A TALE OF TWO OBSERVERS

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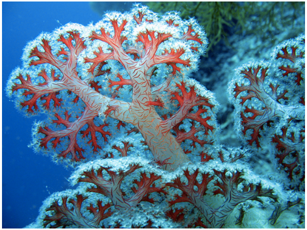
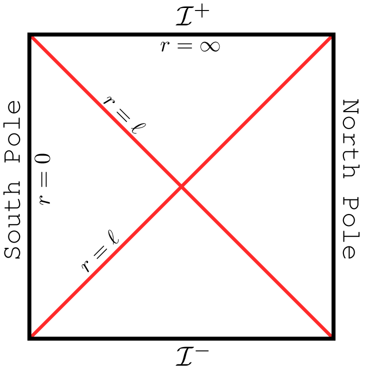


Outline

- Static Patch Curiosities
- Global de Sitter Space
- State Space at $\mathcal{I}^+$/Cosmic Corals
- Outlook

STATIC PATCH CURIOSITIES



The Static patch observer - Cosmic horizons

Observers surrounded by COSMOLOGICAL HORIZON. Geometry is given by:

$$ds^2 = -\left(1 - r^2/\ell^2\right) dt^2 + \left(1 - r^2/\ell^2\right)^{-1} dr^2 + r^2 d\Omega_2^2 \ .$$

How to describe the de Sitter observer remains an open question. Matrix theory [Banks; Susskind;Verlinde]?

Gibbons-Bekenstein Hawking entropy of cosmological horizon? What is it counting [Silverstein,et al.]? What are the microstates?

Basic Confusion: No asymptotic boundaries, no sharp observables.

Consider Green functions.

Boundary conditions are no incoming flux from past horizon and smoothness near $r = 0 \implies$ Sollipsistic, i.e., they capture response of pulses sent by isolated observer:

$$G_R(r, r') = \frac{f^{n}_{\omega lm}(r) g^{in}_{\omega lm}(r')}{W(\omega)} \text{ for } r < r', \quad W(\omega) = f^{n}_{\omega lm} \partial_r g^{in}_{\omega lm} - g^{in}_{\omega lm} \partial_r f^{n}_{\omega lm} .$$

Zeroes of $W(\omega)$ are QNM: $\quad \omega_n = -2\pi i T_{dS} \left(2\Delta_{\pm} + n + 2l\right)$.

More generally one might consider case with non-vanishing flux from past horizon...

Near the worldline

Green functions have hidden $SL(2,\mathbb{R})$ symmetry, which is not a dS_4 isometry.

e.g. for $m^2\ell^2 = 2$ scalar and 4d gravitons near the worldline:

$$G_R^l(t) \sim \theta(t)\left(\frac{1}{\sinh(t/(2\ell))}\right)^{2(l+1)}$$

(Structure of conformal quantum mechanics, related to Poschl-Teller potential)

More general case (i.e. $m^2\ell^2 \neq 2$) it is a product (in frequency space) of two $G_R(\omega)$'s:

$$G_R^l(\omega) = \frac{\Gamma\left(\tilde{\Delta} - i(\ell\omega - ix)/2\right)}{\Gamma\left(1 - \tilde{\Delta} - i(\ell\omega - ix)/2\right)} \frac{\Gamma\left(\tilde{\Delta} - i(\ell\omega + ix)/2\right)}{\Gamma\left(1 - \tilde{\Delta} - i(\ell\omega + ix)/2\right)}$$

where $x = \sqrt{d^2/4 - \ell^2 m^2}$ and $\tilde{\Delta} = l/2 + d/4$.

Related to conformal transformation from $\mathrm{dS}_4 \times S^1$ to $\mathrm{BTZ}\times S^2$. Boundary of BTZ is mapped to worldline.

Black Holes

Cannot build an arbitrarily large black hole. Nariai black holes are maximally large, ($\mathrm{dS}_2 \times S^2$ geometry). Angular momentum is also capped.

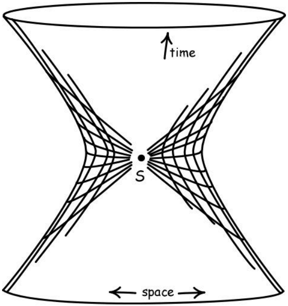
Ratio of Nariai entropy to static patch entropy $= 1/3$. This is universal ratio for Einstein gravity with positive Λ.

Analogy: Most negative mass hyperbolic black holes in AdS_4, ($\mathrm{AdS}_2 \times \mathcal{H}_2$ geometry).

Asymptotic symmetries at $\mathcal{I}^+_{RN}$ of rotating Nariai geometry (S^2 fibered over dS_2) is Virasoro algebra with real positive c_L [D. A.,Hartman;D.A.,Anous]. Note: $\mathcal{I}^+_{RN} \neq \mathcal{I}^+$, in fact it just grazes the future horizon of dS_2.

Is this Virasoro structure connected to static patch $SL(2,\mathbb{R})$?

GLOBAL DE SITTER SPACE: THE METAOBSERVER



More globally, is at late times asymptotically de Sitter has FG expansion:

$$ds^2 = \frac{\ell^2}{\eta^2}\left(-d\eta^2 + g^{(0)}_{ij}dx_i dx_j + \eta^2 g^{(2)}_{ij}dx^i dx^j + \eta^3 g^{(3)}_{ij}dx_i dx_j\right)$$

with $\mathcal{I}^+$ living at $\eta \to 0$. Also, Tr $g^{(3)}_{ij} = \nabla^i g^3_{ij} = 0$.

Though mathematically similar to boundary of AdS, $\mathcal{I}^+$ is also crucially different.

We have to consider deformations of conformal metric on $\mathcal{I}^+$ in addition to deformations of subleading terms in η.

Quantum mechanically, we compute the transition amplitude from initial (quantum) state, i.e. Hartle-Hawking.

CONJECTURE [Strominger,Witten,Maldacena]

Boundary-to-boundary correlators at $\mathcal{I}^+$ with Dirichlet (future) boundary conditions are those of a Euclidean (non-unitary) 3d CFT. Late time (non-normalizable) profiles of fields are sources in the CFT.

Partition function of CFT computes transition amplitude from Bunch-Davies vacuum to a particular late time configuration $\Phi(x)$:

$$Z_{CFT}[\Phi] \propto \Psi_{HH}[\Phi] \approx e^{iS_{cl}[\Phi]} \ .$$

CFT correlators are variational derivatives of $\Psi_{HH}[\Phi]$ w.r.t Φ.

Challenge: Computing CFT partition function for finite, space dependent sources (dissordered systems)?

For 4d Vassiliev gravity with infinite tower of even spin modes we have a proposal [D.A.,Hartman,Strominger]. It is the 3d critical $Sp(N)$ model:

$$\mathcal{L}_{CFT} = \int d^3x\sqrt{g}\left(g^{ij}\partial_i\chi^A\partial_j\chi^B\Omega_{AB} + \frac{1}{8}R[g]\Omega_{AB}\chi^A\chi^B + \lambda(\Omega_{AB}\chi^A\chi^B)^2\right)$$

The χ^A are anti-symmetric scalars which transform as vectors in $Sp(N)$. The above theory flows in the IR to a CFT [LeClair] whose correlators conjecturally reproduce those in higher spin gravity. Three-point functions checked [Giombi,Yin].

We must also impose a $Sp(N)$ SINGLET constraint on the operator content. e.g. Graviton is dual to stress tensor.

Perturbatively this is related to $O(N)$ critical model by $N \to -N$.

Some Lessons

Some lessons from higher spin dS/CFT:

dS/CFT can actually work (at least in this setup).

$N \sim \ell^2/\ell_{Pl}^2$ is an INTEGER and hence the bulk cosmological constant is quantized. Brane constructions?

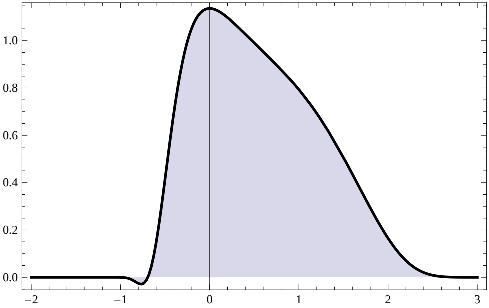
In special cases, we can compute wavefunction (i.e. Z_{CFT}) [D.A.,Denef,Harlow], find interesting structure, possibly (non-perturbative) instabilities even in this simple setup.

For example wavefunction on $S^2 \times S^1$ diverges for small S^1.

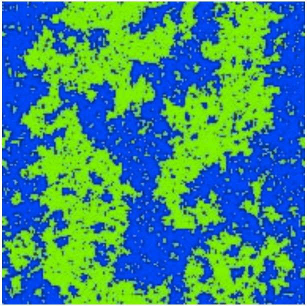
De Sitter is NOT an analytic continuation of AdS beyond perturbation theory.

Example of $Z^{free}_{CFT} = \tilde{\Psi}_{HH}$

On a squashed S^3 with $ds^2 = (d\theta^2 + \cos^2\theta d\phi^2) + e^{-\rho}(d\psi + \sin\theta d\phi)^2$:



COSMIC CORALS



Massless Fields in de Sitter Space

We consider the evolution of a free massless scalar field in a fixed dS_4 background:

$$ds^2 = \frac{\ell^2}{\eta^2}\left(-d\eta^2 + d\vec{x}^2\right) , \quad x^i \sim x^i + L$$

Solution of wave-equation:

$$\phi_{\vec{k}}(\eta) = \frac{1}{\ell(2kL)^{3/2}}\left(A^+_{\vec{k}}(1 - ik\eta)e^{ik\eta} + A^-_{\vec{k}}(1 + ik\eta)e^{-ik\eta}\right) .$$

Quantum fluctuations survive all the way up to $\mathcal{I}^+$ (where $\eta \to 0$) (No cluster decomposition). Large state space at $\mathcal{I}^+$.

Late time profile distributions

Initial quantum state ($=$ Bunch-Davies)
provides a probability distribution for late time configurations in a dS background.

Wavefunctional is given (semiclassically) by Hartle-Hawking. For massless scalar with late time profile ϕ:

$$\Psi_{HH}[\phi] \propto e^{iS_{cl}[\phi]} \implies \mathcal{P}[\phi] \propto |\Psi_{HH}[\phi]|^2 .$$

For massless scalar:

$$\mathcal{P}[\phi] \propto e^{-2\sum_{\vec{k}} \beta_k |\phi_{\vec{k}}|^2} , \quad \beta_k = \frac{1}{2}(Lk)^3 \ell^{d-1}.$$

QUESTION: How are late time spatial configurations organized?

Distances in Field Space

Inspired by spin glasses, we define a notion of overlap between late time profiles. We propose a regularized Euclidean distance:

$$d[\phi_1, \phi_2] = \frac{1}{L^3} \int d^3x \left[(\phi_1(\vec{x}) - \overline{\phi_1(\vec{x})}) - (\phi_2(\vec{x}) - \overline{\phi_2(\vec{x})}) \right]^2 .$$

Overline = Spatial average $\implies$ subtraction of zero mode.

We further subtract the divergent late time average (w.r.t. $\mathcal{P}[\phi]$) from d_{12}.

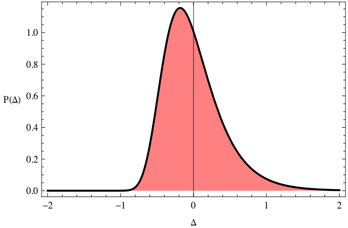
Our task is to compute the single and triple overlap distribution. e.g.

$$P(\Delta) = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 |\Psi_{HH}(\phi_1)|^2 |\Psi_{HH}(\phi_2)|^2 \delta\left(\Delta - d[\phi^{(1)}, \phi^{(2)}]\right) .$$

Single Overlap Distribution

For single overlap distribution, $P(\Delta) = \langle \delta(\Delta - d_{12}) \rangle$ we find the GUMBEL distribution:

$$P(\Delta) = \frac{\kappa^{\kappa}}{\Gamma(\kappa)} \exp\left[-\kappa\left(\Delta' + e^{-\Delta'}\right)\right] \ .$$



Also appears in extreme value physics. Asymmetry $\Longrightarrow$ There are more dissimilar configurations. (Also found by Benna using different 'distance'.)

Triple Overlap Distribution I

Similarly, we can compute the triple overlap distribution:

$$P(\Delta_1, \Delta_2, \Delta_3) \equiv \langle \delta(\Delta_1 - d_{23})\delta(\Delta_2 - d_{13})\delta(\Delta_3 - d_{12}) \rangle_{n=3}$$

$P(\vec{\Delta})$ peaks at $\Delta_3 = \max\{\Delta_1, \Delta_2\}$.

This is a clear signal of ultrametricity directly analogous to that found for spin glasses.