

# A Complete Analytical Description of Few Cycle Focused Electromagnetic Pulses

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With few cycle light pulses available and development progressing at a high rate, detailed vector models of the fields are now needed to describe experiments using them. Molecular dynamics in few cycle pulses, for example, have recently been shown to depend directly on the electric field and not simply the intensity distribution. Additionally, to correctly describe even an intensity driven process such as ionization, the energy distribution within the laser focus must be known very accurately, which is dramatically affected by the inclusion of the laser bandwidth in few cycle pulses. The need for accurate and robust field descriptions for all pulse lengths including even the relatively small off-polarization field components has also been underscored many times both theoretically and experimentally in laser-plasma and direct laser-electron interactions.

To meet these needs, using the angular spectrum of plane waves technique, the exact solution for the fields of a focused laser is derived for any spot size and pulse duration. For tightly focused laser pulses ( $w < 10\lambda$ ), the fields are derived in terms of Gegenbauer polynomials. This has the attractive property that the series converges faster as the laser spot size,  $w$ , decreases requiring roughly  $20w/\lambda$  terms [1]. For laser waists larger than ten wavelengths, the field is instead expanded using spherical Hankel functions. Using these monochromatic models, the pulsed fields are then derived without approximation.

As the pulse duration decreases below ten cycles, significant changes are observed. For example, for a pulse focused to a wavelength, the inclusion of longer wavelengths reduces the fraction of the laser energy in the focus from the monochromatic value of 86.5% to 83.5% for a 5 fs Ti:Sapphire laser and to 72.7% in a single cycle pulse. This is shown directly in Fig. 1, which plots the fraction of the laser energy in the focus as a function of the pulse duration for several waists. The horizontal line indicates the limiting case of a pure Gaussian.

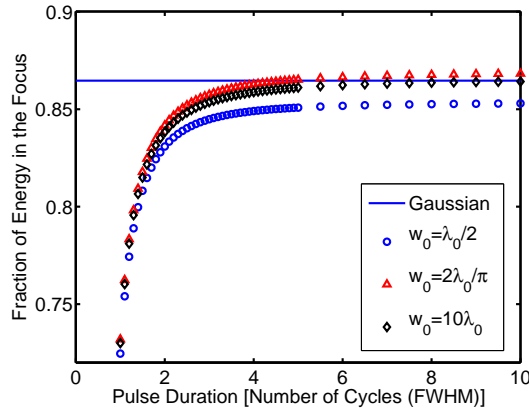


FIG. 1: The fraction of energy in the focus as a function of the pulse duration. The line indicates the Gaussian limit.

This model produces a complete vector description of each of the electromagnetic field components of a focused laser pulse converging to an arbitrary spot size and having any pulse length. Modeling of a host of physical phenomena can now include not only simple plane wave laser fields but also the true focused character of the realistic laser pulses used in experiments.

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[1] S. Sepke and D. Umstadter, “Exact analytical solution for the vector electromagnetic field of Gaussian, flattened Gaussian, and annular Gaussian laser modes,” *Opt. Lett.* **31**, 1447 (2006).

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