BITS & BRANES, MAY 24, 2012

ON THE EMERGENCE OF SPACE-TIME

ERIK VERLINDE

Emergent Space-time

- Gravity and space-time physics, as described by GR + QFT, are likely to be emergent.
- String theory gives important hints, but itself still needs to be developed into a general framework.

Questions

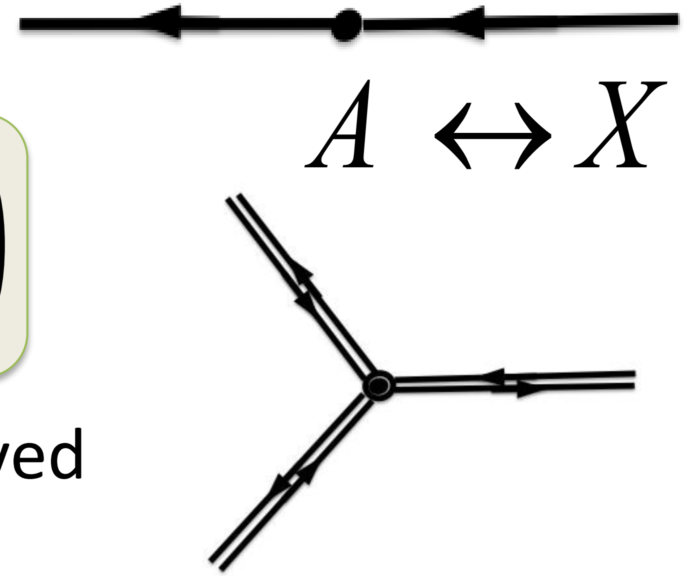
- Is there a universal scenario for the emergence of space-time and gravity?
- What are the basic principles and mechanisms?

Holography & Cosmology

- A proposed scenario should explain the value of the Bekenstein-Hawking entropy.
- And ideally also give a hint towards the origin of the de Sitter entropy?

- What are the implications of the emergence of space-time and gravity in a cosmological setting?
- Does it tell us something about the origin of dark energy and possibly even dark matter?

Open String Field Theory

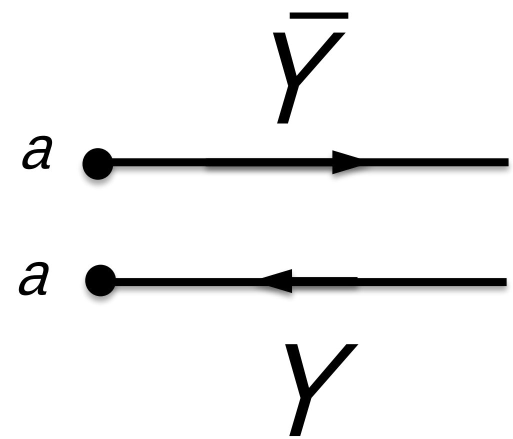


$$k\,\mathrm{Tr}\left(AQ * A + \tfrac{2}{3} A * A * A\right)$$

Can open string field theory be derived from the anomaly of an

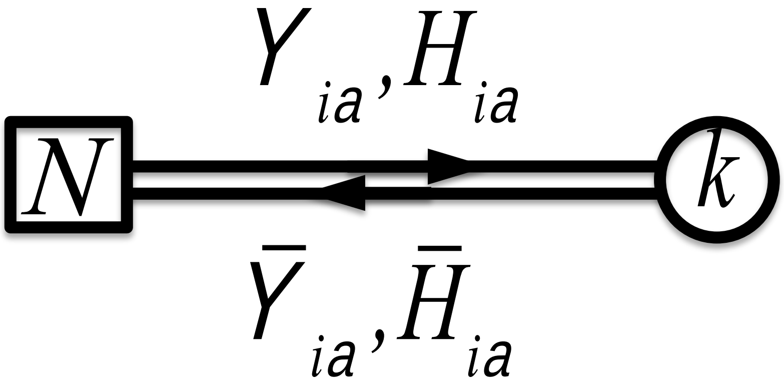
"Half-String" Field Theory

$$\mathrm{Tr}\left(\bar{Y}_a (Q_L + A) \times Y_a\right)$$



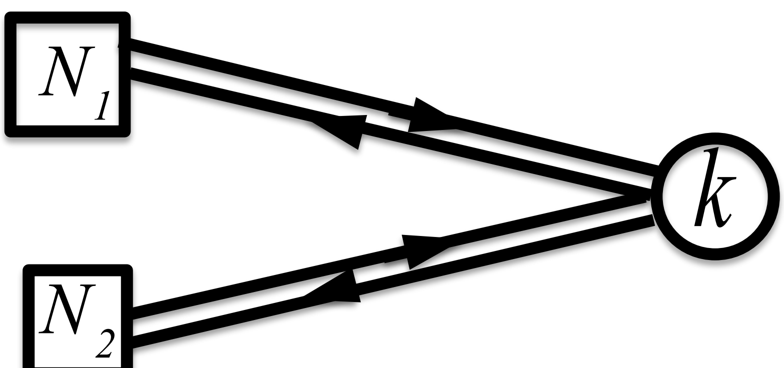
$a = 1,..,k$

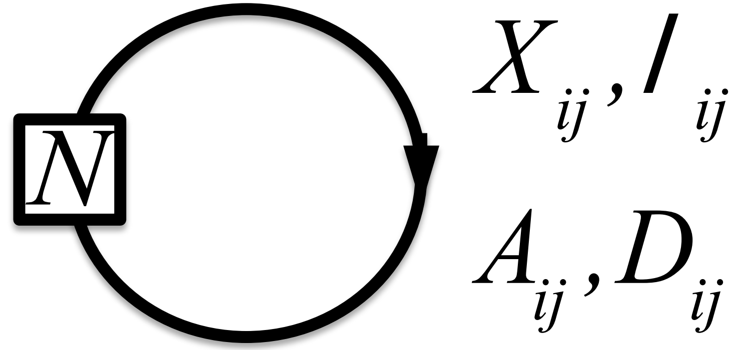
‘ADHM’ Quiver QM = “atoms of space-time”



Bifundamental

Hypermultiplets

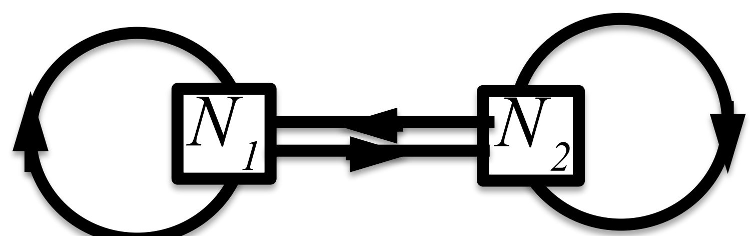




Adjoint

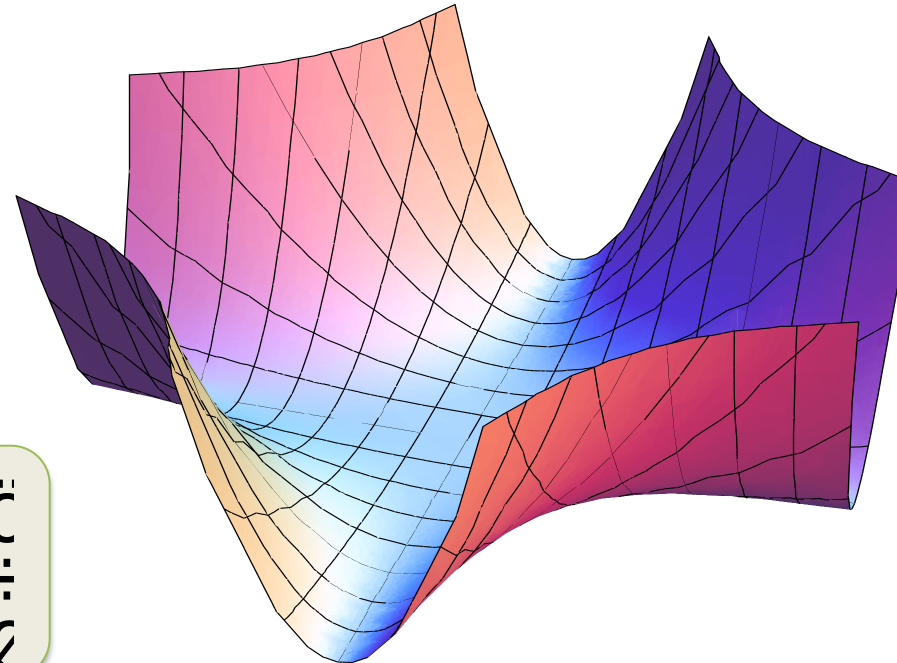
Vectormultiplets

N



$$\left|X_{ij}H^{j}_{a}\right|^{2} + \bar{Y}^{i}_{a} X_{ij} \cdot g\, Y^{j}_{a}$$

$$\mathrm{Tr}\left(\left[X^{I}, X^{J}\right]^{2} + \bar{\lambda}\, \gamma_{I} \left[X^{I}, \lambda\right] \right)$$

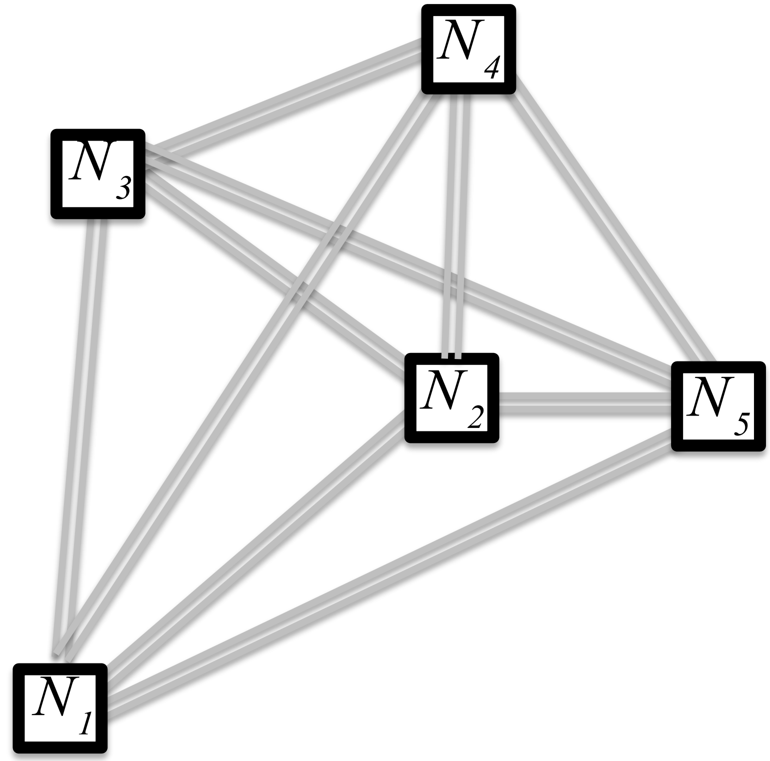


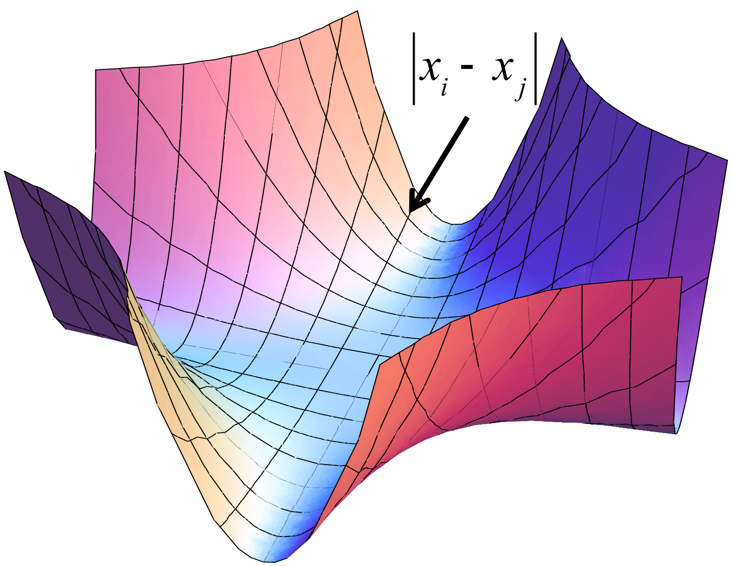
Either *H* or *X* has large but slow variations while the other has fast but small variations. The same holds for the eigenvalues and "off-diagonal" modes of *X*. Space time corresponds to the moduli space of these eigenvalues.

M(atrix) theory

Matter is described as bound states of N_i 'eigenvalues'.

Gravity arises due to integrating out the off-diagonal modes.





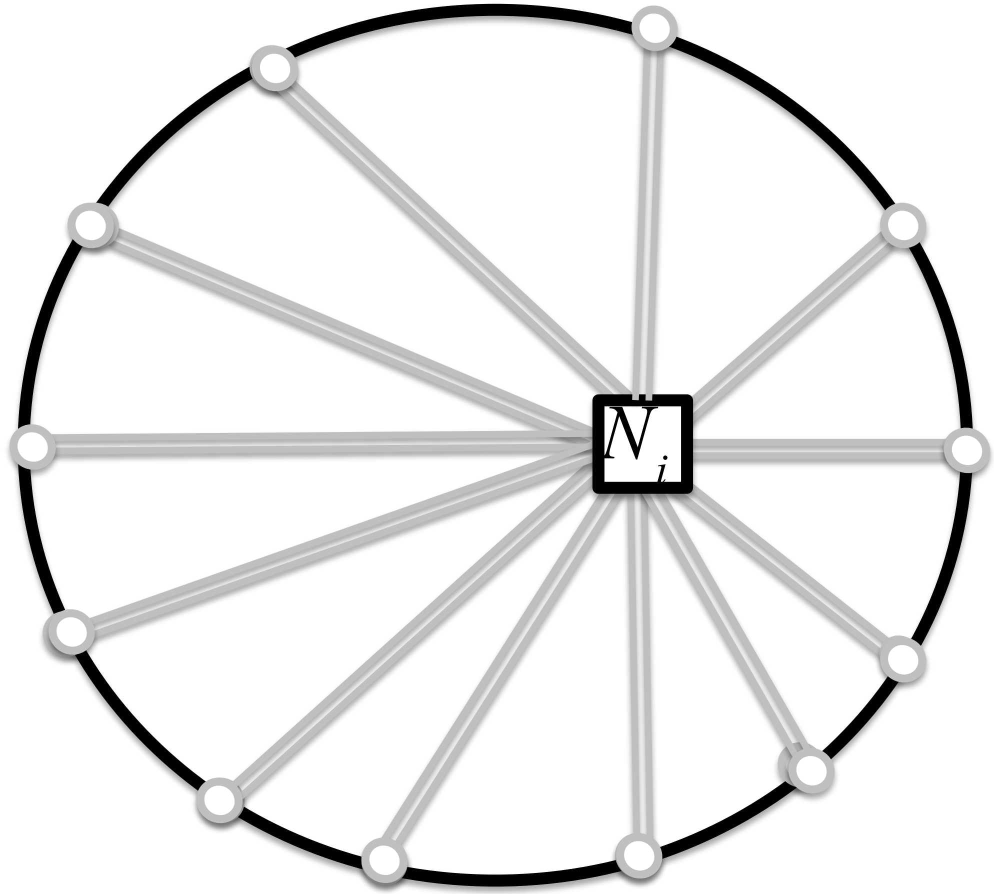
$$\mathrm{Tr}\left(\left[X^I, X^J\right]^2 + \bar{\lambda}\, \gamma_I \left[X^I, \lambda\right] \right)$$

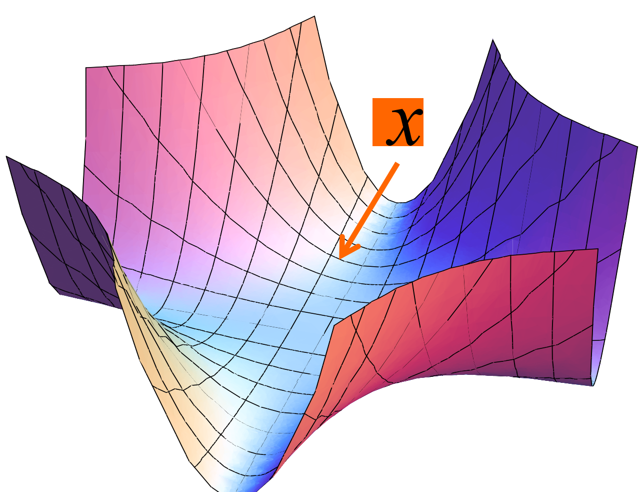
$$\approx \ldots.. + |x_i - x_j|^2 \, |X_{ij}|^2 + |x_i - x_j| \, |\lambda_{ij}|^2 \ldots$$

In the coulomb phase the fermions "condense:

$$\bar{Y}^{i}_{a} Y^{j}_{a} \; 1 \; 0$$

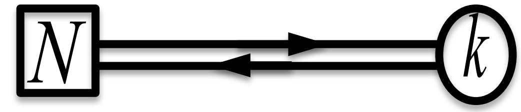
The eigenvalues of *X* get drawn towards the fermi surface.





$$\left| X_{ij} H^{j}_{a} \right|^{2} + \bar{Y}^{i}_{a} X_{ij} \times g \, Y^{j}_{a}$$

Counting of states:



The phase space is characterized by *c=6k* and *N.*

and roughly can be thought of as a symmetric orbifold

$$M_N \approx M^N / S^N \quad \text{with} \quad c(M) = c$$

The dimension of the Hilbert space is typically

$$\dim\left(H_N\right) = c\left(M_N\right)$$

and obeys Cardy's formula

$$\log \dim\left(H_N\right) = 2\pi\sqrt{\frac{c}{6}\left(N - \frac{c}{24}\right)}$$

What values do N and c=6k have?

AdS-Schwarschild Black Holes

$$S = 2\pi\sqrt{\frac{c}{6}\left(N - \frac{c}{24}\right)}$$

$$\frac{c}{12} = \frac{A\ell}{8\pi GR}$$

$$N = M\ell$$

In general we expect $\ell \gg R$

Schwarschild Black Holes

Various authors have derived a Virasoro algebra with

$$\frac{c}{12} = \frac{A}{8\pi G}$$

$$N = MR$$

For black holes these are equal so that

$$S = 2\pi\sqrt{\frac{c}{6}\left(N - \frac{c}{24}\right)}$$

(Carlip, Padmanabhan)

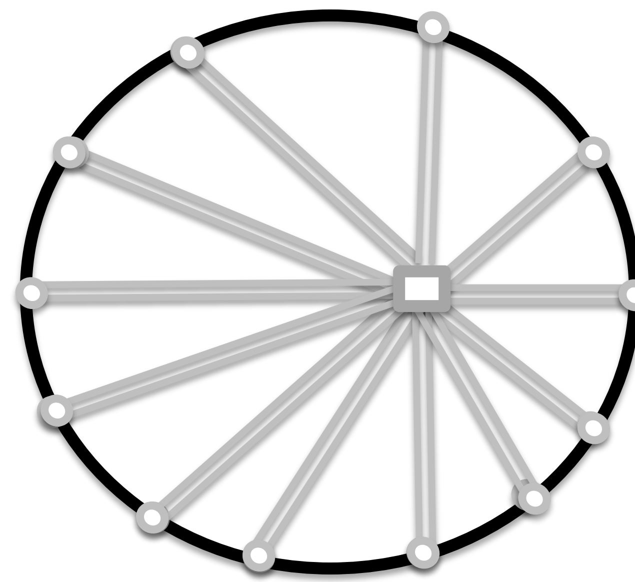
‘de Sitter space’

$$ds^2 = -(1 - H_0^2 R^2)dt^2 + \frac{dR^2}{1 - H_0^2 R^2} + R^2 d\Omega^2$$

Represents a dynamical “quiver system” again with

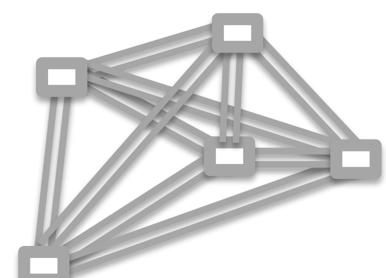
$$\frac{c}{12} = \frac{A_{hor}}{8\pi G}$$

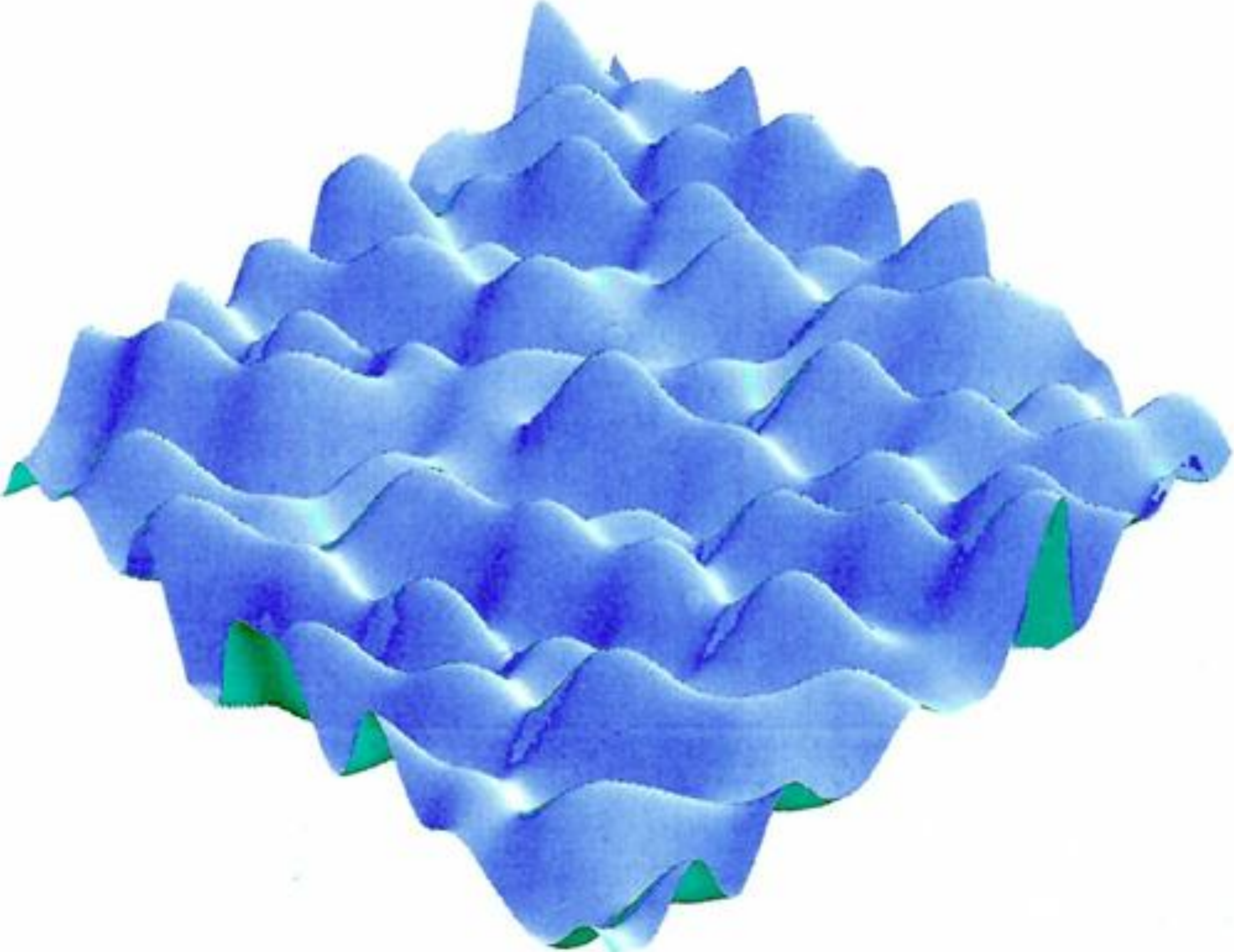
$$N = ER$$

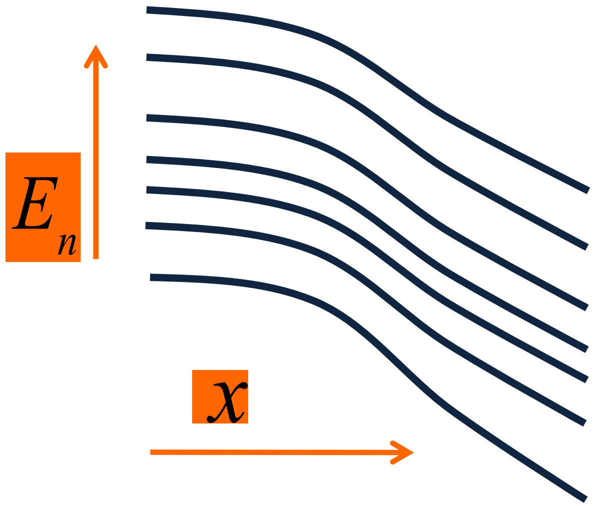


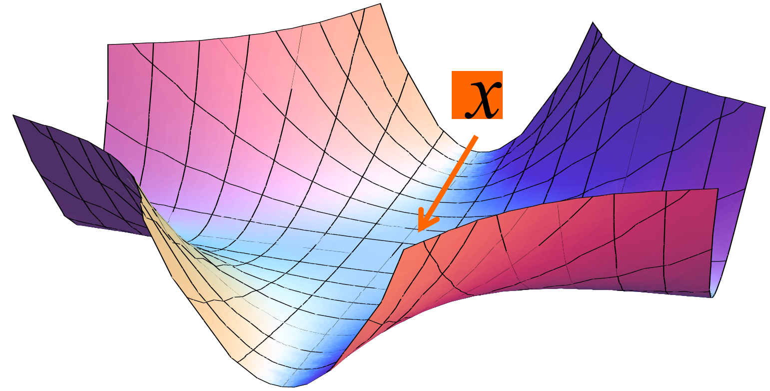
Leading to the correct temperature and entropy (density)

$$T = \frac{H_0}{2\pi} \qquad s \approx \frac{H_0}{G}$$







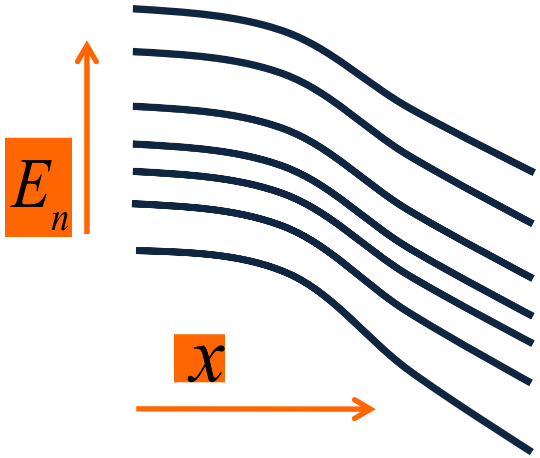


$$H(p_i, q_i; x) \approx \sum_i \left(p_i^2 + \omega_i^2(x) q_i^2 \right) + ..$$

The gravitational (= inertial) force arises as an adiabatic reaction (=Born-Oppenheimer/Casimir) force.

Corrections are given by:
Magnetic forces due to (non-abelian) Berry phases.
Dynamics of “off-diagonal” modes.

$$F = -\left(\frac{\partial E_n}{\partial x} \right)$$



When the separation of time scales between the fast and slow variables is large the phase space volume

$$\Omega(E,x) = \int d^N p \, d^N q \, \Big|_{H(p,q;x) \le E}$$

is preserved

.

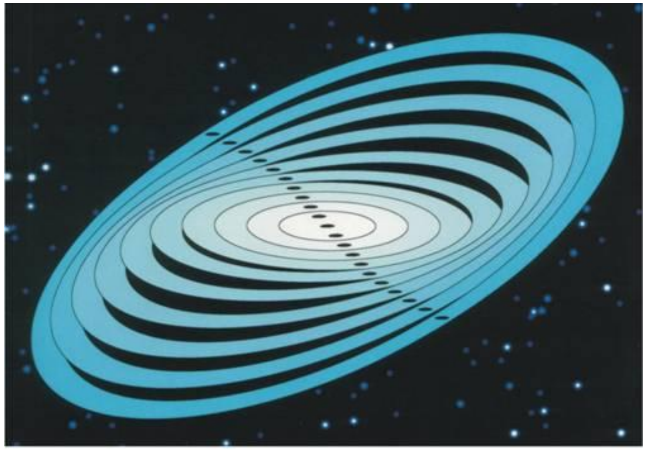
The force can then be written in the form:

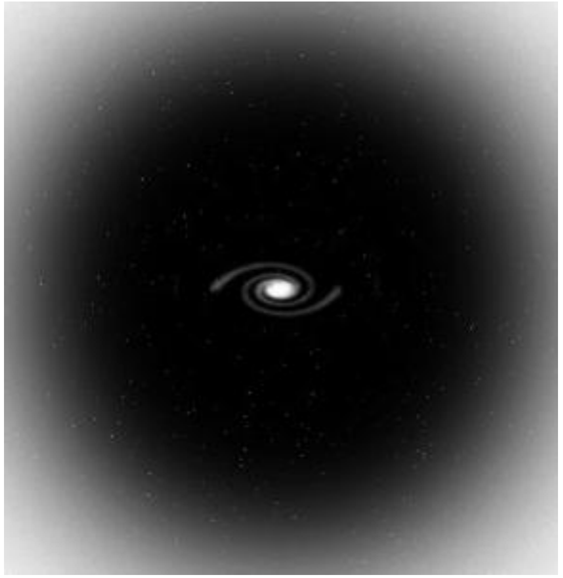
$$F = -\left(\frac{\partial E}{\partial x}\right)_S = T\left(\frac{\partial S}{\partial x}\right)_E$$

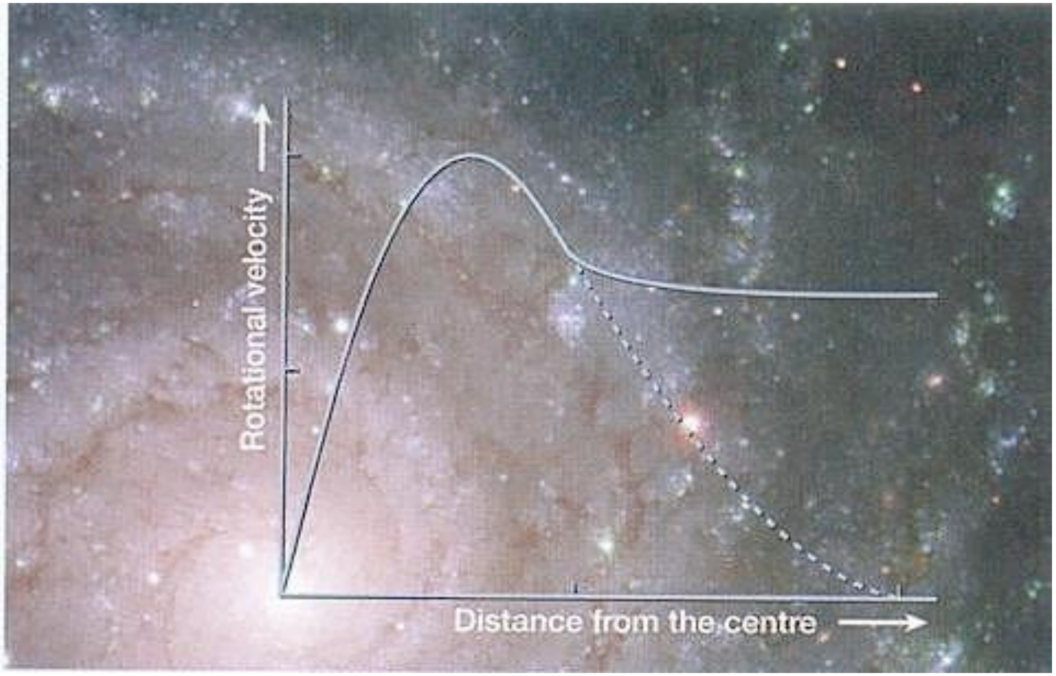
$$S = \log W$$

$$\frac{1}{T} = \frac{\partial_E W}{W}$$

Dark matter In Galaxies

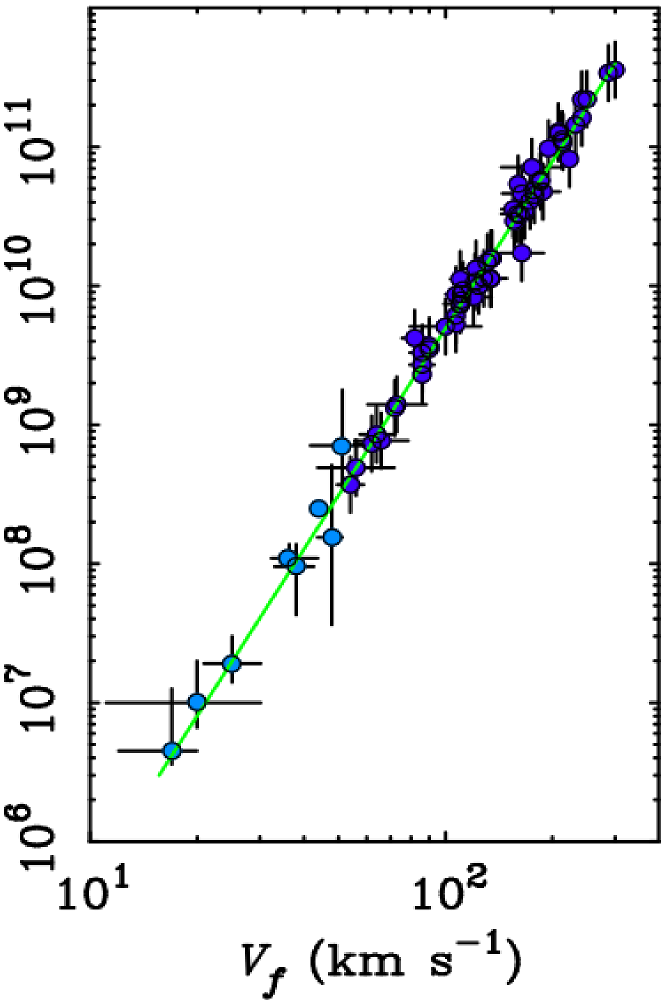


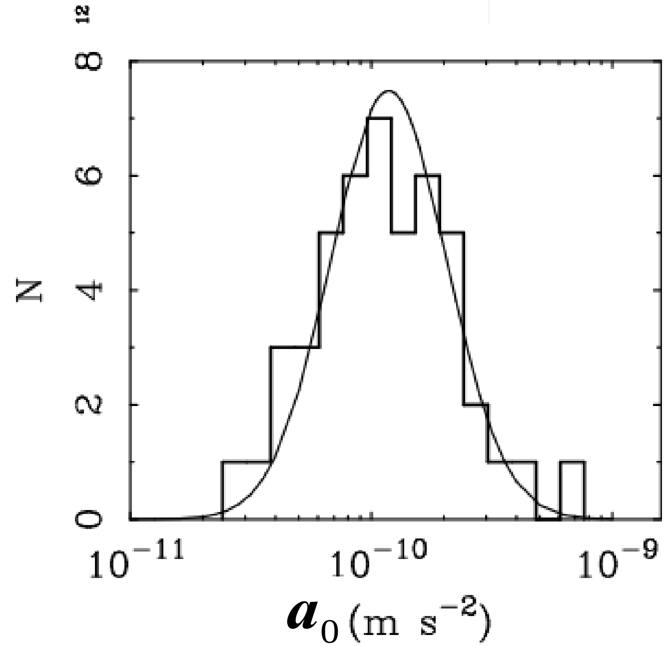




Baryonic Tully-Fisher

$$M_b = M_\star + M_{gas}$$





line:

$$\log M_b = 4 \log V_f + 1.7$$

(McGaugh 2005)

$$a_0 = 1.24 \pm 0.14 \times 10^{-10} \; m/s^2$$

$$V_f^4 » GM_b a_0$$

$$a_0 \approx \frac{cH_0}{2\pi}$$

Suppose we may define a microscopic phase space volume

$$\Omega\big(E;\rho(x)\big)$$

for a given matter density ρ and interpret Newton's potential Φ as the field dual to ρ in a canonical ensemble

$$Z\big(\beta;\Phi\big) = \int [d\rho]\, \Omega\big(E;\rho\big)\, e^{-\beta\left(E-\int \Phi\rho\right)}$$

What would this say about the gravitational force?

The saddle point equations give the relations

$$\Phi = -T\frac{d}{d\rho}\log\,\Omega(E,\rho) \qquad\qquad \frac{1}{T} = \frac{\partial}{\partial E}\log\,\Omega(E,\rho)$$

The gravitational force can thus be written as

$$F = \int r \nabla \mathrm{F} = - T \int \left(r \nabla \frac{d}{dr} \log \mathrm{W}(E, r) \right)$$

which after partial integration and by inserting

$$r(x) = \underset{i}{\text{å}}\, m_i d(x - x_i)$$

gives the following suggestive equation

$$F_i = m_i \nabla F(x_i) = T\, \nabla_i S(E, \{x_i\})$$

where S(E,$\{x_i\}$) equals the microscopic entropy.

=> defining equation for an **adiabatic reaction force**!

Let us go back to the relation

$$\left\langle \nabla F \right\rangle = - T \nabla \frac{d}{dr} \log W(E, r)$$

"linear response"

And use the fact that we know that $\left\langle \nabla^2 F \right\rangle = 4pGr$

To compute the fluctuations

$$\left\langle | \nabla F |^2 \right\rangle = \left(T \nabla \frac{d}{dr} \right)^2 \log W(E, r) = 4pG\, d^{(3)}(0)$$

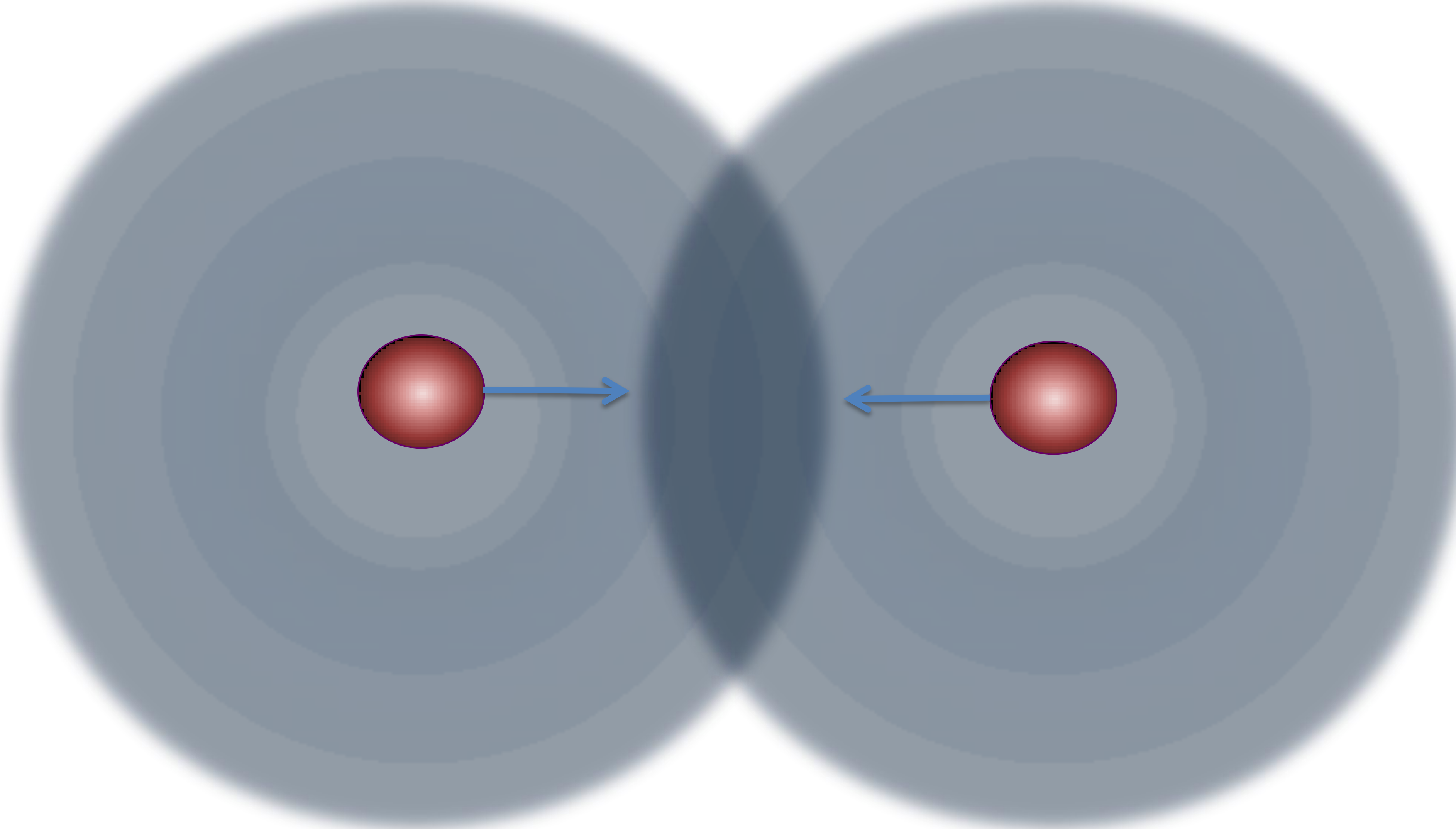
"fluctuation -dissipation"

Or integrated

$$\frac{1}{8pG} \int_V \left\langle | \nabla F |^2 \right\rangle = \frac{1}{2} N k_B T$$

N = number of modes contained in volume V

Note: the size of fluctuations is determined by the UV cut off.



$$E_g = \frac{1}{8\pi G}\int \left|\nabla\Phi\right|^2 - \int \Phi\ \rho$$

Alternative derivation: consider

$$Z[\beta;\rho] = \int [d\Phi]\, e^{-\beta\left(\frac{1}{8\pi G}\int_V |\nabla\Phi|^2 - \int_V \Phi\rho\right)}$$

and compute the one and two point functions. $\langle \nabla\Phi \rangle$ gives the Newtonian acceleration for ρ

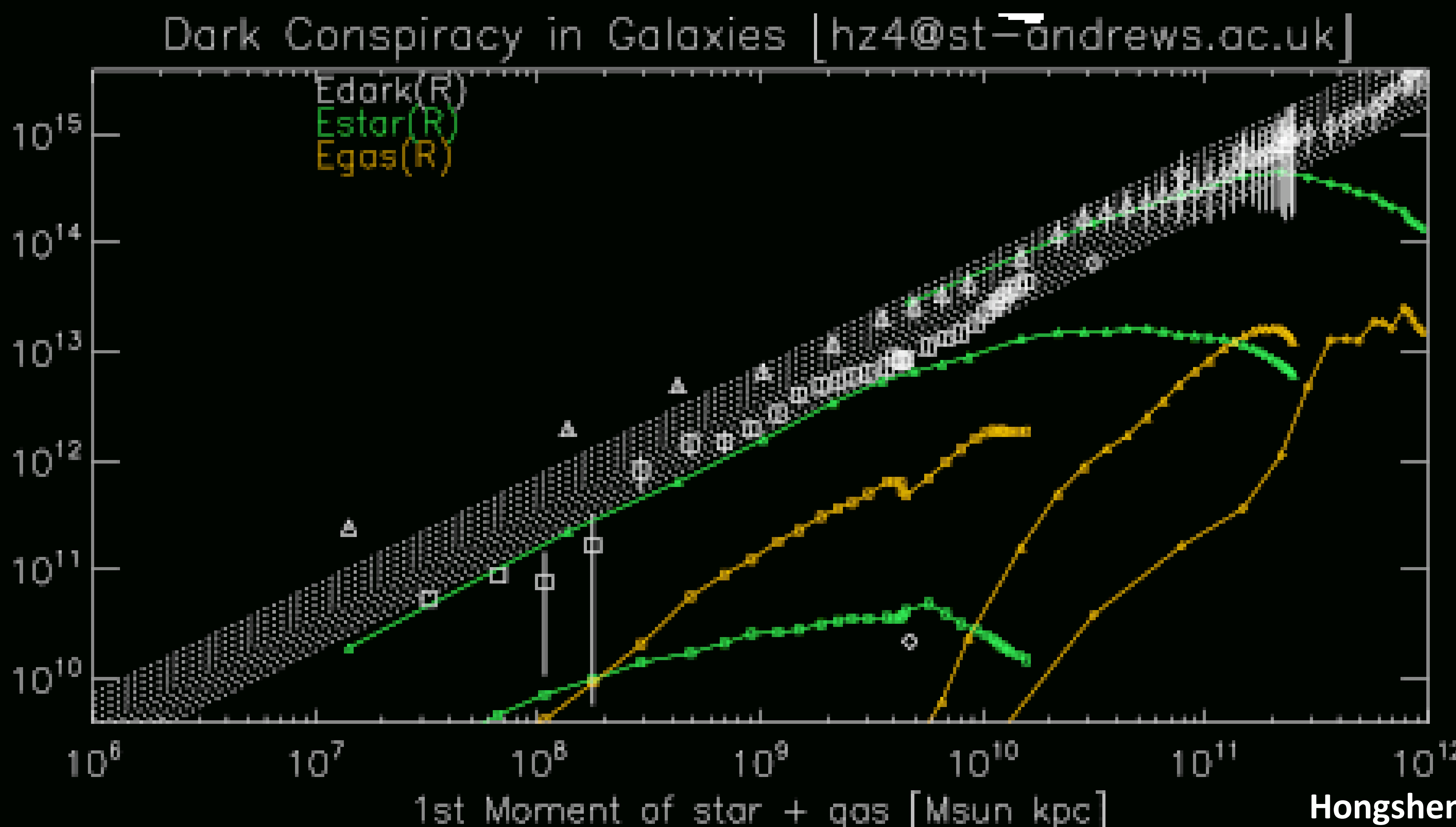
while

$$\frac{1}{8\pi G}\int_V \left\langle |\nabla\Phi|^2 \right\rangle = \frac{1}{2} N k_B T$$

where N is the number of modes of Φ inside the volume V.

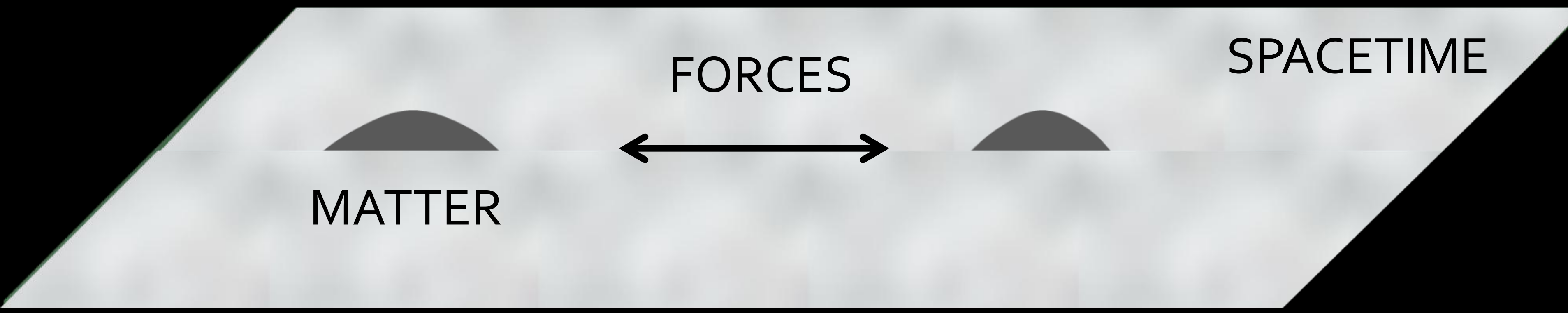
$$E_g(R) = \frac{1}{8\pi G}\int_0^R \left|\nabla \Phi_D\right|^2 = \frac{1}{2} N k_B T$$

$$k_B T = \frac{\hbar H_0}{2\pi} \qquad N = \frac{M_B c R}{\hbar}$$



Hongsheng Zhao

EMERGENT



DARK
MATTER

DARK
ENERGY