Heavy ion acceleration at quasi-parallel shocks

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The theoretical study of alpha particle acceleration at a quasi-parallel shock due to interaction with Alfven waves self-consistently excited in both upstream and downstream regions was conducted using a scale-separation model. The model uses conservation laws and resonance conditions to find where waves will be generated or dumped and hence particles will be pitch--angle scattered as well as the change of the wave energy due to instability or damping. It includes in consideration the total distribution function (the bulk plasma and high energy tail), so no any assumptions (e.g. seed populations, or some ad-hoc escape rate of accelerated particles) are required. In previous studies heavy ions were treated as perfect test particles, they only experienced the Alfven turbulence excited by protons and didn't contribute to turbulence generation. In contrast to this approach, we consider the ion scattering on hydromagnetic turbulence generated by both protons and ions themselves. It is important for alpha particles with their relatively large mass-loading parameter $P = n_\alpha m_\alpha / n_p m_p$ *that defines efficiency of the wave excitation by alpha particles. The energy spectra of alpha particles are found and compared with those obtained in test particle approximation.*

The idea of diffusive shock acceleration (DSA) at quasi-parallel shocks [*Krymsky*, 1977; *Axford et al*., 1977; *Bell*, 1978a,b; *Blandford and Ostriker*, 1978] based on the first order Fermi mechanism of acceleration of charged particles by clouds of Alfven waves excited on both sides of a shock wave was successful in explanations of the main characteristics of accelerated particles. Following the work of *Skilling* [1975a,b,c] and *Bell* [1978a,b], *Lee* [1983] and *Gordon et al.* [1999] developed the quasi-linear model that describes self-consistently the ion acceleration and the wave excitation at interplanetary shocks. It was assumed that some flux of protons (seed population) is injected at the shock front into the upstream plasma and dynamics of only this population due to interaction with waves was studied. As a result the cyclotron instability [*Sagdeev & Shafranov*, 1960], Alfven waves are excited which scatter some particles to the downstream region creating the conditions for the first order Fermi mechanism. Similarly to *Skilling* [1975a,b,c] and *Bell* [1978a,b], it was assumed as well that the particle pitch angle scattering due to cyclotron interaction is the fastest process, thus they have always an almost isotropic distribution function. The pitch-angle averaged distribution function is a solution of the so-called convection-diffusion equation with right-hand side describing the ion source term (see e.g. *Malkov & Drury* [2001]). It was assumed that the flux density of the seed particles in the source term could be found by using results of the shock observations.

It was discussed long ago that the thermal plasma can be considered as a source of the accelerated protons and there was considerable evidence from observations and simulations that shocks can directly accelerate ambient thermal particles (see reviews by *Drury* 1983 and by *Jones & Ellison* 1991 and references therein). *Malkov & Volk* 1995 have shown that the wave fields needed for proton acceleration can be excited by a beam of the downstream tail protons injected into the upstream region. Since the DSA theoretical models didn't consider the thermal plasma dynamics, they couldn't, in principle, include this mechanism in the *macroscopic* acceleration picture and describe the process of proton acceleration from the thermal plasma. Another shortcoming of the DSA models: They don't include the back reaction of the accelerated particles on the shock wave that can limit the process of acceleration. And again, this shortcoming is connected with limitations of DSA approach that evaluates only the dynamics of the high energetic tail of proton distribution.

Galinsky & Shevchenko (2007) proposed a new theoretical/numerical macroscopic model of the proton acceleration at quasi-parallel shocks that automatically includes both the thermal plasma injection scenario and modification of the shock structure due to the reaction of the accelerated protons. Similarly to the analytical consideration by *Lee* (1982) and *Gordon et al.* (1999), this model assumed also that resonant wave-particle interaction is the fastest process of the problem.

However, there are important differences between the approach used by *Lee* (1982) and *Gordon et al.* (1999) and the one used in the new model. In contrast to *Lee* (1982) and *Gordon et al.* (1999), plasma protons is not divided into two classes (i) resonant protons and (ii) thermal plasma and evolution of the entire gyro-phase averaged proton distribution function is analyzed in the model. Stability of the velocity distribution in the total interval of possible parallel velocities $v_{\|}$ is investigated for each distance from the shock front at each time step. Using quasi-linear approach, the energy exchange between particles and waves for each interval of resonant parallel velocities $\Delta v_{\|}$ is analyzed and new velocity distribution function at the interval $\Delta v_{\|}$ as well as the corresponding wave power spectrum is found. As a result, a dynamics of the entire proton distribution and the wave power spectrum is studied as function of time at each distance from the shock front.

Equations of the model

We consider a planar shock wave moving along z-axis from the $+z$ direction obliquely to the magnetic field with angle θ between the magnetic field and z-axis in upstream region and will work in the wave front system of reference. We assume that initially the shock front at $z = z_0$ divides the upstream and downstream plasmas with Maxwellian distributions with parameters that satisfy Rankine- Hugoniot boundary conditions (see e.g. [*Jones & Ellison*, 1991]).

The boundary conditions are only needed at time t=0, since in the new model the spatial-temporal problem of plasma–wave dynamics is solved by considering the upstream and downstream plasmas as one plasma with inhomogeneous parameters.

As in Lee [1983], Gordon et al. [1999], we do not use any external forces and rely only on resonant wave--particle interaction self--consistently included in the model to excite Alfven waves and accelerate particles. We introduce wave actions

$$W^{\pm}(t, z, \omega_k) = |B_k^{\pm}|^2 / 8\pi\omega_k \qquad (1)$$

which describe the wave packets propagating parallel (+) and anti-parallel (-) to the external magnetic field in a medium with varying parameters that are calculated from momentum of the particle distribution function. $|B_k|^2$ is a spectral density of the wave magnetic field.

The resonance conditions for interaction with such waves have the form

$$\omega_k - kv_{||} \mp \omega_c = 0 \qquad (2)$$

Here ω_k and k are the frequency and wave number of Alfven waves, ω_c is the proton cyclotron frequency. The $\mp$ signs in Equation (2) correspond to interactions at normal and anomalous Doppler resonances with left- and right-handed waves.

Cyclotron resonant interaction leads to pitch angle scattering of particles, so resonant protons interacting at both possible resonances (2) with each of broadband packets field $W^{\pm}(t, z, \omega_k)$ diffuse along the lines [Vedenov et al., 1962; Rowlands et al., 1966; Kennel and Engelman, 1966]

$$w^{\pm} = v_{\perp}^2 + (v_{||} - v_{ph}^{\pm})^2 = const \qquad (3)$$

where $v_{ph}^{\pm} = \pm v_A$ is the phase velocity of waves propagating parallel (+) and anti-parallel (-) to the magnetic field.

As a result of resonance interaction, a shell-like distribution of protons $f = f(w)$ is formed in the interval of resonant velocities v_z.

We will study the dynamics of the wave excitation and particle acceleration relying on the thermal plasma as a source of so called seed population that excites waves needed for particle acceleration. Thus, we will analyze stability of the entire gyro-phase averaged proton distribution function. By averaging the proton kinetic equation over period of oscillations we obtain in quasi-linear approximation the equation for the so-called background distribution function of protons $f(t, z, v_{||}, v_{\perp})$:

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} = QL_f(f, W^+, W^-) \qquad (4)$$

Equations for wave actions in quasi-linear approximation have the form:

$$\frac{\partial W^{\pm}}{\partial t} + \frac{\partial}{\partial z}(v_{gz}^{\pm} W^{\pm}) = QL_{W}^{\pm}(f, W^{\pm}) \tag{5}$$

Here $v_{gz}^{\pm}$ is z-component of the group velocity for waves propagating in parallel and anti-parallel directions.

Right-hand-side terms in equations (4)-(5) are quasi-linear operators that take into account all four possible interactions with left- and right-hand polarized MHD waves propagating in both directions along the magnetic field.

The equations (4)-(5) describe processes of the wave excitation and particle acceleration in the system under consideration. Since these equations are non-stationary and non-homogeneous nonlinear equations, we solve them numerically. The region over z where processes of the wave excitation and particle acceleration take place stretches in both directions from the shock front. The size L of the region is chosen rather large so all waves are kept in this region during the simulation. We will check that this is the case during numeric solution of equations. We divide the region L into small intervals with locations

$$z_i, i = 1,2,3,\ldots N_z \tag{6}$$

and solve equations (4)-(5) at each interval for each time step.

To do this, we introduce the wave spectrum with a wide band of possible wave numbers

$$k_i, i = 0,1,...N_k \tag{7}$$

to assure resonant cyclotron interaction with protons that have any parallel velocity from the entire possible interval of resonant velocities $v_{\|}$ on their distribution function.

In the new model a scale-separation techniques is used to solve equations (4)-(5) numerically by assuming that characteristic temporal and spatial scales of pitch-angle diffusion are smaller than the time step and size of each spatial interval. That means that steady state of the plasma-wave system is developed at each time step at any distance from the shock front. The steady state is settled in two cases: (i) when the pitch-angle averaged distribution function is formed $\partial f / \partial v_{\|}\big|_{w=const} = 0$ or (ii) when the resonant waves are totally absorbed $W_k = 0$ at any interval of resonant velocities. Thus, at the end of each time step, the right-hand-side terms in equations (4)-(5) are equal zero.

For the purpose of calculating the input from the resonant particles to the waves and vice-versa, we divide the entire region of possible parallel and perpendicular velocities into small intervals with grid locations

$$v_{\|}^{i}, i = 1,2,3,...N_{v_{\|}} \tag{8}$$

$$v_{\perp}^{i}, i = 1,2,3,...N_{v_{\perp}} \tag{9}$$

To calculate input from waves to resonant particles and vice-versa, we need to find the free energy available in the particle distribution for each resonant velocity interval. The amount of free energy available in nth-interval of resonant velocities $v_{\|} \in [v_{\|}^{n} - \delta v_{\|}, v_{\|}^{n} + \delta v_{\|}]$ is just a difference between the energy contained in the current particle distribution function (PDF) and the energy contained in the pitch-angle scattered PDF both taken over the resonance interval. We find the pitch-angle average function at each time step using conservation of the proton number along diffusion lines (3):

$$\left\langle f(t,z,v_{\|},v_{\perp}) \right\rangle^{\pm} = \int_{S_n^{\pm}} f(t,z,w^{\pm},\alpha-\alpha')d\alpha' \Big/ \int_{S_n^{\pm}} d\alpha' \tag{10}$$

The α coordinate is directed along lines $w^{\pm} = const$ and $S_n^{\pm}$ is the n-interval of resonant velocities $v_{\|} \in [v_{\|}^{n} - \delta v_{\|}, v_{\|}^{n} + \delta v_{\|}]$. The amount of particle free energy for each resonance region is obtained as a difference of kinetic energy in final and initial states in a frame of reference where the bulk of the plasma (with a bulk velocity $u(t,z)$) is at rest

$$\Delta F_n^{\pm}(t,z) = \frac{m_p}{2} \int_{S_n^{\pm}} \left[\left\langle f(t,z,v_{\|},v_{\perp}) \right\rangle^{\pm} - f(t,z,v_{\|},v_{\perp}) \right] \left\{ v_{\perp}^2 + [v_{\|} - u(t,z)]^2 \right\} v_{\perp} dv_{\perp} dv_{\|} \tag{11}$$

We calculate a balance of energy between waves and particles for each resonance interval as

$$\Delta E_n^{\pm}(t,z) = \sum_{k\in\{n\}} E^{\pm}(t,z,\omega_k) - \Delta F_n^{\pm} \tag{12}$$

Where $\{n\}$ represents all the waves belonging to the n-th resonant interval that can be found from the resonance condition (2); $\Delta E^{\pm}(t,z)$ is change of the total wave energy density in nth resonance region with the quantity $E^{\pm}(t,z,\omega_k)$ defined by

$$E^{\pm}(t,z,\omega_k) = -W_k \frac{\partial D}{\partial \omega_k} \frac{\omega_k^2}{k^2c^2} \tag{13}$$

$$D(\omega_k, k) = k^2c^2 + \omega_{pp}^2 \frac{\omega_k}{\omega_k - \omega_{cp}} + \omega_{pe}^2 \frac{\omega_k}{\omega_k + \omega_{ce}} \tag{14}$$

$E^{\pm}(t,z,\omega_k)$ is the sum of potential and kinetic energy density of the wave with frequency ω_k that is defined from equation

$$D(\omega_k, k) = 0$$

In our case they are equal each other and $E^{\pm}(t,z,\omega_k) = |B_k^{\pm}|^2 / 4\pi$.

Algorithm of the numerical solution

The algorithm for numeric solution of equations (4)-(5) can be described as the following procedures:

1. It accordance with assumption that the steady state of the plasma-wave system is developed at each time step at any distance from the shock front, we update the distribution function as well as the wave spectrum at the beginning of each time step by integrating equations (4)-(5) with zero right-hand sides. We obtain the new distribution function at every $z \times v_{||} \times v_{\perp}$ grid location by using flux conservation and assuming one dimensional streaming of plasma in force-free environment. The waves are updated for every z_i by using conservation of their energy flux in streaming medium with locally varying parameters.
2. We use the new distribution function to find the proton density, temperature as well as the local rest frame of reference for every z_i and calculate the pitch angle lines (3).
3. By using equation (10), we obtain the pitch angle scattered distribution function in every resonant region $v_{||} \in [v_{||}^n - \delta v_{||}, v_{||}^n + \delta v_{||}]$.
4. After that, using equation (11), we check if the energy of particles is increased or decreased, that is if the distribution function is

stable or unstable with respect to wave generation in the current resonance interval.

4.1 If it is unstable, we use the pitch angle scattered particle distribution function as a new one for the current resonance interval. We assign the available energy to the waves in this interval by using equation (12) and proceed to next resonant interval.

4.2 If the particle distribution function is stable and waves are present in the current resonant region, they will damp and transfer their energy to the particles. The new particle distribution function and the new wave level in the resonant region are found by using the same equations (10)-(12).

4.3 Since the levels of the stable functions can be different for neighboring resonant intervals that can lead to instability, the entire procedure is repeated iteratively until we arrive to the (quasi)-stationary partitioning of energy between waves and particles.

In the case when initially there are waves in the plasma, we should first perform procedure 4.2 by using equations (10)-(12) and proceed after that with solution of equations at the first time step using the described above numerical algorithm of solution.

Results of tests of the model

DSA theory predicts the power-law spectra for high-energy tails on the PDF in upstream region:

$$f(v_{||}) \sim v_{||}^{-\beta} \ , \quad \beta = 3r/(r-1), r = u_1/u_2$$

u_1, u_2 are the plasma speed in upstream and downstream regions.

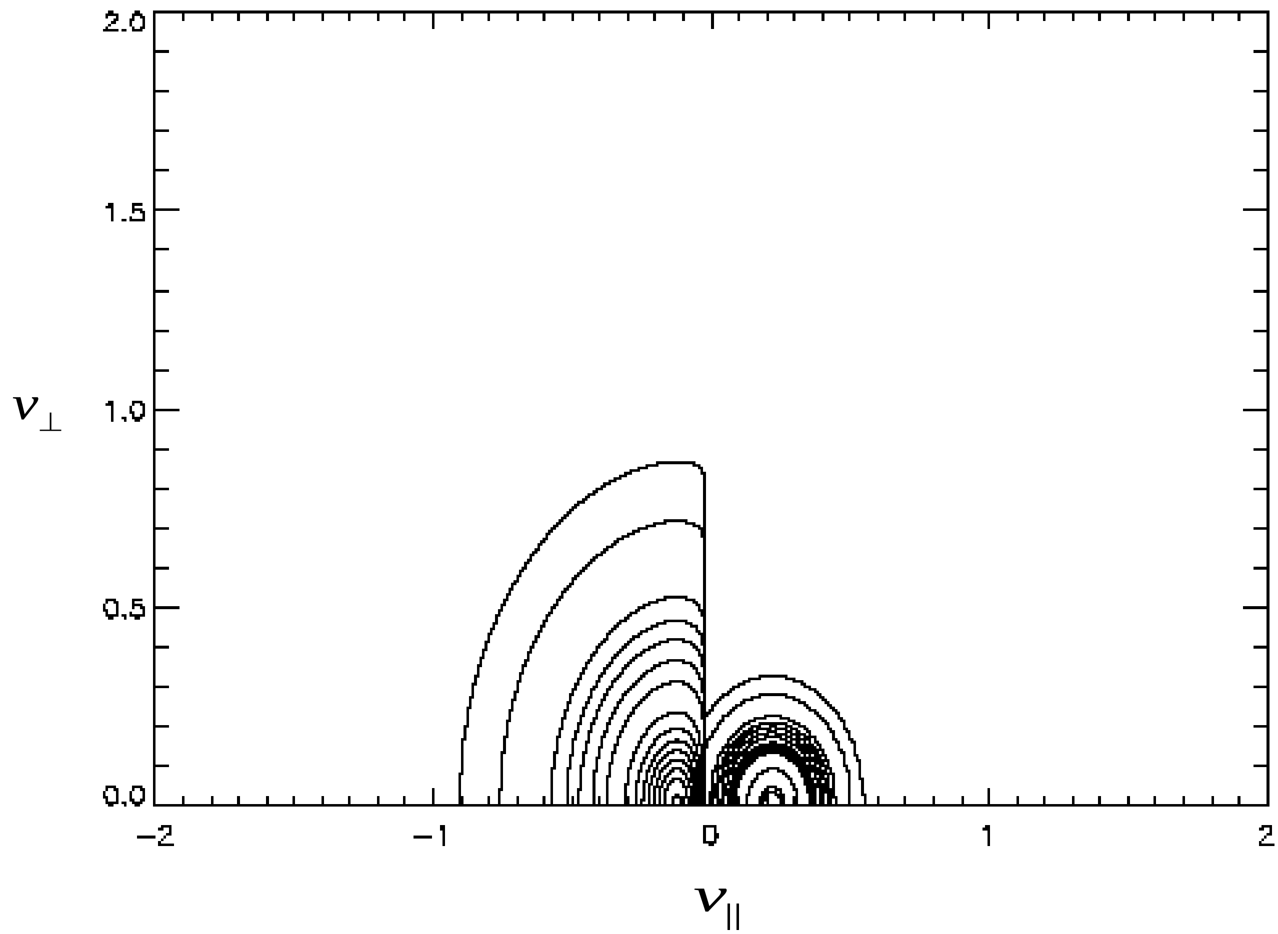
We used the model to study the particle acceleration by interplanetary shock, November 11-12, 1978 observed by Kennel et al., 1986.

Parameters:

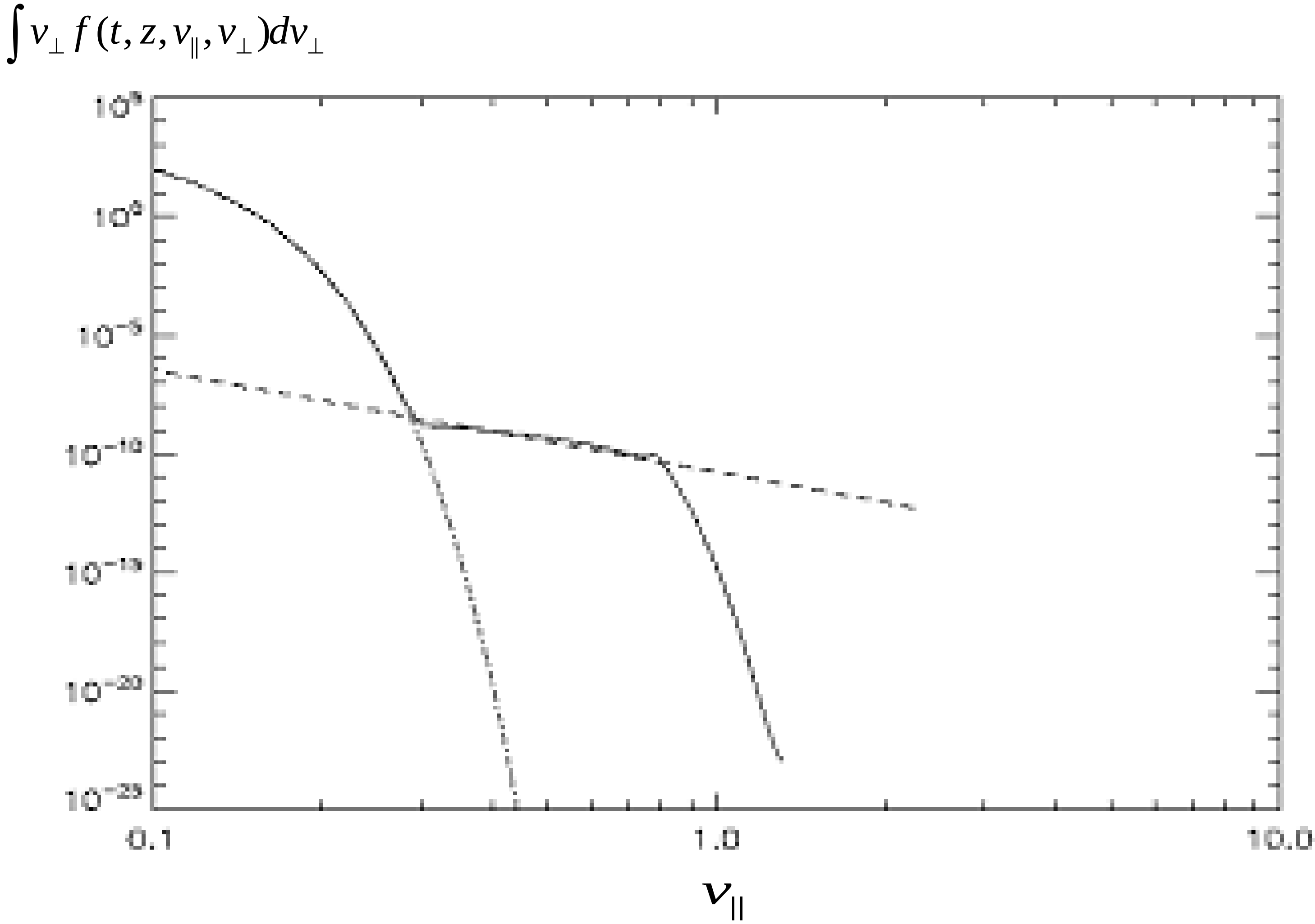
$B_0 = 6.85nT$, $n_1 = 4cm^{-3}$, $u_1 = 2.4\times 10^7 cm/s$

$\theta = 41^\circ$, $\beta = 4.3$

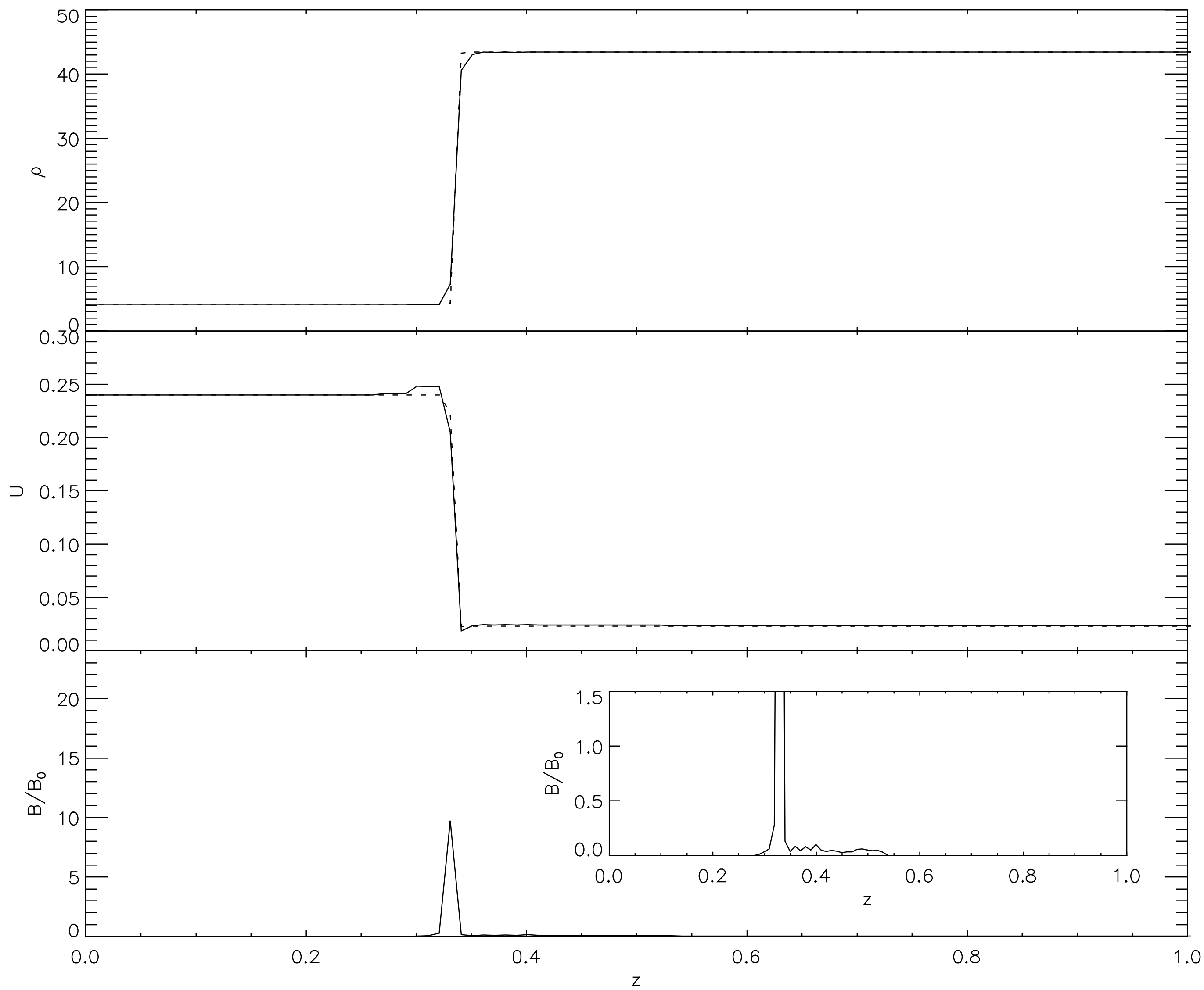
- **No seed population**
- **No wave fluctuations at initial time**
- **Maxwellian distributions of protons in both upstream and downstream regions**



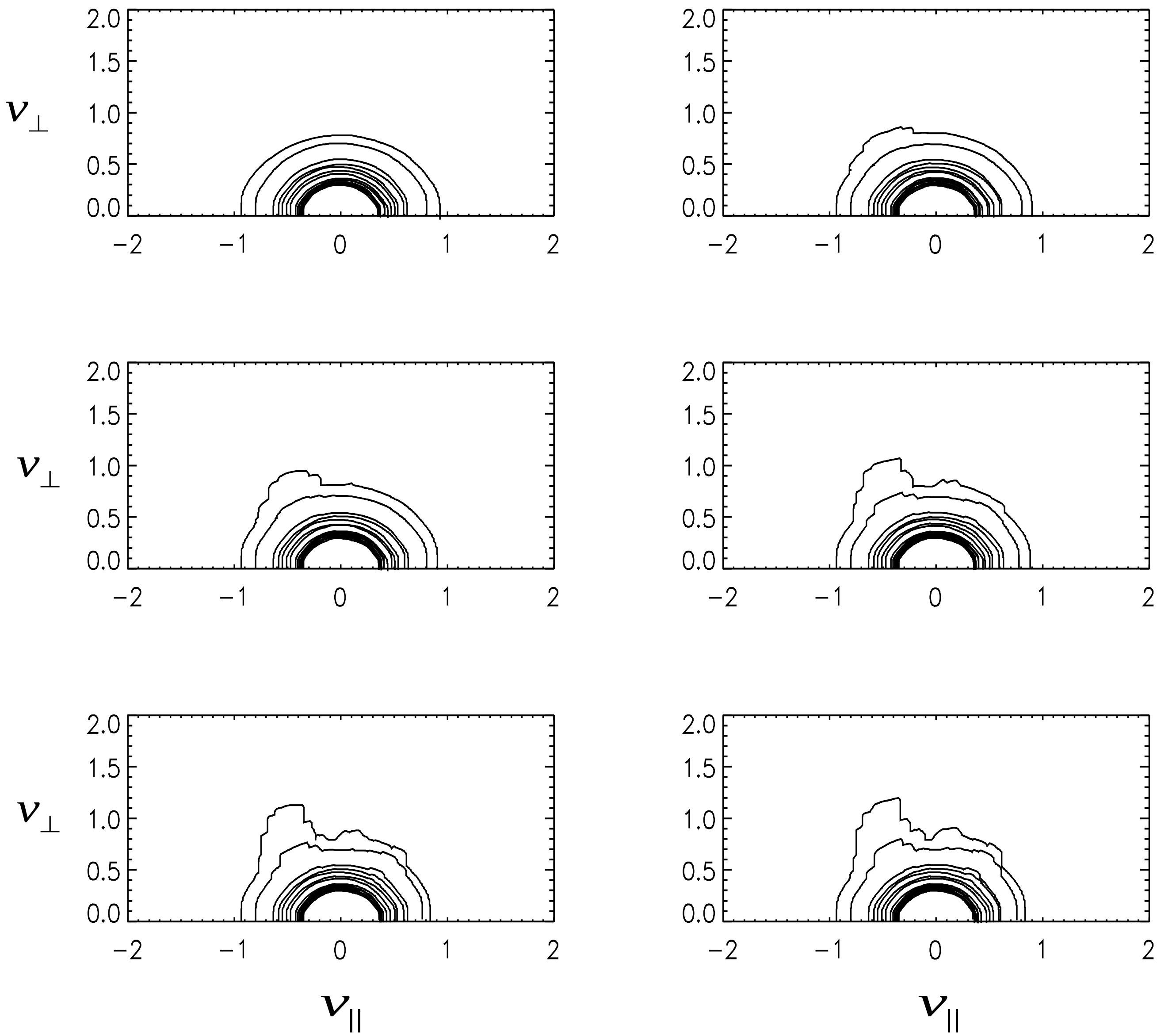
Contour plots of the upgraded upstream particle distribution function (PDF) at the initial time step. It can be seen from the figure that energetic tails of the downstream distribution penetrate in the upstream region and create a beam-plasma like distribution that is unstable and will generate an upstream waves. Thus, it is quite clear that the beam-like part of the particle distribution created from the thermal core itself will provide a source of the energetic (seed) population.



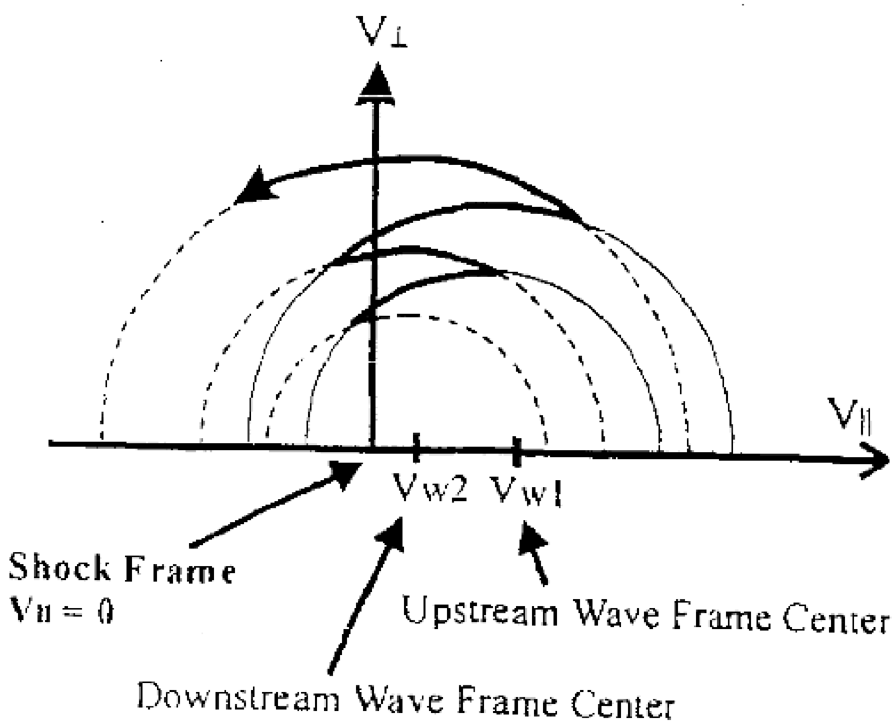
Logarithmic plots of the PDF integrated over $v_\perp$ as a function of $v_\parallel$ for several distances from the shock front (in arbitrary units) for shock parameters taken from ISEE-3 data (Kennel et al, 1986). Dashed line is the line $v_\parallel^{-\beta}$ for the same parameters.



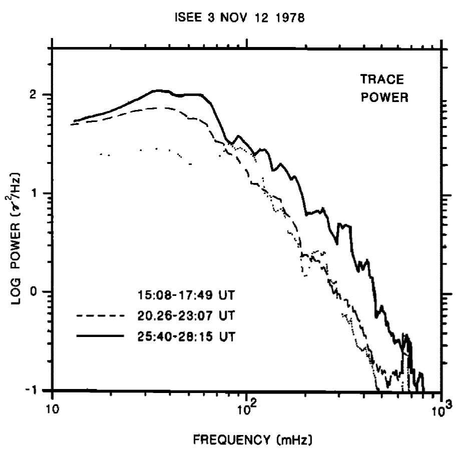
Profiles of density, bulk velocity and wave amplitude (from top to bottom, ordinate units are arbitrary, z is in units of 2106 r_B, where r_B is the proton gyroradius). In the first two panels the solid lines show calculated profiles and dotted lines correspond to the initial shock discontinuity.

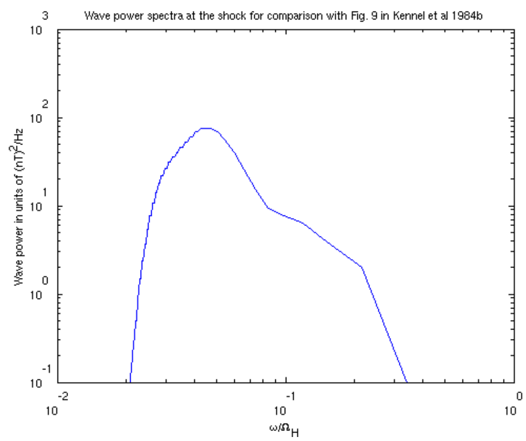


Time sequence of contour plots of PDF in downstream region in close vicinity of the shock front (from left to right, then from top to bottom, units are arbitrary).



Ions interacting with waves in the upstream region move along the solid circles. After crossing the shock front to the downstream region, interaction with downstream waves makes them to move along the dashed circles. If ions change the parallel velocity to a negative value (i.e. directed upstream) as they interact with downstream waves, they can return to the shock front and escape to the upstream region (Sugiyama & Terasawa, 1999).





Upstream wave spectra near the shock. Insert shows results of analysis of ISEE-3 data (from Kennel et al, 1986) (100 mHz frequency in spacecraft frame corresponds to 10^{-1} ω/Ω_H in solar wind frame).

Acceleration of heavy ions

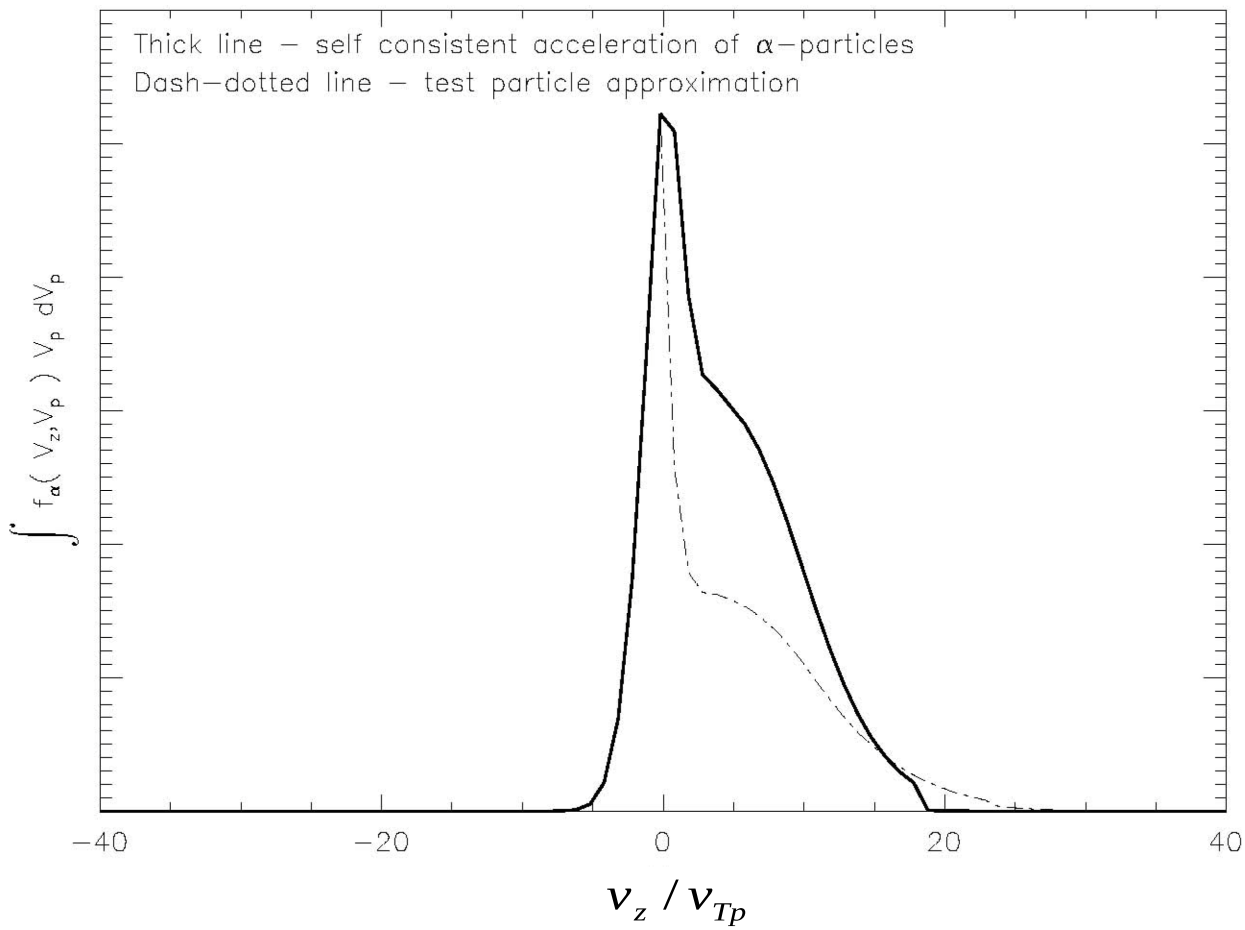
In previous studies heavy ions were treated as perfect test particles, they only experienced the turbulence excited by protons and didn't contribute to turbulence generation. In contrast to this approach, we will consider the ion scattering on hydromagnetic turbulence generated by both protons and ions themselves. This means that we should have two equations (4) for protons and heavy ions and should add second term in the right hand side of equation (5) that describes generation/absorption of waves by ions. The equations that were used in our scale-separation approach have the dimensionless form:

$$\frac{\partial f_i}{\partial t} + v_z \frac{\partial f_i}{\partial z} = 0 \quad (i = p, \alpha) \tag{15}$$

$$\frac{\partial W^{\pm}}{\partial t} \pm \frac{\partial}{\partial z}\left(\frac{v_A}{v_{A1}} W^{\pm}\right) = 0 \tag{16}$$

$$\begin{aligned}
&\Delta \tilde{B}^{2\pm}_{n}(t,z) = \tilde{B}^{2\pm}_{n}(t,z) \\
&- \pi\beta_1 \int_{S_n^{\pm}} \left[\left\langle f_p(t,z,v_{\|},v_{\perp}) \right\rangle^{\pm} - f_p(t,z,v_{\|},v_{\perp}) \right] \left\{ v_{\perp}^2 + [v_{\|} - u_p(t,z)]^2 \right\} v_{\perp} dv_{\perp} dv_{\|} \\
&- P\pi\beta_1 \int_{S_n^{\pm}} \left[\left\langle f_\alpha(t,z,v_{\|},v_{\perp}) \right\rangle^{\pm} - f_\alpha(t,z,v_{\|},v_{\perp}) \right] \left\{ v_{\perp}^2 + [v_{\|} - u_\alpha(t,z)]^2 \right\} v_{\perp} dv_{\perp} dv_{\|}
\end{aligned}$$

Here v_{A1}, $\beta_1 = v_{T1}^2 / v_{A1}^2$ are correspondingly the initial Alfven speed and gas kinetic to magnetic pressure ratio in the upstream region; $P = n_\alpha m_\alpha / n_p m_p$ is the mass-loading parameter. We introduced here the magnetic variance $\tilde{B}^{2^{dls}} = \tilde{B}^2 / B_{01}^2$, where $\tilde{B}^2 = \sum |B_k|^2$. The last term describes input of alphas in energy exchange with waves, it is proportional to the mass-loading parameter and is sufficient in comparison with proton input.


Thick line – self consistent acceleration of α–particles
Dash–dotted line – test particle approximation
$\int f_\alpha(V_z, V_p)\, V_p\, dV_p$
−40
−20
0
20
40
v_z / v_{Tp}

SUMMARY

- The theoretical study of alpha particle acceleration at a shock due to interaction with Alfven waves self-consistently excited in both upstream and downstream regions was conducted using a new theoretical scale-separation model.

- The main difference of the model from DSA models is that it does not treat the accelerated particles as a separate substance but, similar to shock simulations, considers them as an integral part of the plasma distribution.

- The model automatically includes an injection scenario in the macroscopic picture of the particle acceleration at shocks and confirms that neither additional population of suprathermal seed particles nor external wave turbulence are required for the acceleration process to operate.

- The numerical analysis based on the model shows quite good agreement with DSA models in predicting power spectra of accelerated particles. However, it was shown that there is a break of the power spectra at large energies of particles.

- Signatures of acceleration of thermal core particles by a mechanism similar to the second-order Fermi acceleration at close proximity to the shock front were obtained.

- It was shown that in case when the hydromagnetic turbulence is generated by both protons and alphas themselves, the number of accelerated alpha particles is much larger than in case when they are treated in test particle approximation.