Turbulence and waves ``but how much mixing?”

and dissipation?

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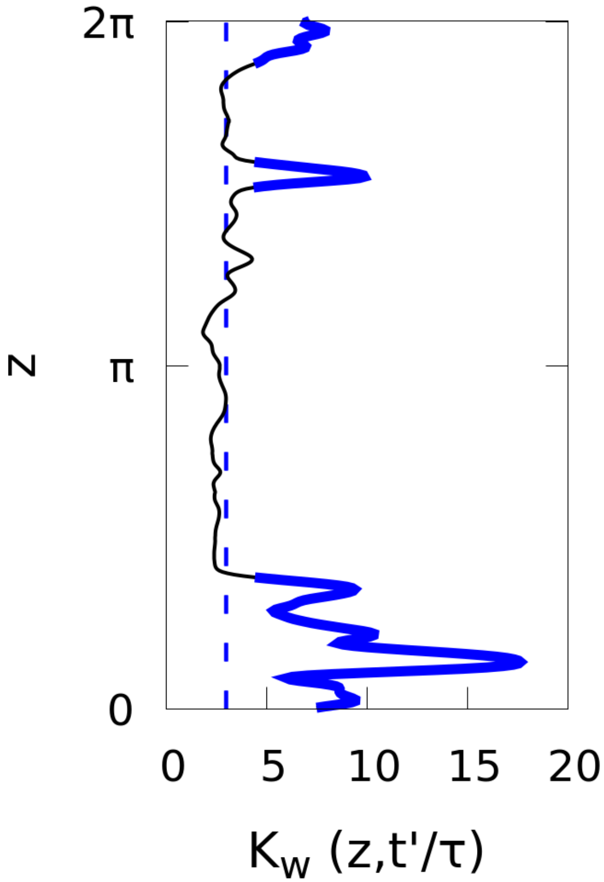
NSF/NCAR/Yellowstone: 76 decay or forced RST runs @1024^3 res., 6 forced runs @2048^3 ; ~ 85% background (free) time
DOE/Titan: Decay RST run @ 4096^3 point resolution, a few outputs of which are on the *John Hopkins turbulence data base*

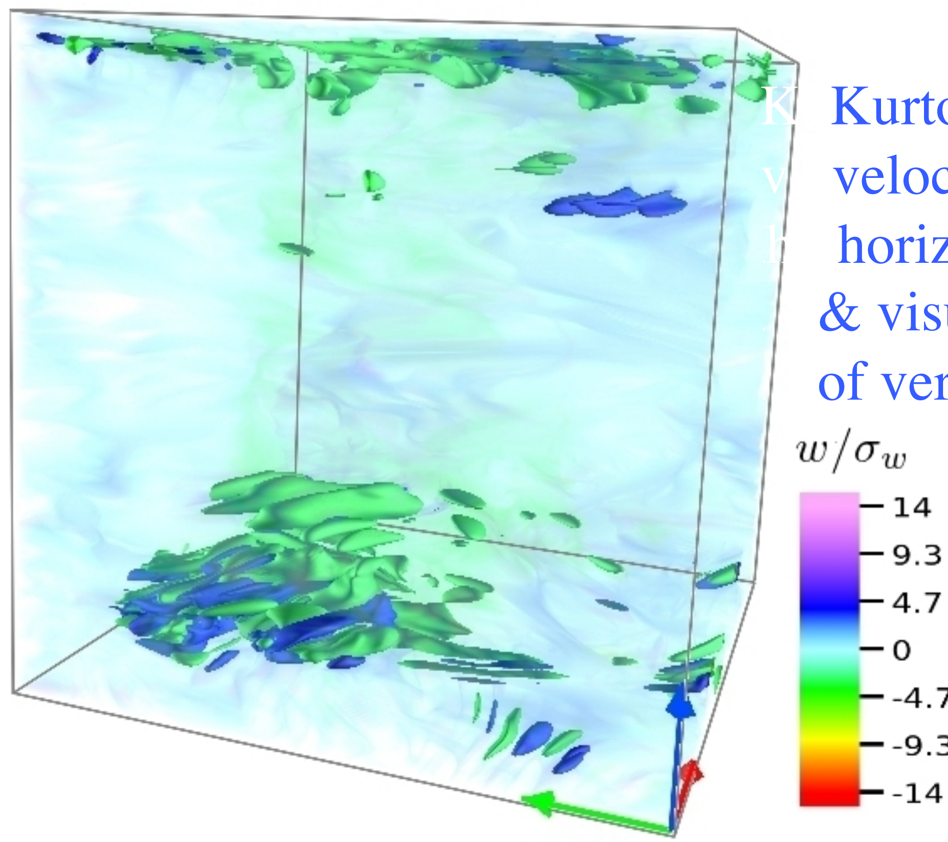
Scaling laws for mixing & dissipation in unforced rotating stratified turbulence, *J. Fluid Mech*. **844**, 519, 2018
Variations of characteristic time scales in rot. strat. turb. using a large parametric numerical study, *Eur. Phys. J-E* **39**, 8, 2016
Evidence for Bolgiano-Obukhov scaling in rotating stratified turbulence using high-resolution Direct Num. Sim., *Phys. Fluids* **27**, 055105, 2015

BLayers18, KITP, Santa Barbara, June 4, 2018

pouquet@ucar.edu

Forced 512^3 run,
Fr~ 0.08, Re~ 3800,
No rotation *(Newton-Calabria)*



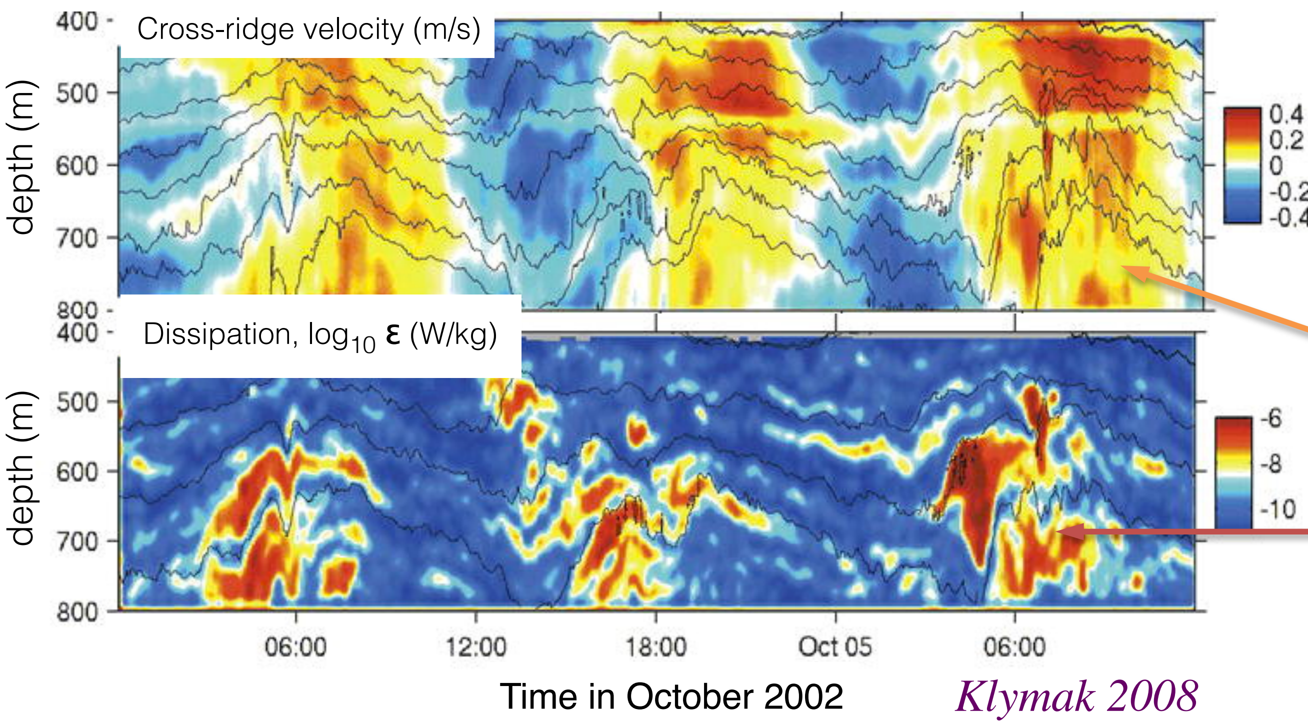


Kurtosis of vertical velocity w, horizontally averaged, & visualization of vertical velocity w

Strong intermittency of the vertical *velocity*
Model through the Vieillefosse system with gravity waves, forcing and dissipation

Feraco+, Vertical drafts and mixing in stratified turbulence: sharp transition with Froude number. Submitted to *Eur. J. Phys. Lett.*. ArXiv/ 1806.00342
(see also Rorai+, Turbulence comes in bursts in stably stratified flows, Phys. Rev. E ***89****, 043002, 2014)*

b) Breaking internal tides over tall steep topography



Hawaiian ridge

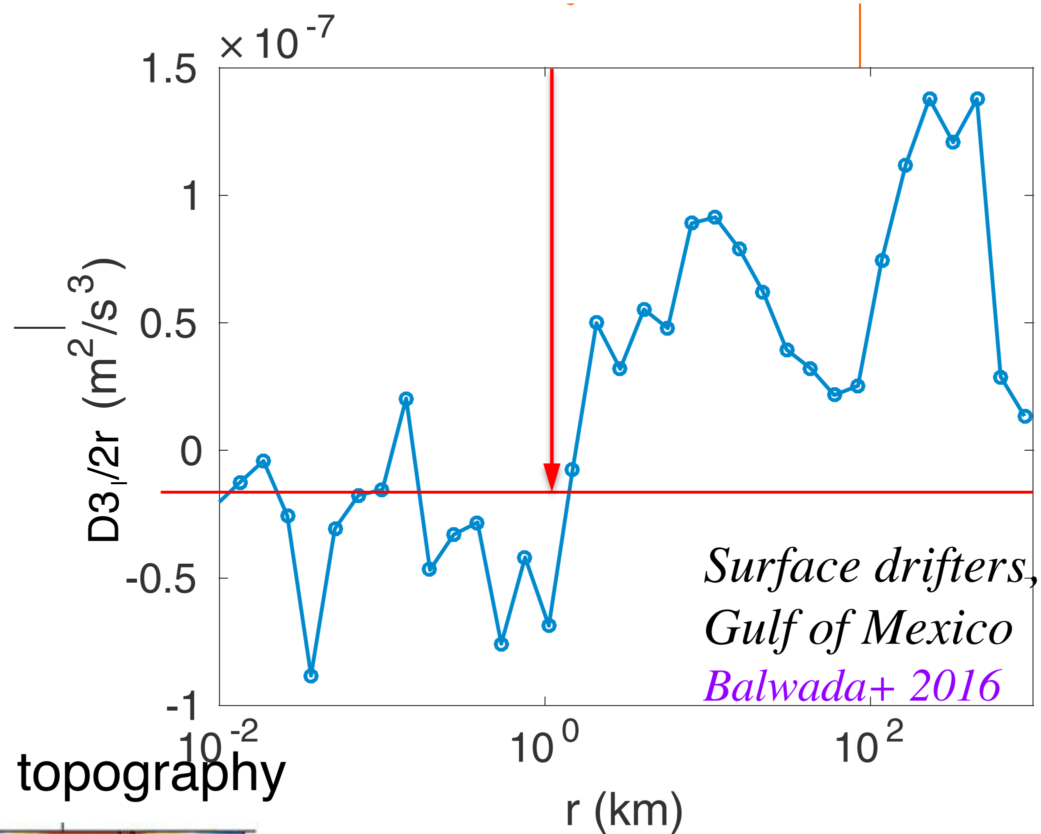
$U=0.1m/s$, $L=1000m$

$\tau_{NL} = L/U \sim 3\ hrs$

$N=0.001s^{-1}$, $Fr \sim 0.1$

$\varepsilon_V \sim 10^{-6} W \sim \varepsilon_D = U^3/L$

$Re \sim 10^8$, $R_B \sim 10^6$



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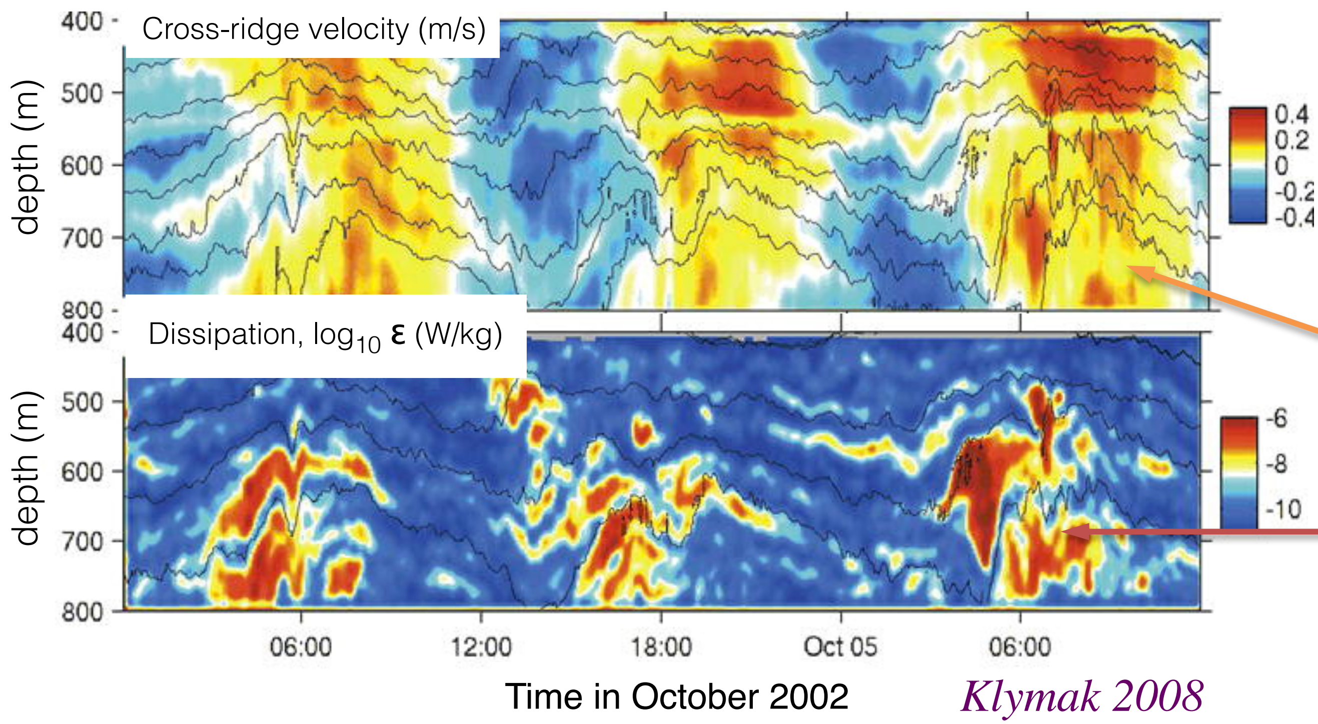

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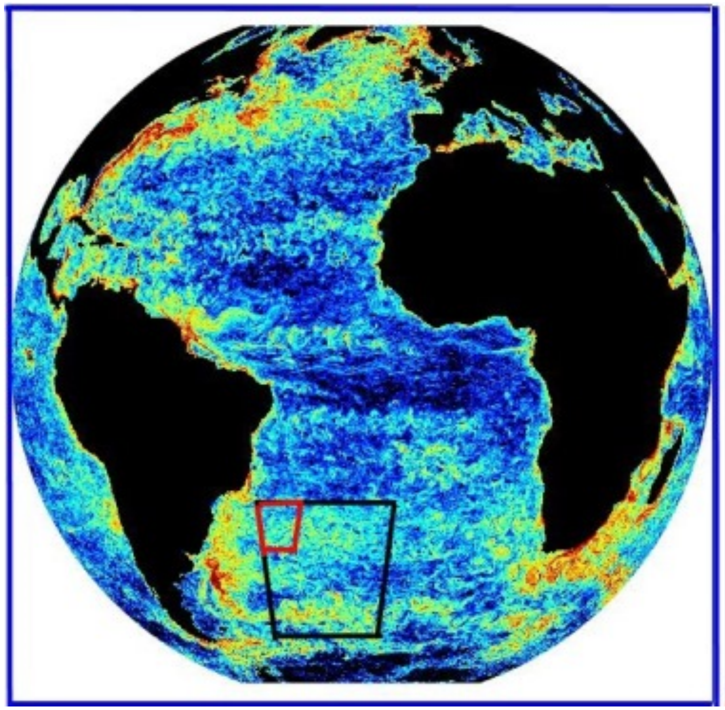
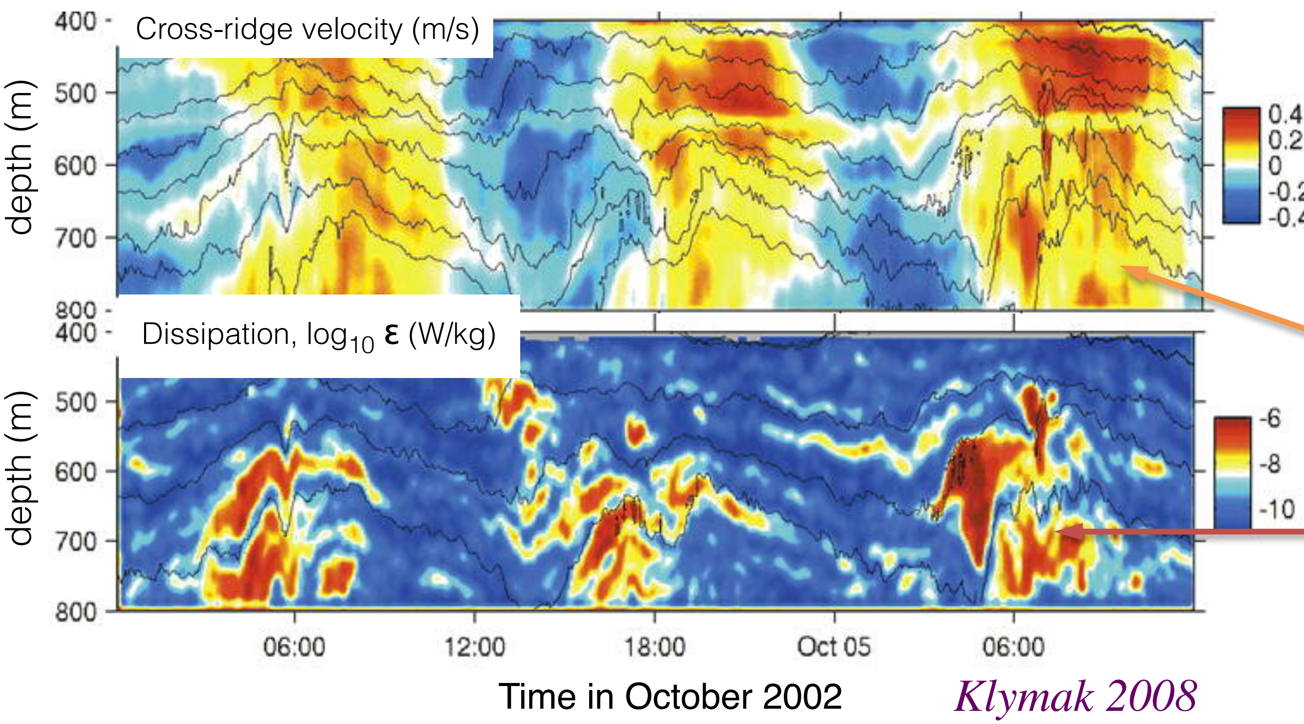


Ocean model,
smallest resolved scale: 20km

Kinetic energy dissipation

Pearson+2018

b) Breaking internal tides over tall steep topography



Klymak 2008

Hawaiian ridge

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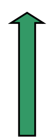
Outline

- Introduction
- Equations and characteristic scales
- Parametric study with direct numerical simulations

- Three constitutive laws for normalized E_P , E_w , ε_V

- Consequences of  for scaling of mixing efficiency+

- Discussion, Conclusions and Perspectives

Incompressible Boussinesq equations + rotation
3D cubic box, periodic boundary conditions

$$\frac{\partial \mathbf{u}}{\partial t} + \omega \times \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -N\vartheta \hat{e}_z - \nabla \mathcal{P} + \nu \nabla^2 \mathbf{u},$$

$$\frac{\partial \vartheta}{\partial t} + \mathbf{u} \cdot \nabla \vartheta = N w + \kappa \nabla^2 \vartheta,$$

$[\theta] = [L\, T^{-1}]$

Governing dimensionless parameters:

Reynolds, Froude, Rossby, Prandtl=1

$$Re = \frac{U_0 L_0}{\nu}, \quad Fr = \frac{U_0}{L_0 N}, \quad Ro = \frac{U_0}{L_0 f}, \quad Pr = \frac{\nu}{\kappa}, \qquad f = 2\Omega$$

Fr < 1 , together with Re >> 1

N/f = Ro/Fr > 2.5 (ocean, atmosphere)

SCALES: Purely stratified flow (f=0): $Fr = U_0/[NL_0] < 1$

Scale at which $Fr = 1$?

Purely stratified turbulence (f=0): Fr $= U_0/[NL_0] < 1$

Scale at which Fr =1?

→ $L_B = U_0/N$, buoyancy scale

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l ? such that Fr(l)=1, for a Kolmogorov spectrum: $u(l) \sim \varepsilon_v^{1/3} l^{1/3}$

→ $l = l_{Oz} = [\varepsilon_v/N^3]^{1/2}$, Ozmidov scale

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Buoyancy Reynolds number: $R_B \equiv \varepsilon_v/[\nu N^2] = [l_{Oz}/\eta]^{4/3}$

$R_B = 1$ for $l_{Oz} = \eta = [\varepsilon_v/\nu^3]^{-1/4}$ *(η: Kolmogorov dissipation scale)*

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Numerical conendrum: large R_B, small Fr for geophysical flows

With rotation: Ro also small; *Ro/Fr= N/f ~ 5 (ocean) or 100 (atmosphere)*

Purely stratified flow (f=0): $Fr = U_0/[NL_0] < 1$

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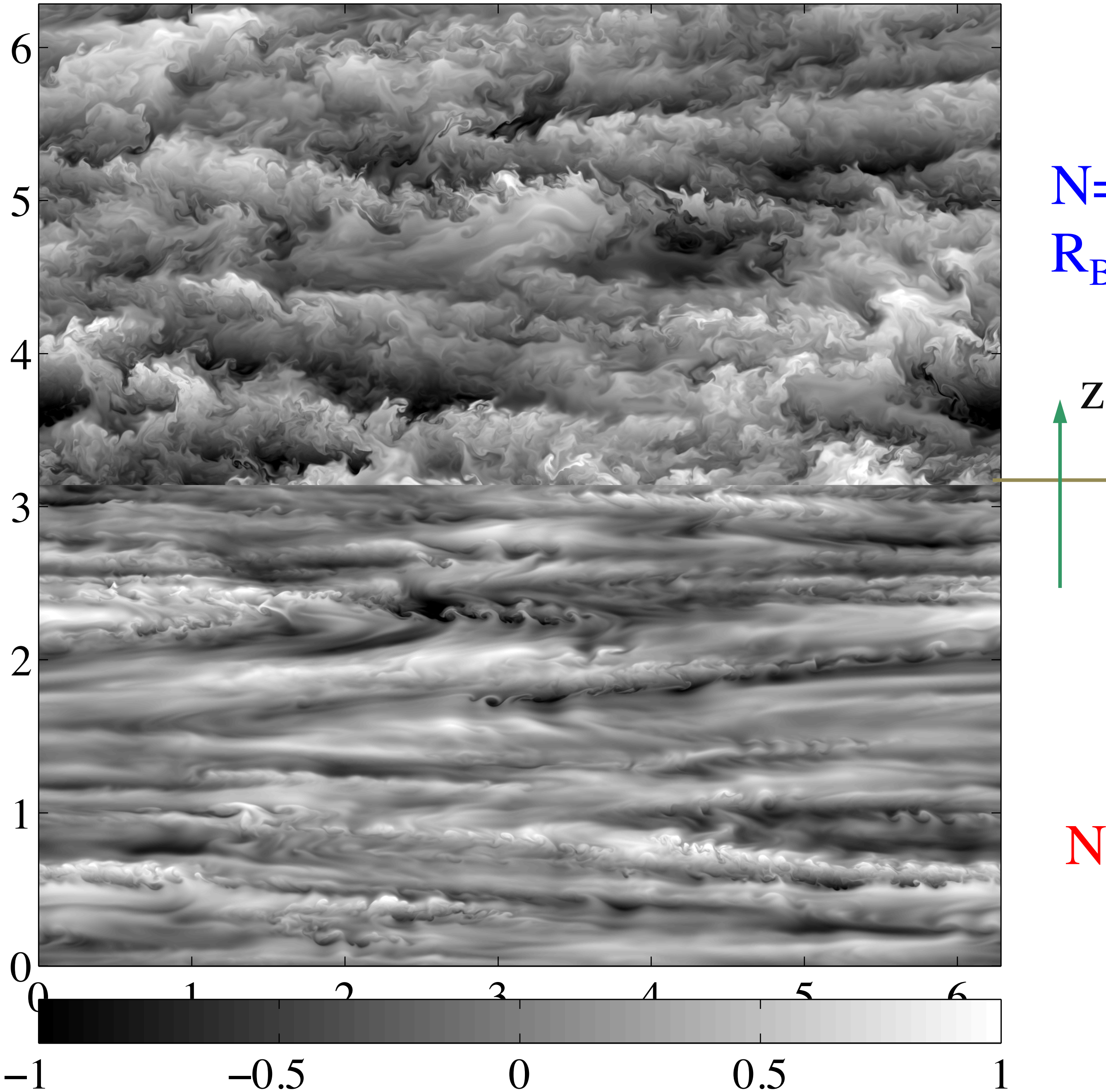
Stratification, no rotation, large scale forcing, Re~ 24000, 2048^3 grid:

Temperature fluctuations, xz slice



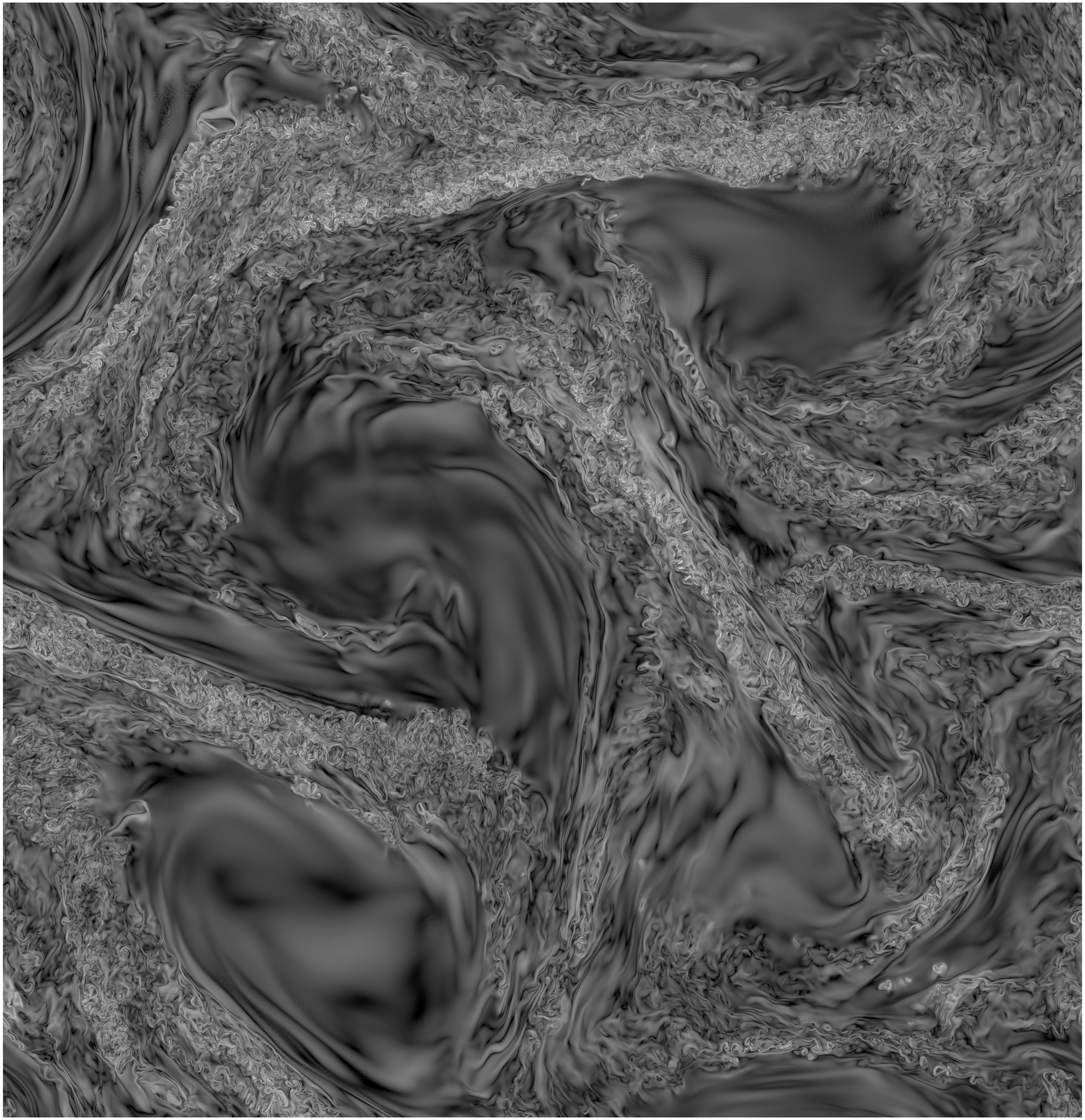
N=4, Fr ~ 0.11

$R_B = ReFr^2 \sim 300$

N=12, Fr ~ 0.03

$R_B \sim 22$

Rorai et al., 2014



Vertical vorticity

.at peak of dissipation

($\omega_{z\text{-mag}}$, *horizontal cut*):

Eddies & lanes

Plot @ full 4096^2 res.

GHOST pseudo-spectral code (DOE/titan, 2014)

Log scale

f=2.7, ω_{rms} ~17

Re=55000 ``***ocean***”

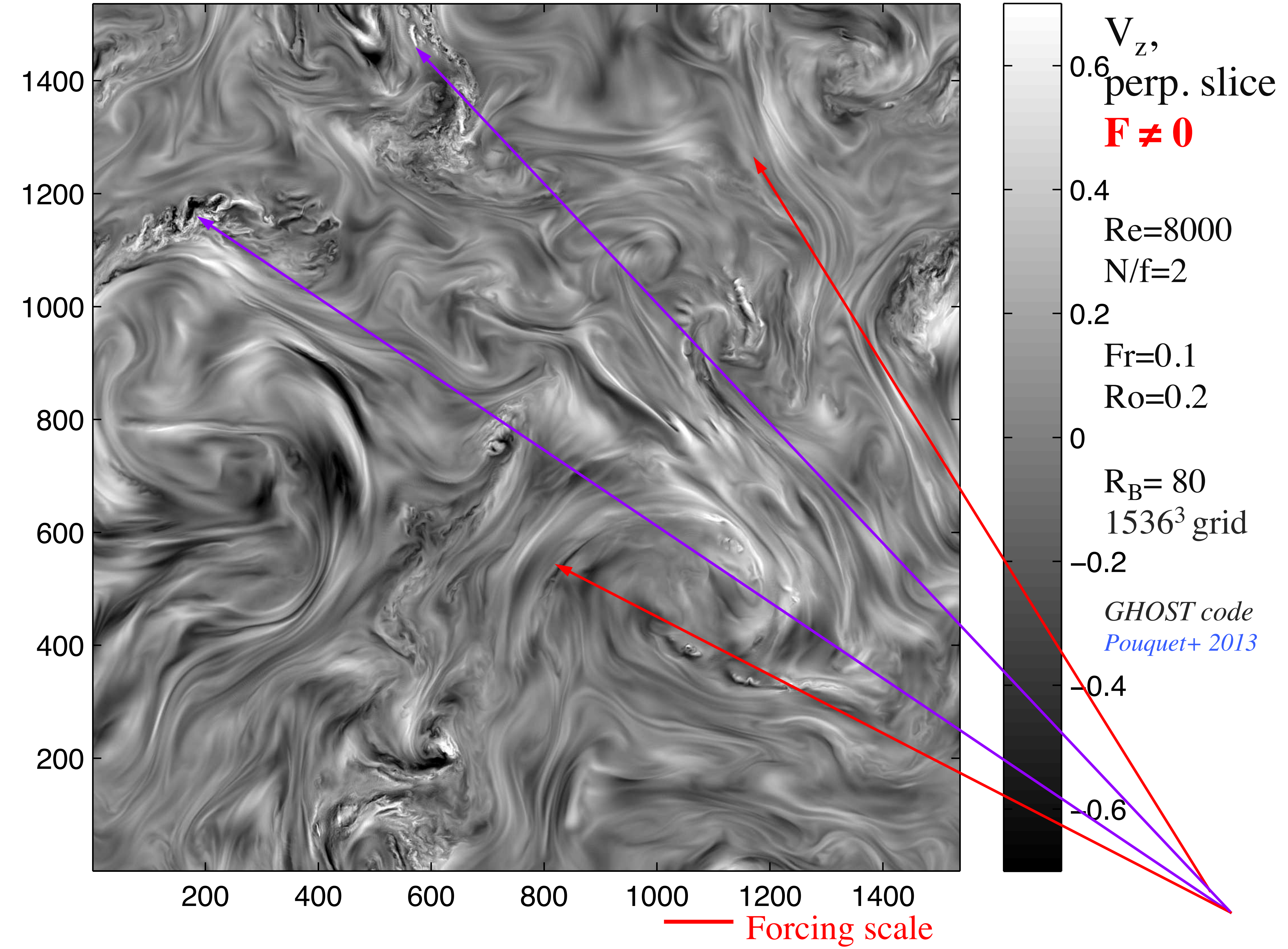
Fr=0.024, N/f= 5

R_B=32, $k_{max}\eta$ ~2

No forcing, k_0~2.5

Bolgiano-Obukhov scaling

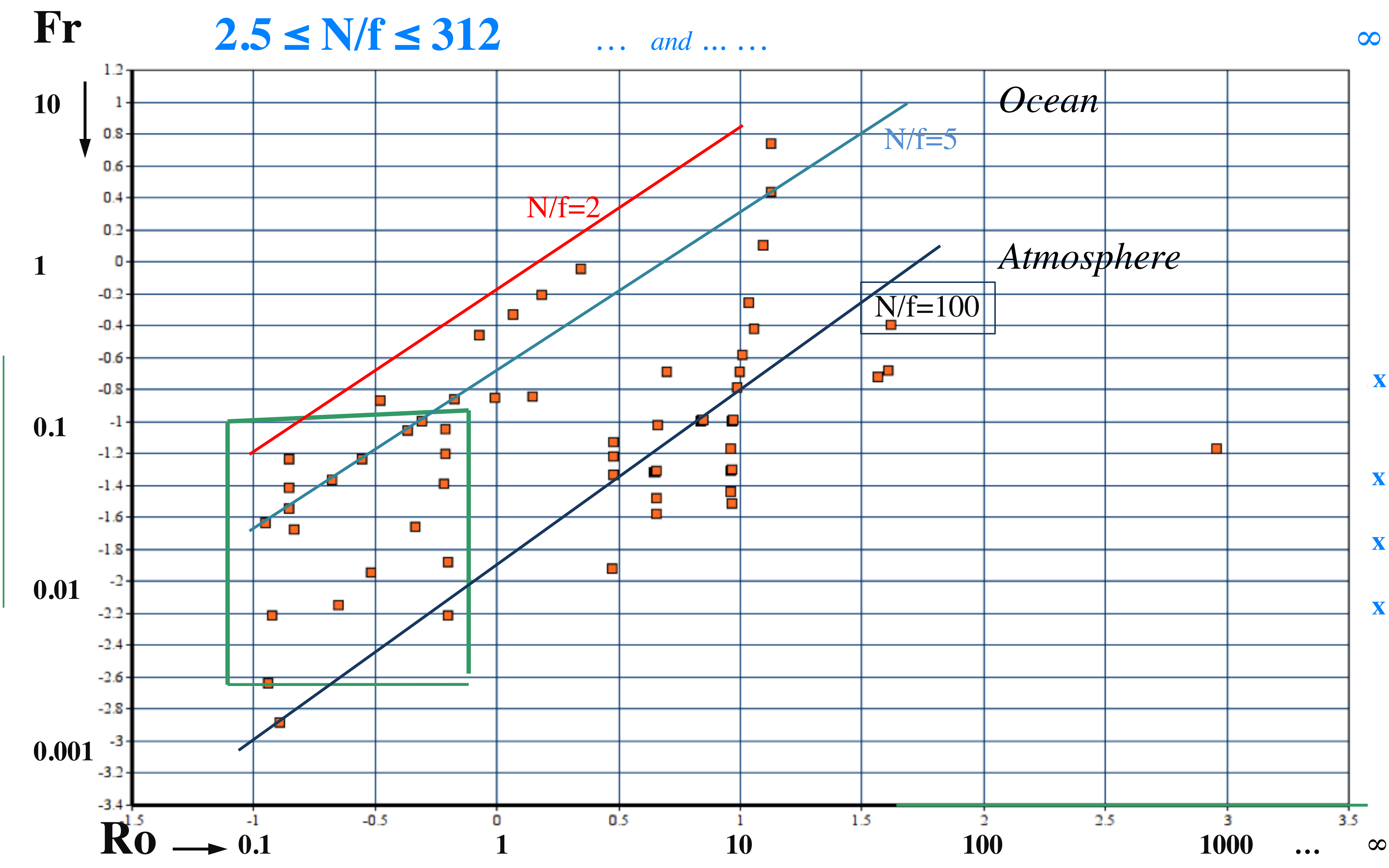
Rosenberg+ 2015



1400
1200
1000
800
600
400
200
200
400
600
800
1000
1200
1400
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
V_z, perp. slice
F ≠ 0
Re=8000
N/f=2
Fr=0.1
Ro=0.2
R_B= 80
1536^3 grid
GHOST code
Pouquet+ 2013
Forcing scale

THE PARAMETRIC STUDY

1024^3, Re~1.2×10^4, $\theta_{(t=0)}=0$, F=0, k_0~2.5, isotropic (I); GHOST code.

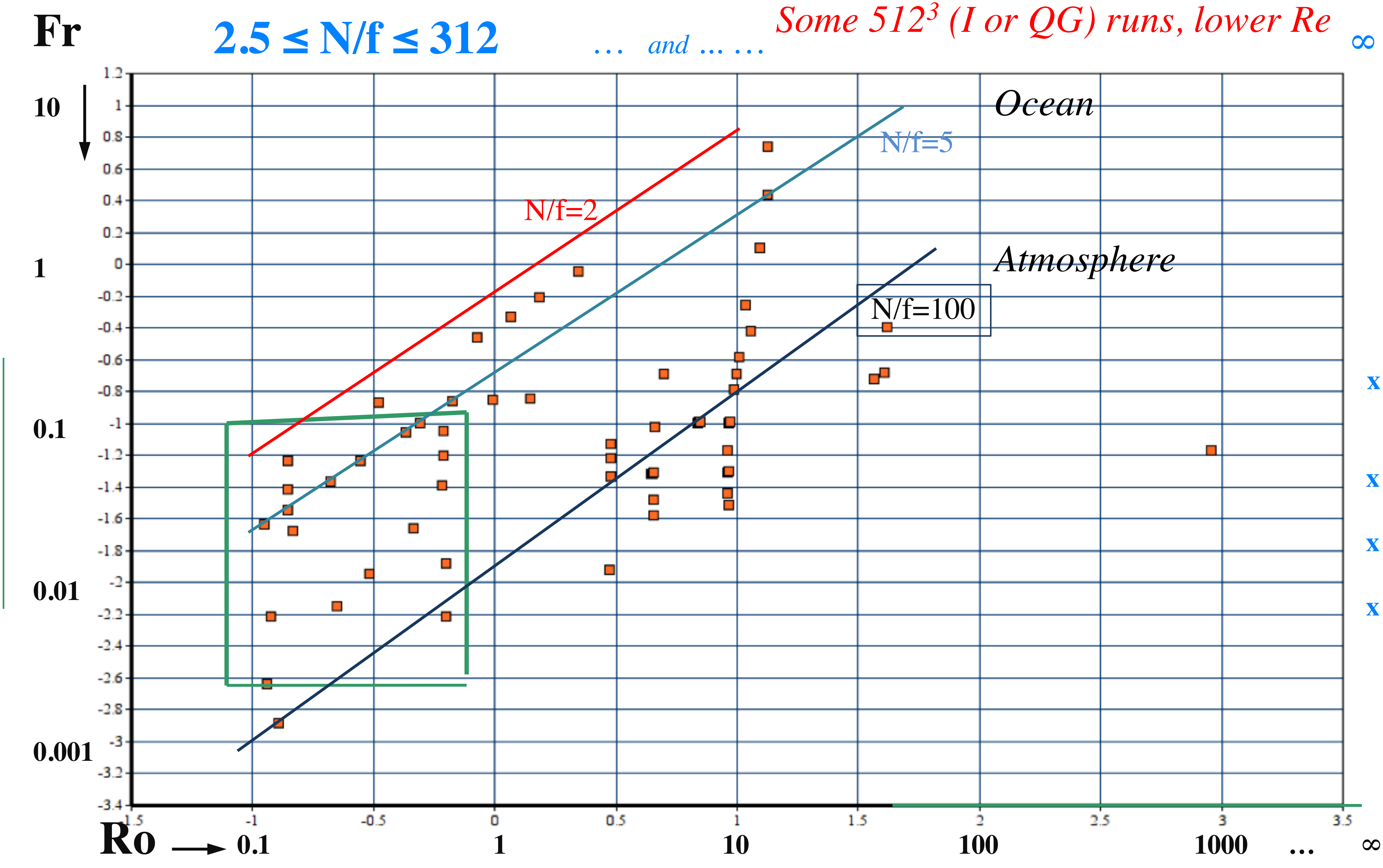


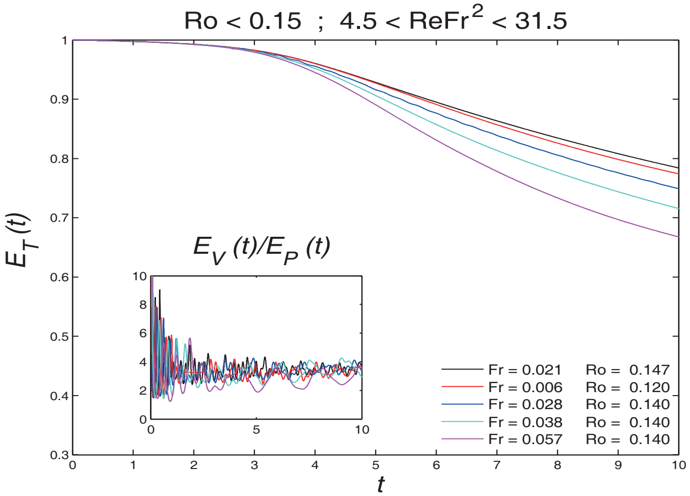
THE PARAMETRIC STUDY

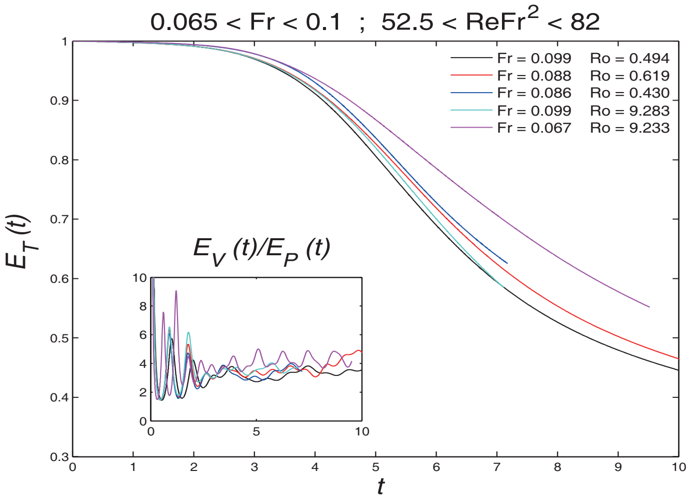
1024^3, Re~1.2×10^4, $\theta(t=0)=0$, F=0, k_0~2.5, isotropic (I); GHOST code.

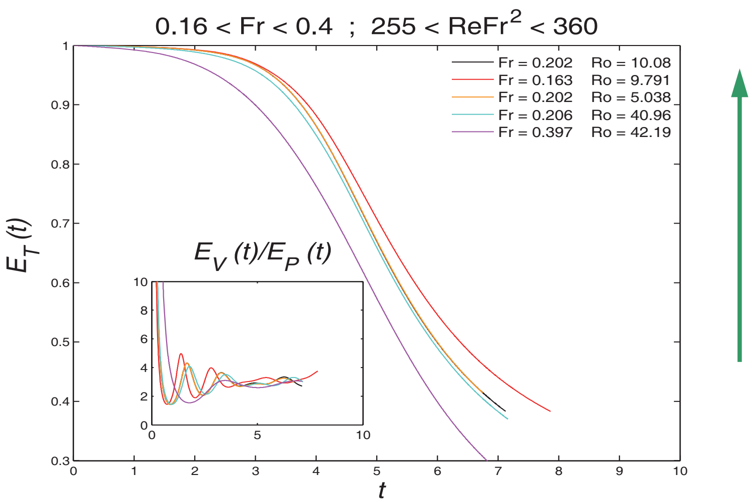
Some 512^3 (I or QG) runs, lower Re

$2.5 \le N/f \le 312$ *… and … …*









$E_T = f(t)$ and
$E_v/E_p = f(t)$

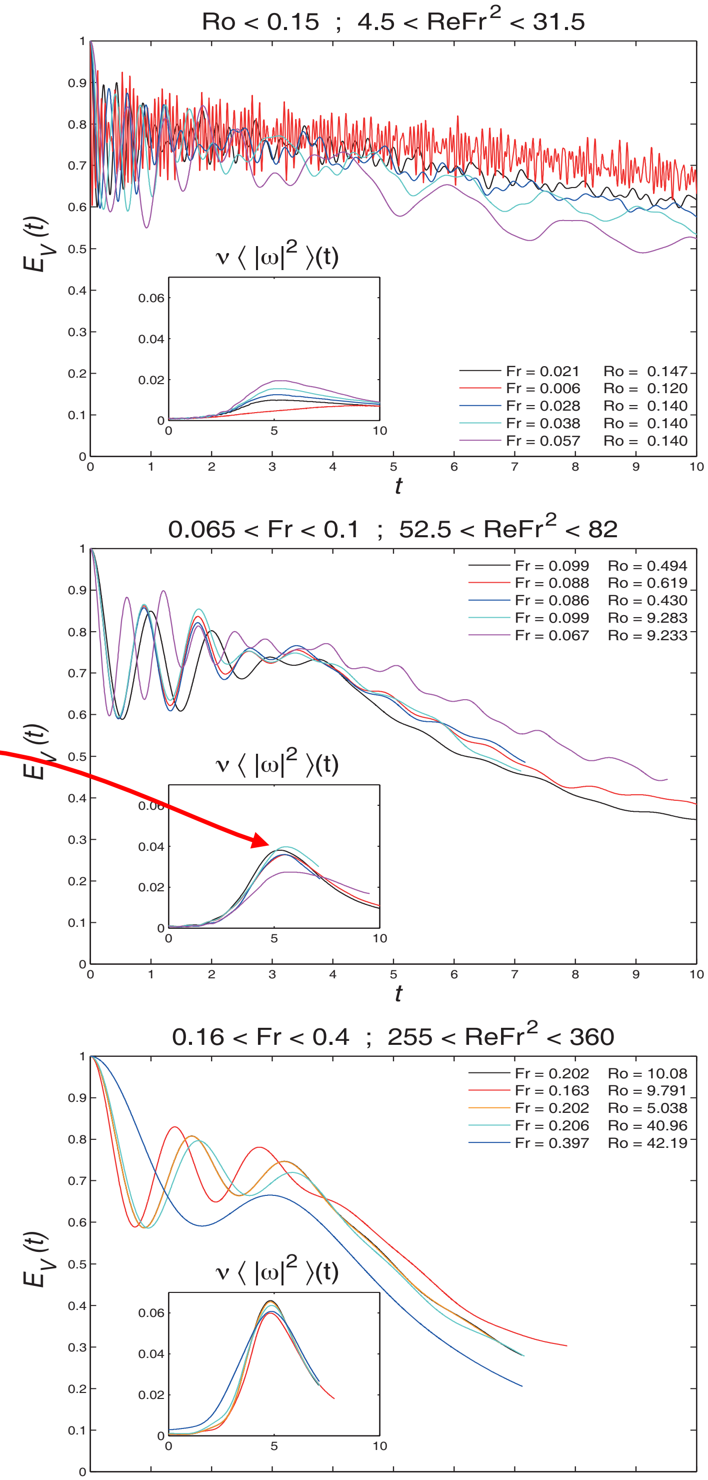
← Low Fr, low Ro
Fr~0.03

← Intermediate range
Fr~0.09

← High R_B
Fr~ 0.2

$E_v = f(t)$

and kinetic energy dissipation $\varepsilon_V = f(t)$



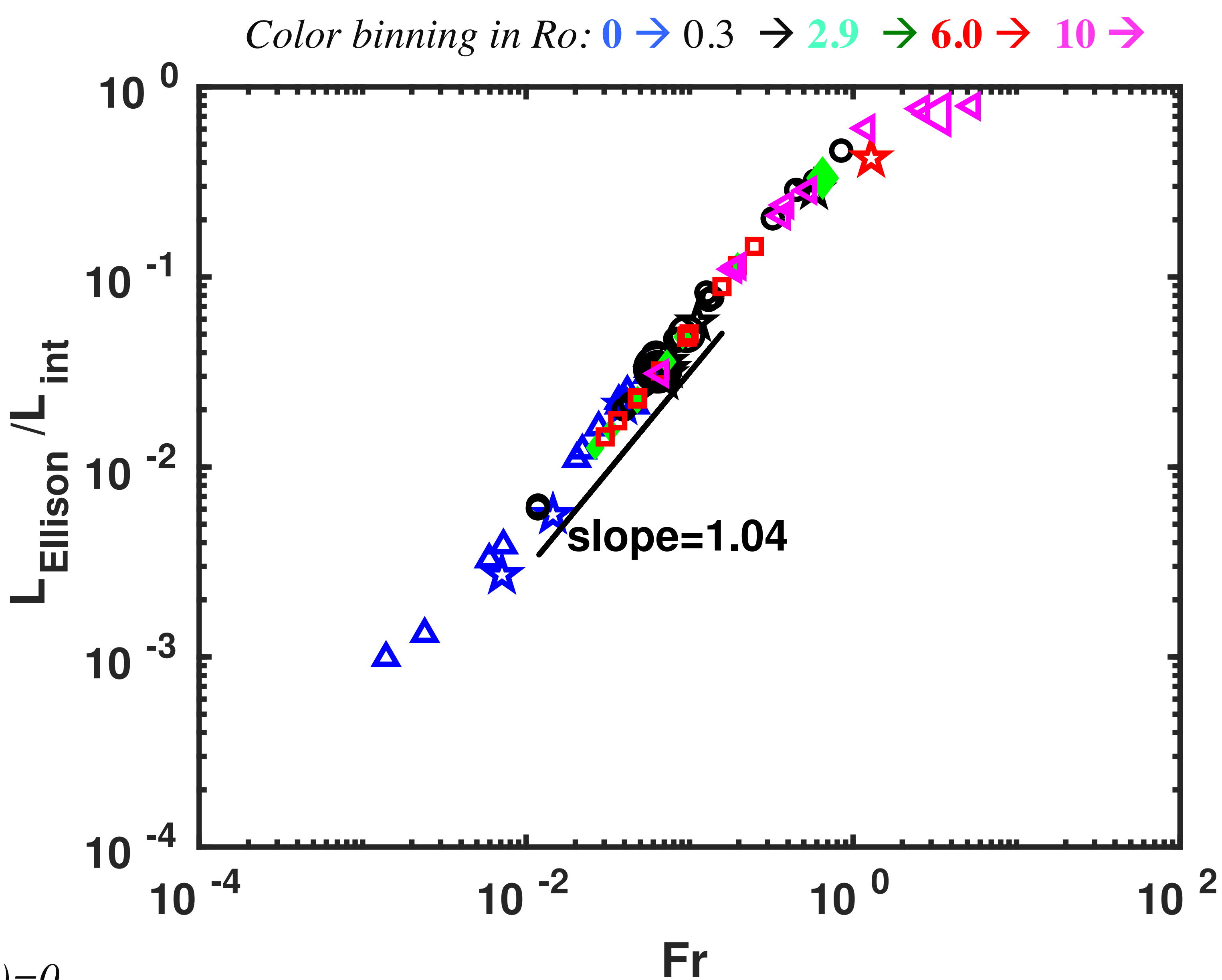
Statistics taken at peak of dissipation

with a temporal averaging within

variations of $\pm 0.025\, \varepsilon_V$

Ellison scale
$L_E = \sqrt{E_P}/N$

Integral scale
$L_{int} = f(E_V)$



Initial conditions:
Stars: QG
Otherwise, HIT, θ(t=0)=0
Size of symbol: proportional to viscosity

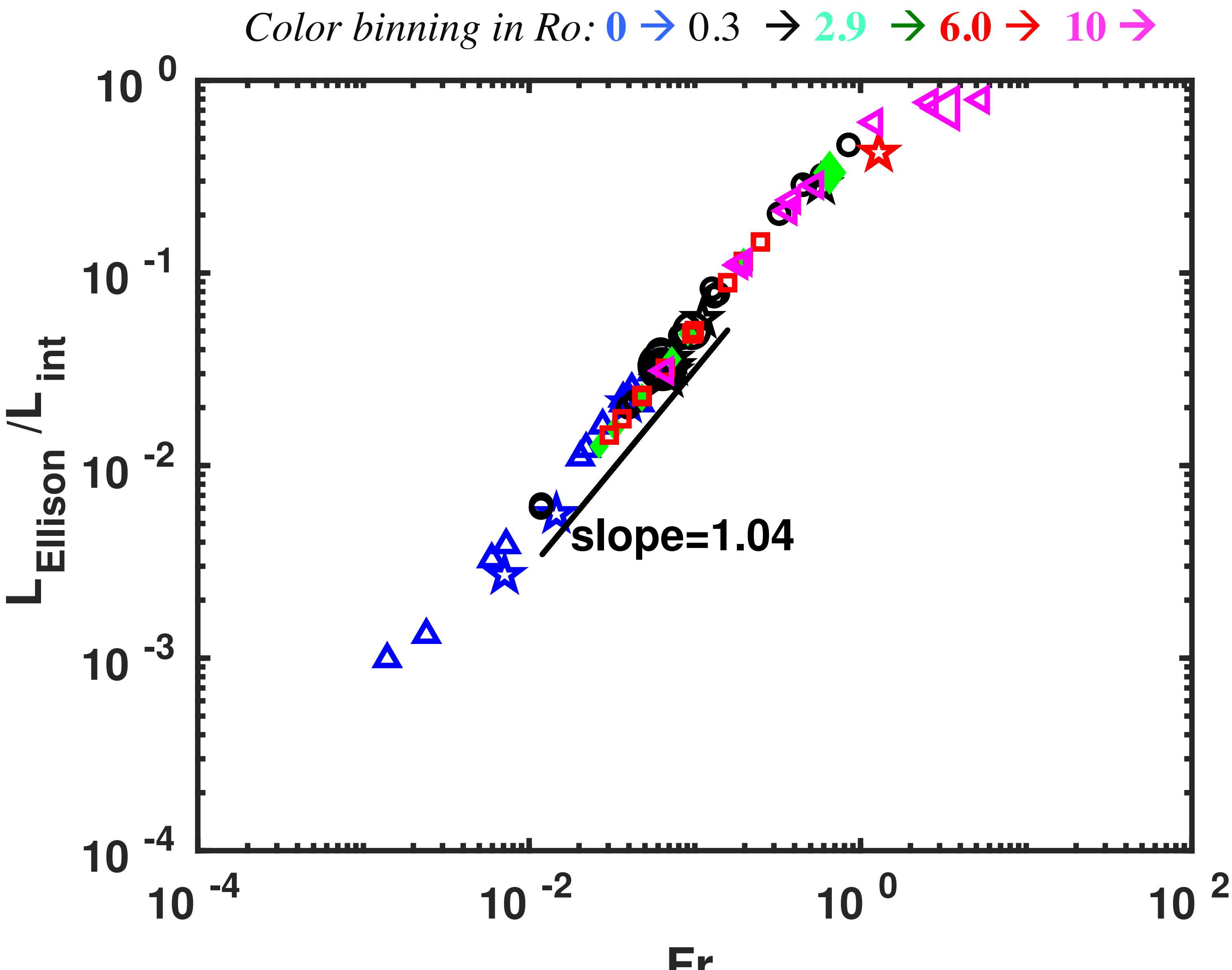
$L_E / L_{int} \sim Fr \sim U_{rms}/[L_{int}N]$

Ellison scale

$L_E = \sqrt{E_P} / N$

Integral scale

$L_{int} = f(E_V)$

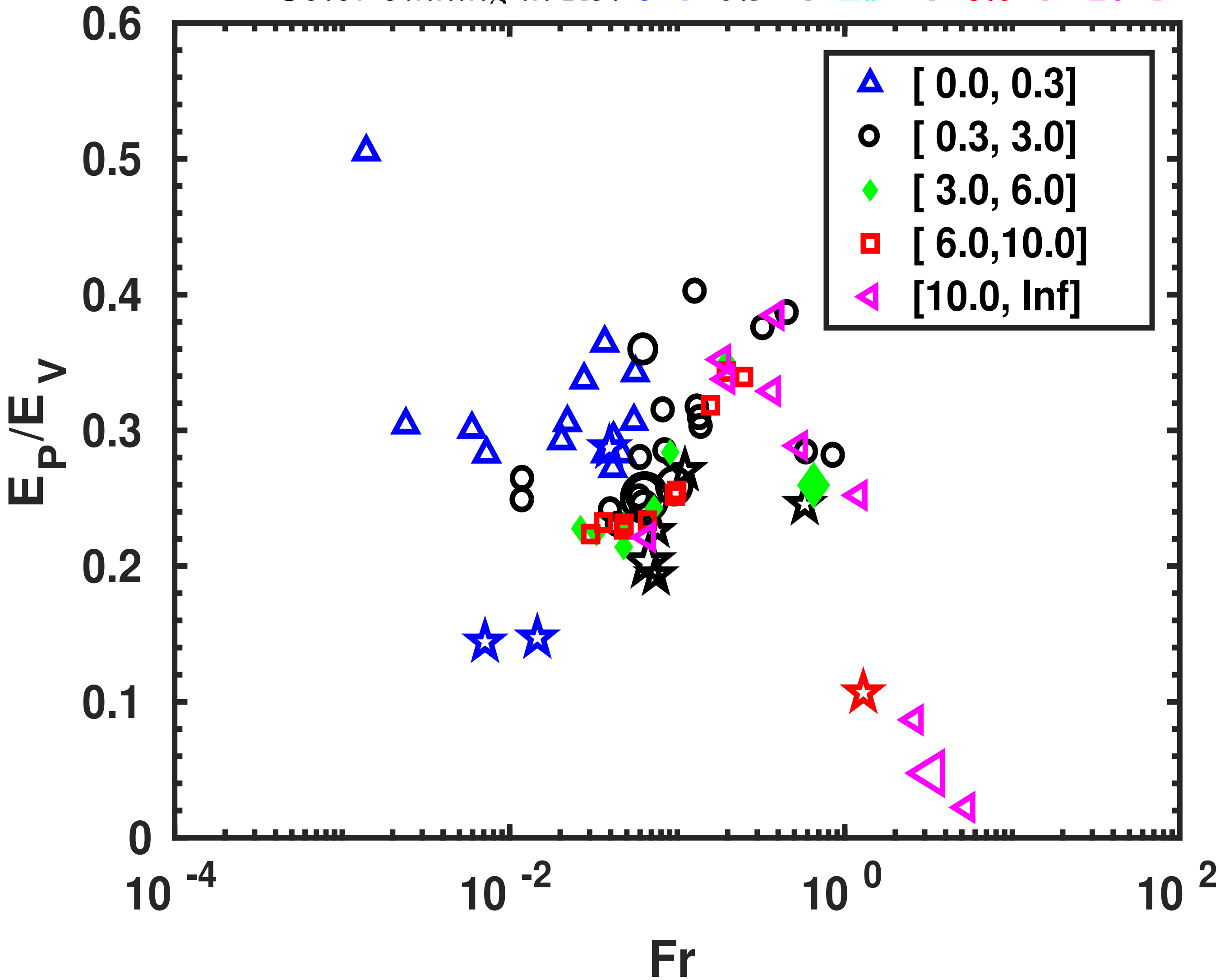


$L_E / L_{int} \sim Fr \sim U_{rms}/[L_{int}N]$

→ $L_E \sim L_B = U_{rms}/N$

Ratio of potential to kinetic energy

Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →



Initial conditions:

Stars: QG

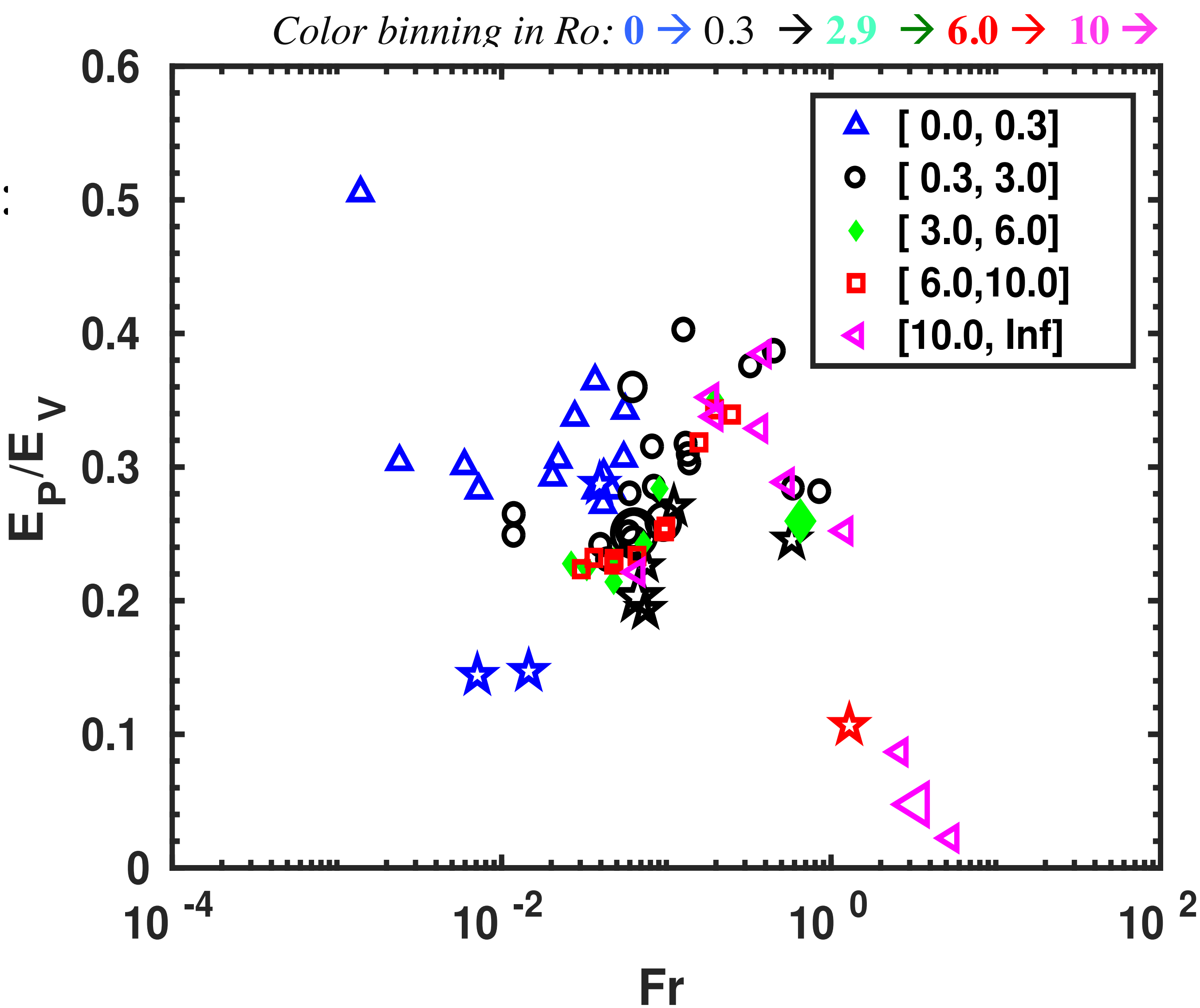
Otherwise, u_{perp}~ w and θ=0

Size of symbols: Roughly inversely proportional to numerical resolution, that is, in fact, to Reynolds number

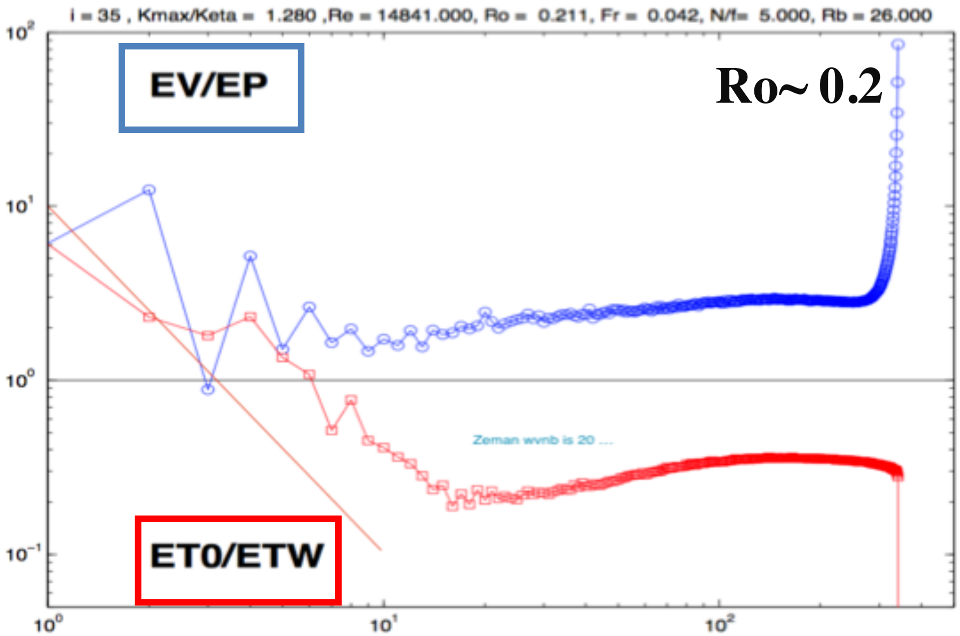
Ratio of potential to kinetic energy

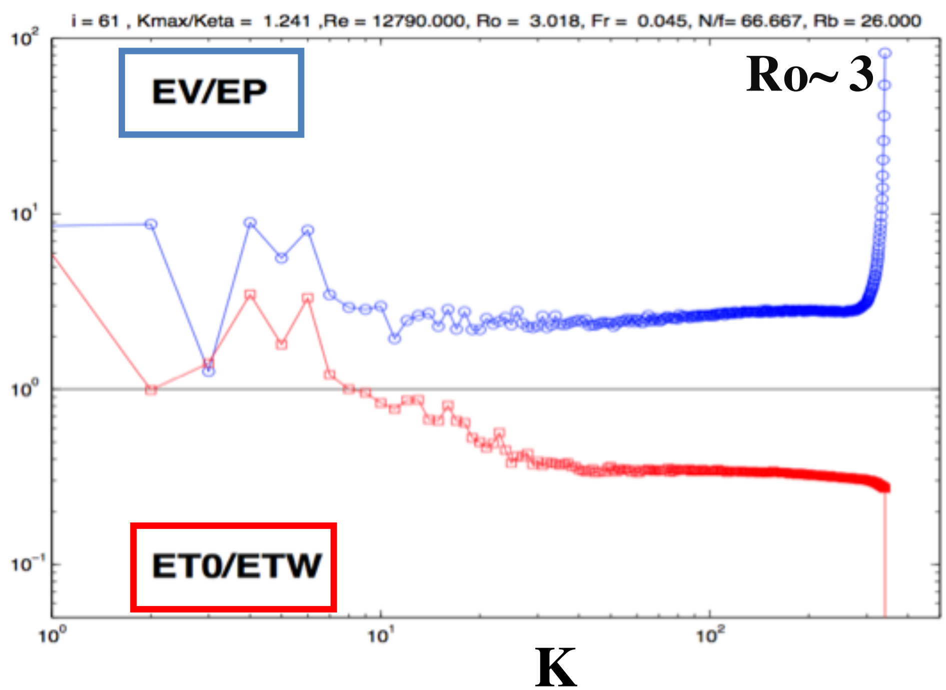
Intermediate regime:
$E_P \sim E_V$
Or

$\Theta_{rms} \sim U_0$ (1)



Stars: Quasi-geostrophic ICs





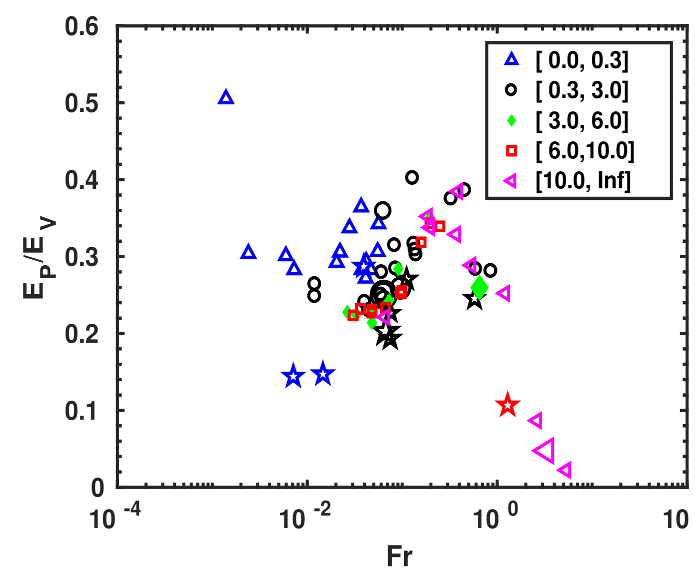
Ratios of energy **spectra** at peak:

Kinetic to potential

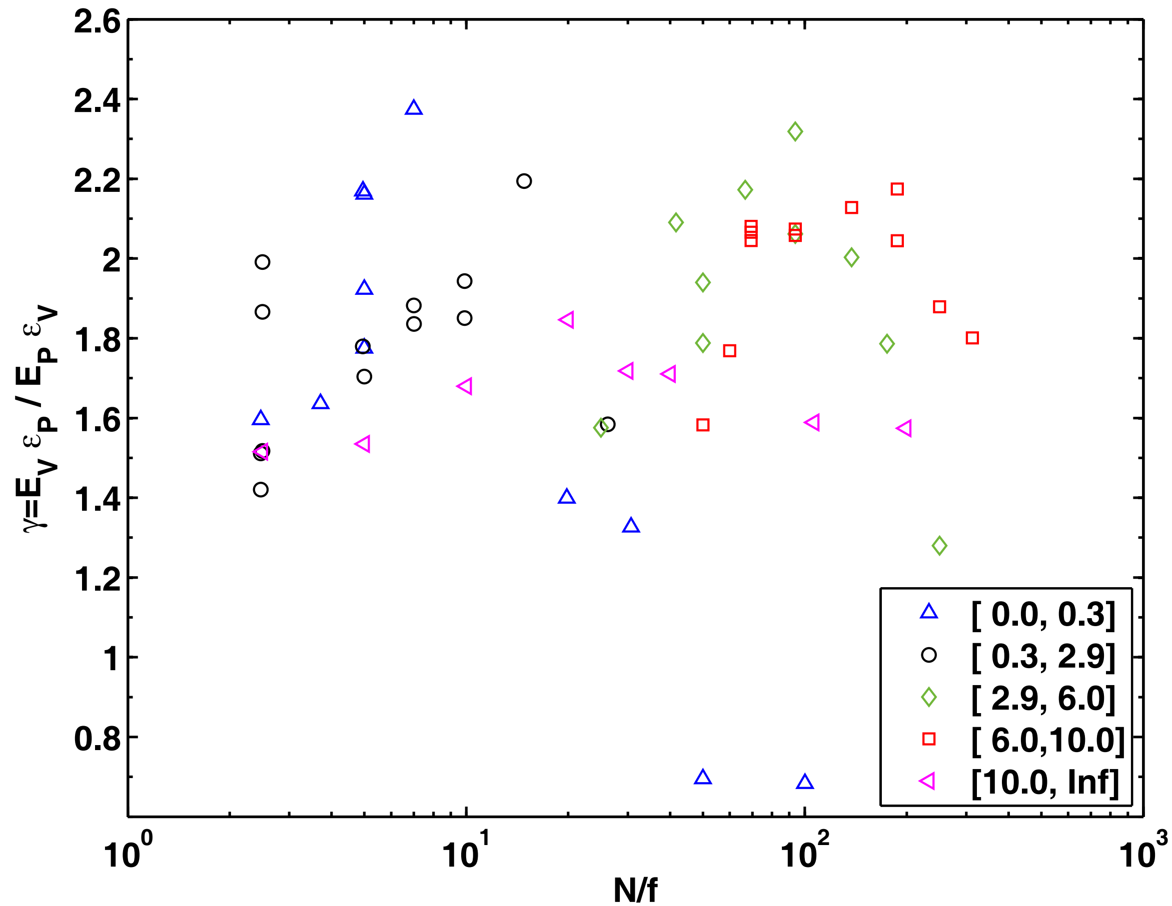
and

zero to wave mode

Both runs: Fr=0.04 , R_B~ 26

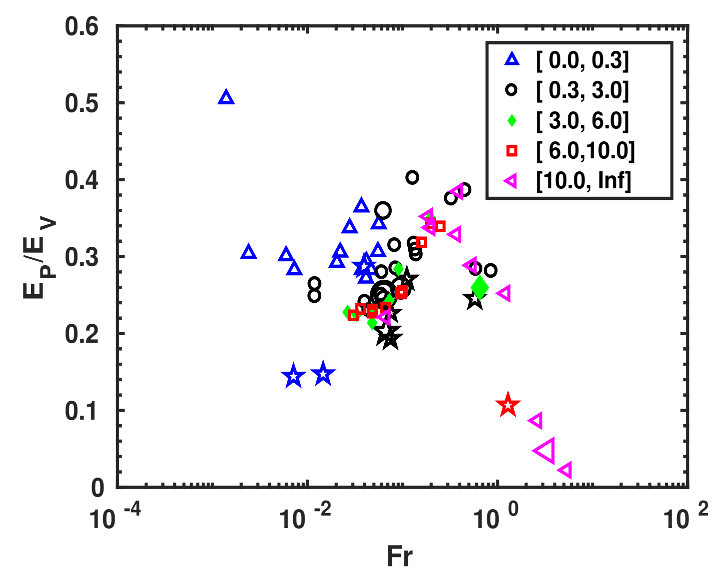


Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →



Ratio of kinetic to potential energy **effective dissipation** times

$$T_V/T_P = [E_V/E_P]*[\varepsilon_P/\varepsilon_V]$$



Rosenberg+ EPJ-E 2016, 2017

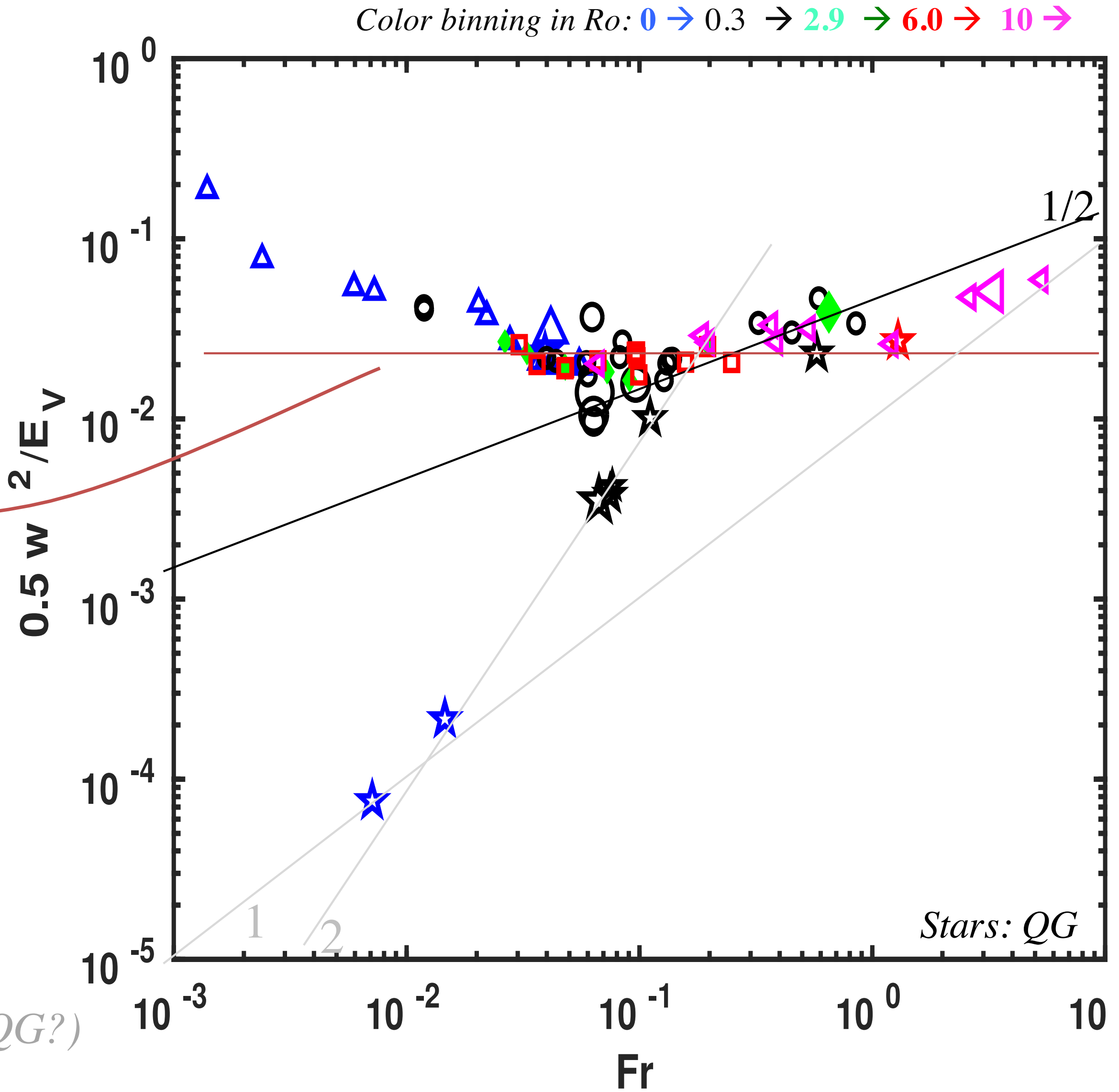
Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

Low Fr:
. influence of initial conditions

Vertical velocity
around peak of dissipation

Intermediate values:
$w^2/ 2E_V \sim Fr^0$
Or

$w/U_0 = a\ Fr^0$ (2a)



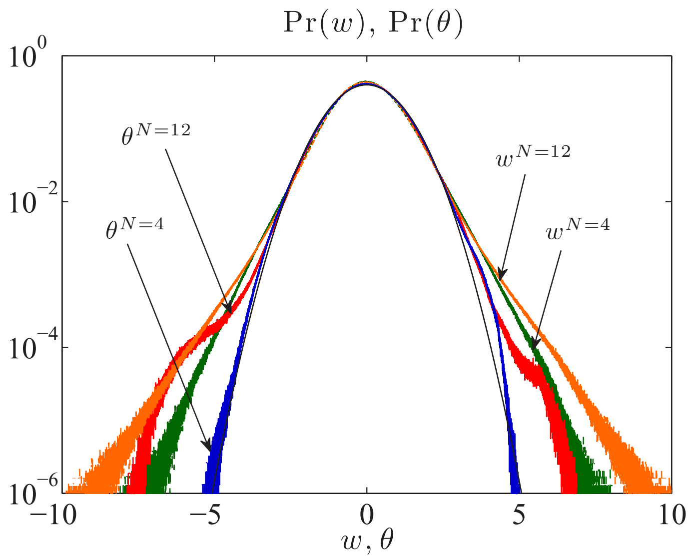
Other possible scaling:
$w/U \sim Fr^{1/4}$ (2b, data?),
or $w/U \sim Fr^{1/2}$ *(2c, Maffioli+ 2016),*
.or $\sim Fr$ *(2d, incompressibility; & QG?)*

Vertical velocity

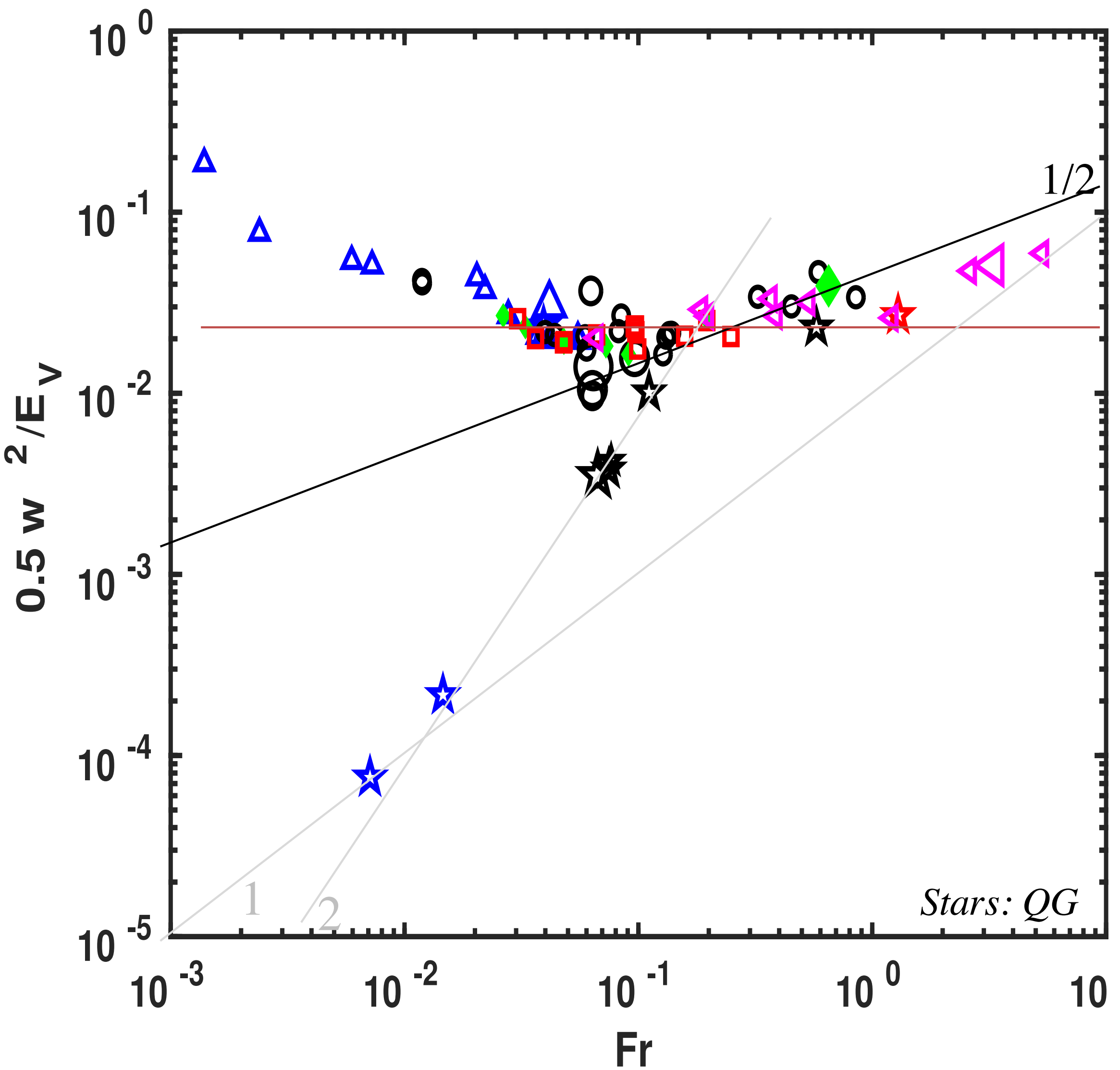
around peak of dissipation

Rorai+ PHYSICAL REVIEW E **89**, 043002 (2014)

2048³ forced



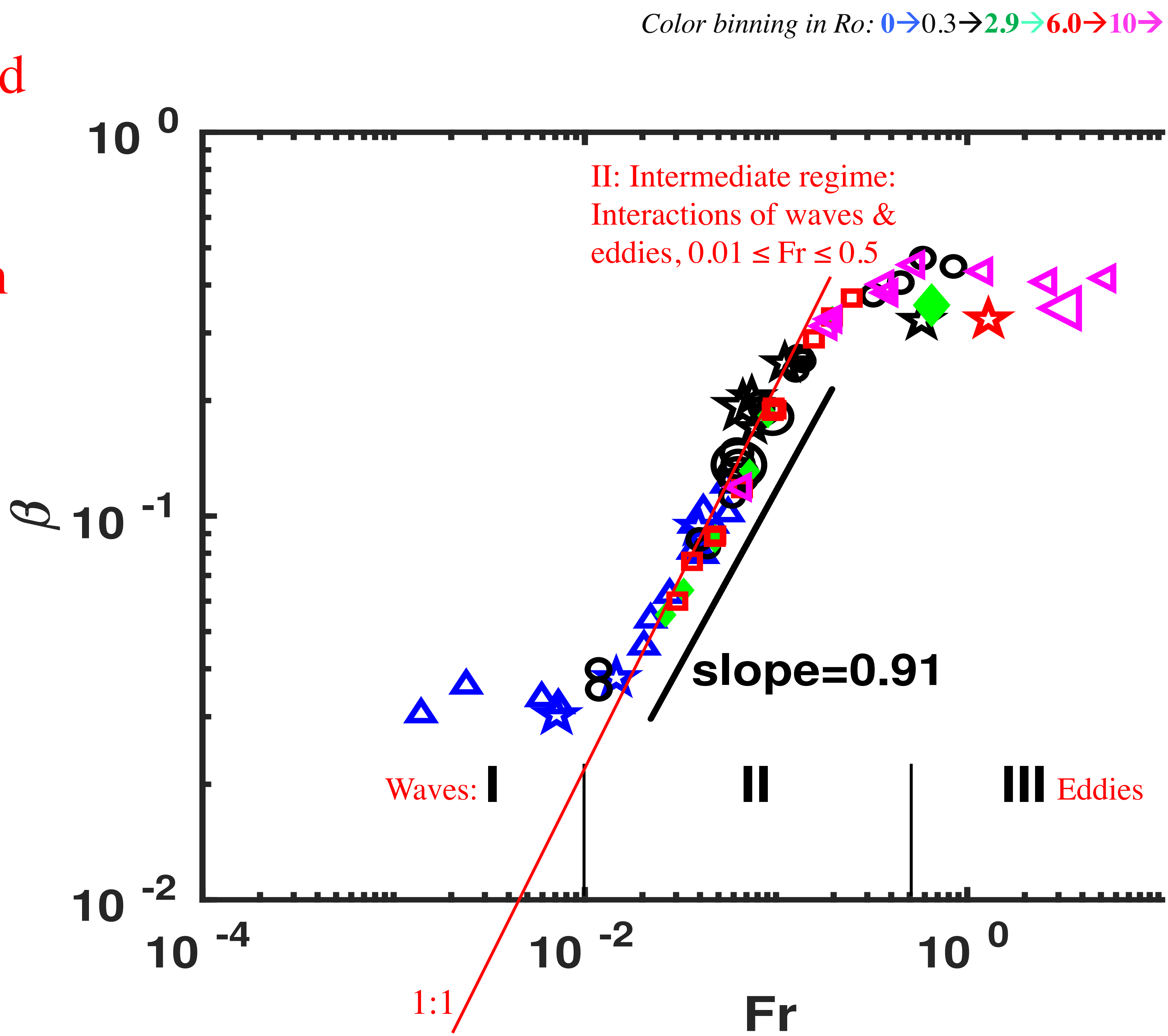
also Feraco+ 2018



Normalized kinetic energy dissipation rate

$\beta = \varepsilon_V / \varepsilon_D$

→ 3 regimes

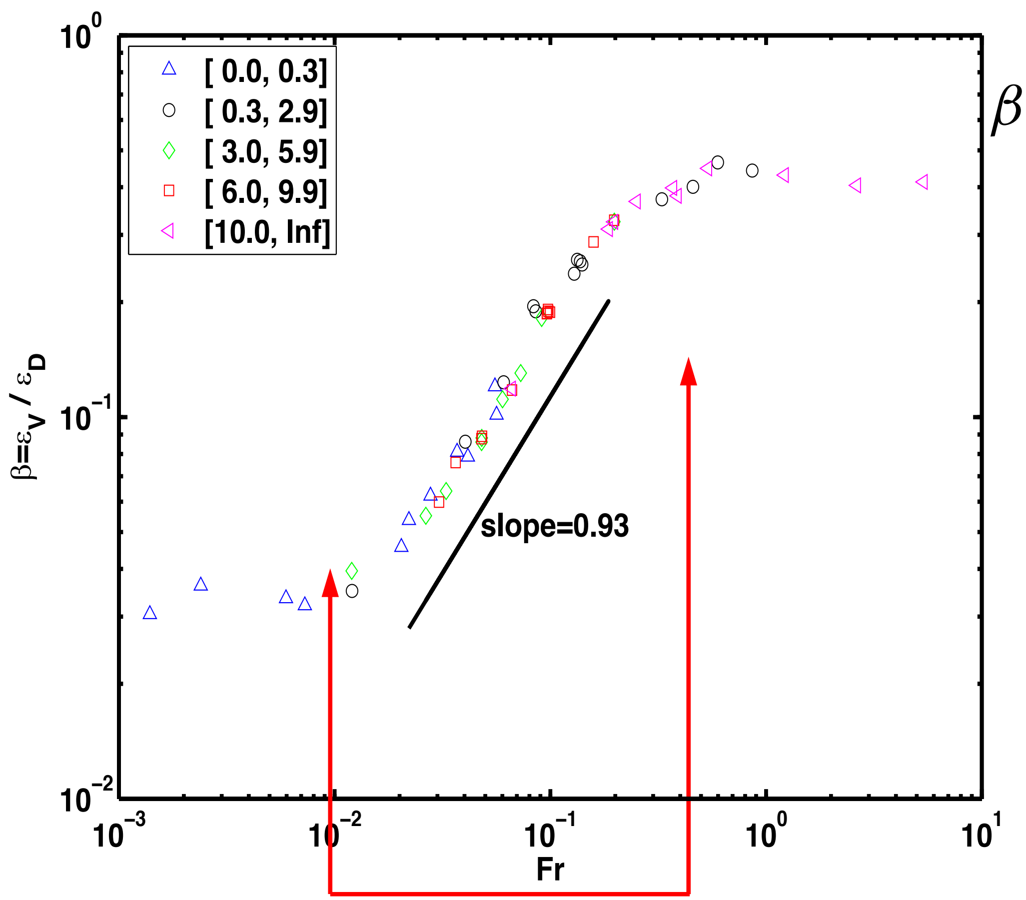


Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

Normalized kinetic energy dissipation rate

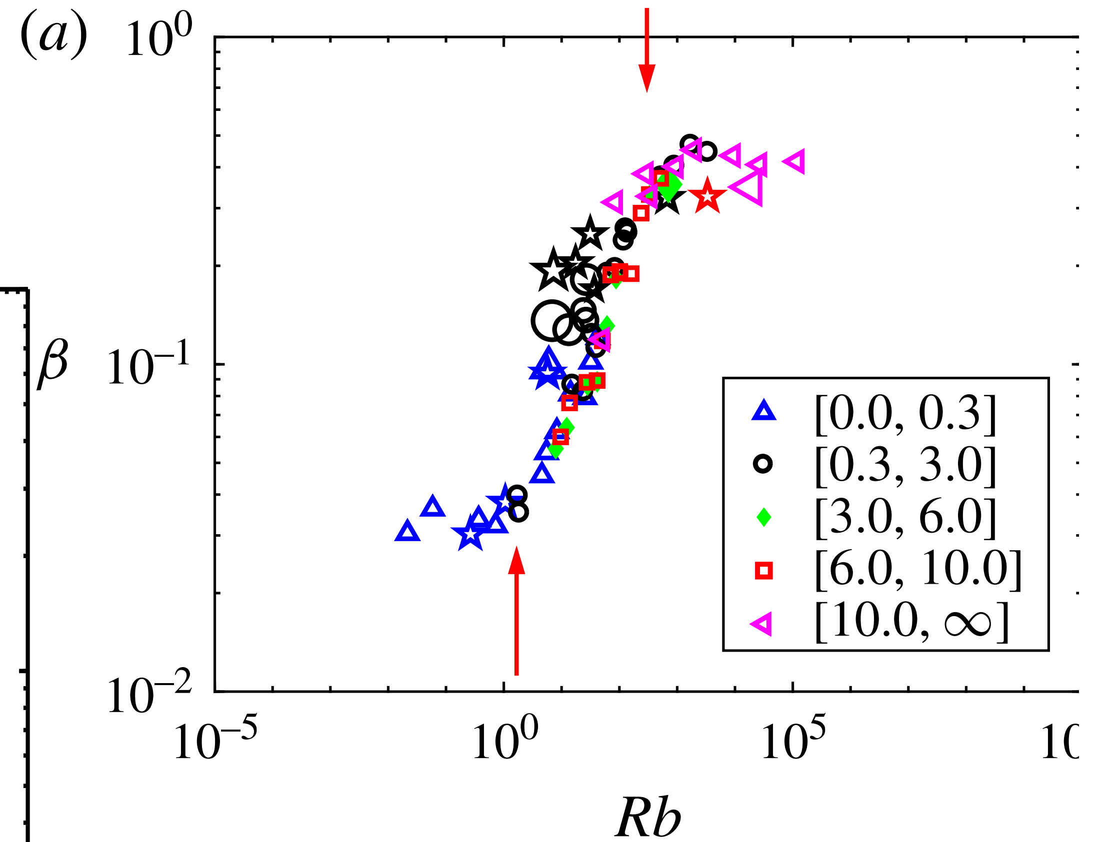
$\varepsilon_V/\varepsilon_D$ → 3 regimes

No QG, no low Re runs



II: Intermediate regime:

$0.010 \leq Fr \leq 0.5$



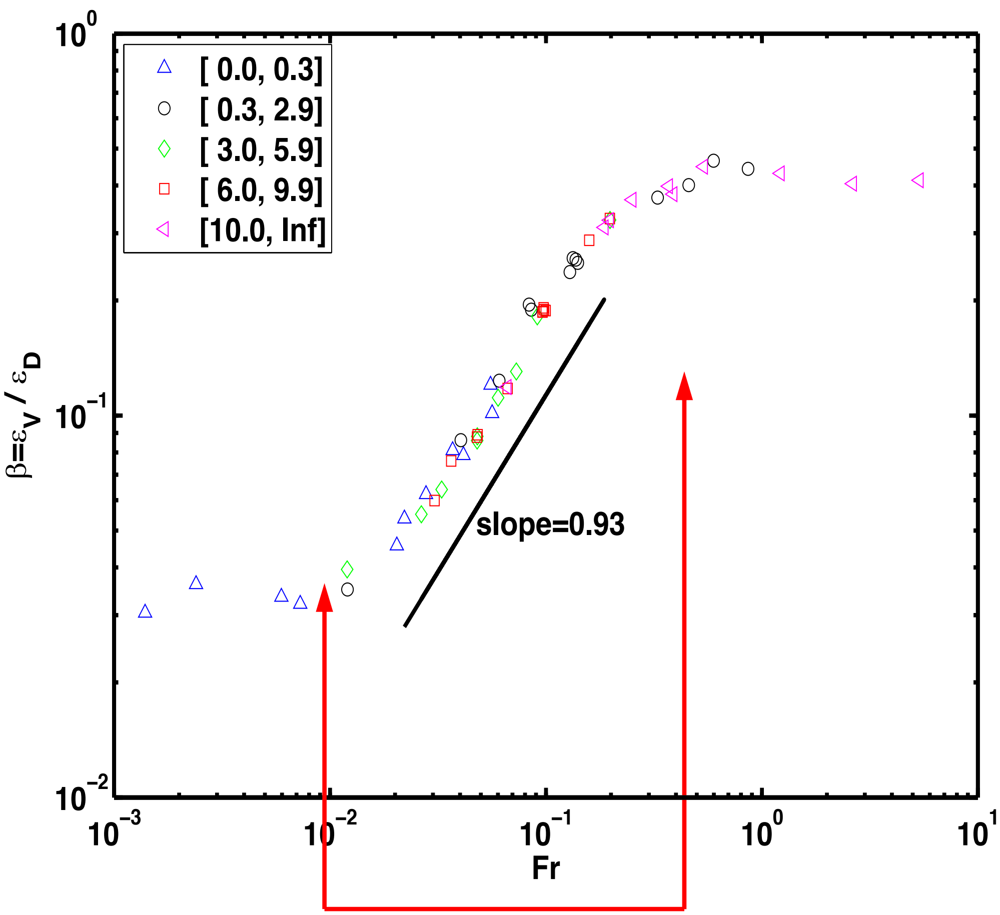
or $1 \leq R_B \leq 200$

Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

Normalized kinetic energy dissipation rate

$\varepsilon_V/\varepsilon_D$ → 3 regimes

No QG, no low Re runs



II: Intermediate regime:
$0.010 \le Fr \le 0.5$

Classical model of **weak,** wave turbulence: energy transfer **slower** than eddy $\tau_{NL}=L_0/U_0$:

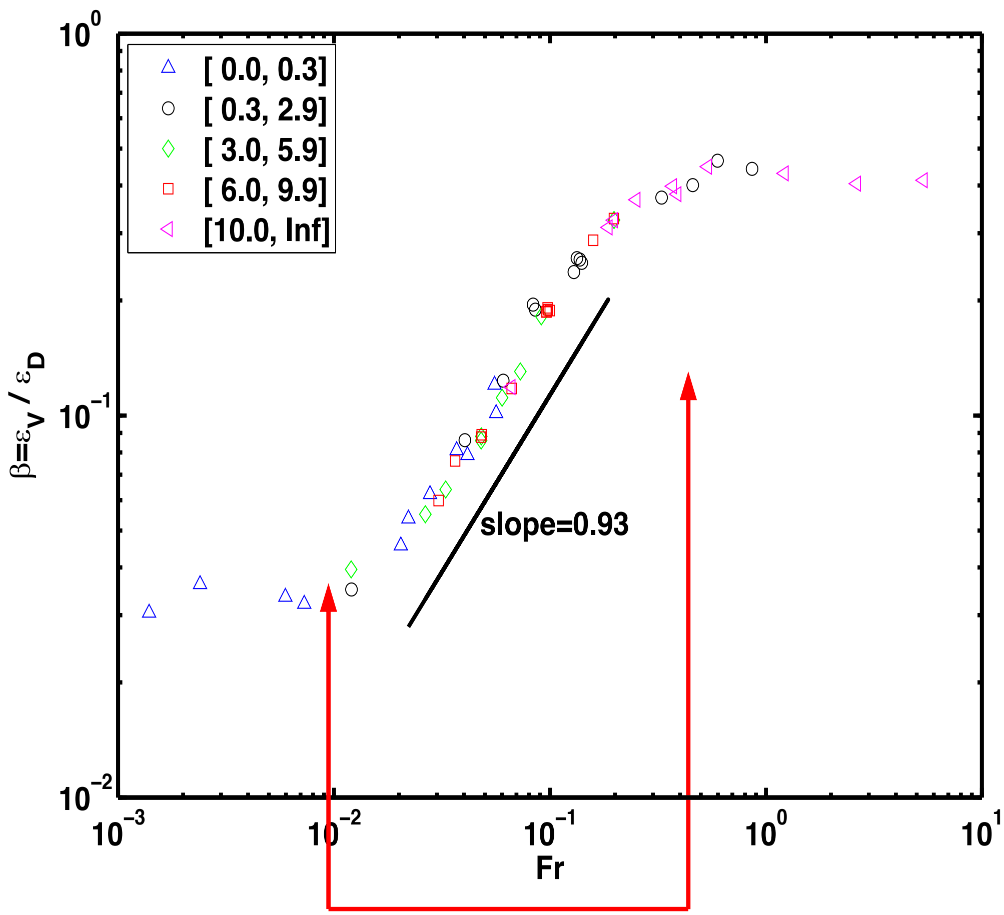
$\tau_{transfer} \sim \tau_{NL} / Fr$

Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

Normalized kinetic energy dissipation rate

ϵ_V/ϵ_D → 3 regimes

No QG, no low Re runs



II: Intermediate regime:
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$$\tau_{transfer} \sim \tau_{NL} / Fr$$
$$\sim \tau_{NL} * [\tau_{NL} / \tau_W]$$

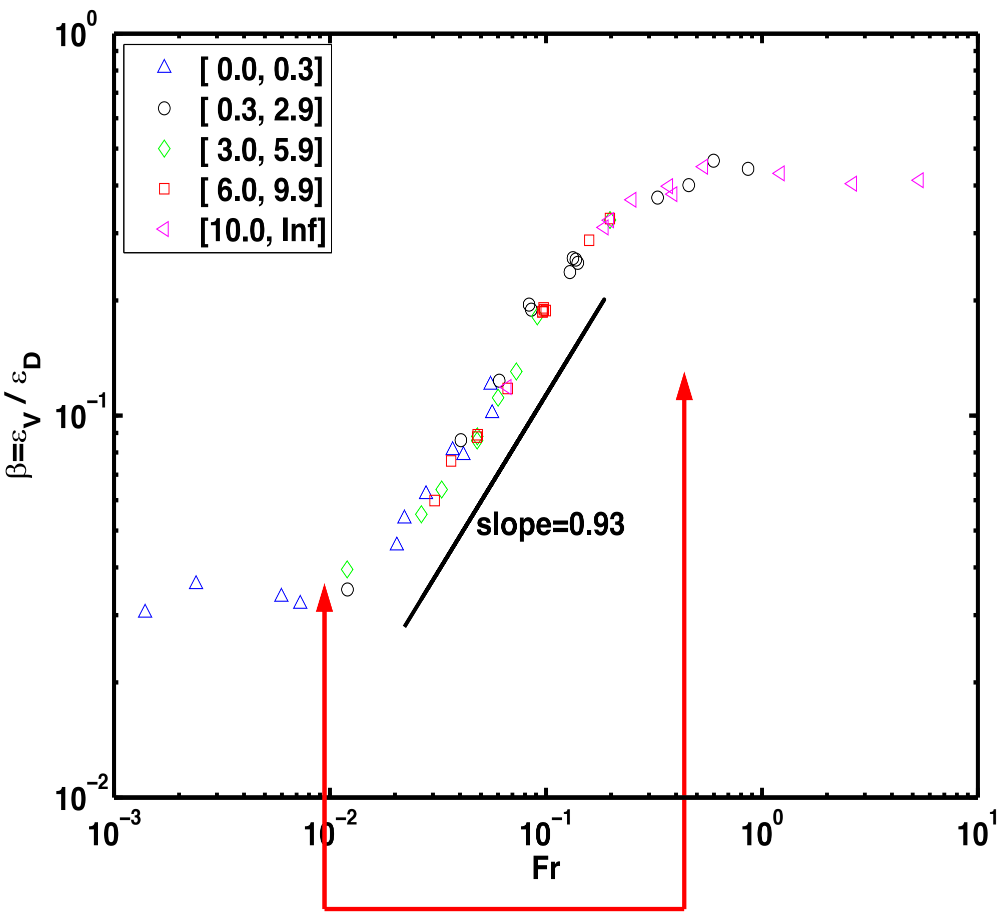
(MHD: Iroshnikov-Kraichnan, ~ '60s, Zakharov+ '80s, weak/wave turbulence)

Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

Normalized kinetic energy dissipation rate

$\varepsilon_V/\varepsilon_D$ → 3 regimes

No QG, no low Re runs



II: Intermediate regime:
$0.010 \leq Fr \leq 0.5$

Classical model of **weak,** wave turbulence: energy transfer **slower** than eddy $\tau_{NL}=L_0/U_0$:

$\tau_{transfer} \sim \tau_{NL} / Fr$

$\sim \tau_{NL} * [\tau_{NL} / \tau_W]$

(MHD: Iroshnikov-Kraichnan, ~ '60s, Zakharov+ '80s, weak/wave turbulence)

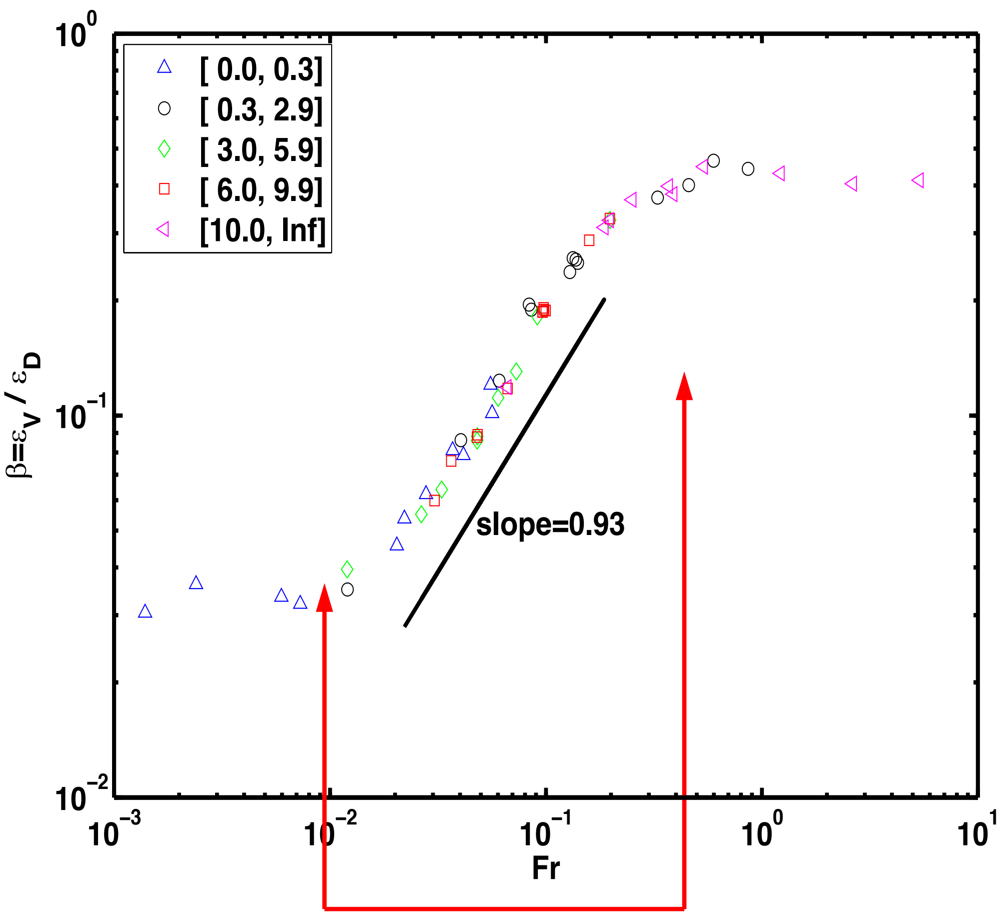
$$\varepsilon_v = dE_v/dt \sim E_v/\tau_{transfer}$$

Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

Normalized kinetic energy dissipation rate

$\varepsilon_V/\varepsilon_D$ → 3 regimes

No QG, no low Re runs



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(MHD: Iroshnikov-Kraichnan, ~ '60s, Zakharov+ '80s, weak/wave turbulence)

$$\boldsymbol{\varepsilon_v = dE_v/dt \sim E_v/\tau_{transfer}}$$

$$\varepsilon_v \sim Fr\,\varepsilon_D = Fr\,U_0^3/L \quad (3)$$

Or: $\beta = \varepsilon_v / \varepsilon_D \sim Fr$ *as ~ observed*

Role of anisotropy?

Three laws → Energy balance & mixing: Γ_f, $R_{f;}$ & Γ_D, R_D

$D_t E_v = - B_f + \varepsilon_v$
$D_t E_p = +B_f + \varepsilon_p$

$B_f = \langle N\theta w\rangle$: *Vertical buoyancy flux*

Three laws → Energy balance: Γ_f, R_f; & Γ_D, R_D

$D_t E_v = - B_f + \varepsilon_v$

$D_t E_p = +B_f + \varepsilon_p$

$B_f = <N\theta w>$: *Vertical buoyancy flux*

$E_v = ½ <|\mathbf{u}|^2>$, $E_p = ½ <\theta^2>$

$\varepsilon_v = \nu<\omega^2>$, $\varepsilon_p = \varkappa<|\text{grad } \theta|^2>$, $E_T = E_v + E_p$, $\varepsilon_T = \varepsilon_v + \varepsilon_p$

$B_f /\varepsilon_v = \Gamma_f = R_f/[1-R_f]$: Mixing efficiency *(momentum equation)*

$R_f = B_f /[B_f+\varepsilon_v]$: Flux Richardson number \in [0,1]

$\varepsilon_p/\varepsilon_v = \Gamma_D = R_D/[1-R_D]$, $R_D = \varepsilon_p/\varepsilon_T$ \in [0,1] *(coupled equations)*

Scaling?

$D_t E_v = - B_f + \varepsilon_v$
$D_t E_p = +B_f + \varepsilon_p$

$B_f = \langle N\theta w \rangle$: *Vertical buoyancy flux*

$E_v = ½ \langle |\mathbf{u}|^2 \rangle$, $E_p = ½ \langle \theta^2 \rangle$
$\varepsilon_v = \nu \langle \omega^2 \rangle$, $\varepsilon_p = \varkappa \langle |\text{grad}\ \theta|^2 \rangle$, $E_T = E_v + E_p$, $\varepsilon_T = \varepsilon_v + \varepsilon_p$

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$\varepsilon_p/\varepsilon_v = \Gamma_D = R_D/[1-R_D]$, $R_D = \varepsilon_p/\varepsilon_T$ \in [0,1] *(coupled equations)*

High R_B: $\Gamma_f = 0.2$ *(Osborn-Cox '80)* *vs.* $\Gamma_f \sim R_B^{-1/2}$ *(Lozovatsky & Fernando 2013)* *vs.* ?

→ Prediction of scaling for mixing efficiency, flux Richardson number and dissipation in the two regimes of wave-eddy interactions and of strong eddies

using the three constitutive scaling laws for temperature, vertical velocity and dissipation efficiency versus Froude number:

$$\Theta_{rms} \sim U_0 \qquad (1)$$

$$w/U_0 = a\, Fr^0 \qquad (2a)$$

$$\varepsilon_V \sim Fr\, \varepsilon_D = Fr\, U_0^3/L \qquad (3)$$

[0 → ¼? (2b) Or → 1 (2d)?]

Intermediate regime II, $Fr \leq 1$, using the 3 scaling laws:

$B_f = \langle N\theta w \rangle$, $\theta_{rms} \sim U_0$, $w \sim Fr^0 U_0$, $\varepsilon_v \sim Fr\, \varepsilon_D$, $\varepsilon_D \sim U_0^3/L$

$R_B > 1$, *Re>>1 but irrelevant otherwise*

→ $\Gamma_f^{II} = B_f/\varepsilon_V = N\langle w\,\theta \rangle/\varepsilon_V \sim 1/Fr^2 \sim [R_B]^{-1}$ *(observed)*

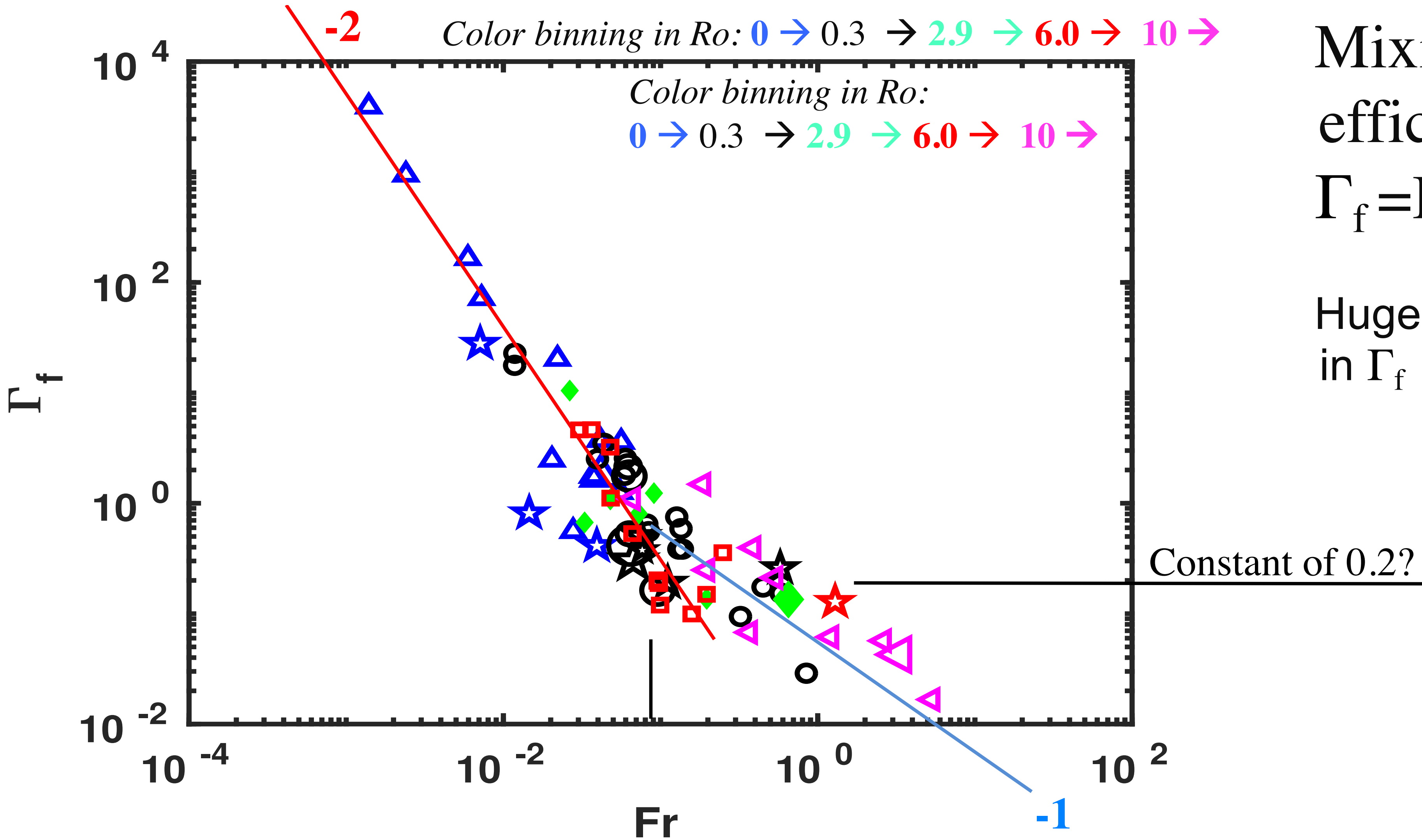
Higher regime III, Fr ≥1:

$\theta_{rms} \sim U_0$, $w \sim Fr^0 U_{0,\, Fr=1} = U_0$, $\varepsilon_v \sim Fr\, \varepsilon_D$ *at* $Fr = 1$, $\varepsilon_v \sim \varepsilon_D$

$R_B > 1$, *Re>>1 but irrelevant otherwise*

→ $\Gamma_f^{III} = B_f/\varepsilon_V = N\langle w\,\theta \rangle/\varepsilon_V \sim 1/Fr \sim [R_B]^{-1/2}$ *(observed)*

→ *Our numerical data?*

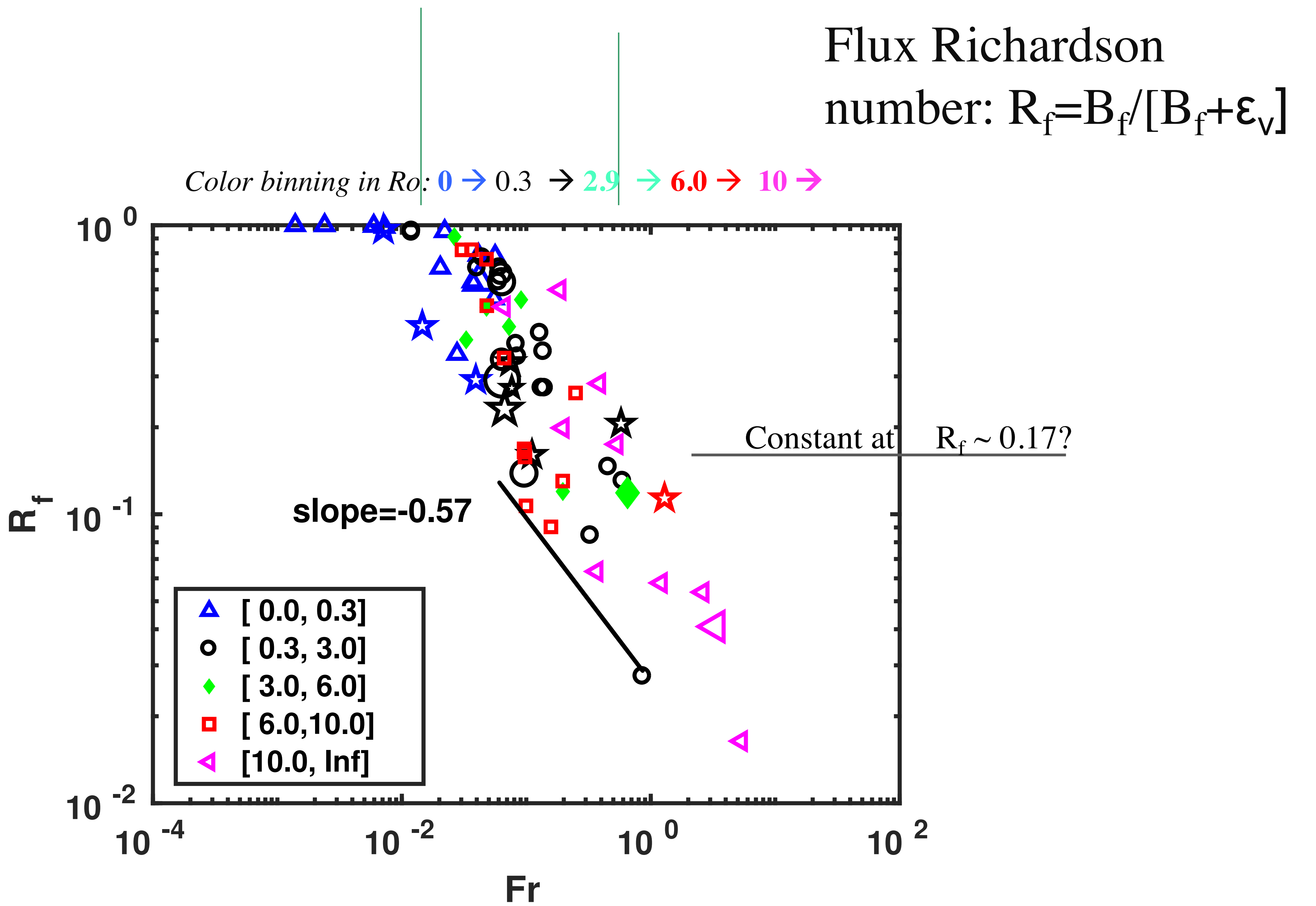


Mixing efficiency $\Gamma_f = B_f/\varepsilon_v$

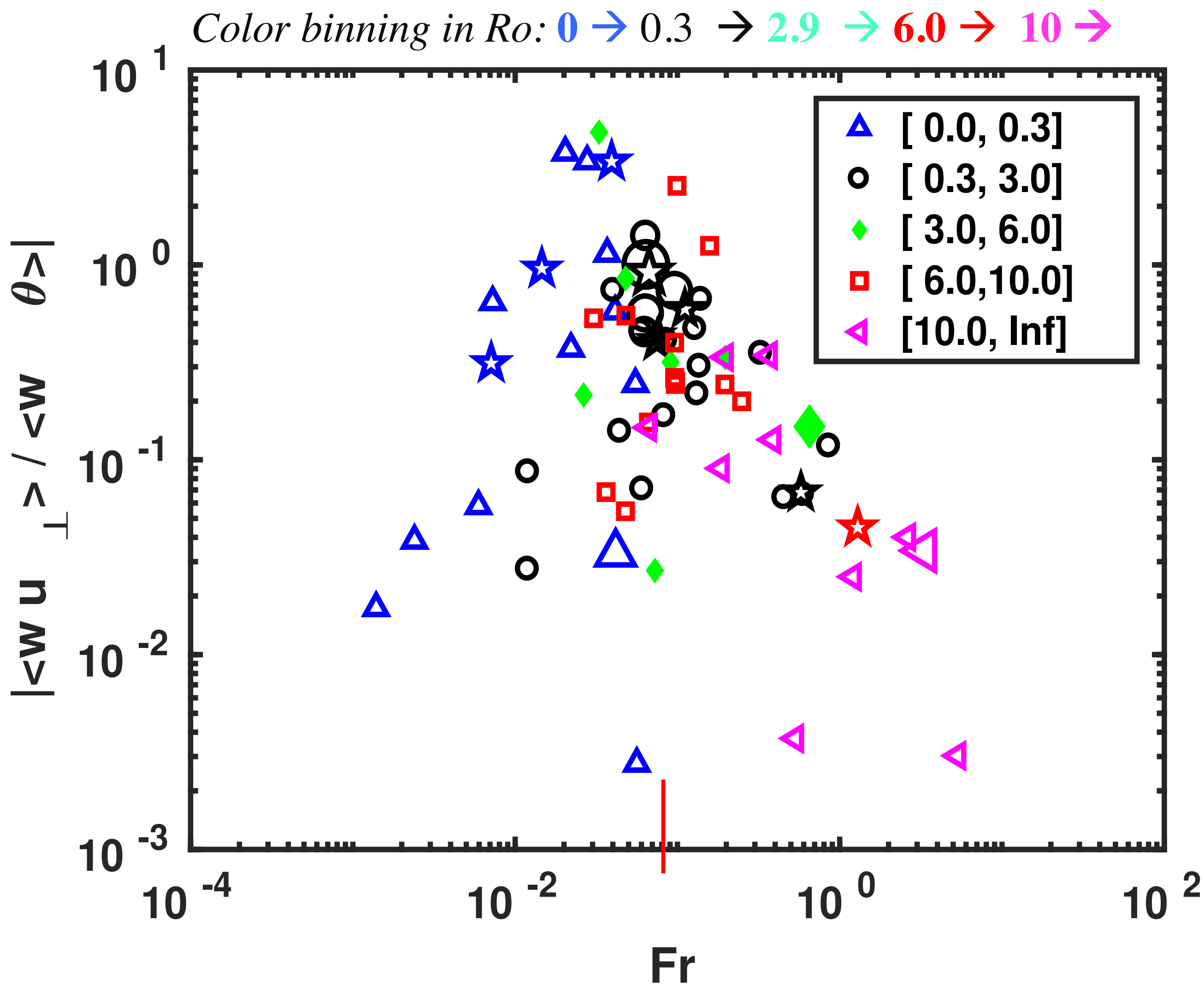
Huge range in Γ_f

$N\langle w\,\theta\rangle/\varepsilon_V$ = $\Gamma_f \sim 1/Fr^2 \sim [R_B]^{-1}$ or for Fr~1: $\Gamma_f \sim 1/Fr \sim [R_B]^{-1/2}$

(regime I & II?) *(regime III)*



Flux Richardson number: $R_f=B_f/[B_f+\varepsilon_v]$
Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →
Constant at $R_f \sim 0.17$?
slope=-0.57
[0.0, 0.3]
[0.3, 3.0]
[3.0, 6.0]
[6.0,10.0]
[10.0, Inf]
R_f
Fr
10^0
10^{-1}
10^{-2}
10^{-4}
10^{-2}
10^0
10^2

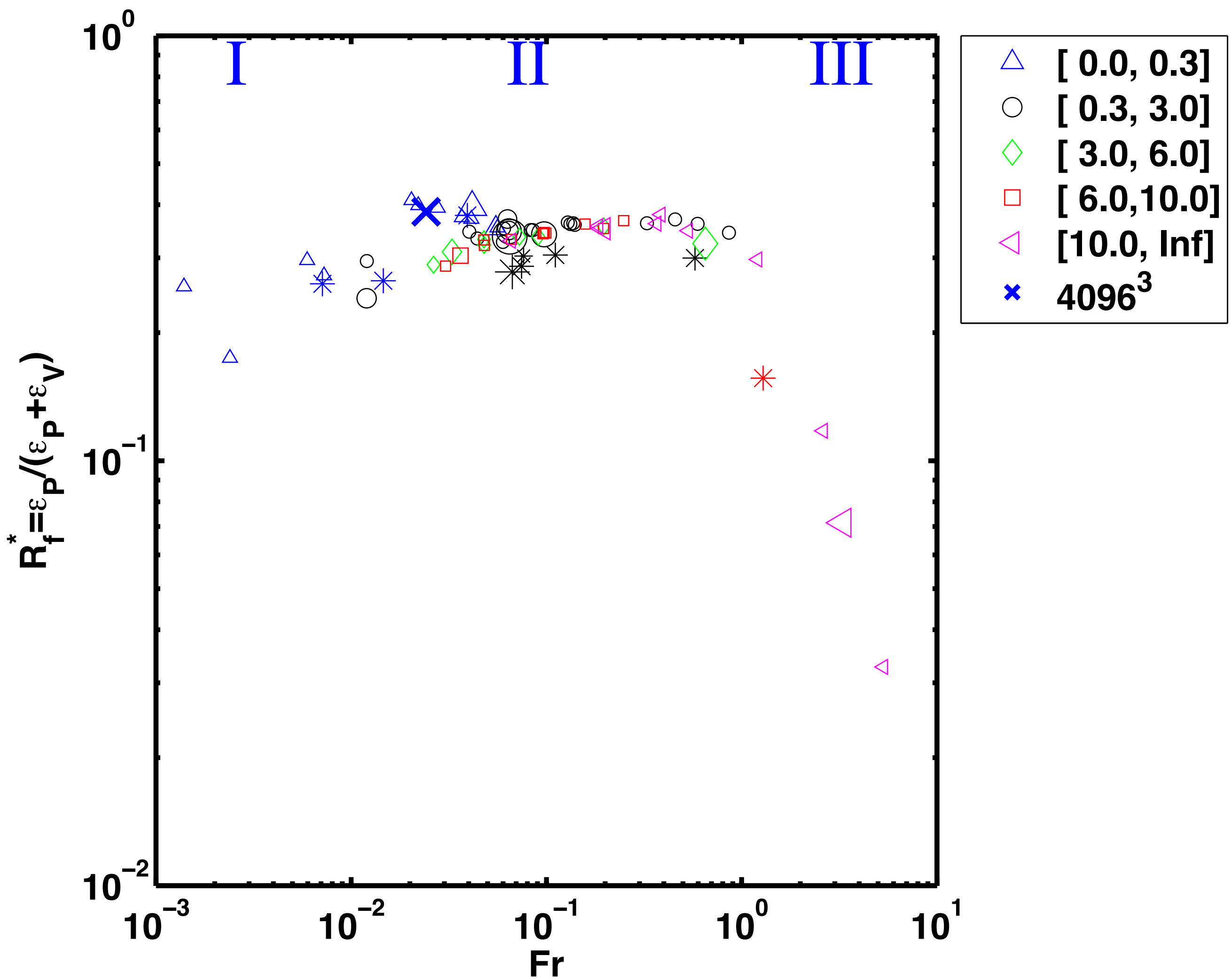


Ratio of vertical fluxes
of horizontal velocity
to that of
temperature fluctuations
versus
Froude Number

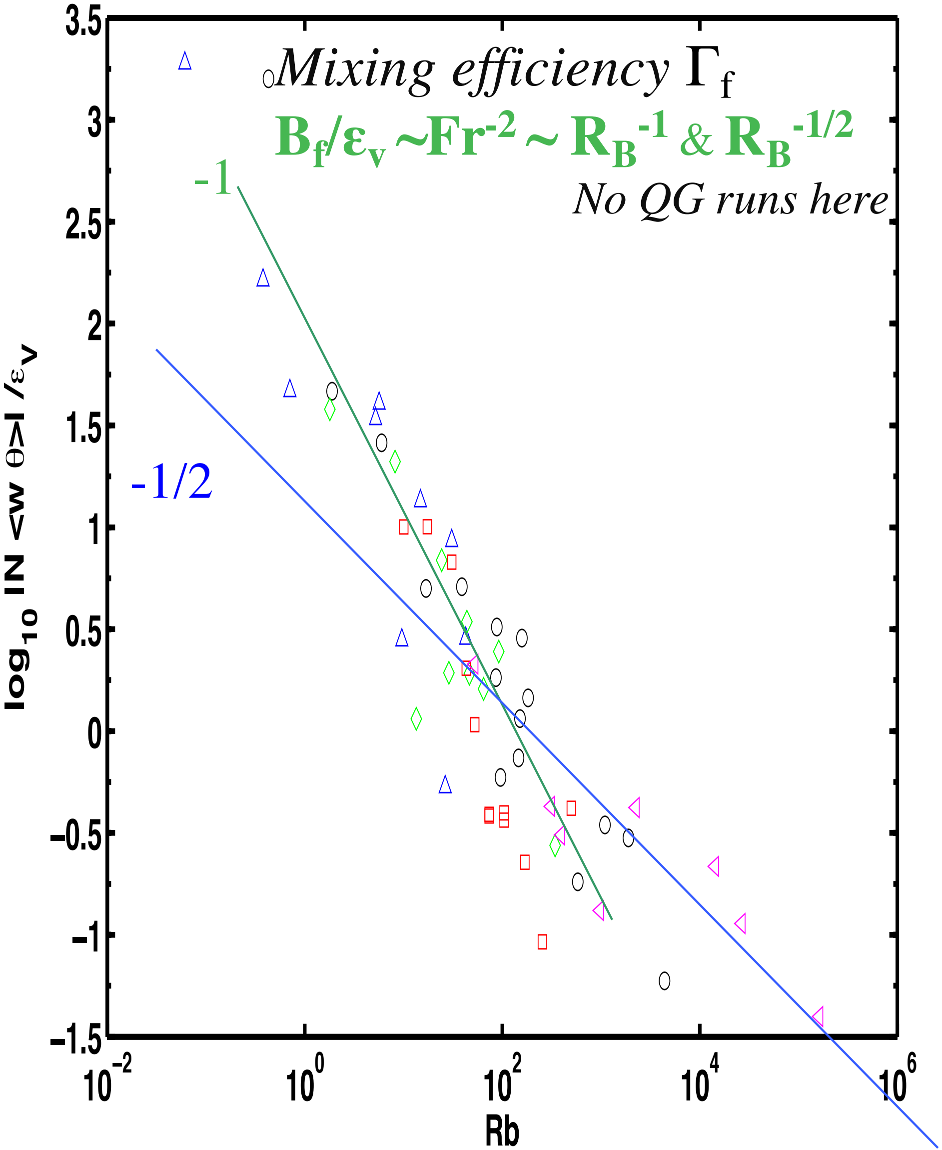
Non-monotonic, with a plateau/peak around Fr~0.07 ?

Ratio of potential to total energy dissipation

$\epsilon_p/[\epsilon_p + \epsilon_v]$



Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →



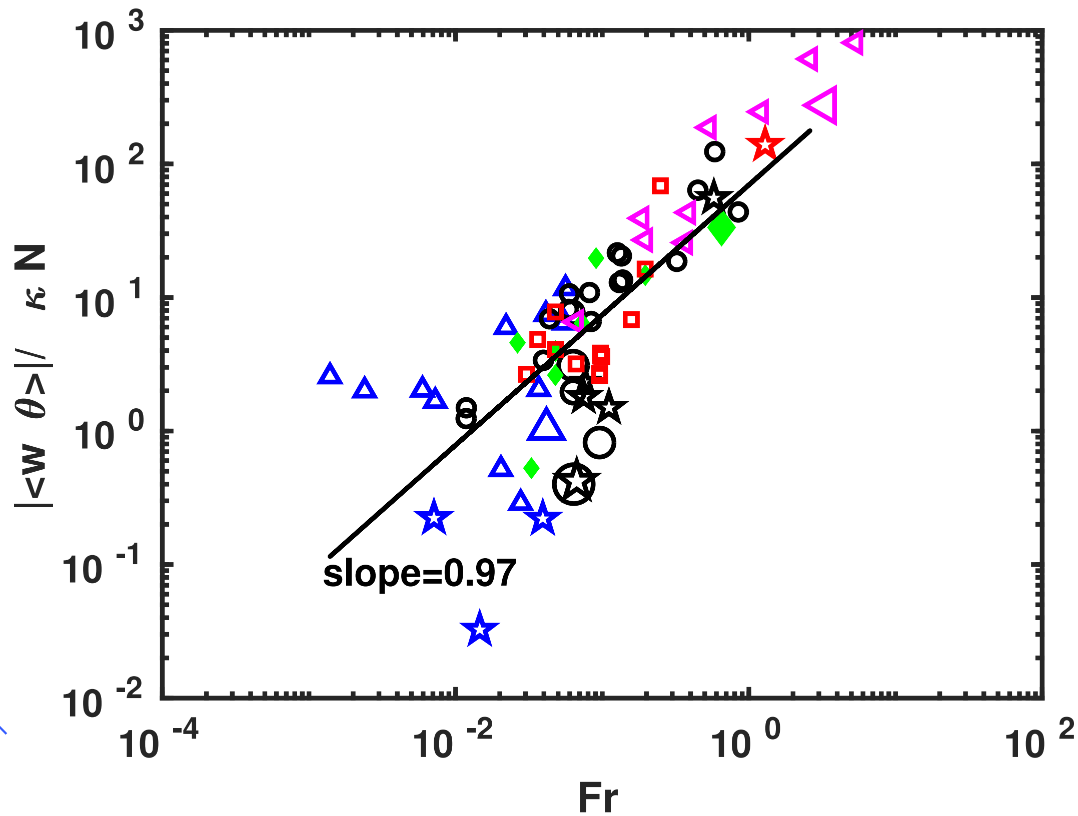
Buoyancy flux $B_f = <N\ w\ \theta>$

with two different normalizations for the

← ***momentum*** *and* ***buoyancy*** *equations*

↓

$B_f/[\varkappa_0 N^2] \sim Fr$



Normalized anomalous diffusivity $\varkappa_\theta$

$B_f/[\varkappa N^2] = \varkappa_\theta/\varkappa \sim Fr$ in regimes II & III,

$\varkappa_\theta \sim U_{rms} L_{int}\ Fr$

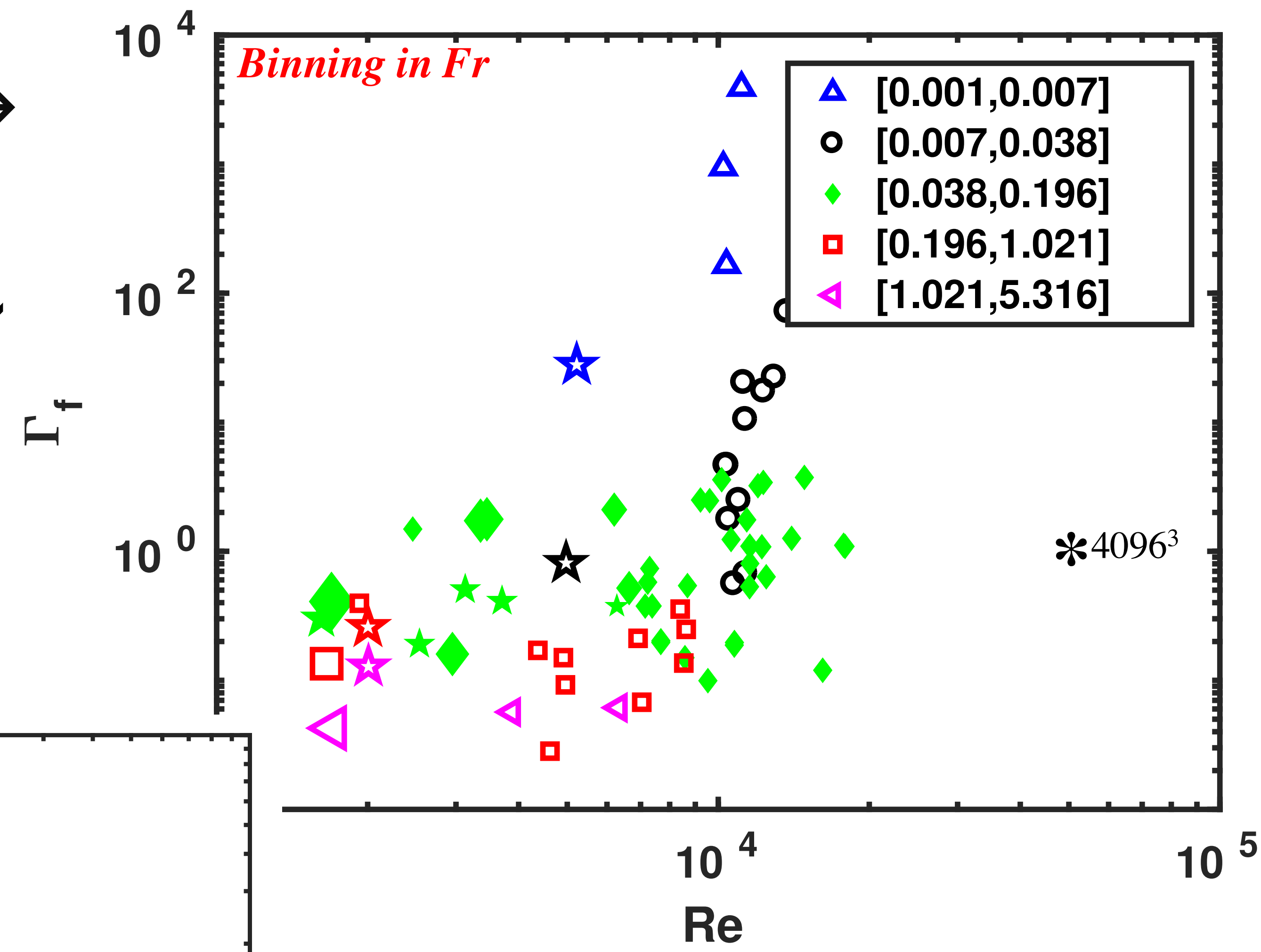
Mixing efficiency →

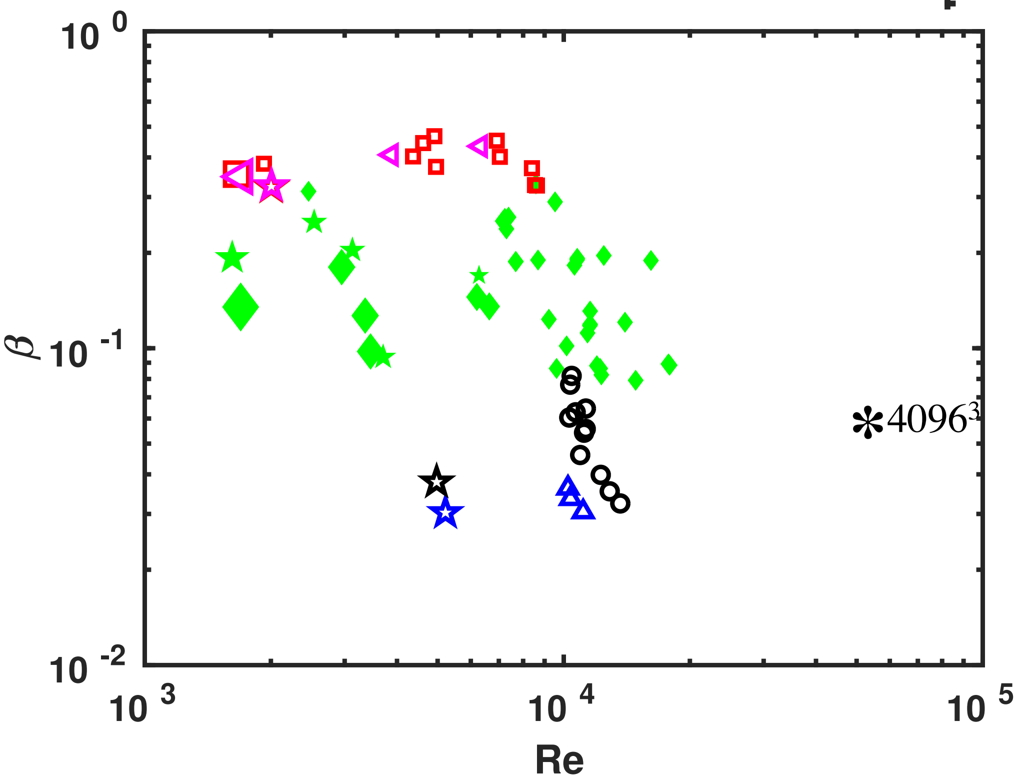
&

Dissipation efficiency ↓

versus

Reynolds Number





Questions and Perspectives

- Role of rotation?
- Role of $k_0 \sim 2.5$, poor large-scale statistics & weaker resonances?
- Importance of QG?
- Role of lack of stationarity due to lack of forcing?

→ Add forcing and large-scale friction → Temporal averaging

→ But, what about anisotropy – dispersion relation and – forcing?

- Role of boundary conditions?

- Will small aspect ratio help, role of vertical shear?

- Approach through small-scale modeling: will *Artificial Intelligence & Machine Learning* help?

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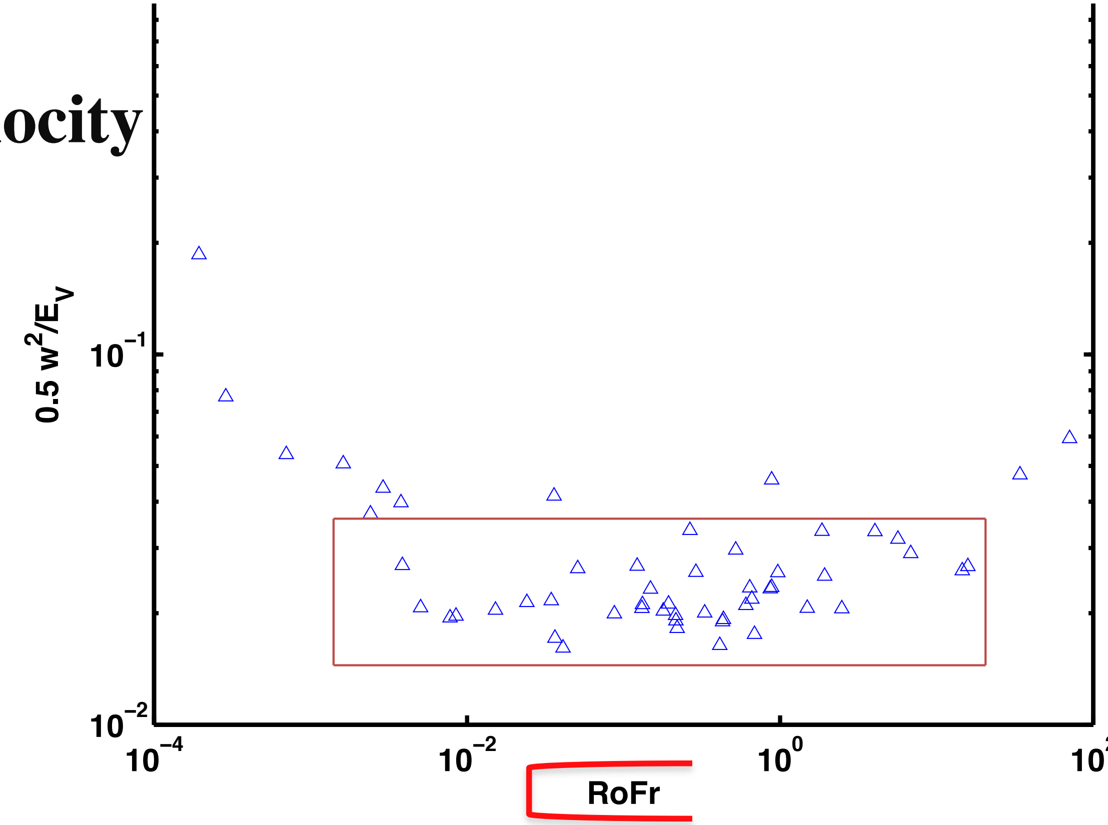
Perspective (1): Vertical velocity

in the presence of rotation

Intermediate regime:
w/ $U_0 \sim$ [Ro.Fr]$^{\sim 0}$ (3b)

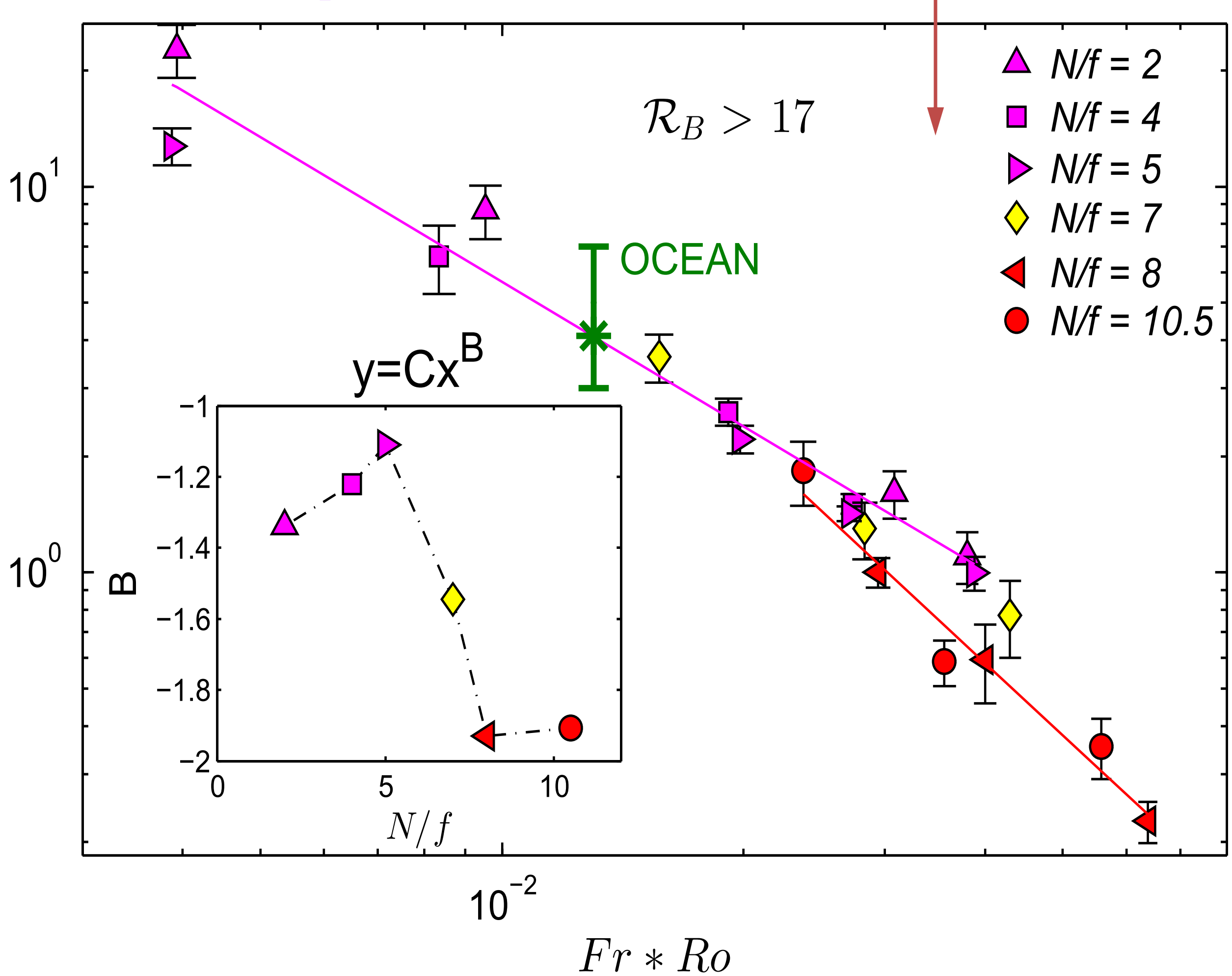
→ $\Gamma_f = \langle N\theta w \rangle / \varepsilon_v \sim Fr^{-2}$

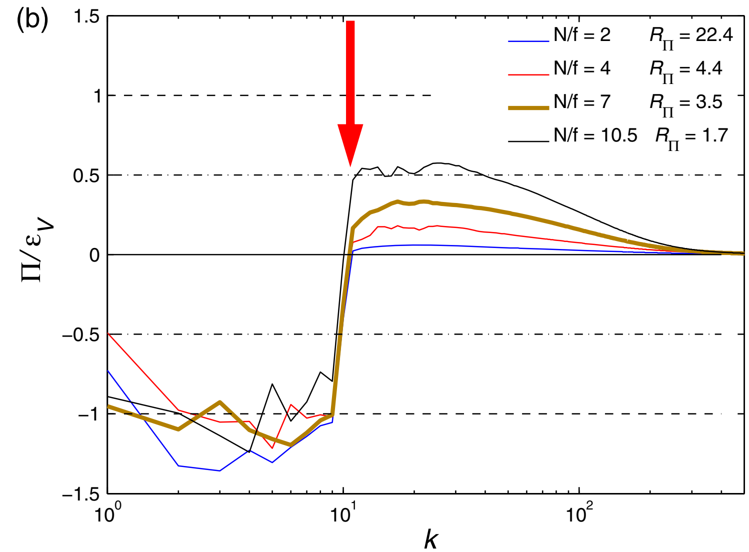


Perspective (2): Forced runs

• Analyze runs of rotating stratified turbulence with isotropic forcing at intermediate scale (k_F~10): what are their mixing properties?

Pouquet+ 2013 ; Marino+ 2015 (rot+strat)





$R_\Pi = \varepsilon_{LS}/\varepsilon_{ss}$

$\varepsilon_{ss} \sim \varepsilon_D \, Fr$, $\varepsilon_D \sim U^3/L$

$\varepsilon_{LS} \sim [1/Ro]\, \varepsilon_D$

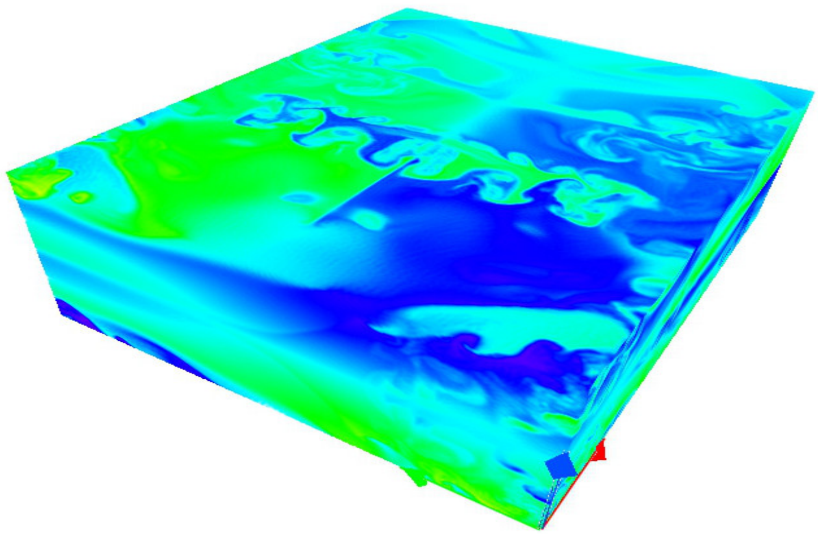
→ $R_\Pi = \varepsilon_{LS}/\varepsilon_{ss} \sim [Ro.Fr]^{-1}$

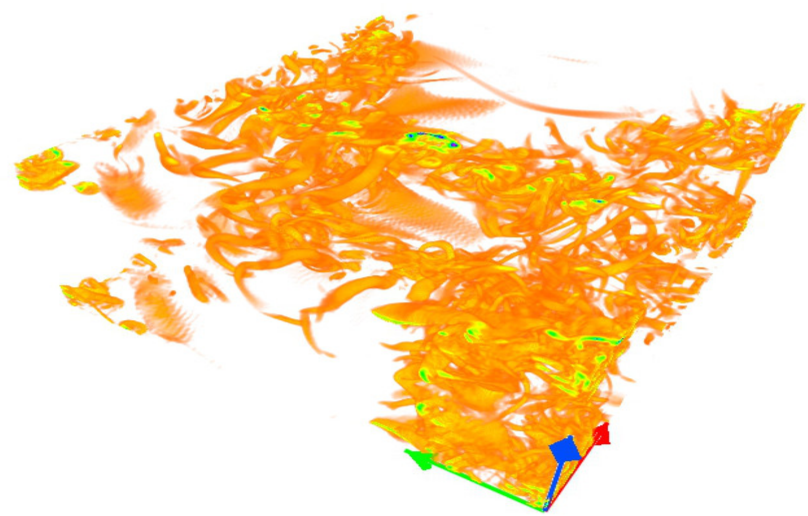
with transition at Ro~ 0.45

Perspective (3): Shallow fluids

- Explore the dynamics in match boxes:
- Example; 2048^2 X 256 points with Taylor-Green forcing, resulting locally in strong vertical shear, and strong dissipation, with fronts and filaments

Temperature & vorticity *(Oks+, arXiv* 1706.10287)





Summary and perspectives:

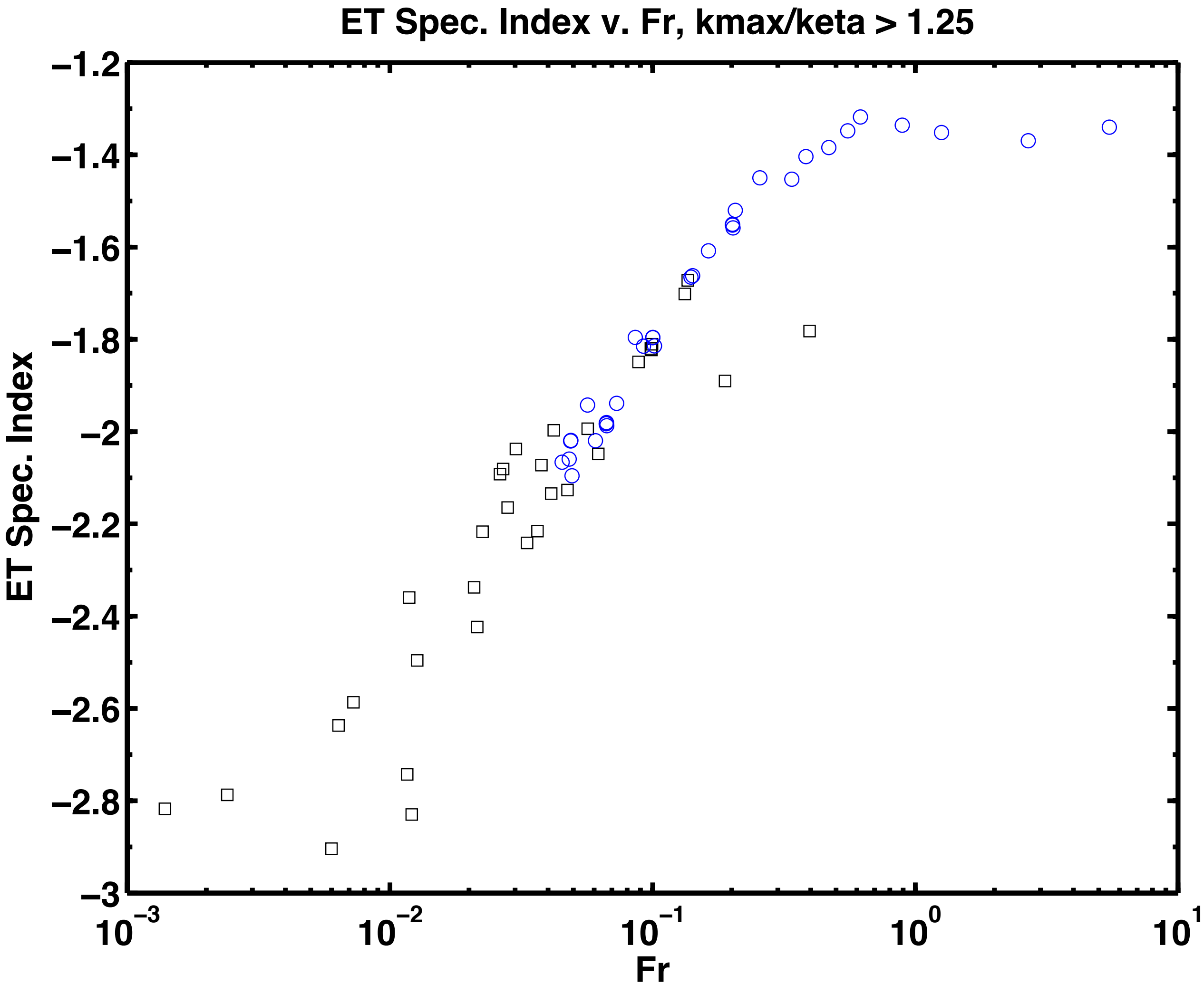
→ There are three distinct regimes in rotating stratified turbulence, determined by the Froude number for high enough Reynolds number, with somewhat different thresholds when analyzing different fields: Strong waves, eddy-wave interactions and full turbulence

→ Mixing efficiency and flux Richardson number vary measurably in the intermediate regime, $0.01 \leq Fr \leq 0.5$ *($1 \leq R_B \leq 1000$)*, *and with scaling laws*

→ Dissipation increases as Froude, ``weak'' turbulence regime, for $R_B \approx 10\text{-}10^3$

→ Lower values of mixing efficiency at high R_B → *the velocity and temperature fluctuations are only weakly coupled (passive scalar regime)*

- Together, large numerical resolution and large parametric study allow for some scale separation and for understanding separately the roles of some of the players
- Local instabilities and very intense local small-scale dynamics *(more so than in FDT)*
- Role of Reynolds number in mixing, in the intermediate regime in particular?
- Roles of I.C. or forcing (3D *vs.* 2D, θ or not, vortices *vs.* waves, balanced or not …), of non-local interactions, of large-scale instabilities & of large-scale friction?
- Intermittency high kurtosis and non-monotonicity *(Rorai+ 2014; Feraco+ ArXiv 2018)*



Spectral index of total energy spectra:

Again 3 regimes and going *smoothly* from ``-3'' to ``-5/3''

Machine learning and modeling

- Use data to guess the functional form for the role of small scales in order to write sub-grid scale models

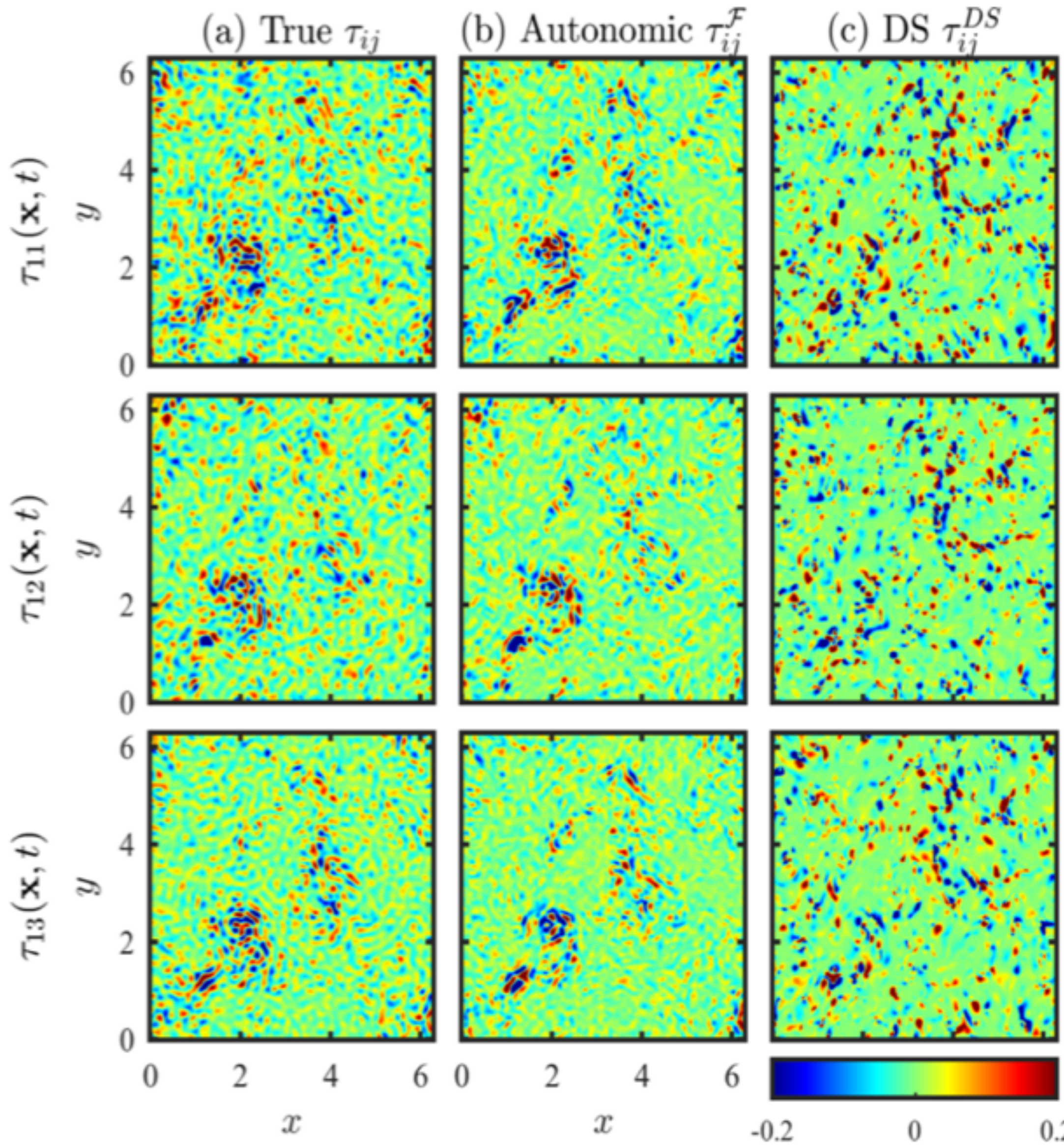


FIG. 2. Coarse-grained turbulent stress fields $\tau_{11}(\mathbf{x},t)$ (top row), $\tau_{12}(\mathbf{x},t)$ (middle row), and $\tau_{13}(\mathbf{x},t)$ (bottom row), showing results for (left column) the true stress $\tau_{ij}(\mathbf{x},t)$, (middle column) the autonomic closure $\tau_{ij}^{\mathcal{F}}(\mathbf{x},t)$, and (right column) the dynamic Smagorinsky model $\tau_{ij}^{DS}(\mathbf{x},t)$.

King+2016

Machine learning and modeling

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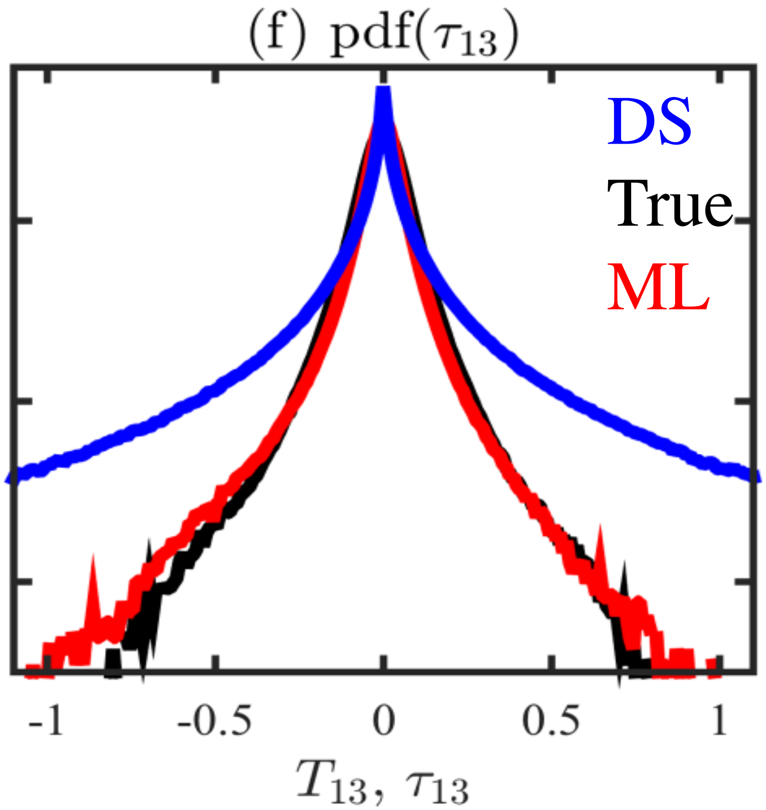

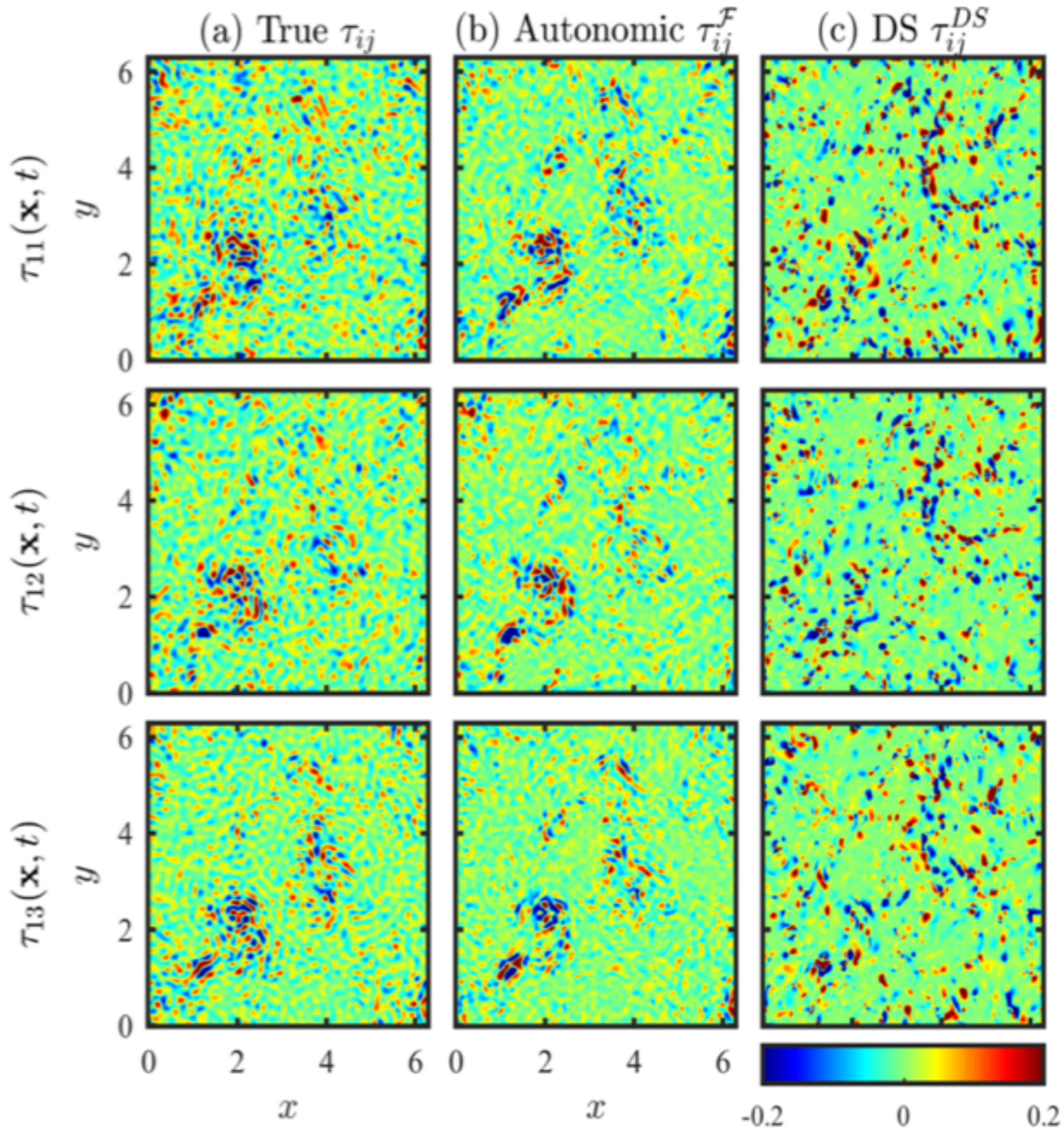

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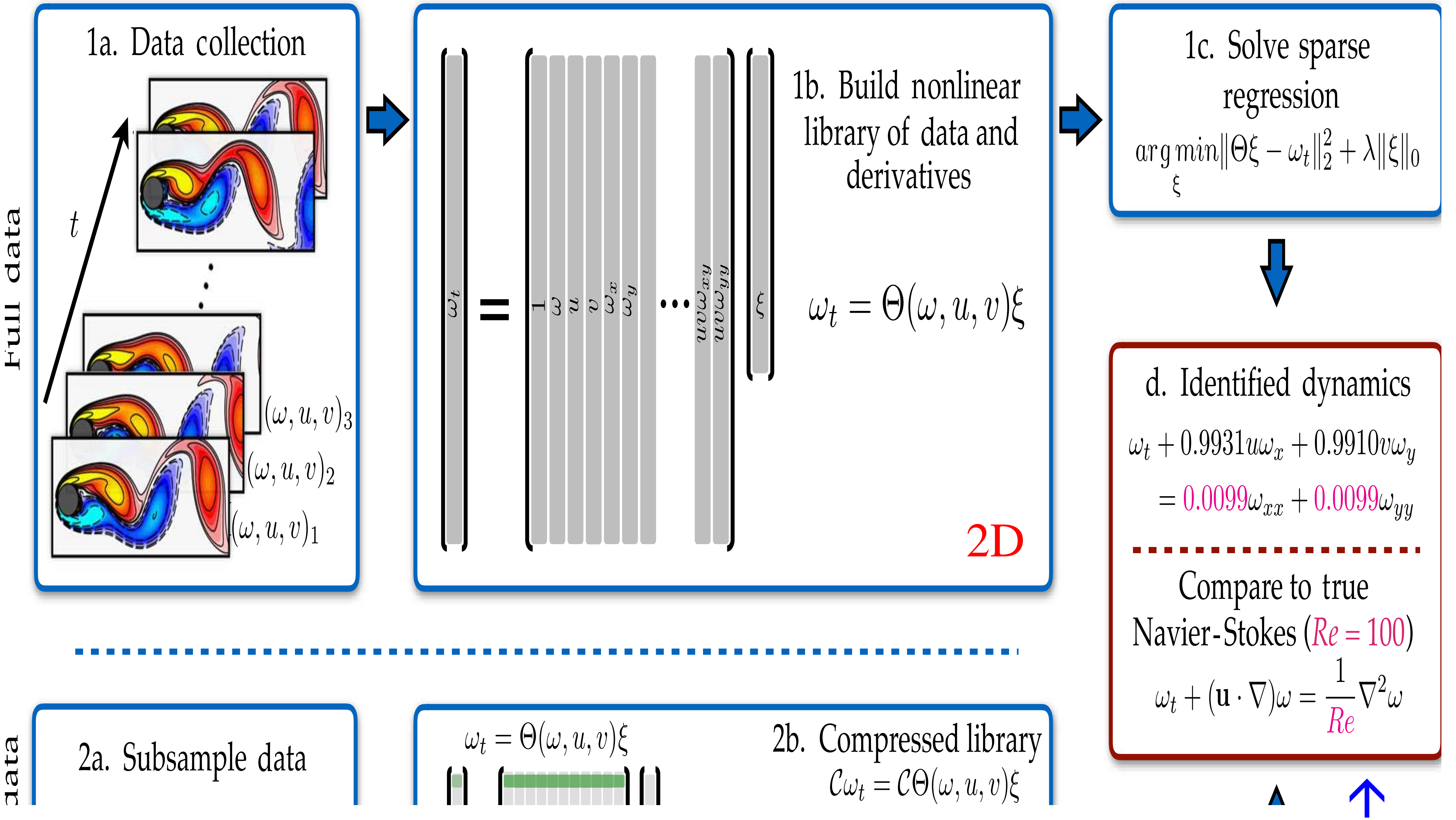
King+2016



Full data
1a. Data collection
t
(ω, u, v)3
(ω, u, v)2
(ω, u, v)1
ω_t
=
1
ω
u
v
ω_x
ω_y
...
uω_xy
uω_yy
ξ
1b. Build nonlinear library of data and derivatives
ω_t = Θ(ω, u, v)ξ
2D
1c. Solve sparse regression
arg min_ξ ‖Θξ − ω_t‖_2^2 + λ‖ξ‖_0
d. Identified dynamics
ω_t + 0.9931uω_x + 0.9910vω_y
= 0.0099ω_xx + 0.0099ω_yy
Compare to true
Navier-Stokes (Re = 100)
ω_t + (u · ∇)ω = (1/Re)∇^2ω
data
2a. Subsample data
ω_t = Θ(ω, u, v)ξ
2b. Compressed library
Cω_t = CΘ(ω, u, v)ξ

Rudy+ 2017

Thank you for your attention

Some references

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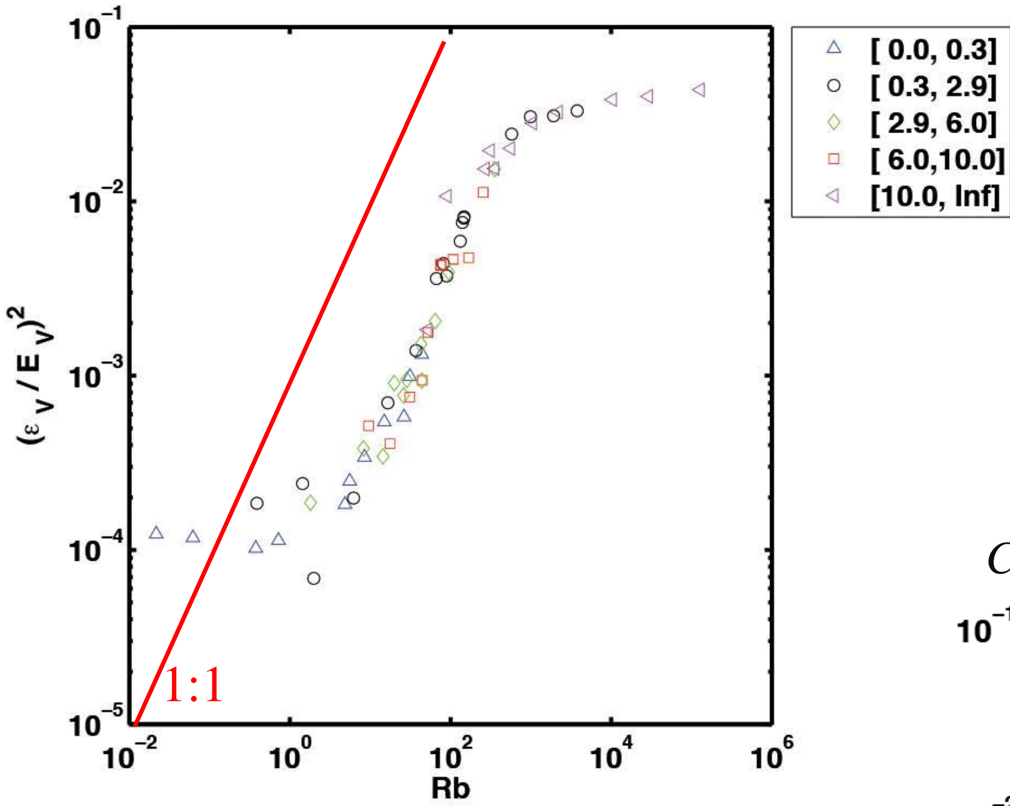
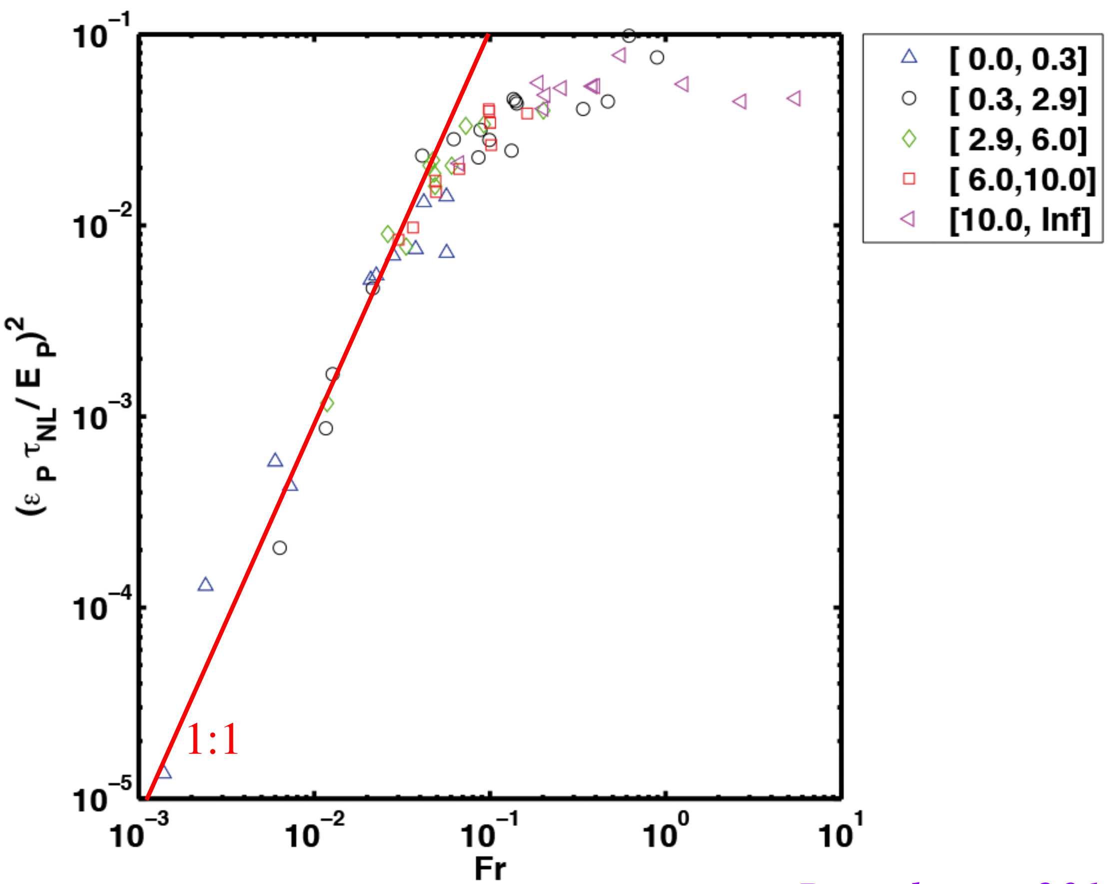
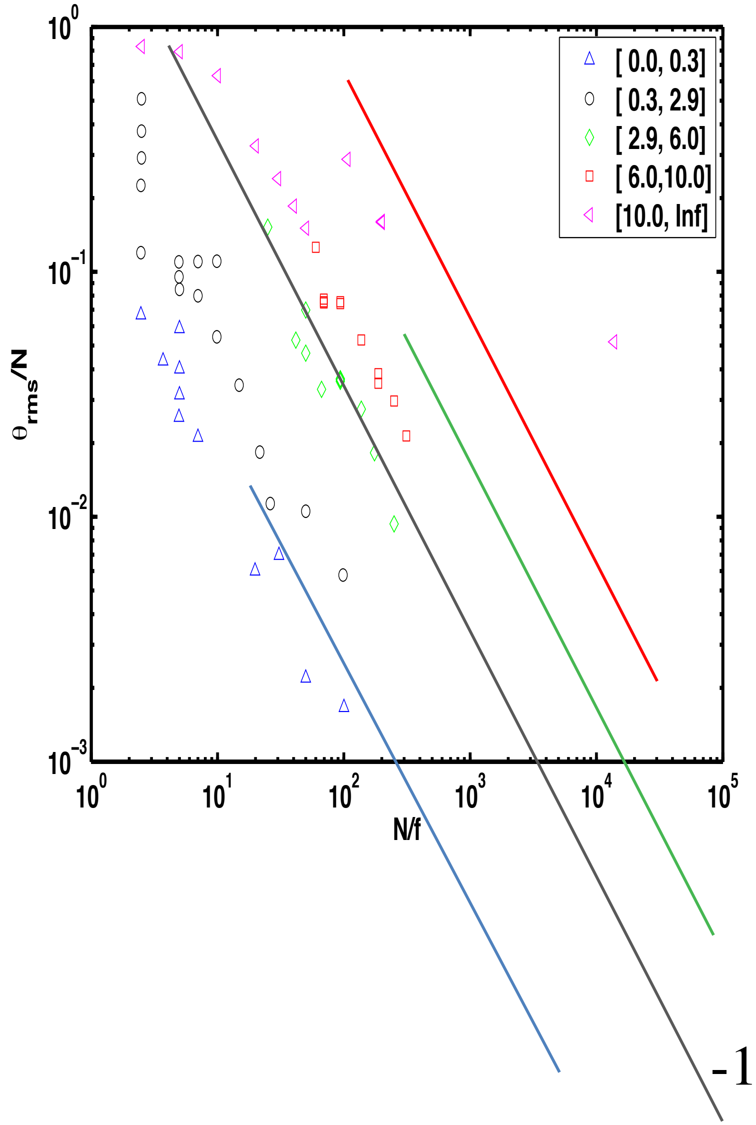


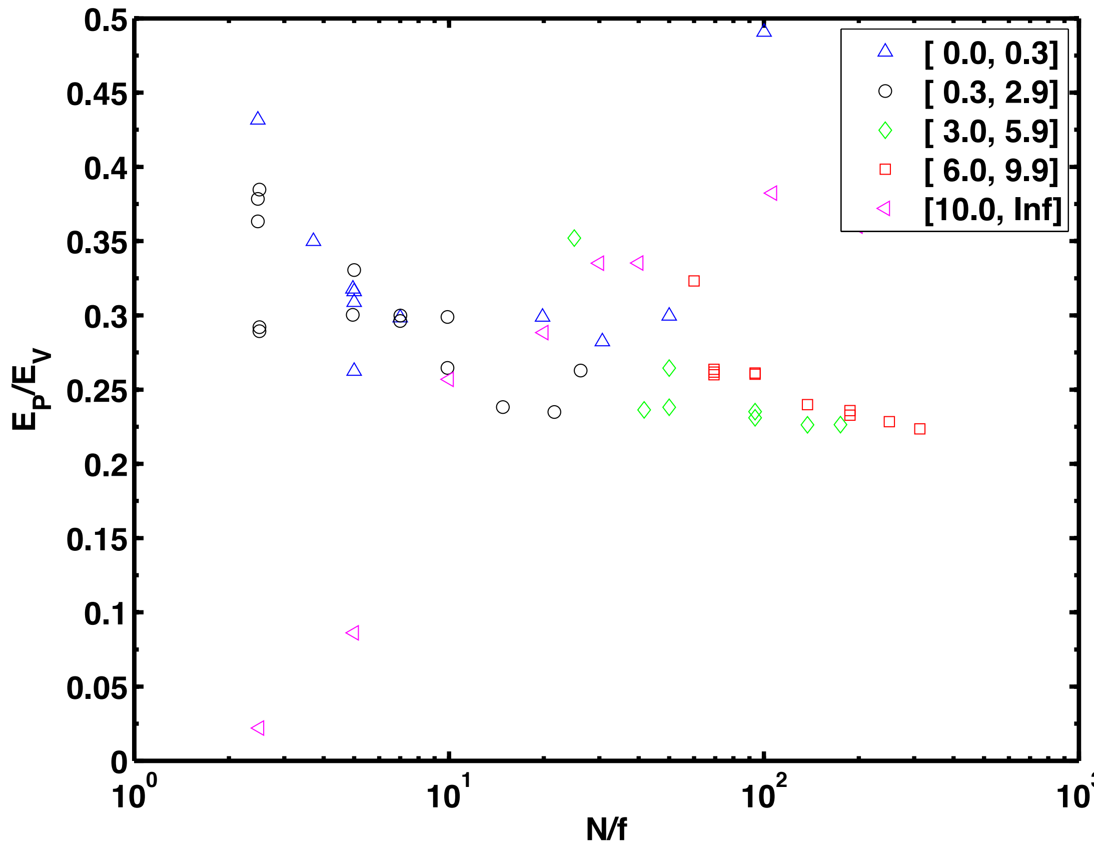
Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

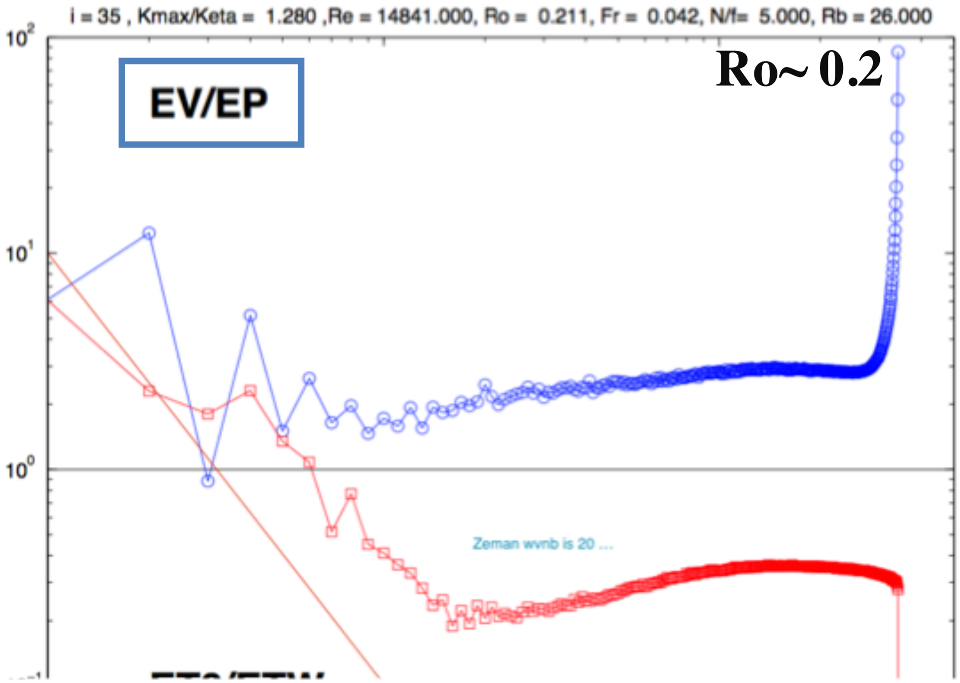


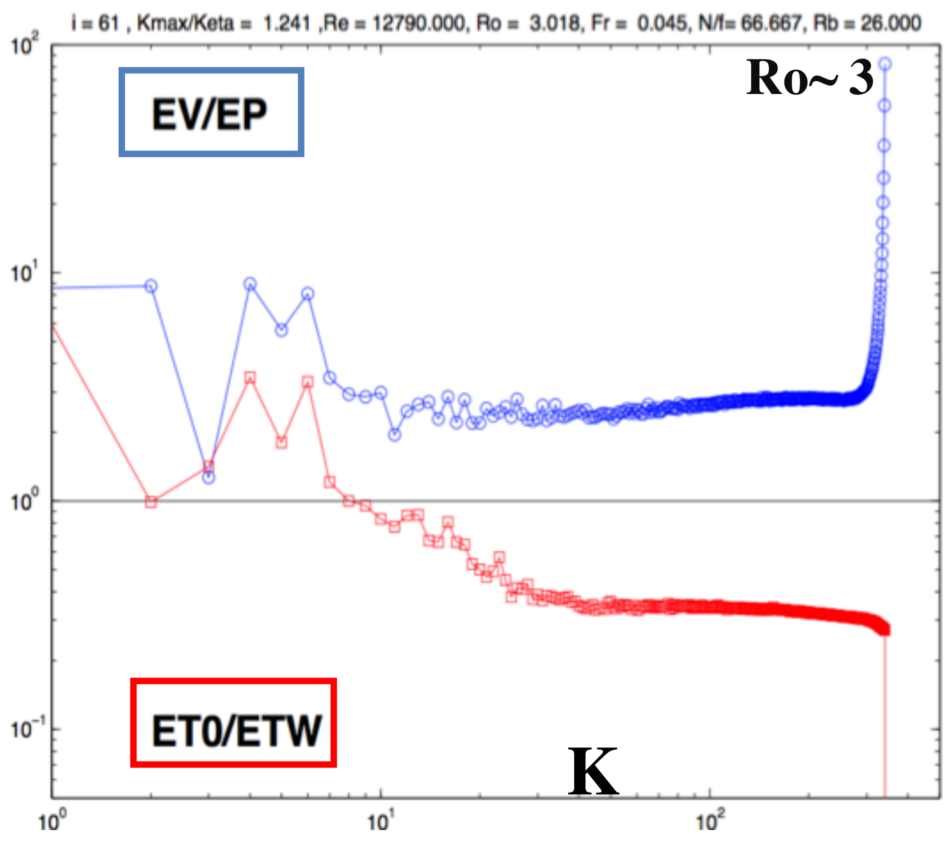
Rosenberg+ 2016

Variations with N/f of
←Ellison scale
and
E_P/E_V ↓









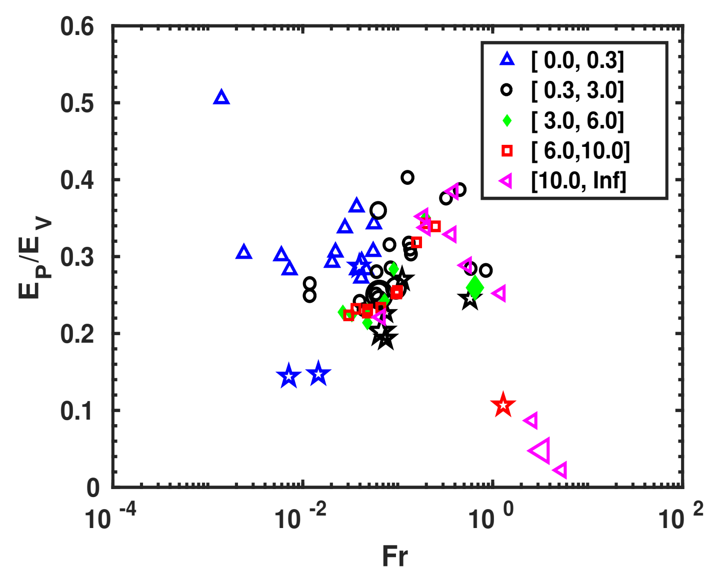
Ratios of energy **spectra** at peak:

Kinetic to potential

and

zero to wave mode

← Fr=0.04 , R_B~ 26



Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

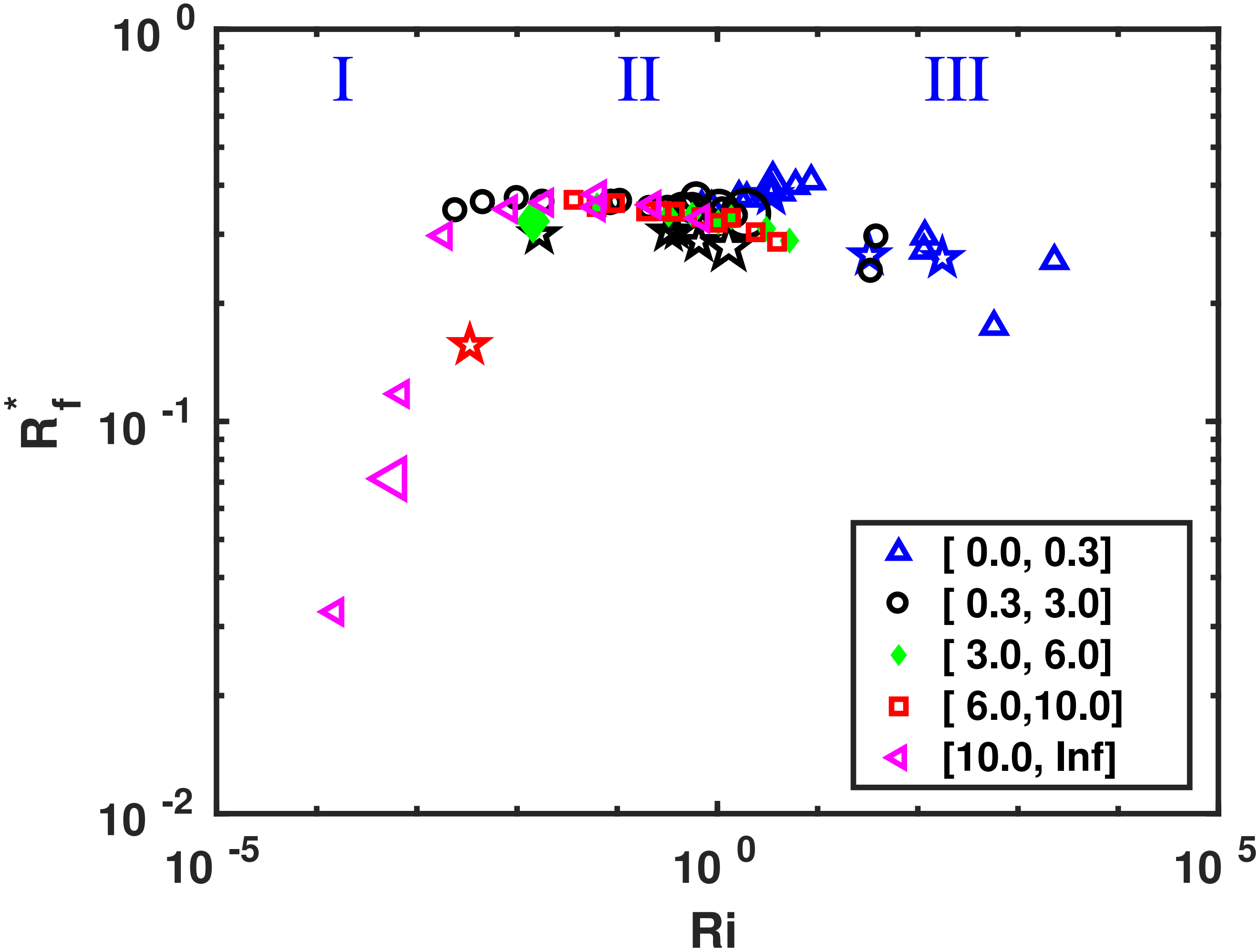
Ratio of potential to total energy dissipation

$\varepsilon_p / [\varepsilon_p + \varepsilon_v]$

versus

Richardson Number

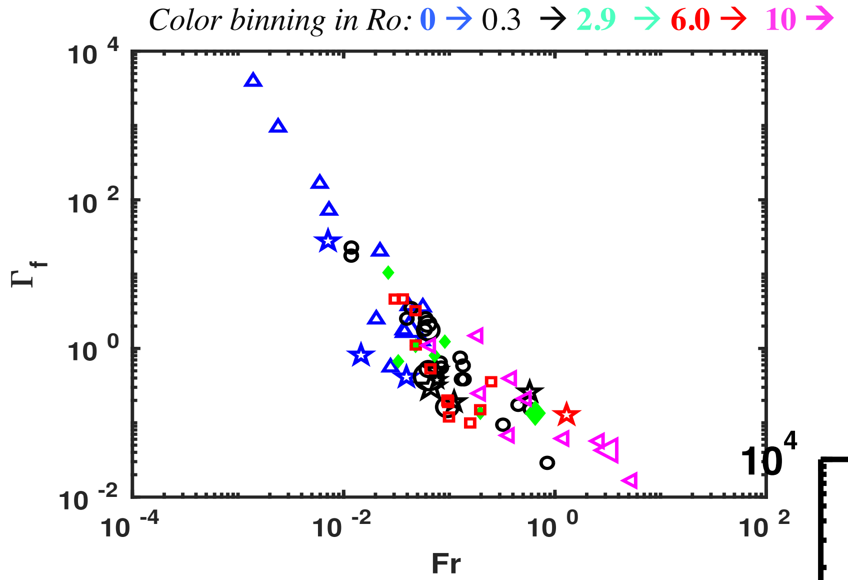
$$Ri \equiv [N / \langle \partial_z u_\perp \rangle]^2$$



Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

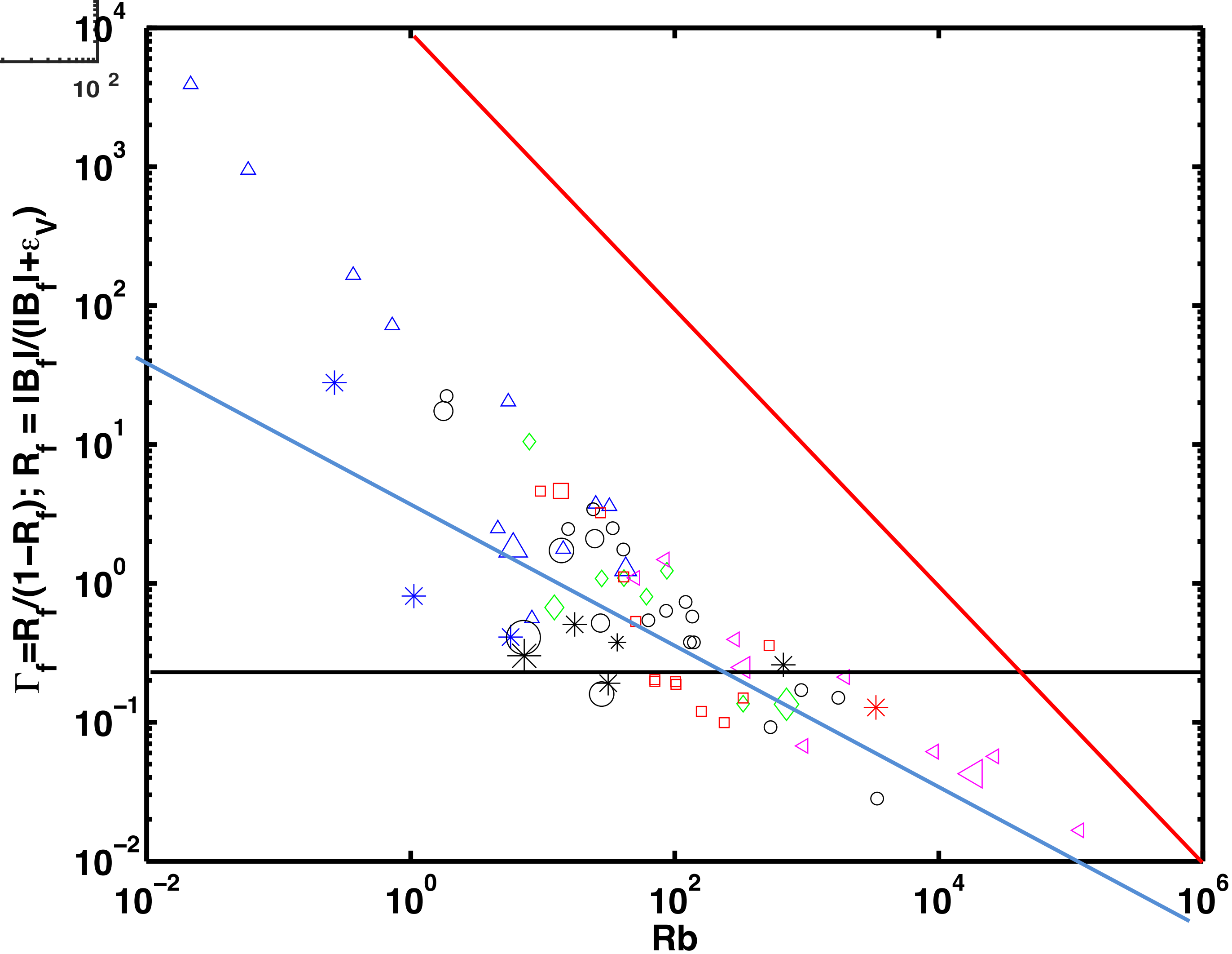
Rather constant in regime II,

but slight effect of rotation (blue triangles)

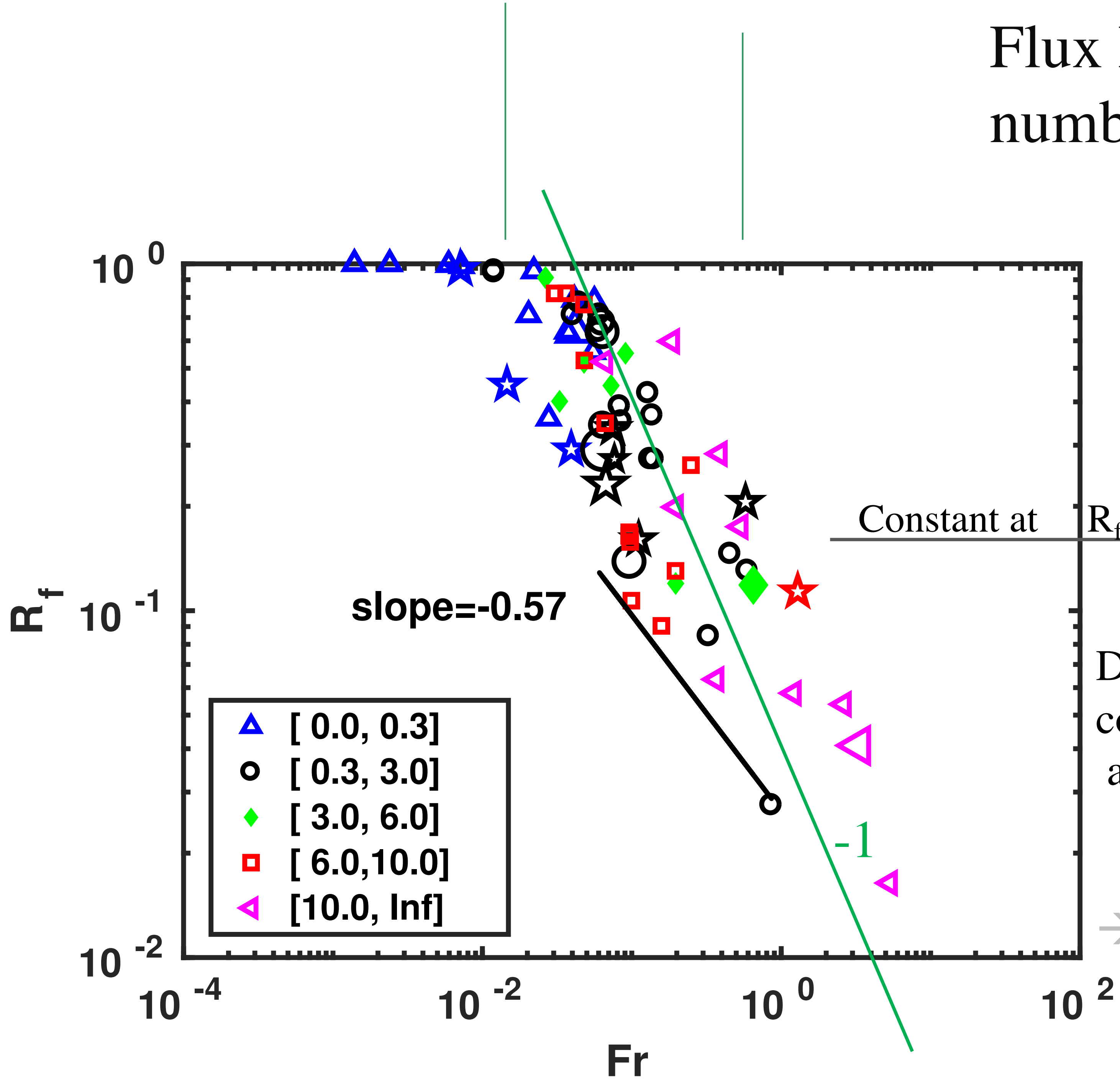


Mixing efficiency $\Gamma_f = B_f/\varepsilon_v$

in terms of R_B



Flux Richardson number: $R_f = B_f/[B_f + \varepsilon_V]$



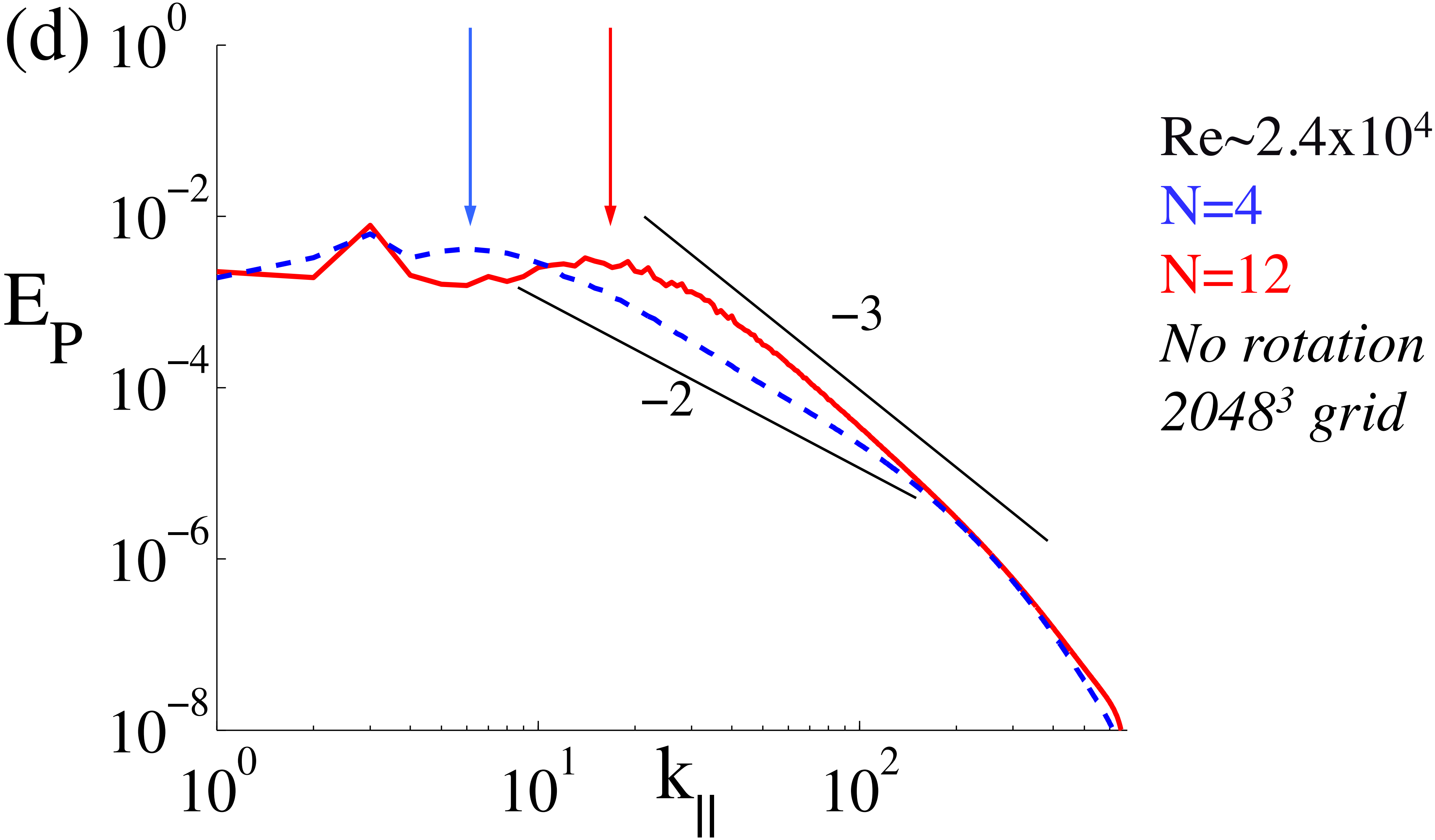
Constant at $R_f \sim 0.17$? *Perhaps for QG??*

Dependency on initial conditions for θ (small), and on N/f (large)?

vshw~[N/f]$d_\perp$ θ

→ θ~U[f/N] <1

Color binning in Ro: 0 → 0.3 → 2.9 → 6.0 → 10 →

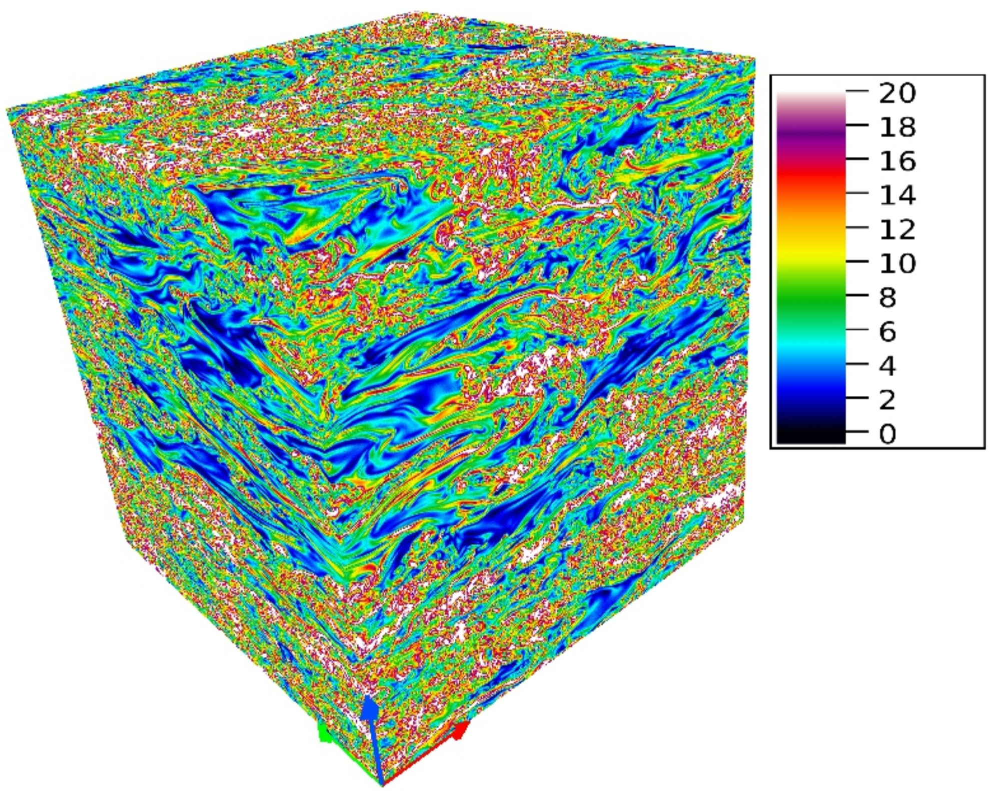


Potential energy=f(k_z):
Plateau until k_B~N/U,
the buoyancy wvnb.

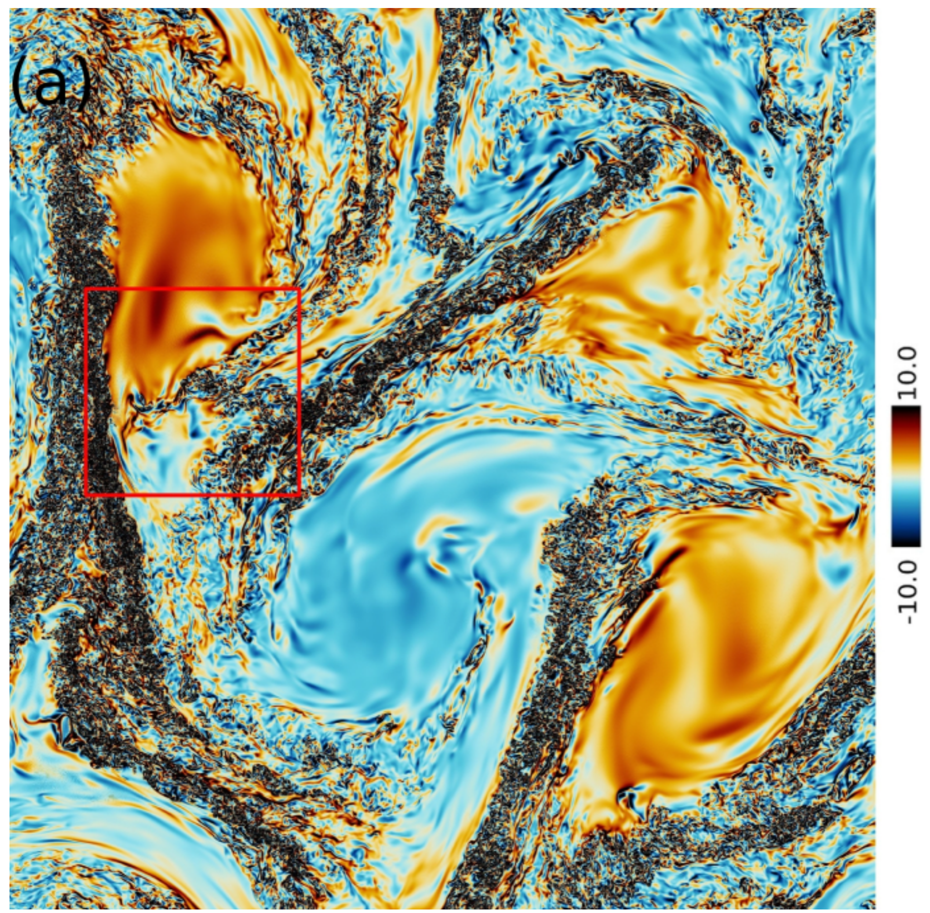
Rorai et al. 2014

Incompressible Boussinesq equations + rotation

Vorticity, *3D rendering ``atmosphere"*
Ro = 9.2, Fr = 0.067, Re ≃ 12000, R_B~53, **N/f=137**, 1024^3 grid, … *(Rosenberg+ 2016)*

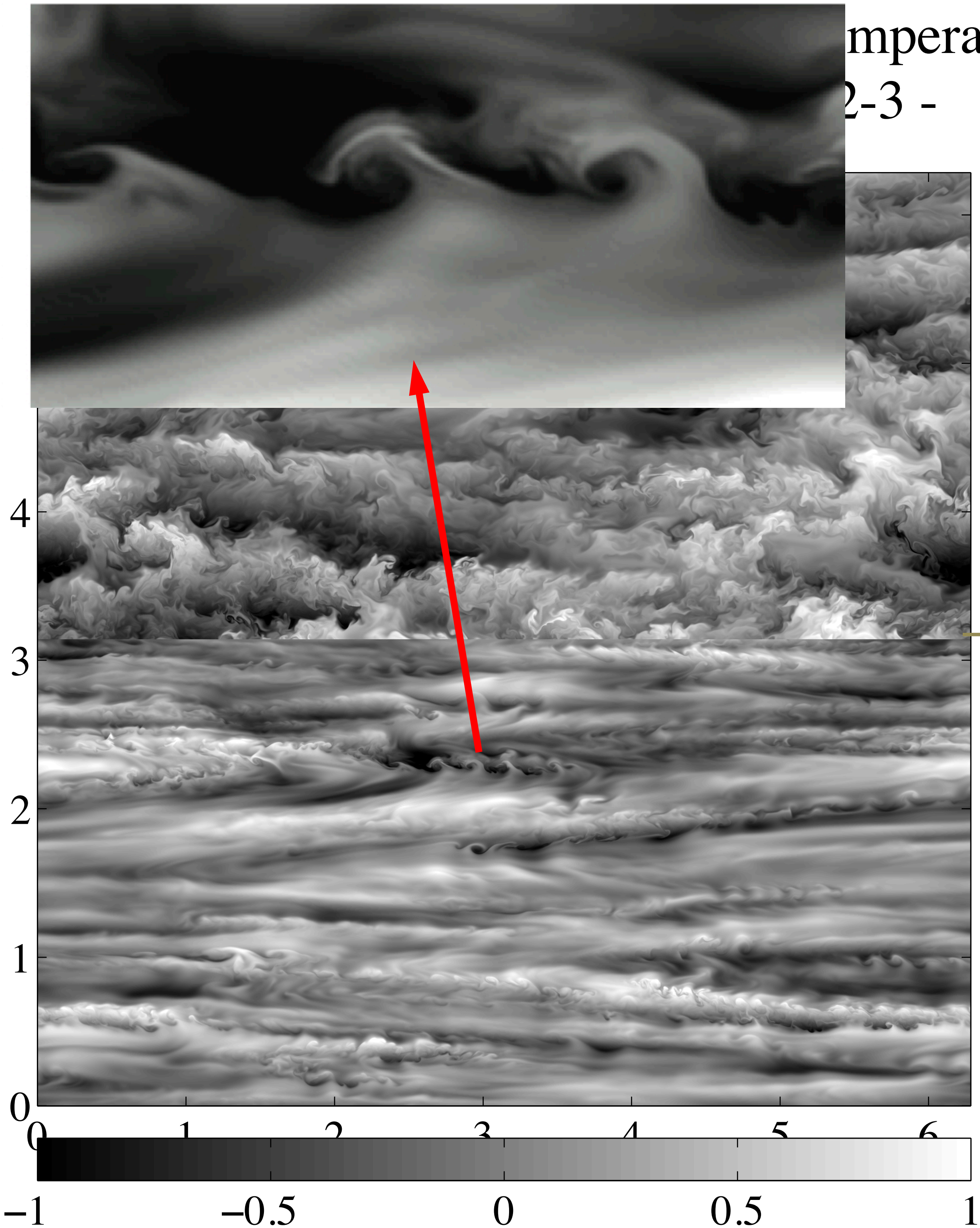


Vorticity, *2D cut, ``ocean"*, Ro ~ 0.12, Fr ~ 0.024, Re ~ 54000 , R_B~32, **N/f=5,** 4096^3 grid, decaying, resolved & strongly intermittent *(Rosenberg+ 2015)*



mperature fluctuations, xz slice

2-3 - **Re~ 24000, 2048^3 grids, f=0**



N=4, Fr ~ 0.11

R_B = $ReFr^2$ ~ 300

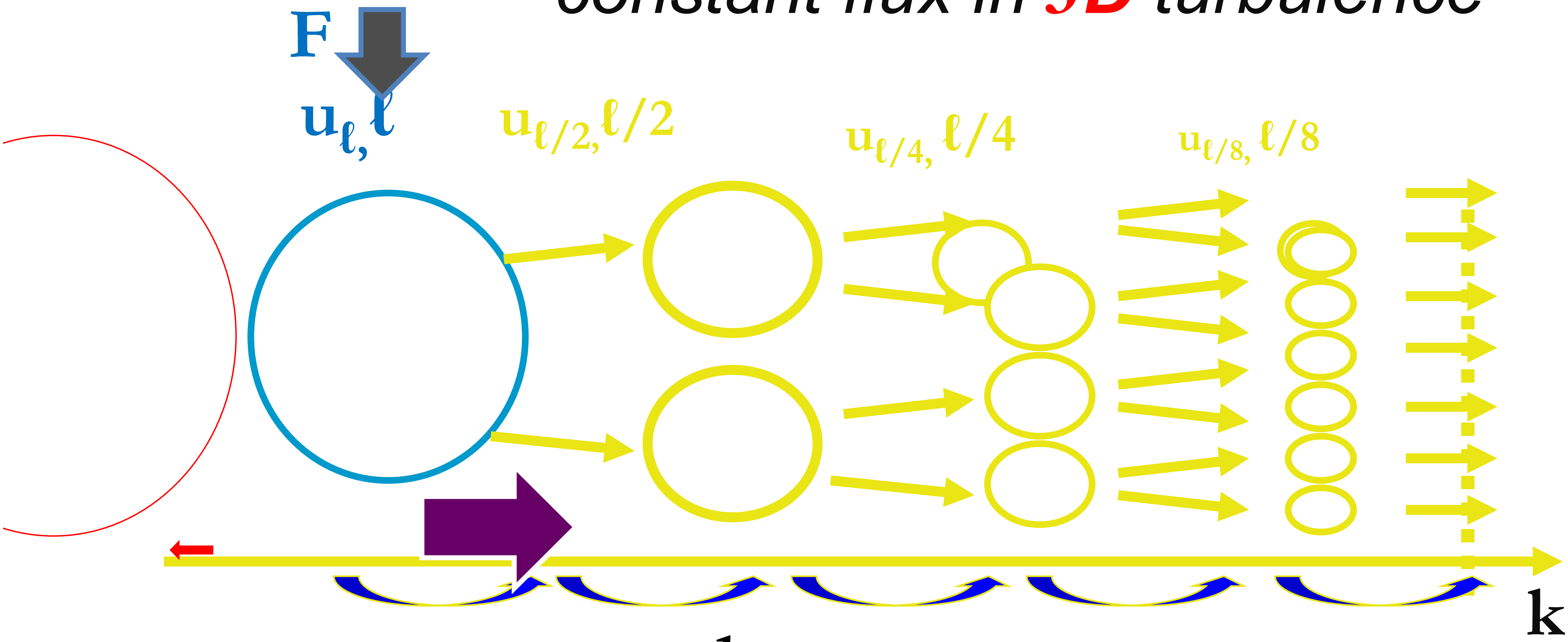
z

N=12, Fr ~ 0.03

R_B ~ 22

Rorai et al., 2014

Classical energy cascade with *constant flux in **3D** turbulence*



$$\sin a \cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))$$

Advection

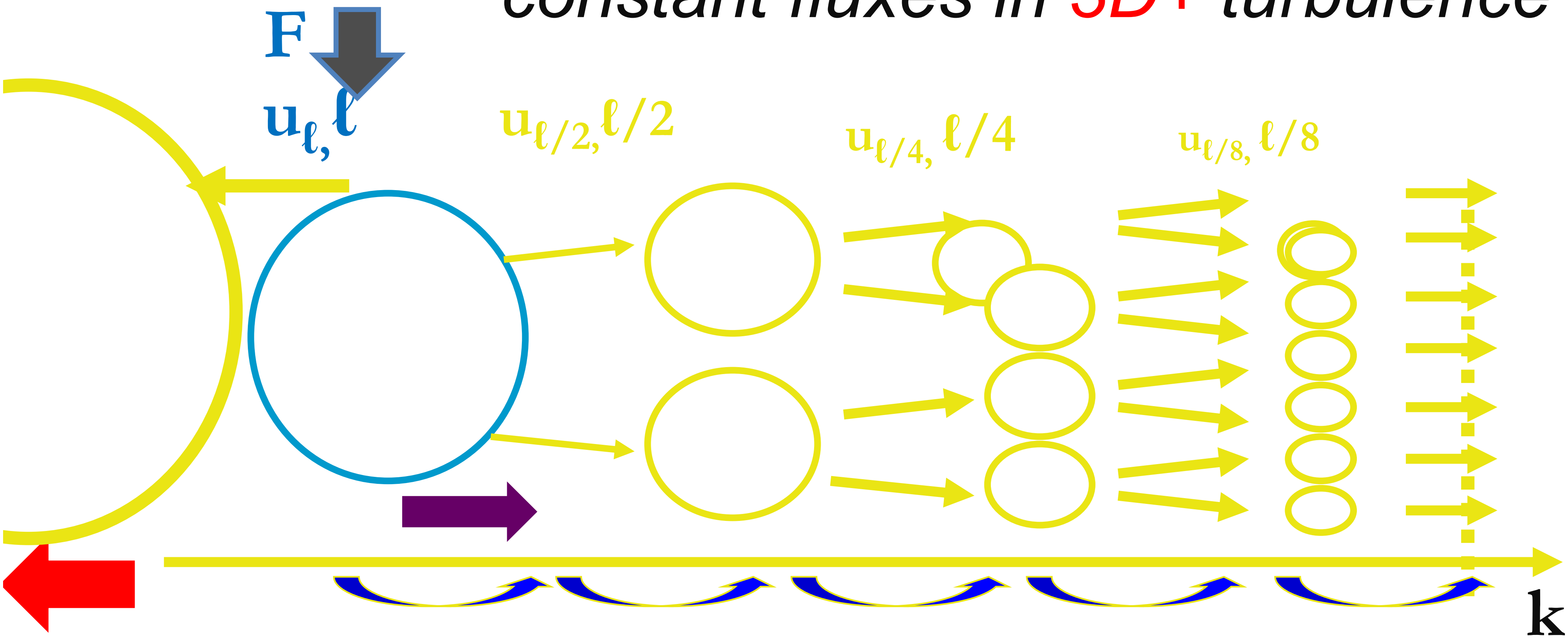
u.grad(u) →Fourier : E goes to small & large scales

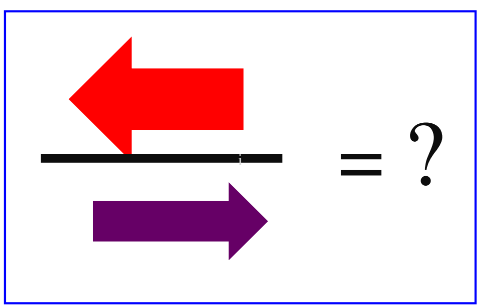
→ Convolution

ν

Eddy viscosity & eddy noise

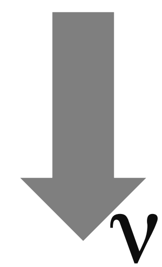
Dual *energy cascades* with dual *constant fluxes in 3D+ turbulence*





Rotation
with or without
stratification

And/or magnetic field

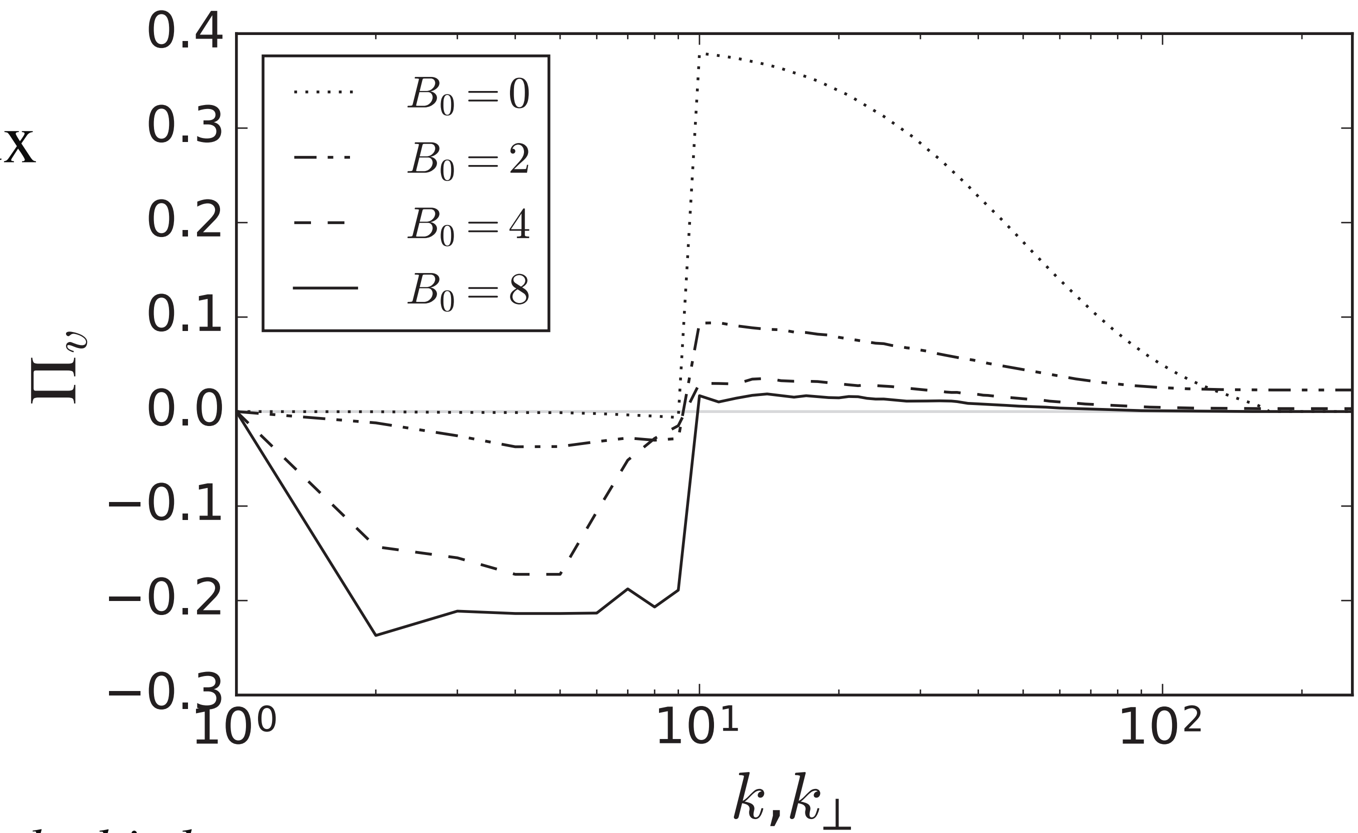


3D-MHD + imposed external field B_0 + F_V but with $F_M = 0$

(Alexakis 2011; Sujovolsky+ 2016)

TRIDIMENSIONAL TO BIDIMENSIONAL TRANSITION IN ...

Kinetic energy flux

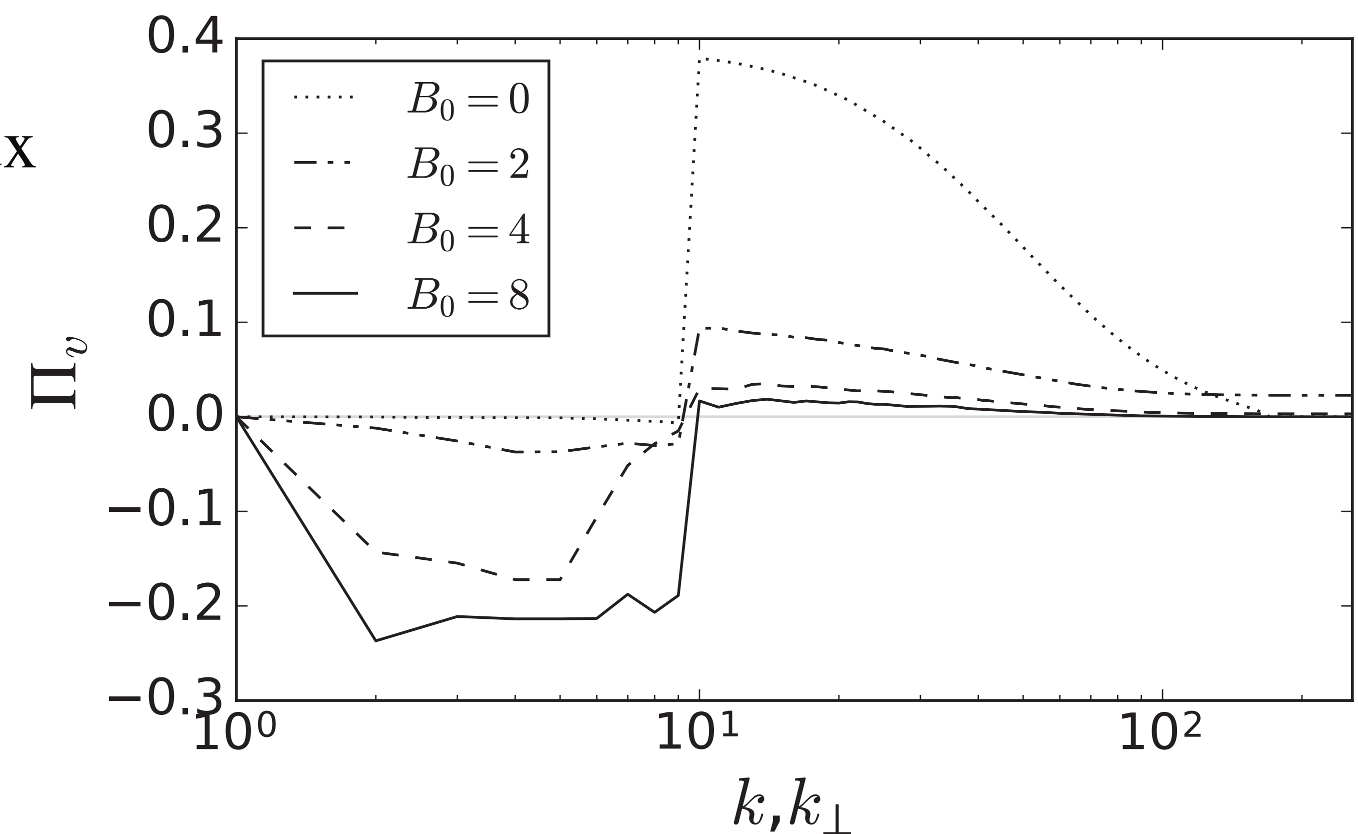


Also: kinetic Alfven and whistler waves in the ``Solar Wind'' *(Che+ 2014)*

3D-MHD + imposed external field B_0
+ F_V but with $F_M = 0$ *(Alexakis+ 2009; Sujovolsky+ 2016)*

TRIDIMENSIONAL TO BIDIMENSIONAL TRANSITION IN . . .

Kinetic energy flux

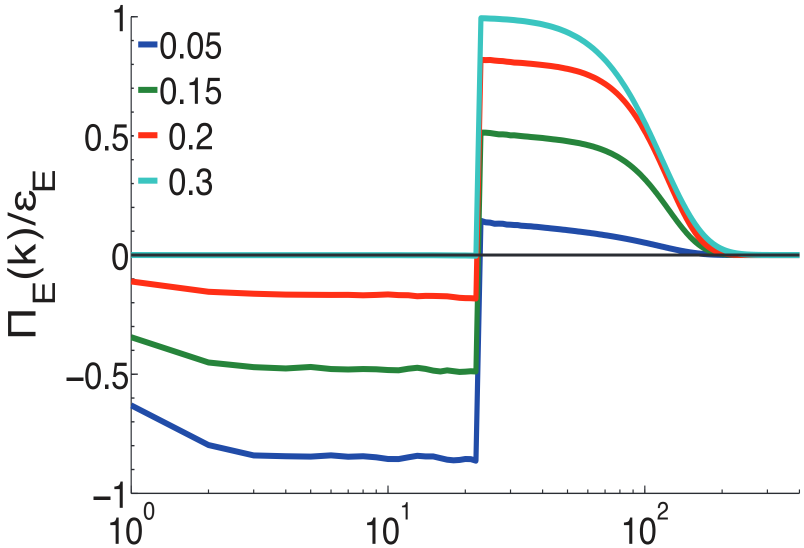


Also: kinetic Alfven and whistler waves in the ``Solar Wind'' (Che+ 2014)

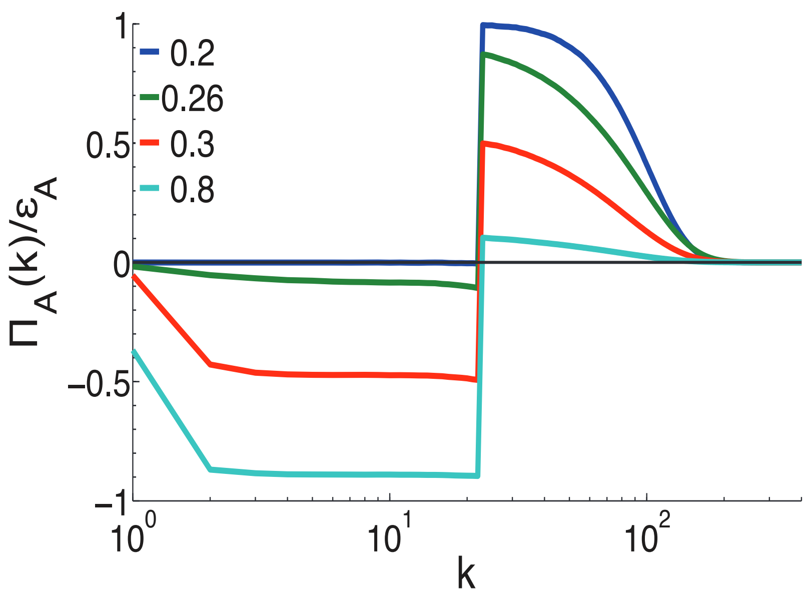
Possibility of lab. experiment?

Strictly two-dimensional forced MHD

Control parameter: $\mu = F_M/F_V$



Energy ↑ and magnetic potential ↓



FLUXES

$$\epsilon_E^- \propto (\mu_c - \mu)^{\gamma_E}$$

$$\epsilon_A^- \propto (\mu - \mu_c)^{\gamma_A}$$

$\gamma_E \sim 0.82$

$\gamma_A \sim 0.24$

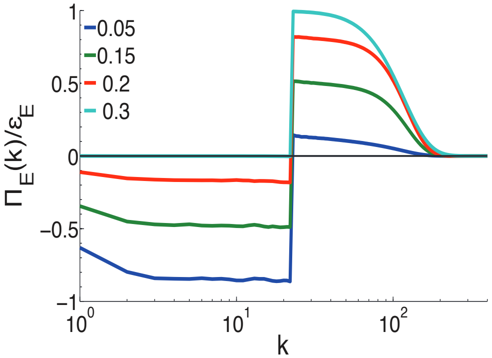
$\mu_c \sim 0.22_{(E)}$ *or* $0.25_{(A)}$

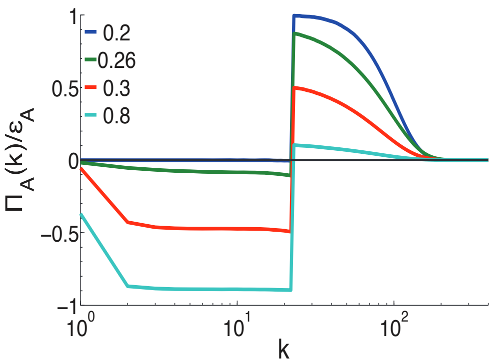
Seshasanayan+ 2014, 2016

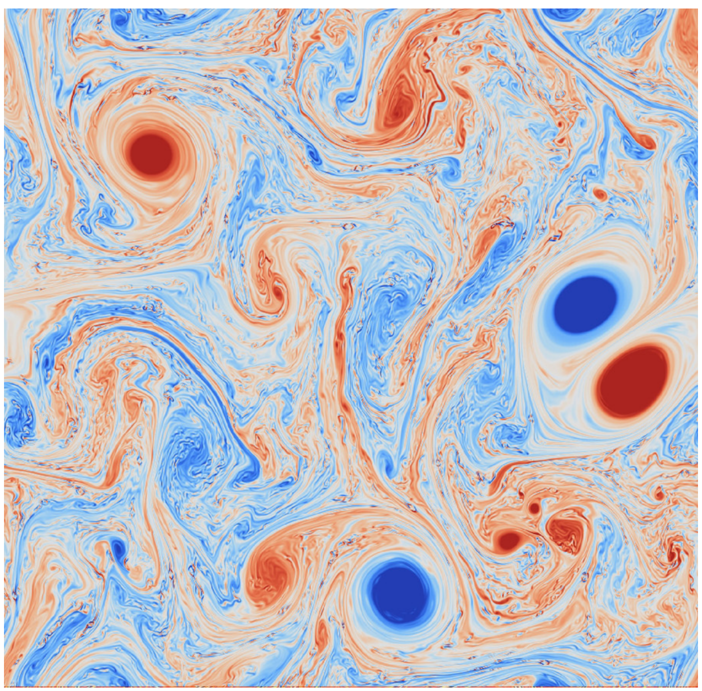
Strictly 2D forced MHD,
Control parameter $\mu = F_M/F_V$

$$E_A(k) = k^{-2}\, E_M(k)$$

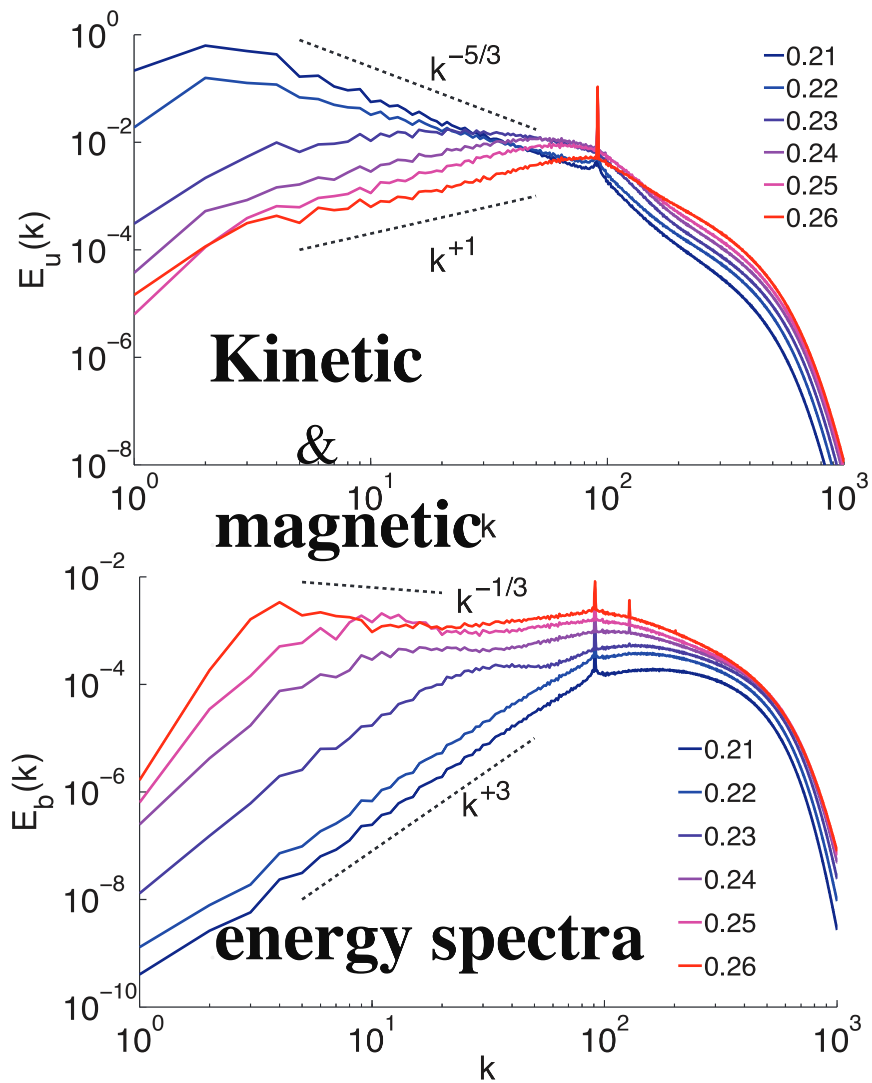
SESHASAYANAN, BENAVIDES, AND ALEXAKIS *(2014, 2016)*







Vorticity, forcing scale 1/20: _

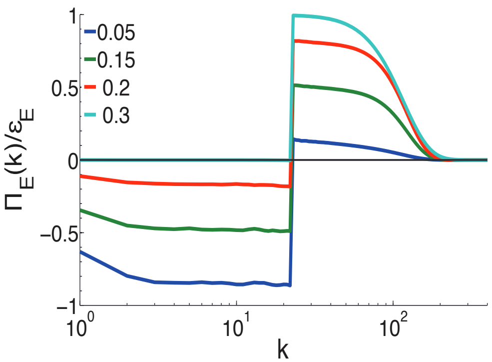


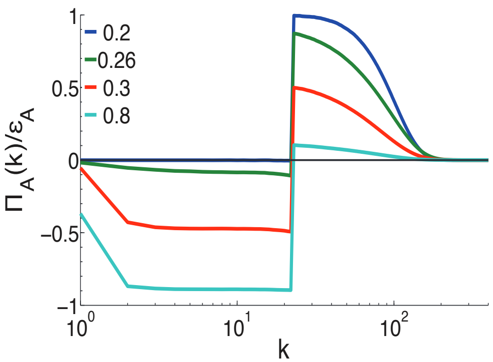
Strictly 2D forced MHD,

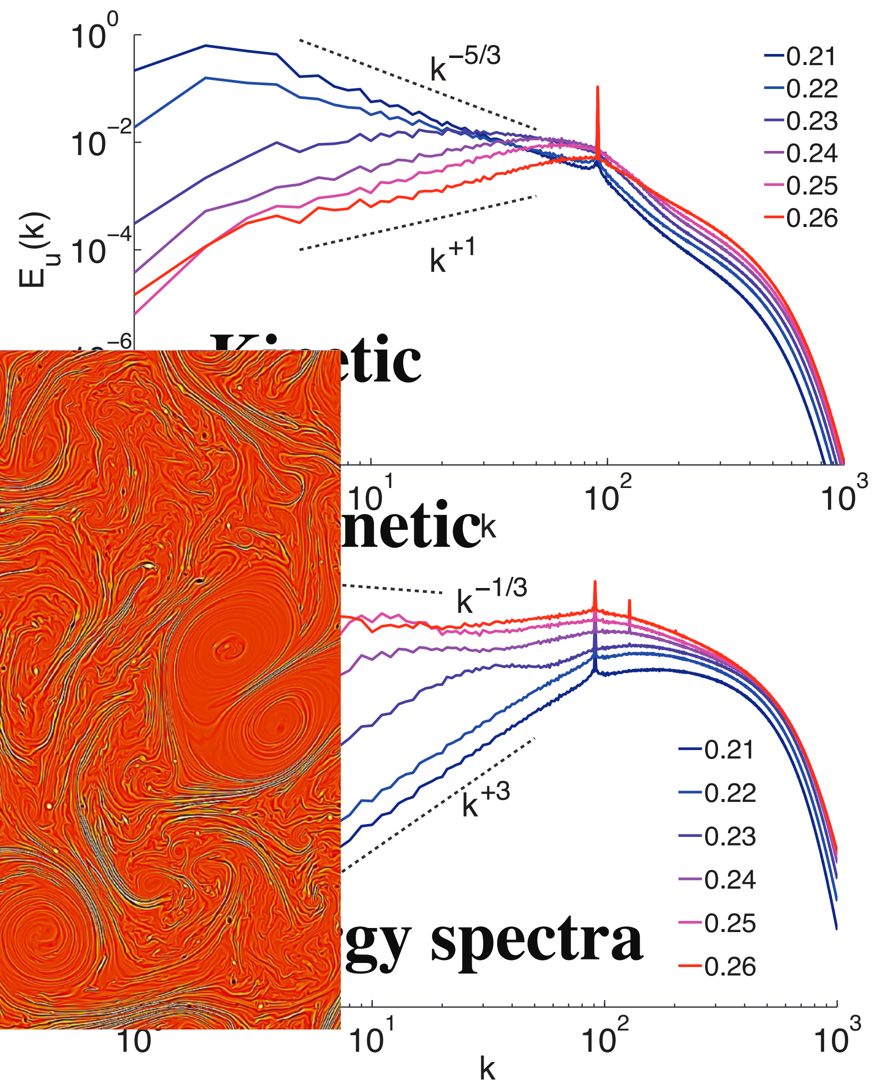
Control parameter $\mu = F_M/F_V$

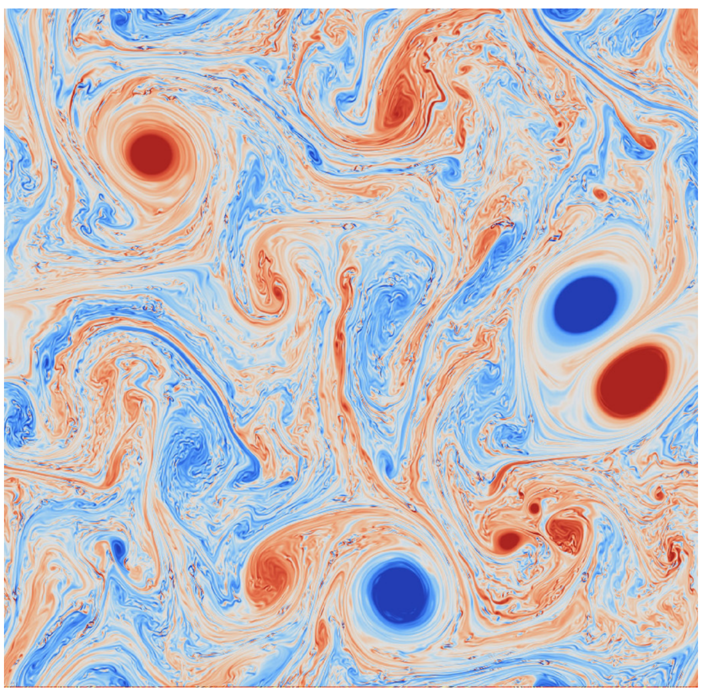
$$E_A(k)=k^{-2}\,E_M(k)$$

SESHASAYANAN, BENAVIDES, AND ALEXAKIS *(2014, 2016)*









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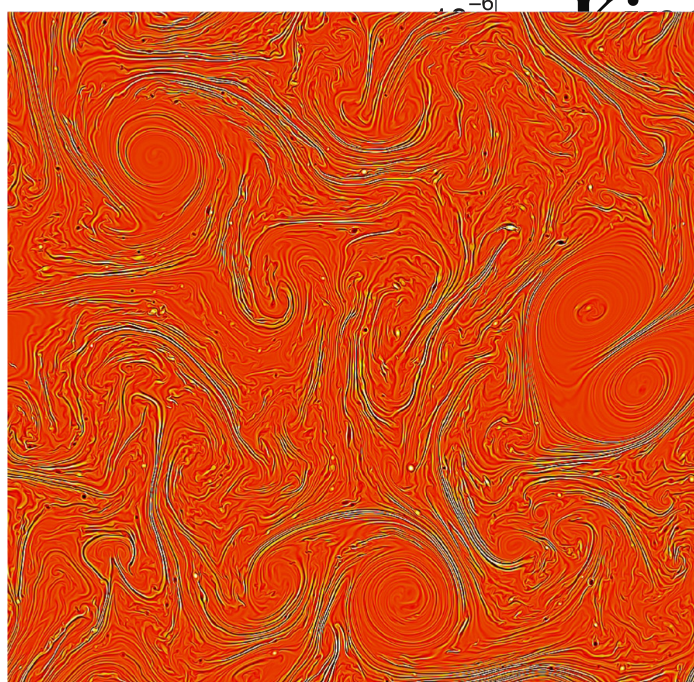

Current