Models of Magnetic Field Amplification in Nonlinear Shock Simulations

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Goals

- We are building a model of a nonlinear shock that will incorporate the **plasma physics from numerical simulations and theoretical analysis**, and match the observational estimates of shock parameters.
- Here, I will demonstrate a nonlinear shock model that incorporates magnetic field amplification (MFA) by **two different mechanisms (resonant and non-resonant)**.
- Our **analytical description** of MFA contains nonlinear elements suggested by theory and simulations.
- The results allow us to **compare and understand the impact** of the different mechanisms.
- The key element of the model is **self-consistency**: particle acceleration, magnetic field amplification and nonlinear shock back-reaction are all coupled to each other.

Method

The Nonlinear Model

- Particle transport modeled with a **Monte Carlo simulation**;
- **Analytic description** for magnetic field amplification;
- Fundamental **conservation laws** used to iteratively derive a nonlinear shock modification that conserves mass, momentum and energy;

Reasoning

- We describe a **large dynamic range** in turbulence scales and particle energies;
- Elements of the model **tested** against spacecraft observations of heliospheric shocks;
- Works for highly **anisotropic** particle distributions (particle escape and injection; large gradients of u and B).
- Ability to incorporate **non-diffusive** particle transport (future work).

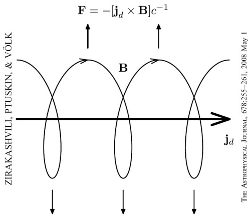
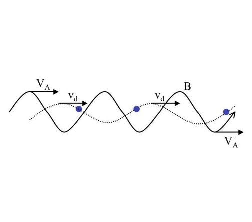
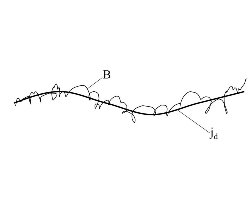


FIG. 1.—Explanation of the nonresonant streaming instability. The magnetic spiral (*thin solid line*) is stretched by the Lorentz force $F = -(j_d \times B)c^{-1}$ that appears due to the diffusive cosmic-ray electric current j_d.





Bell's nonresonant current-driven instability works on small scales by stretching magnetic field lines (Bell, 2004).

Resonant CR streaming instability. Alfvén waves gain energy from resonant particles streaming faster than the waves. (Skilling, 1975).

Nonresonant long-wavelength instability arises from the back-reaction of the amplified waves on the current of the streaming CRs (Bykov et al., 2009).

Other Possible Sources of Magnetic Fields

- **Two-stream** instability in the subshock vicinity (very short scale turbulence, important for thermal particles); **Weibel** instability for unmagnetized shocks;
- MHD amplification (**dynamo**) – requires a solenoidal velocity component (various possible sources: acoustic instability, clumps, etc.);
- **Magnetosonic** instability (e.g., Malkov & Diamond 2009, Dorfi & Drury 1985);
- Cold beam of protons due to charge exchange – amplifies B only downstream;
- etc.

Evolution of Waves in the Precursor

Definitions

We describe turbulence by $W(x, k)$ – spectral energy density of turbulent fluctuations, and separate it into

$$W = W_M + W_K = \sum_{i \in \text{modes}} W_M^{(i)} + \sum_{i \in \text{modes}} W_K^{(i)}.$$

Index M – magnetic fluctuations, K – associated plasma velocity fluctuations, and (i) runs over two types of waves (A – Alfvén waves, B – Bell's modes).

Equations

Evolution for each mode is given by the equation for $W^{(i)} = W_M^{(i)} + W_K^{(i)}$:

$$u \frac{\partial W^{(i)}}{\partial x} = \gamma^{(i)} W^{(i)} - L^{(i)} + \left[-\alpha^{(i)} W^{(i)} + \frac{\partial}{\partial k} \left(k W^{(i)} \right) \right] \frac{du}{dx} - \frac{\partial \Pi^{(i)}}{\partial k} \qquad (1)$$

Re-Normalizations for Strong Turbulence

Scale Separation

Given a wavenumber k, we separate the fluctuations in the large-scale $(k' < k)$ and the small-scale $(k' > k)$ parts, and define

$$\frac{B_{\rm ls}^2(x,k)}{8\pi} = \frac{B_0^2}{8\pi} + \int_0^k W_m(x,k')dk', \qquad \frac{B_{\rm ss}^2(x,k)}{8\pi} = \int_k^\infty W_m(x,k')dk'.$$

Re-normalized Resonance Condition

For the wavenumber k we define a 're-normalized' resonant momentum as

$$p = \frac{eB_{\rm ls}(x,k)}{ck}$$

Correlation Length

The correlation length of the small-scale fluctuations is

$$l_{\rm cor}(x,k) = \frac{\int_k^\infty W_m(x,k')/k' \, dk'}{\int_k^\infty W_m(x,k') \, dk'}.$$

Our Current Recipe for Nonlinear Wave Growth

In order to describe the growth of the instabilities in the $\Delta B \gg B_0$ regime, in the results presented here, we assume that:

- **No spectral transfer** (cascading) occurs in any mode ($\Pi = 0$);
- **Quasi-linear growth** rate γ applies for $\Delta B > B_0$, possibly with a replacement $B_0 \to B_{\rm ls}(x, k)$ in the expression;
- A **nonlinear dissipation** mechanism ($L \propto W^{1+\sigma}$, $\sigma > 0$) operates.

Magnetic field amplification (MFA) is also **self-regulated** through the nonlinear shock structure (i.e., too large $\Delta B \to$ lower $R_{\rm tot} \to$ weaker particle acceleration $\to$ lower ΔB).

Passage of Turbulence through the Subshock

- Equation (1) applies only when $u/|(\nabla u)| \gg k^{-1}$ (WKB limit).
- Flow discontinuities lead to reflection, partial transmission and transformation of the waves.
- The effect of subshock on **Alfvén waves** reduces the total compression ratio R_{tot} (Caprioli+, 2008, MNRAS).
- Transmission of **Bell's modes** is being studied by simulations (e.g., Zirakashvili+);
- Interaction with **entropy waves** may lead to subshock rippling, affects injection? (Bykov, 1982).

Present Solution: All or Nothing

- All same order effects must be accounted for **simultaneously**;
- The **non-linear case** $\Delta B \gg B_0$ must be simulated;
- We **ignore** these effects and apply the equation (1) at the subshock, which leads to $W(x=-0,\, k) = R_{\text{sub}}^{\alpha} W(x=+0,\, k/R_{\text{sub}})$;
- This is works in the right direction, but is not the correct solution.

Resonant Streaming Instability

Quasi-Linear Theory

Growth rate at a wavenumber k is

$$\gamma^{(\mathrm{A})} W^{(\mathrm{A})} = v_A \left[\frac{\partial P_{\mathrm{cr}}(x,p)}{\partial x} \left| \frac{dk}{dp} \right| \right]_{p=\frac{eB_{\mathrm{ls}}}{ck}} .$$

The amplified harmonics are Alfvén waves characterized by

$$W_M^{(\mathrm{A})} = W_K^{(\mathrm{A})}$$

Fluxes of momentum and energy ($v_A \ll u_0$):

$$\Phi_P = \frac{1}{2} W^{(\mathrm{A})},$$
$$\Phi_E = \frac{3}{2} W^{(\mathrm{A})} u_0 .$$

Strong Fluctuations (Nonlinear Theory)

- We include transit time damping (Achterberg & Blandford, 1986) at large amplitudes:

$$L^{(\mathrm{A})} = \sqrt{\frac{\pi}{2}} k r_{g,\,\mathrm{th}} \frac{\left[W^{(\mathrm{A})}\right]^2}{B_0^2/(8\pi)} \omega_B .$$

- Saturation at $\Delta B \approx$ a few B_0 may occur (Lucek & Bell 2000). We do not include this effect.

Nonresonant Current Driven Instability (Bell)

Quasi-Linear Theory

Growth rate at k is

$$\gamma^{(\mathrm{B})} = v_A k \sqrt{\frac{4\pi j_d}{c B_0 k} - 1},$$

applicable for $1/r_{\mathrm{g\,min}} < k$. Amplified modes have a small phase speed and

$$W_M^{(\mathrm{B})} = \frac{1}{4} W_K^{(\mathrm{B})}$$

for the fastest growing k. Fluxes are:

$$\begin{aligned} \Phi_P &= \frac{1}{4} W^{(\mathrm{B})}, \\ \Phi_E &= \frac{6}{5} W^{(\mathrm{B})} u_0. \end{aligned}$$

Strong Fluctuations (Nonlinear Theory)

Simulations[a] show for $\Delta B \gtrsim B_0$:

- Growth slows down,
- Dominant k decreases (dissipation at large k, inverse cascade, or both)
- Saturation at $r_{\mathrm{g\,min}} \approx k$

We assume, following Riquelme & Spitkovsky 2009, dissipation

$$L^{(\mathrm{B})} = C \left[W^{(\mathrm{B})} \right]^{\frac{3}{2}} \rho^{-\frac{1}{2}} k^{\frac{3}{2}}$$

and set $\gamma^{(\mathrm{B})} = 0$ for $k > r_{\mathrm{g\,min}}$.

[a] Bell, Reville+, Zirakashvili+, Niemec+, Riquelme+

Calculating the Particle Mean Free Path

Prescription

- Mean free path of a particle with momentum p is calculated as

$$\lambda = \frac{r_{\rm ss}^2}{l_{\rm cor}},$$

where $r_{\rm ss}$ is the gyroradius in $B_{\rm ss}$.

- If $d\lambda/dp < 0$ for $p < p_c$, we set $\lambda = l_{\rm cor}$ for $l_{\rm cor}$ calculated for $k' > k(p_c)$.

Rationale

This prescription reproduces:

- The $\lambda \propto p^2$ scattering in **small-scale** fluctuations for $B_{\rm ss} \gg B_0$ (highest energy particles)
- The **resonant scattering** regime $\lambda \propto p^{2-s}$ for $W \propto k^{-s}$ (intermediate energies)
- A $\lambda = \rm const$ regime for the **smallest** particle energies

Conservation Laws with Turbulence

Momentum and energy fluxes of all modes are calculated from $W^{(i)}$ according to the relationship between $W_M^{(i)}$ and $W_K^{(i)}$. Then substituted into the conservation laws

$$\begin{aligned} \rho u &= \rho_0 u_0, \\ \rho u^2 + P_{\rm th} + P_{\rm cr} + \sum_{(i)} \Phi_P^{(i)} &= \rho_0 u_0^2 + P_{\rm th0} + \Phi_{\rm P0}, \\ \frac{1}{2}\rho u^3 + w_{\rm th} u + w_{\rm cr} u + \sum_{(i)} \Phi_E^{(i)} &= \frac{1}{2}\rho_0 u_0^3 + w_{\rm th0} u_0 + \Phi_{\rm E0} + Q_{\rm esc}. \end{aligned}$$

to find the self-consistent shock structure ($u(x)$, $R_{\rm tot}$, $R_{\rm sub}$, etc.).

Dissipation of Turbulence

The energy drained from the turbulence due to dissipation (term $L^{(i)}$ in equation (1)) is put into the thermal particle population by calculating $P_{\rm th}$ from

$$\frac{u\rho^{\gamma}}{\gamma-1}\frac{d}{dx}\left(P_{\rm th}\rho^{-\gamma}\right)=\sum_i L^{(i)}.$$

This $P_{\rm th}$ enters the conservation laws and, in our thermal leakage injection model, affects the low-energy CR spectrum.

Simulation Results

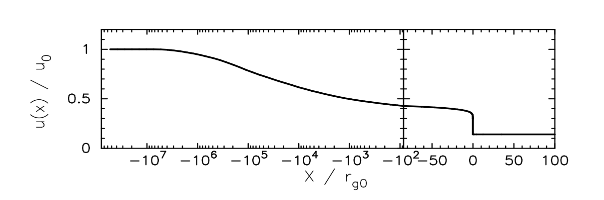
We ran a series of simulations with the following parameters:

Physical Parameters

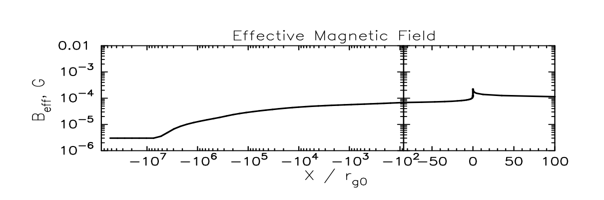
- Shock speeds from u_0=500 km/s to u_0 =20,000 km/s
- Free escape boundary at $x_{\rm FEB} = -0.03$ pc
- Pre-existing uniform magnetic field B_0=3 μG
- Density n_0=0.3 μG
- Unshocked plasma temperature T_0=10^4 K

Assumptions

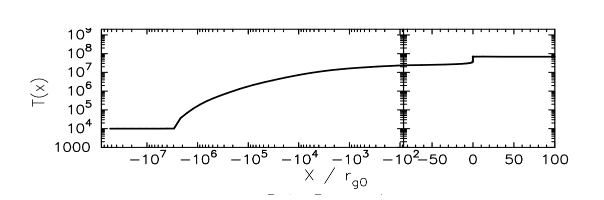
- **Resonant** streaming instability limited by transit time damping;
- **Bell's** instability limited by $k \approx r_{g\,\rm min}$ saturation and non-linear damping;
- **Quasi-linear** growth rates with B_0 (also one case with $B_{\rm ls}$ will be shown);
- Steady-state, planar shock, parallel geometry ($\mathbf{u}_0 \parallel \mathbf{B}_0$);
- **Mean free path** calculated by the model with field re-normalization.



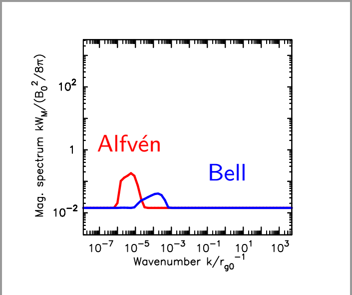
The case with $u_0 = 5,000$ km/s. Total compression $R_{\rm tot} \approx 7$.



Effective magnetic field amplified to $B_{\rm tot} \approx 130$ μG.

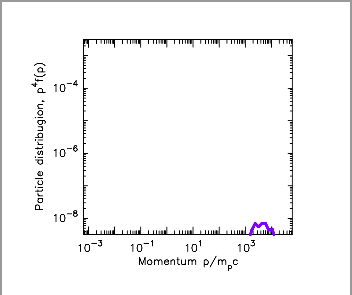


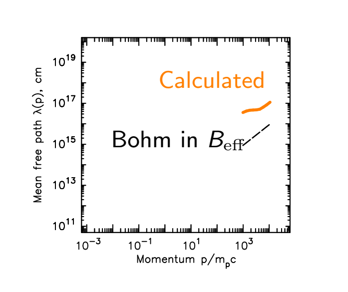
Strong heating in the precursor; subshock has $M_s \approx 2.1$, and $T_2 \approx 7 \cdot 10^7$ K.

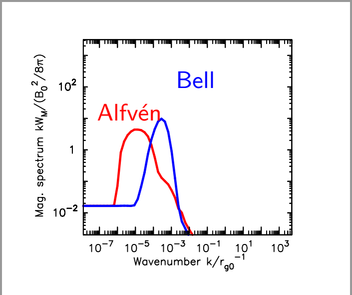


Particles and turbulence in the 5,000 km/s shock.

Far upstream, $x = -L_{\mathrm{FEB}}$

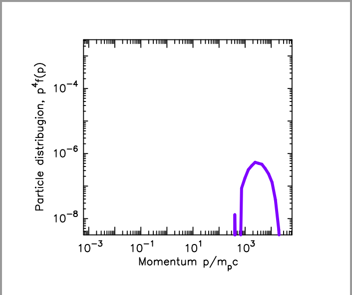


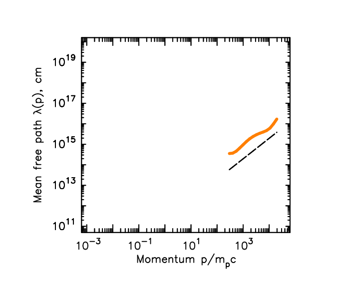


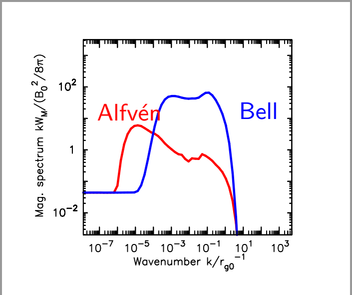


Particles and turbulence in the 5,000 km/s shock.

Upstream, $x \approx -10^{-1}\, L_{\mathrm{FEB}}$

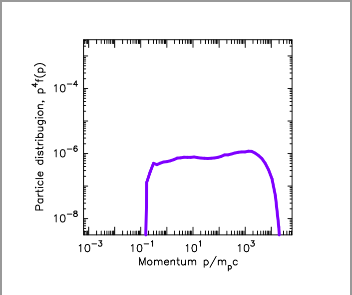


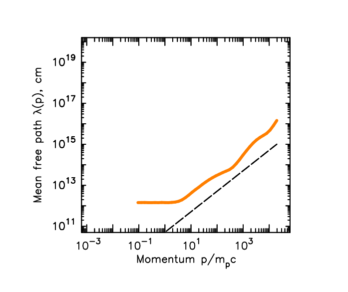


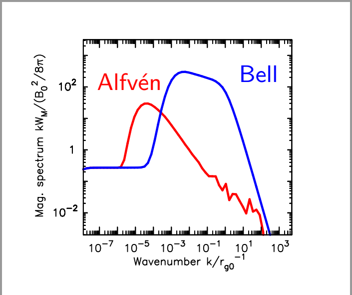


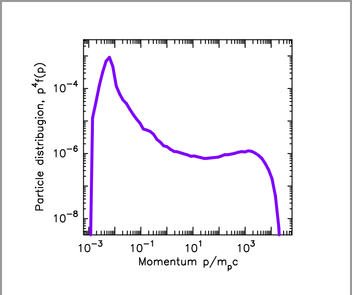
Particles and turbulence in the 5,000 km/s shock.

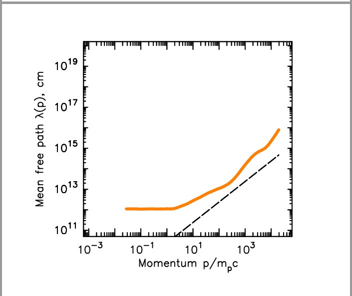
Upstream, $x \approx -10^{-4}\, L_{\mathrm{FEB}}$





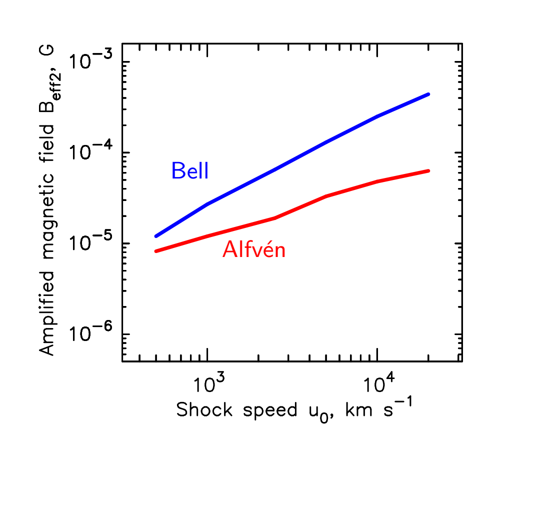






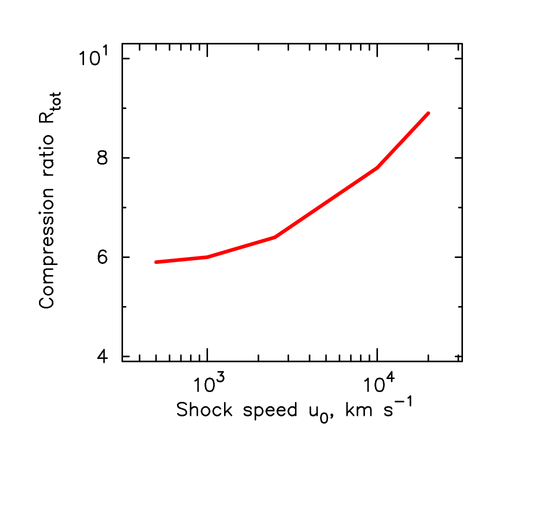
Particles and turbulence in the 5,000 km/s shock.

Downstream, $x = +35\, r_{g0}$



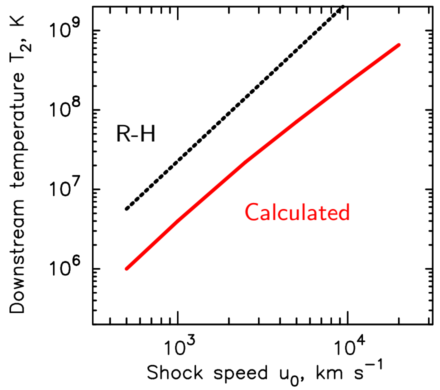
Trend: Magnetic Field

- Bell's nonresonant instability dominates turbulent energy for all studied shock speeds;
- Resonant instability produces a smaller B_{eff}, but provides stronger scattering for the most energetic particles (see further).



Trend: Compression Ratio

- The self-consistent total compression ratio varied from $R_{\rm tot} = 6$ to $R_{\rm tot} = 8$ – in the ballpark of observational estimates for some historic SNRs (Warren+, Cassam-Chenai+).
- These values correspond to rather low ($\approx$ a few percent) escaping energy fluxes.

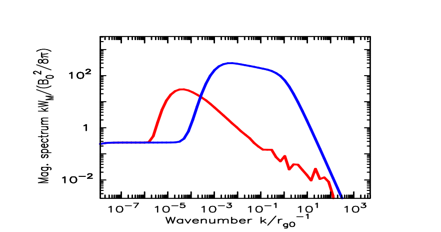


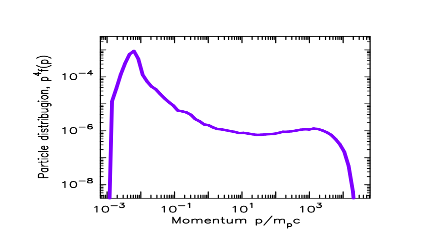
Trend: Temperature

- The downstream temperature is reduced by CR acceleration by approximately an order of magnitude compared to the Rankine-Hugoniot adiabat.
- The dependence of the shocked gas temperature on the shock speed is slightly weaker than $T_2 \propto u_0^2$.

Nonresonant + Resonant

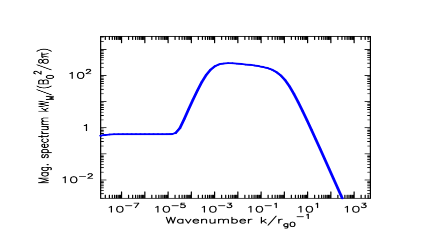
The same case as presented a few slides before (5000 km/s).

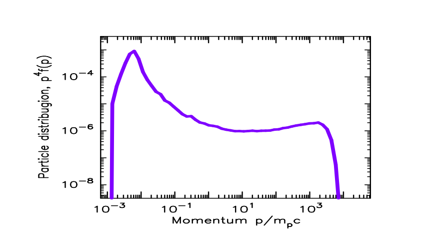


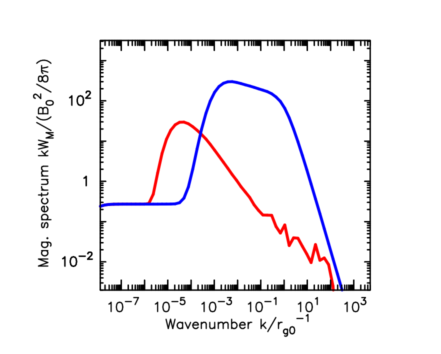


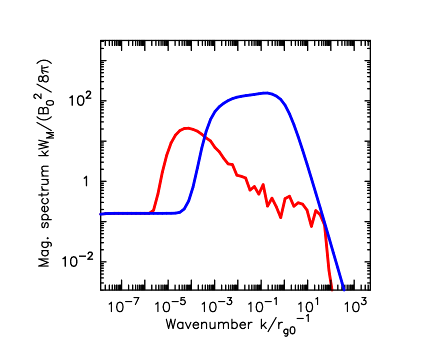
Only Nonresonant

Note the difference in the high energy turn-over.









Quasi-linear Growth for $\Delta B > B_0$

Same 5000 km/s result as before, B_0 used in the non-linear growth rate.
$R_{\rm tot} \approx 7$, $B_{\rm eff} = 130\,\mu$G,
$T_2 \approx 7 \cdot 10^7$ K.

Re-normalized Nonlinear Growth

Here, $B_{\rm ls}$ was in used the non-linear growth rate.
$R_{\rm tot} \approx 5$, $B_{\rm eff} = 100\,\mu$G,
$T_2 \approx 1.5 \cdot 10^8$ K.

Future Work

- The analytical description of the **nonlinear regime** ($\Delta B \gg B_0$) needs improvement. This can be done by performing and analyzing the corresponding **plasma simulations**;
- Other instabilities may operate. The **nonresonant long-wavelength instability** (Bykov+, 2009) may be important for the highest energy particles;
- Particle **transport model** may need improvement for low energies ($r_g \ll l_{\rm cor}$);
- **Subshock**: interaction with turbulence, effect on injection.

Summary of the Model

Model

- **Self-consistently** couples particle acceleration, MFA and nonlinear flow modification;
- Combines **resonant streaming** instability **and Bell's nonresonant** instability;
- Describes MFA analytically using estimates of **nonlinear** behavior based on theory and simulations;
- Calculates particle **transport based on the turbulence** energy spectra.

Results

- The self-consistently determined R_{tot}, T_2 and B_{eff} are in the ballpark of the recent observations.
- The solutions depend on the assumptions for the $\Delta B \gg B_0$ regime.

Bell's instability:

- Produces a **large** B_{eff},
- Amplifies **small wavelengths** ($\lambda \ll r_g$),
- Determines **synchrotron** radiation and **magnetic pressure**,
- Contributes to precursor plasma **heating** (important for R_{tot} and T_2).

Resonant streaming instability:

- Produces a **smaller** B_{eff}
- Amplifies **large wavelengths** ($\lambda \approx r_g$);
- Determines the **high-energy cutoff** of particle spectra;
- Contributes to plasma **heating** (important for R_{tot} and T_2).