A CFT dual of General Black Holes

A long-term program

Finn Larsen

Michigan Center for Theoretical Physics

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Counting Black Holes in D=4,5.

- ***BPS black holes***: very good quantitative understanding in terms of *chiral* 2D CFTs.

- ***Other Extremal:*** some understanding in terms of *chiral* 2D CFTs.

- ***Near-BPS:*** good qualitative understanding in terms of *non-chiral* 2D CFTs.

- ***Generic:*** clues suggest understanding in terms of *non-chiral* 2D CFTs.

The ***goal of this talk***: review some of these clues.

Setting

- ***Setting*** (in this talk): $D = 4$ black holes in $N = 4$ string theory.

- ***Note:*** no further restriction, so setting includes Schwarzchild, Kerr....; and also BPS black holes.

- ***The proposal:*** a dual description in terms of a 2D SCFT with $(0, 4)$ SUSY.

- ***Some references:***

FL, arXiv: 9702153.

M. Cvetic and FL, arXiv: 9705192, 1106.3341, 1112.4846.

A. Castro, A. Maloney, and A. Strominger, arXiv:1004.0996.

Entropy Phenomenology (Kerr)

- The Entropy of Kerr black holes

$$S = 2\pi \left(G_4 M^2 + \sqrt{G_4^2 M^4 - J^2} \right) \ .$$

- The form of the entropy suggests a ***chiral split***

$$S = S_L + S_R \ .$$

- The two chiral halves of the CFT interact weakly, at least for the purpose of semi-classical counting.

- Model for ***Angular Momentum:*** angular momentum is identified with the R-charge of the (0,4) SCFT.

  Physics: only R-movers in the 2D CFT have the ability to carry J.

Parametric Charges

- The addition of charges in the context of $N = 4$ SUGRA involves the introduction of parametric mass and angular momentum m, a and parametric charges δ_i:

$$
\begin{aligned}
G_4 M &= \frac{1}{4} m \sum_{i=1}^{4} \cosh 2\delta_i \ , \\
G_4 Q_i &= \frac{1}{4} m \sinh \delta_i \ , \ (i = 1, 2, 3, 4) \ , \\
G_4 J &= m a (\Pi_c - \Pi_s) \ .
\end{aligned}
$$

- Abbreviations

$$
\Pi_c \equiv \prod_{i=1}^{4} \cosh \delta_i \ , \quad \Pi_s \equiv \prod_{i=1}^{4} \sinh \delta_i \ .
$$

Chiral Entropy

- The entropy is much more complicated after general charges have been added

$$S_L = \frac{2\pi m^2}{G_4}(\Pi_c + \Pi_s) \ ,$$
$$S_R = 2\pi\sqrt{\frac{m^4}{G_4^2}(\Pi_c - \Pi_s)^2 - J^2} \ .$$

- An ***encouraging feature***: the dependence on angular momentum can still be accounted for by the physical assumption that only right movers are able to carry angular momentum.

Diffeomorphism Anomaly

- Define ***physical temperatures for left and right movers independently*** by the generalized first laws

$$dS_L = \beta_L dM \quad , \quad dS_R = \beta_R dM \ .$$

(It follows that the Hawking temperature is $\beta_H = \beta_L + \beta_R$) .

- Model chiral contributions to black hole entropy as a 1D gas on a circle with radius $\mathcal{R}$:

$$S_{R,L} = \frac{\pi}{6} c T_{R,L} \times 2\pi\mathcal{R} \ .$$

- An ***encouraging feature***: the central charges determined this way are not chiral ($c_R = c_L$).

  So there is ***no diffeomorphism anomaly*** in the semi-classical theory, as expected.

Level Matching

- The model for the semiclassical entropy employes the Cardy form:

$$S = 2\pi\sqrt{kh_L} + 2\pi\sqrt{kh_R}$$

  Notation: c=6k for $(0,4)$ SCFTs.

- An ***encouraging feature***: there is a quantization condition

$$k(h_L - h_R) = J^2 + J_4\ .$$

  that can be interpreted as level matching of the 2D CFT dual to the general black holes.

  Notation: the quartic invariant is the integer

$$J_4 = \vec{q}^2\vec{p}^2 - (\vec{q}\cdot\vec{p})^2$$

- ***Moduli Independence***: the charge vectors $\vec{q}$, $\vec{p}$ depend on moduli but J, J_4 do not. This is a generalized ***attractor mechanism***.

Strategy

- ***Goal***: find further evidence for 2D conformal geometry.
- ***Strategy (for now)***: analyze details of the geometry.
- ***Inspiration***: analysis of BTZ black holes (using AdS_3 geometry), analysis of Kerr black holes using near horizon symmetry....
- ***Strategy (medium term)***: precise description by tensoring of chiral CFT's that succesfully describe extremal black holes, using level matching to constrain 0-modes.

The Black Hole Geometry

The ***explicit geometry*** with all charges turned on:

$$ds_4^2 = -\Delta^{-\frac{1}{2}} G (dt + \mathcal{A}_\phi d\phi)^2 + \Delta^{\frac{1}{2}} \left(\frac{dr^2}{X} + d\theta^2 + \frac{X \sin^2\theta}{G} d\phi^2 \right) ,$$

where

$$\begin{aligned}
X &= r^2 - 2mr + a^2 , \\
G &= r^2 - 2mr + a^2 \cos^2\theta , \\
\mathcal{A} &= \frac{2ma \sin^2\theta}{G} \left[(\Pi_c - \Pi_s) r + 2m\Pi_s \right] d\phi , \\
\Delta_0 &= \prod_{i=1}^{4} (r + 2m \sinh^2\delta_i) + 2a^2 \cos^2\theta [r^2 + mr \sum_i \sinh^2\delta_i \\
&\quad + 4m^2 (\Pi_c - \Pi_s)\Pi_s + 2m^2 \sum_{i<j<k} \sinh^2\delta_i \sinh^2\delta_j \sinh^2\delta_k + \frac{1}{2} a^2] .
\end{aligned}$$

Abbreviation: $\mathcal{A}_{\rm red} = \frac{G}{a \sin^2\theta} \mathcal{A}_\phi$ depends on r alone.

The Laplacian

The entropy formula suggests a dual 2D CFT. But the corresponding features in the scalar wave equation are much more precise.

The general Laplacian:

$$\Delta_0^{-\frac{1}{2}}[\partial_r X \partial_r - \frac{1}{X}(\mathcal{A}_{\rm red}\partial_t - \partial_\phi)^2 + \frac{\mathcal{A}_{\rm red}^2 - \Delta_0}{G}\partial_t^2 + \frac{1}{\sin\theta}\partial_\theta \sin\theta \partial_\theta + \frac{1}{\sin^2\theta}\partial_\phi^2]$$

Recall: Kerr black holes allow separation of variables. The addition of charges (largely) maintains this property:

$$\frac{\mathcal{A}_{\rm red}^2 - \Delta_0}{G} = r^2 + 2mr\left(1 + \sum_{i=1}^{4} s_i^2\right) + 8m^2(\Pi_c - \Pi_s)\Pi_s$$
$$-4m^2 \sum_{i<j<k} s_i^2 s_j^2 s_k^2 + a^2\cos^2\theta \ .$$

Hypergeometric Structure?

- The resulting radial equation is "almost" ***hypergometric***.

- If it were precisely hypergeometric we could find an $SL(2)^2$ symmetry which perhaps would be ***enhanced to conformal symmetry***.

- The ***obstacle*** is that the effective potential

$$V_{\text{eff}} = \frac{\mathcal{A}^2_{\text{red}} - \Delta_0}{G}$$

  rises too quickly far from the black hole: $V_{\text{eff}} \sim r^2$ because $\Delta_0 \sim r^4$ corresponds to asymptotically flat space and $G \sim r^2$.

- ***Interpretation***: we should *enclose the black hole in a box* that regulate asymptotic behavior.

The Subtraction Prescription

- ***Prescription:*** Modify the conformal factor so that $\Delta \sim r$ instead of $\Delta_0 \sim r^4$.

- ***Consequence:*** the scalar wave equation in the subracted geometry is ***exactly*** a hypergeometric equation.

- ***Technical Assumptions:*** determine Δ uniquely by assuming separability is maintained, and also that Δ is analytical in r, θ.

$$\Delta = (2m)^3(\Pi_c^2 - \Pi_s^2)r + (2m)^4\Pi_s^2 - (2m)^2(\Pi_c - \Pi_s)^2 a^2 \cos^2\theta$$

- ***Interpretation:*** Δ encodes the ***environment*** of the black hole, not its internal structure.

General Causal Structure

- Recall the ***full metric***:

$$ds_4^2 = \Delta^{\frac{1}{2}}\left(\frac{dr^2}{X} + \frac{X\sin^2\theta}{G}d\phi^2\right) + \left[\Delta^{\frac{1}{2}}d\theta^2 - \Delta^{-\frac{1}{2}}G(dt + \mathcal{A}_\phi d\phi)^2\right]$$

Assume $X = X(r)$, $G = X - a^2\sin^2\theta$
(and $\mathcal{A}_\phi$, Δ general functions of r, θ).

- The ***ergosphere*** (inside static limit):

$G = 0$.

- The ***event horizon*** (t, ϕ subspace changes signature):

$X = 0$.

General Thermodynamics

- Remain in the ***general context*** where $X, \mathcal{A}_{\rm red}, \Delta$, are not specified.

- The ***thermodynamics*** determined for these black holes:

$$\begin{aligned} \Omega_H &= \frac{a}{(\mathcal{A}_{\rm red})_{\rm hor}} \, , \\ \beta_H &= \frac{4\pi(\mathcal{A}_{\rm red})_{\rm hor}}{(\partial_r X)_{\rm hor}} \, , \\ S &= \frac{\pi(\mathcal{A}_{\rm red})_{\rm hor}}{G_4} \, . \end{aligned}$$

- The ***striking feature***: the conformal factor Δ does not appear in the thermodynamics.

  It also did not appear in the causal structure.

- ***Interpretation:*** Δ is a feature of the embedding into an ambient spacetime.

Supporting Matter

- ***Issue:*** the subtracted geometry (with modified Δ) does not satisfy the equations of motion with the original matter.

- ***Interpretation:*** the sources supporting the modified Δ represents the physical matter that realizes the box enclosing the black hole.

- In situations (such as near extremality) where the offending terms can be approximated away the ”additional” matter is negligible.

- In general the ***matter is sensible***: it satisfies “all” energy conditions.

- ***A simple realization*** of the matter: 5D minimal SUGRA, KK reduced to 4D.

A 5D Lift

- It is illuminating to ***lift the 4D geometry to a 5D geometry***, via an auxiliary coordinate α:

$$\begin{aligned} ds_5^2 &= \Delta(d\alpha + \mathcal{B})^2 + \Delta^{-1/2} ds_4^2 \\ &= -\frac{X}{\rho} dt^2 + \frac{dr^2}{X} + \frac{\rho}{4m^2(\Pi_c - \Pi_s)^2}(d\alpha + \frac{\mathcal{A}_{\rm red}}{\rho} dt)^2 + d\Omega_2'^2 \,. \end{aligned}$$

The linearly shifted radial coordinate:

$$\rho = 8m^3[r(\Pi_c^2 - \Pi_s^2) - \frac{a^2}{2m}(\Pi_c - \Pi_s)^2 + 2m\Pi_s^2] \,.$$

- In the 5D form ***separability is manifest.***

- ***Bonus***: *massive* scalar fields that couple minimally to the 5D geometry are also separable.

Virasoro

- The full 5D space is ***locally*** $\mathrm{AdS}_3 \times S^2$.

- So there is Virasoro^2 symmetry and also an $SU(2)$ R-symmetry.

- Except: the reduction of the 5D space to the physical 4D spacetime introduces ***global issues***.

- Concretely: physical deformations are independent of α so the full Virasoro is not realized on the physical space.

Central Charge

- The ***phenomenological model*** determines the CFT central charge in units of the CFT radius $\mathcal{R}$:

  $$c\mathcal{R} = \frac{24m^3}{G_4}(\Pi_c^2 - \Pi_s^2) \ .$$

  The cubic dependence on mass generalizes the MSW formula far off extremality.

- The ***scale*** $\mathcal{R}$ relates the physical spacetime temperature to the dimensionless CFT temperature:

  $$T_{L,R}^{\rm CFT} = T_{L,R}^{\rm phys}\mathcal{R} \ .$$

- The ***lift to 5D*** leaves $\mathcal{R}$ arbitrary *a priori*.

Summary

Progress towards conformal symmetry of general black holes:

- The ***phenomenology of the black hole entropy formula*** suggests a 2D CFT origin.
- Black hole thermodynamics in a manner that exhibits ***independence of the conformal factor***.
- The ***subtraction procedure*** isolates a part of the geometry that has $SL(2)^2$ symmetry.
- Rudiments of a ***full Virasoro symmetry*** even for very general black holes.