta F_z - \varepsilon_K ,$$

$$\frac{DE_\theta}{Dt} + \nabla \cdot \mathbf{\Phi}_\theta = -(\mathbf{F} \cdot \nabla)\Theta - F_z \frac{\partial \overline{\Theta}}{\partial z} - \varepsilon_\theta ,$$

$$\frac{DF_i}{Dt} + \frac{\partial}{\partial x_j} \Phi_{ij}^{(F)} = \beta \delta_{i3} \langle \theta^2 \rangle - \frac{1}{\rho_0} \langle \theta \nabla_i \, p \rangle - \tau_{ij} \frac{\partial (\Theta + \overline{\Theta})}{\partial x_j} - F_j \frac{\partial U_i}{\partial x_j} - \varepsilon_i^{(F)} ,$$

$$\tau_{ij} \equiv \langle u_i \, u_j \rangle = \delta_{ij} \frac{\langle \mathbf{u}^2 \rangle}{3} - K_M \left(\nabla_i U_j + \nabla_j U_i \right),$$

$$\varepsilon_K = \frac{E_K}{t_T}, \quad \varepsilon_i^{(F)} = \frac{F_i}{C_F \, t_T}, \quad \varepsilon_\theta = \frac{E_\theta}{C_P \, t_T},$$

EFB-Theory for CBL-Core for Shear-Free Convection

$$\frac{E_K}{E_U} = 3C_\tau A_z \left(\frac{l}{L_z}\right)^2 \frac{\Phi_1(A_*)}{1-\hat{F}},$$

$$\hat{F} \equiv \frac{\beta F_z t_T}{E_K} = \frac{(2\pi C_\tau A_z)^2}{\sigma \, \mathrm{Pr}_T} \frac{l^2}{L_z^2} \Phi_2(A_*),$$

$\langle \ldots \rangle_V$ **implies the averaging over the volume of the semi-organized structure.**

$$\frac{E_\theta}{E_\Theta} = \frac{C_P C_F A_r}{7A_*^2} \left(\frac{l}{L_z}\right)^2,$$

$$U_D = \left[\left(F_z + \langle \Theta U_z \rangle_V\right) \beta L_z\right]^{1/3}$$

$$\frac{F_z}{\langle \Theta U_z \rangle_V} = \frac{1}{40} \left(\frac{l}{L_z}\right)^2.$$

$$\Theta_D = \left(F_z + \langle \Theta U_z \rangle_V\right) / U_D$$

$$\frac{E_\Theta}{\Theta_D^2} = \frac{6C_P C_F C_\tau A_r A_z}{\left(1-\hat{F}\right)^{1/3}} \left(\frac{l}{L_z}\right)^{10/3} \left(1 - \frac{F_z}{F_{\mathrm{tot}}}\right)^{4/3} \Phi_8(A_*),$$

EFB-Theory for CBL-Core for Shear-Free Convection

The kinetic energy of the semi-organized structures (cloud cells):

$$E_U \equiv \frac{1}{2} U_{z0}^2 = \frac{1}{3 C_\tau A_z} \left(\frac{L_z}{l} \right)^{4/3} \left(\langle \Theta U_z \rangle_V \beta L_z \right)^{2/3} \frac{\Phi_9(A_*)}{\Phi_1(A_*)} \left(1 - \hat{F} \right)^{1/3},$$

The thermal energy of the semi-organized structures (cloud cells):

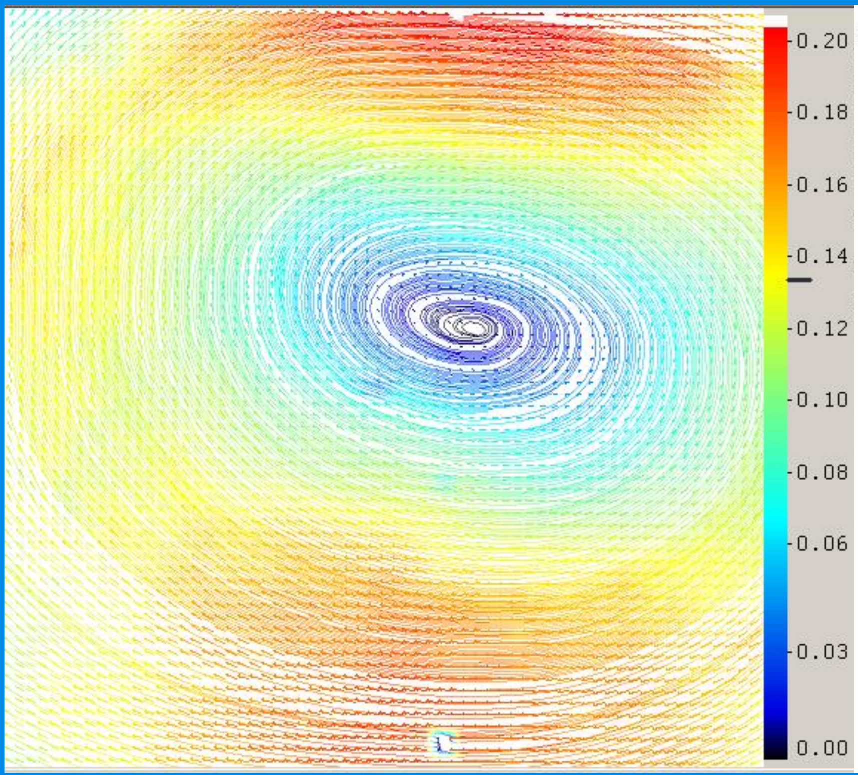
$$E_\Theta \equiv \frac{1}{2} \Theta_0^2 = \frac{83}{2} \left(\frac{l}{L_z} \right)^{4/3} \left(\frac{\langle \Theta U_z \rangle_V^2}{\beta L_z} \right)^{2/3} C_\tau A_z \left(1 - \hat{F} \right)^{-1/3} A_*^2 \, \Phi_8(A_*).$$

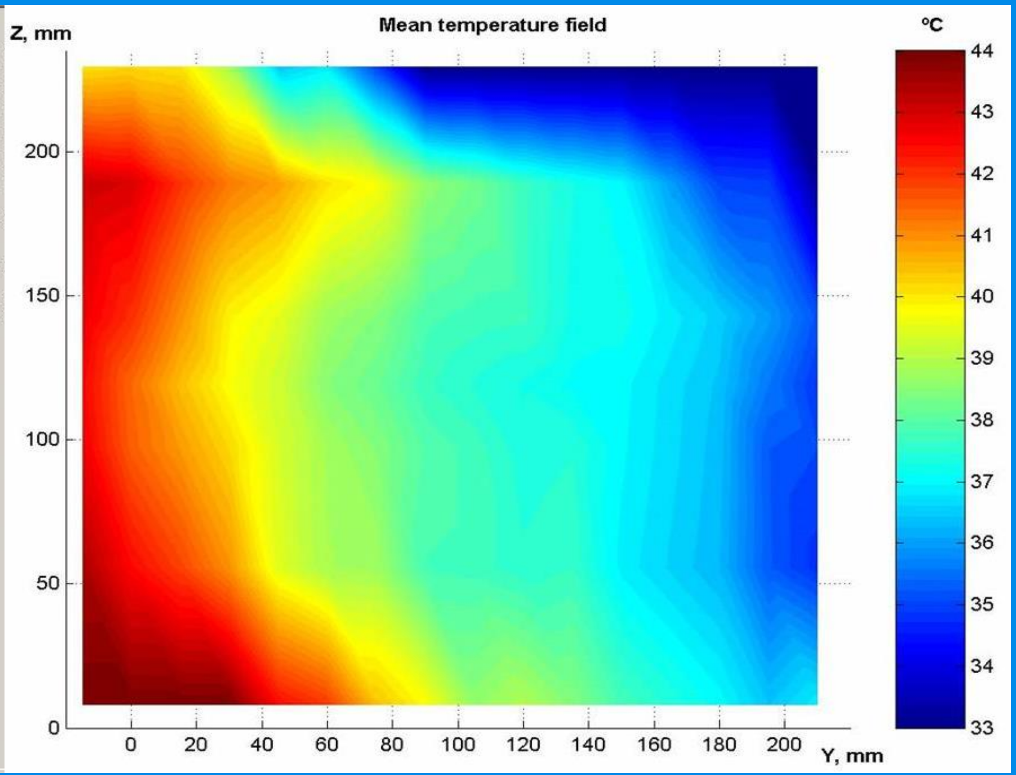
The mean vertical temperature gradient:

$$A_* = \pi R / \lambda L_z$$

$$\frac{\partial \overline{\Theta}}{\partial z} = 12.7 \frac{\left(\langle \Theta U_z \rangle_V l \right)^{2/3}}{\beta^{1/3} L_z^2} \frac{C_F A_r}{A_z} \left(1 - \hat{F} \right)^{1/3} \Phi_6(A_*) \left[1 - \frac{A_z^2 \, \mathrm{Pr}_T \, \Phi_7(A_*)}{3 A_r \sigma (1 - \hat{F})} \right],$$

Unforced Convection: A = 1





$\overline{U}(y,z)$

$\overline{T}(y,z)$

EFB-Theory for CBL-Core for Shear-Free Convection

The vertical flux of entropy transported by the semi-organized structures:

$$\langle \Theta U_z \rangle_V = \frac{1}{2} \Theta_0 \, U_{z0} \, J_2^2(\lambda) = \left(C_\tau A_z\right)^{3/2} \frac{l^2 \, U_{z0}^3}{\beta L_z^3} \frac{\Phi_4\left(A_*\right)}{\left(1-\hat{F}\right)^{1/2}},$$

The ratio of fluxes of entropy :

$$\frac{\langle \Theta U_z \rangle_V}{F_z} = \frac{\sigma \, \mathrm{Pr}_T}{\left(C_\tau A_z\right)^2} \frac{L_z^2}{l^2} \left(1-\hat{F}\right) \Phi_5\left(A_*\right),$$

Sheared Convection (CBL-core)

$$\frac{D\boldsymbol{\omega}}{Dt} = K_M \Delta\,\boldsymbol{\omega} - \beta(\mathbf{e}\times\nabla)\Theta + (\boldsymbol{\omega}\cdot\nabla)\,\mathbf{U} + (\boldsymbol{\omega}^{(s)}\cdot\nabla)\,\mathbf{U},$$

$$\frac{D\Theta}{Dt} = -\nabla\cdot\mathbf{F}\ ,$$

$$\nabla\cdot\mathbf{F} = -t_T\left(\sigma F_z^*(\mu\Delta_h - \Delta_z)U_z + \frac{8}{15}\mathbf{F}_x^*\cdot(\mathbf{e}\times\nabla)\omega_z\right) - K_H\,\Delta\Theta,$$

The shear velocity:

$$\mathbf{U}^{(s)} = (Sz,0,0)$$

$$\boldsymbol{\omega}^{(s)} = (0,S,0)$$

$$A_* = L_y / L_z$$

The solution of linearized equations:

$$U_x = -U_{x0}\cos\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$U_y = -A_* U_{z0}\sin\left(\frac{\pi y}{L_y}\right)\cos\left(\frac{\pi z}{L_z}\right),$$

$$U_z = U_{z0}\cos\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\Theta = \Theta_0\cos\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\frac{U_{x0}}{U_{z0}} = \frac{L_y^2 S}{\pi^2 K_M}\left(1+A_*^2\right)^{-1},$$

$$\frac{\Theta_0}{U_{z0}} = \frac{L_y^2 S^2}{\pi^2 \beta K_M}\left(1+A_*^2\right)^{-1}\left[1+\frac{\pi^4 K_M^2}{L_y^4 S^2}\left(1+A_*^2\right)^2\right].$$

Vorticily of Cloud Streets

$$\omega_x = -\frac{\pi U_{z0}}{L_y}\left(1 + A_*^2\right)\sin\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right),$$

$$\omega_y = -\frac{\pi U_{x0}}{L_z}\cos\left(\frac{\pi y}{L_y}\right)\cos\left(\frac{\pi z}{L_z}\right),$$

$$\omega_z = -\frac{\pi U_{x0}}{L_y}\sin\left(\frac{\pi y}{L_y}\right)\sin\left(\frac{\pi z}{L_z}\right).$$

A large-scale helicity produced by the semi-organized structures:

$$\chi_U \equiv \mathbf{U}\cdot\boldsymbol{\omega} = \frac{\pi U_{x0} U_{z0}}{2L_y} A_*^2 \sin\left(\frac{2\pi y}{L_y}\right) = \frac{\pi U_{z0}^2 A_*^2}{2C_* l \hat{E}_K^{1/2}(1 + A_*^2)} .$$

Budget Equations for Sheared Convection

$$\frac{DE_K}{Dt} + \nabla \cdot \mathbf{\Phi}_K = -\tau_{ij} \frac{\partial U_i}{\partial x_j} + \beta F_z - \varepsilon_K ,$$

$$\frac{DE_\theta}{Dt} + \nabla \cdot \mathbf{\Phi}_\theta = -(\mathbf{F} \cdot \nabla)\Theta - F_z \frac{\partial \overline{\Theta}}{\partial z} - \varepsilon_\theta ,$$

$$\frac{DF_i}{Dt} + \frac{\partial}{\partial x_j} \Phi_{ij}^{(F)} = \beta \delta_{i3} \langle \theta^2 \rangle - \frac{1}{\rho_0} \langle \theta \nabla_i p \rangle - \tau_{ij} \frac{\partial (\Theta + \overline{\Theta})}{\partial x_j} - F_j \frac{\partial U_i}{\partial x_j} - \varepsilon_i^{(F)} ,$$

$$\tau_{ij} \equiv \langle u_i u_j \rangle = \delta_{ij} \frac{\langle \mathbf{u}^2 \rangle}{3} - K_M (\nabla_i U_j + \nabla_j U_i),$$

$$\varepsilon_K = \frac{E_K}{t_T}, \quad \varepsilon_i^{(F)} = \frac{F_i}{C_F t_T}, \quad \varepsilon_\theta = \frac{E_\theta}{C_P t_T},$$

Production in Sheared Convection

The production of turbulence is caused by three sources:

a) the shear of the semi-organized structures:

$$\Pi^{(cs)} = -\left\langle \tau_{ij}\, \partial U_i / \partial x_j \right\rangle_V = K_M \left\langle S_{ij}\, S_{ji} \right\rangle_V$$

b) the background wind shear:

$$\Pi^{(s)} = K_M\, S^2$$

c) the buoyancy:

$$\hat{E}_K \left(\frac{l}{L_y} \right)^2 >> 1 .$$

$$\hat{E}_K = E_K / L_y^2\, S^2 ,$$

$$\frac{E_K}{E_U} = \frac{C_*}{4(1-\hat{F})} \left(\frac{l}{L_y} \right)^2 \left(\left(1 + A_*^2\right)^2 - 3\, A_*^2 \right) ,$$

$$C_* = 2\pi^2 C_\tau A_z ,$$

$$\frac{E_\theta}{E_\Theta} = \pi^2 C_P C_F\, A_z \left(\frac{l}{L_z} \right)^2 .$$

$$\frac{\left\langle \Theta U_z \right\rangle_V}{F_z} = \frac{\pi^2 \sigma \Pr_T}{4C_*} \left(\frac{U_{z0}}{L_y\, S} \right)^2 \left(\frac{A_*^2 - \mu}{1 + A_*^2} \right) ,$$

EFB-Theory for CBL-Core for Sheared Convection

The kinetic energy of the semi-organized structures (cloud streets):

$$E_U \equiv \frac{1}{2} U_{z0}^2 = \frac{2^{1/3}}{C_*} \left(\frac{L_y}{l} \right)^{4/3} U_D^2 \left(1 - \hat{F}\right)^{1/3} \Phi_{10}(A_*),$$

The thermal energy of the semi-organized structures (cloud streets):

$$E_\Theta \equiv \frac{1}{2} \Theta_0^2 = C_* \left(\frac{l}{2 L_y} \right)^{4/3} \left(\frac{U_D^2}{\beta L_y} \right)^2 \left(1 - \hat{F}\right)^{-1/3} \Phi_{12}(A_*),$$

The Deardorff velocity scale:

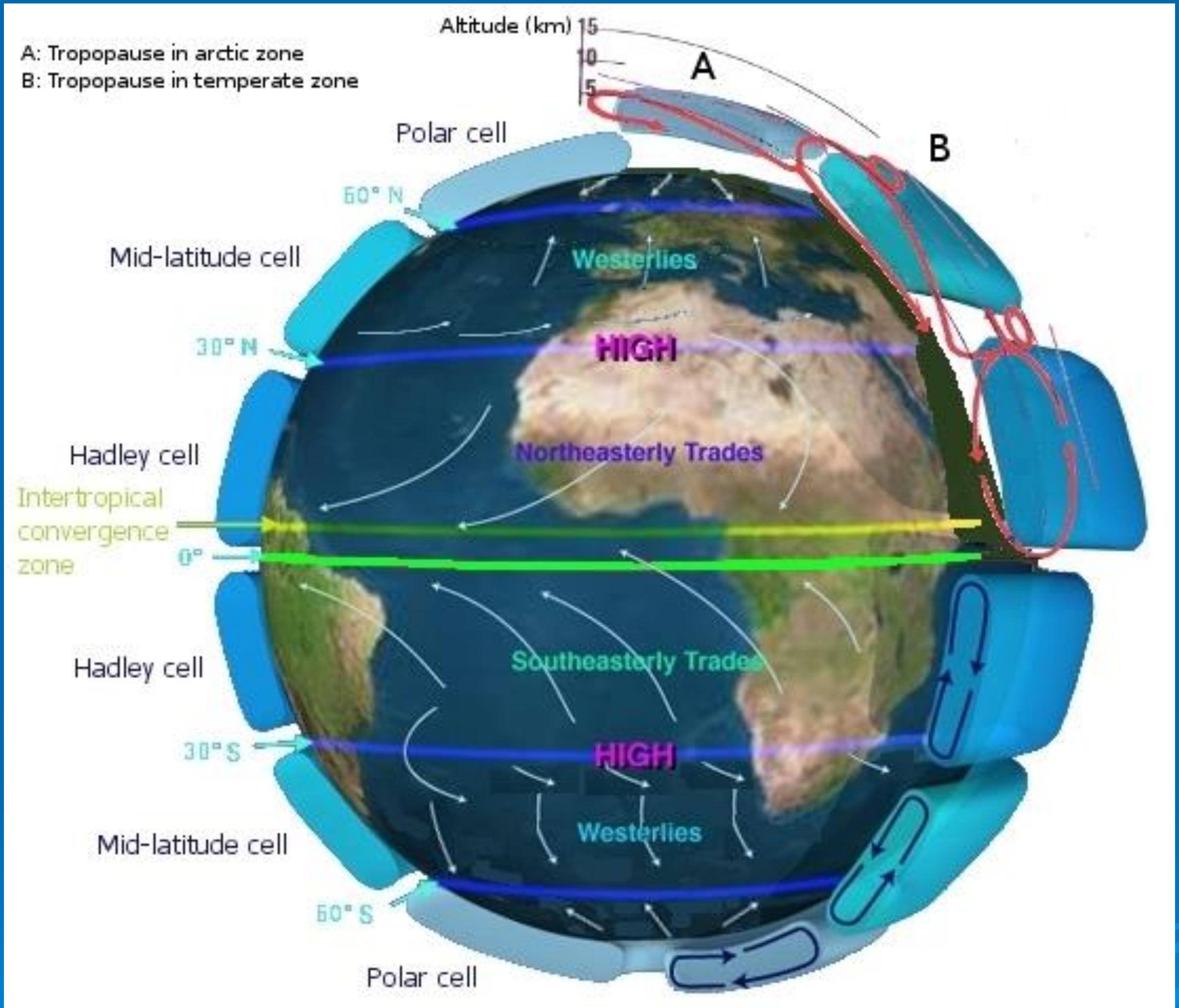
$$U_D = \left(\langle \Theta U_z \rangle_V \beta L_z \right)^{1/3}$$

$$\hat{F} = \frac{C_*^2}{\pi^2 \sigma \Pr_T \hat{E}_K} \left(\frac{l}{L_y} \right)^2 \frac{\left(1 + A_*^2\right)^2}{(A_*^2 - \mu)},$$

The vertical turbulent flux of entropy:

$$F_z = \frac{2^{2/3} C_*^2}{\pi^2 \sigma \Pr_T} \left(\frac{l}{L_y} \right)^{4/3} \left(\frac{U_D L_y S^2}{\beta} \right) \left(1 - \hat{F}\right)^{-1/3} \Phi_{11}(A_*),$$

Convection in Planetary Scales



According to the observations, 6 large-scale convective cells are observed in the Earth's atmospheric large-scale circulation:

the Hadley cell, the Mid-latitude cell and the Polar cell

Southern Oscillations

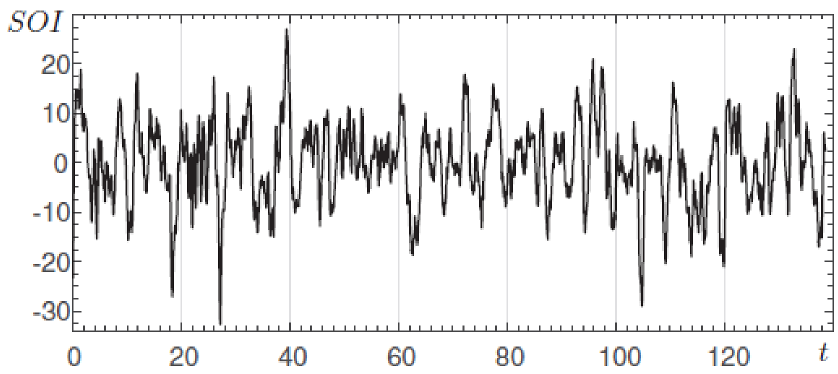


FIG. 1. Time dependence of the southern oscillation index (SOI) after 5 month window averaging, where the time is measured in years, $t = 0$ corresponds to the year 1878, and the total time interval of the observations of the SOI is 138 years. The data are taken from [29].

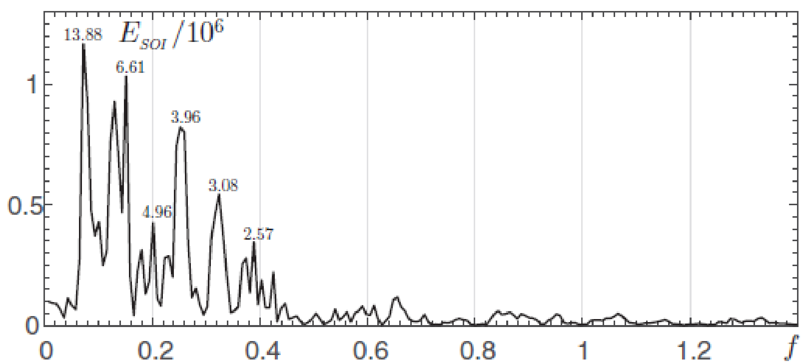


FIG. 2. The spectrum $E_{SOI}(f)$ of the SOI. The frequency is measured in the units of inverse years. The shown periods of oscillations (13.88; 6.61; 4.96; 3.96; 3.08; 2.57) are measured in years.

The Southern Oscillation is an oscillation of surface air pressure between the tropical eastern and the western Pacific Ocean.

The strength of the Southern Oscillation is characterized by the Southern Oscillation Index (SOI), and is determined from fluctuations of the surface air pressure difference between Tahiti and Darwin, Australia.

http://www.bom.gov.au/climate/current/soi2.shtml

Rossby Waves

Classical 2D Rossby waves

$$\omega_R^{(2D)} = \frac{2\Omega m}{\ell(\ell+1)}$$

In the beta-plane approximation

$$\omega = -\frac{\beta_f k_x}{k_x^2 + k_y^2 + R_d^{-2}}$$

$$f = f_0 + \beta_f y \qquad R_d = \sqrt{gH}/f_0$$

The periods of the classical planetary 2D Rossby waves do not exceed 100 days.

Slow 3D Rossby waves in planetary convection

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla\left(\frac{p}{\rho}\right) - \beta\theta + 2\mathbf{v}\times\mathbf{\Omega},$$

$$\frac{\partial \theta}{\partial t} + (\mathbf{v}\cdot\nabla)\theta = -(\mathbf{v}\cdot\nabla)\theta_{\rm eq},$$

$$\nabla\cdot(\rho\,\mathbf{v}) = 0,$$

$$\omega_R^{(3D)} = \frac{8m\,\Omega\,H_\rho}{R\,(1+\sigma^2)} \qquad \sigma = 2k\,H_\rho$$

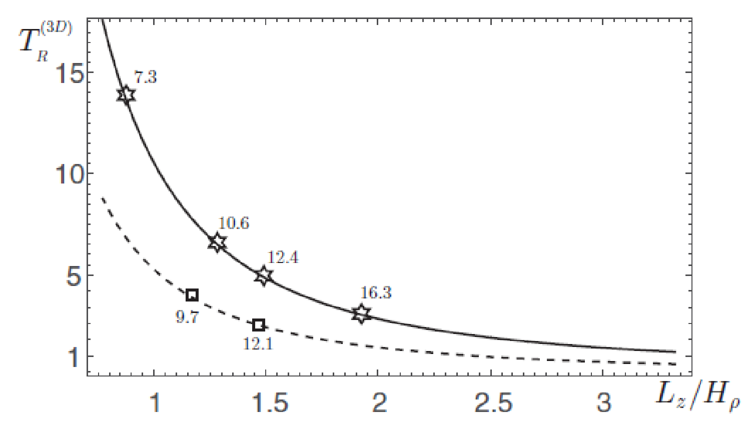


FIG. 3. The period $T_R = 2\pi/|\omega_R^{(3D)}|$ (measured in years) of the slow 3D Rossby waves vs the characteristic scale L_z/H_ρ, where $L_z = \pi/k$ for $m=1$ (solid) and $m=2$ (dashed). The stars (for $m=1$) and squares (for $m=2$) correspond to the observed periods of the SOI; see Fig. 2. The vertical sizes of these observed modes, $L_z = 7.3; 10.6; 12.4; 16.3$ km, determine the wave numbers k of the slow 3D Rossby waves; see Eqs. (23) and (25).

Slow 3D Rossby Waves in Planetary Convection

$$U = \sqrt{\rho}\, v,\ W = \sqrt{\rho}\, w \text{ and } \Theta = \sqrt{\rho}\,\theta$$

$$\mathbf{\Lambda} = -\nabla\rho/\rho = \text{const},$$

$$\left(\frac{\Lambda^2}{4} - \Delta\right)\frac{\partial U_r}{\partial t} = (2\Omega\cdot\nabla + \Omega\cdot\Lambda)W_r + 2(\mathbf{\Omega}\times\nabla)_r\,(\mathbf{\Lambda}\cdot\boldsymbol{U}) + r^{-1}\,\Omega_\theta\,(2\nabla_r + \Lambda)U_\varphi - \beta\Delta_\perp\Theta, \quad (16)$$

$$\frac{\partial W_r}{\partial t} = (2\mathbf{\Omega}\cdot\nabla - \Omega\cdot\mathbf{\Lambda})U_r + \frac{2\Omega}{r}\,U_\vartheta\,\sin\vartheta, \quad (17)$$

$$\frac{\partial\Theta}{\partial t} = -\frac{\Omega_b^2}{\beta}U_r - U_\vartheta\nabla_\vartheta\Theta_{\rm eq}, \quad (18)$$

$$\nabla\cdot\boldsymbol{U} = \frac{1}{2}\mathbf{\Lambda}\cdot\boldsymbol{U}, \quad (19)$$

$$\boldsymbol{U} = \sum_{\ell,m} \exp(\lambda t - i k r)[\hat{A}_{\rm r}\boldsymbol{Y}_{\rm r} + \hat{A}_{\rm p}\boldsymbol{Y}_{\rm p} + \hat{A}_{\rm t}\boldsymbol{Y}_{\rm t}]$$

$$\boldsymbol{Y}_{\rm r} = \boldsymbol{e}_r Y_\ell^m(\vartheta,\varphi), \quad \boldsymbol{Y}_{\rm p} = r\,\nabla Y_\ell^m(\vartheta,\varphi),$$

$$\boldsymbol{Y}_{\rm t} = (\boldsymbol{r}\times\nabla)Y_\ell^m(\vartheta,\varphi),$$

$$\ell^2 \gg 1$$

$$\sigma = 2k\,H_\rho$$

$$Y_\ell^m(\vartheta,\varphi) = A_{\ell,m}\,P_\ell^{|m|}(\vartheta)\,\exp(im\varphi),$$

$$\left(\frac{\Lambda^2}{4} - \Delta\right)\frac{\partial U_r}{\partial t} \sim 2(\mathbf{\Omega}\times\nabla)_r\,(\mathbf{\Lambda}\cdot\boldsymbol{U}).$$

$$\left(\lambda - 2i\omega_R^{(2D)}\right)\left(\lambda + i\omega_R^{(3D)}\right) + 2\Omega^2\left(1 + \frac{3i\sigma}{\ell^2(1+\sigma^2)}\right) + \frac{\Omega_b^2 H_\rho}{R}\left(\frac{i\omega_R^{(3D)}}{\lambda} + \frac{4\ell^2 H_\rho}{R(1+\sigma^2)}\right) - \omega_R^{(2D)}\left(\omega_R^{(2D)} + \omega_R^{(3D)}\right) = 0, \quad (22)$$

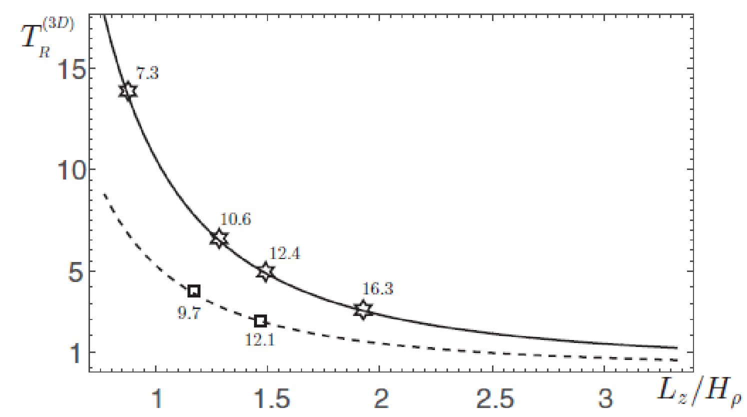


FIG. 3. The period $T_R = 2\pi/|\omega_R^{(3D)}|$ (measured in years) of the slow 3D Rossby waves vs the characteristic scale L_z/H_ρ, where $L_z = \pi/k$ for $m = 1$ (solid) and $m = 2$ (dashed). The stars (for $m = 1$) and squares (for $m = 2$) correspond to the observed periods of the SOI; see Fig. 2. The vertical sizes of these observed modes, $L_z = 7.3$; 10.6; 12.4; 16.3 km, determine the wave numbers k of the slow 3D Rossby waves; see Eqs. (23) and (25).

$$\omega_R^{(3D)} = \frac{8m\,\Omega\,H_\rho}{R\,(1+\sigma^2)}$$

$$\gamma_R = \frac{3\sigma\,\omega_R^{(3D)}}{\ell^2(1+\sigma^2)}$$

Generation of slow 3D Rossby Waves in Planetary Convection

$$\gamma_R = \frac{3\sigma\,\omega_R^{(3D)}}{\ell^2(1+\sigma^2)}$$

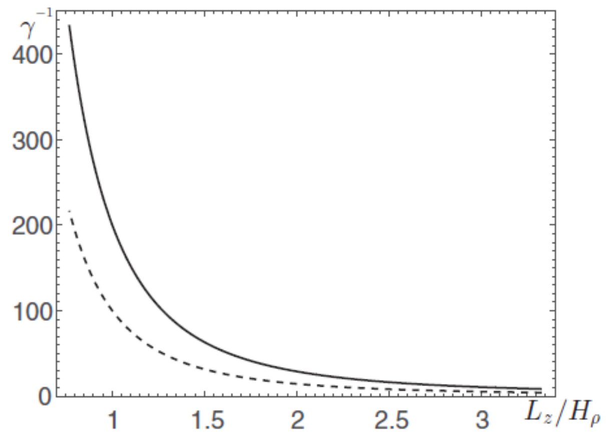


FIG. 4. The characteristic time $T_{\rm inst} = \gamma_R^{-1}$ (measured in years) of the excitation of the 3D Rossby waves vs the characteristic scale L_z/H_ρ of the convective cells for $m = 1$ (solid) and $m = 2$ (dashed).

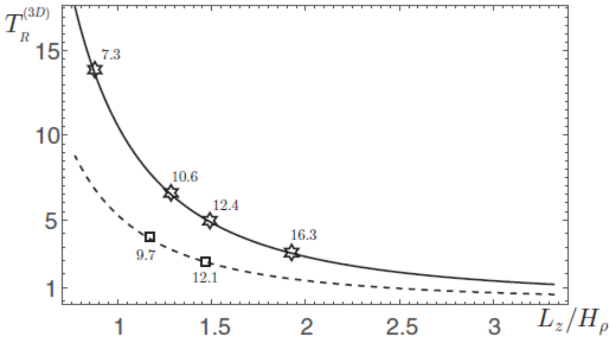


FIG. 3. The period $T_R = 2\pi/|\omega_R^{(3D)}|$ (measured in years) of the slow 3D Rossby waves vs the characteristic scale L_z/H_ρ, where $L_z = \pi/k$ for $m = 1$ (solid) and $m = 2$ (dashed). The stars (for $m = 1$) and squares (for $m = 2$) correspond to the observed periods of the SOI; see Fig. 2. The vertical sizes of these observed modes, $L_z = 7.3; 10.6; 12.4; 16.3$ km, determine the wave numbers k of the slow 3D Rossby waves; see Eqs. (23) and (25).

$$\omega_R^{(3D)} = \frac{8m\,\Omega\,H_\rho}{R\,(1+\sigma^2)}$$

$$\ell^2 \gg 1$$

$$\sigma = 2k\,H_\rho$$

$$\left(\frac{\Lambda^2}{4} - \Delta\right)\frac{\partial U_r}{\partial t} \sim 2(\boldsymbol{\Omega}\times\boldsymbol{\nabla})_r\,(\boldsymbol{\Lambda}\cdot\boldsymbol{U}).$$

The instability causes excitation of the 3D Rossby waves interacting with the convective mode and the inertial wave mode.

The energy of this instability is supplied by thermal energy of convective motions.

Conclusions

- Mechanism of formation of cloud cells in shear-free convection
- Mechanism of formation of cloud streets in sheared convection
- EFB theory for shear-free convection
- EFB theory for sheared convection
- Convection in Planetary Scales,
  3D Slow Rossby waves and Southern Oscillations

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