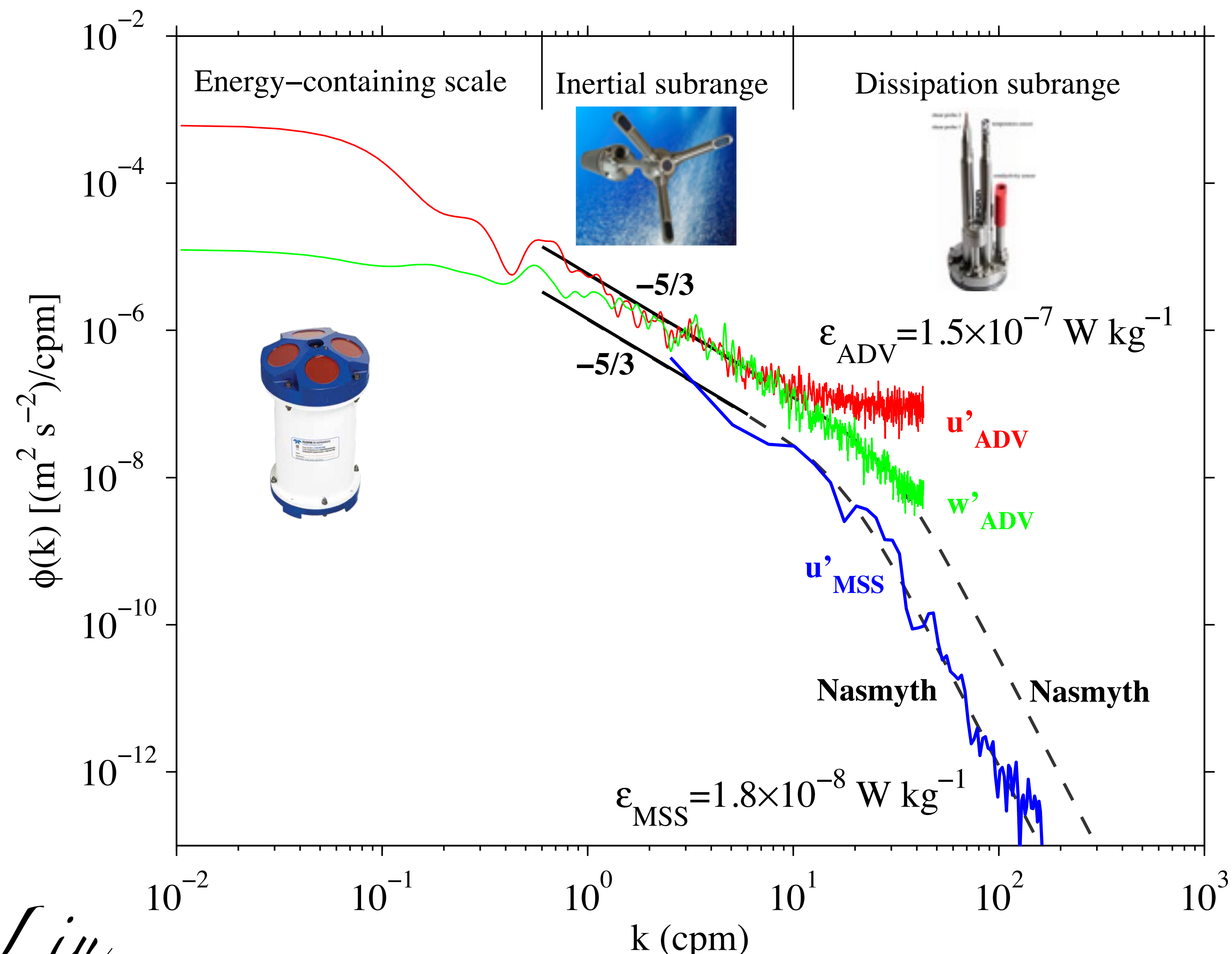
Measuring (small-scale) **Turbulence in the Ocean**



Zhiyu Liu

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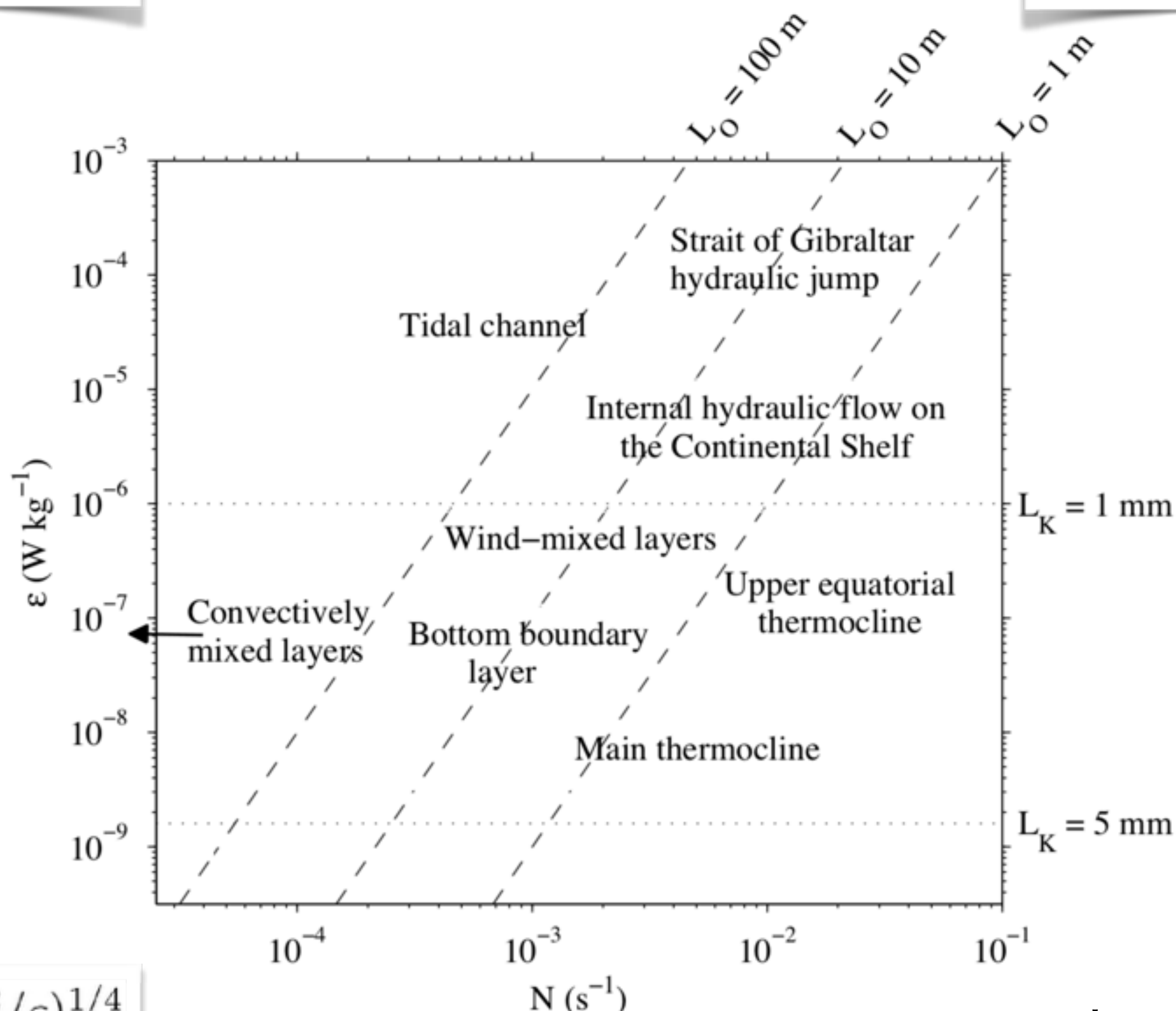
State Key Laboratory of Marine Environmental Science (Xiamen University)

Length Scales of Turbulence

$$L_O = (\varepsilon / N^3)^{1/2}$$

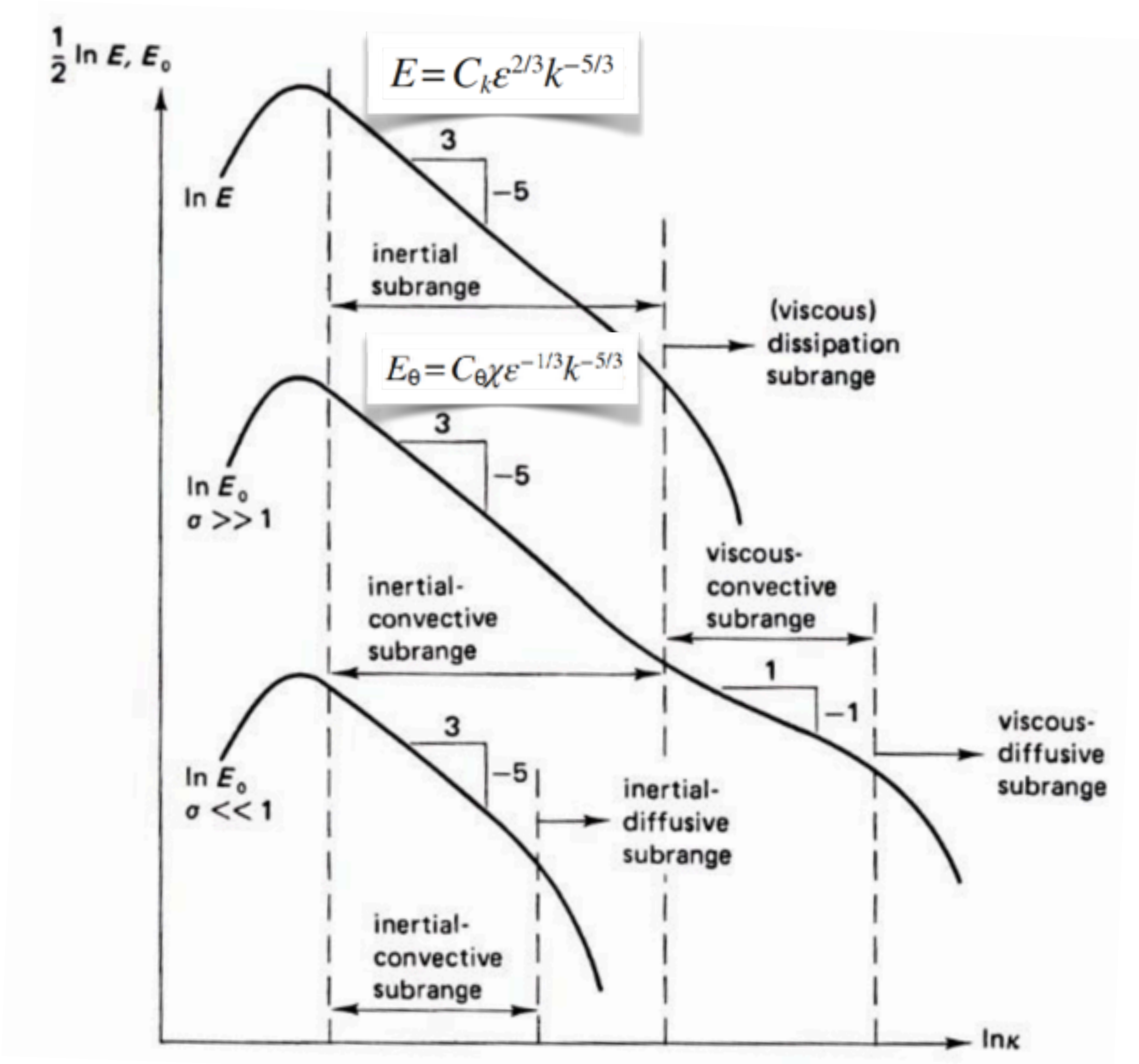
$$L_K = (\nu^3 / \varepsilon)^{1/4}$$



$$L_B = (\nu \kappa^2 / \varepsilon)^{1/4}$$

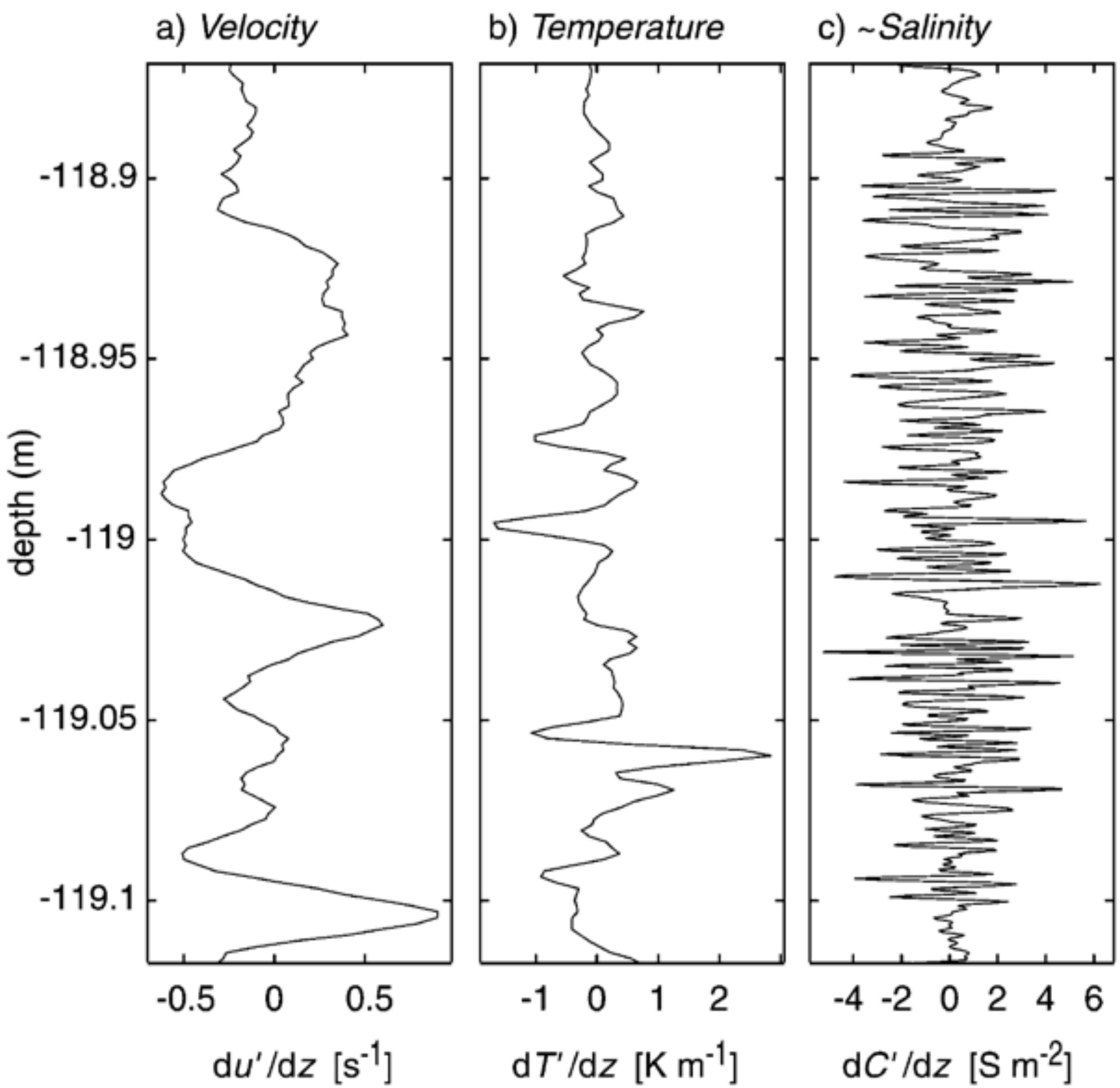
Smyth & Moum (2001)

Theoretical Spectra of Turbulence



Tennekes & Lumley (1972)

Ocean Microstructure



Nash & Moum (2002)

First Confirmation of the Kolmogorov Theory

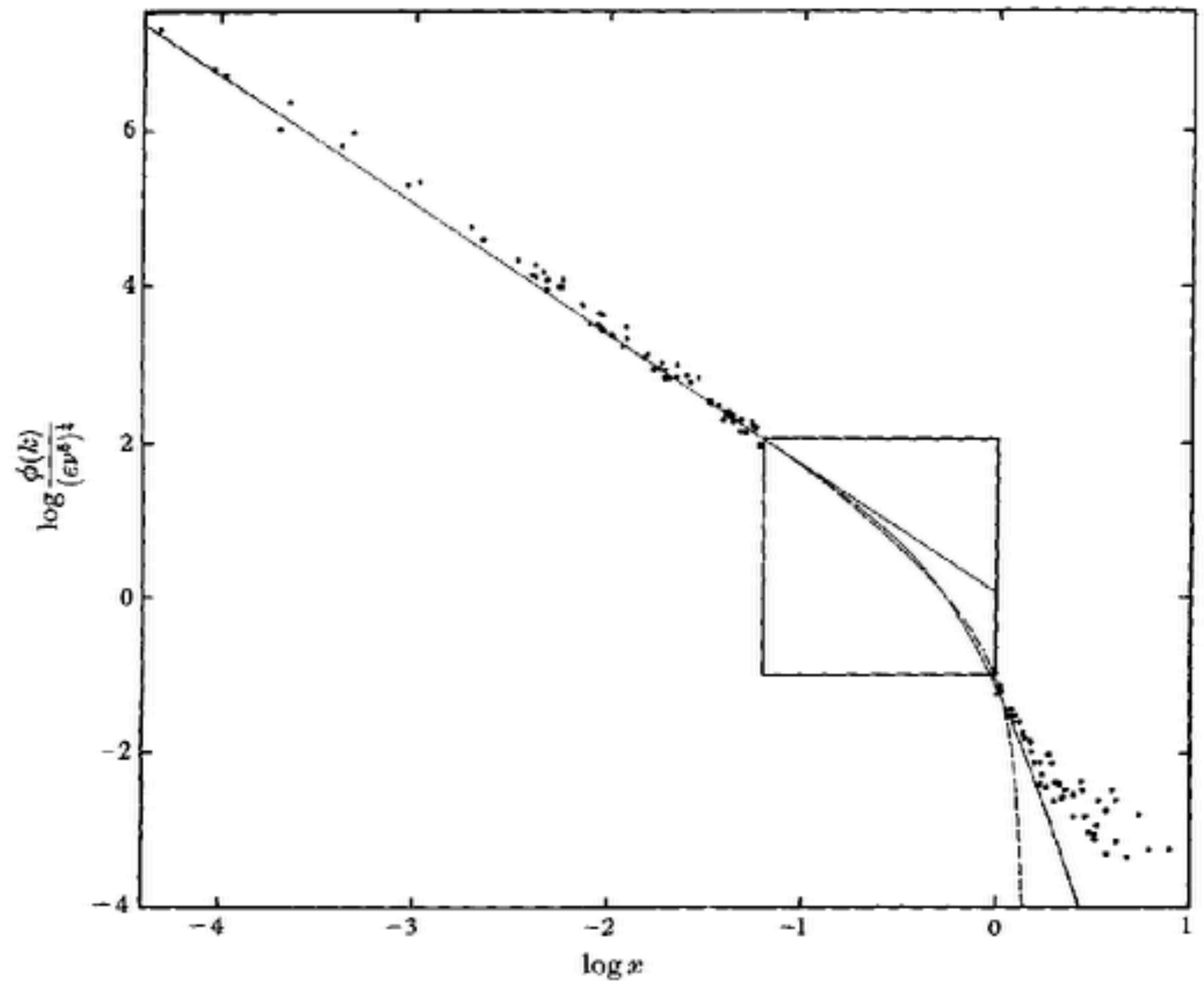
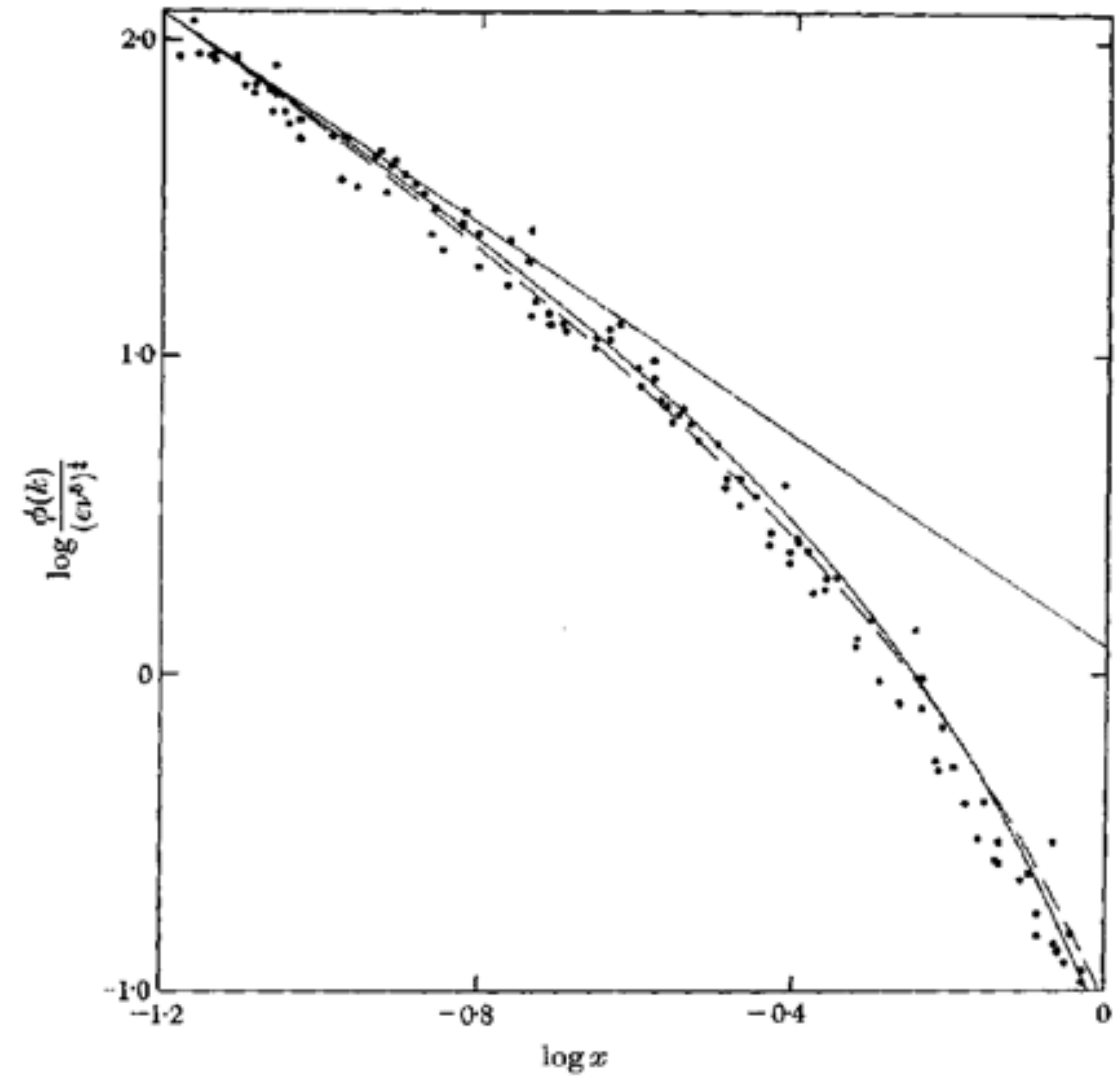


FIGURE 12. Seventeen spectra compared to the theories of Kolmogoroff, Heisenberg and Kovasznay. The straight line has a slope of $-\frac{5}{3}$, the curved solid line is Heisenberg's theory and the dashed line is Kovasznay's theory. Within the square, the observations are too crowded to display on this scale and they are shown in figure 13.



Turbulence spectra from a tidal channel

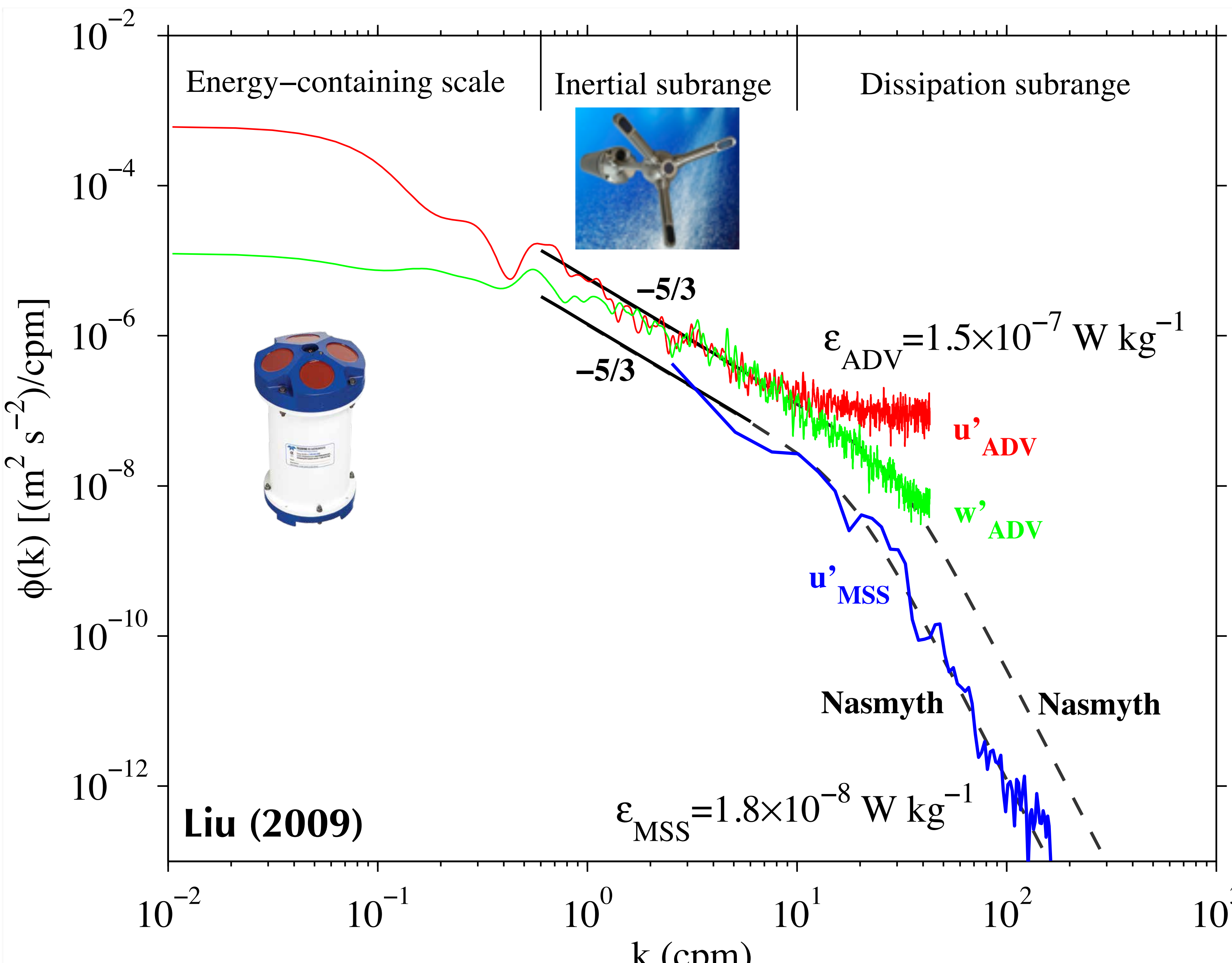
By H. L. GRANT, R. W. STEWART† AND A. MOILLIET

Pacific Naval Laboratory of the Defence Research Board of Canada, Esquimalt, B.C., Canada

Grant et al. (1962)

Measuring Turbulence in the Ocean

$$\underbrace{\frac{\partial k}{\partial t}}_{\substack{\text{Local}\\\text{derivative}}} + \underbrace{\bar{u}_j \frac{\partial k}{\partial x_j}}_{\text{Advection}} = \underbrace{-\frac{1}{\rho_o}\frac{\partial \overline{u_i' p'}}{\partial x_i}}_{\substack{\text{Pressure}\\\text{diffusion}}} \underbrace{-\frac{1}{2}\frac{\partial \overline{u_j' u_j' u_i'}}{\partial x_i}}_{\substack{\text{Turbulent}\\\text{transport}\\\mathcal{T}}} + \underbrace{\nu \frac{\partial^2 k}{\partial x_j^2}}_{\substack{\text{Molecular}\\\text{viscous}\\\text{transport}}} \underbrace{-\overline{u_i' u_j'}\frac{\partial \bar{u}_i}{\partial x_j}}_{\substack{\text{Production}\\\mathcal{P}}} - \underbrace{\nu \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}}_{\substack{\text{Dissipation}\\\varepsilon_k}} - \underbrace{\frac{g}{\rho_o}\overline{\rho' u_i'}\delta_{i3}}_{\substack{\text{Buoyancy flux}\\b}}$$



Measuring Turbulence in the Ocean

(with local isotropy assumption)

$$\varepsilon = \frac{15}{2}\nu\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2} \quad \text{for} \quad i \neq j, \tag{4}$$

where ν is the kinematic viscosity. It follows that

$$\varepsilon = \frac{15}{2}\nu\int_0^\infty \psi(k)\,dk, \tag{5}$$

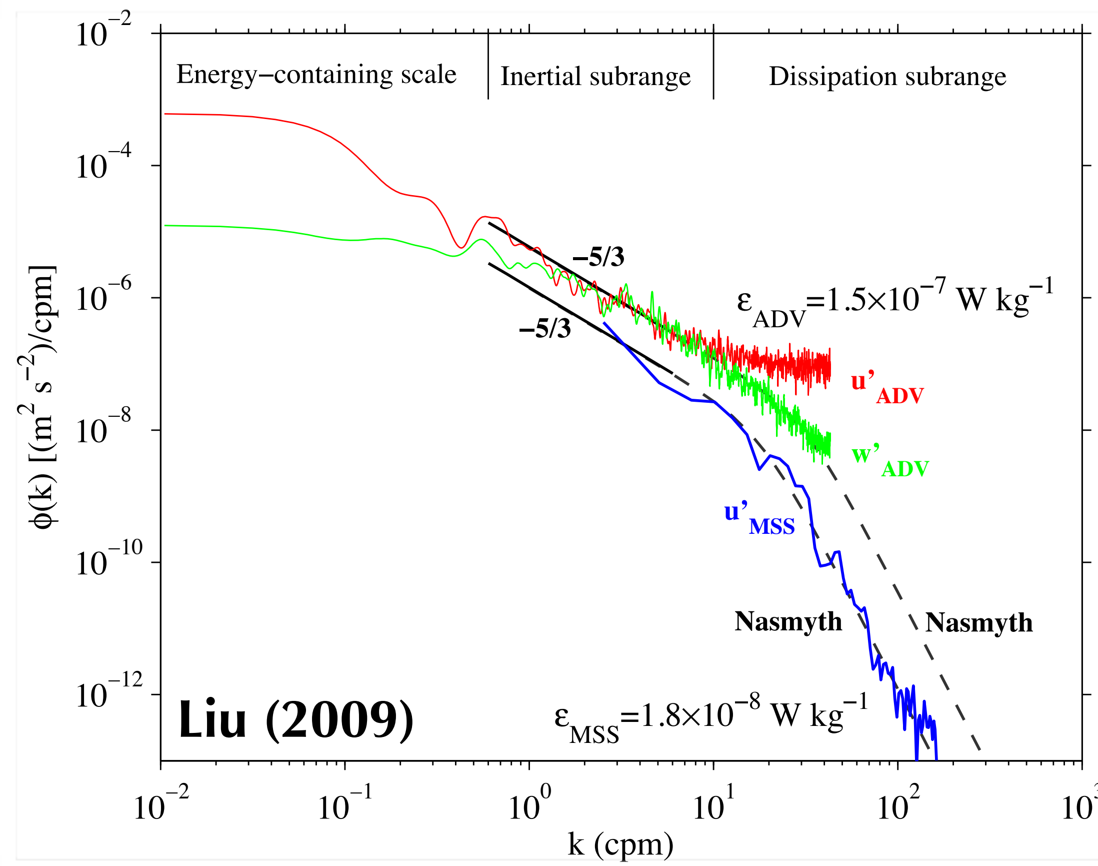
where $\psi(k)$ is the spectral density of the velocity shear. An empirical function governing ψ was developed by Nasmyth (1970) and formally published by Oakey (1982). An analytical fit to the function was initially obtained by Wolk et al. (2002); however, a slightly modified version that ensures that the integral of the shear spectrum preserves the variance of the signal is given by Lueck (2015) as

$$\psi(\tilde{k}) = \left(\frac{\varepsilon^3}{\nu}\right)^{1/4}\frac{8.05\tilde{k}^{1/3}}{1 + (20.6\tilde{k})^{3.715}}, \tag{6}$$

where $\tilde{k}$ is a nondimensional wavenumber given by $\tilde{k} = 2\pi\eta k$ and η is the Kolmogorov microscale defined by

$$\eta \equiv \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}. \tag{7}$$

The form of the shear spectrum given by Eq. (6) spans both the inertial subrange and the (viscous) dissipation range.



$$\phi_{uu} = C_u\varepsilon^{2/3}k^{-5/3} \quad \text{and} \tag{2}$$

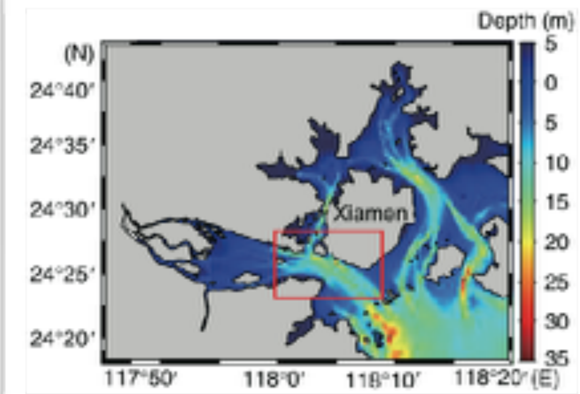
$$\phi_{ww} = C_w\varepsilon^{2/3}k^{-5/3}, \tag{3}$$

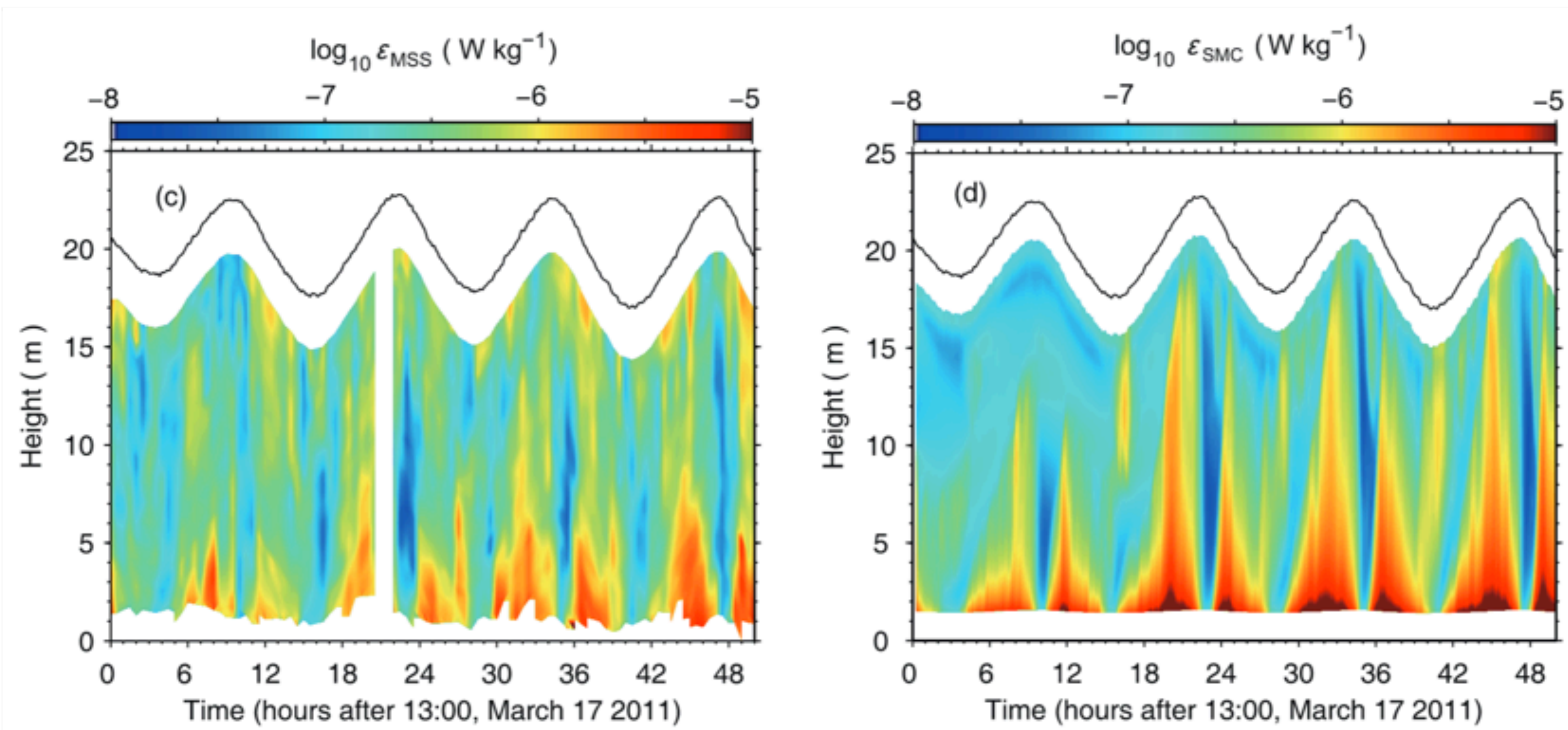
where ε is the rate of dissipation; k is the radian wavenumber in the streamwise direction; and C_u and C_w are the Kolmogorov constants, given by 0.5 and 0.67, respectively (Sreenivasan 1995).

McMillan et al. (2016)

Measuring Turbulence in the Ocean

An example in the "tidal channel"





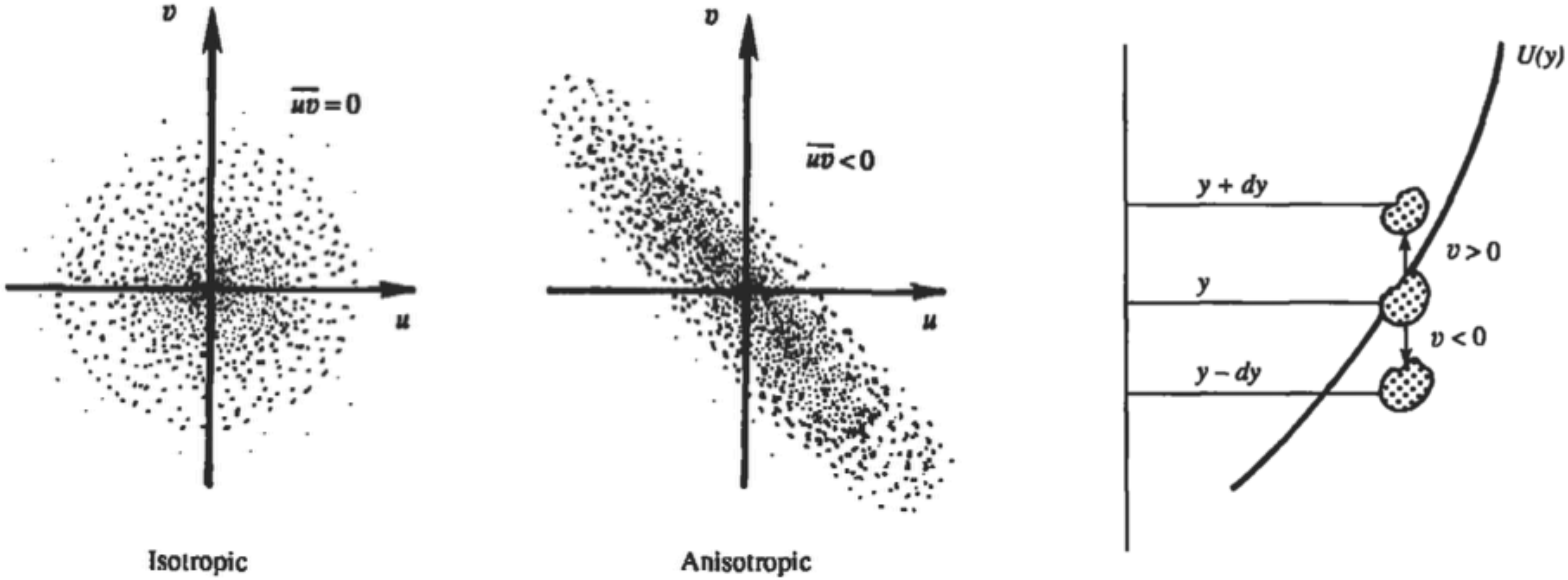
Lian & Liu (2015)

Wave-Turbulence Decomposition & Reynolds Stress Estimation in Wavy Aquatic Environment

Bian, Liu, et al. (2018)

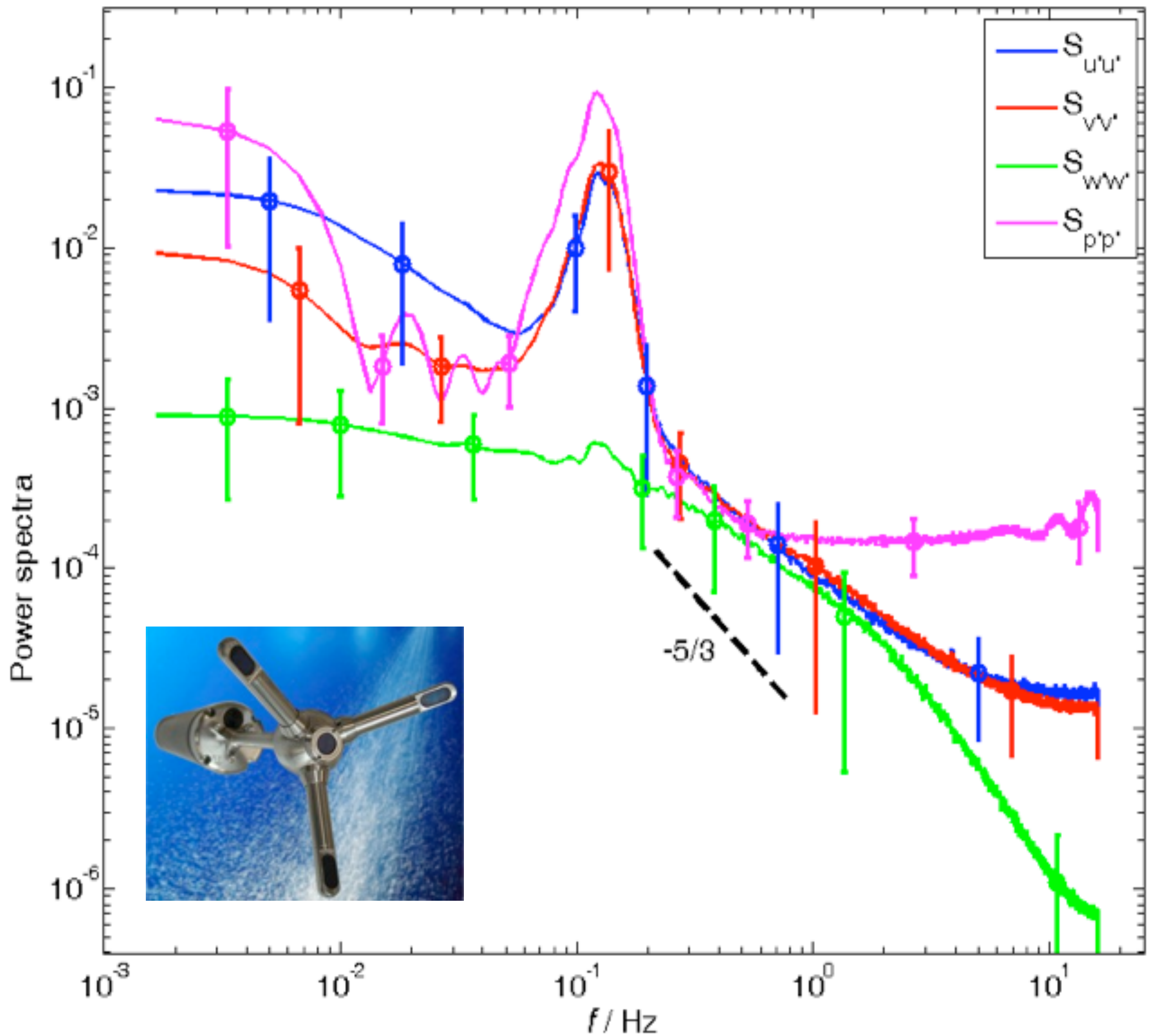
Reynolds (Shear) Stress

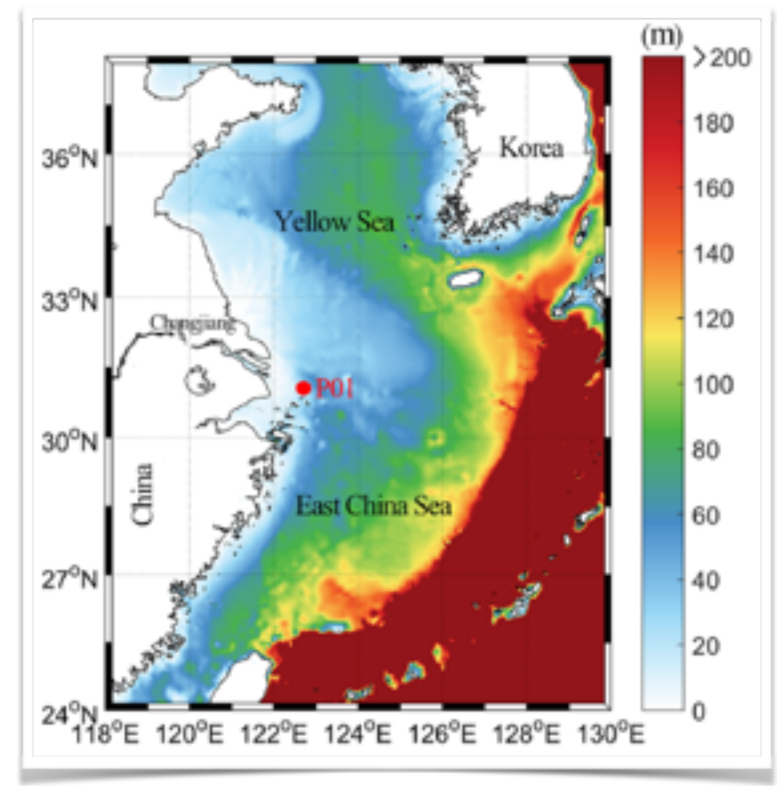
(consequence of anisotropy)



Kundu & Cohen (2002)

Turbulence in wavy aquatic environment





Bian, Liu, et al. (2018)

Reynolds Stress Estimation: Cohenrence Method

(Benilov & Filyushkin, 1970)

$$S_{ii}^{(w)}(\omega) = \gamma_i^2(\omega)\cdot S_{ii}(\omega);$$

$$S_{ii}^{(T)}(\omega) = [1-\gamma_i^2(\omega)]\cdot S_{ii}(\omega);$$

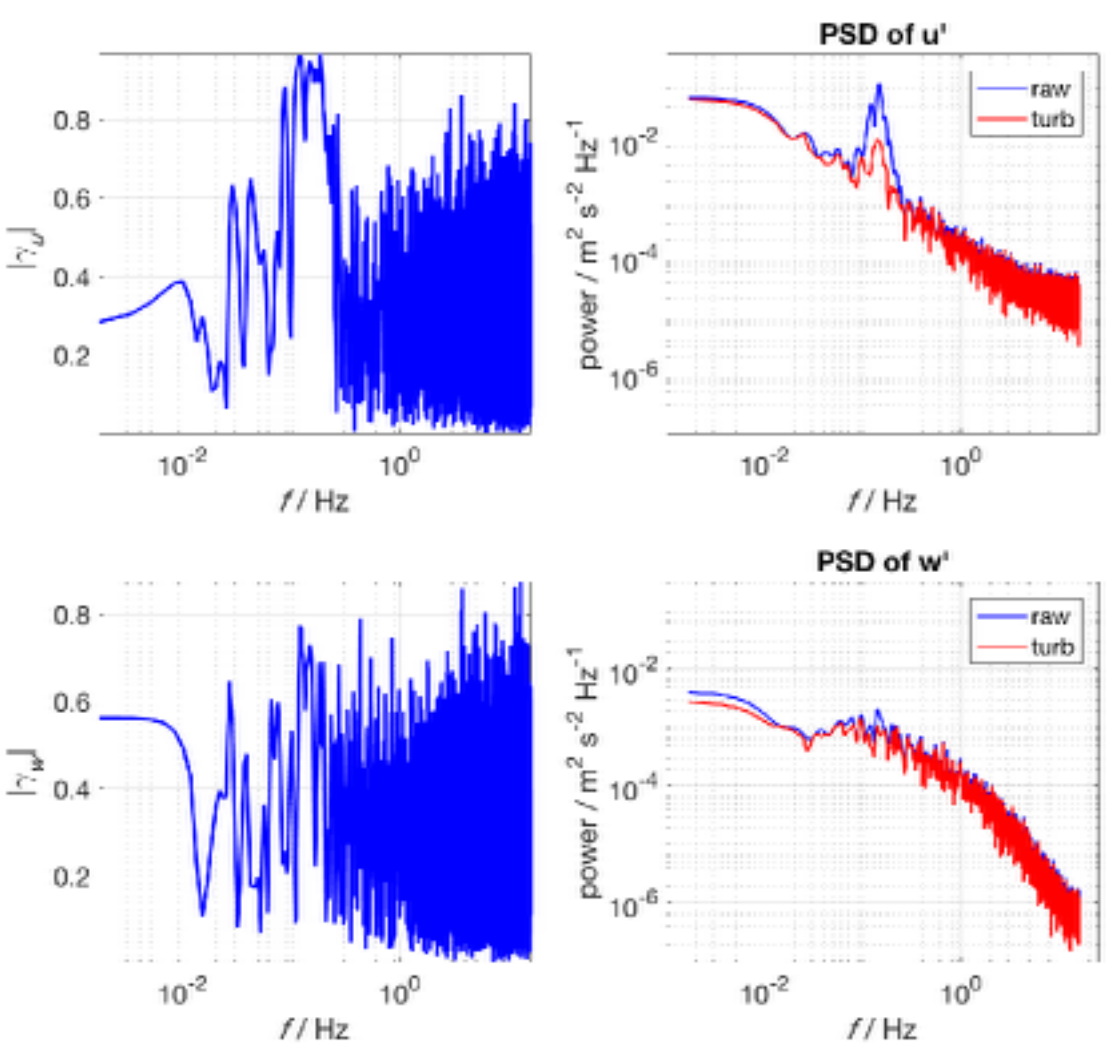
$$S_{ij}^{(w)}(\omega) = \frac{S_{i\eta}(\omega)\cdot S_{j\eta}^*(\omega)}{S_{\eta\eta}(\omega)};$$

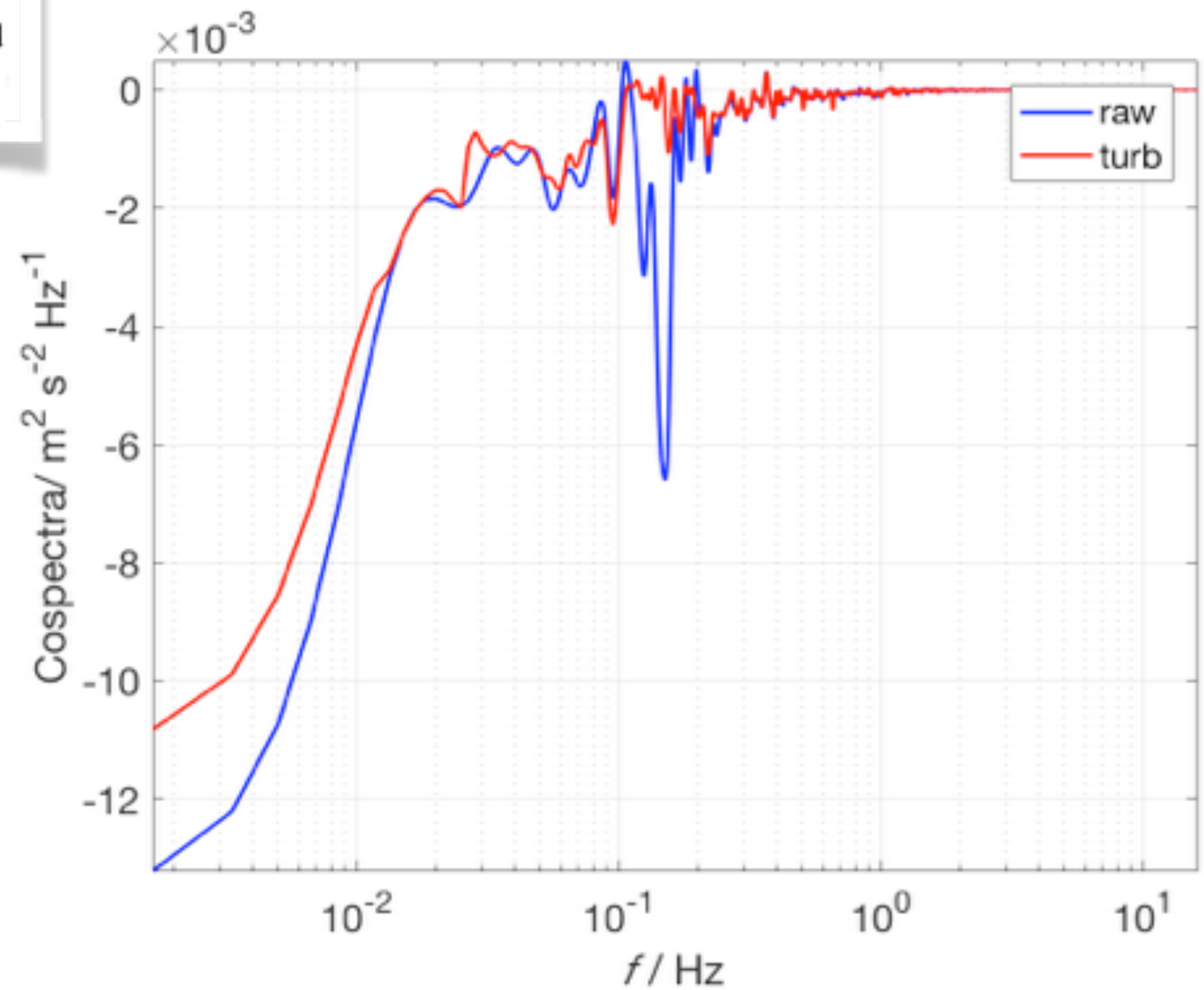
$$S_{ij}^{(T)}(\omega) = S_{ij}(\omega) - S_{ij}^{(w)}(\omega);$$

where

$$S_{\alpha\beta}(\omega) = \frac{1}{2\cdot\pi}\int_{-\infty}^{\infty} e^{-i\omega\tau}\cdot B_{\alpha\beta}(\tau)\cdot d\tau; \quad (\alpha, \beta = u_i, u_j, \eta)$$

is a Fourier transform of the correlation function $B_{\alpha\beta}(\tau)=\overline{\alpha(t+\tau)\beta(t)}$ and $\gamma_i^2(\omega)=S_{i\eta}(\omega)\cdot S_{i\eta}^*(\omega)/S_{ii}(\omega)\cdot S_{\eta\eta}(\omega)$ is the coherence.

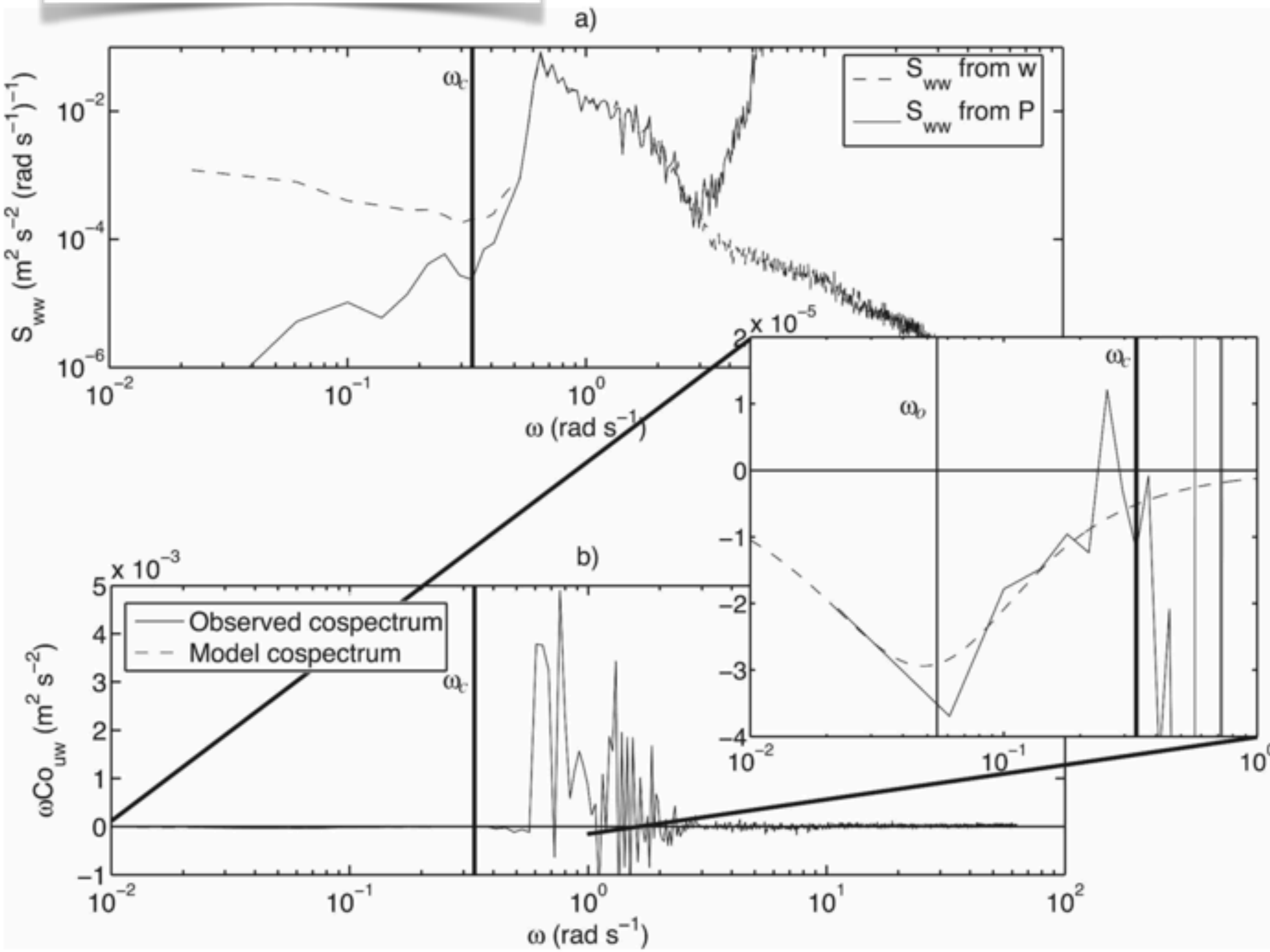




Reynolds Stress Estimation: Cospectra Method

$$F_{13}(k_1) = \frac{2}{3} \frac{\overline{u_1' u_3'}\lambda}{(1 + |k_1|\lambda)^{7/3}}$$

(Kaimal et al., 1972)



Gerbi et al. (2008)

Reynolds Stress Estimation: "Phase" Method

(Bricker & Monismith, 2007)

$$u = \overline{u} + \tilde{u} + u'$$

$$\overline{u'w'} = \overline{uw} - \overline{\tilde{u}\tilde{w}}$$

$$S_{u'w'}(\omega) = S_{uw}(\omega) - S_{\tilde{u}\tilde{w}}(\omega)$$

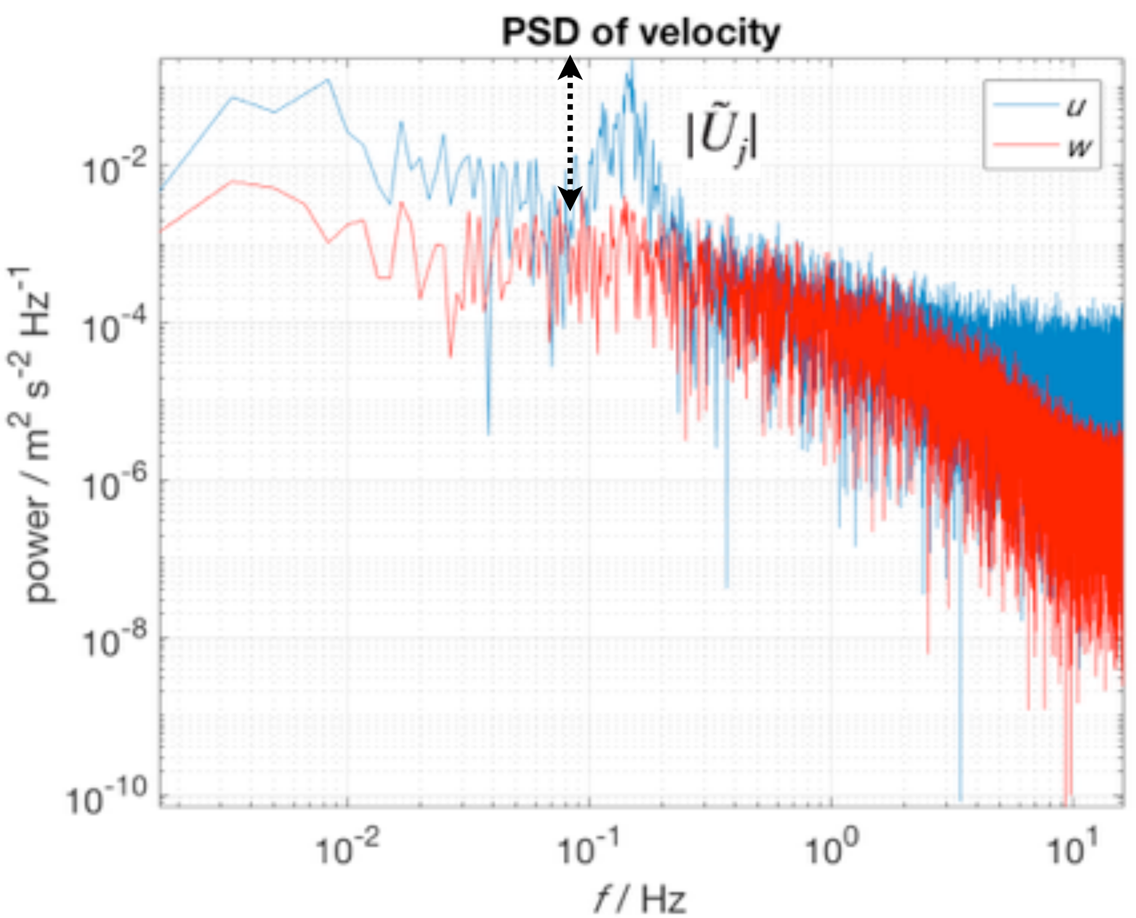
$$\overline{\tilde{u}\tilde{w}} = \int_{-\omega_{\text{Nyquist}}}^{\omega_{\text{Nyquist}}} S_{\tilde{u}\tilde{w}}(\omega)\, d\omega$$

$$\overline{\tilde{u}\tilde{w}} = \sum_{-N/2}^{N/2} \tilde{U}_j^* \tilde{W}_j$$

where $U_j = U(\omega_j)$ is the Fourier transform of $u(t)$ at frequency ω_j, N is the number of data points used in the Fourier transform, and t is time.

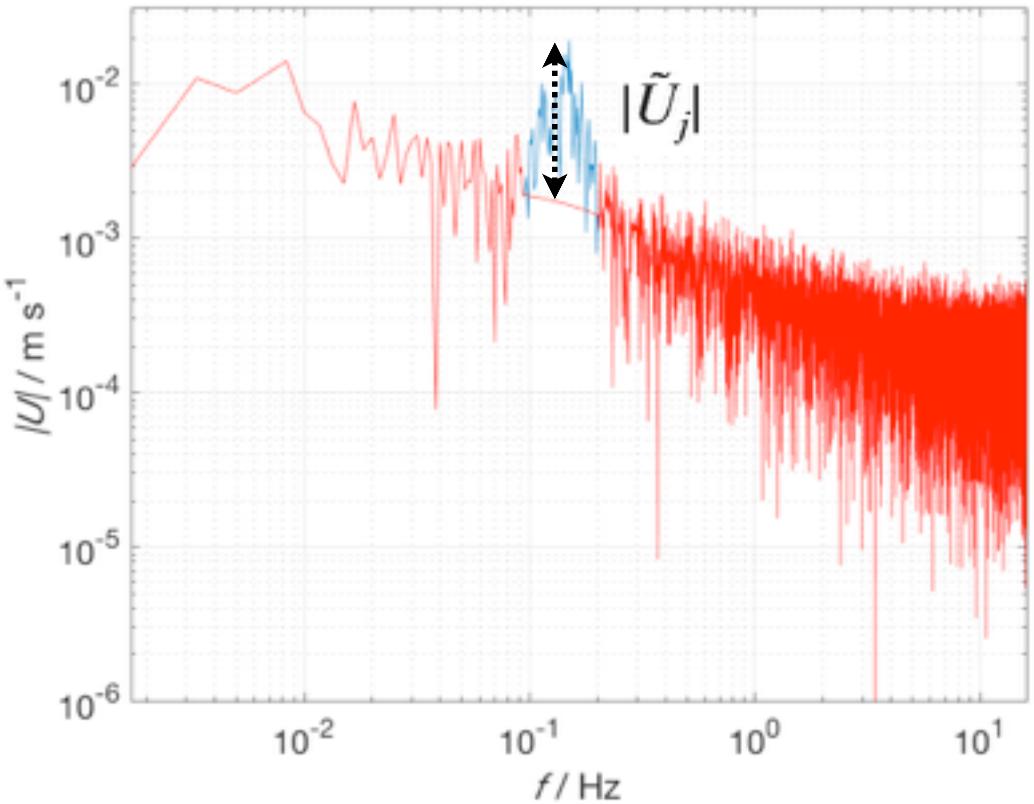
$$U_j^* W_j = |U_j||W_j|[\cos(\angle W_j - \angle U_j) + i \sin(\angle W_j - \angle U_j)]$$

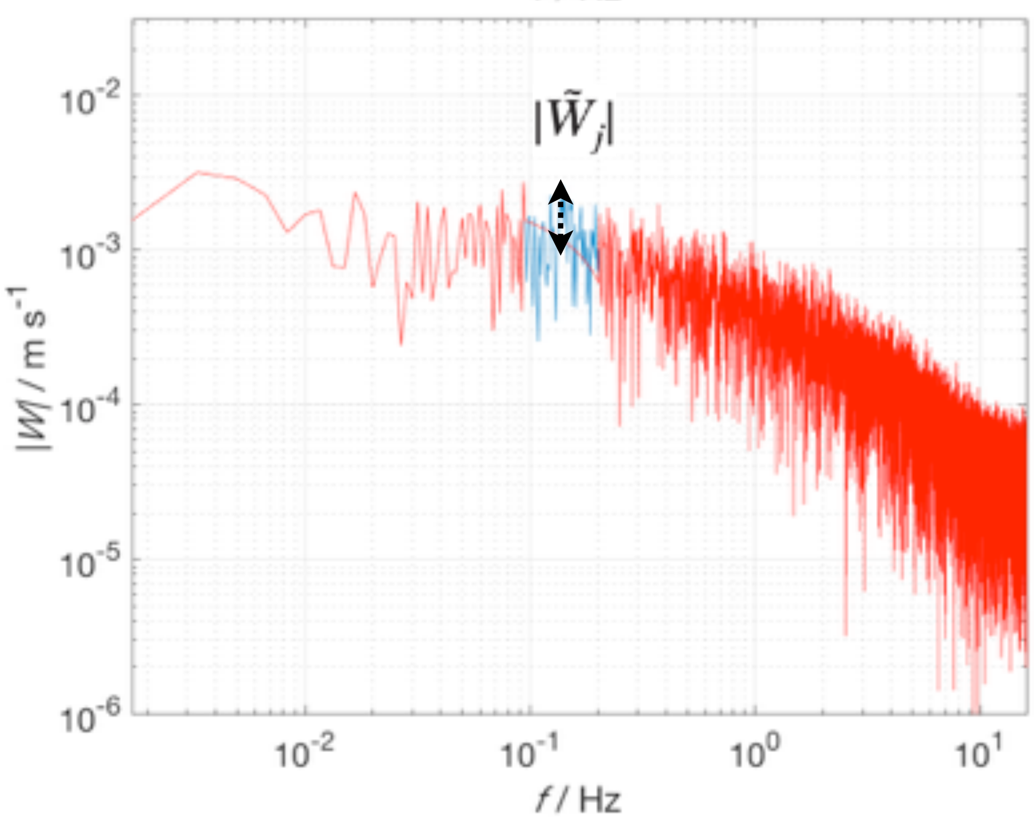
$$\overline{\tilde{u}\tilde{w}} = \sum_{j=\text{wave\_peak}} \tilde{U}_j^* \tilde{W}_j = \sum_{j=\text{wave\_peak}} |\tilde{U}_j||\tilde{W}_j| \cos(\angle W_j - \angle U_j)$$

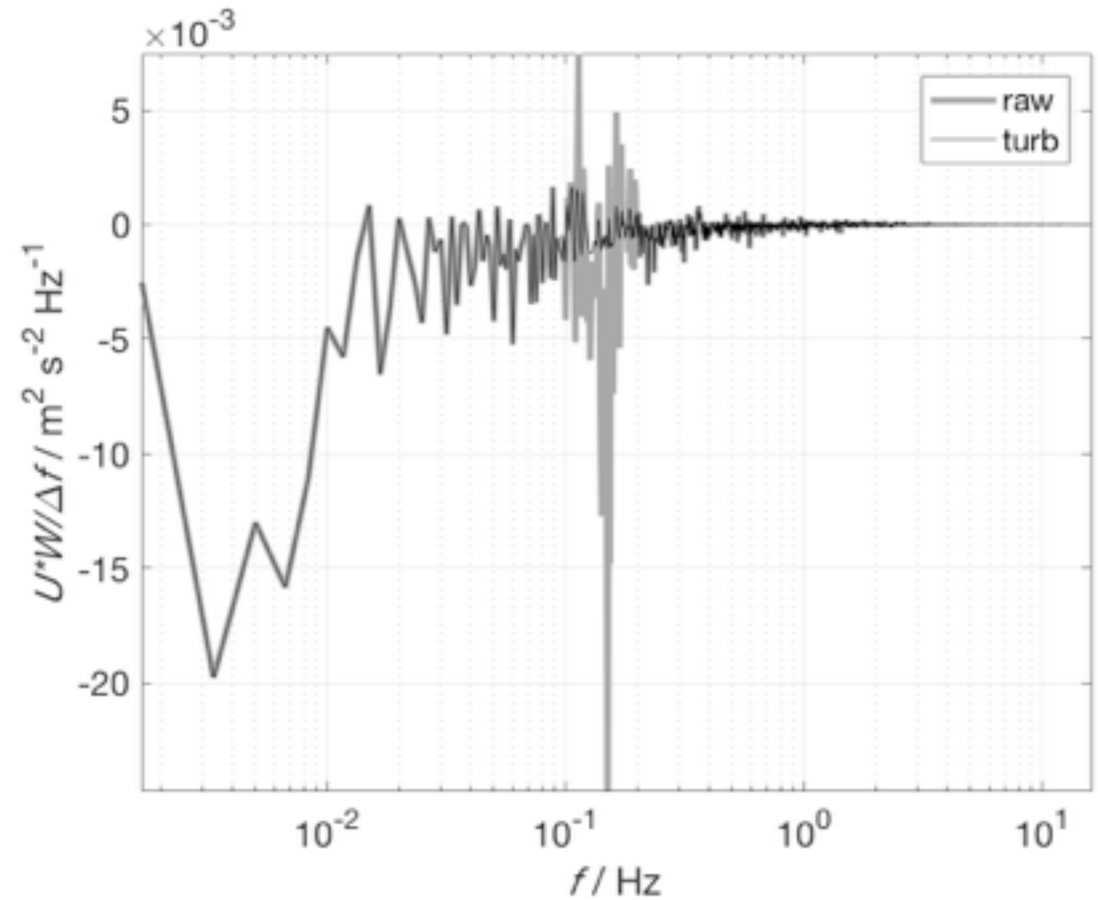


Reynolds Stress Estimation: "Phase" Method

(Bricker & Monismith, 2007)







$$\overline{u'w'} = \overline{uw} - \overline{\tilde{u}\tilde{w}}$$

$$\overline{\tilde{u}\tilde{w}} = \sum_{j=\text{wave\_peak}} \tilde{U}_j^* \tilde{W}_j$$

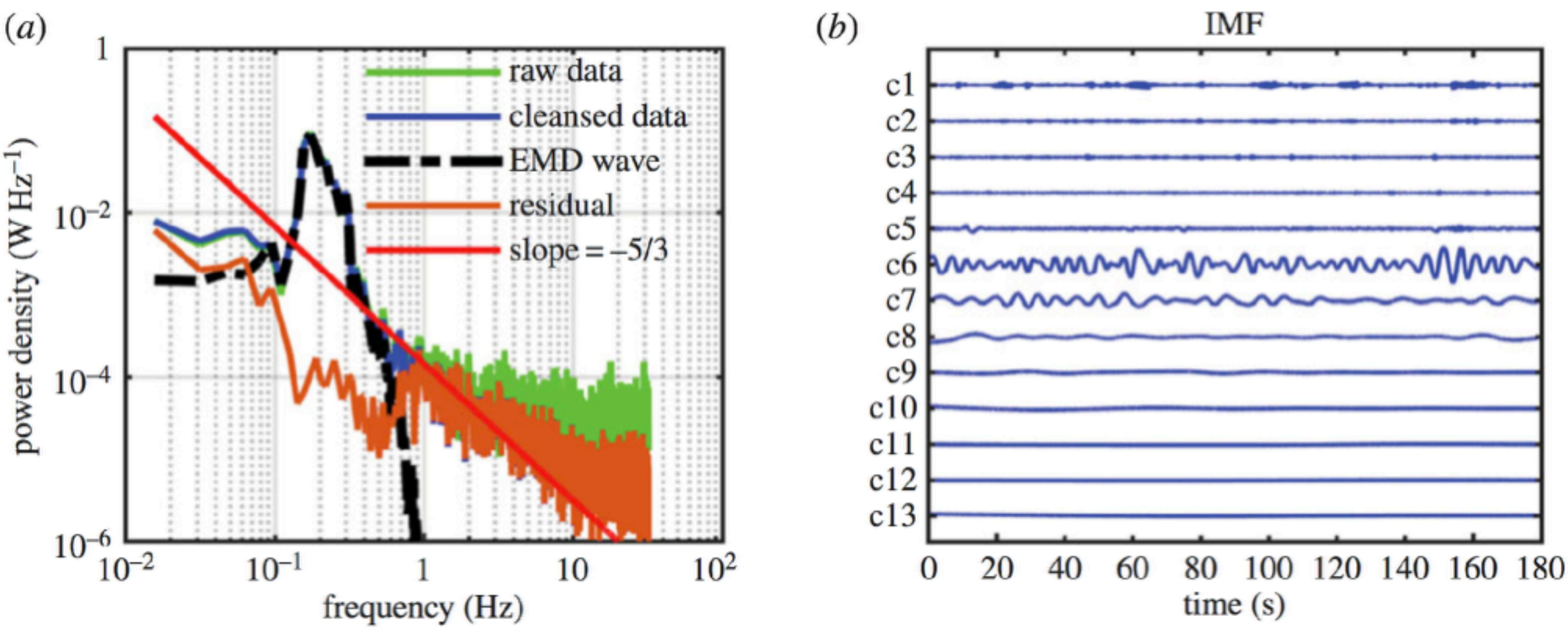
$$= \sum_{j=\text{wave\_peak}} |\tilde{U}_j||\tilde{W}_j| \cos(\angle W_j - \angle U_j)$$

Wave-dominated

Reynolds Stress Estimation: EMD Method

(Huang et al., 1998)

EMD = Empirical Mode Decomposition **IMF = Intrinsic Mode Function**



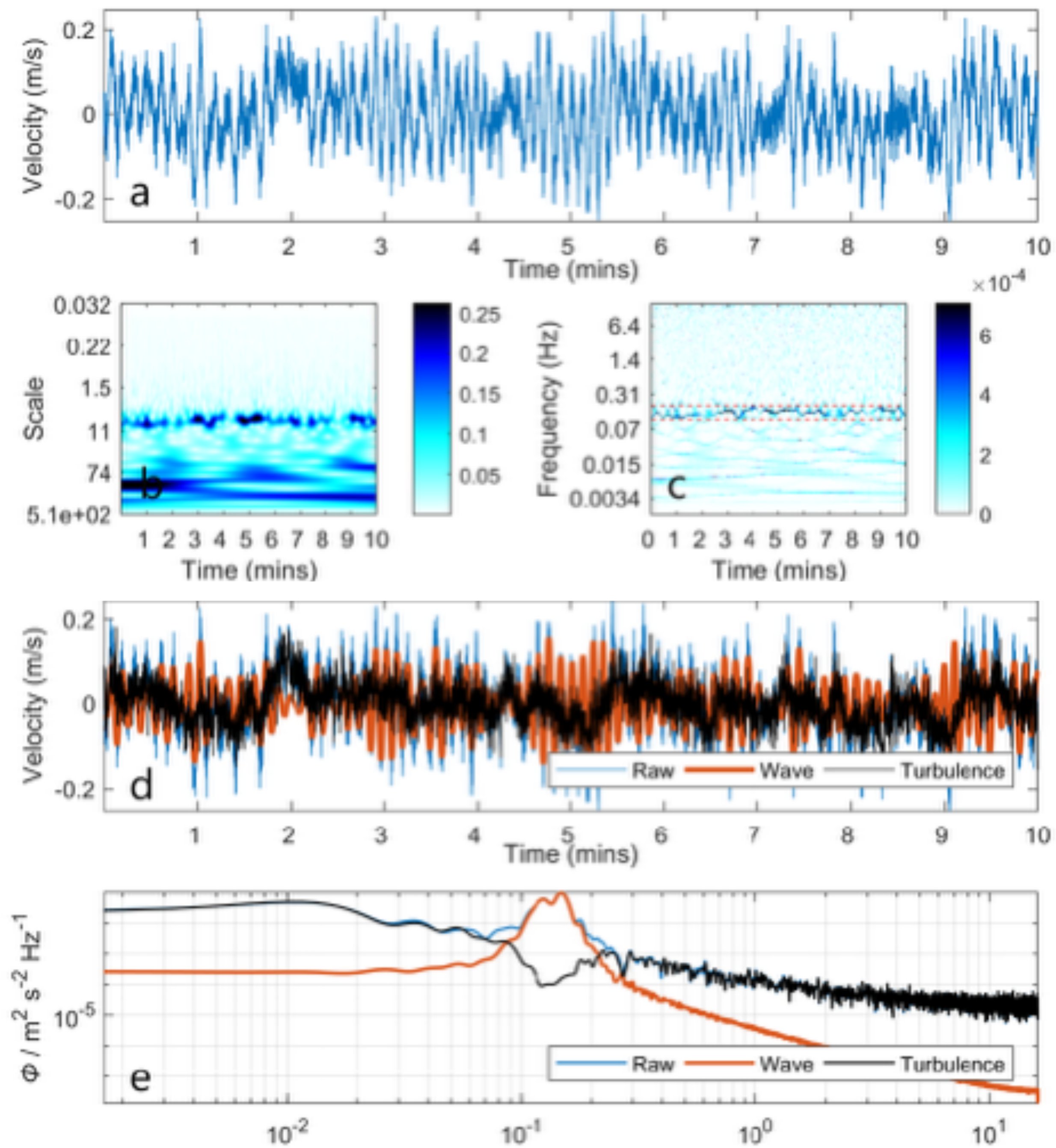
The empirical mode decomposition and the Hilbert spectrum for ...
rspa.royalsocietypublishing.org/content/454/1971/903
by NE Huang - 1998 - Cited by 16905 - Related articles

Qiao et al. (2016)

Reynolds Stress Estimation: SWT Method

(Daubechies et al., 2011)

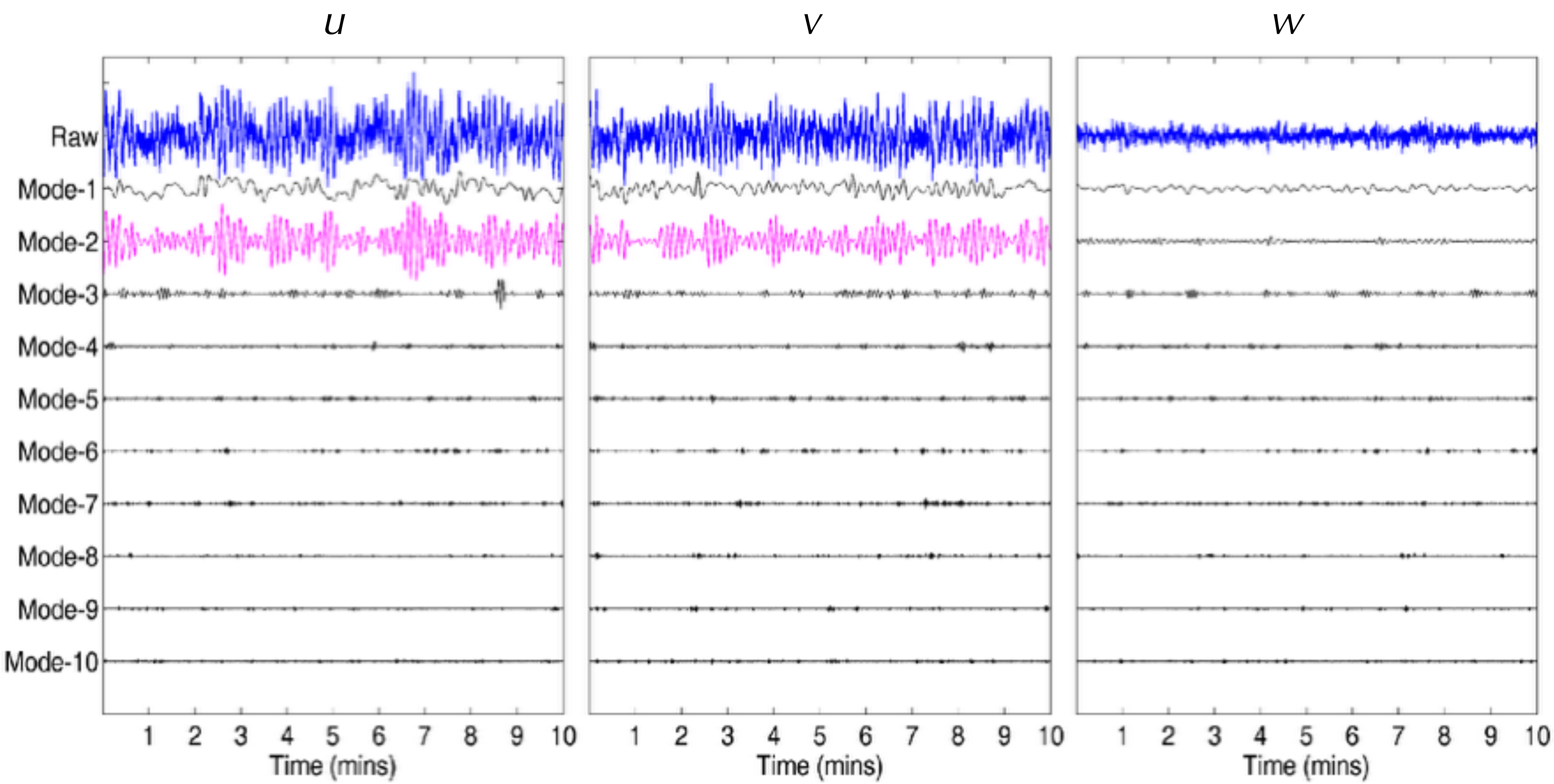
SWT = Synchrosqueezed Wavelet Transform



Synchrosqueezed wavelet transforms: An empirical mode ...
https://www.sciencedirect.com/science/article/pii/S1063520310001016
by I Daubechies - 2011 - Cited by 575 - Related articles

Bian, Liu, et al. (2018)

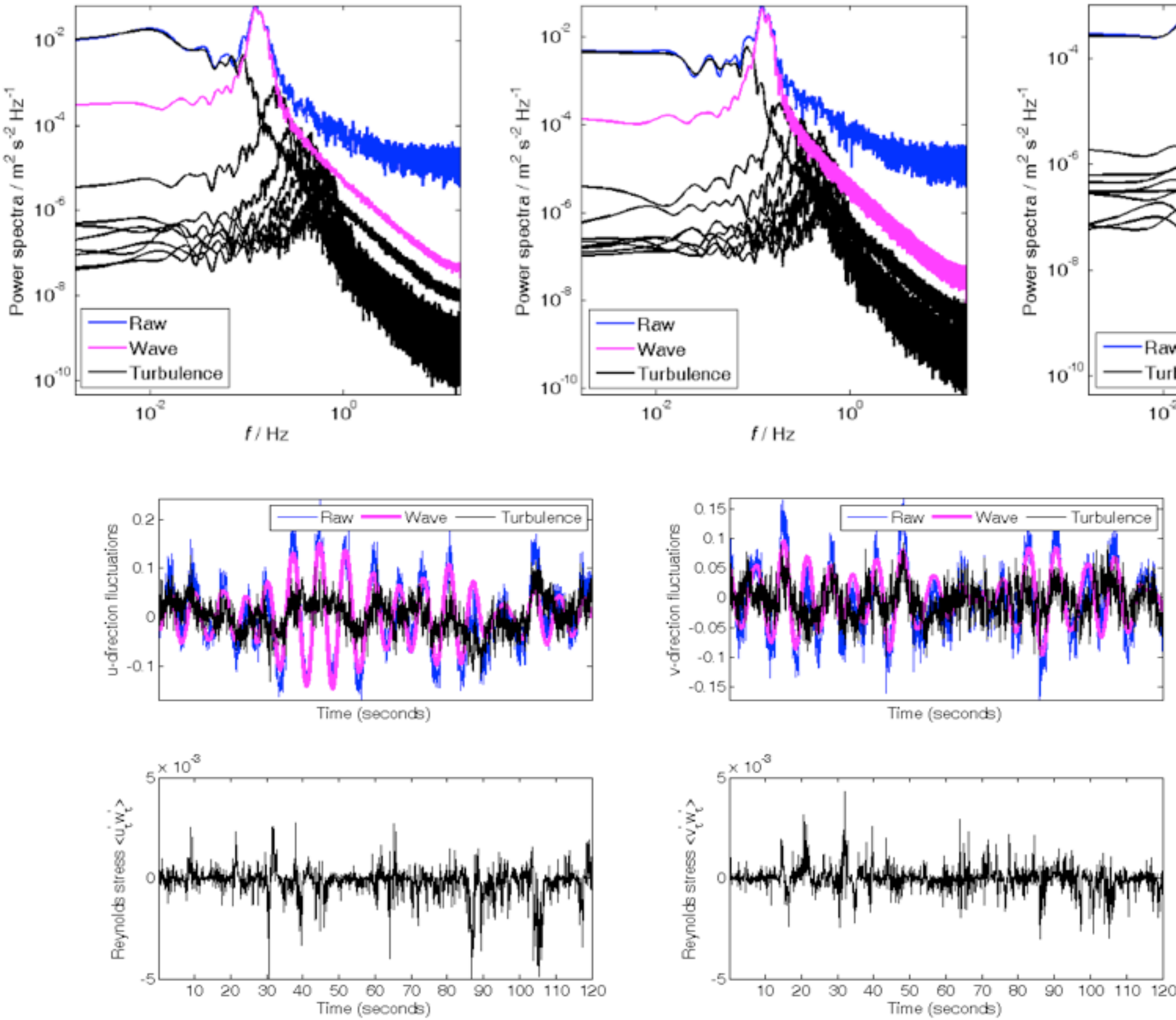
Reynolds Stress Estimation: SWT Method



The raw velocity fluctuations and the first ten IMF modes (224 IMF modes in all) derived from the raw velocity fluctuations using the SWT method: (a) *u*, (b) *v* and (c) *w*.

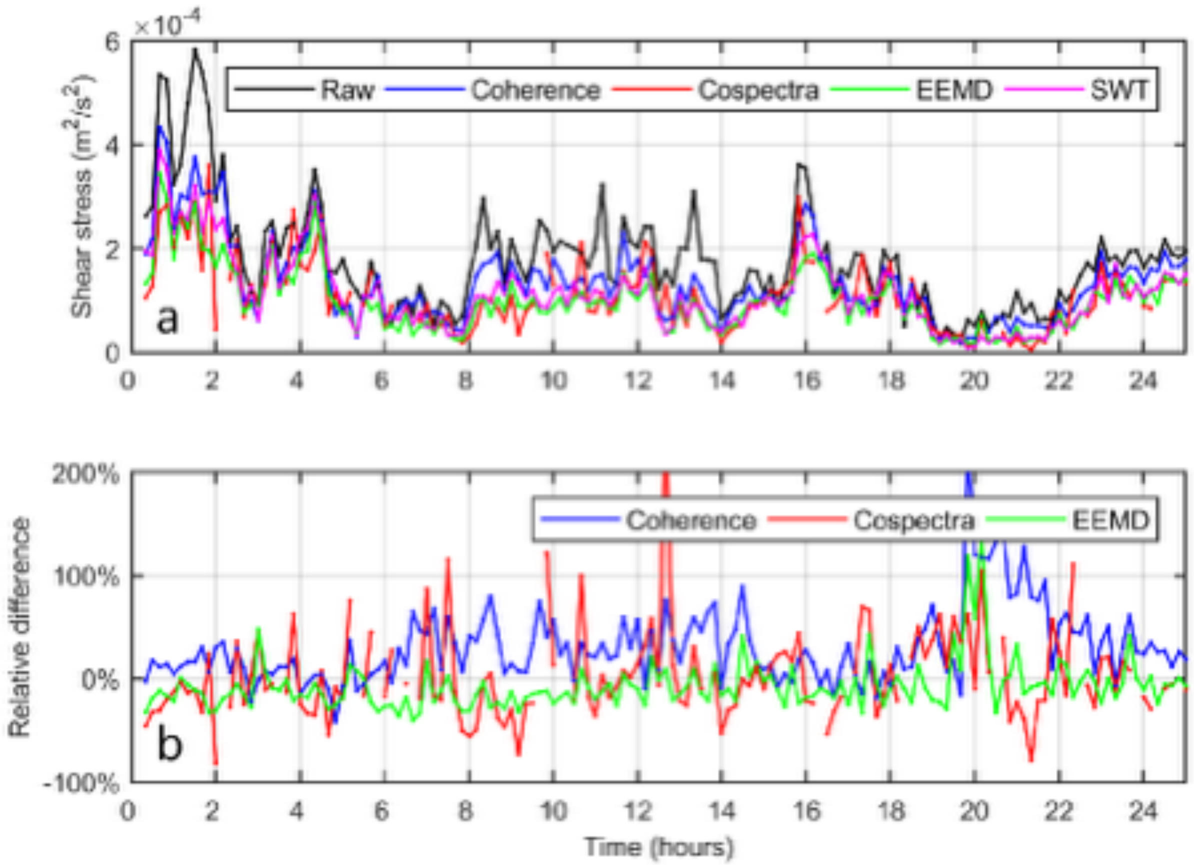
Bian, Liu, et al. (2018)

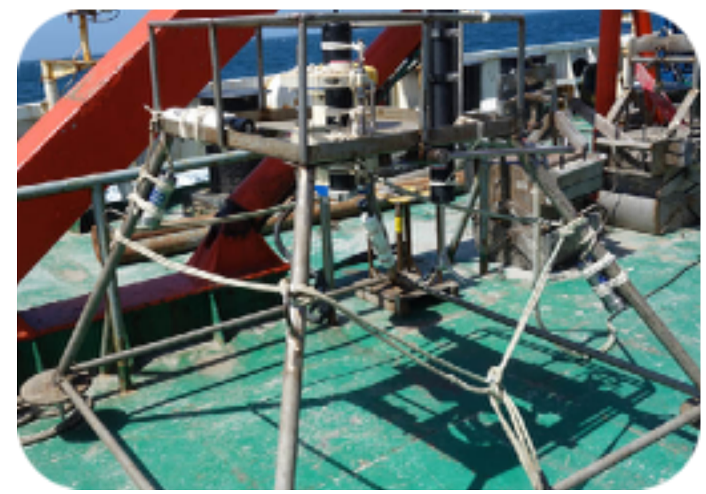
Reynolds Stress Estimation: SWT Method

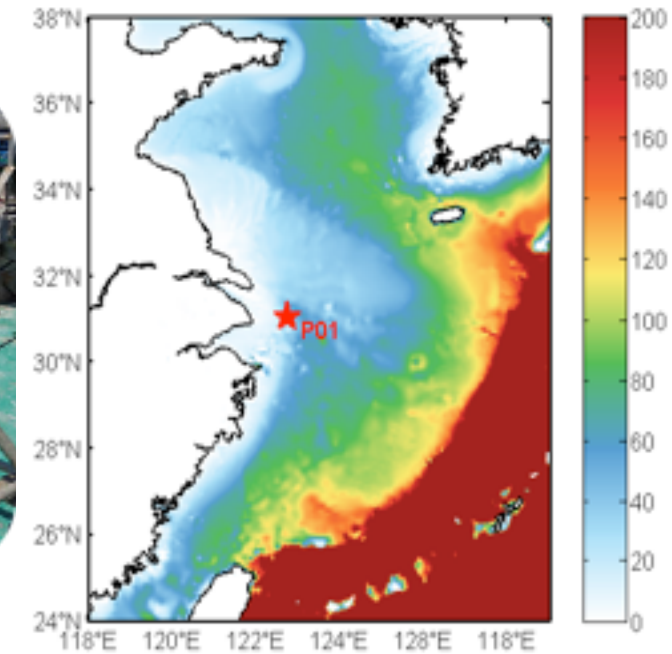


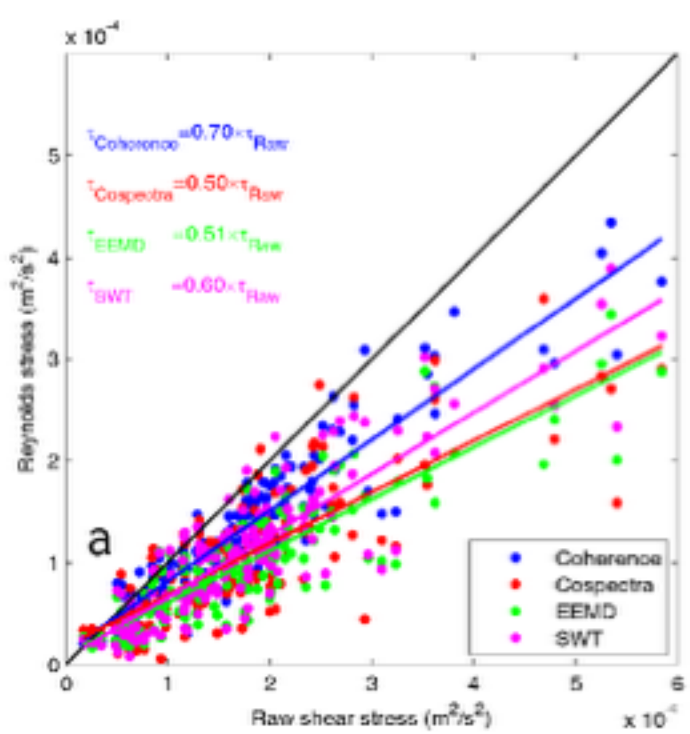
Bian, Liu, et al. (2018)

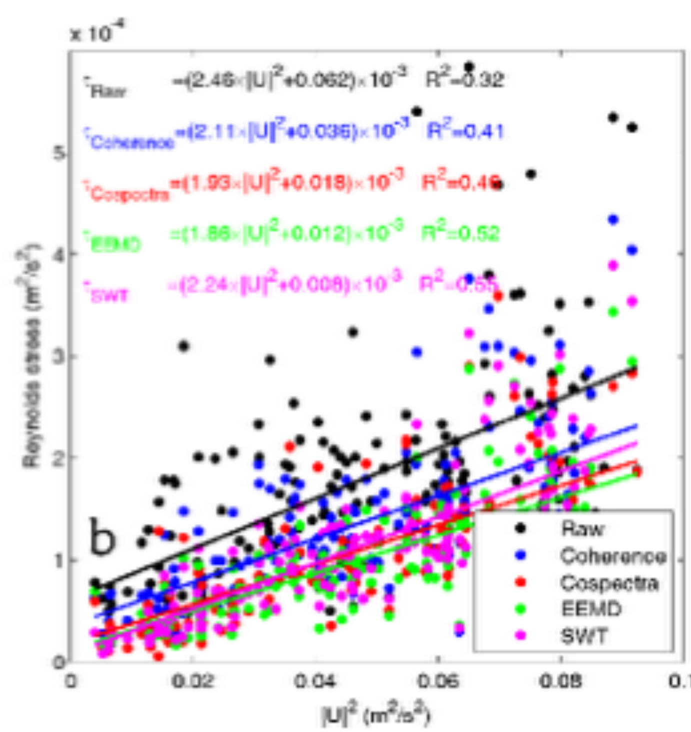
Applications/Assessments





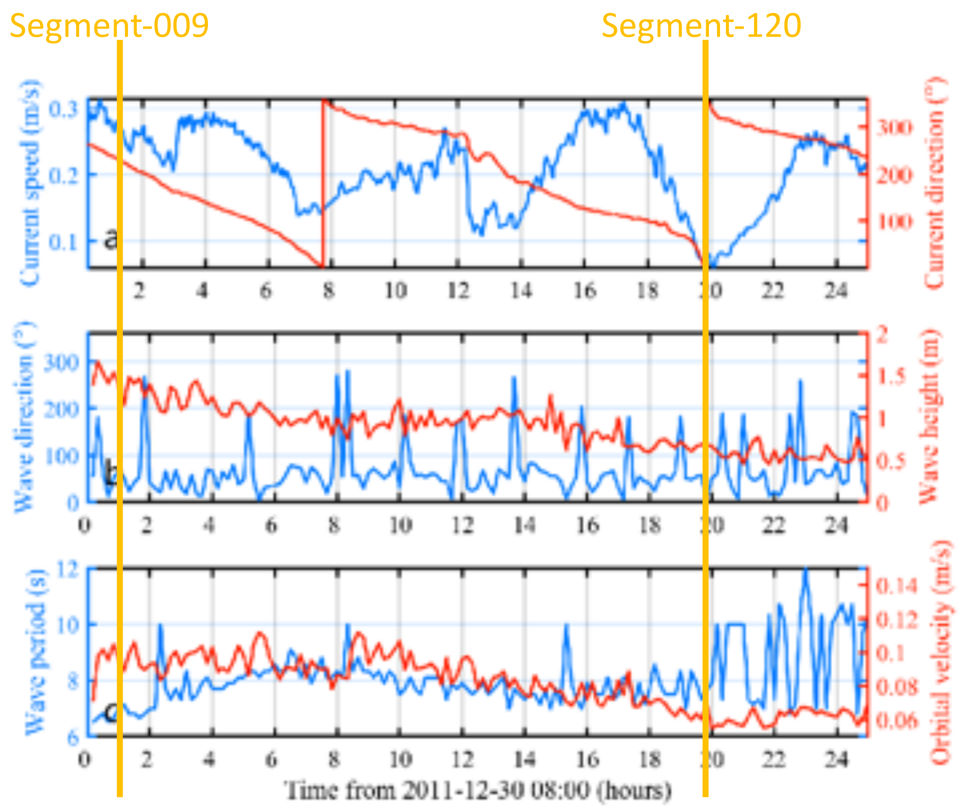


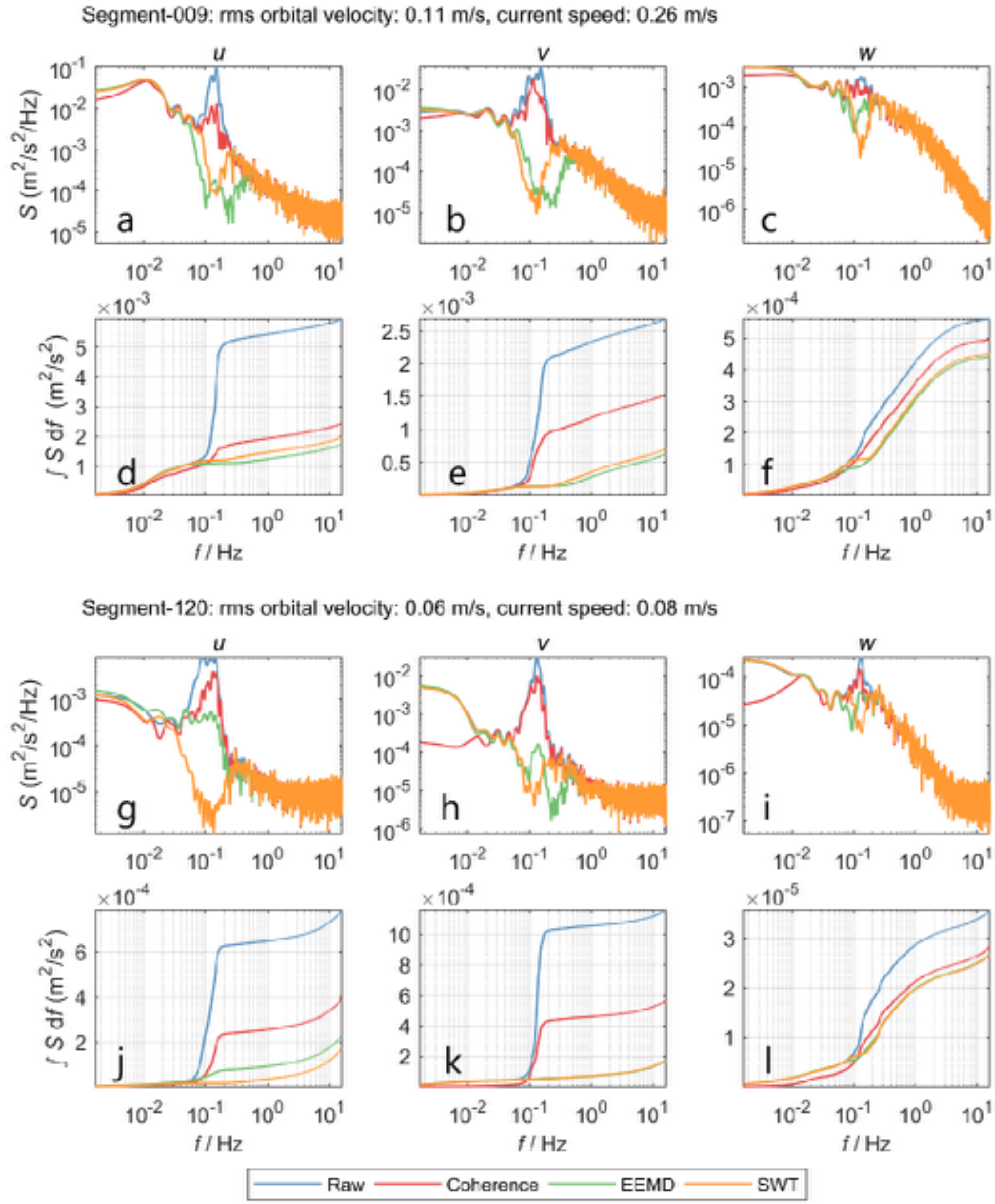




Bian, Liu, et al. (2018)

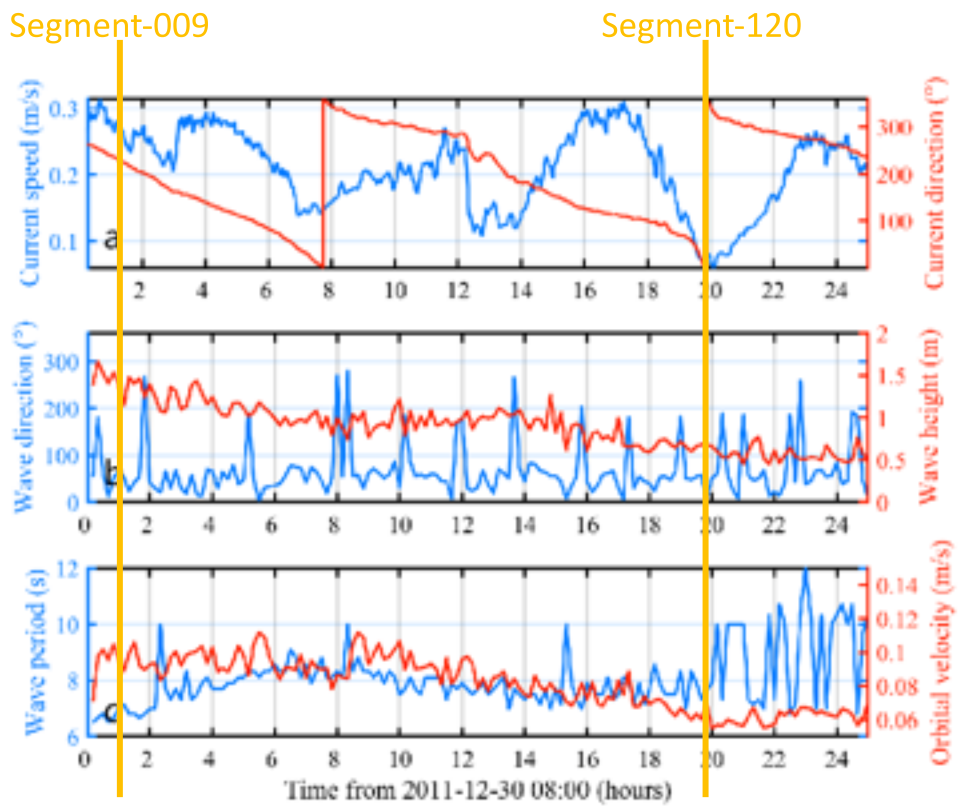
Assessment of method performance: power spectra

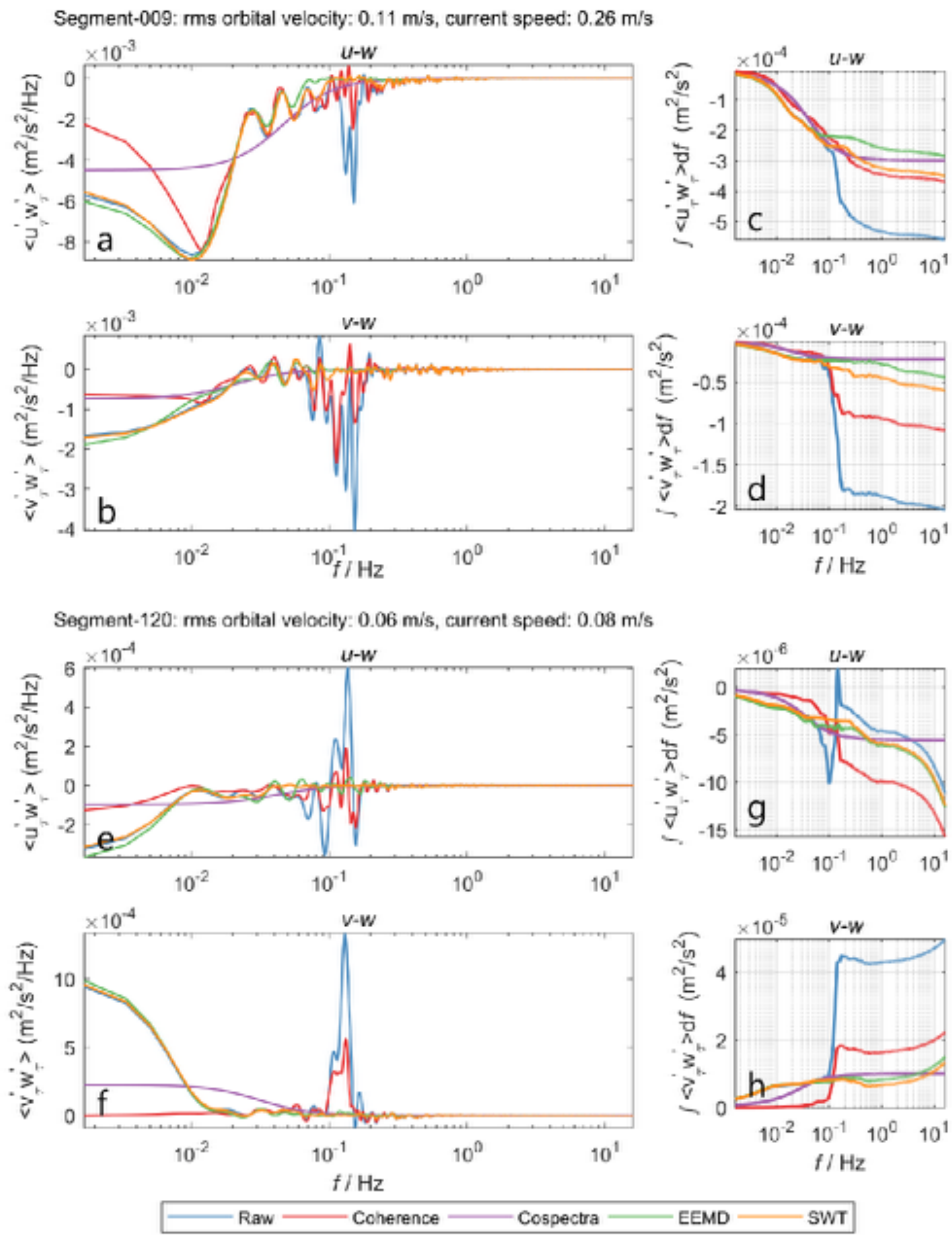




Bian, Liu, et al. (2018)

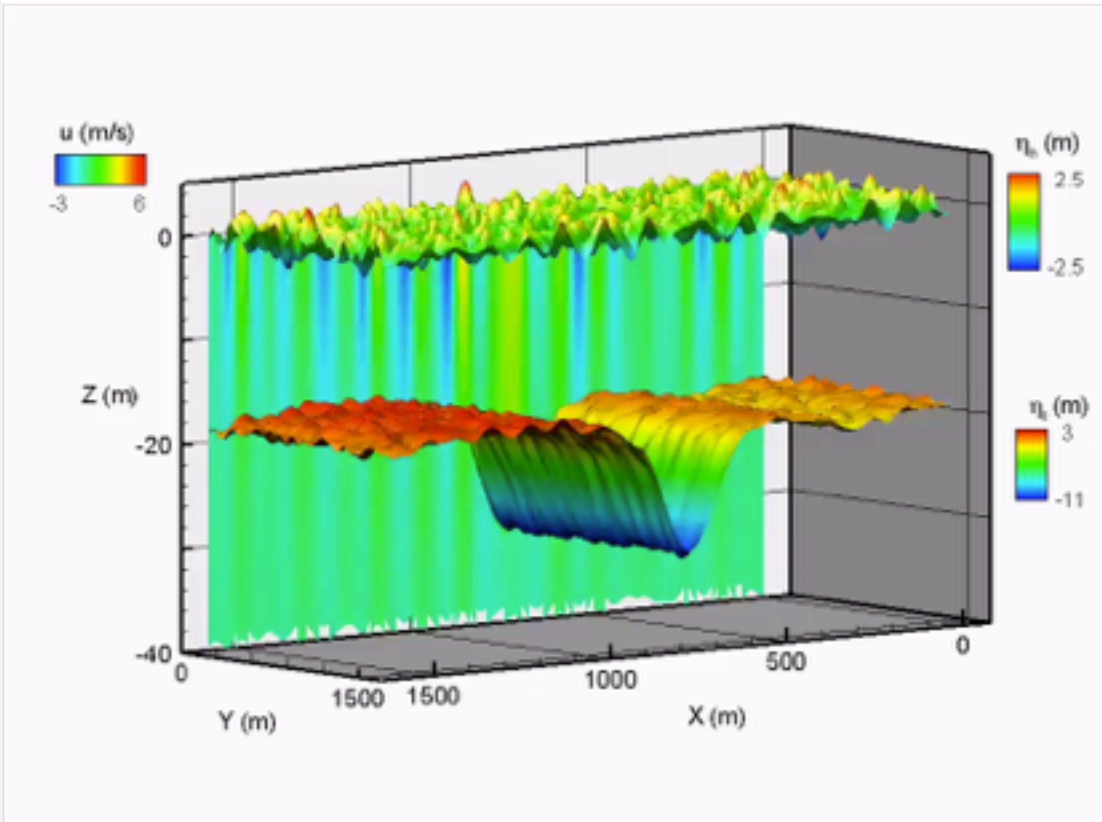
Assessment of method performance: cospectra

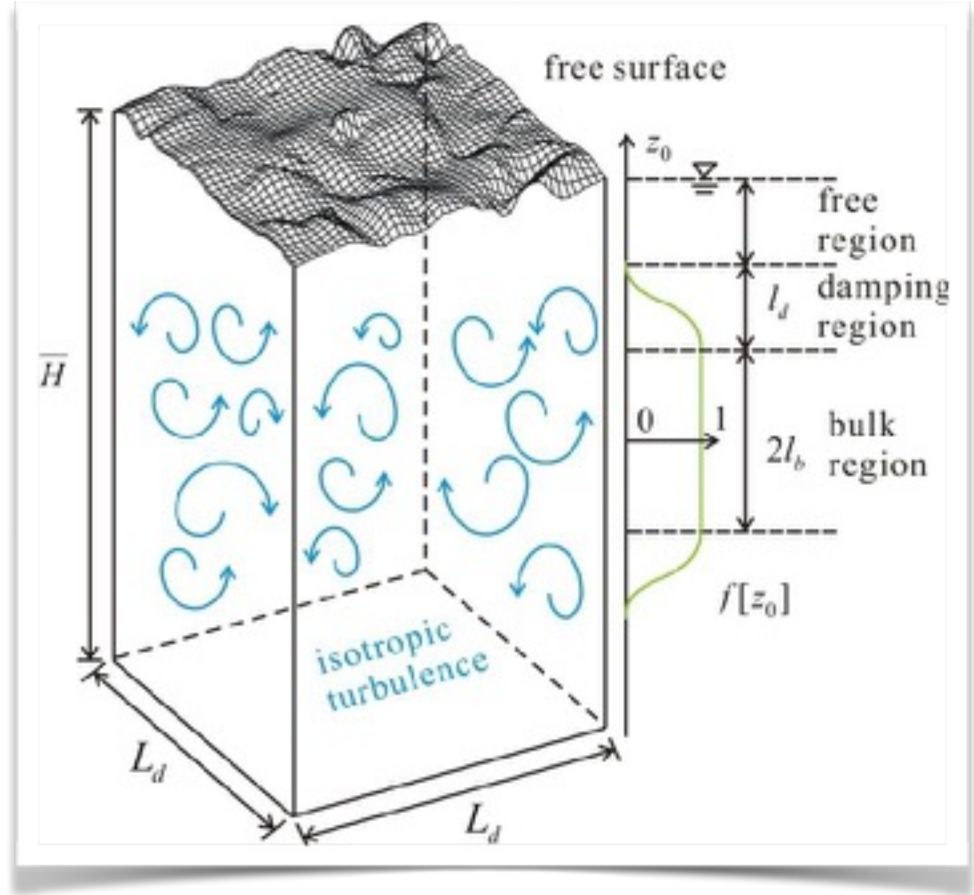


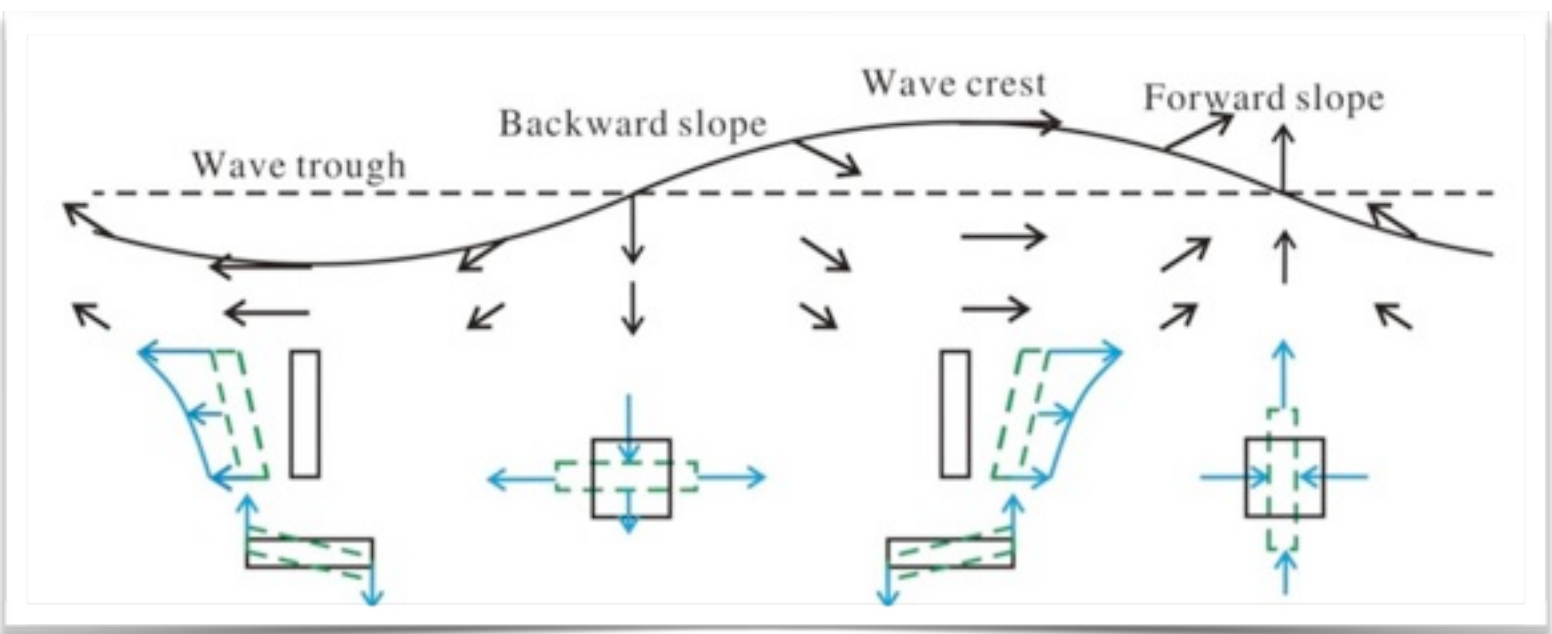


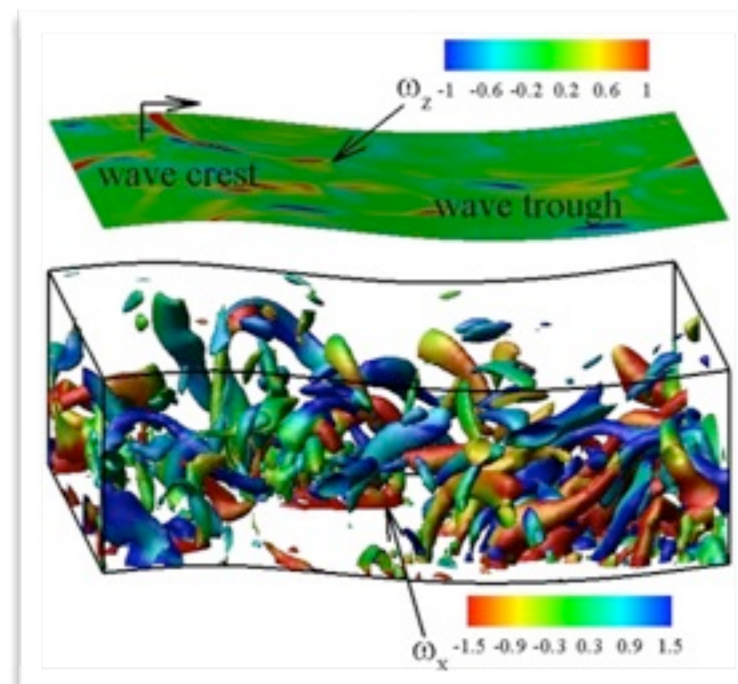
Bian, Liu, et al. (2018)

Wave-Turbulence Interaction



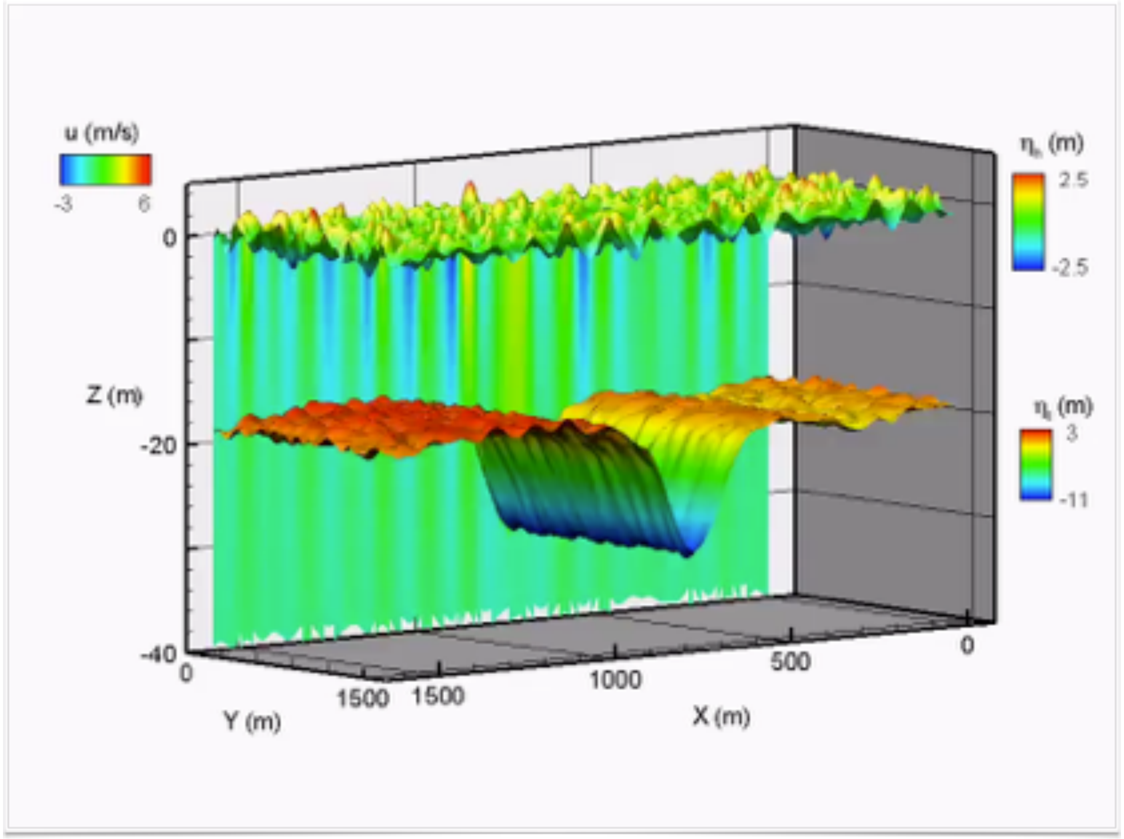


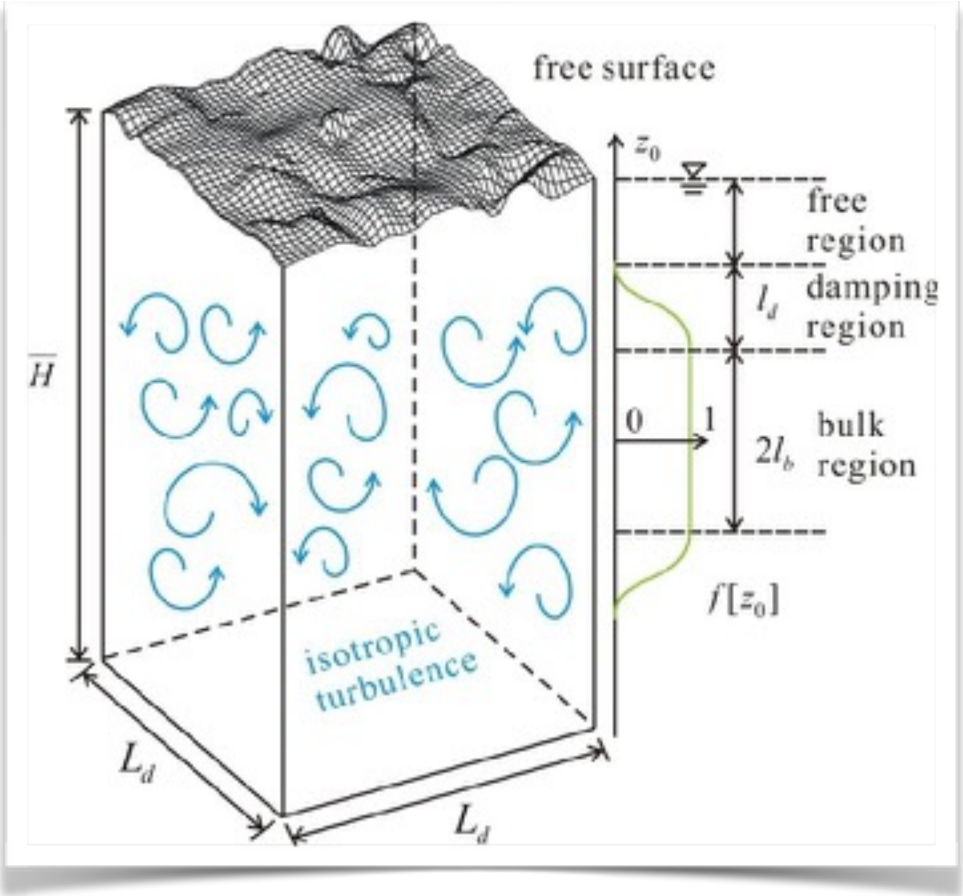


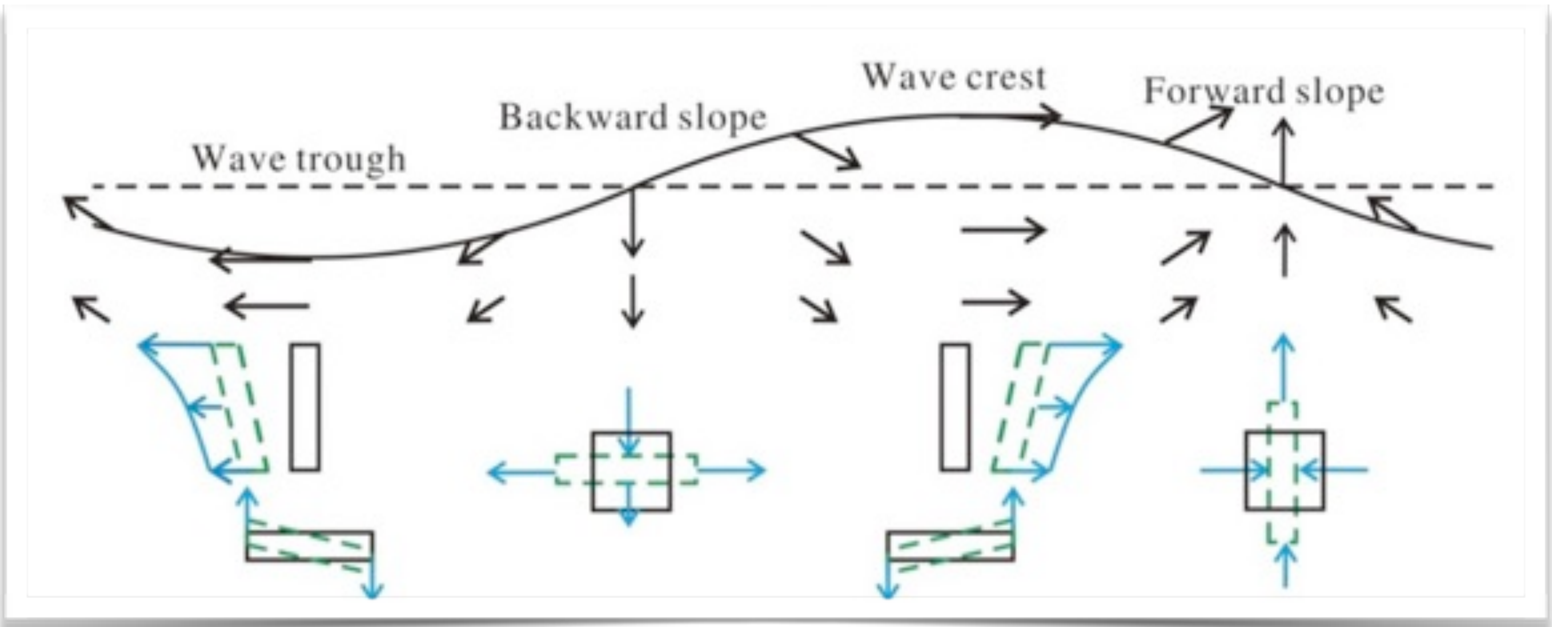


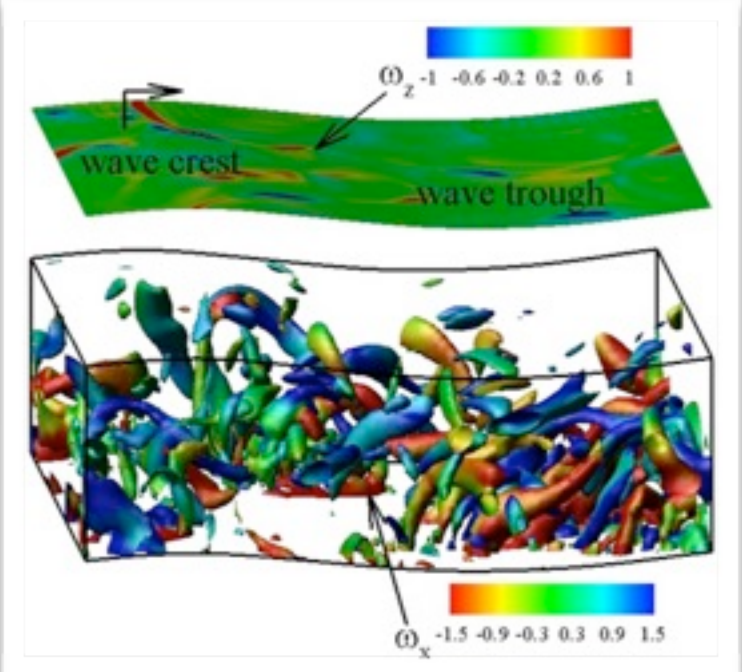
Courtesy of Lian Shen

Wave-Turbulence Interaction









Courtesy of Lian Shen

Thank You!

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State Key Laboratory of
Marine Environmental Science

近海海洋环境科学国家重点实验室（厦门大学）

State Key Laboratory of Marine Environmental Science
(Xiamen University)