Planetary Boundary-Layer Turbulence Modeling with Direct Statistical Simulation

Joseph Skitka[1]

Brad Marston[1]

Baylor Fox-Kemper[2]

1. Brown University, Department of Physics
2. Brown University, Department of Earth, Environmental, and Planetary Science

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Kavli Institute for Theoretical Physics
Program on Planetary Boundary Layers

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Motivation and Concept

- Use CE2/QL to model 3D planetary boundary layer turbulence
- Work towards a general subgrid modeling framework

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- Use CE2/QL to model 3D planetary boundary layer turbulence
- Work towards a general subgrid modeling framework
- Efficiency:
    - choose horizontally homogeneous cases
    - use horizontal averaging (to start): "HQL" / "HCE2"
    - still need further reduction
- Cases of developing turbulence:
    - Thermal Convection (Ait-Chaalal et al. 2015)
    - Langmuir Turbulence (McWilliams et al. 1997)

Thermal Convection

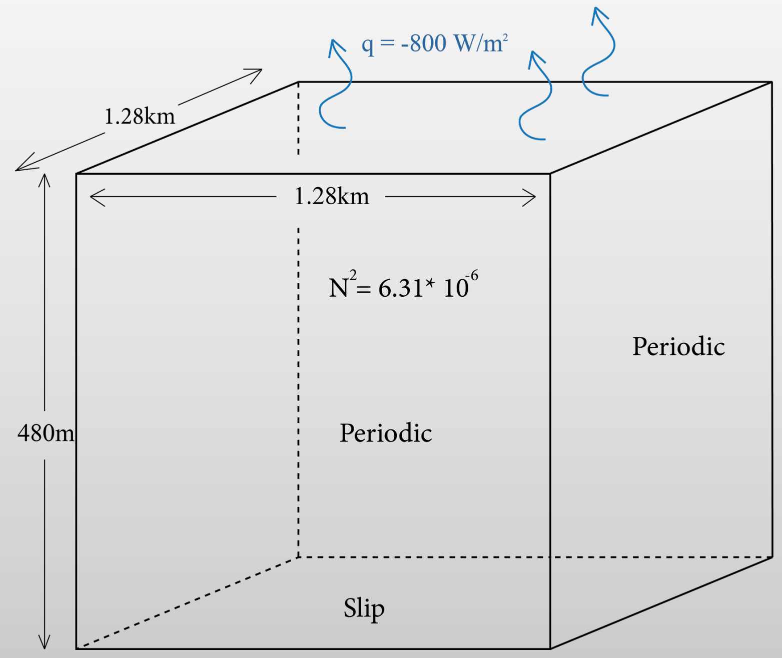
Non-Hydrostatic Boussinesq Equations:

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \mathbf{f}\times\mathbf{u} = -\frac{1}{\rho_0}\nabla p + g\rho_0\alpha(T - T_0) + \nu\nabla^2\mathbf{u}$$

$$\partial_t T + (\mathbf{u}\cdot\nabla)T = w + \kappa\nabla^2 T$$

$$\nabla\cdot\mathbf{u} = 0$$

Regular Cartesian (128x128x48) domain:



[1]F. Ait-Chaalal, T. Schneider, B. Meyer, and J.B. Marston. *Cumulant Expansions for Atmospheric Flows*. New Journal of Physics, **18**, 2016

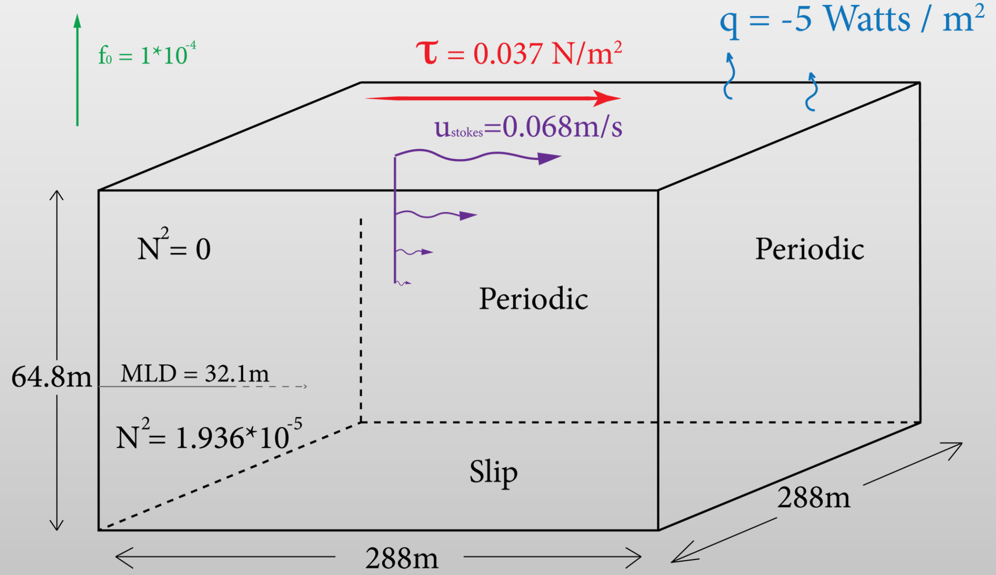
Langmuir Turbulence

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} + \mathbf{f}\times(\mathbf{u}+\mathbf{u_s}) = -\frac{1}{\rho_0}\nabla\left(p + \frac{\rho_0}{2}\left|\mathbf{u}+\mathbf{u_s}\right|^2 - \left|\mathbf{u}\right|^2\right) + \nu\nabla^2\mathbf{u} + \mathbf{u_s}\times\nabla\times\mathbf{u} + \frac{\boldsymbol{\tau}}{\rho_0} + g\rho_0\alpha\,(T - T_0)$$

$$\partial_t T + ((\mathbf{u}+\mathbf{u_s})\cdot\nabla)\,T = w + \kappa\nabla^2 T$$

$$\nabla\cdot\mathbf{u} = 0$$

Stokes drift: $\mathbf{u_s} = u_s e^{2k_s z}\hat{\mathbf{x}}$



$\{N_x, N_y, N_z\} = \{96, 96, 72\}$

J. C. McWilliams, P. P. Sullivan, and C. Moeng. *Langmuir Turbulence in the Ocean*. Journal of Fluid Mechanics, **334**, 1997.

Reduction #2: Model Reduction
Truncated Basis

Choose an energetically optimized basis

Proper Orthogonal Decomposition (POD)

$$\mathsf{C}_{ij} = \langle q_i q_j \rangle$$

$$\mathsf{C}\phi_i = \lambda_i \phi_i$$

Note: POD modes are horizontal Fourier modes

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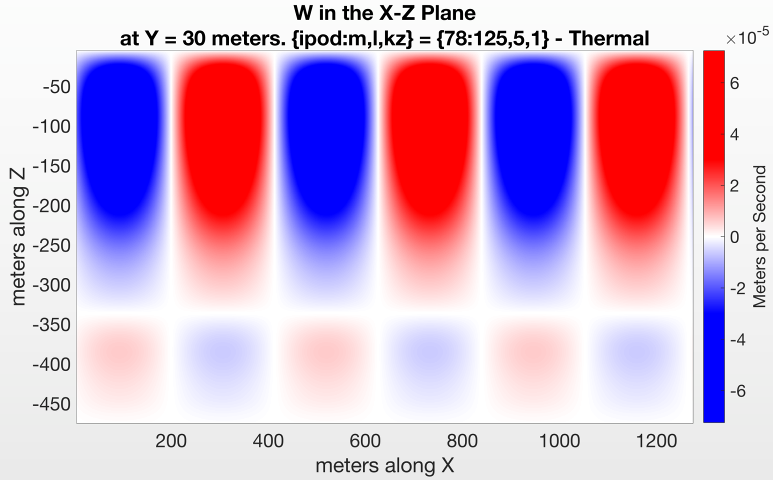
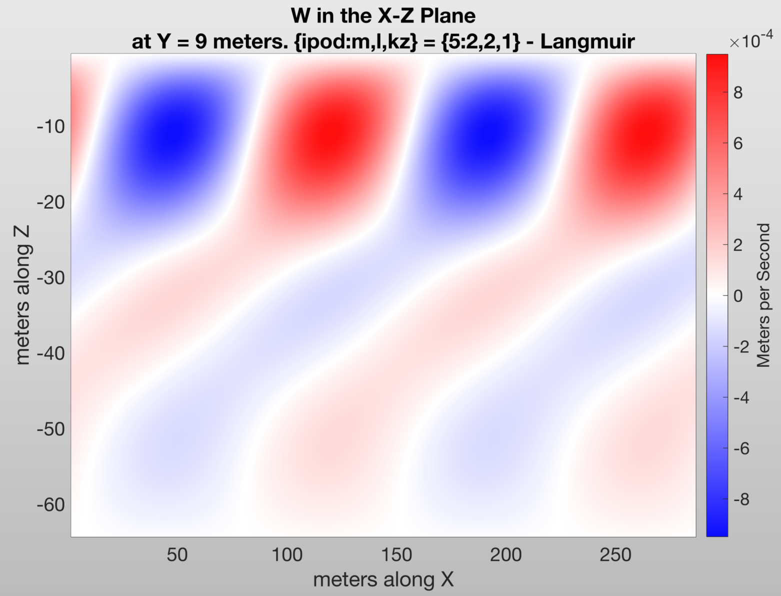


78th POD mode



5th POD mode

Reduced Model Process

24 HQL "DNS" runs on the MITgcm[1]

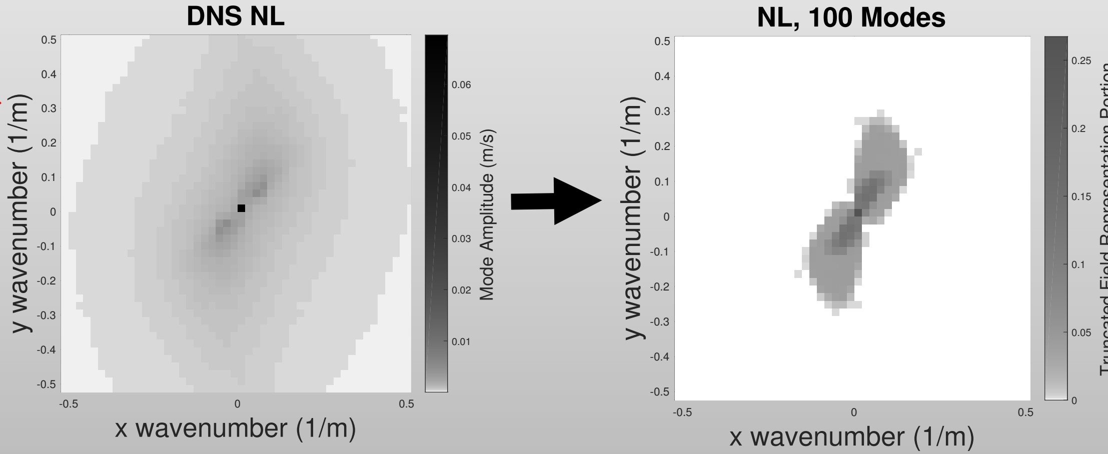
Proper Orthogonal Decomposition (POD)

$$C_{ij} = \langle q_i q_j \rangle$$

$$C = \frac{1}{M} \sum_{1 \leq s \leq M} \boldsymbol{q_s} \boldsymbol{q_s}^{\top}$$

$$C\phi_i = \lambda_i \phi_i$$

Truncation



12 runs of Galerkin Projection of EOMs on new basis (RM HQL)

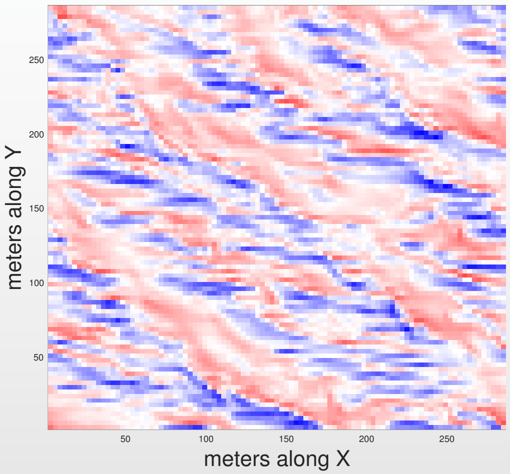
[1] http://mitgcm.org/

Results

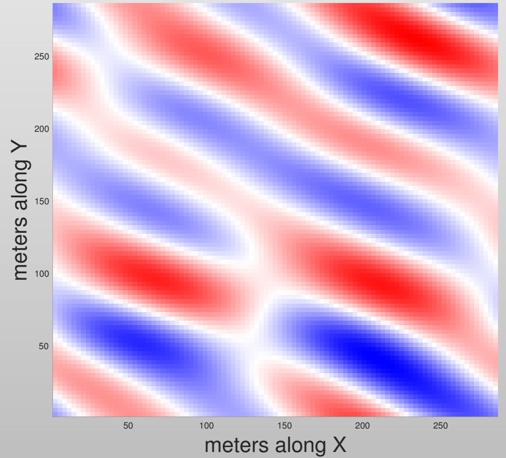
Results: Vertical Velocity Fields
Langmuir Turbulence

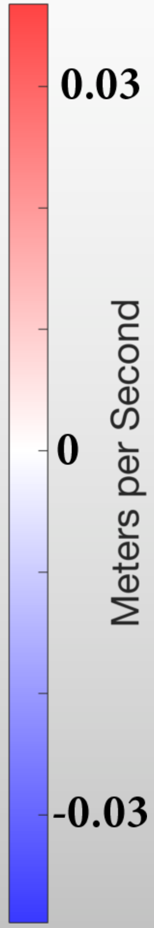
t = 54h
z = -7.2m
Top View

NL DNS



HQL DNS





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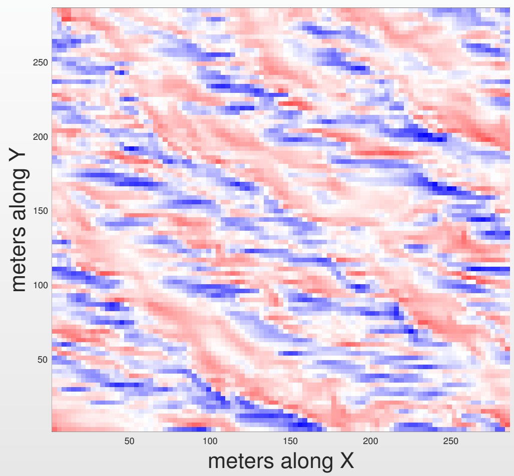
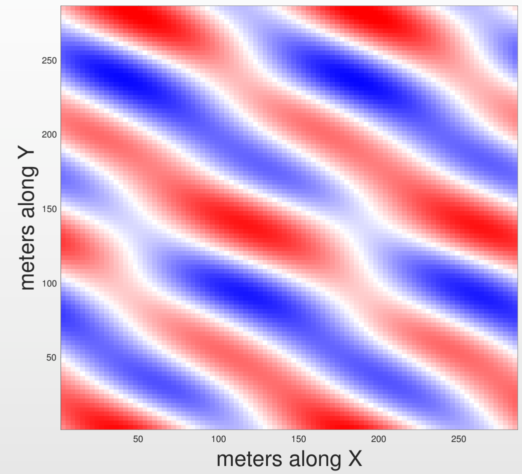


HQL 50 Modes



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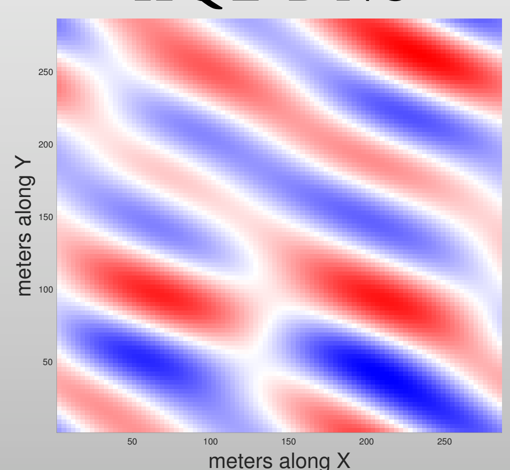

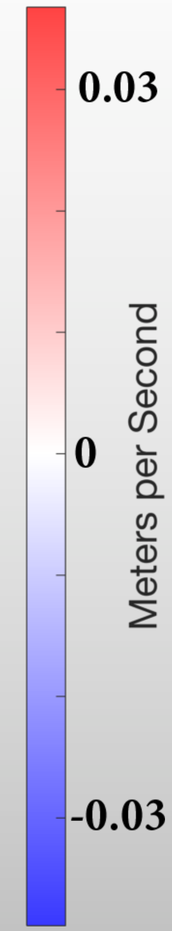

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Results: Vertical Velocity Fields
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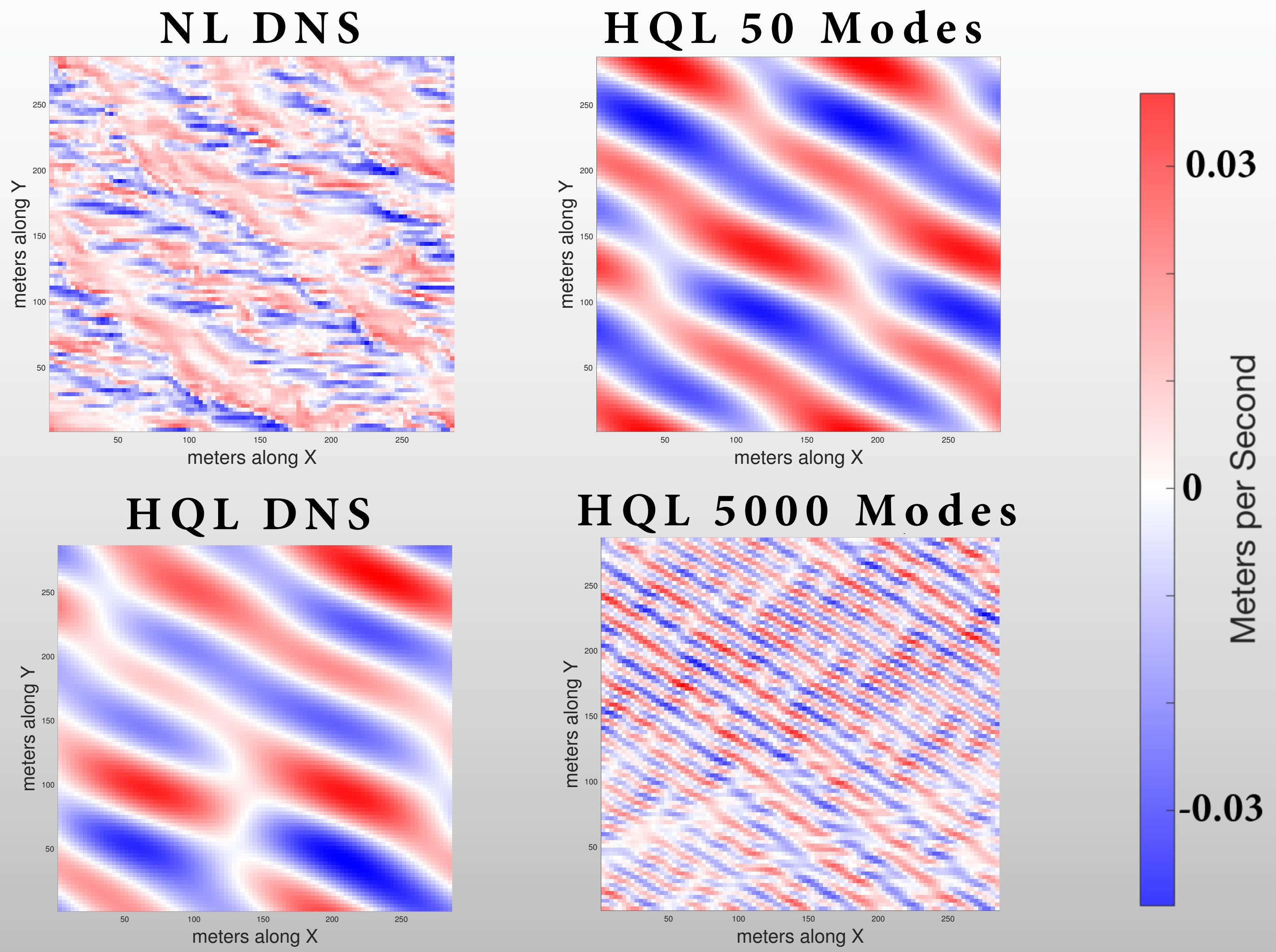

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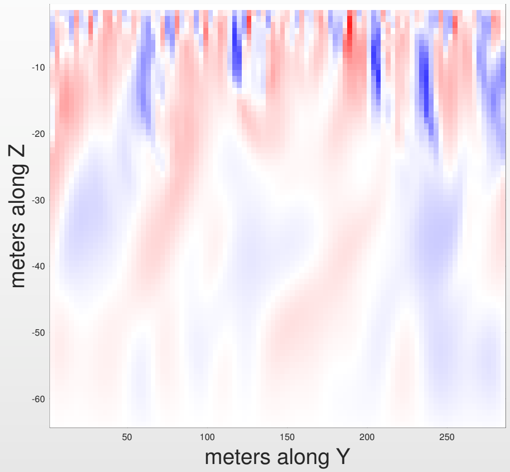
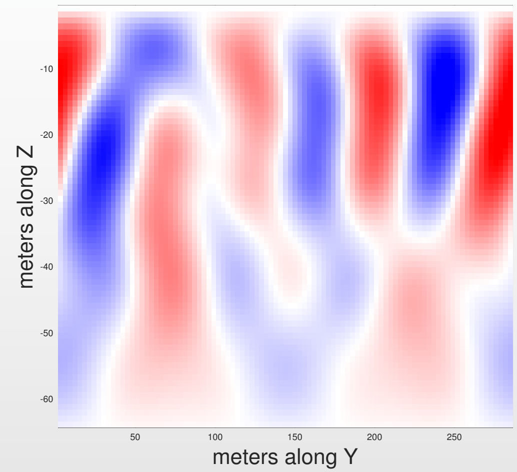
Results: Vertical Velocity Fields
Langmuir Turbulence

t = 54h
Side View

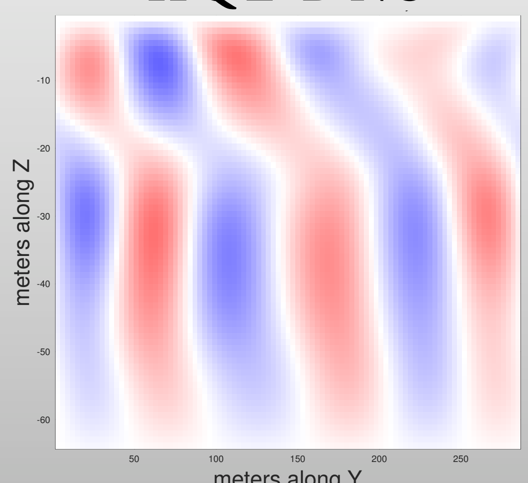
NL DNS



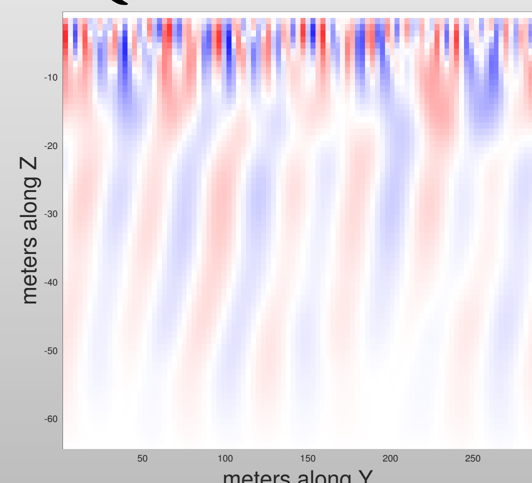
HQL 50 Modes

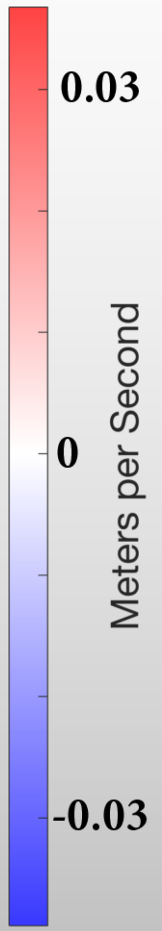


HQL DNS



HQL 5000 Modes





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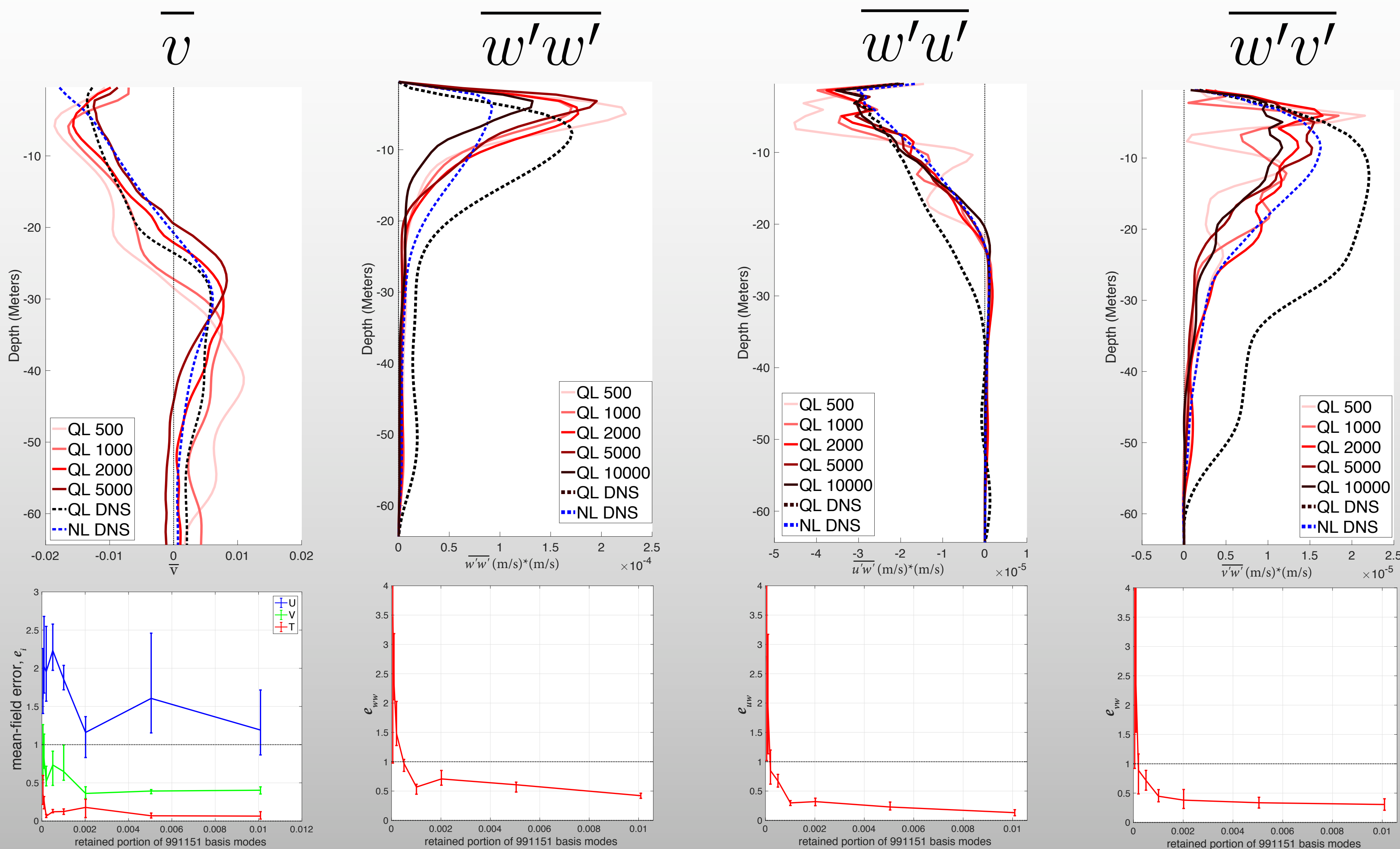
Results: Vertical Profiles and Errors Langmuir Turbulence



Thermal Convection Results

- Similar to Langmuir, except the performance first gets better, around 500 modes, and then worse, around 5000 modes.

Conclusions & Open Questions

Key Conclusions:

- RM HQL exhibits nonuniform convergence
- RM HQL can perform better than HQL "DNS"

HQL Conclusions & Open Questions

Key Conclusions:

- RM HQL exhibits nonuniform convergence
- RM HQL can perform better than HQL "DNS"

Open Questions:

- Can we determine optimal basis truncation?
- Can we predict quality of representation?
- Can we capture localized coherent structures?

Ensemble Averaging

Improvements with Ensemble Averaging?

Challenge with horizontal averaging:

- cannot capture coherent structures

ensemble averaging, EQL/ECE2:

- Fields are larger in memory
- Execution is more expensive
- Mean fields can have coherent structures

Horizontal Averaging	Ensemble Averaging
$\overline{q}\left(\mathbf{m}=\mathbf{0},z\right)$	$\overline{q}\left(\mathbf{m},z\right)$
$q'\left(\mathbf{m}\neq\mathbf{0},z\right)$	$q'\left(\mathbf{m},z\right)$

Improvements with Ensemble Averaging?

- Homogenous IC's, runs the same as HQL/HCE2
- Single instance IC's, runs the same as NL
- Inhomogeneous noise in IC's, inhomogeneous mean field can emerge.

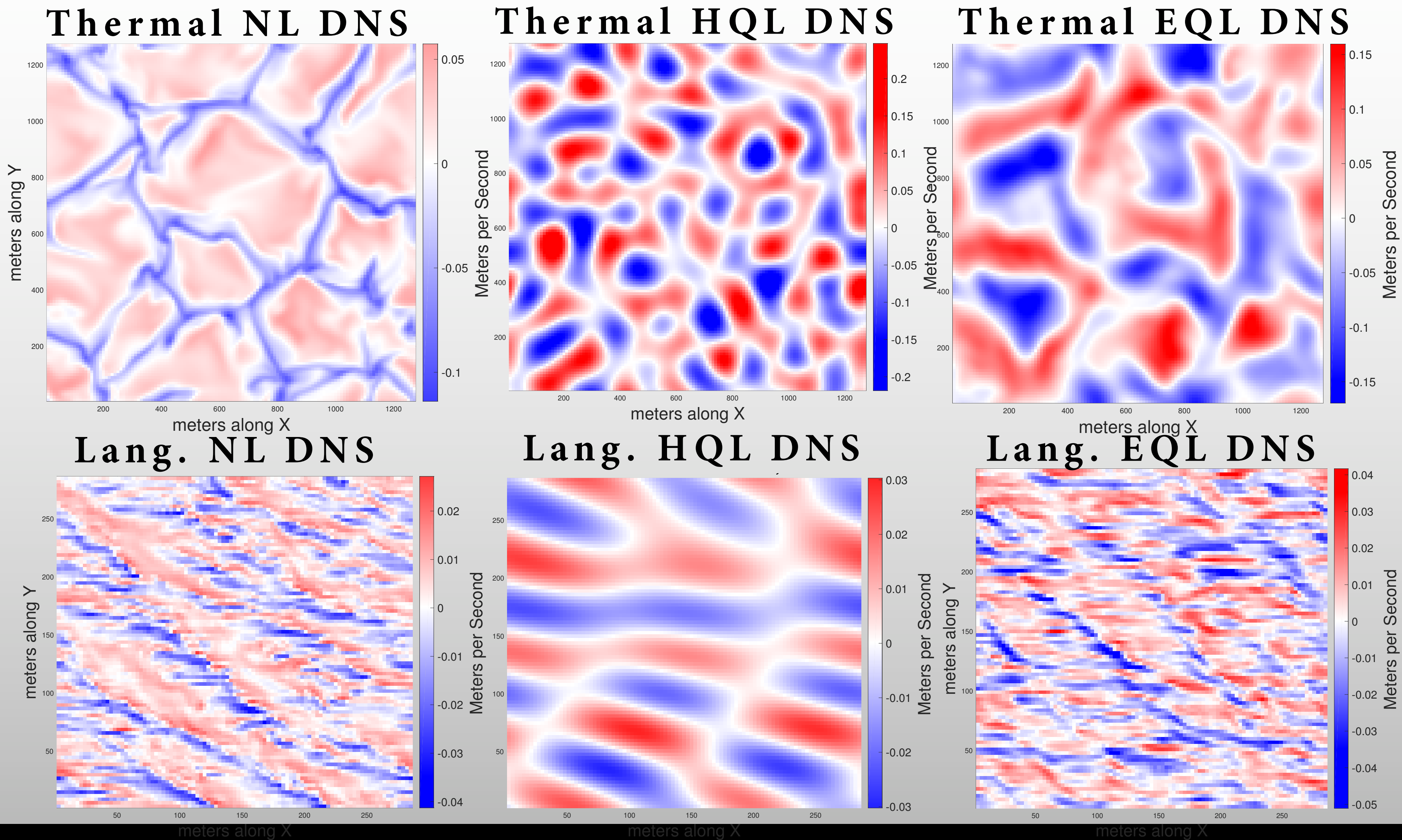
<u>Horizontal Averaging</u>

$$\overline{q}\left(\mathbf{m}=\mathbf{0},z\right)$$
$$q'\left(\mathbf{m}\neq\mathbf{0},z\right)$$

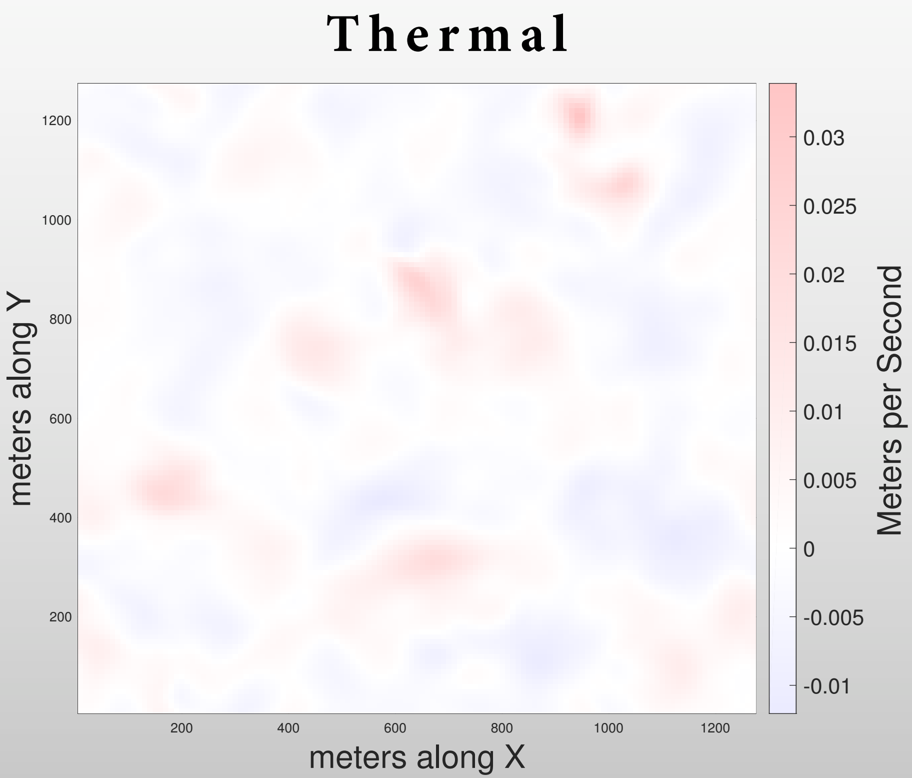
<u>Ensemble Averaging</u>

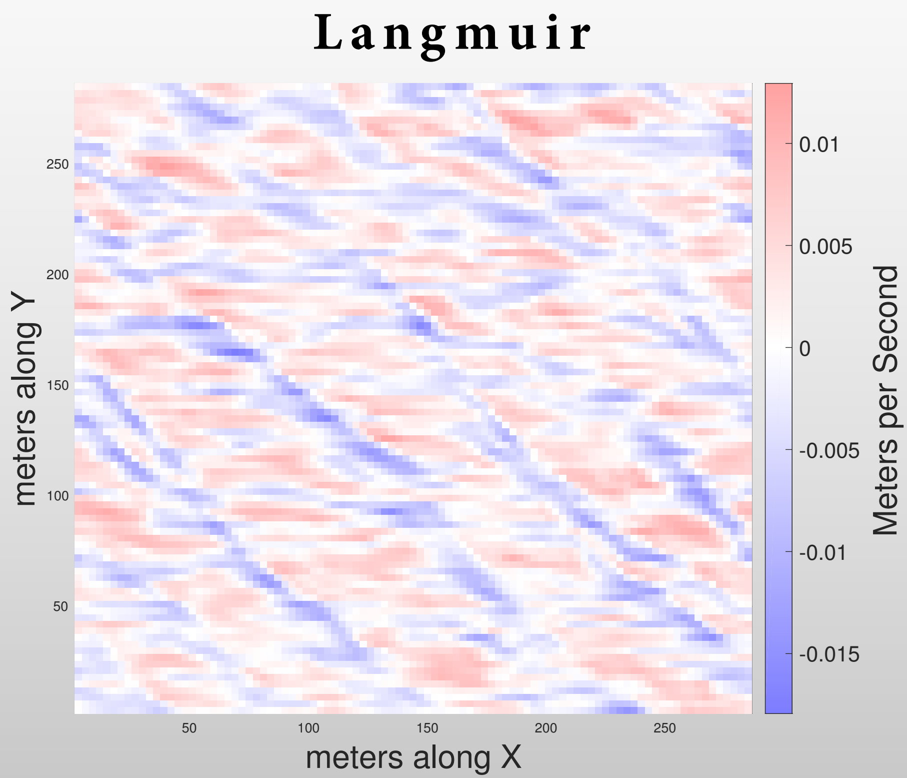
$$\overline{q}\left(\mathbf{m},z\right)$$
$$q'\left(\mathbf{m},z\right)$$

EQL Fields (w top-down)

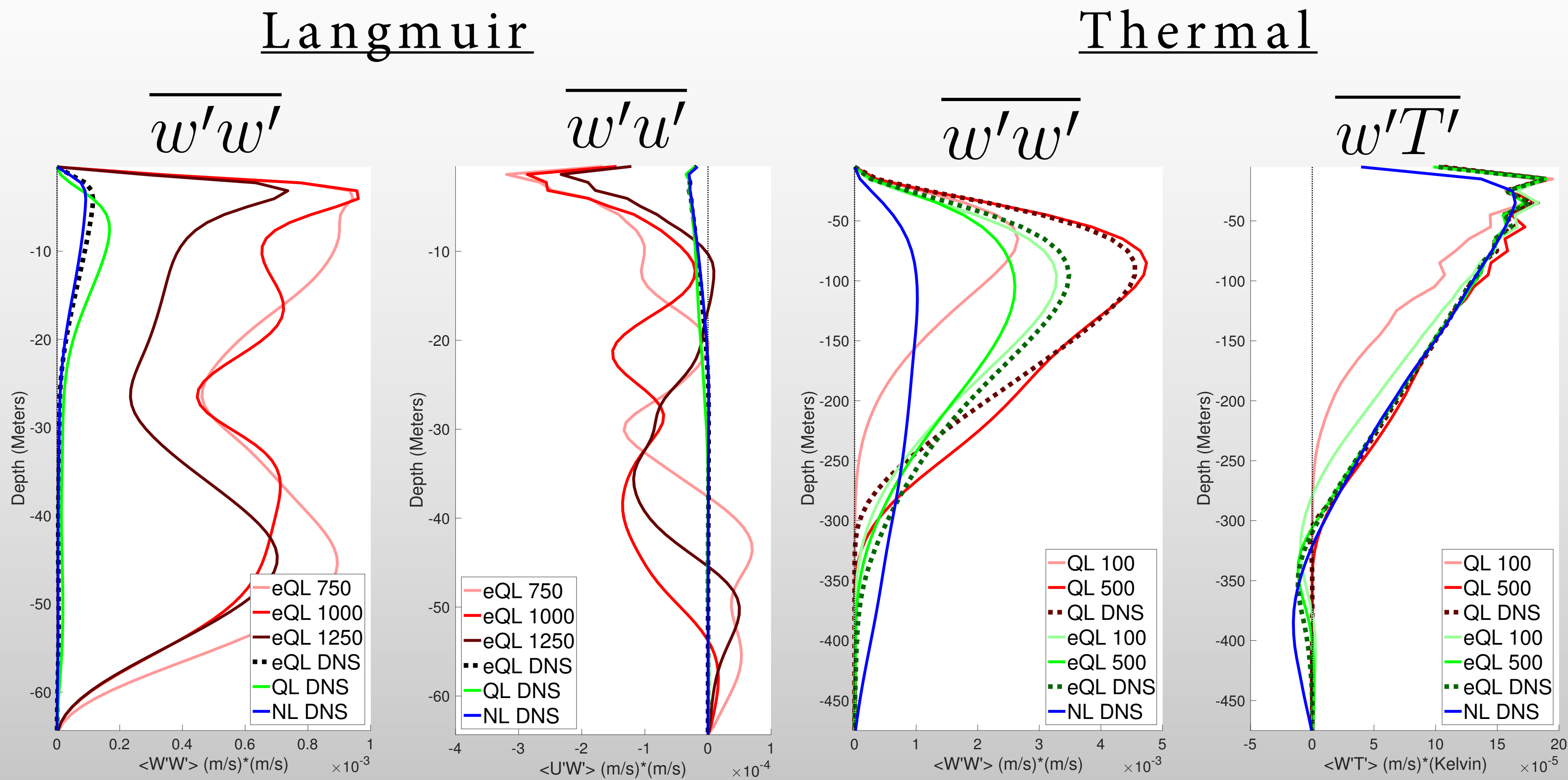


EQL Mean Fields





RM EQL Profiles



EQL Conclusions and Questions

- Very different mean-field behavior that depends on structure symmetries.
- Strong mean-field emergence results in NL-like EQL DNS solutions.
- There is a tradeoff between mean-field emergence and efficient RM modeling.

- **Question**: How can we predict or control mean-field emergence?

Next Steps

Path Forward:

- Develop means of predicting, stimulating, and suppressing mean-field emergence in EQL
- Discerning non-local from local subgrid effects
- Plug this into an overlying model somehow

Thank You; References

Background Reading

[1] Ait-Chaalal, F., Schneider, T., Meyer, B. and Marston, J.B., Cumulant expansions for atmospheric Flows. New Journal of Physics 18.2 (2016): 025019.

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[3] Bakas, N.A. and Ioannou, P.J. Emergence of large scale structure in barotropic β-plane turbulence. Physical review letters 110.22 (2013): 224501.

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[5] Large, W.G., McWilliams, J.C. and Doney, S.C., Oceanic vertical mixing: A review and a model with a nonlocal boundary layer parameterization. Reviews of Geophysics 32.4 (1994)

[6] Skitka, J. M., Marston, J. B. and Fox-Kemper, B. Reduced-Order Quasilinear Ocean Boundary-Layer Turbulence Modeling. In Preparation, 2018.

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