# *NEARSHORE STICKY WATERS*

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# Shallow-Water Oil Fate Model

- Uses:
  - Simulation, prediction, policy/hazards diagnostic tool.
- Components
  - Surface and Subsurface oil ( 1,000 chemicals, 20 droplet sizes)
  - Wind, Sun, SST
  - Ocean Waves, Currents, Turbulence.
  - Chemodynamics
- Resolution:
  - 100s m to 10s Km
  - 10 sec to weeks
- Particular Characteristics:
  - Depth-averaged, Eulerian.
  - High performance code with data assimilative capabilities.

GULF OF MEXICO RESEARCH INITIATIVE

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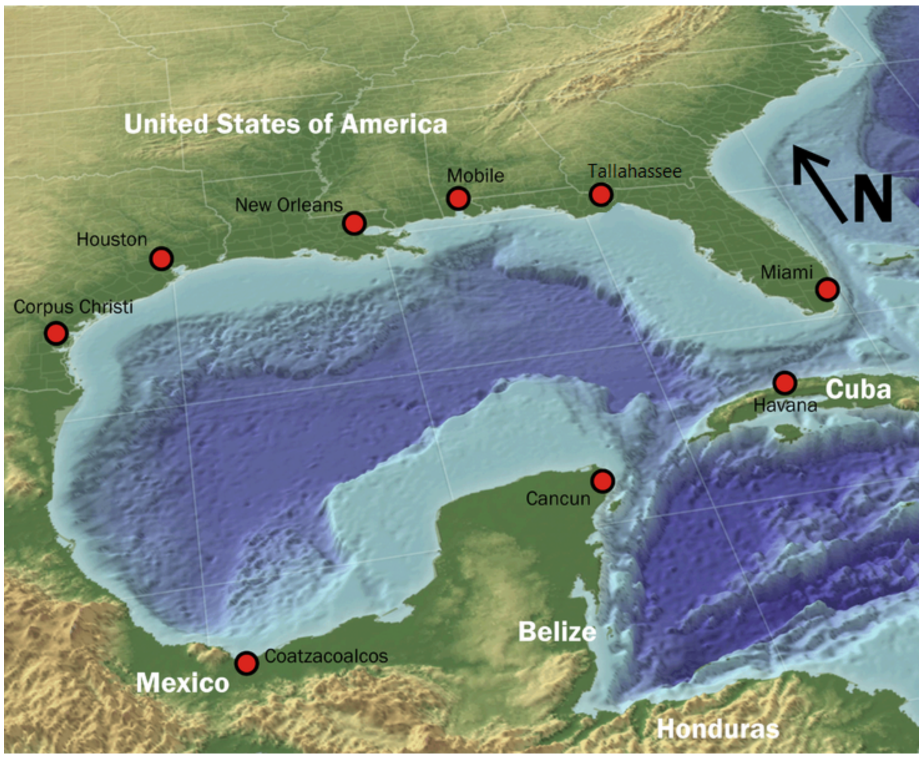
WHERE DISCOVERIES BEGIN

# WHY FOCUS ON SHALLOW WATERS?

## The Gulf Coast Environment




ESRI Data and Maps.

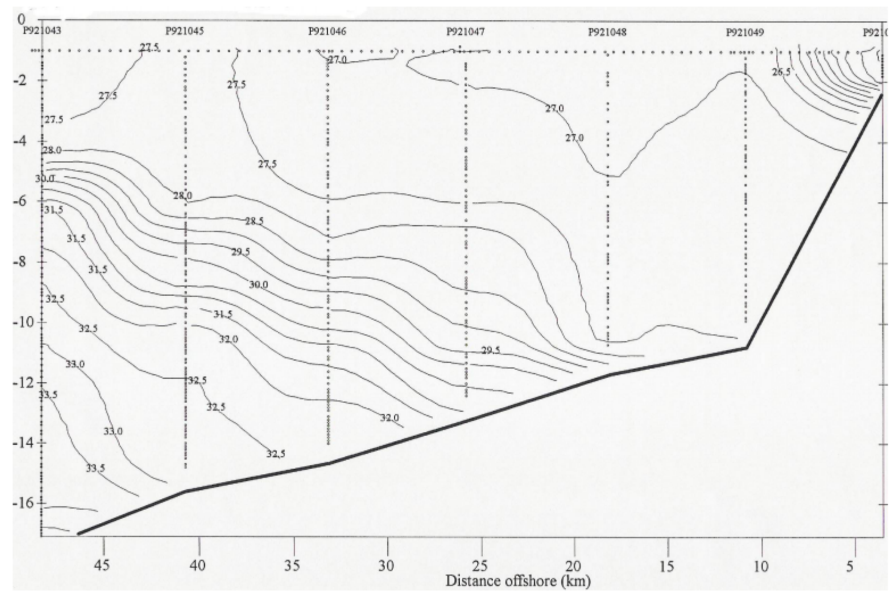
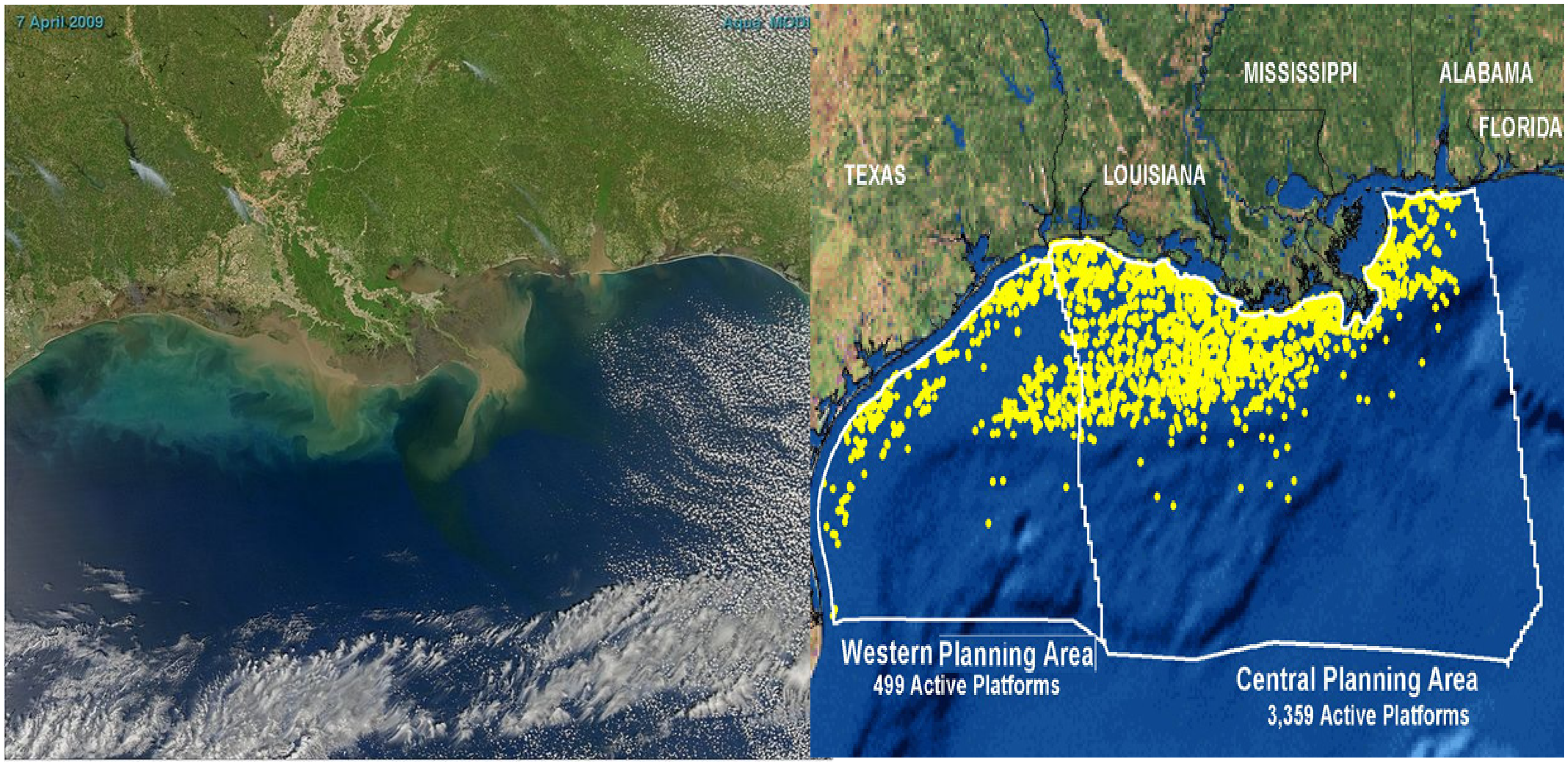



Figure 11. Example of a salinity section across a well-defined coastal front. (The depth scale is in meters.)

Image from Nasa.

# Why Focus on Shallow Waters?




ESRI Data and Maps.

# Surface - Subsurface Oil Transport Model

$$\rho\left[\frac{\partial s_i}{\partial T}+\nabla_\perp\cdot\left(\mathbf{V}s_i+\frac{\boldsymbol{\tau}}{2\mu}s_i^2\right)\right]=\underbrace{\nabla_\perp[\Psi\nabla_\perp s_i]}_{\text{diffusion}}-\underbrace{E^s(s_i)}_{\text{chemistry/evaporation}}+\underbrace{R_i}_{\text{mass exchange}}+\underbrace{P^s}_{\text{photo/biodegradation}}+\underbrace{G^s}_{\text{source/sink}}$$

$$\frac{\partial HC_i}{\partial T}+\underbrace{\nabla_\perp\cdot(H\mathbf{V}C_i)}_{\text{advection}}=\underbrace{\nabla_\perp\cdot[H\Psi\nabla_\perp C_i]}_{\text{diffusion/dispersion}}+\underbrace{E^C(C_i)}_{\text{chemistry/evaporation}}-\underbrace{R_i}_{\text{mass exchange}}+\underbrace{P^C}_{\text{biodegradation}}+\underbrace{G^C}_{\text{source/sink}}$$

The exchange rates $R_i$ depend on:

- Droplet distribution dynamics:
  - FAST: $\frac{\partial n(v,T)}{\partial t}=(B_v-D_v)$,
  - SLOW: $\frac{\partial n(v,T)}{\partial T}=$ lateral advection/dispersion
- wave activity and wall turbulence
- buoyancy (depends on size of droplets)
- surface tension, density, viscosity
- chemistry

*A multiscale challenge*

# Nearshore Sticky Waters

**A "red tide" event off the coast of Florida.**

Image courtesy of P. Schmidt, Charlotte Sun.

# Nearshore Sticky Waters

**Oregon Coast**

# Nearshore Sticky Waters

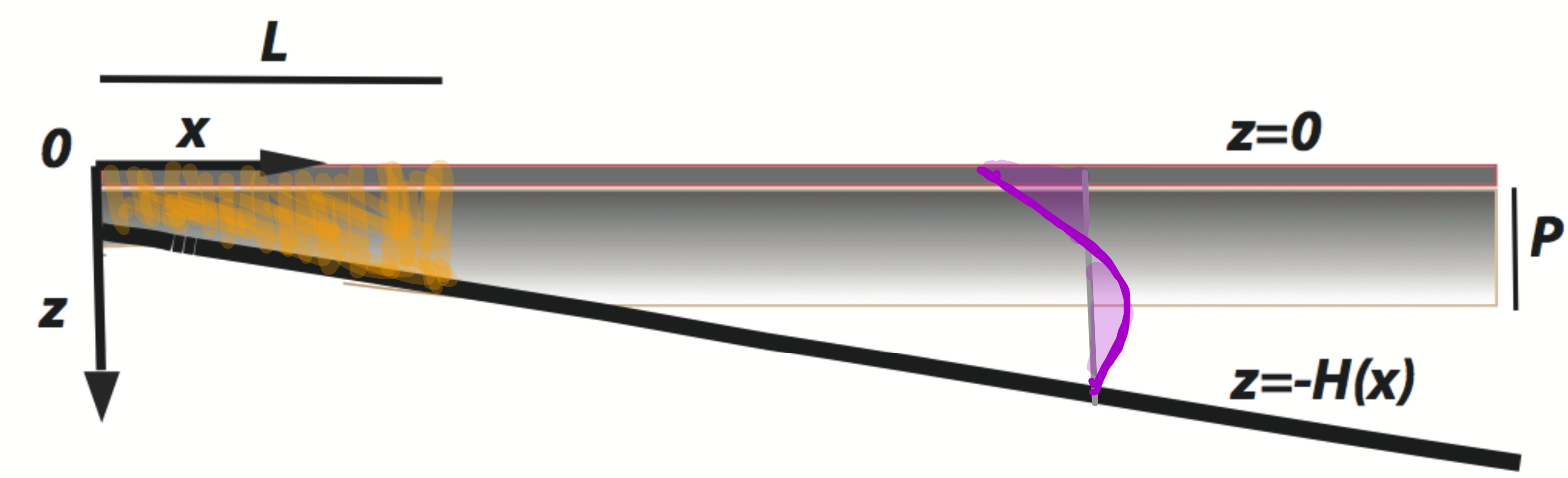
sticky waters

# Nearshore Sticky Waters

- Why and under what circumstances do shore-directed pollutants slow down, or park?
- How do you model beaching pollutants properly?
- How do you formulate numerical boundary conditions for ocean-transport codes?

Restrepo, J. M., Venkataramani, S. C., & Dawson, C. (2014). Nearshore sticky waters. Ocean Modelling, 80, 49-58

# Basic Phenomenology




- advection
- diffusion
- inter-layer interaction
- boundary conditions

# Surface - Subsurface Oil Transport Model

$$\rho\left[\frac{\partial s_i}{\partial T}+\nabla_\perp\cdot\left(\mathbf{V}s_i+\frac{\boldsymbol{\tau}}{2\mu}s_i^2\right)\right]=\underbrace{\nabla_\perp[\Psi\nabla_\perp s_i]}_{\text{diffusion}}-\underbrace{E^s(s_i)}_{\text{chemistry/evaporation}}+\underbrace{R_i}_{\text{mass exchange}}+\underbrace{P^s}_{\text{photo/biodegradation}}+\underbrace{G^s}_{\text{source/sink}}$$

$$\frac{\partial HC_i}{\partial T}+\underbrace{\nabla_\perp\cdot(\mathbf{V}HC_i)}_{\text{advection}}=\underbrace{\nabla_\perp\cdot[H\Psi\nabla_\perp C_i]}_{\text{diffusion/dispersion}}+\underbrace{E^C(C_i)}_{\text{chemistry/evaporation}}-\underbrace{R_i}_{\text{mass exchange}}+\underbrace{P^C}_{\text{biodegradation}}+\underbrace{G^C}_{\text{source/sink}}$$

# REDUCED TRANSPORT MODEL

$$\rho\left[\frac{\partial s}{\partial T} + \nabla_\perp \cdot \left(\mathbf{V}s + \frac{\boldsymbol{\tau}}{2\mu}s^2\right)\right] = \underbrace{\nabla_\perp[\Psi\nabla_\perp s]}_{\text{diffusion}} + \underbrace{R}_{\text{mass exchange}}$$

$$\frac{\partial HC}{\partial T} + \underbrace{\nabla_\perp \cdot (\mathbf{V}HC)}_{\text{advection}} = \underbrace{\nabla_\perp \cdot [H\Psi\nabla_\perp C]}_{\text{diffusion/dispersion}} - \underbrace{R}_{\text{mass exchange}}$$

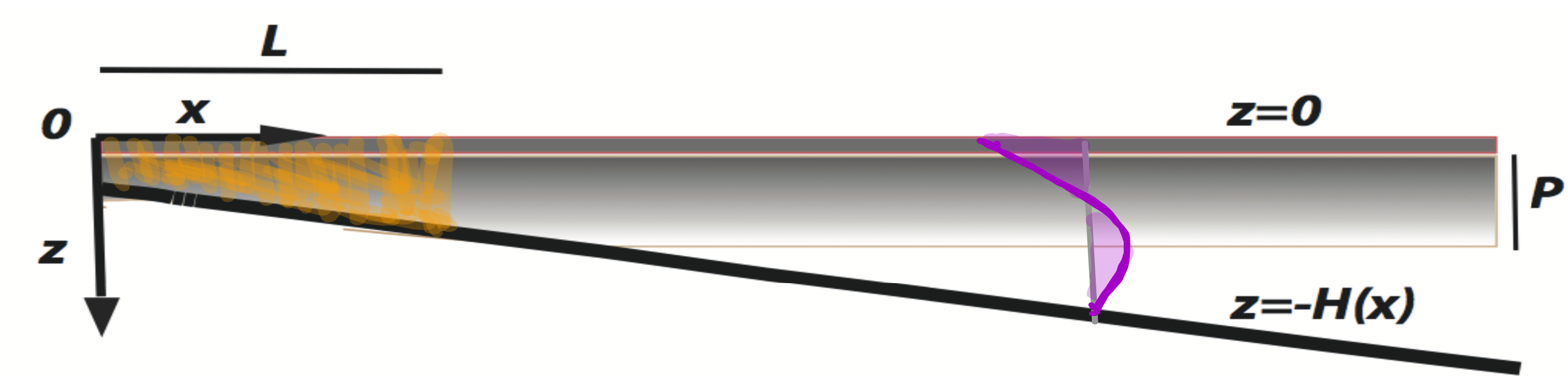
- single chemical species,
- no chemistry,
- no sources/sinks,
- no evaporation/photolysis

# REDUCED TRANSPORT MODEL

$$\frac{\partial s}{\partial t} + \frac{\partial[u_S(x)s]}{\partial x} = \frac{\partial}{\partial x}\left[D(x)\frac{\partial s}{\partial x}\right] + I$$

$$\frac{\partial B}{\partial t} + \frac{\partial[u_B(x)B]}{\partial x} = \frac{\partial}{\partial x}\left[D(x)\frac{\partial B}{\partial x}\right] - I$$

- $b(x,t) = B(x,t)/\zeta(x)$,
- $\zeta(x) = \min(H(x), P)$,
- Interaction term $R = I(\gamma, s, b) = \frac{(1-\gamma)s+\gamma PB}{\tau(x)}$

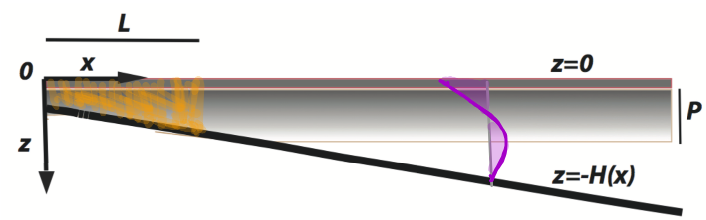


# The Model

$$\frac{\partial s}{\partial t} + \frac{\partial [u_S(x) s]}{\partial x} = -I(\gamma, s, b) + \frac{\partial}{\partial x}\left[D(x)\frac{\partial s}{\partial x}\right],$$
$$\frac{\partial b}{\partial t} + \frac{\partial [v(x) b]}{\partial x} = I(\gamma, s, b) + \frac{\partial}{\partial x}\left[D(x)\frac{\partial b}{\partial x}\right],$$

Boundary Conditions: $u_S(x)s - D(x)\frac{\partial}{\partial x}s = 0, \quad v(x)b - D(x)\frac{\partial}{\partial x}b = 0.$

- $b(x,t) = B(x,t)/\zeta(x)$,
- $\zeta(x) = \min(H(x), P)$,
- $v(x) := u_B(x) + D(x)\zeta'(x)/\zeta(x)$.



# ADVECTION

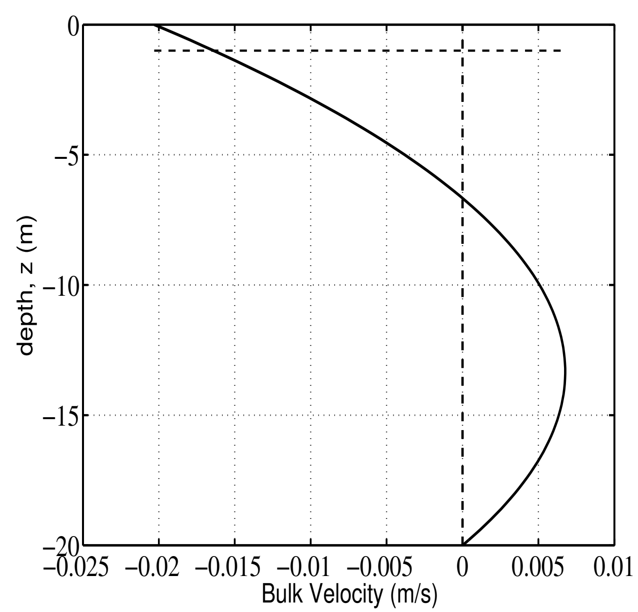
**The Time-Averaged Slick Advection** $u_S s$:

- $u_S = u^{St}$, (Stokes drift) wave-generated residual current.

**The Time-Averaged Bulk Advection** $u_B b$:

$$u_B(x) = u^{St}\left[H(x) - \zeta(x)\right]^2 / H(x)^2.$$

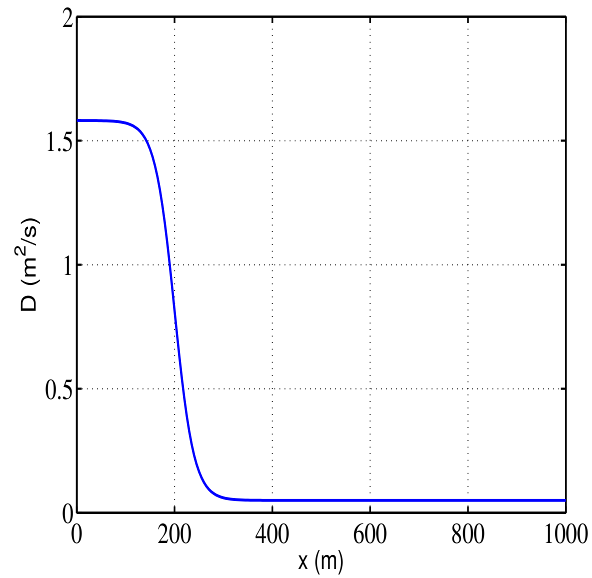
- $u_B$ equals $u^{St}$ at $z = 0$,
- $u_B$ equals zero at $z = -H(x)$
- $u_B$ has zero depth average *i.e.*, $\int_{-H}^{0} u_B \, dz = 0$.



# DIFFUSIVITY

$$D(x) = D_{eddy} + D_{mixing},$$

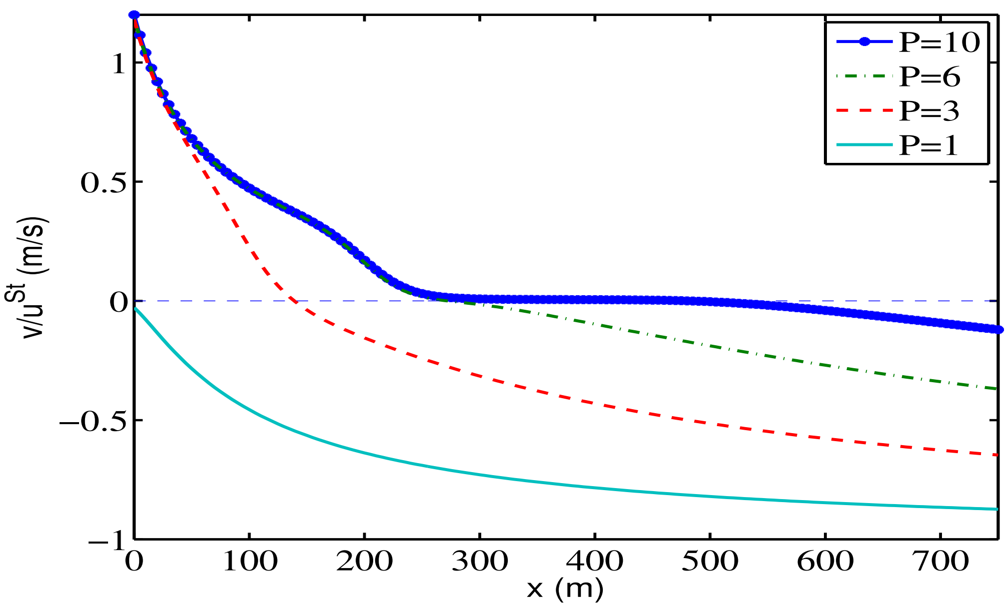
- $L = 200\text{m}$
- $w = 20\text{m}$ is the width of the transition of the sigmoid.
- $D_{eddy} = 0.05\text{m}^2/\text{s}$
- $D_{mixing} = 1.6\ \text{m}^2/\text{s}$
- Diffusive flux on the surface: $-D(x)\frac{\partial}{\partial x}s$ .
- Diffusive flux to the bulk: $-\zeta(x)D(x)\frac{\partial}{\partial x}B.$



$$\frac{\partial s}{\partial t} + \frac{\partial [u_S(x)s]}{\partial x} = -I + \frac{\partial}{\partial x}\left[D(x)\frac{\partial s}{\partial x}\right],$$

$$\frac{\partial b}{\partial t} + \frac{\partial [v(x)b]}{\partial x} = I + \frac{\partial}{\partial x}\left[D(x)\frac{\partial b}{\partial x}\right].$$

$$\zeta(x) = \min\left(H(x), P\right), \qquad v(x) := u_B(x) + D(x)\zeta'(x)/\zeta(x).$$




$v(x)$

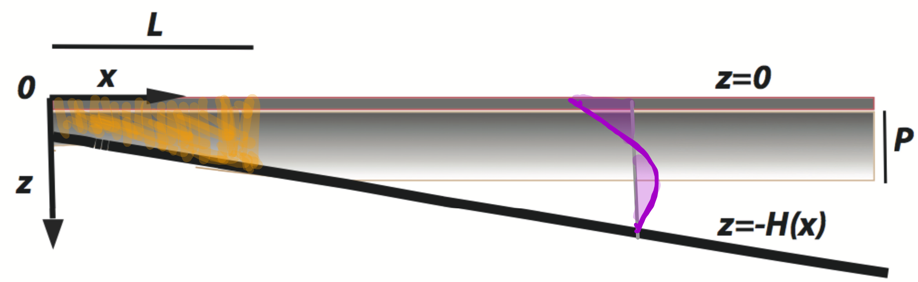
# RESULTS

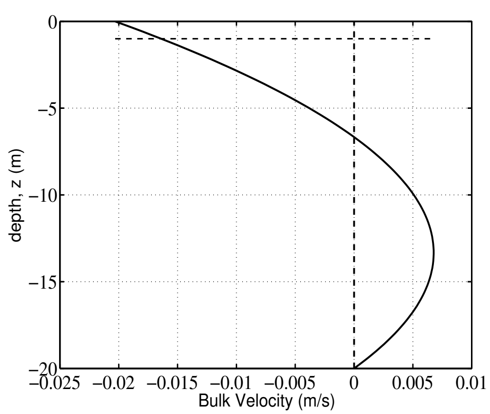
Fixed Initial Conditions and Parameters:

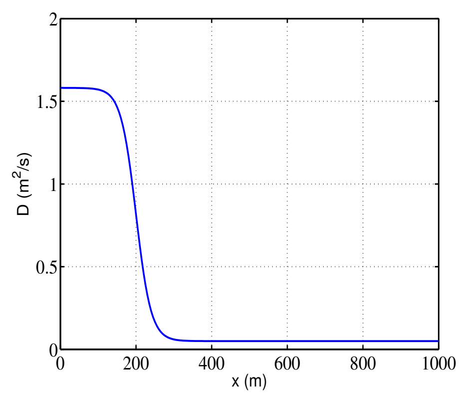
$$s(x,t=0) = \exp(-0.001(x-500)^2)/\sqrt{1000\pi}, \qquad b(x,t=0) = 0.$$

and topography:

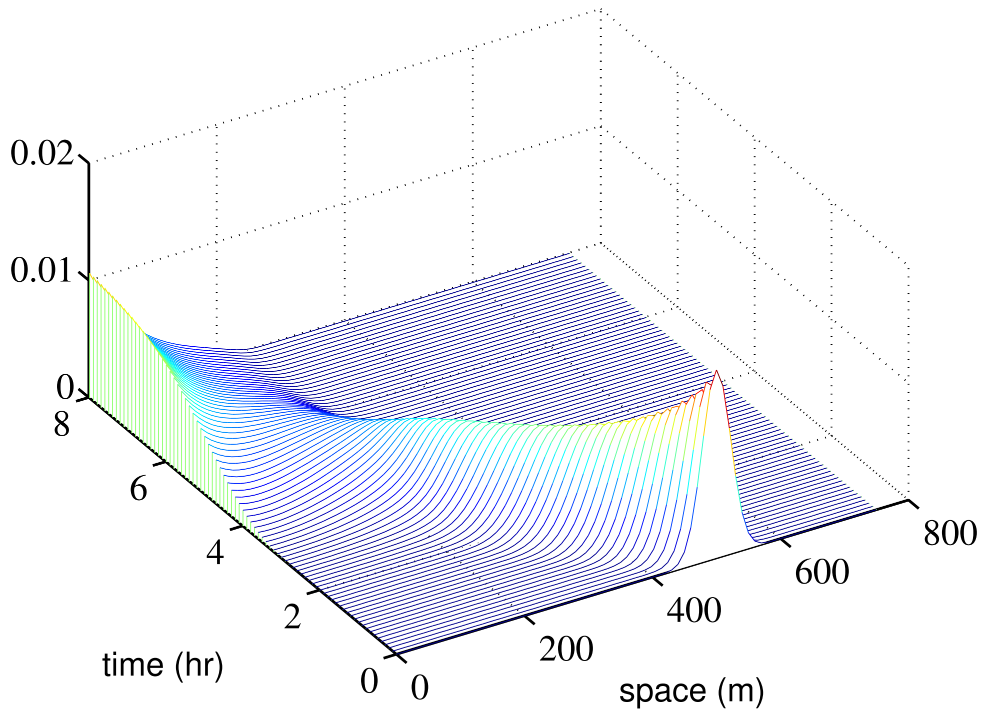
- shore: $H_0 = 1.2$m
- deep end: $H_\infty = 20$m
- length: $X = 1000$m
- breakzone: $L = 200$m



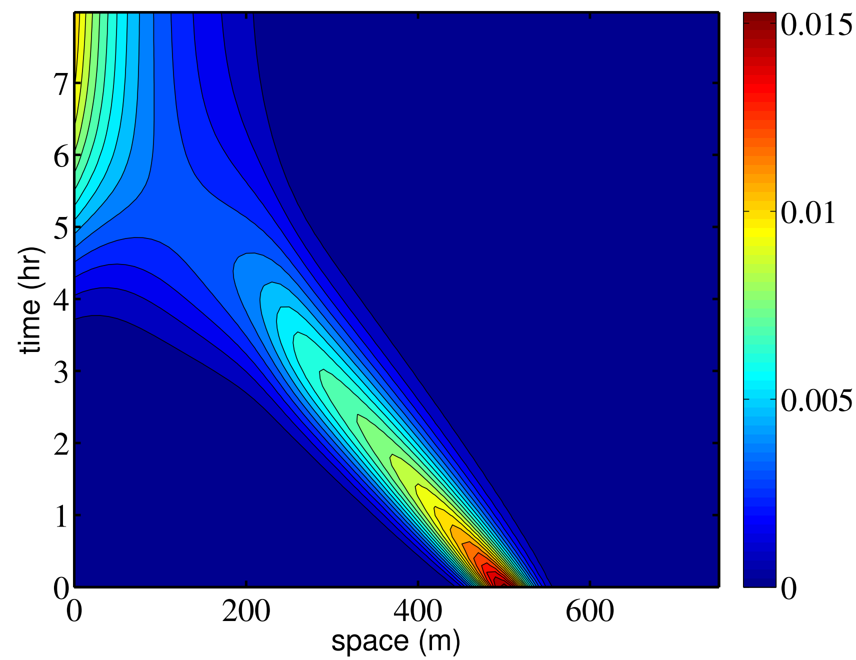



# $P = 1 < H(x)$

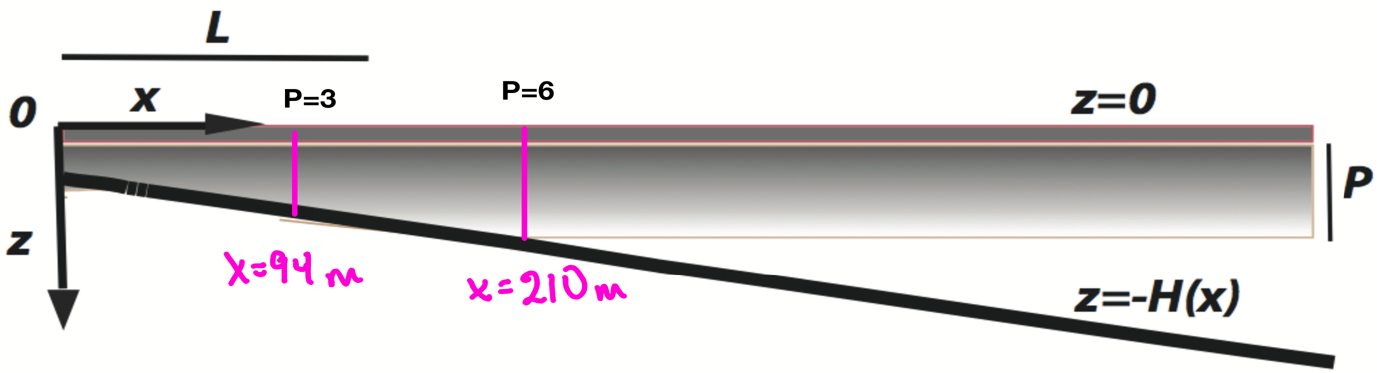


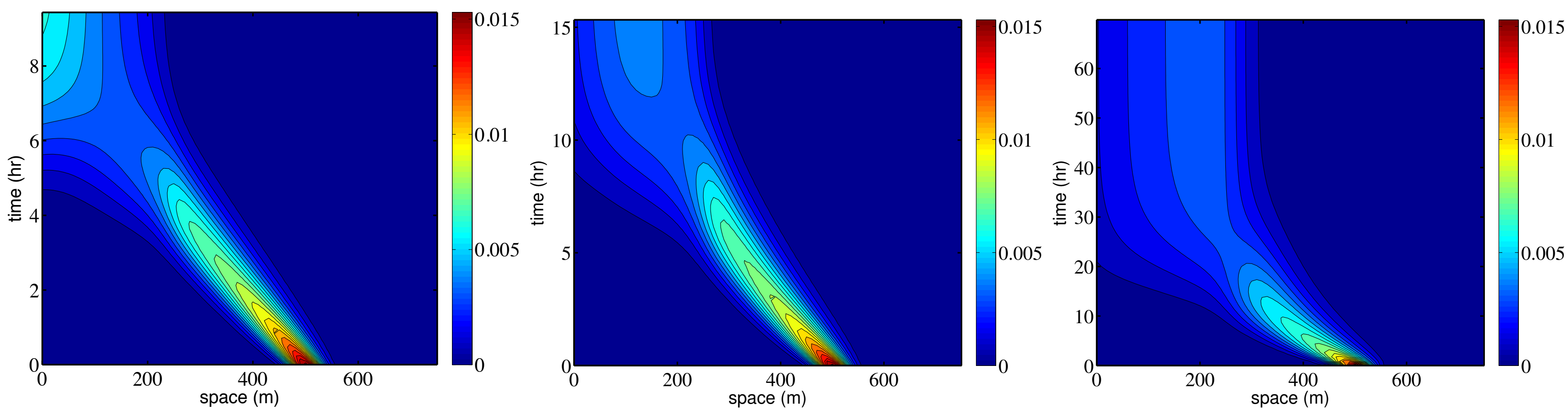
0.02
0.01
0
8
6
4
2
0
time (hr)
0
200
400
600
800
space (m)

time (hr)
0
1
2
3
4
5
6
7
0
200
400
600
space (m)
0.015
0.01
0.005
0

# $b(x, t)$ contours in space/time:




$P = 1$ $P = 3$ $P = 6$

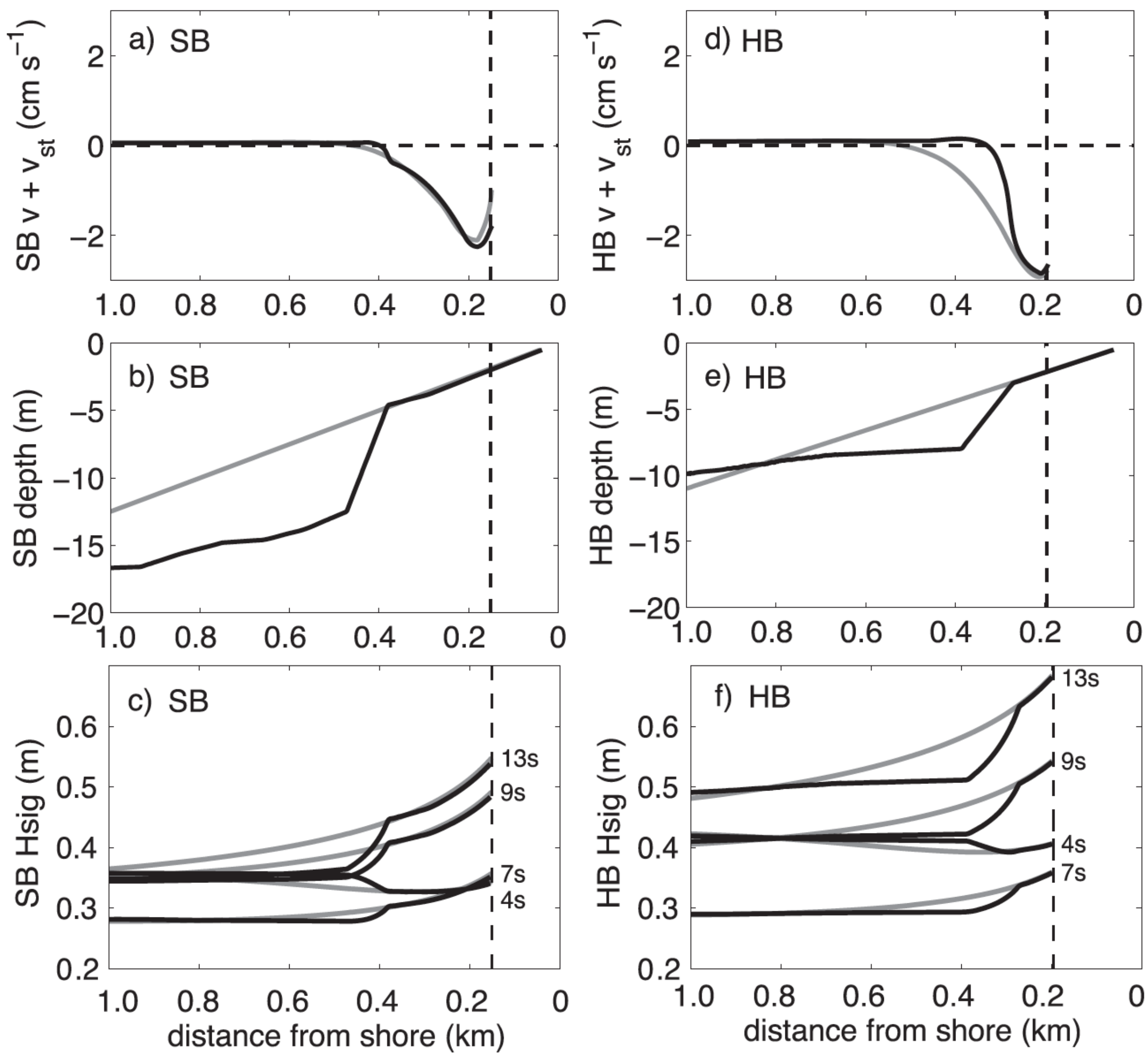



Figure taken from *Lagrangian Observations of Inner-Shelf Motions in Southern California: Can Surface Waves Decelerate Shoreward-Moving Drifters Just outside the Surf Zone?*, J. C. Ohlmann, et al., JPO (2012)

# Reduced Model

Substitute $q \approx \frac{1}{\sqrt{2\pi\sigma^2(t)}} \exp\left[-\frac{(x-\mu(t))^2}{2\sigma^2(t)}\right]$ into

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial x}\left[D(x)\frac{\partial q}{\partial x} - u_e(x)q\right], \qquad u_e(x) = \frac{\gamma P u_S + (1-\gamma)\zeta v(x)}{\gamma P + (1-\gamma)\zeta}.$$

for long times $t \to \infty$, we obtain the *steady state*

$$q \to q_\infty = C\exp\left[\int \frac{u_e(x)}{D(x)}dx\right], \qquad C \text{ is a normalizing constant.}$$

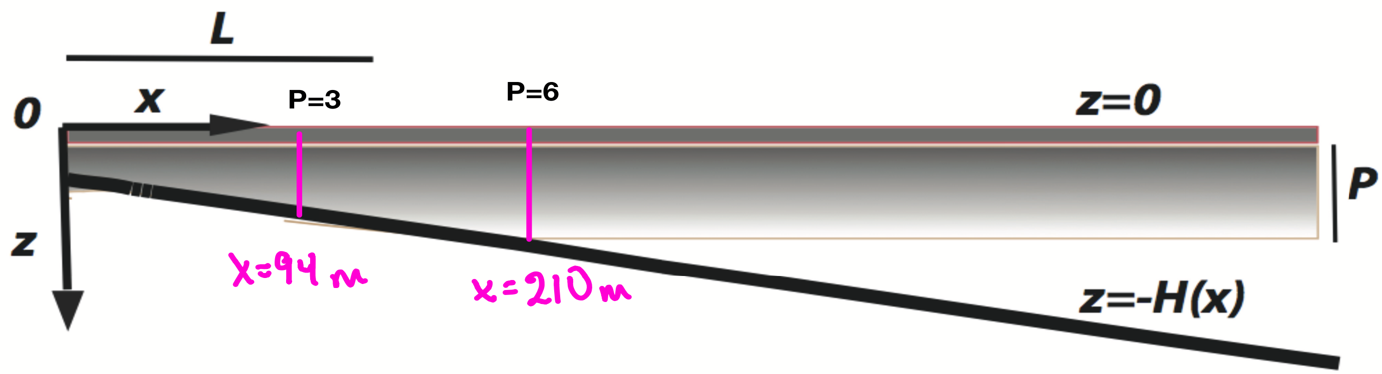
So

$$q(x,t) \approx q_\infty(x) + f(x)e^{-\lambda_1 t}, \qquad \frac{\partial}{\partial x}\left[D(x)\frac{\partial f}{\partial x} - u_e(x)f\right] = -\lambda_1 f,$$

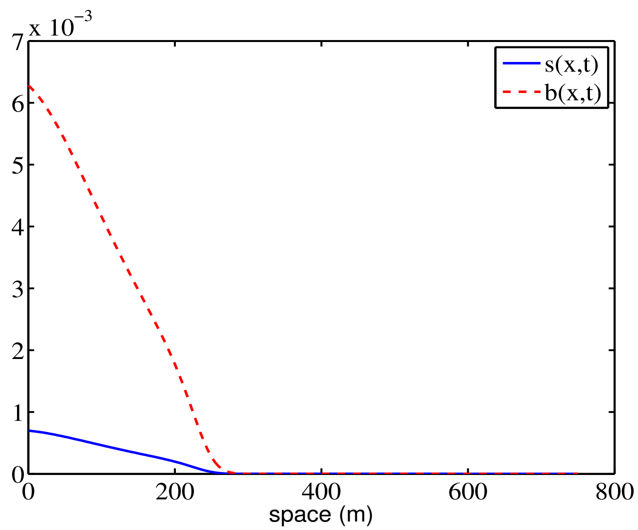
with solution, for $t > t_e$,

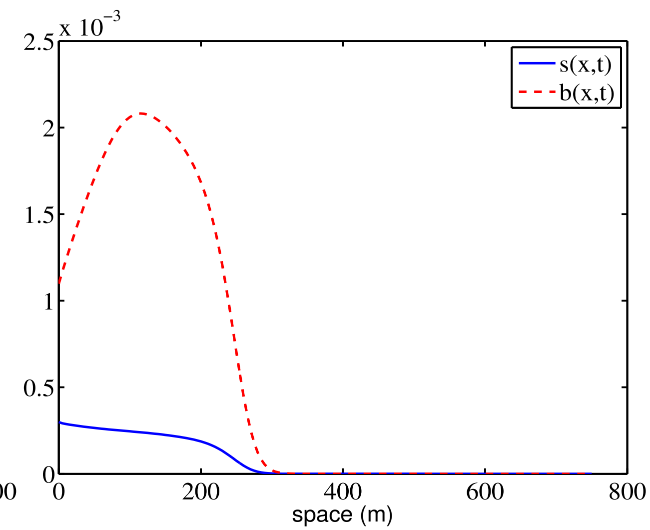
$$q(x,t) \approx q_\infty + \left[\sqrt{\frac{2}{\pi\sigma^2(t_e)}}\frac{1}{1+\mathrm{Erf}(\sqrt{a/2})}\exp\left[-\frac{(x-\mu(t_e))^2}{2\sigma^2(t_e)}\right] - q_\infty\right]e^{-\lambda_1(t-t_e)}.$$

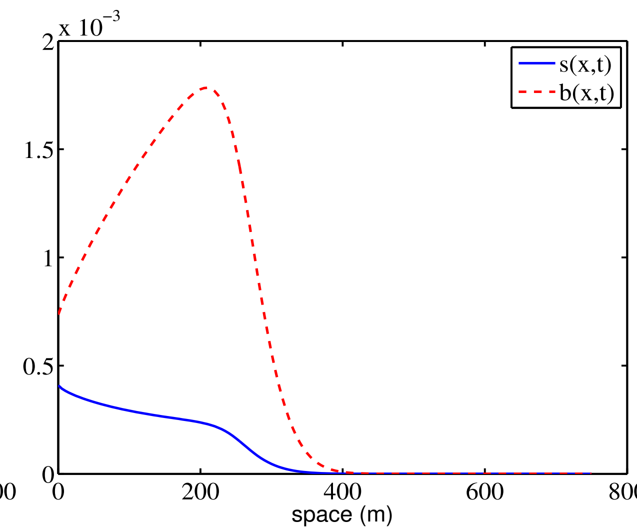
# LONG-TIME CONCENTRATIONS




$P = 1$ $P = 3$ $P = 6$

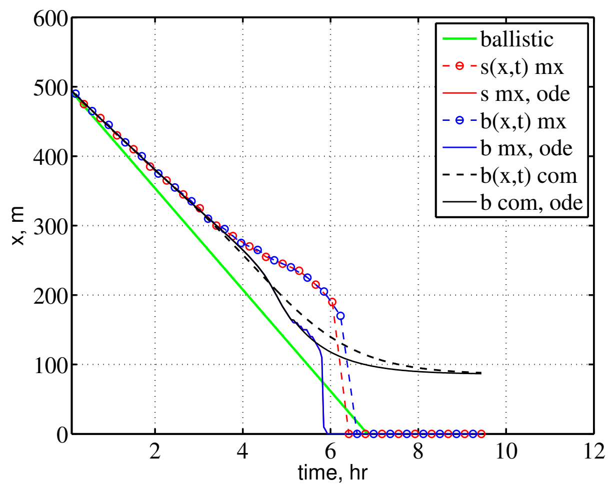


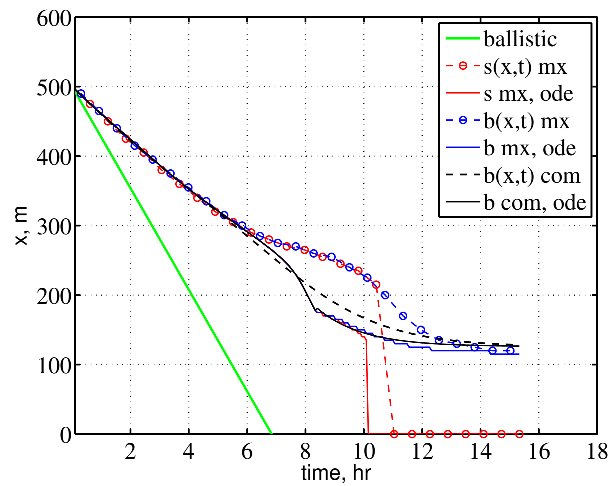



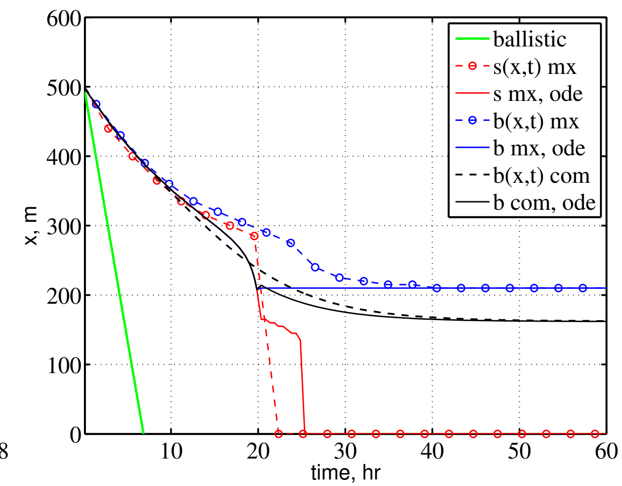



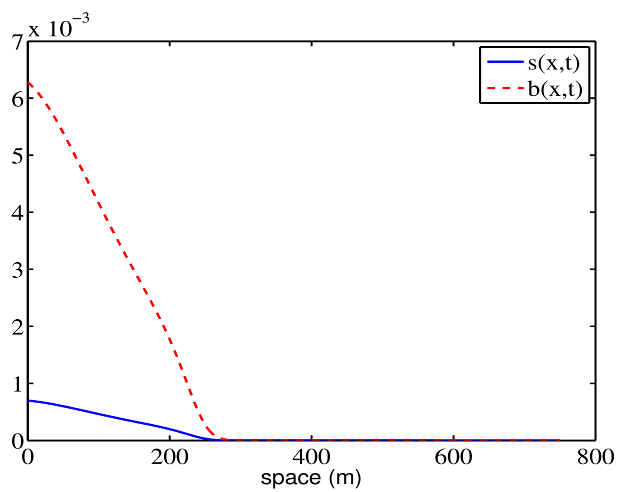
# Long-Time Concentrations

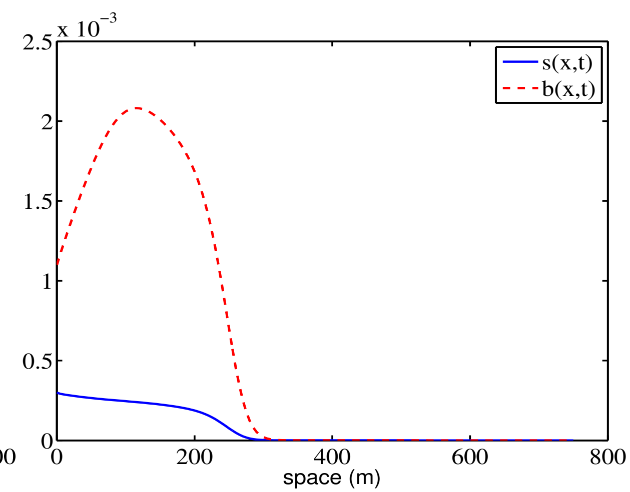
$P = 1$ $P = 3$ $P = 6$

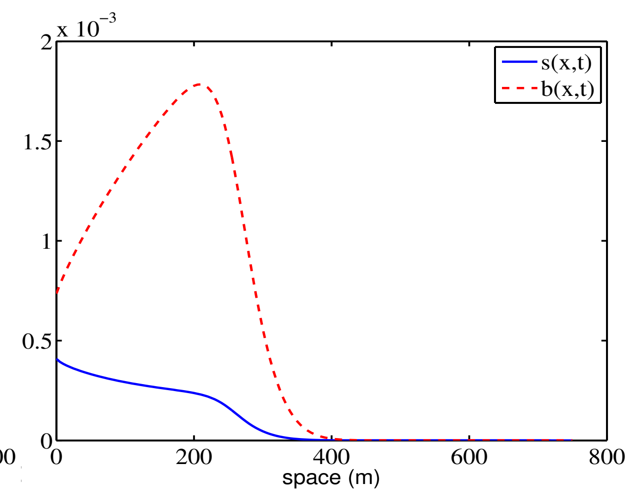


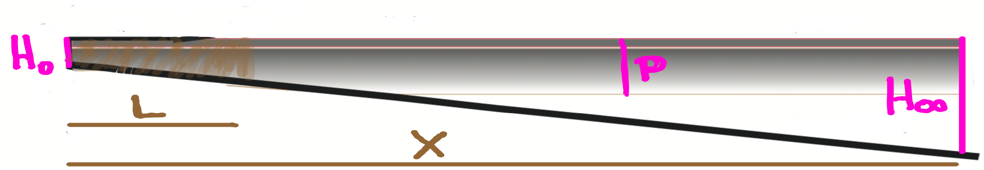



# Nearshore Sticky Waters

Its determining factors are:

- **Larger $\beta$ leads to a more sticky nearshore.**

$$\beta := \left(\frac{P - H_0}{H_\infty - H_0}\right)\frac{X}{L}.$$




  - If $\beta < 0$ then $P < H(x)$ for all $x$.
  - If $0 \leq \beta \leq 1$, the point where $P = H(x)$ is in the break zone,
  - if $\beta > 1$, this point is outside the break zone.
- **Decreasing $\gamma$ leads to more stickiness.** $I(\gamma, s, b) = \frac{(1-\gamma)s+\gamma PB}{\tau(x)}$

# The Future

- More realistic model
- Laboratory experiments
- Field experiments
- Extensions of model that address biogeochemical issues.
- Washed up flotsam

**Reprints/Preprints:**


- J.M.R., S.C. Venkataramani & C. Dawson (2014), *Nearshore Sticky Waters*. Ocean Modelling, **80**, 49-58.
- J.M.R., J. Ramirez & S. Venkataramani (2015), *An Oil Fate Model for Shallow Waters*, J. Mar. Science and Eng. **3**, pp1504-1543
- J. Ramirez, S. Moghimi, J.M.R. & S. Venkataramani (2018), *Modelling the Mass Exchange Dynamics of Oceanic Surface and Subsurface Oil*, Ocean Modellng


**Further information:**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`http://www.math.oregonstate.edu/~restrepo`



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