New entropy formula – old opinions – some questions

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Entanglement entropy as the entropy of a spacetime region

One can attach an entropy to any spacetime region R

Formula works for **free gaussian scalar** field

Works in both **continuum** and **causal set**

• Will let us compute entanglement entropy in causal set

• Hopefully will simplify calculation of entropy of **non-equilibrium** black hole

• Perhaps can also simplify CFT calc

(Causet affords Lorentz-respecting cutoff ℓ; how will S depend on ℓ ?)

Define the QFT via operators $\phi(x)$ and expectation $\langle \cdot \rangle$

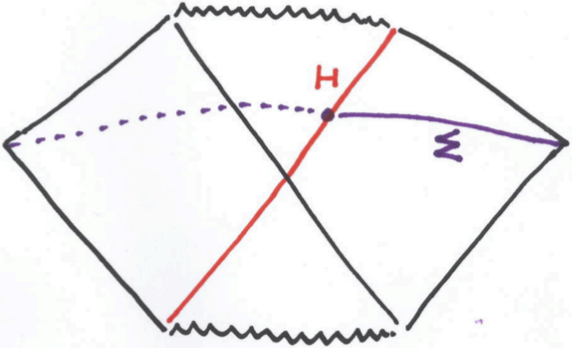
Assume Wick's rule with $\langle \phi \rangle = 0$:

$$\langle \phi\phi \cdots \phi \rangle = \sum \langle \phi\phi \rangle \cdots \langle \phi\phi \rangle$$

To each region $R \longleftrightarrow \mathfrak{A}(R)$ and ω = restriction of $\langle\rangle$ to $\mathfrak{A}(R)$

Will derive S algebraically from $\mathfrak{A}, \omega$

Now consider a black hole spacetime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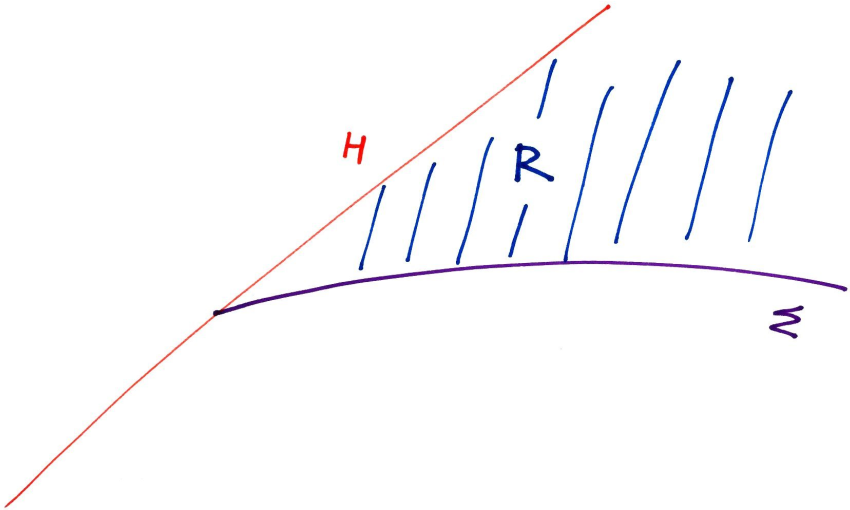
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ϕ, $\dot{\phi}$ on Σ generate $\mathfrak{A}(\Sigma)$

ϕ in R generate $\mathfrak{A}(R)$

Clearly $\mathfrak{A}(R) \supseteq \mathfrak{A}(\Sigma)$

Also $\mathfrak{A}(R) \subseteq \mathfrak{A}(\Sigma)$ since $(\Box - m^2)\phi = 0$

Hence $S(R) = S(\Sigma) = S(\mathfrak{A}, \omega)$

To compute $S(\mathfrak{A}, \omega)$ we represent $\mathfrak{A}$ **irreducibly** in $\mathfrak{H}$ (if possible) and find $\rho \in L(\mathfrak{H})$ such that $\omega(A) = \mathrm{Tr}\,\rho A$

Then $S = \mathrm{Tr}\,\rho \log \rho^{-1} = \langle \log \rho^{-1} \rangle$

Can then compute S from $W(x, y) = \langle \phi(x)\phi(y) \rangle$

Let $R = \mathrm{Re}\,W$, $\Delta = 2\,\mathrm{Imag}\,W$.

If $\ker \Delta \neq \ker R$ then $S = \infty$

Otherwise can work in $\mathrm{im}\,\Delta = \mathrm{im}\,R$

Then the eigenvalues of $R\Delta^{-1}$ come in pairs $\pm i\sigma \quad (\sigma \geq 1/2)$

$$S = \sum (\sigma + \frac{1}{2}) \log(\sigma + \frac{1}{2}) - (\sigma - \frac{1}{2}) \log(\sigma - \frac{1}{2})$$

(Can also write result as $S = \mathrm{Tr}\,L \log |L|$ where $L = -iW\Delta^{-1}$)

Two (differing) opinions and two (neglected) questions

1. Quantum gravity is not unitary **anyway** so why should black hole evaporation be?

2. The entropy “resides” on the horizon, which acts like a dissipative membrane.
It is not inside. (cf. area law, Oppenheimer-Snyder)

3. Why does S increase? (counting is only **half** the story)

and need to consider black hole **out of equilibrium**

4. Has the horizon a fractal structure, and does F-D theorem apply to it? (eg teleology)

In light of 3, non-unitarity is welcome: it can help to prove GSL, making black hole thermodynamics more satisfactory than ordinary thermodynamics
(with nonunitary evolution, $\mathrm{Tr}\,\rho\log\rho$ genuinely can change.)