A new Additional Theorem for the time resolved terms of Maxwell's Equations in the X-ray spectrum

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A new Additional Theorem, arrived at from a mathematical transform equation, allows the terms of Maxwell's Equations to be solved for in the time domain, while considering electromagnetic frequencies in the *X-ray spectrum*. The new Additional Theorem is the area under the curve for any integrand variable *f(t)y*, as it varies in terms of a differential variable *f(t)x*, exactly equals the time integral of the product for the integrand and first derivative of the differential. Below are Maxwell's Equations and their corresponding new Additional Theorem equivalencies, which are expressed in the time domain:

$$\text{Gauss's Law} \oint \mathrm{E} \cdot dA = \frac{Q}{\varepsilon_o} = \oint (E(t)\frac{dA}{dt})dt$$

$$\text{Gauss's Law Magnetism} \oint B \cdot dA = 0 = \oint (B(t)\frac{dA}{dt})dt$$

$$\text{Faraday's Law} \oint E \cdot ds = -\frac{d\Phi_m}{dt} = \oint (E(t)\frac{dx}{dt})dt$$

$$\text{Ampere - Maxwell Law} \oint B \cdot ds = \mu_o I + \varepsilon_o \mu_o \frac{d\Phi_e}{dt} = \oint (B(t)\frac{dx}{dt})dt.$$

Demonstrations of the new Additional Theorem involving a cycloid are given online at Math Words and Paul's Math notes in the Parametric Integrals section. It is shown that the new additional theorem is valid in the phase domain θ as well.

As an example, if we consider an electromagnetic plane wave in free space and use the Ampere-Maxwell equation,

$$\oint B \cdot ds = \varepsilon_0 \mu_0 \frac{d\Phi_e}{dt}.$$

Substitute on the left and right hand sides equivalent expressions:

$$\oint B Cos\theta \, ds = \varepsilon_0 \mu_0 \frac{d\oint E Cos\theta \, dA}{dt}.$$

In the most ideal case, to keep the introductory example simple, *cos θ =1*:

$$\oint Bds = \varepsilon_0 \mu_0 \frac{d\oint EdA}{dt}$$
.

Then, substitute, on the left and right hand side of the equation, the corresponding equivalent expressions, using the new Additional Theorem. Thus, we have

$$\oint \left(B(t) \frac{dx}{dt} \right) dt = \varepsilon_0 \mu_0 \frac{d\oint \left(E(t) \frac{dA}{dt} \right) dt}{dt}$$
.

Simplifying the right hand side gives us

$$\oint \left(B(t) \frac{dx}{dt} \right) dt = \varepsilon_0 \mu_0 \; E(t) \frac{dA}{dt}$$
.

Substituting for *dx/dt*, and *dA/dt, v(t)* and *A'*, respectively, we have

$$\oint \left(B(t) v(t) \right) dt = \varepsilon_0 \mu_0 E(t) A'$$
.

Differentiating both sides of the equation gives us

$$B(t)v(t) = \varepsilon_o \mu_0 A' \frac{dE(t)}{dt} + \varepsilon_o \mu_o E(t) \frac{dA'}{dt}$$
.

Again, in order to simplify the introductory example, we take $dA'/dt = 0$:

$$B(t)v(t) = \varepsilon_o \mu_0 A' \frac{dE(t)}{dt}$$
.

Because we are considering electromagnetic waves in a vacuum, we substitute the expressions for sinusoidal waves in free space on both sides of the equation, where x is held constant, and kx is some multiple of 2π. On the right hand side of the equation, we also take the partial derivative of the electric field:

$$B_0 \cos(-\omega \cdot t) v(t) = \varepsilon_o \mu_o A' \frac{\partial E_0 \cos(-\omega \cdot t)}{\partial t}.$$

Simplifying the right side of the equation we have

$$B_0 \cos(-\omega \cdot t) v(t) = \varepsilon_o \mu_o A' \omega \cdot E_0 \sin(-\omega \cdot t).$$

Dividing both sides of the equation by the magnetic field gives us

$$v(t) = \frac{\varepsilon_o \mu_o A' \omega \cdot E_0 \sin(-\omega \cdot t)}{B_0 \cos(-\omega \cdot t)}.$$

Simplifying the right side of the equation gives

$$v(t) = \frac{\omega \cdot A'}{c} \tan(-\omega \cdot t).$$

The above is the contour velocity in units of meters per second. When c*os(-wt)=0*, the velocity is undefined (infinity), so a constant average magnetic field *Bav*, can be added to the magnetic field:

$$v(t) = \frac{\omega \cdot A'}{c} \cdot \frac{\sin(-\omega \cdot t)}{\cos(-\omega \cdot t) + B_{av}}.$$

When *cos(-wt)=0* (the phase is $\pi/2$), the *absolute value* of the peak velocity, instead of being undefined, could be the following:

$$v(t) = \frac{\omega \cdot A'}{c} \cdot \frac{1}{B_{av}}.$$

The preceding is not difficult to imagine if you consider that, at the Max Planck Institute, it was discovered that an array of micrometer sized rings could individually carry a small constant current, and thus a corresponding small magnetic field, in the absence of an *emf.* It should then follow that a magnetic field in a vacuum could, on average, also have a constant magnetic field, however small, to satisfy the equation.
The above equations represent the contour velocity, which can be applied to electromagnetic waves, in the *X-ray spectrum*. Using algebra, one can solve for *dA/dt* as

well. The introductory example was made simple by taking *cosθ =1* and *dA/dt = a constant*. The new Additional Theorem works in harmony with problems involving more complex integrands and differentials, including the necessary Relativistic calculations, by substituting equivalent Relativistic expressions for the factors and terms in the equations and performing the same steps. When working with more dynamical systems involving electromagnetic waves moving through various media, where *cosθ =1* is some function *f(t),* and *dA/dt* is also some time function *f(t)*, results can be obtained using the new Additional Theorem, but with more terms.

In addition, considering electromagnetic waves in a vacuum:

$$\oint E \cdot ds = -\frac{d\Phi_m}{dt}.$$

Substituting equivalent expressions on the left and right hand sides we have

$$\oint E Cos\theta\, ds = -\frac{d\oint B Cos\theta\, dA}{dt}.$$

In the most ideal case, like in the example outlined above that keeps the introductory example simple, *Cosθ =1*:

$$\oint E ds = -\frac{d\oint B dA}{dt}.$$

Then, substituting on the left and right hand side of the equation the corresponding equivalent expressions using the new Additional Theorem, we have

$$\oint \left(E(t)\frac{dx}{dt} \right) dt = -\frac{d\oint \left(B(t)\frac{dA}{dt} \right) dt}{dt}.$$

Simplifying the right side of the equation containing the derivative of the integral gives us

$$\oint \left(E(t) \frac{dx}{dt} \right) dt = -B(t) \frac{dA}{dt}$$
.

If we substitute for *dx/dt* and *dA/dt, v(t)* and *A'*, respectively, we then have

$$\oint \left(E(t) v(t) \right) dt = -B(t) A'$$
.

Taking the time derivative of both sides of the equation gives us

$$E(t)v(t) = -A' \frac{dB(t)}{dt} - B(t) \frac{dA'}{dt}$$
.

If we then follow the same steps given above in the first example, we can arrive at an expression for the contour integral velocity in units of meters per second for electromagnetic waves in the *X-ray spectrum*. The preceding involves Maxwell's equations using Faraday's Law of Induction and can be applied to various problems, which can also involve necessary Relativistic adjustments along with additional dynamic considerations, where $Cos\theta = 1$ is some function *f(t)*. *dA/dt* is also some time function *f(t)*, which involves the electromagnetic *X-ray spectrum*. Interestingly, it was recently discovered in Japan that an equivalent analogous antenna to that of the *Yagi-Uda Antenna* could be constructed, within the nanometer dimension, in order to harness electromagnetic radiation in the light spectrum. Perhaps, in the future, the same can be done to harness the electromagnetic radiation in the *X-ray spectrum* for improved control and use of *X-ray radiation*.

As a theoretical analogy, perhaps some *X-ray* radiation emitted by stars from their interior or near their surface could be closely modeled using the following Ampere-Maxwell Equation:

$$\oint B \cdot ds = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi}{dt}$$
.

If we consider the magnetic field normal to the path of the contour integral and substitute for the displacement current a sinusoidal function, we have the following:

$$\oint Bds = I_d + I_0 Sin(\omega \cdot t + \theta)$$
.

Substituting an equivalent expression, using the Additional Theorem on the left side of the equation, gives

$$\oint \left(B(t)\frac{dx}{dt} \right) dt = I + I_0 Sin(\omega \cdot t + \theta)$$

Differentiating both sides with respect to time and substituting dx*/dt=v(t)* results in

$$B(t)v(t) = I_0 \cdot \omega \cdot \cos(\omega \cdot t + \theta)$$

Substituting *B(t)* with an equivalent sine function gives us

$$B_0 \sin(\omega \cdot t + \theta)v(t) = I_0 \cdot \omega \cdot \cos(\omega \cdot t + \theta)$$

Dividing both sides by *B(t)* solves for the contour integral velocity:

$$v(t) = \frac{I_0 \cdot \omega}{B_0} \cdot \frac{\cos(\omega \cdot t + \theta)}{\sin(\omega \cdot t + \theta)}$$

$$v(t) = \frac{I_0 \cdot \omega}{B_0} \cot(\omega \cdot t + \theta)$$

The velocity is undefined when *(wt+θ)=0*. Considering a constant average magnetic field term *Bav*, we have

$$v(t) = \frac{I_0 \cdot \omega \cdot \cos(\omega \cdot t + \theta)}{B_0 \sin(\omega \cdot t + \theta) + B_{av}}$$

With the addition of a constant average magnetic field term *Bav*, the velocity is defined at *(wt+θ) = 0:*

$$v(t) = \frac{I_0 \cdot \omega}{B_{av}}$$

The above equation is the theoretical peak velocity in meters per second that is analogous to the current in an enclosed region in the interior or near the surface of the sun in the *X-ray spectrum*. In reality, one could consider, perhaps, the above integral, if we set the phase angle *θ = 0*, on a limit of integration that corresponds to *0 to π /2 radians* in the phase domain. In such an interval, as the current and magnetic field approach their maximum values, the corresponding calculated contour velocity starts from its peak value and then decreases toward zero. Interestingly, the contour velocity for a constant current *I*, using Ampere's law, is zero. In other words, as currents approach some steady state value, the contour velocity tends toward zero (in reality there are fluctuations even for the most ideal situation, which translate into a non-absolute zero derivative for a unit step current, which means there is a non-absolute zero velocity, however small, which satisfies the equation). As another example, if one considers a linear or ramp function with a small enough slope to allow for a very slow increase toward some maximum steady state value, for both the current and magnetic field in the above example, then the contour velocity would be inversely proportional to time (*1/t*), which translates into a decreasing contour velocity as both the magnetic field and current approach some maximum steady state. When considering the *X-ray spectrum*, the time periods and wavelengths are very small, but the regions over which stars produce them are vast. The above example and the aforementioned examples involving the sinusoidal function, for the ¼ cycle of the period and the linear or ramp function, would occur in very small time frames, producing very small wavelengths. However, given the immense surface area of the sun, and due to superposition of the electromagnetic waves, they provide coherent sources of *X-rays*, which emanate over immense regions of our sun.

Summary:
The new Additional Theorem, which was derived from a mathematical transform equation, can be used to find the exact area under the curve for any integrand variable *f(t)y* as it varies in terms of some differential variable *f(t)x*. In the above examples, we used Maxwell's Equations, with the new Additional Theorem, to solve for the contour integral velocity in the time domain while considering electromagnetic waves in the *X-ray spectrum*. The above expressions can also be adjusted to account for necessary Relativistic effects in order to arrive at the correct Relativistic expressions by substituting the equivalent Relativistic expressions for the factors and terms in the equations.