Counting the entropy of any causal horizon

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To take away

- Causality, causality, causality (relativistic)
- Black hole entropy and DeSitter entropy are examples of "causal horizon entropy" and the Ur-horizon is Rindler.
- The microscopic degrees of freedom in quantum gravity are discrete causal relations
- Kinematically, the entropy of a horizon is the number of nearest neighbour causal relations that cross the horizon (c.f. ``entropy of a gas is the number of molecules").

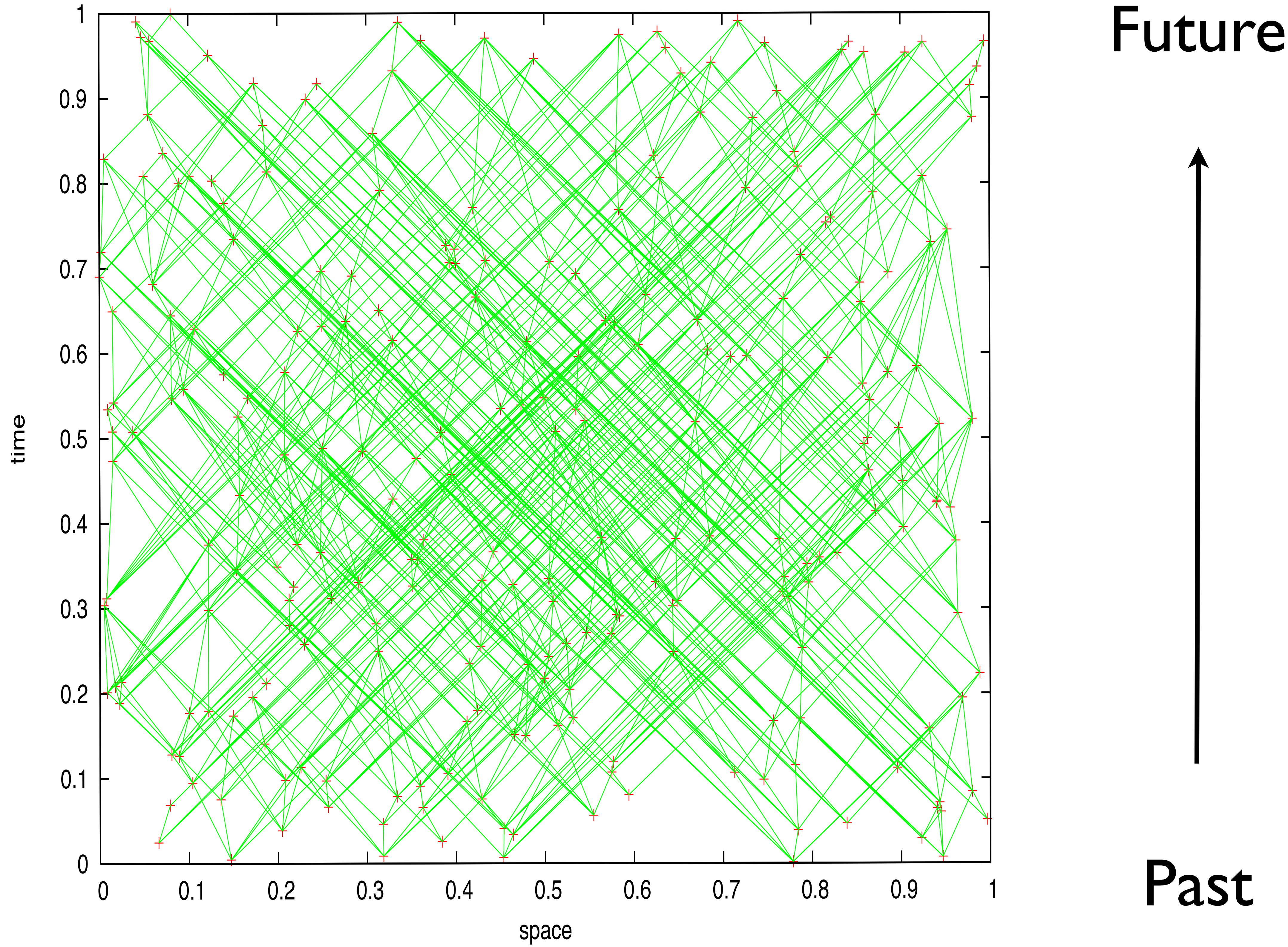
There is prima facie evidence that:

- Black hole thermodynamics is due to the **causal** nature of the horizon. In particular the Generalised Second Law: Hawking's area theorem; violations of GSL without relativistic causal structure **(Eling, Foster, Jacobson & Wall)**; violation of GSL for apparent or dynamical horizons **(Wall).**
- **All causal horizons** obey the laws of thermodynamics where a causal horizon is the boundary of the causal past of a future inextendible timelike curve. e.g. Black hole horizon, DeSitter horizon, Rindler horizon. We should think fundamentally about "horizon entropy" not black hole entropy **(Jacobson & Parentani).** Consequently, the horizon entropy does not count "internal states" but is associated to the horizon itself **(Sorkin).**
- Without a **physical cutoff** the entropy of a black hole would be infinite, and the finite physical value of black hole entropy tells us the cutoff scale is the Planck scale **(Sorkin)**

Seeking the statistical mechanics of horizon thermodynamics

- Spacetime has no structure on subPlanckian length scales and causal order is meaningful => go **discrete** but **hold fast to** relativistic causal order (no lattices allowed, no "order one Lorentz violation in the UV")
- In fact, the causal order encodes the geometry of a (continuum, strongly causal) Lorentzian spacetime up to a conformal factor. The spacetime causal order **unifies** within itself Topology, Differentiable Structure **(Hawking)** and 9/10 of the Metric.
- Adding the spacetime volume measure provides the missing metric information: Order + Volume = Geometry
- In a discrete manifold, you get that information for free because you can count elements **(Riemann)**: Order + Number = Geometry **(Sorkin)**
- So a Lorentzian spacetime can be encoded in a **discrete causal order** or **causal set** **('tHooft; Myrheim; Bombelli, Lee, Meyer & Sorkin).**

For Example



This is what Minkowski spacetime is (like)

Causal sets

- Causal sets are discrete and Lorentz invariant. Proposal: discrete causal relations are the microscopic degrees of freedom in quantum gravity.

- For every (causal) Lorentzian spacetime, (M, g) there are many causal sets to which M is a continuum approximation, gotten (mathematically, not physically) by a random Poisson process of “sprinkling” into M at density $\rho = l^{-d}$ .where l is a fundamental length of order the Planck length.

- Causal set quantum gravity in one line:

$$Z(N) = \sum_{\mathcal{C}} e^{iS(\mathcal{C})}$$

“The primal struggle between Action and Entropy”

- Not Loop Quantum Gravity (that has no causal order). Not Spinfoams (they and other simplicial complex based approaches are “latticelike”). Not Causal Dynamical Triangulations (that’s defined in the continuum limit. Causal sets keep the discreteness scale finite).

Kinematics of horizons

- A continuum spacetime with a causal horizon is an approximation to a causal set which is the underlying microstructure ("things are made of atoms")
- In a causal set, C, the analogue of a timelike curve is a chain
- A causal horizon can be identified with a **partition** of C into the past of a future inextendible chain and the complement of this past set.
- We'll assume that the hypersurface Sigma, which intersects the horizon and on which we want to evaluate the entropy is similarly identifiable with a partition of C into the past and future of Sigma.
- To investigate e.g. a Rindler horizon, consider all causal sets that Minkowski space approximates: all the sprinklings. Everything I will say will be about **averages** over sprinklings (fluctuations very important but ignored here).

What does horizon entropy count?

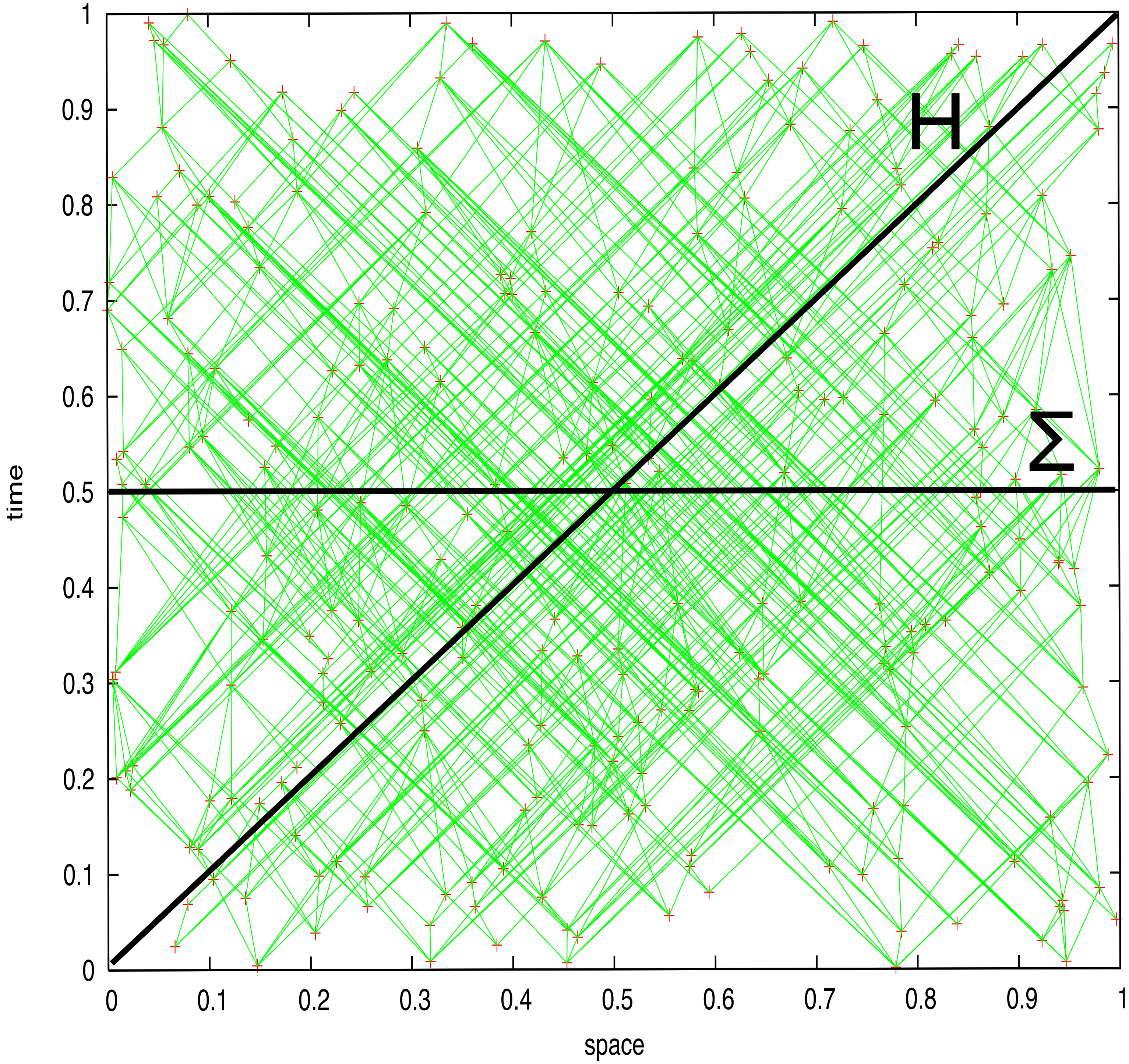
(**Kinematics only**: "the entropy of a gas is the number of molecules")

- Dou and Sorkin: A spacetime with a horizon is actually a causal set and the horizon entropy counts **links** straddling the horizon. So count the links across H in the vicinity of a hypersurface Sigma and see if we get, in the limit of large area, for **all** causal horizons:

$$\langle \text{Number of links across horizon} \rangle = c \, \frac{A}{l^2}$$

c should be of order one and would tell us the Planck length in terms of the fundamental length (up to dynamical factor of order one)

Nonlocality => infinitely many links



- Dou & Sorkin suggest one scheme to localise the counting
- There's a new proposal, based on the causal set action

Causal set actions **(Dionigi Benincasa & FD)**

$$\frac{\mathcal{S}^{(2)}(\mathcal{C})}{\hbar} := \zeta^{(2)}\left(\frac{1}{2}N - N_1 + 2N_2 - N_3\right)$$

$$\frac{\mathcal{S}^{(4)}(\mathcal{C})}{\hbar} := \zeta^{(4)}\left(N - N_1 + 9N_2 - 16N_3 + 8N_4\right)$$

$$\zeta^{(4)} = \frac{2}{\sqrt{6}}\left(\frac{l}{l_p}\right)^2$$

- There are causal set actions in all other dimensions **(FD & Lisa Glaser)**
- We have reason to believe that the average over sprinklings into (M,g) is related to the continuum action: e.g. in continuum limit

$$\left\langle \frac{\mathcal{S}^{(4)}}{\hbar} \right\rangle \to \frac{1}{2l_p^2}\int_M d^4x\sqrt{-g}\,R(x) + \text{boundary terms}$$

Nonlocality

Action is **bilocal** (a blast from the past for me!)

Let $\mathcal{C} = X \cup Y$ be a partition of $\mathcal{C}$. Then

$$\mathcal{S}(\mathcal{C}) \neq \mathcal{S}(X) + \mathcal{S}(Y)$$

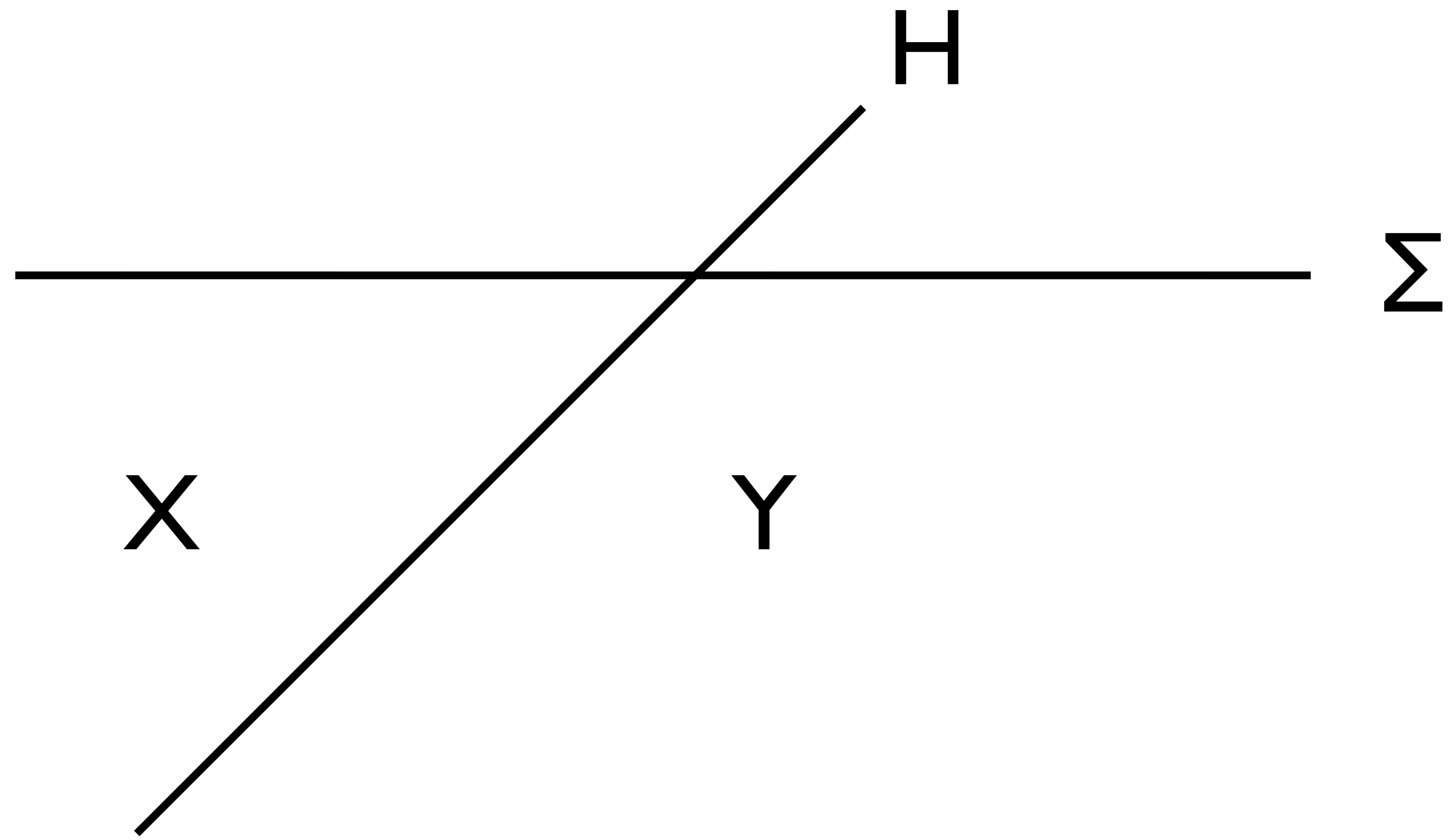
Define $$\mathcal{I}(X,Y) := \frac{1}{\hbar\zeta}\Big(\mathcal{S}(X) + \mathcal{S}(Y) - \mathcal{S}(X \cup Y)\Big)$$

Comparing with the mutual information

$$I(A,B) = H(A) + H(B) - H(A \cup B)$$

we call $\mathcal{I}(X,Y)$ the "Spacetime Mutual Information" (SMI)

The conjecture



The spacetime mutual information of (X,Y) is equal to the entropy of the horizon on Σ

$$\mathcal{I}^{(2)}(X,Y) = N_1(X,Y) - 2N_2(X,Y) + N_3(X,Y)$$

$$\mathcal{I}^{(4)}(X,Y) = N_1(X,Y) - 9N_2(X,Y) + 16N_3(X,Y) - 8N_4(X,Y)$$

Normalisation chosen for "link counting"

Does it "work"?

Does it give, on average, for all causal horizons and in the limit of large horizon area:

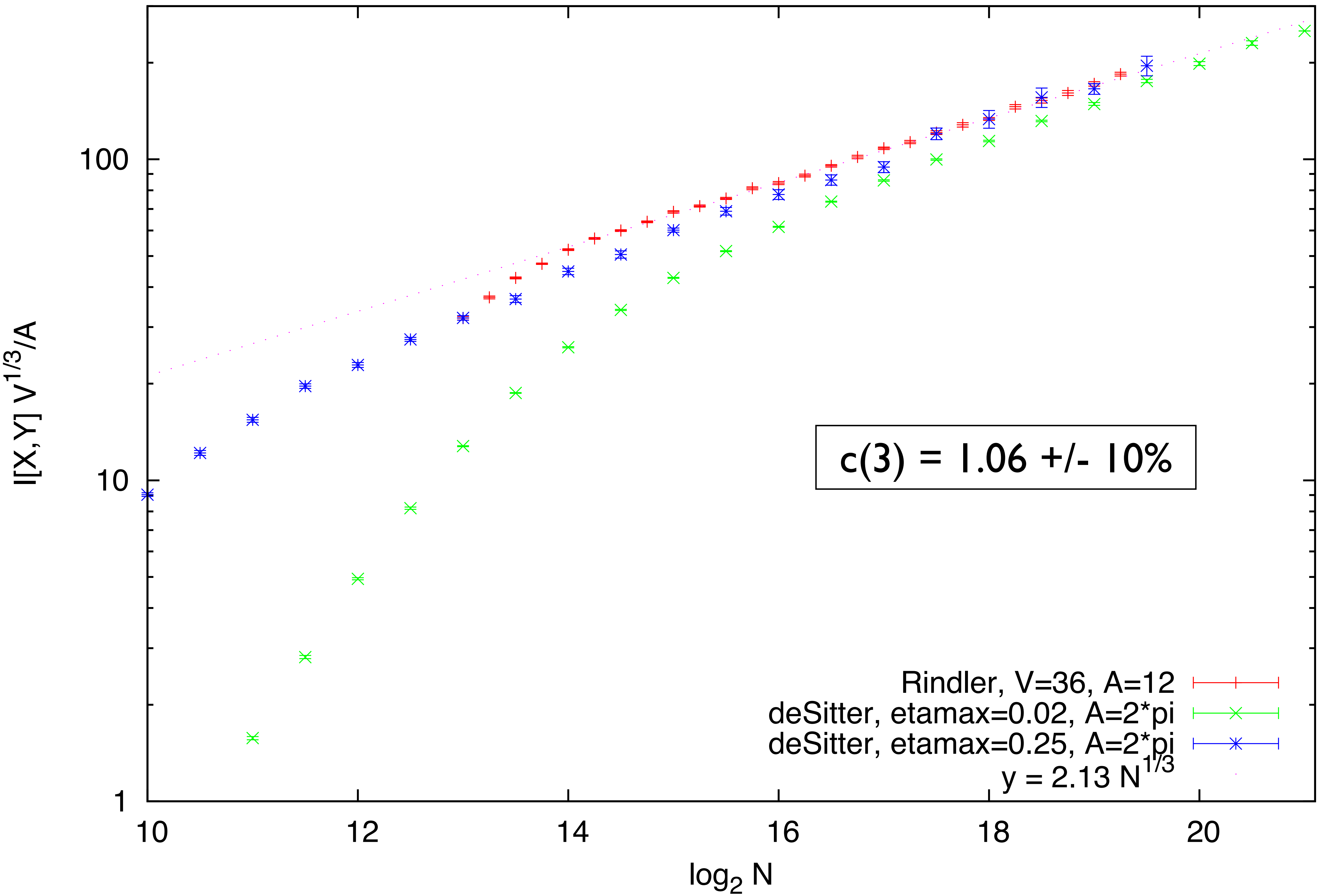
$$c_{(d)} \frac{A}{l^{d-2}}$$

with $c_{(d)}$ of order one.

Does it **fail** to give this for surfaces that are not causal horizons?

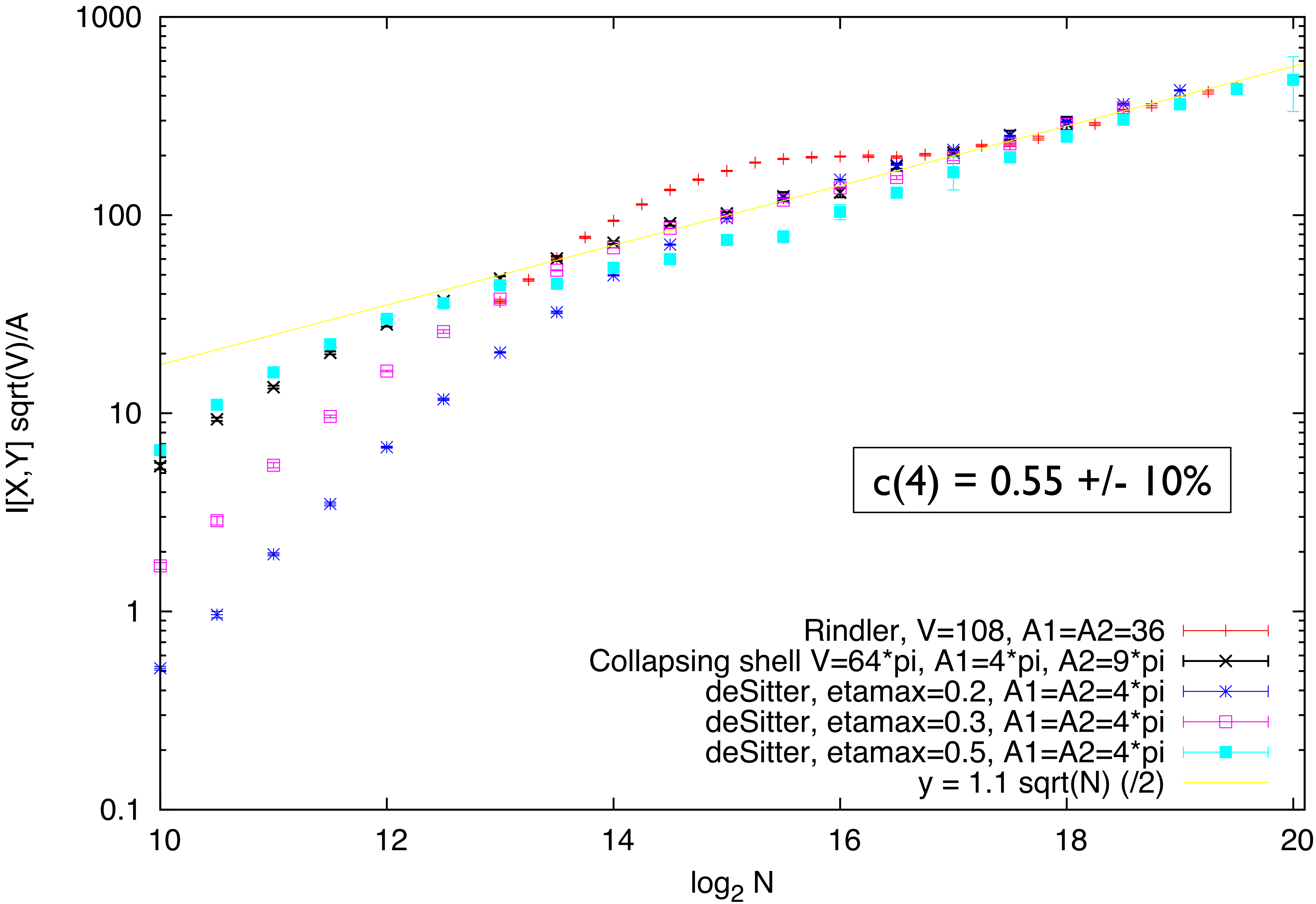
3d Results

SMI of Rindler and dS horizons



4d Results

SMI of Rindler, Collapsing shell, and dS horizons



More Questions than Answers

- Why is it "working"? It must be effectively localising at the horizon even though the definition is global. Counting the links, with cancellations?
- When the boundary surface between X and Y is timelike, the SMI grows like the co-dimension one volume of the surface. When the boundary surface is spacelike we have evidence we get the wrong right answer again. A failure?
- A null surface that is not a horizon is yet to be studied. We'd like it to fail but the above suggests it would work for this too. What if it also fails to fail?
- We need to show there is an actual entropy that is equal to this quantity. Hints: mutual information, looks like "information flow" across H.
- Is there a "counting of quantum states" or an "entanglement entropy" that is numerically equal to the SMI? Most importantly, we would have to prove a GSL from the microscopic, quantum theory. (This has to be done in any approach -- something that is mostly ignored).
- Trying to prove a GSL might lead us to a spacetime ("histories") quantum mechanical understanding of entropy **(Gell-Mann & Hartle, Sorkin)**.

Final Comment

Jacobson and Parentani say, "One of the key questions is whether the **local** notion of horizon entropy density is a valid concept, or whether something essentially **global** is involved." The suggestion here is that horizon entropy could be global in the sense of pertaining to a causally defined dichotomy of spacetime and yet still be effectively localised.

The answer to Jacobson and Parentani's key question would then be, "**Yes and yes**."