Time evolution and density operator

1) Schrödinger picture: $\hat{O} = \hat{O}(t_0)$, $\forall$ operator

$$|\psi(t)\rangle = \hat{U}(t t_0)|\psi(t_0)\rangle$$

with $i\partial_t \hat{U} = \hat{H}(t)\hat{U}$, $\hat{U}(t_0 t_0) = \hat{1}$

Density operator: $\hat{\rho}(t) = \sum_i W_i |\psi^{(i)}(t)\rangle\langle\psi^{(i)}(t)|$

$$= \hat{U}(t t_0)\hat{\rho}(t_0)\hat{U}^+(t t_0)$$

2) Expectation value $\langle\hat{O}\rangle(t) = \mathrm{Tr}\,\hat{U}\hat{\rho}(t_0)\hat{U}^+\hat{O}(t_0)$

cyclic inv of trace: $= \mathrm{Tr}\,\hat{\rho}(t_0)\underbrace{\hat{U}^+\hat{O}(t_0)\hat{U}}$

$$= \mathrm{Tr}\,\hat{\rho}(t_0)\,\hat{O}_H(t)$$

$\rightarrow$ Recover result in Heisenberg picture where $\hat{\rho}$ is time-independe

This was used in the formulation of the Nonequil Green's functions theory in the lecture.

3) Time dependence of W_i does not arise from dynamics driven by $\hat{H}$, but e.g. from time-dependent coupling to the environment. In (local) equilibrium e.g. via $\mu(t)$, $\beta(t)$.

I thank Daniela and Robin for their comments