# The Kozai Resonance
# And The Evolution of Binary
# Supermassive Black Holes

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Large galaxies have undergone many mergers over the history of the universe, and these mergers are likely to repeatedly introduce binary and multiple supermassive black hole systems in their nuclei (e.g. Kauffmann & Haehnelt 2000). Binary black holes harden with time due to interactions with the surrounding stars and gas (Begelman, Blandford, & Rees 1980), but it is unclear whether these binaries harden to the point where gravitational radiation causes them to merge before a subsequent galaxy merger introduces a third black hole into the system. If such a multiple black hole system evolves to the point where there are strong dynamical interactions between its members, then some or all of the black holes may be ejected from the system (e.g. Valtonen et al. 1994).

Repeated encounters between the third black hole and the binary can pump up the eccentricity of the binary, accelerating its merger by gravitational radiation (Makino & Ebisuzaki 1994). Rather than a series of encounters, the third black hole and the binary may also become bound to form a hierarchical trinary which itself hardens due to interactions with surrounding material. If the mutual inclination between the inner and outer binaries which form this trinary is sufficiently high ($> 39°$), then tidal forces exerted on the inner binary by the outer black hole can induce eccentricity oscillations (Kozai 1962). An initially circular binary will therefore cycle through periods of higher eccentricity (Figure 1) and as a result will undergo accelerated merging by gravitational radiation (Figure 2).

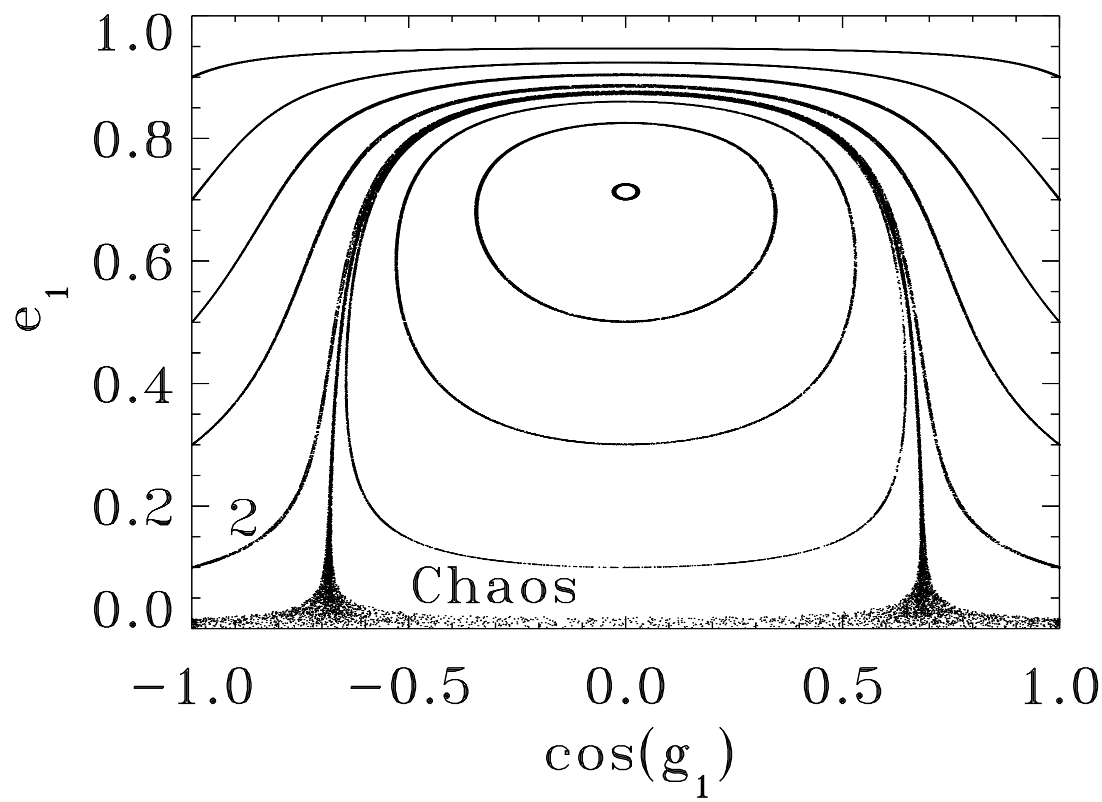


Fig. 1. Trajectories in the phase space of inner eccentricity $e_1$ vs. inner argument of periastron $g_1$ for trinaries with inner black hole masses of $10^6$ and $2 \times 10^6$ $M_\odot$ and outer black hole mass $10^6$ $M_\odot$, neglecting gravitational radiation. Each curve corresponds to a different initial inner eccentricity $e_1$ and mutual inclination angle $i$ chosen to keep the total angular momentum of the trinary fixed. The curve labeled 2 corresponds to the same initial conditions as that of the trinary evolution depicted in figure 2.

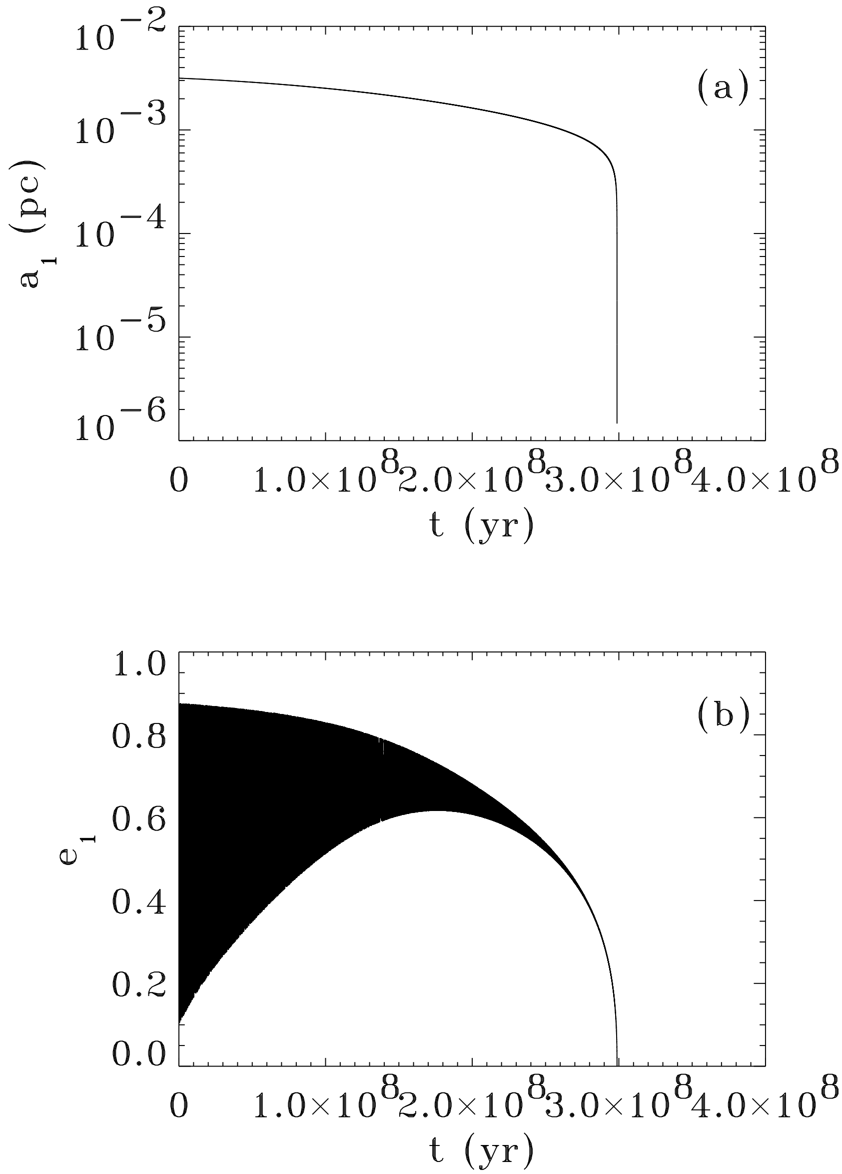


Fig. 2. Evolution of (a) the semimajor axis $a_1$ and (b) the eccentricity $e_1$ of the inner binary with time for a trinary consisting of inner black hole masses $2 \times 10^6$ $M_\odot$ and $10^6$ $M_\odot$ and outer black hole mass $10^6$ $M_\odot$. The inner binary starts out with $a_1 = 3.16 \times 10^{-3}$ pc and $e_1 = 0.1$. The outer black hole orbits with a semimajor axis ten times larger at an initial inclination of 80°. The Kozai resonance causes large amplitude eccentricity oscillations, and this greatly accelerates the merger of the inner binary: in the absence of eccentricity oscillations, the binary would take $9.3 \times 10^9$ yr to merge.

In order for the Kozai mechanism to operate, the outer black hole must be sufficiently close that the characteristic time scale for eccentricity oscillations be less than the time scale for general relativistic periastron precession of the inner binary. Figure 3 illustrates this effect.

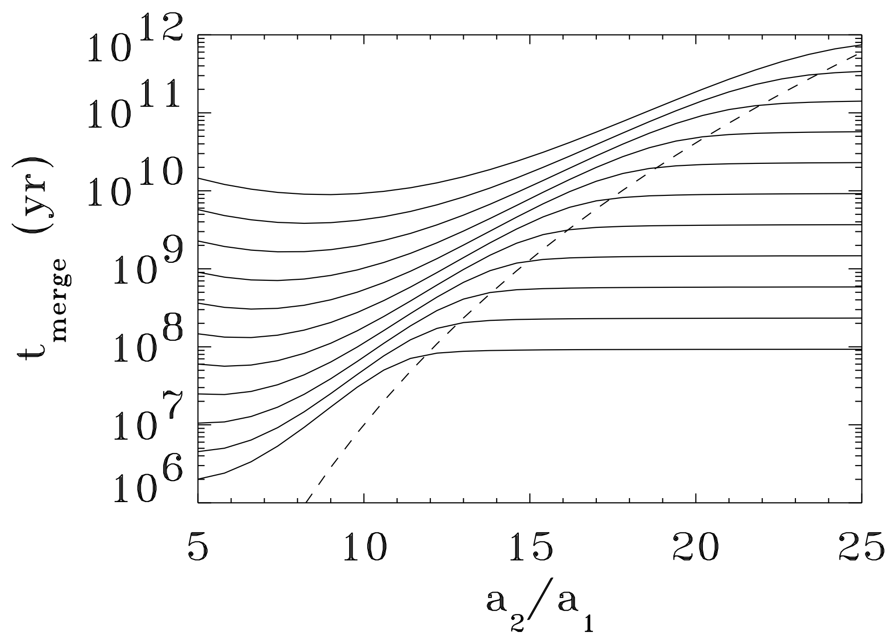


Fig. 3. Merger time of an inner binary black hole system with masses $2 \times 10^6$ $M_\odot$ and $10^6$ $M_\odot$ in a hierarchical trinary with outer black hole mass $10^6$ $M_\odot$, as a function of the initial semi-major axis ratio $a_2/a_1$ of the trinary. The trinary initially has $e_1 = 0.1$ and $i = 80°$. From bottom to top, the different solid curves show results for different initial semimajor axes of the inner binary, spaced at equal logarithmic intervals: $a_1 = \{1.00, 1.26, 1.58, 2.00, 2.51, 3.16, 3.98, 5.01, 6.31, 7.94, 10.0\} \times 10^{-3}$ pc. The dashed curve separates the region on the left where the Kozai resonance exists (at least initially) from that on the right where general relativistic precession destroys the eccentricity oscillations. In that region on the right, the inner binary's merger behavior is the same as that of an isolated binary.

Figure 4 illustrates the dependence of the inner binary merger time on the initial mutual inclination angle of the trinary. Significant reduction in the merger time only occurs when the inclination angle is high enough that the Kozai resonance is present: $\cos i < 0.77$. That reduction can be substantial. If inclinations are distributed randomly in solid angle, then merger times can be reduced by greater than a factor of ten in more than 30 percent of all cases in trinaries consisting of roughly equal mass black holes with a near circular inner binary.

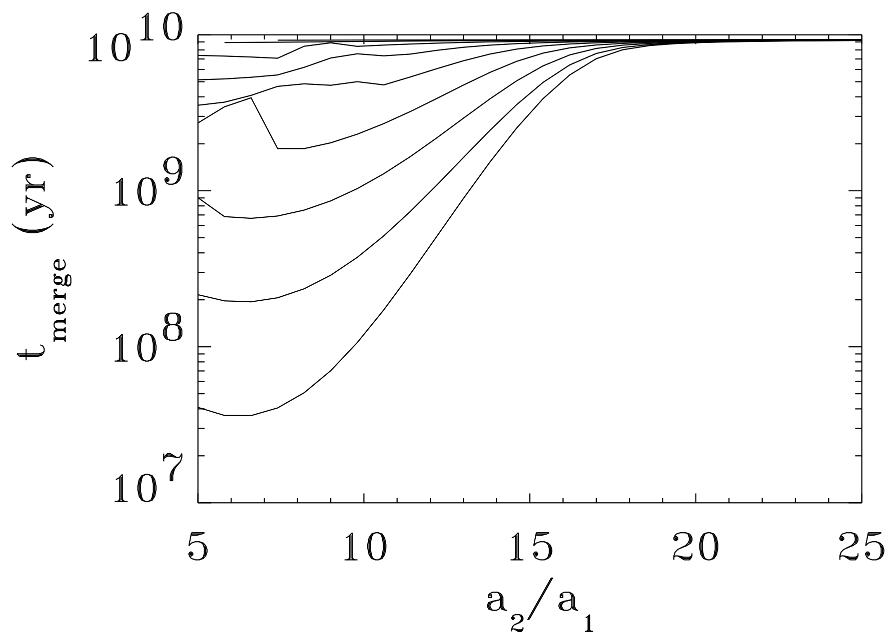


Fig. 4. Merger time of an inner binary black hole system with masses $2 \times 10^6$ and $10^6$ $M_\odot$ in a hierarchical trinary with outer black hole mass $10^6$ $M_\odot$, as a function of the initial semimajor axis ratio $a_2/a_1$ of the trinary. The different curves correspond to different initial mutual inclinations of the binary: from bottom to top, $\cos i$ ranges from 0.1 to 0.9 in steps of 0.1. (The 0.8 and 0.9 curves lie almost on top of each other.) For the curves shown here, the inner binary always starts with $a_1 = 3.16 \times 10^{-3}$ pc and $e_1 = 0.1$.

We have also investigated other mass combinations. For example, increasing the outer black hole mass results in even faster merger times commencing at larger semimajor axis ratios, as shown in figure 5. Such a system might arise from the merger of a small galaxy containing a stalled binary with a larger galaxy containing a bigger black hole.

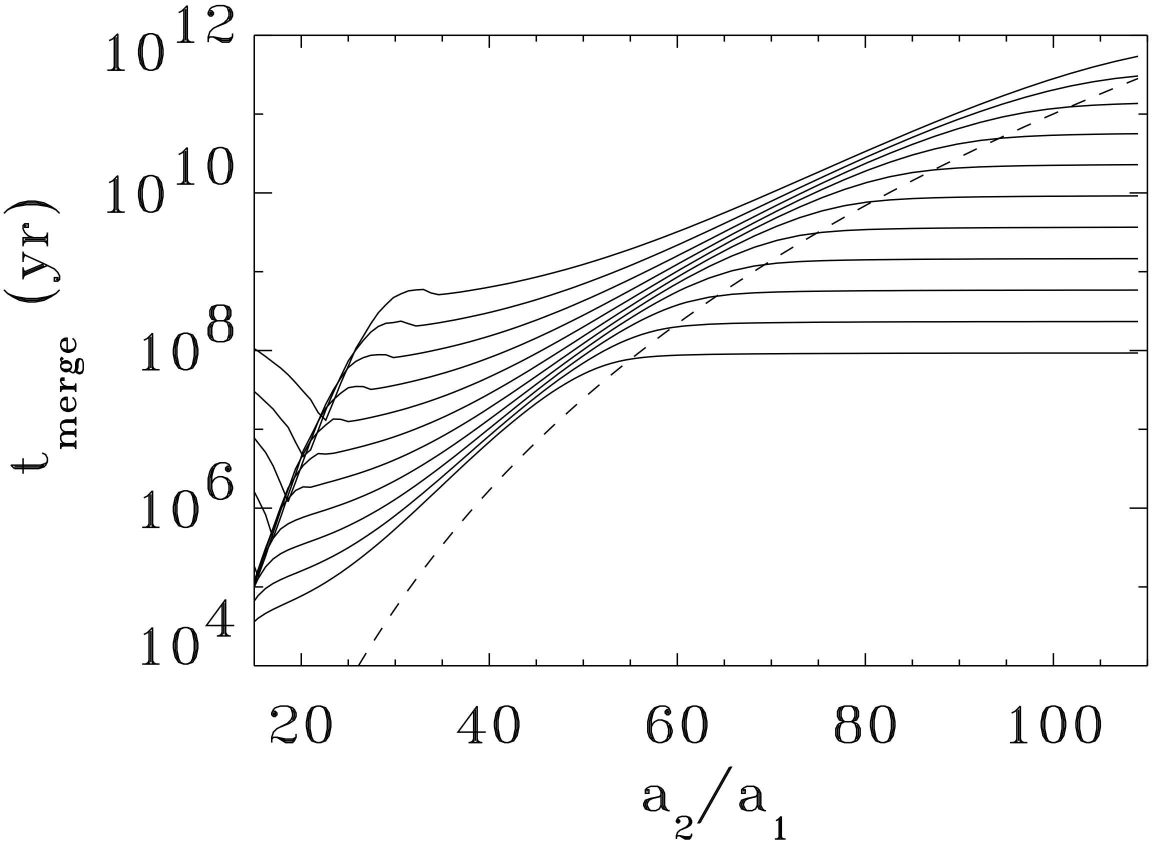


Fig. 5. Same as figure 3 except for an outer black hole mass of $10^8$ $M_\odot$.

Other sources of tidal perturbations on a binary supermassive black hole can also produce eccentricity oscillations due to the Kozai mechanism. For example, an aspherical distribution of stars and/or gas around the binary will induce eccentricity oscillations provided the exterior quadrupole moment $\rho_{2m}$ satisfies

$$\rho_{2m} \gtrsim 10^6 \mathrm{M}_\odot \ \mathrm{pc}^{-3} \left( \frac{M}{10^6 \mathrm{M}_\odot} \right)^2 \left( \frac{a_1}{10^{-2} \mathrm{pc}} \right)^{-4} (1 - e_1^2)^{-3/2},$$

where $M$ is the total mass of the binary. If this is true, then relativistic periastron precession is not effective in destroying the Kozai resonance. Note the strong scaling with the semimajor axis $a_1$ of the binary. Such high quadrupole moments might be possible to achieve given the actual stellar mass densities observed in galactic nuclei (e.g. Faber et al. 1997).

## References

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