Applied Math
UNIVERSITY OF COLORADO **BOULDER**

Investigations of rapidly rotating, stably stratified and non-hydrostatic flows

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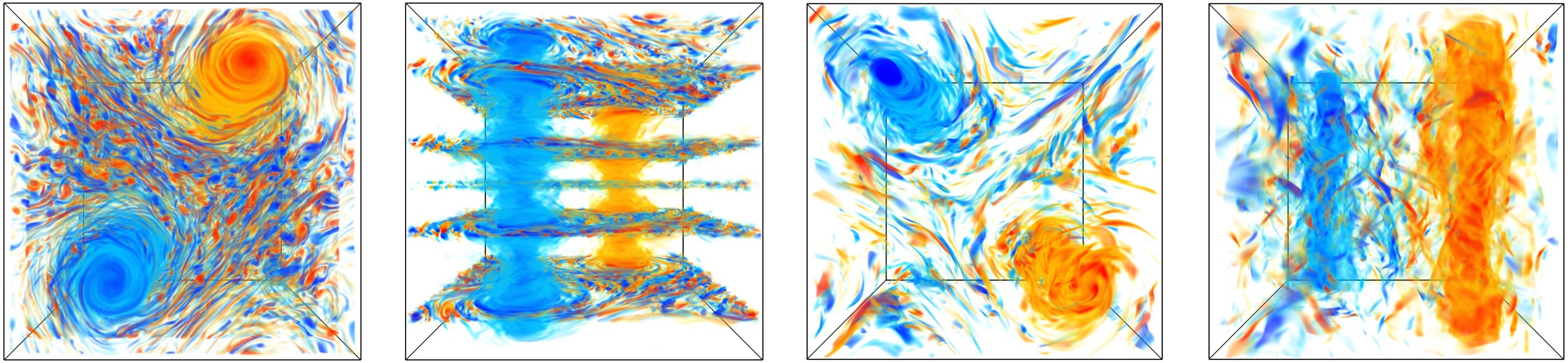
Nieves, Grooms, J. & Weiss (2016) *Journal of Fluid Mechanics, vol. 801*



Motivation:

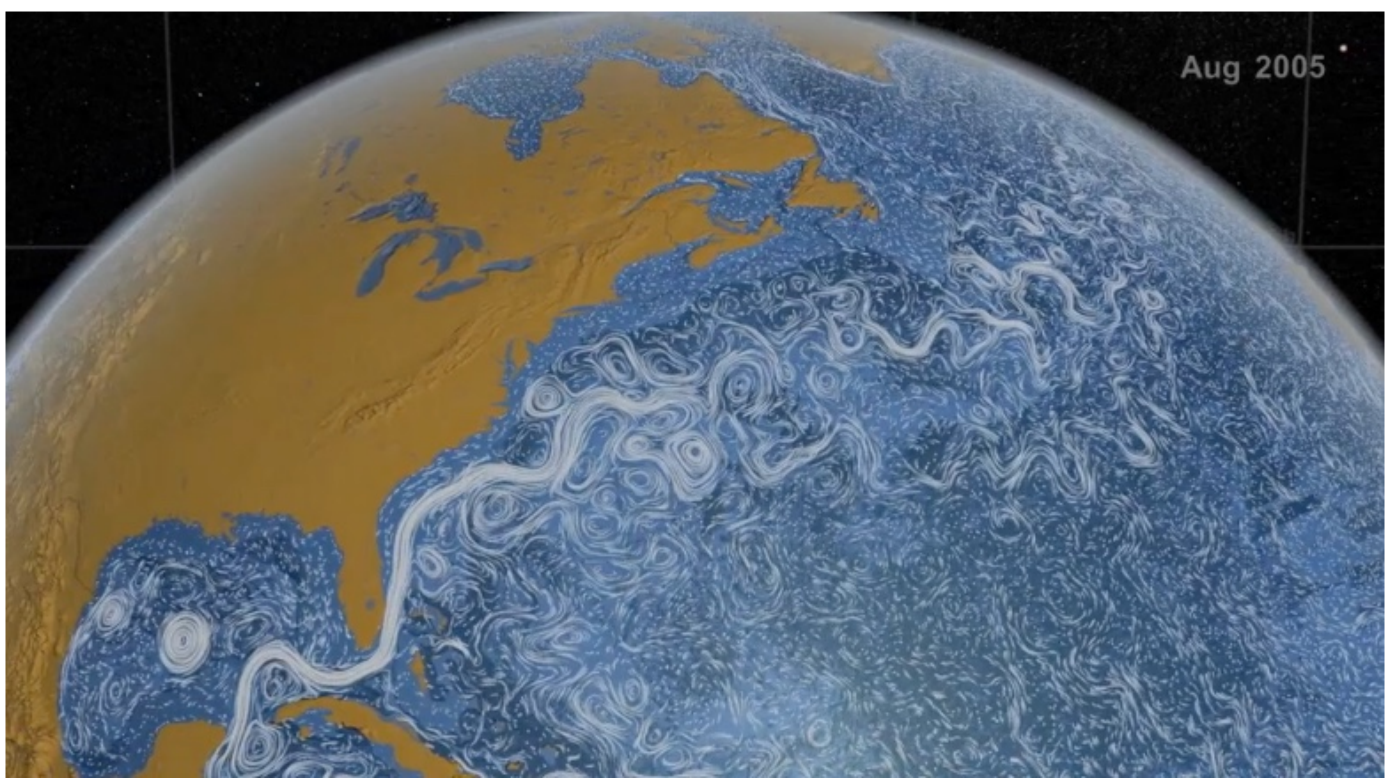
- ***Strong system rotation*** & ***stable density stratification*** characterize many geophysical flows
  - Primary balance: geostrophy where $Ro_f \ll 1$
  - Primary balance: hydrostacy where $Fr_f \ll 1$
  - Strong spatial anisotropy $L_f \gg H$

$$Ro_f = \frac{U_f^*}{2\Omega L_f^*} \quad \text{Rossby}$$

$$Fr_f = \frac{U_f^*}{N_0^* L_f^*} \quad \text{Froude}$$

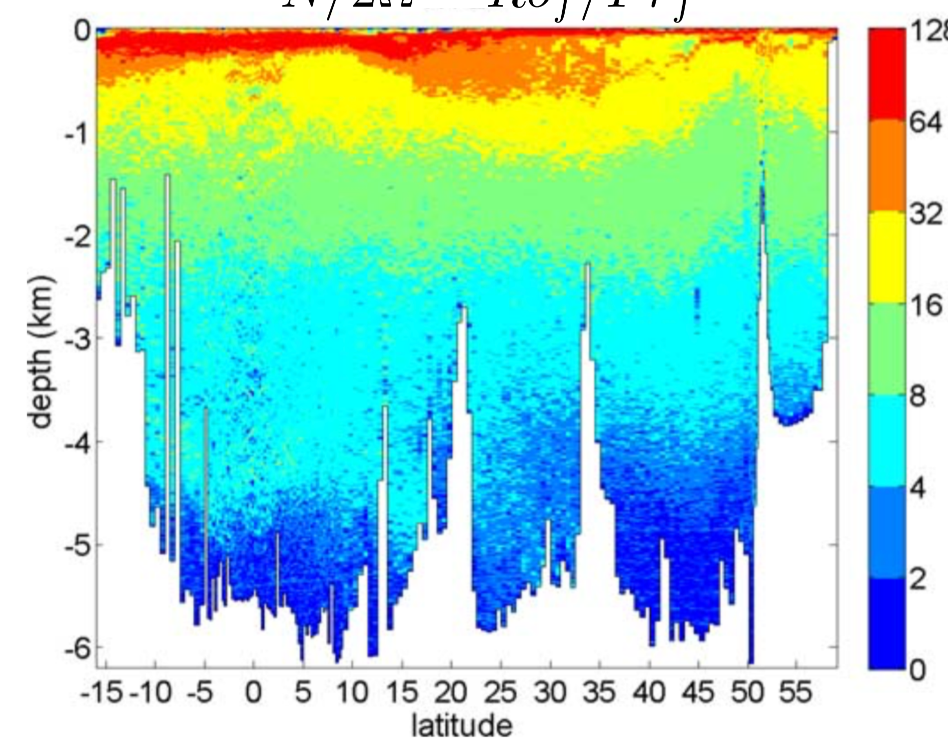
$$Re_f = \frac{U_f^* L_f^*}{\nu} \quad \text{Reynolds}$$

$$Fr_f \ll Ro_f \ll 1$$



Source: NASA (GCM synthesizing satellite and in-situ data)

$$N/2\Omega = Ro_f/Fr_f$$



North-South section of Pacific ocean at 179° E
Gerkema et al. Rev of Geophysics 2004

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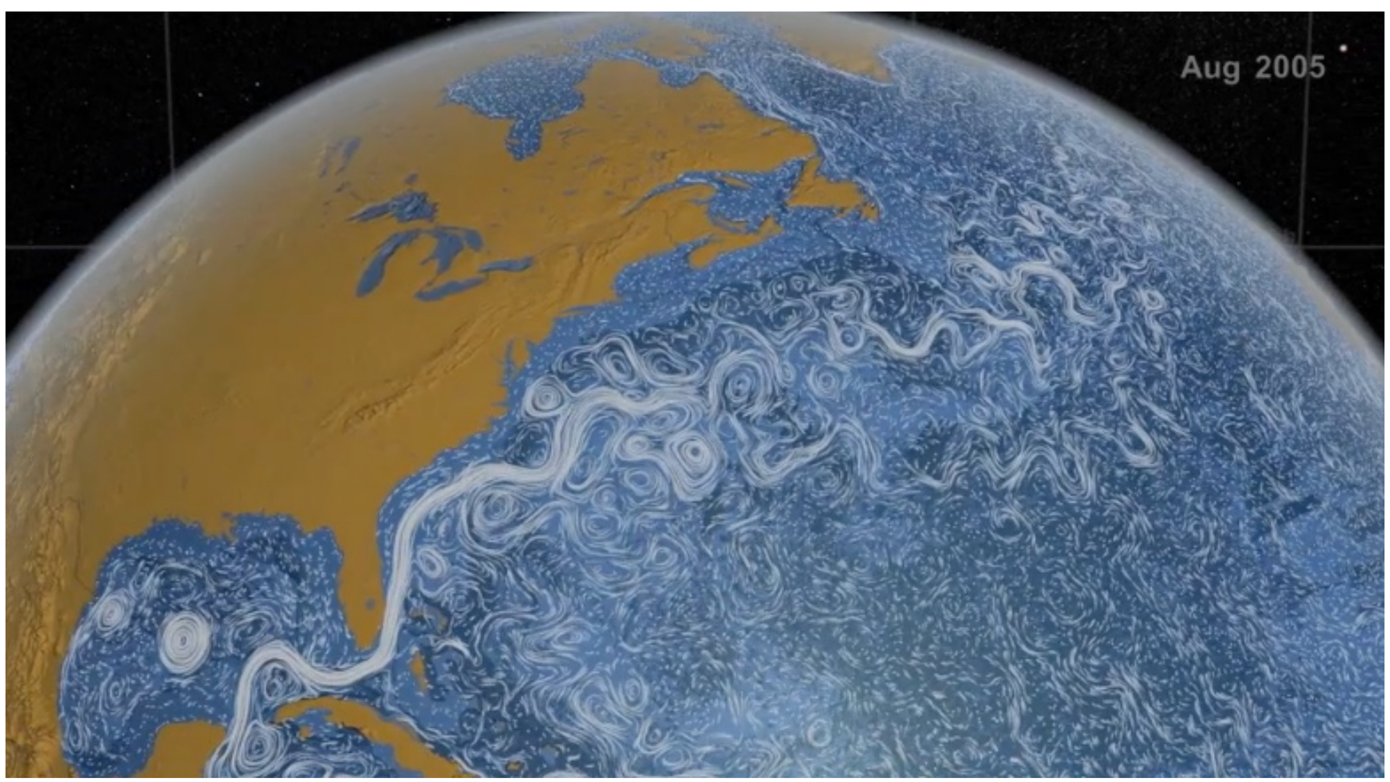

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Consider normal modes:
temporal separation btw wave & eddies

$$\propto \exp[i(\omega t + \mathbf{k}_\perp \cdot \mathbf{x}_\perp + k_z z)]$$

$$\omega_{\text{wave}}^2 = \frac{1}{Fr_f^2} \sin^2 \theta_{\boldsymbol{k}} + \frac{1}{Ro_f^2} \cos^2 \theta_{\boldsymbol{k}},$$

$$\omega_{\text{eddy}}^2 = 0$$

$$\tan \theta_k = \frac{k_\perp}{k_z}$$

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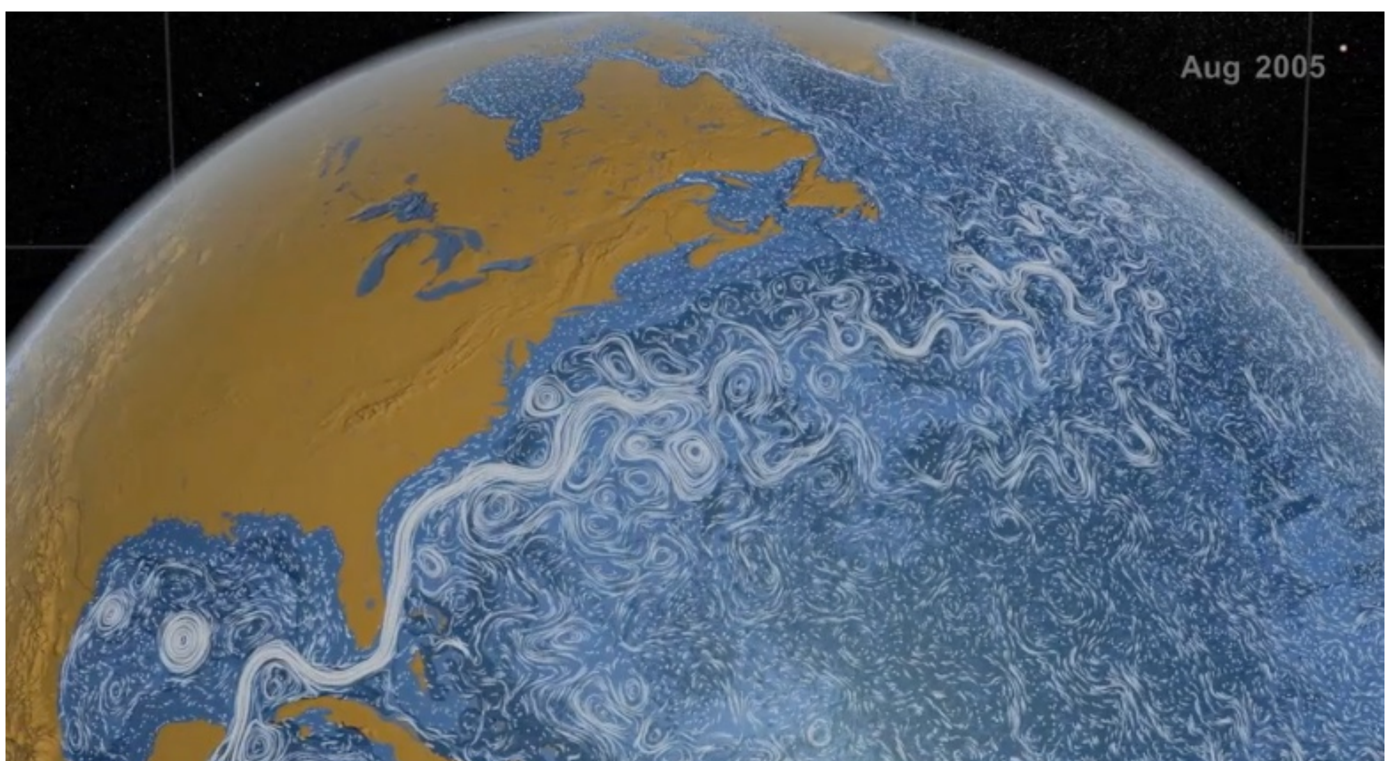

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$$\omega_{\text{wave}} \sim \mathcal{O}\left(\min\left\{\frac{1}{Fr_f}, \frac{1}{Ro_f}\right\}\right) \gg \omega_{\text{eddy}}$$

Motivation: Hydrostatic-QG

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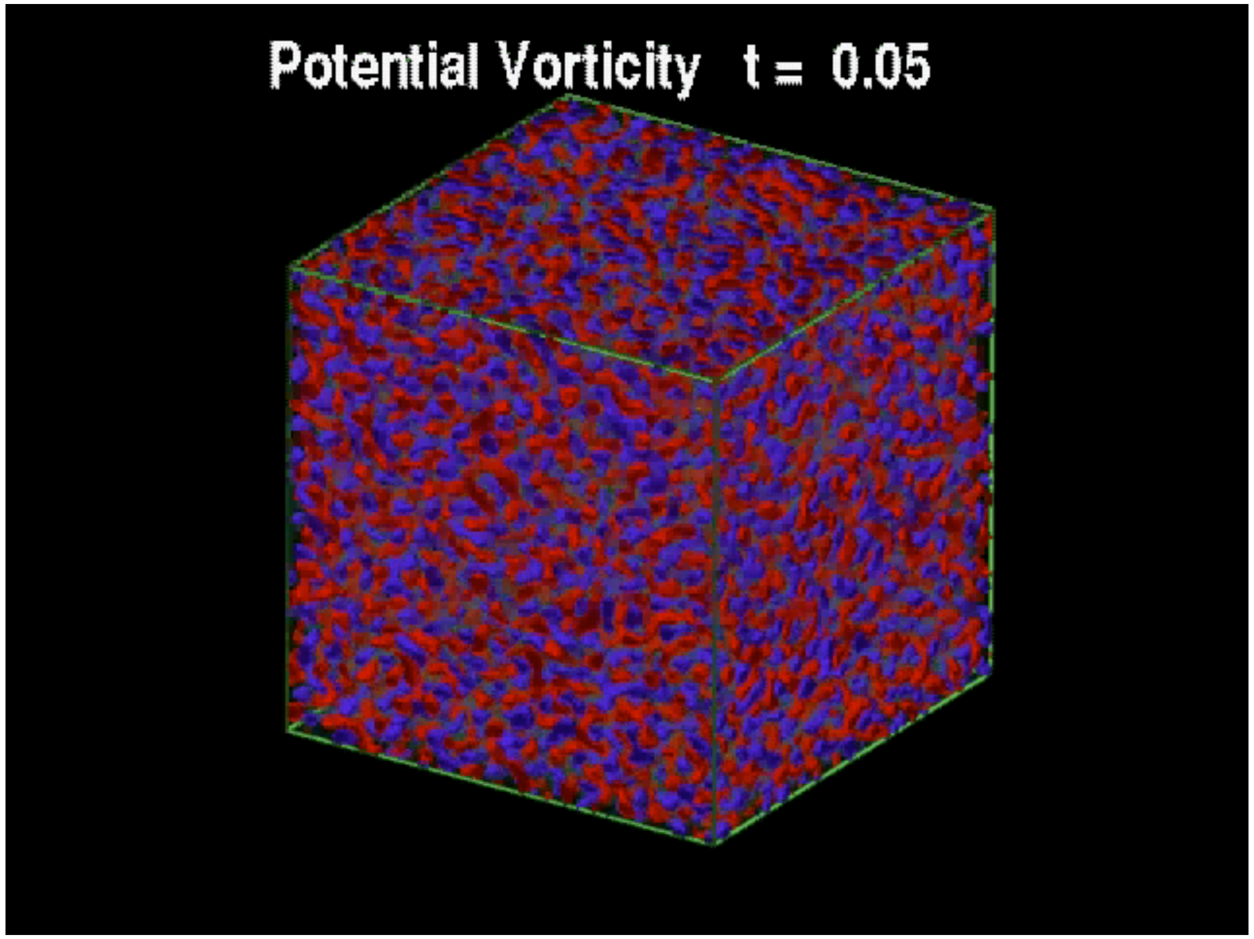
- Result classical QG balance filtering fast wave dynamics



McWilliams, Weiss & Yavneh JFM 1999

Classical QG (Charney '48,71): reduction to PV

$$p = \Psi, \ \boldsymbol{u}_\perp = \nabla_\perp \times \Psi \hat{\boldsymbol{z}}, \ b = \partial_z \Psi \ \ w = \mathcal{O}\left(\frac{H}{L_f} Ro_f\right)$$

$$D_t^\perp \left(\nabla_\perp^2 \Psi + \frac{4\Omega^2}{N_0^{*2}} \partial_{zz} \Psi \right) = f + d$$

Inverse energy cascade $E(k) \approx k^{-5/3}, k < k_f$

Direct enstrophy cascade $\approx k^{-3}, k > k_f$

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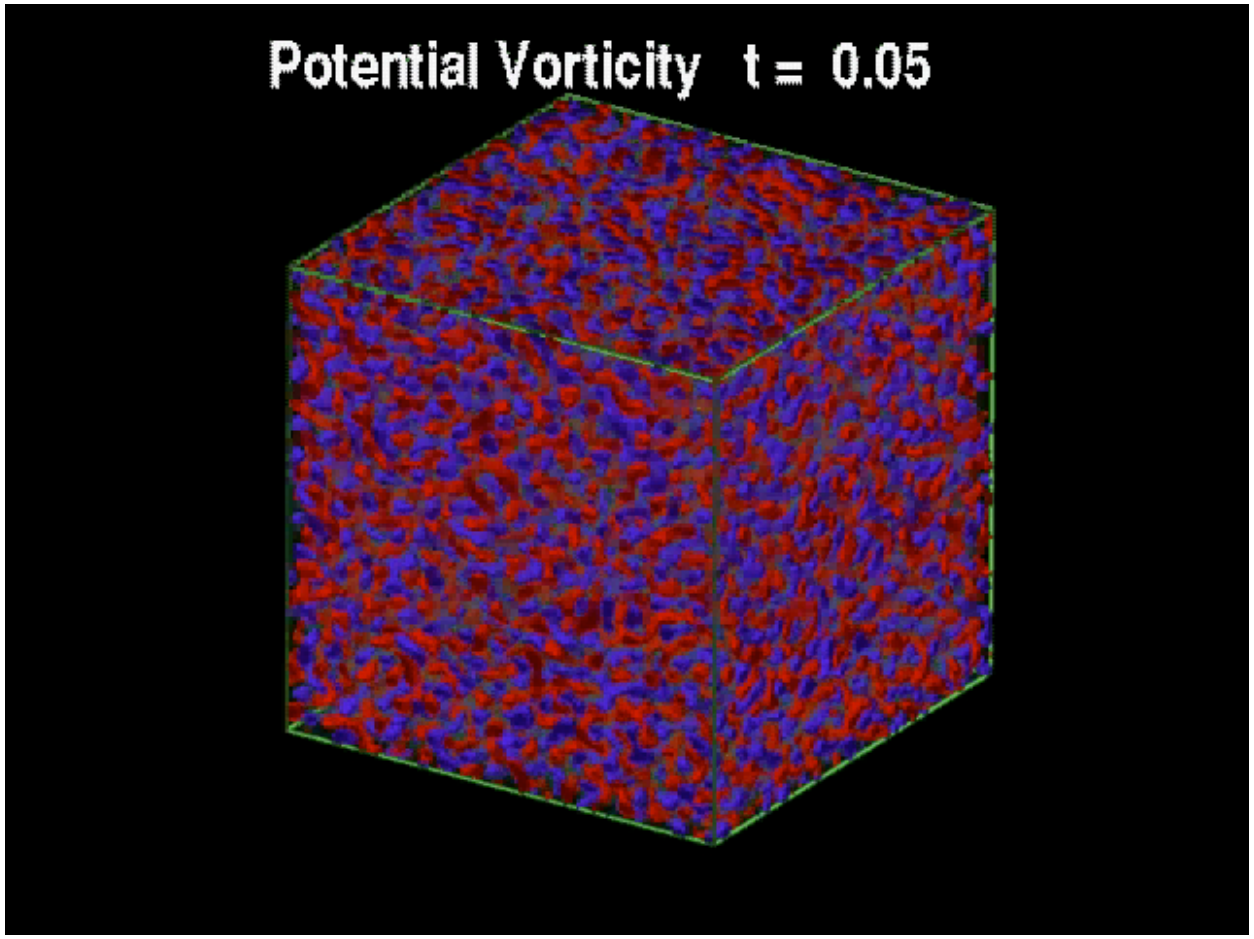

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Validity of QG - Embid & Majda GAFD 1998
- Temam & Wlrosoetisno JAS '10,'11

Evidenced in DNS - Smith & Waleffe JFM '02
Marino et al EPL '13
Whitehead & Wingate JFM '14

Motivation:

- ***Strong system rotation*** & ***weak stable density stratification*** also prevalent in geophysical flows

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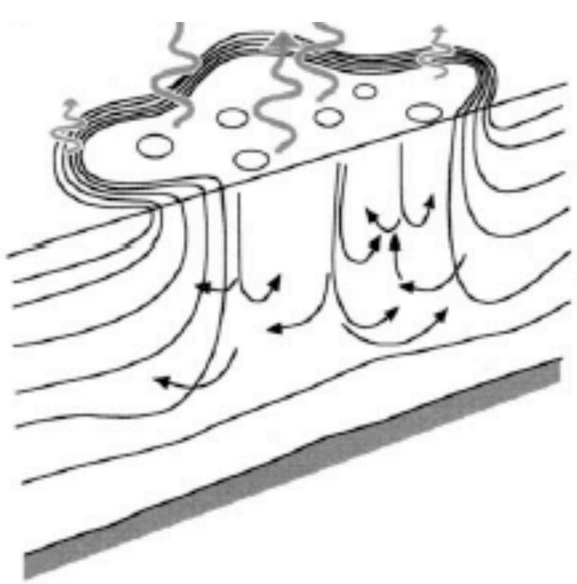
Current focus

- Primary balance: geostrophy where $Ro_f < 1$
- weak stable stratification where $Fr_f = \mathcal{O}(1)$

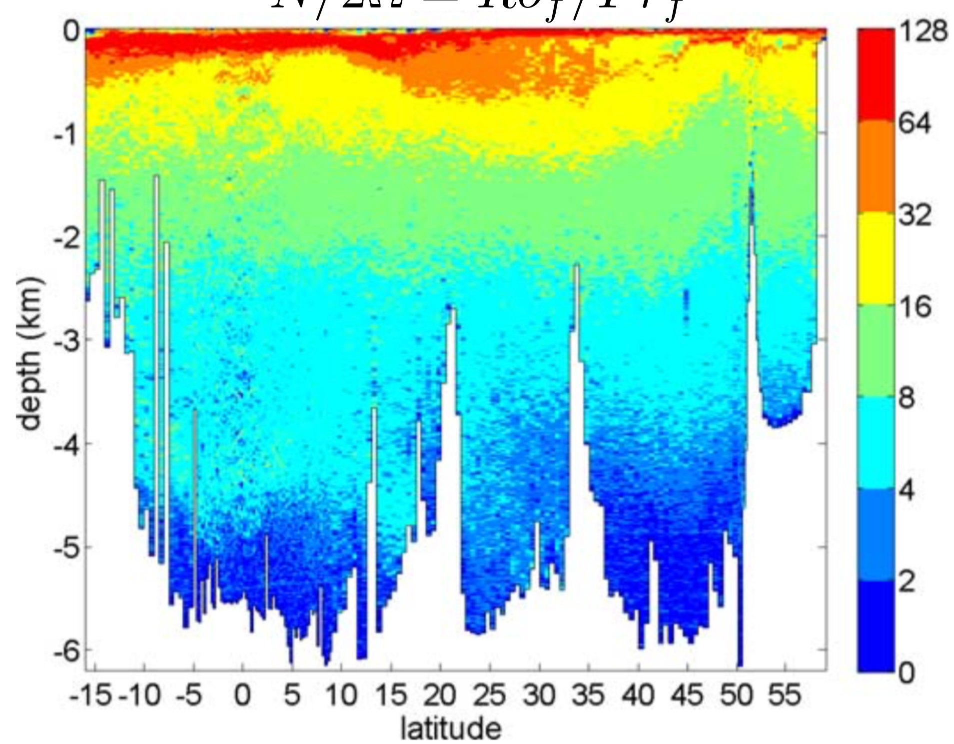
Examples

- Open ocean deep convection, convective overshoot
- High latitude abyssal oceans (not well studied)



Marshall & Schott RG 1999

$$N/2\Omega = Ro_f/Fr_f$$



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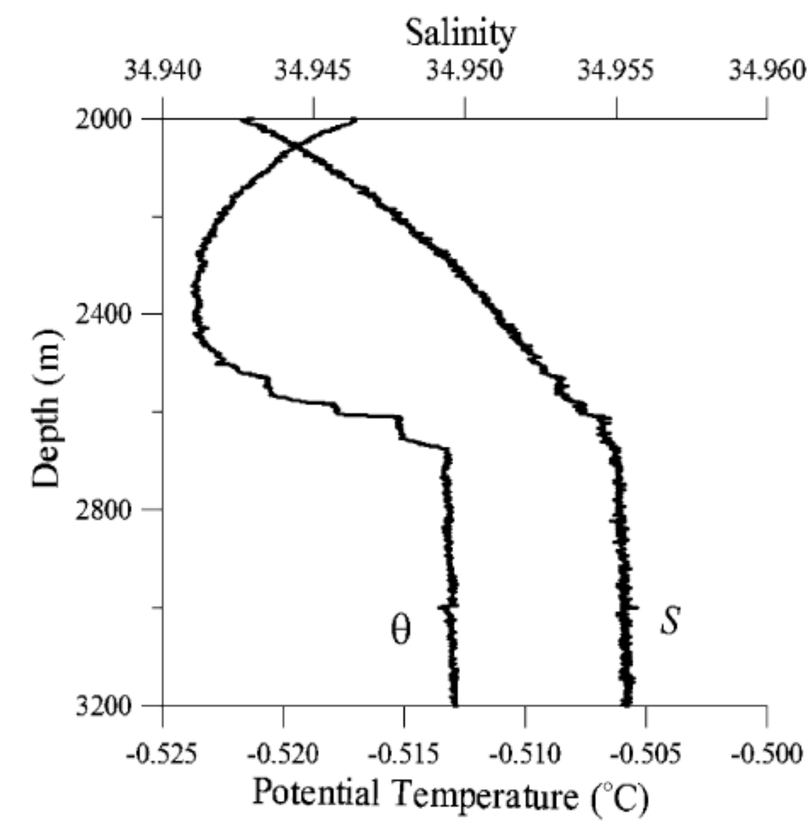

Timmermans et al. DSR 2003

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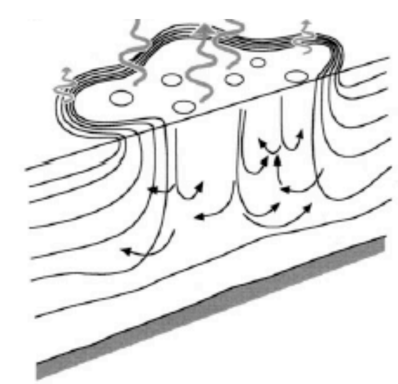
- High latitude abyssal oceans (not well studied)
- Result: no separation between eddy and wave dynamics



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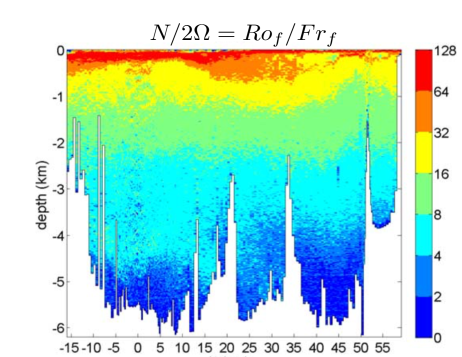
$$\tan\theta_k = \frac{k_\perp}{k_z}$$

$$A = \frac{k_\perp}{k_z} = \mathcal{O}(Ro_f^{-1}) \implies \theta = \pm\left(\frac{\pi}{2} - \mathcal{O}(Ro_f)\right)$$



Considering anisotropy: columnar dynamics is slow

Governing equations (*non-dimensional* Boussinesq equations):

$$D_t\mathbf{u} + \frac{1}{Ro_f}\hat{\mathbf{z}} \times \mathbf{u} = -Eu_f \nabla p + b\hat{\mathbf{z}} + \frac{1}{Re_f}\nabla^2\mathbf{u}$$

$$D_t b + \frac{1}{Fr_f^2} S\left(z\right) w = \frac{1}{\sigma Re_f}\nabla^2 b$$

$$\nabla \cdot \mathbf{u} = 0$$

imposed system rotation

$$\mathbf{\Omega} = \Omega\hat{\mathbf{z}}$$

imposed ambient stratification

$$N_0^* \equiv |g\rho_0^{*-1}(\partial_{z^*}\delta\hat{\rho}^*(z^*))_{max}|$$

$$S(z) \equiv -\partial z \delta\hat{\rho}$$

non-dimensional numbers

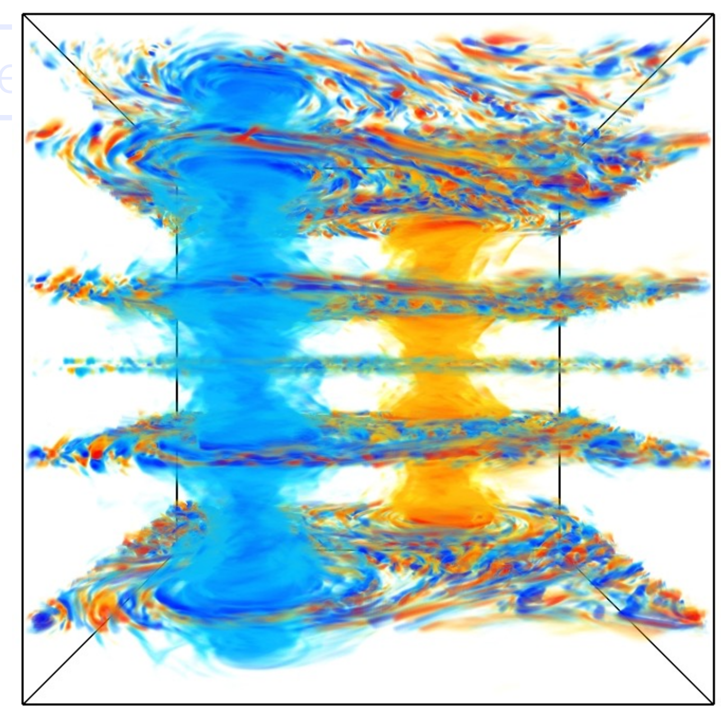
$$Ro_f = \frac{U_f^*}{2\Omega L_f^*}$$ Rossby number

$$Fr_f = \frac{U_f^*}{N_0^* L_f^*}$$ Froude number

$$Eu_f = \frac{\delta p_0}{\rho_0 U_f^{*2}}$$ Euler number

$$Re_f = \frac{U_f^* L_f^*}{\nu}$$ Reynolds number

$$\sigma = \frac{\nu}{\kappa}$$ Prandtl number



3D periodic rotating box
stratification - weak to strong

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energy injection: required to excite fluid motions
vertical motions only

ϵ_f^* injection rate

L_f^* injection scale

characteristic scales

$$U_f^* = \left(\epsilon_f^* L_f^*\right)^{1/3}$$

$$T_f^* = \left(L_f^{*2}\epsilon_f^{*-1}\right)^{1/3}$$

Internal length scales (partitioning of parameter space)

$L_K \equiv Re_f^{-3/4}$ Kolmogorov scale - equivalence of viscous and eddy turnover times

$L_D \equiv \frac{A Ro_f}{Fr_f}$ Deformation radius - PE to KE conversion by baroclinic instability

$L_\Omega \equiv Ro_f^{3/2}$ Zeman scale* - Coriolis (buoyancy) force negligible

$L_N \equiv Fr_f^{3/2}$ Ozmidov scale

aspect ratio

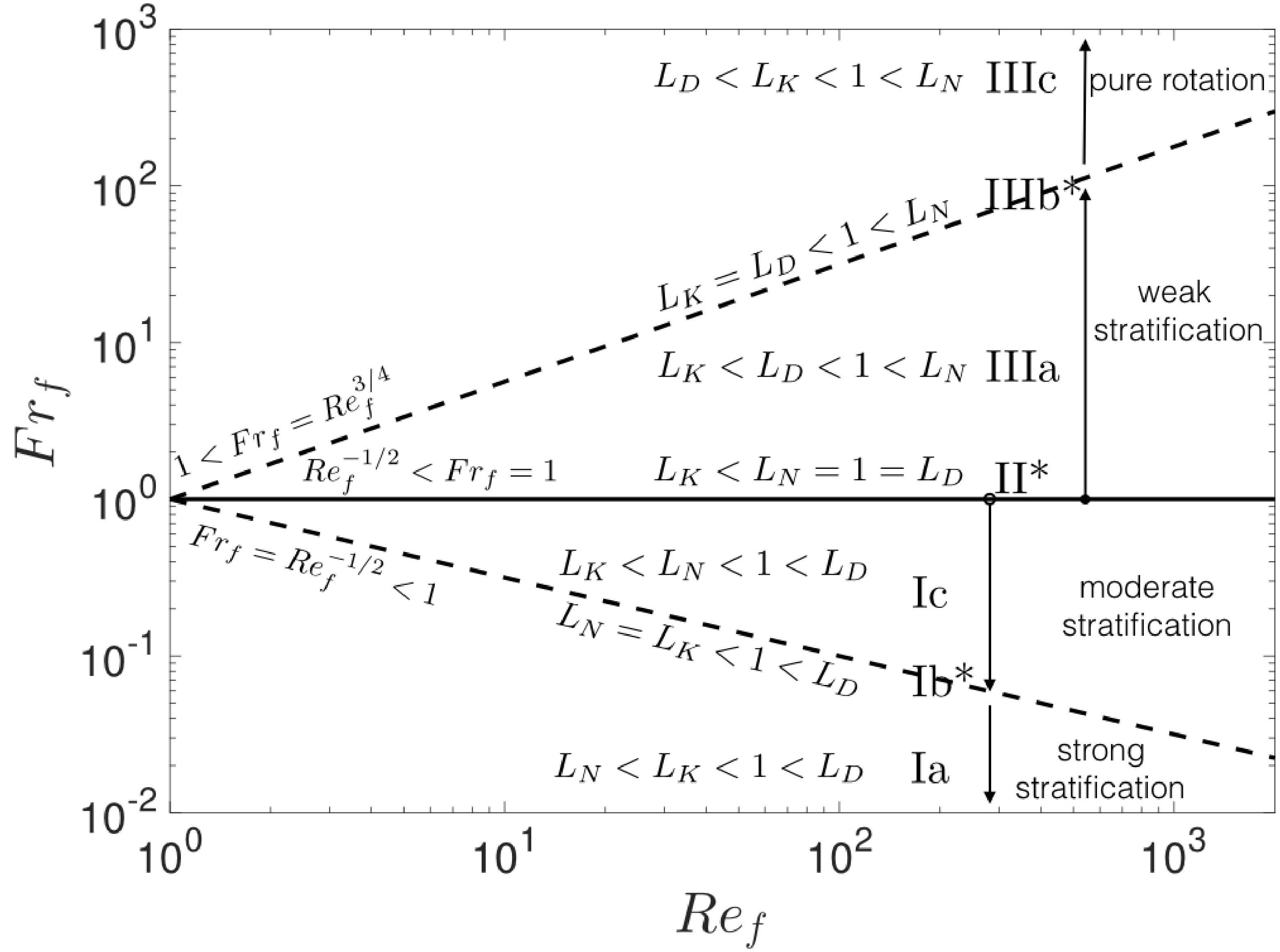
$$A = \frac{H^*}{L_f^*} \sim Ro_f^{-1}$$

We focus on flows with

$$Ro_f = o\left(Re_f^{-1/2}\right)$$

$$L_\Omega = o\left(L_K\right)$$

partitioning of parameter space for $Ro_f \ll 1$



Governing equations (*non-dimensional* Boussinesq equations):

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$$D_t b + \frac{1}{Fr_f^2} S(z)\, w = \frac{1}{\sigma Re_f}\nabla^2 b$$

$$\nabla \cdot \mathbf{u} = 0$$

$$A = Ro_f^{-1} \gg 1$$

$$w = \mathcal{O}\left(A Ro_f\right) \sim 1$$

leading order properties:

$$\hat{\mathbf{z}} \times \mathbf{u}_0 + \nabla_\perp p_0' = 0 \qquad \mathbf{u}_0' = -\nabla_\perp \times \psi_0' \hat{\mathbf{z}} + w_0' \hat{\mathbf{z}} \quad \text{(geostrophic solution)}$$

$$\nabla_\perp \cdot \mathbf{u}_0 = 0 \qquad p_0' = \psi_0' \quad \text{(geostrophic streamfunction)}$$

Non-hydrostatic quasi-geostrophic (NH-QG) equations

$$\partial_t \zeta_0 + J[\psi_0, \zeta_0] - \partial_Z w_0 = \frac{1}{Re_f} \nabla_\perp^2 \zeta_0$$ vertical vorticity

$$\partial_t w_0 + J[\psi_0, w_0] + \partial_Z \psi_0 = b_0 + \frac{1}{Re_f} \nabla_\perp^2 w_0 + f_{w_0}$$ vertical velocity

$$A = Ro_f^{-1} \gg 1$$
$$w = \mathcal{O}\left(A Ro_f\right) \sim 1$$

$$\left.\begin{aligned} \partial_t b_0 + J[\psi_0, b_0] + w_0 \left(\partial_Z \overline{b}_{-1} + \frac{1}{Fr_f^2} S(Z) \right) &= \frac{1}{\sigma Re_f} \nabla_\perp^2 b_0 \\ \partial_T \overline{b}_{-1} + \partial_Z \left(\overline{w_0 b_0} \right) &= \frac{1}{\sigma Re_f} \partial_Z^2 \overline{b}_{-1} \end{aligned}\right\}$$ buoyancy

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$$\nabla_\perp \cdot \mathbf{u}_0 = 0 \qquad p_0' = \psi_0'$$ (geostrophic streamfunction)

$$J[\psi_0, \cdot] \equiv \mathbf{u}_0' \cdot \nabla_\perp$$ (vertical advection is a subdominant effect)

Linearization for inviscid NH-QG equations about S(Z) = 1 gives

$$\omega_{\text{wave}}^2 = \frac{1}{Fr_f^2} + \frac{k_Z^2}{k_\perp^2} \qquad k_Z = \mathcal{O}(1)$$ (wave-eddy interactions occur on order-one timescale)
(inviscid conserved quantity - energy not enstrophy)

Julien et al. JFM 2006, Sprague et al. JFM 2006

Numerical simulation of NH-QG equations

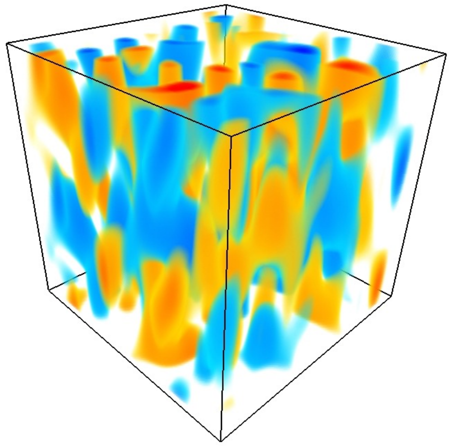
$$\partial_t \zeta_0 + J[\psi_0, \zeta_0] - \partial_Z w_0 = \frac{1}{Re_f}\nabla_\perp^2 \zeta_0$$

$$\partial_t w_0 + J[\psi_0, w_0] + \partial_Z \psi_0 = b_0 + \frac{1}{Re_f}\nabla_\perp^2 w_0 + \boxed{f_{w_0}}$$

} stochastic forcing of vertical velocity used to excite fluid motions (waves)

$$\partial_t b_0 + J[\psi_0, b_0] + w_0 \left(\partial_Z \overline{b}_{-1} + \frac{1}{Fr_f^2} S(Z)\right) = \frac{1}{\sigma Re_f}\nabla_\perp^2 b_0$$

$$\partial_T \overline{b}_{-1} + \partial_Z \left(\overline{w_0 b_0}\right) = \frac{1}{\sigma Re_f}\partial_Z^2 \overline{b}_{-1}$$



time-stepping:

- deterministic dynamics evolved using 3rd-order Runge-Kutta
- white-noise forcing is nowhere differentiable in time stochastic dynamics evolved using Euler-Maruyama

$$u_{n+1}^* = u_{n+1} + \sqrt{\epsilon_f \Delta t}\chi_n$$

$$\mathbb{E}\left[u_{n+1}^{*2}\right] = \mathbb{E}\left[u_{n+1}^2\right] + \epsilon_f \Delta t$$

numerical resolution:

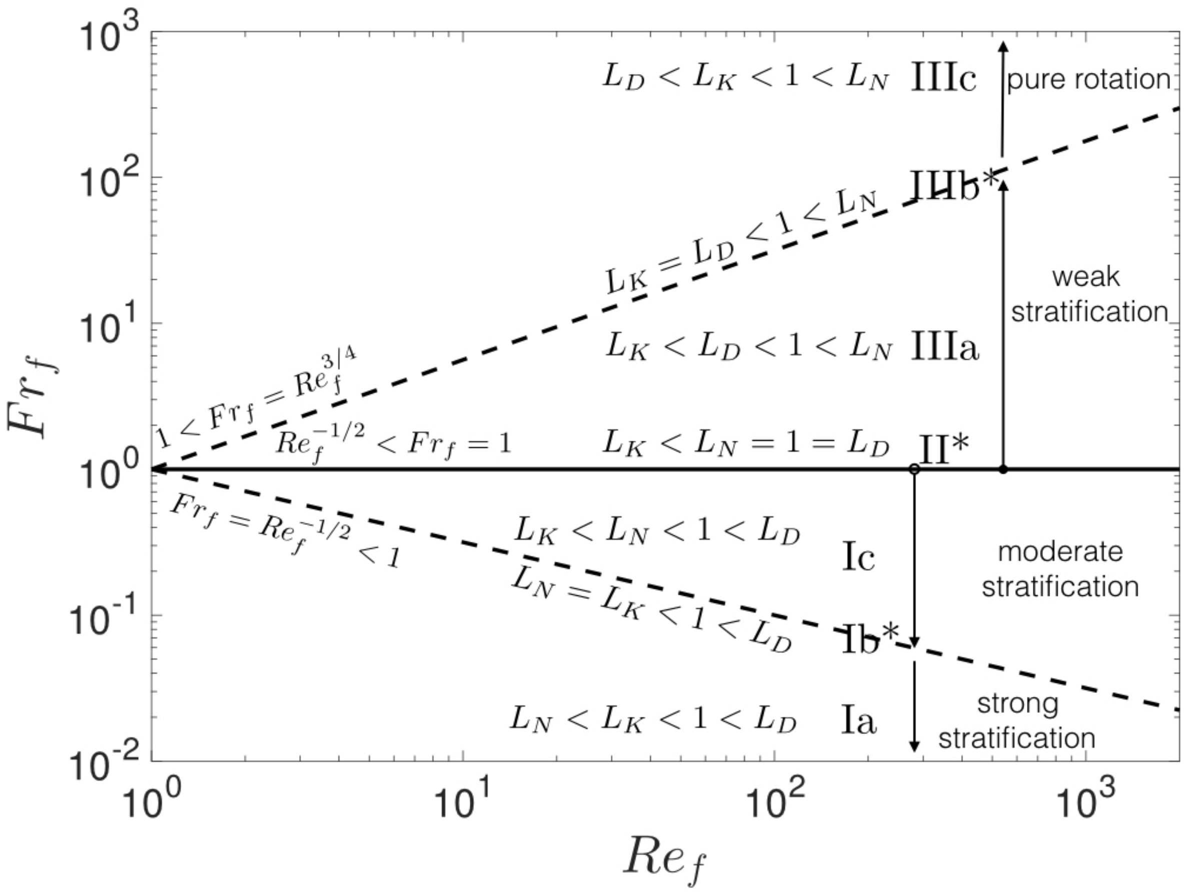
$$\Delta x = 2L_K, \qquad L_K = Re^{-3/4}$$

$$N_{x,y,z} = \frac{L_b}{2} Re^{3/4}, \qquad L_b = 10$$

Domain size: $L_b \times L_b \times 1$

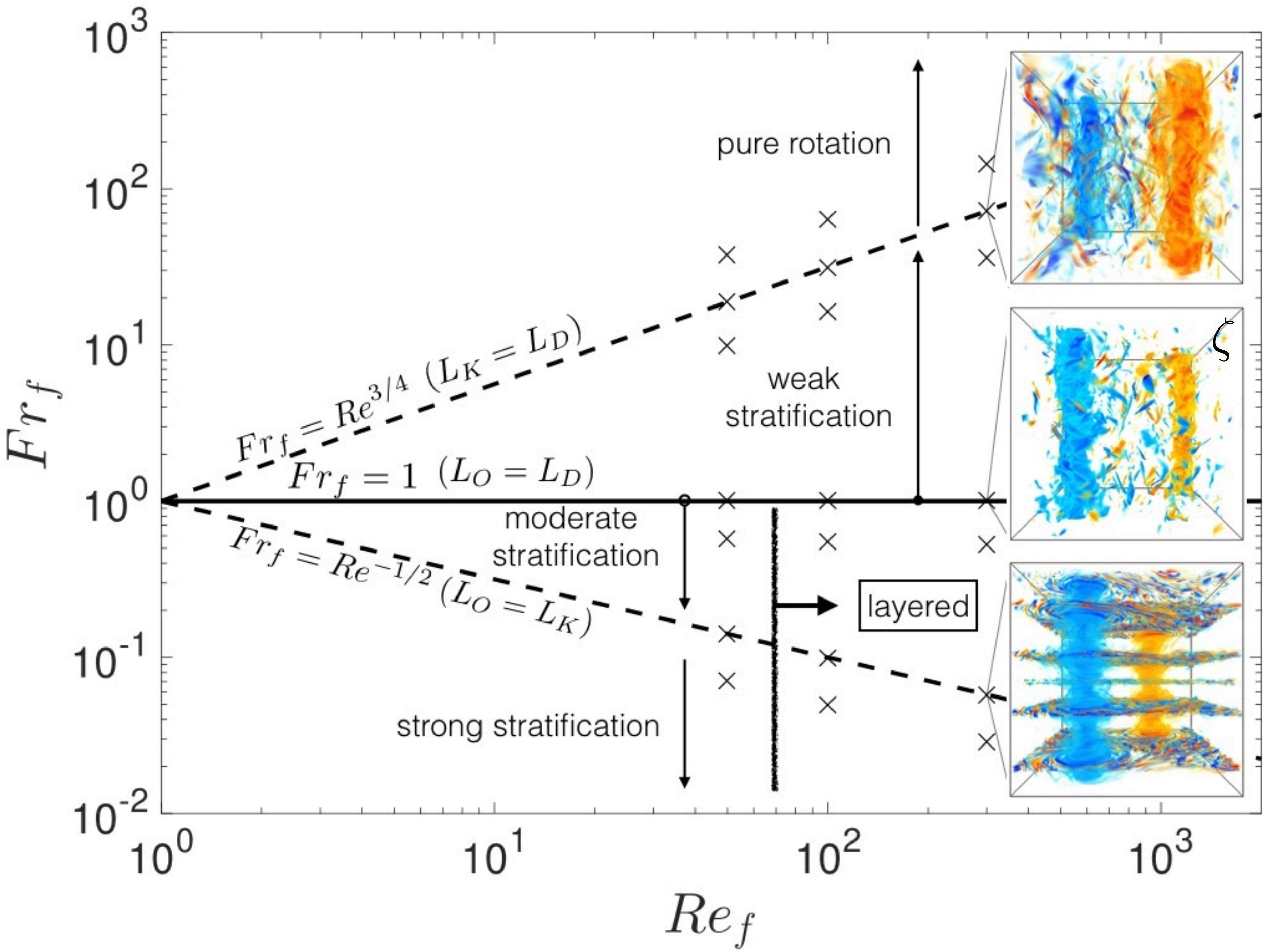
Numerical Survey

partitioning of parameter space for $Ro_f \ll Re_f^{1/2} < 1$



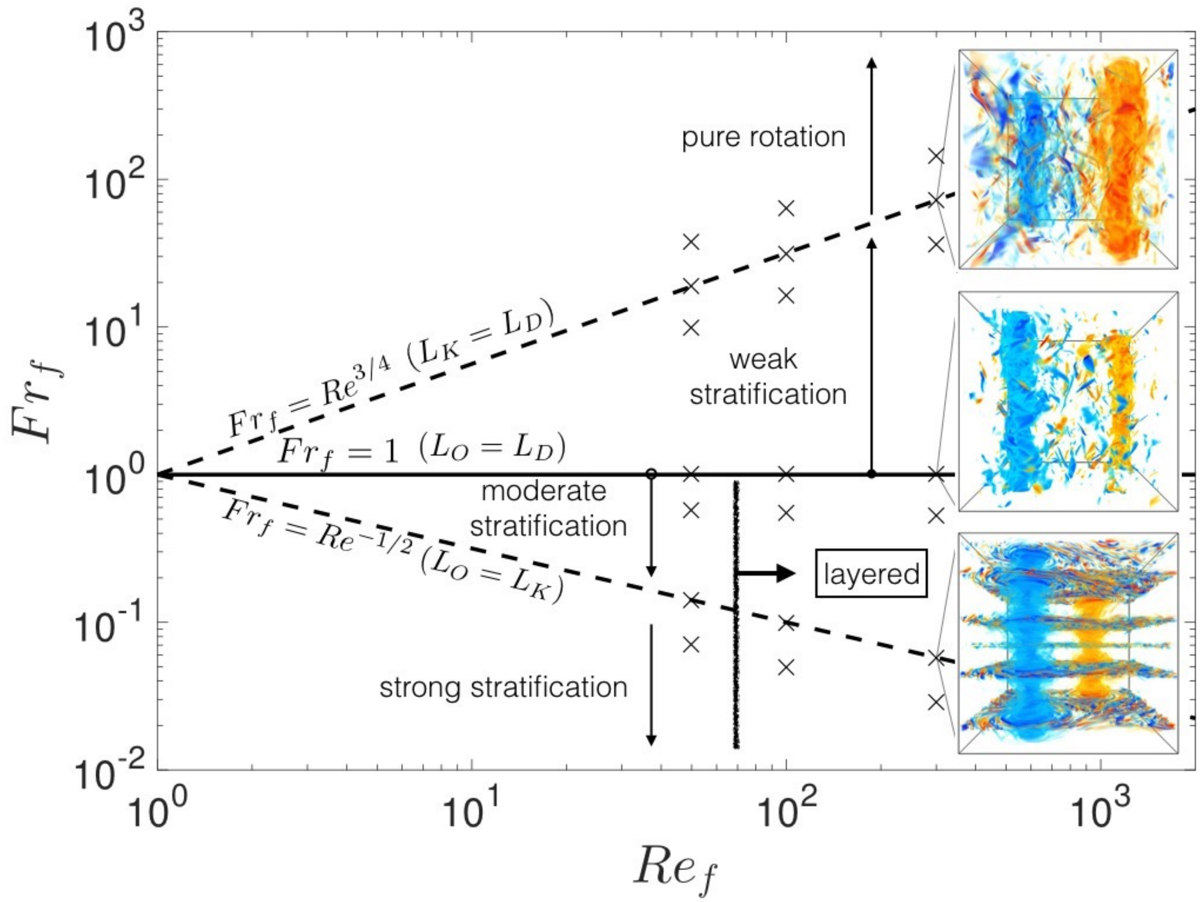
Regime	$Fr_f(Re_f)$	$Fr_f(Re_f = 50)$	$Fr_f(Re_f = 100)$	$Fr_f(Re_f = 300)$
Ia	$\frac{1}{2}Re_f^{-1/2}$	0.0707	0.0500	0.0289
Ib*	$Re_f^{-1/2}$	0.1414	0.1000	0.0577
Ic	$\frac{1}{2}(1 + Re_f^{-1/2})$	0.5707	0.5500	0.5289
II*	1	1	1	1
IIIa	$\frac{1}{2}(1 + Re_f^{3/4})$	9.9015	16.311	36.542
IIIb*	$Re_f^{3/4}$	18.803	31.623	72.084
IIIc	$2Re_f^{3/4}$	37.606	63.246	144.17
Grid resolution	$N_x \times N_y \times N_z$	$96 \times 96 \times 96$	$192 \times 192 \times 192$	$384 \times 384 \times 384$

Results (qualitative): large scale barotropic mode



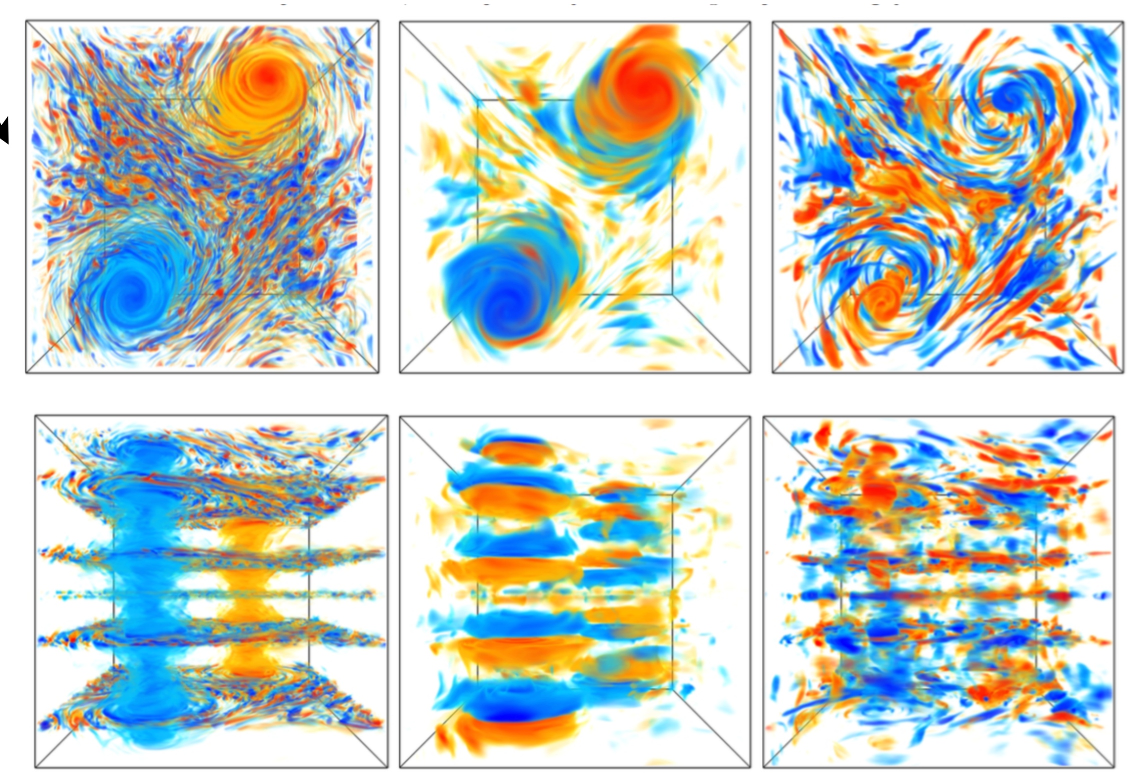
Flow organizes into a barotropic dipole surrounded by small scale geostrophic turbulence

Results (qualitative): large scale barotropic mode



- *tendency to form well-defined layers*
- *turbulence significantly enhanced within layers*
- *provided Re sufficiently large*
- *layering not observed in classical QG (coupling to waves absent*

$Re_f = 300, \quad Re_c = 4290 \quad Fr_f < 1$



(a) Vertical vorticity, ζ (b) Buoyancy, b (c) Vertical velocity, w

Results: Layering

mixing leads to locally reduced stratification

$$\partial_Z \left(\overline{w_0 b_0} - \frac{1}{Pe} \partial_Z \overline{b}_{-1} \right) = 0$$

vorticity RMS profiles:

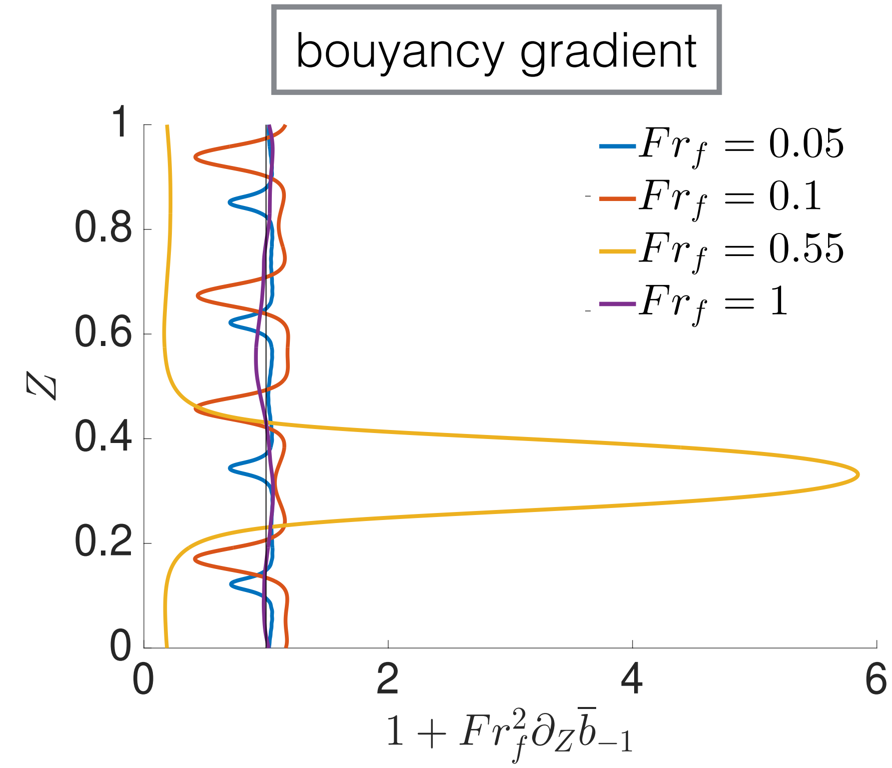
- *layering: regions of enhanced vortex stretching*
- *evidence of sublayers (thickness: distance b/w maxima)*
- *locally increased kinetic dissipation rate* $Re_f^{-1}\overline{\zeta^2}^{\mathcal{A}}$

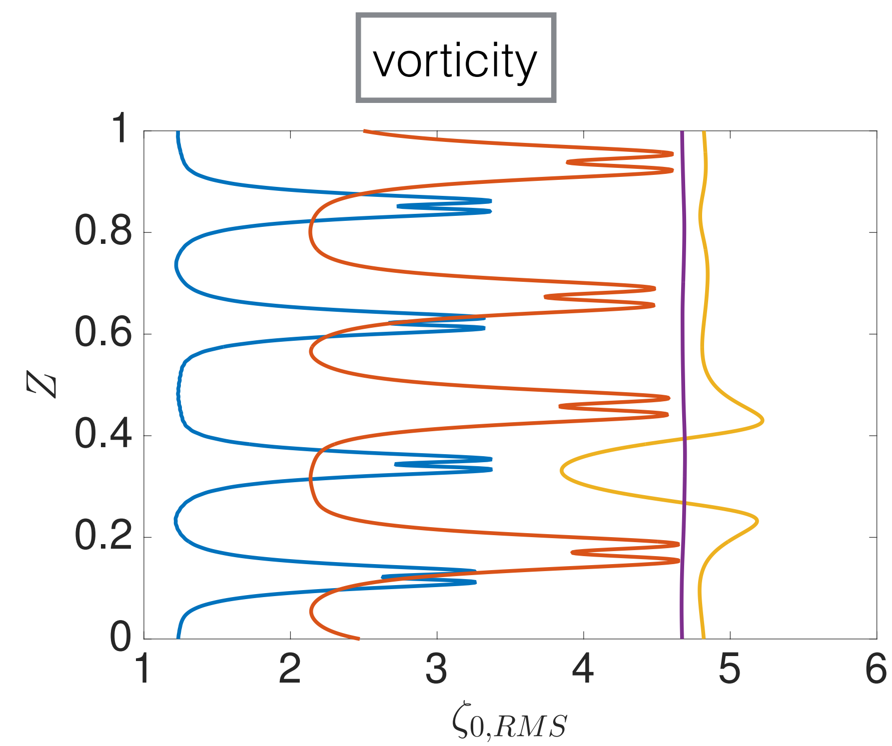
layer location:

- *minima of mean-buoyancy gradient*
- *local minima in vorticity RMS profile*

layering here is dissimilar to

- *layering in purely stratified turbulence (Kimura & Herring)*
- *doubly-diffusive systems with steep buoyancy gradients*
- *K-H shear layer*





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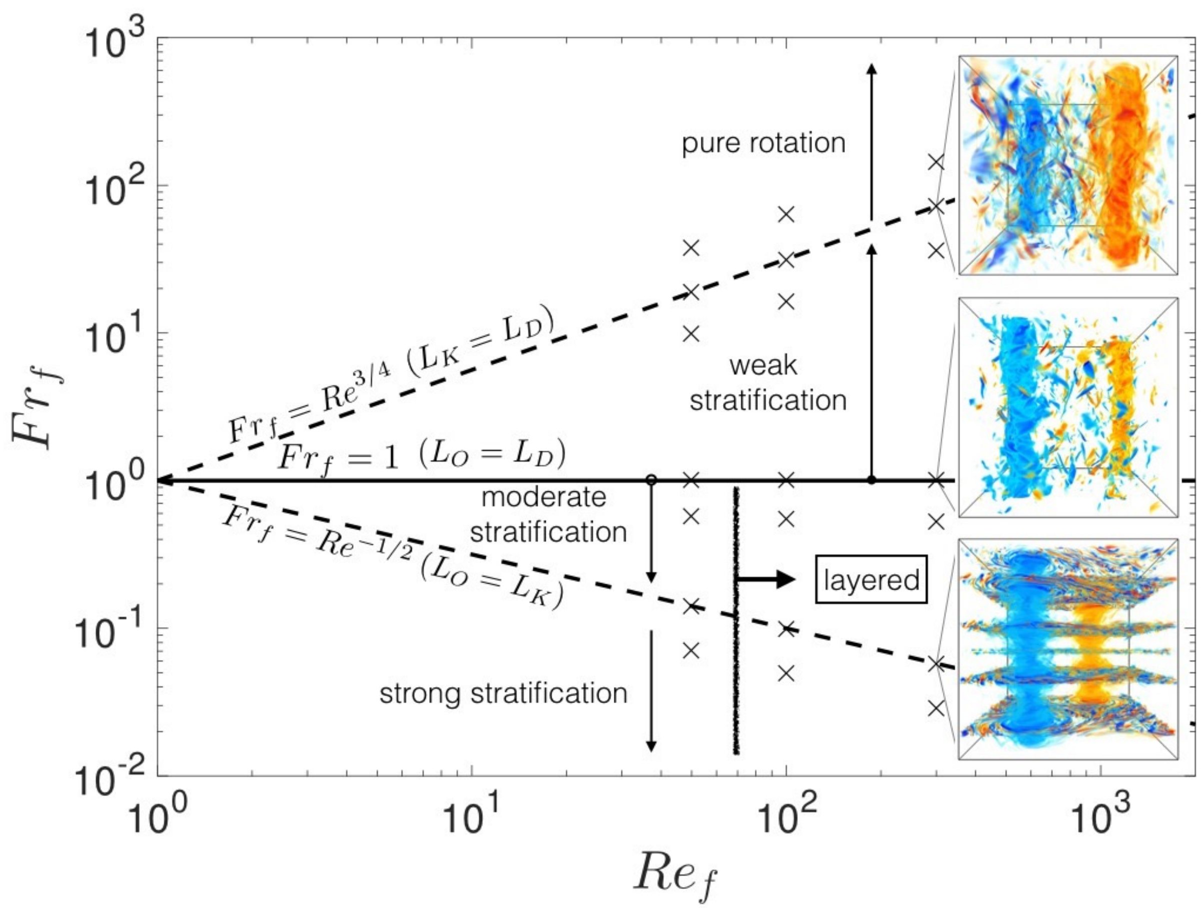
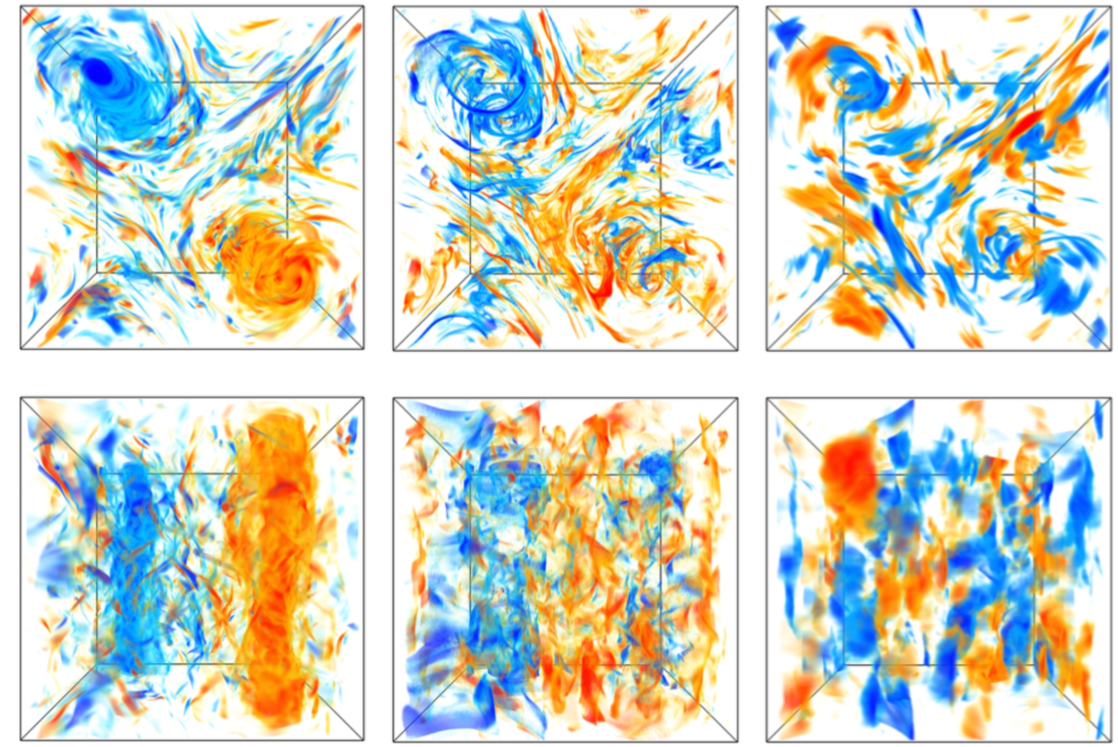
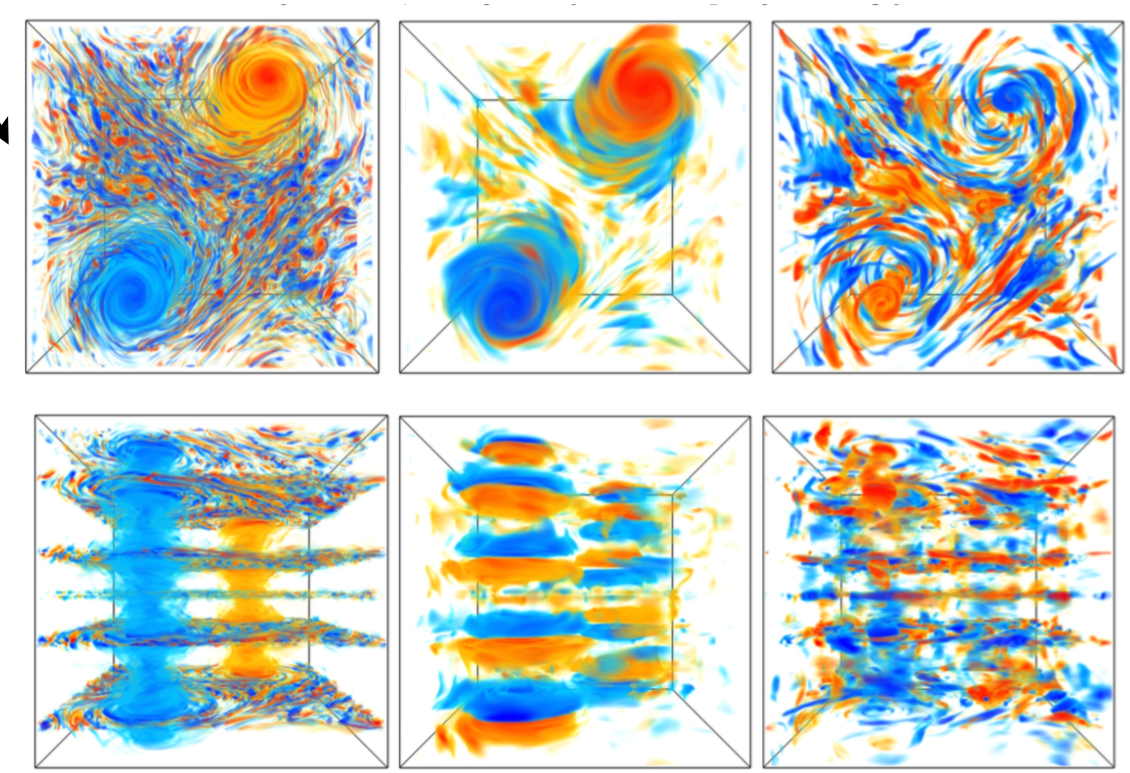


$Re_f = 300, \quad Re_c = 5742 \quad Fr_f > 1$



(a) Vertical vorticity, ζ (b) Buoyancy, b (c) Vertical velocity, w

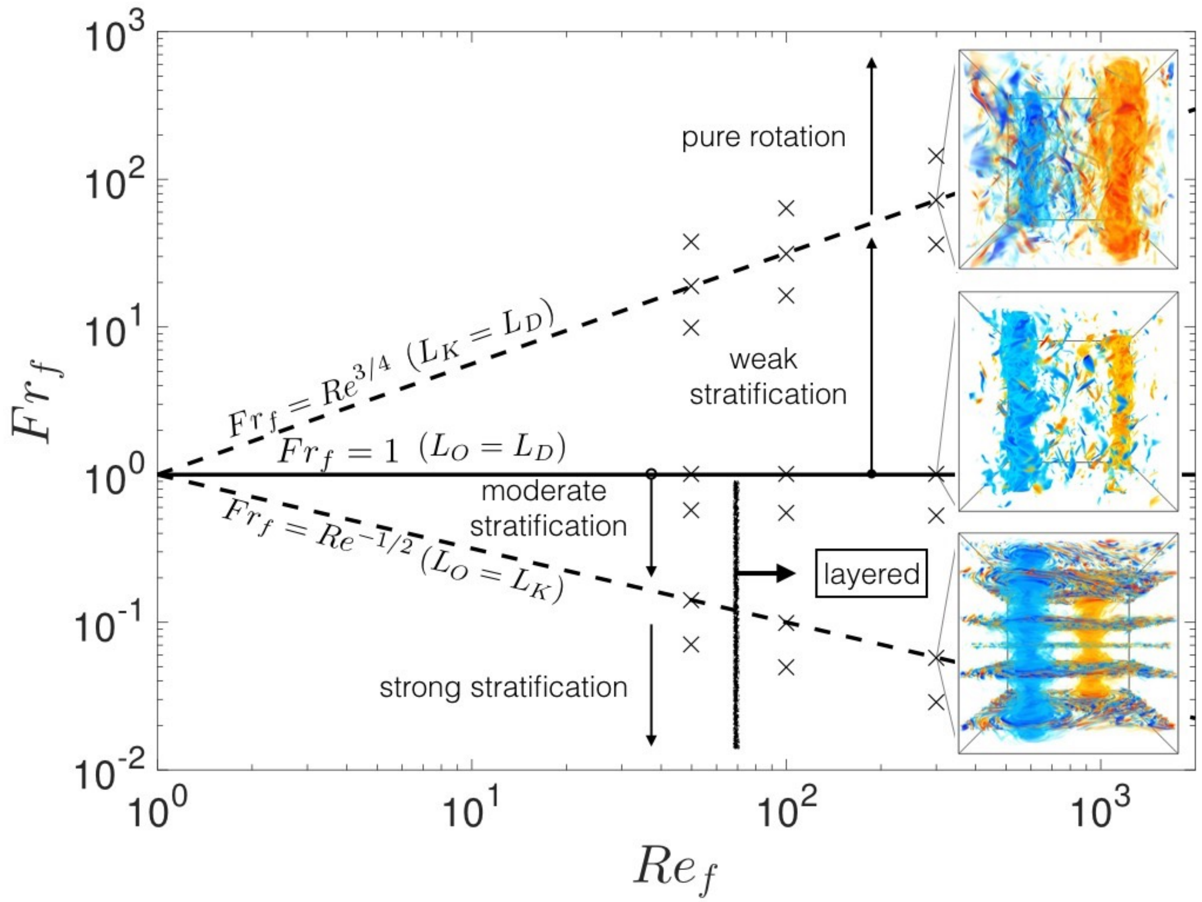
$Re_f = 300, \quad Re_c = 4290 \quad Fr_f < 1$



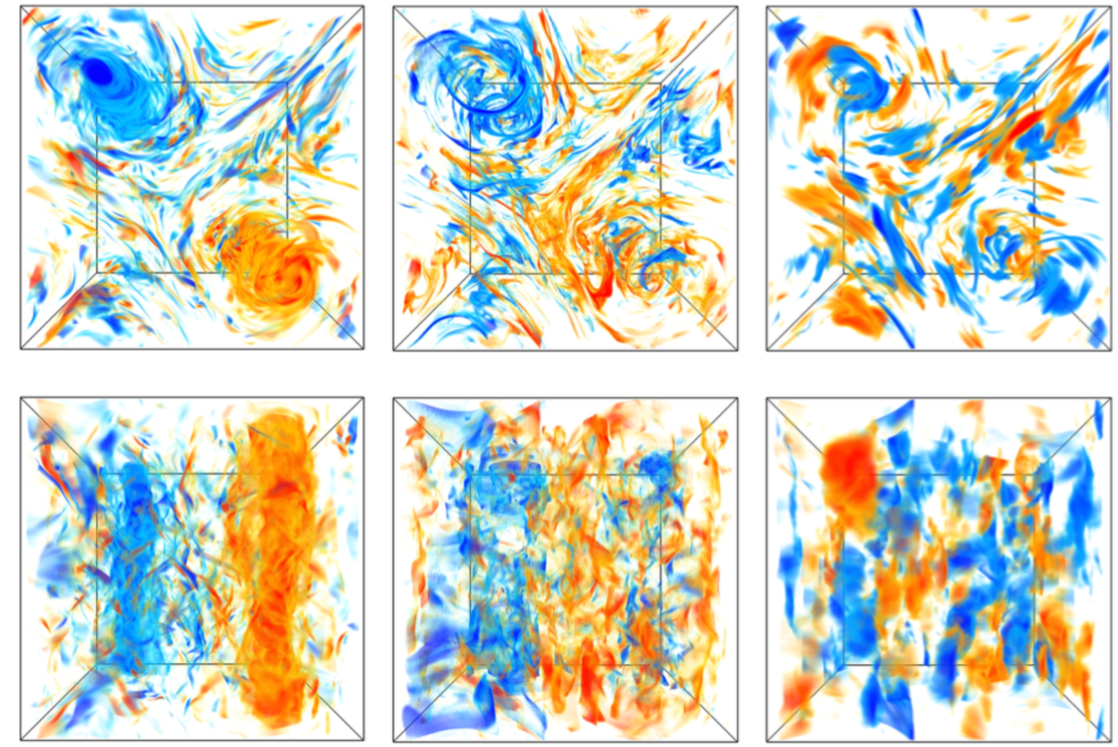
(a) Vertical vorticity, ζ (b) Buoyancy, b (c) Vertical velocity, w

- *Turbulence unobstructed by layers*
- *Does final state preserve a priori classification?*

Results (qualitative): large scale barotropic mode

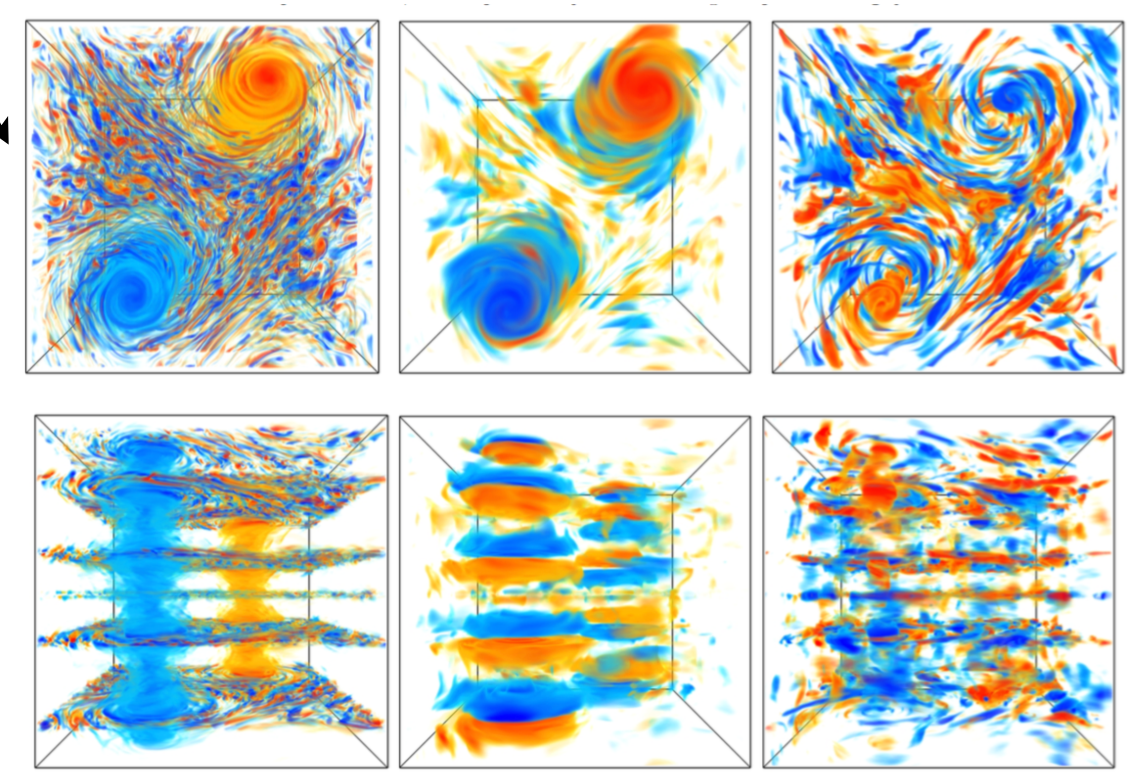


$Re_f = 300, \quad Re_c = 5742 \quad Fr_f > 1$



(a) Vertical vorticity, ζ (b) Buoyancy, b (c) Vertical velocity, w

$Re_f = 300, \quad Re_c = 4290 \quad Fr_f < 1$



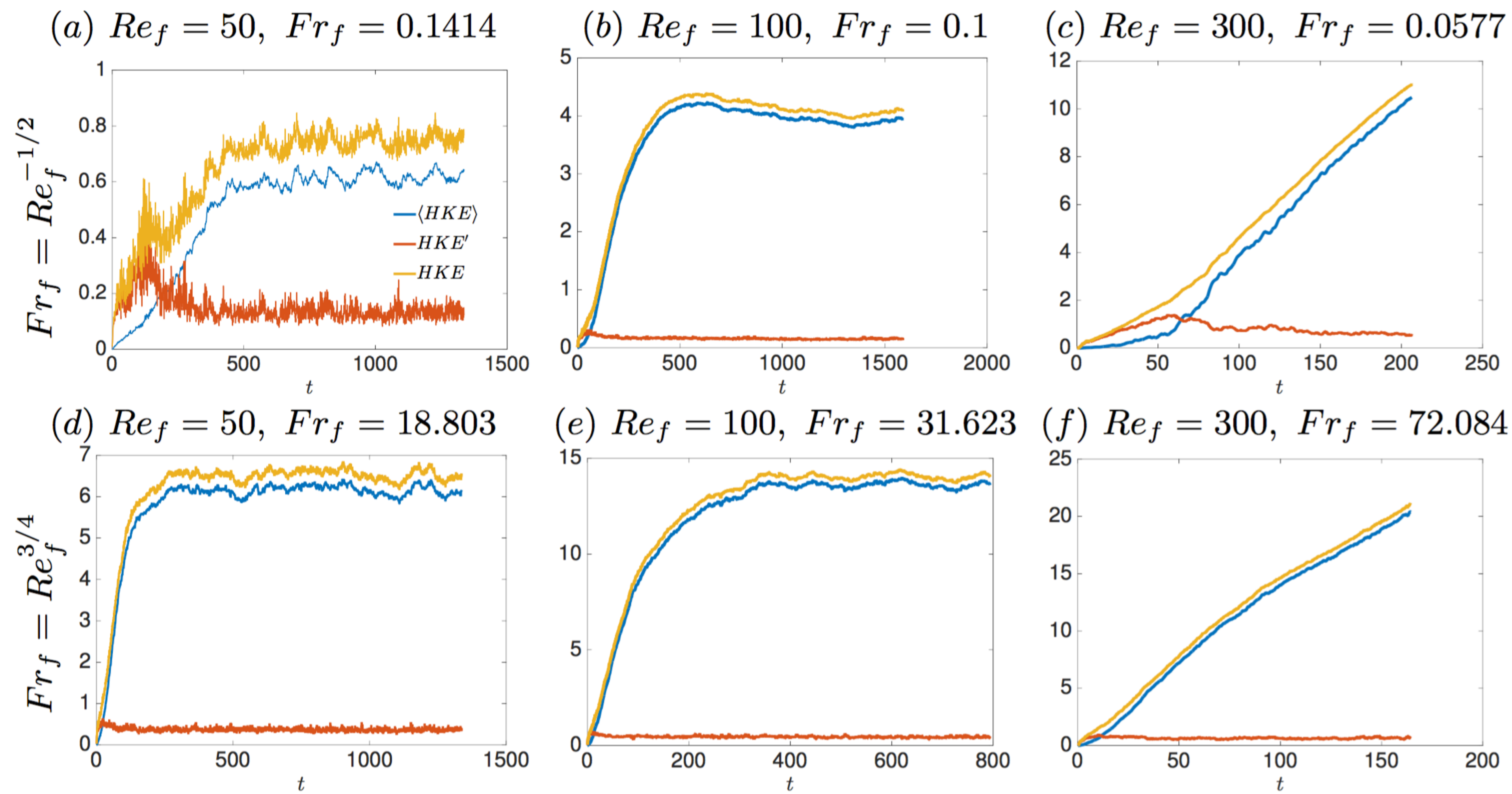
(a) Vertical vorticity, ζ (b) Buoyancy, b (c) Vertical velocity, w

measured quantities based on energy centroid

$$k_c = \frac{\int kE(k)dk}{\int E(k)dk} \qquad Re_c = Re_f U_c L_c, \quad Fr_c = Fr_f \frac{U_c}{L_c}$$

Re_f	Re_c	Fr_f	Fr_c	L_c	U_c
50	150	0.1414	0.0118	6.0	0.5
100	980	0.1000	0.0200	7.0	1.4
300	4290	0.0577	0.0195	6.5	2.2
50	604	18.80	4.50	7.1	1.7
100	2190	31.62	12.99	7.3	3.0
300	5742	72.08	31.67	6.6	2.9

Results: Energetics and equilibrium



strong stratification

weak stratification

- *baroclinic & barotropic decomposition:*

$$\psi = \langle \psi \rangle + \psi'$$

$$\mathbf{u}_{bt} = \nabla^{\perp} \langle \psi \rangle \qquad (w_{bt}, b_{bt}) = 0$$

$$\mathbf{u}_{bc} = \nabla^{\perp} \psi' \qquad (w_{bc}, b_{bc}) = (w, b)$$

- *horizontal kinetic energy (HKE)*

$$\partial t \langle HKE \rangle := \partial t \left[\frac{1}{2} \overline{|\nabla_{\perp} \langle \psi_0 \rangle|^2}^{\mathcal{A}} \right] = \overline{\langle \psi_0 \rangle \langle J[\psi_0', \zeta_0'] \rangle}^{\mathcal{A}} \cdots\cdots\cdots\cdots\cdots$$

(baroclinic forcing of barotropic mode, c.f. 2D BVE)

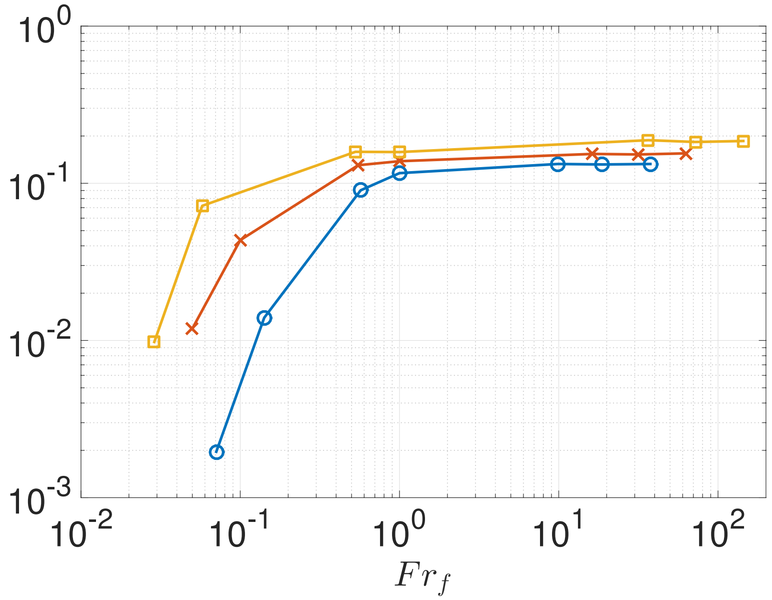
$$\partial t HKE' := \partial t \left[\frac{1}{2} \langle \overline{|\nabla_{\perp} \psi_0'|^2}^{\mathcal{A}} \rangle \right] = -\overline{\langle \psi_0 \rangle \langle J[\psi_0', \zeta_0'] \rangle}^{\mathcal{A}} + \langle \overline{w_0' \partial Z \psi_0'}^{\mathcal{A}} \rangle \cdots\cdots$$

(vortex stretching: strictly baroclinic)

Results: energy conversion

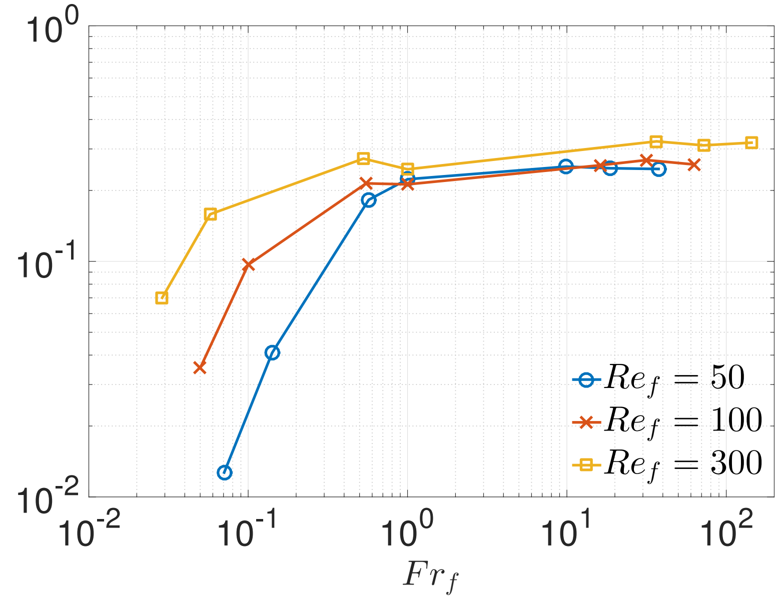
baroclinic forcing

$$\overline{\langle\psi_0\rangle\langle J[\psi_0', \zeta_0']\rangle}^{\mathcal{A}}$$



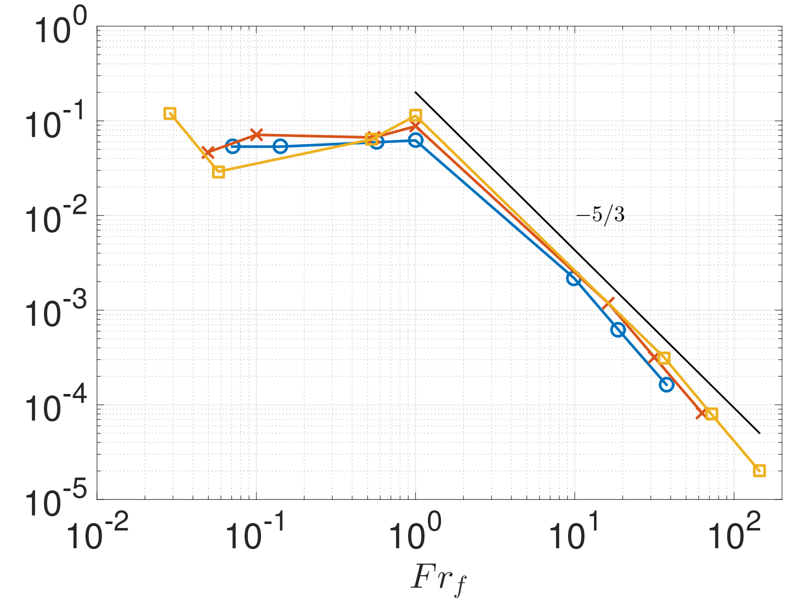
vortex stretching

$$\langle\overline{w_0\partial_Z\psi_0}^{\mathcal{A}}\rangle$$

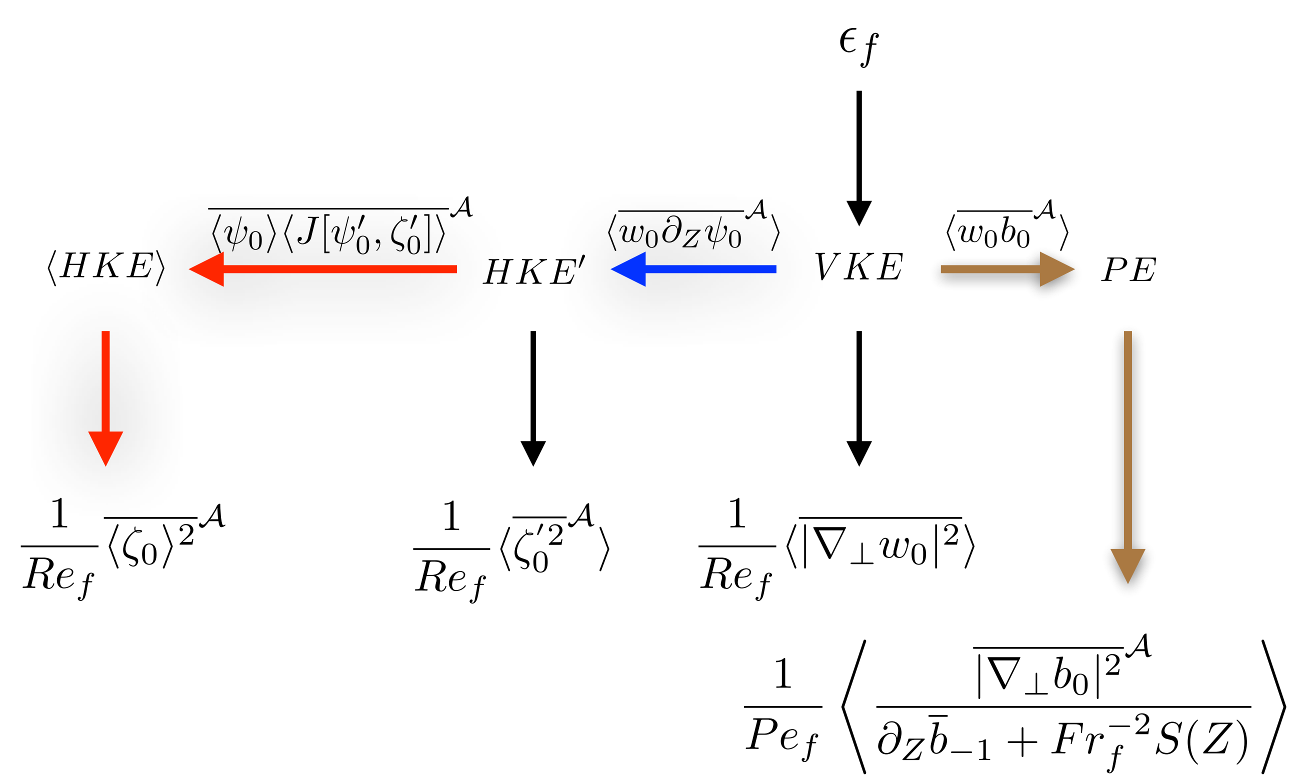


vertical buoyancy flux

$$-\langle\overline{w_0 b_0}^{\mathcal{A}}\rangle$$



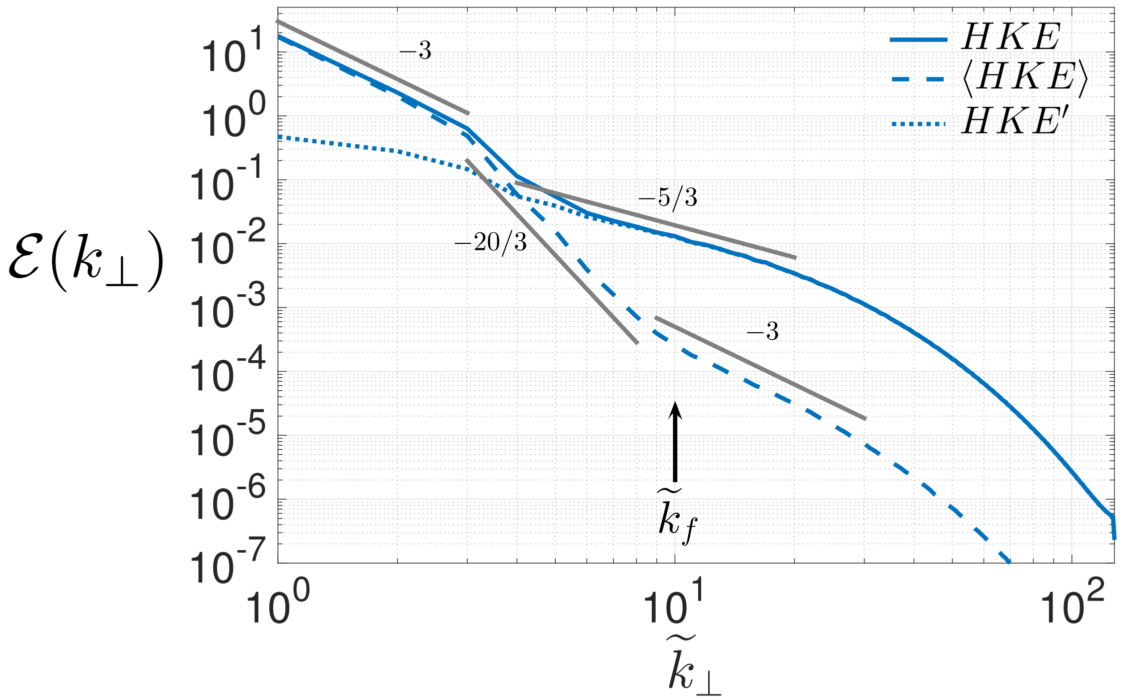
energy conversions



Results: Energy spectra for energy containing scales (inverse cascade)

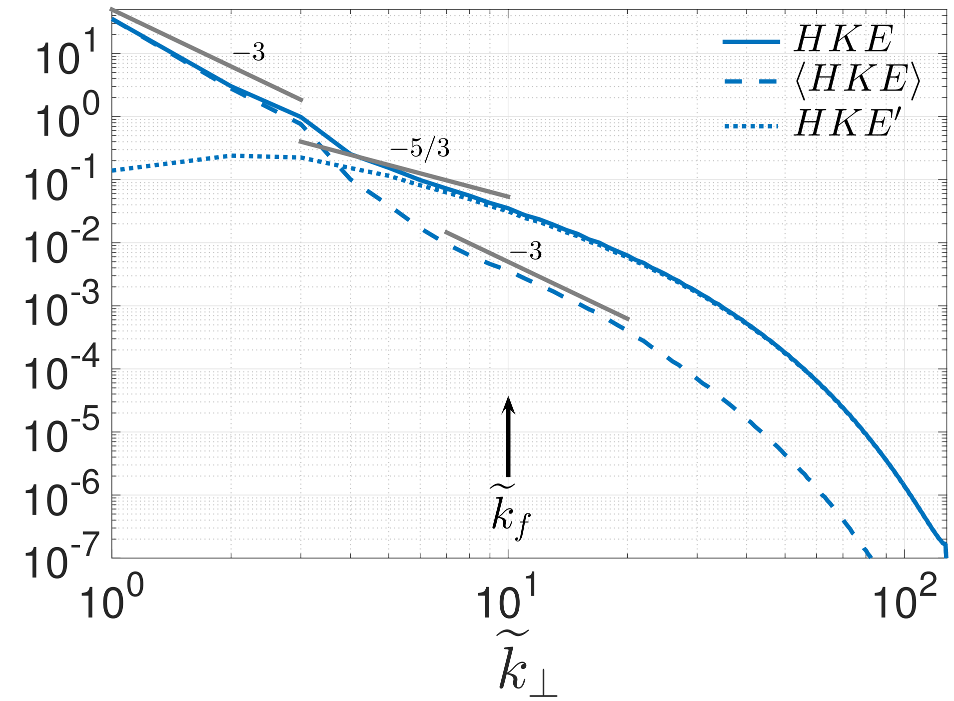
strong stratification

$$(-\psi_0 \cdot \zeta_0), \; Fr_f = Re_f^{-1/2} = 0.0577$$



weak stratification

$$(-\psi_0 \cdot \zeta_0), \; Fr_f = Re_f^{3/4} = 72.08$$



$$Re_f = 300$$

- *baroclinic & barotropic decomposition:*

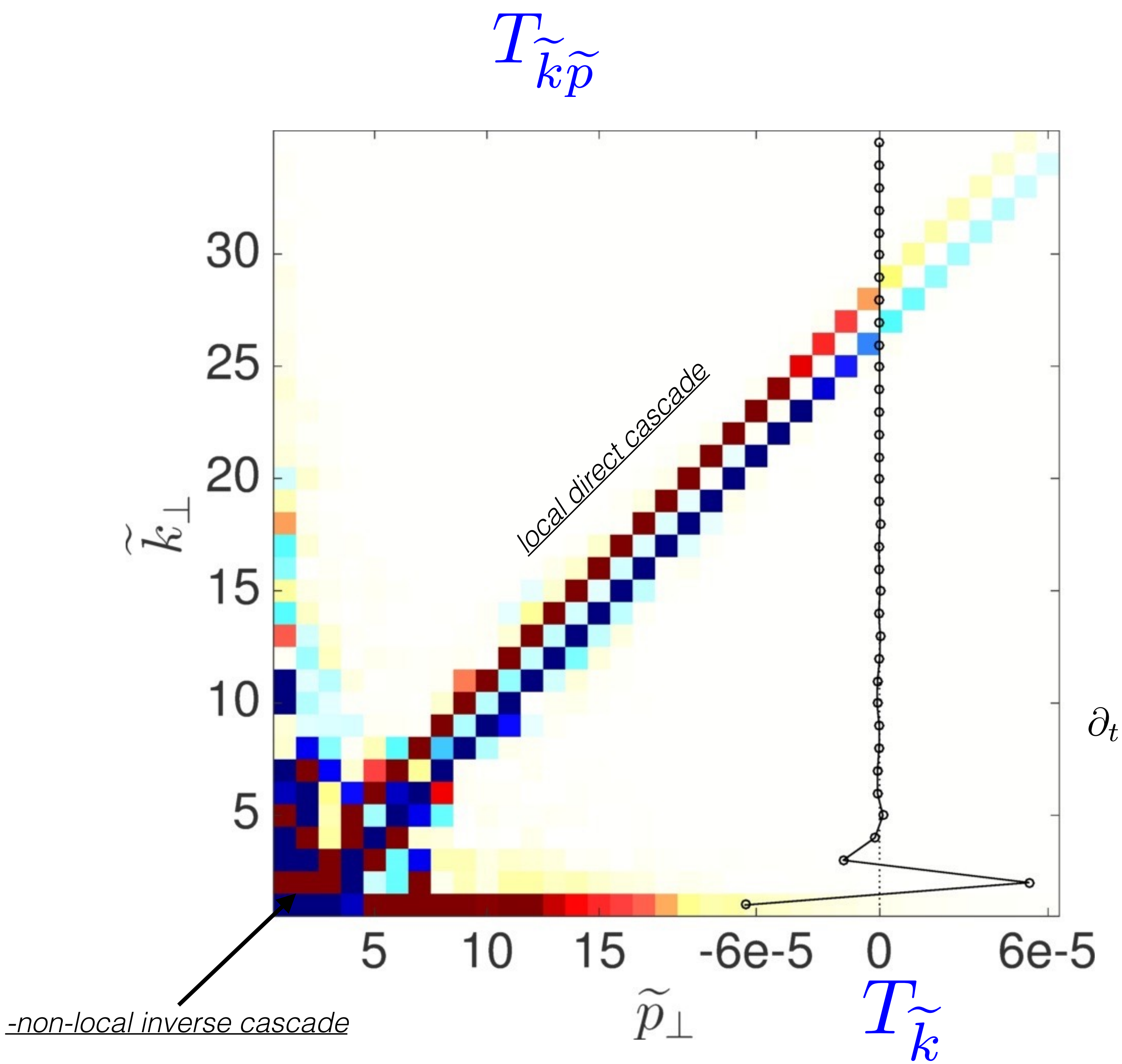
$$\psi = \langle \psi \rangle + \psi'$$

$$\mathbf{u}_{bt} = \nabla^\perp \langle \psi \rangle \qquad (w_{bt}, b_{bt}) = 0$$

$$\mathbf{u}_{bc} = \nabla^\perp \psi' \qquad (w_{bc}, b_{bc}) = (w, b)$$

- *Steep transition in strong stratification regime result of reduced VKE-HKE conversion*
- *-3 regime at high wavenumber indicative of forward enstrophy cascade*
- *-5/3 regime observed in baroclinic HKE - indicative of forward energy cascade*

Results: Nonlocal (barotropic) inverse cascade



$$\partial_t \langle \zeta_0 \rangle + J[\langle \psi_0 \rangle, \langle \zeta_0 \rangle] = -\langle J[\psi_0', \zeta_0'] \rangle + \frac{1}{Re_f} \nabla_\perp^2 \langle \zeta_0 \rangle,$$

barotropic triad interactions

$$\partial_t \langle HKE \rangle_{\boldsymbol{k}} = \sum_{\boldsymbol{k}=\boldsymbol{p}+\boldsymbol{q}} T_{\boldsymbol{kpq}} + \sum_{\boldsymbol{k}=\boldsymbol{p}+\boldsymbol{q}} F_{\boldsymbol{kpq}} + D_{\boldsymbol{k}}$$

- *Transfer map illustrating energy conversion to mode k from mode p*
- *Local forward cascade co-exist with a nonlocal inverse cascade (through baroclinic interactions)*

Summary

- Exploration of *rapidly rotating* and *weakly* stratified geophysical flows
- Two regimes identified:
    1. layered regime (strong stratification $Fr_f < 1$)
    2. columnar regime (strong stratification $Fr_f \geq 1$)
- Distinct from classical QG (where inertia-gravity waves are absent)
- Quantified regimes by energetics
    - observed energy equilibration for moderate Reynolds
    - diagnosed scales active in energy conversion
- ?? DNS verification

Nieves, Grooms, J. & Weiss (2016) *Journal of Fluid Mechanics, vol. 801*