Inverse energy transfer via magnetic reconnection

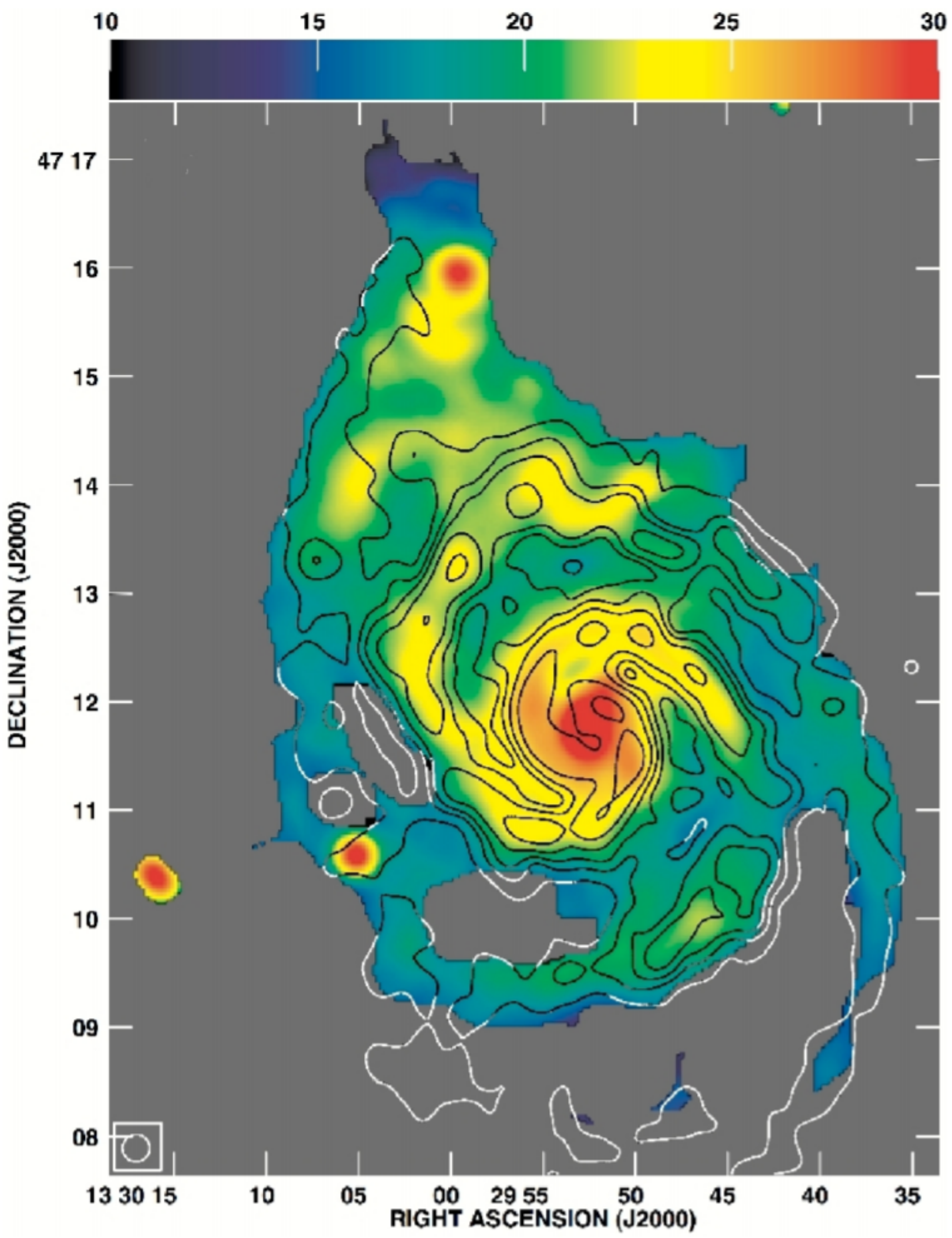
Muni Zhou[1], Nuno F Loureiro[1], Dmitri A Uzdensky[2]
Pallavi Bhat[1,3] , Greg R Werner[2]

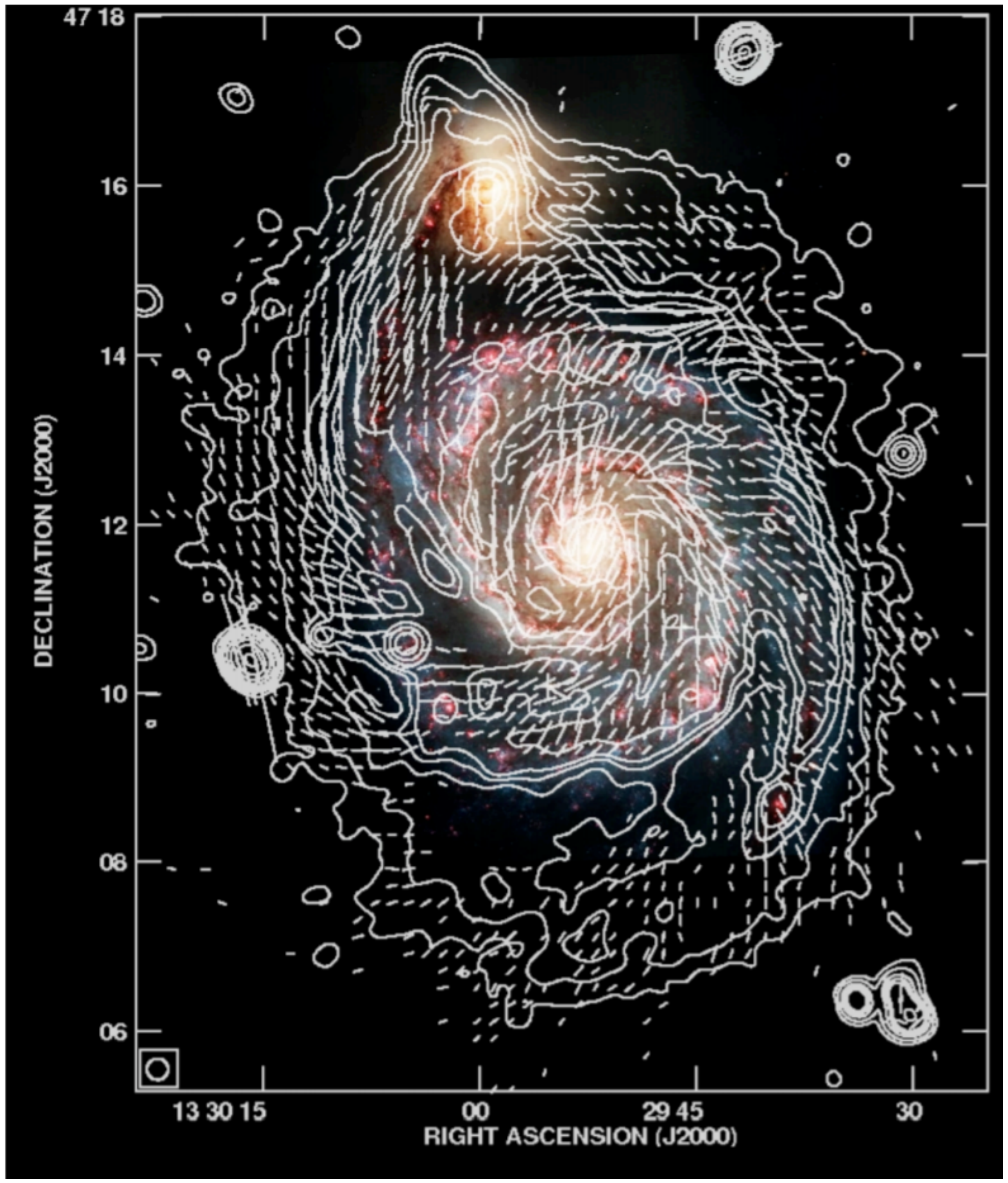
[1]MIT, [2]University of Colorado – Boulder,
[3]University of Leeds

PSFC University of Colorado Boulder

Cosmic magnetic fields

Large coherent structure and strong fields are observed



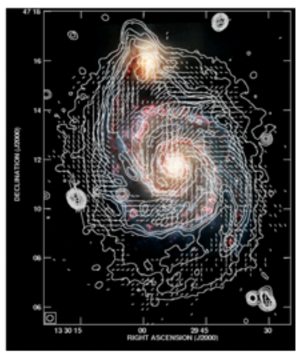


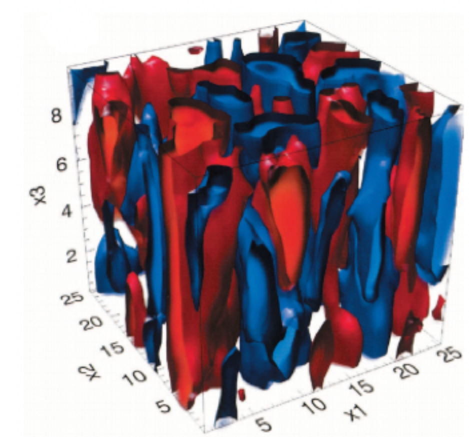
Magnetic fields of galaxy M51. [Fletcher et al. 2011]

Left: Total magnetic field strength, (color scale in μG).

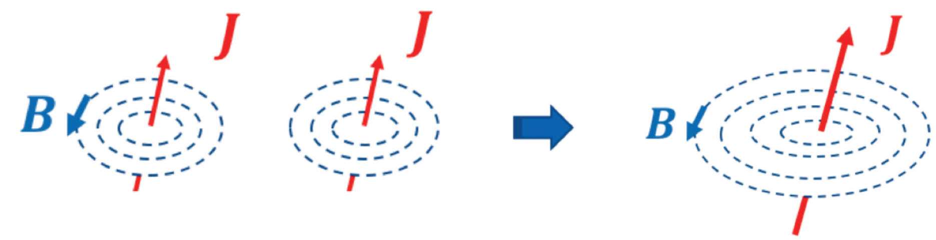
Right: the B-vectors imposed on optical image.

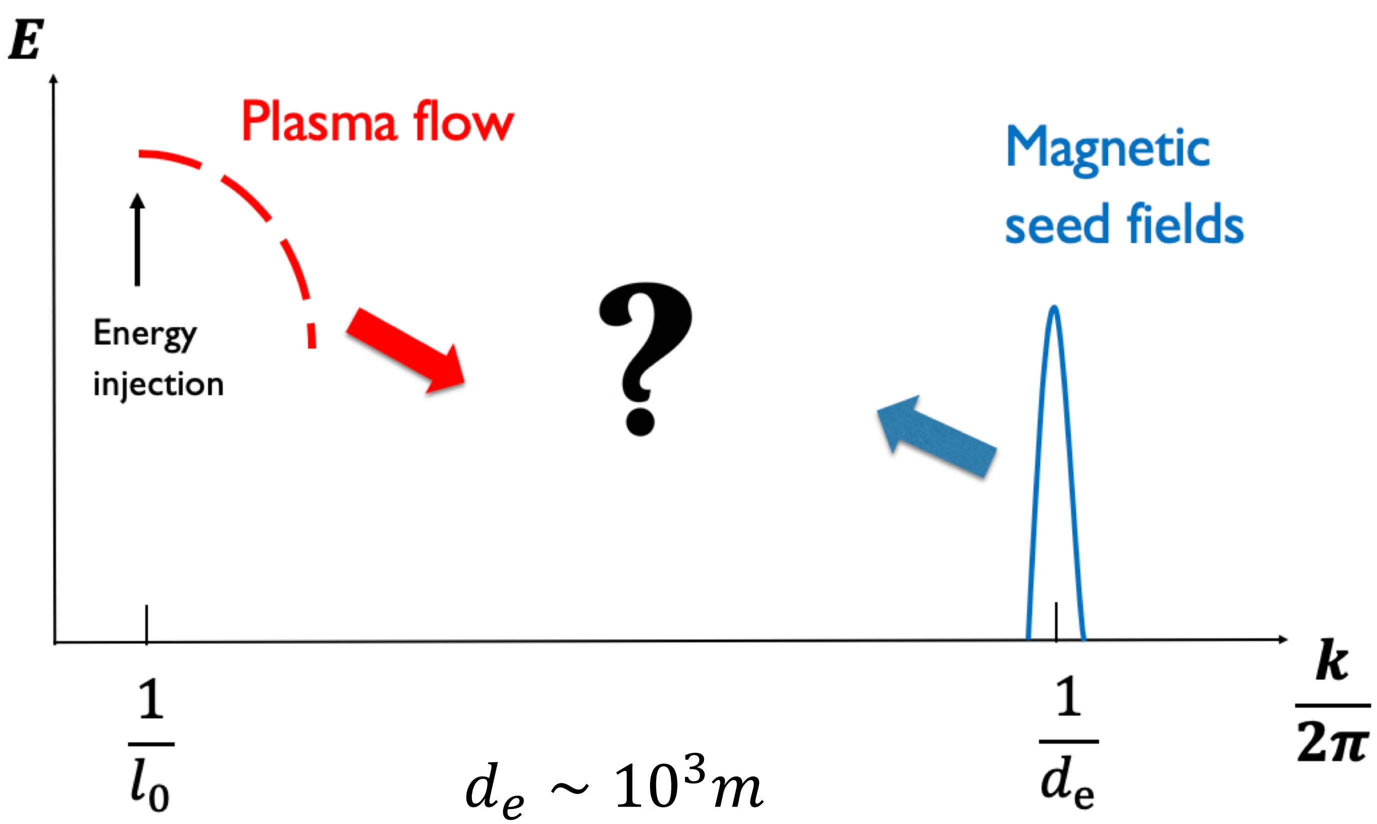
One possible route





Coalescence instability [Finn & Kaw 1977]





$d_e \sim 10^3 m$

$l_0 \sim 10^{18} m$

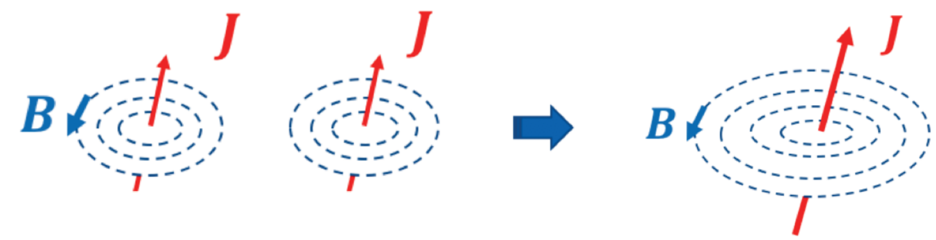
Weibel instability

[Silva et al APJ 2003]

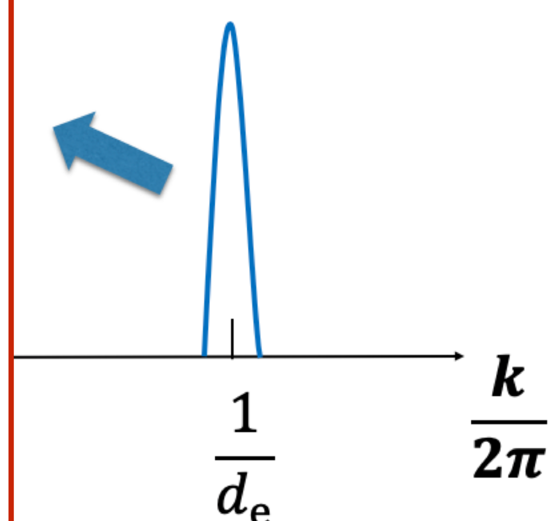
One possible route

- Propose a dynamical model based on magnetic reconnection
- Test the model in the MHD regime
- Use artificial setup of magnetic seed field

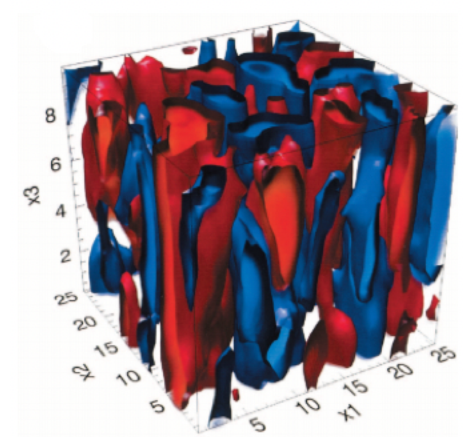
Coalescence instability [Finn & Kaw 1977]



Magnetic seed fields



Weibel instability



[Silva et al APJ 2003]

Hierarchical merger model

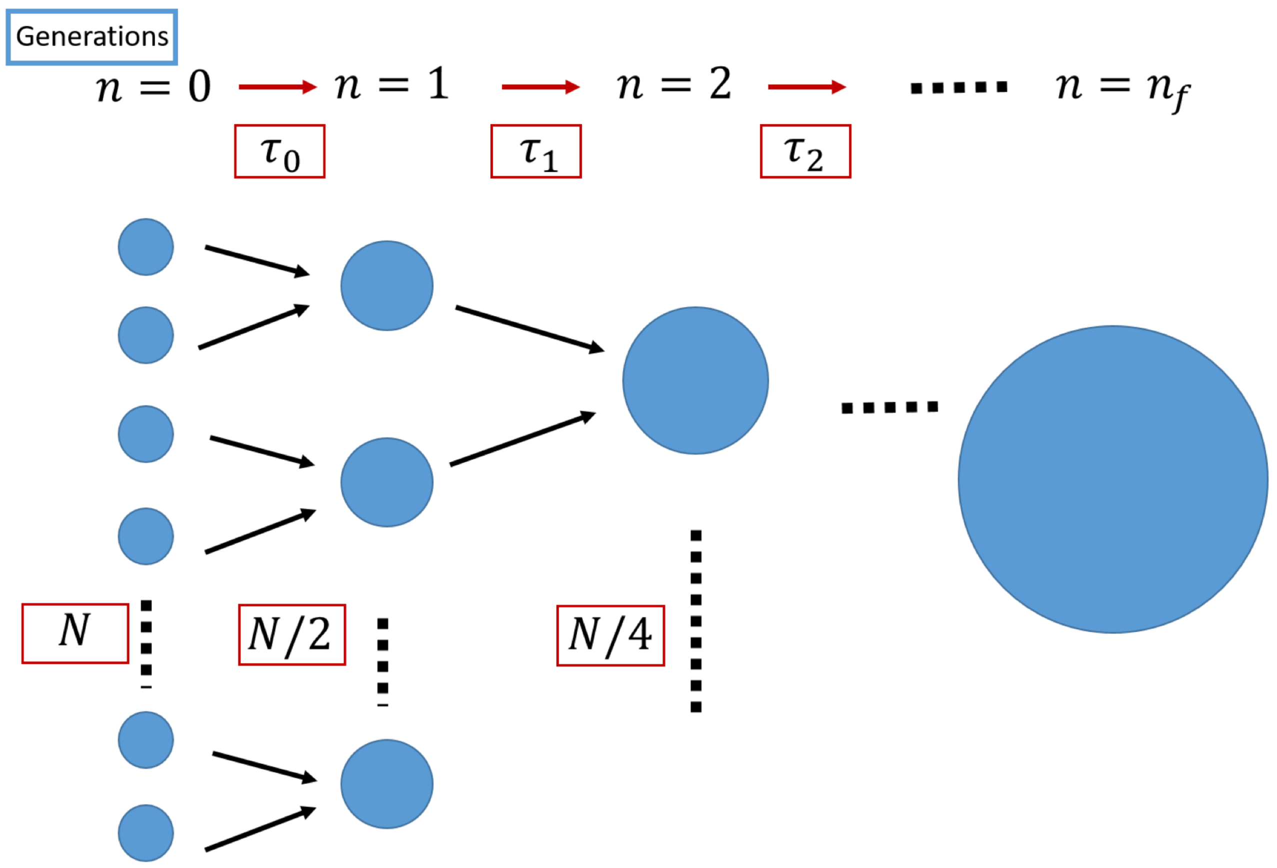
Minimal model for successive magnetic structures mergers

Assumptions

- Consider an ensemble of **identical** magnetic structures
- **Hierarchical** fashion

  → structures merge successively
- Merge in **discrete** steps

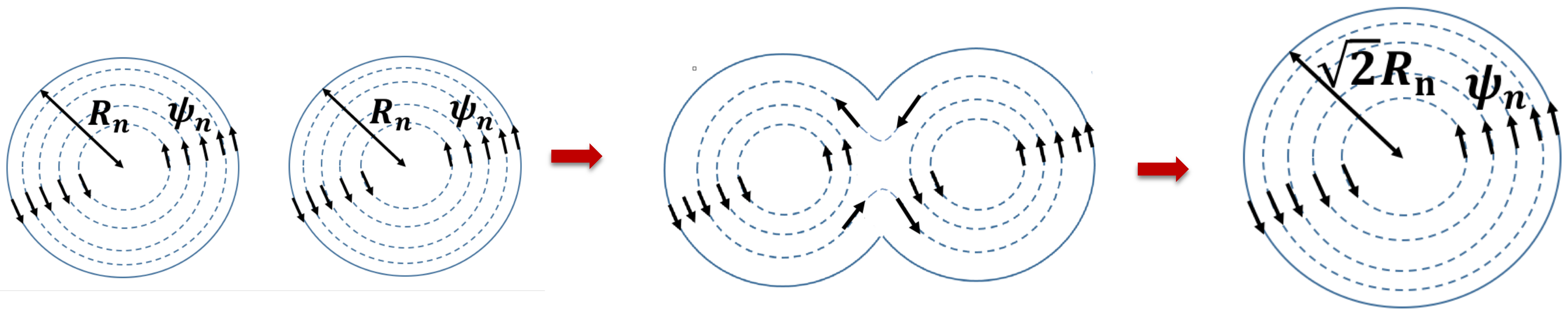
  → generation of structures denoted by n
- Merge **in pairs**



2D: Merger conserves area and flux

Transition from one generation to the next

As islands merge, physical quantities evolve with n



Characterize the nth-generation islands:

- Flux enclosed in an island ψ_n
- Typical magnetic field in an island

$$B_n = \psi_n / R_n$$

- Magnetic energy density

$$\mathcal{E}_n = B_n^2 / 8\pi$$

Conservation laws of merger

1. Mass conserved, assume **incompressibility**, area conserved

$$R_{n+1} = \sqrt{2} R_n$$

2. Flux conserved: $\psi_{n+1} = \psi_n$

And hence:

$$B_{n+1} = B_n / \sqrt{2} \quad \mathcal{E}_{n+1} = \mathcal{E}_n / 2$$

[Fermo et al, 2010; Zrake et al, 2017; Lyutikov et al, 2017]

Magnetic reconnection during mergers

Lundquist number and reconnection rate are conserved

In MHD regime: $S_n = R_n v_{A,n}/\eta \propto R_n B_n \propto \psi_n$ **is preserved, $\boldsymbol{\beta_{rec}}$ is preserved**

In collisionless regime: $\beta_{\rm rec} \approx 0.1$ is preserved

Merging process remains in the same reconnection regime in which it starts initially

Merger time for n-th generation islands:

$$\tau_n \approx R_n/v_{\rm rec,n}$$

Reconnection velocity $v_{rec,n}$ and $v_{A,n}$ are related by dimensionless reconnection rate:

$$\beta_{{\rm rec},n} \equiv v_{{\rm rec},n}/v_{A,n}.$$

Merger time evolves as $\tau_{n+1} = 2\tau_n$

Scaling laws from the hierarchical model:

$$k = k_0\tilde{t}^{-1/2}, \quad B = B_0\tilde{t}^{-1/2},$$

$$\mathcal{E} = \mathcal{E}_0\tilde{t}^{-1}, \quad N = N_0\tilde{t}^{-1}, \quad \psi = \psi_0$$

$$\tilde{t} \equiv t/\tau_0$$

Is the reconnection times scale

2D MHD simulation

Done with pseudo-spectral code Viriato [Loureiro et al, 2016]

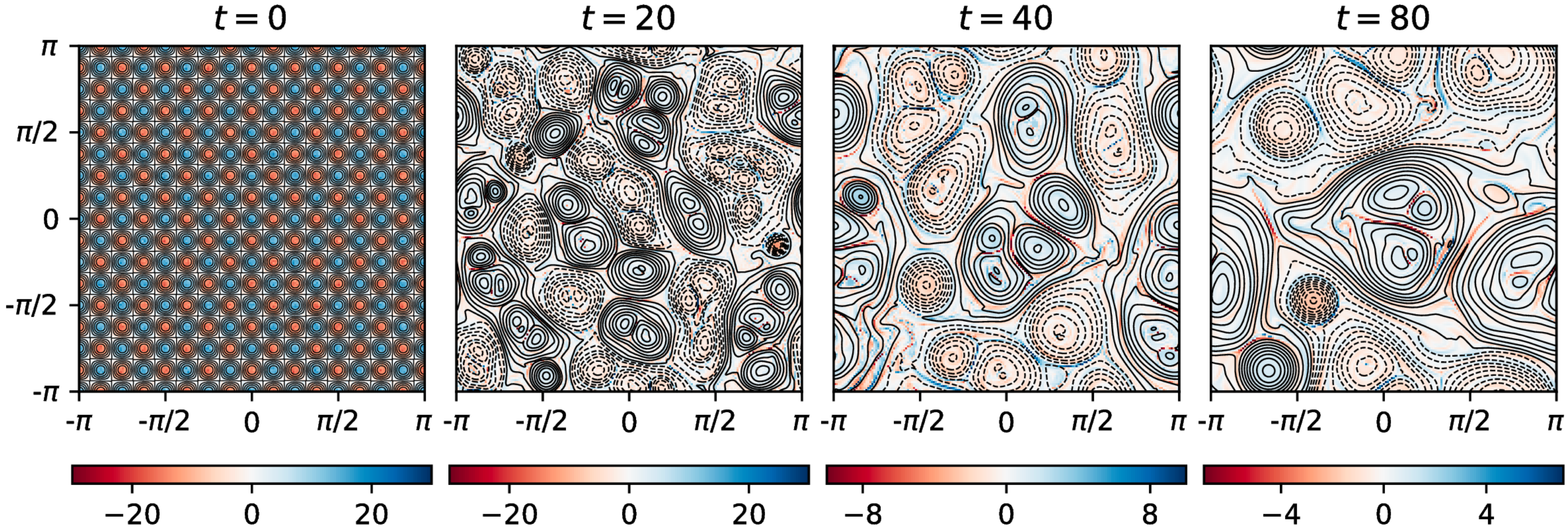
$$\partial_t \psi + \mathbf{v}_\perp \cdot \nabla \psi = \eta \nabla^2 \psi$$ Induction equation

$$\partial_t \omega + \mathbf{v}_\perp \cdot \nabla \omega - \mathbf{B}_\perp \cdot \nabla j = \nu \nabla^2 \omega$$ Momentum equation

Initial condition

$$\psi(x,y) = \psi_0 \cos(k_x x)\cos(k_y y)$$

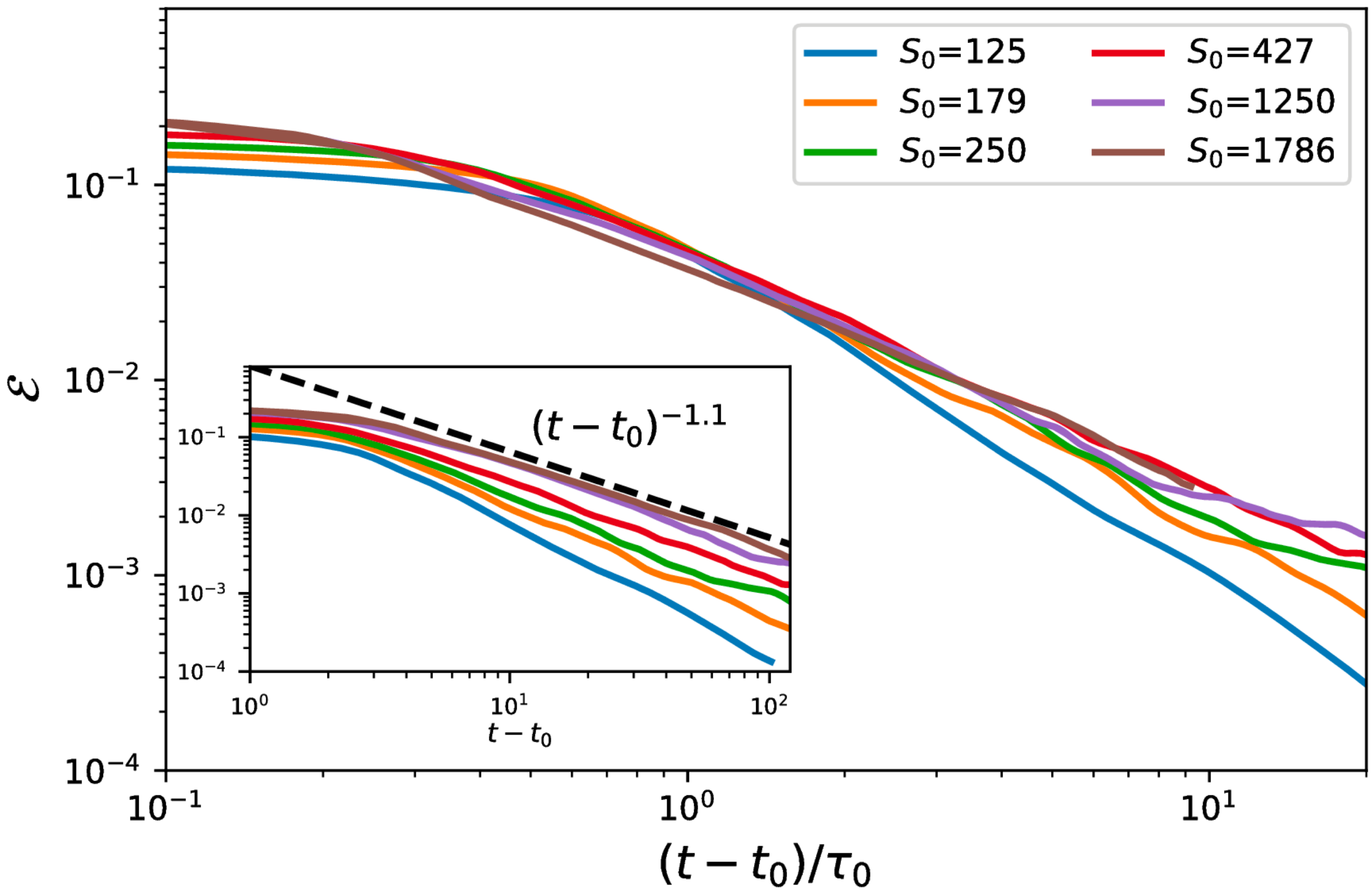
$$\phi(x,y) = 0$$



2D MHD simulation

Magnetic energy decay at reconnection time scale

Rescaled (to τ_0) energy decay curves overlap



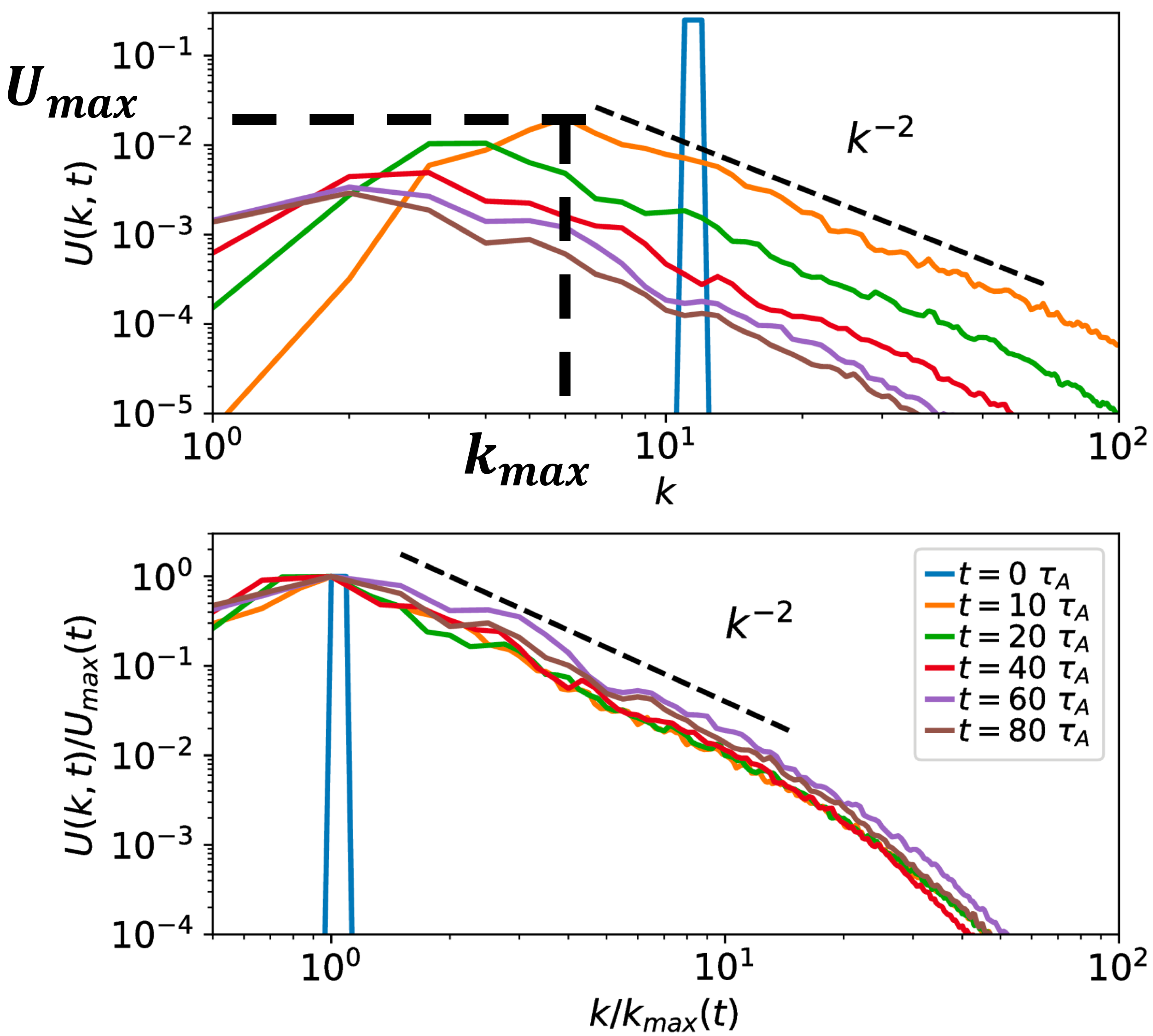
2D MHD simulation

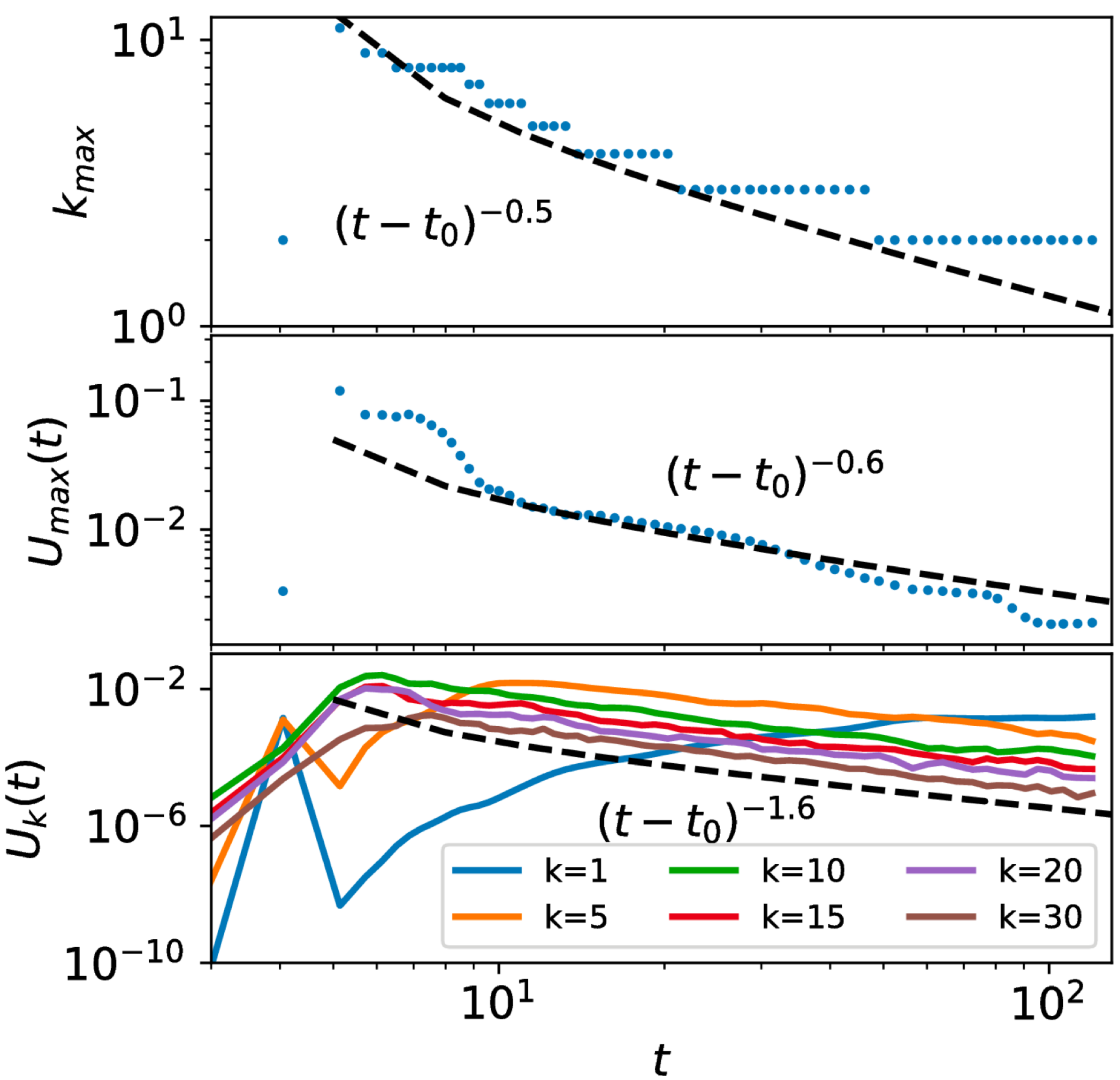
Self-similar evolution of the system

self-similar time evolution $B(k/l, l^2\tilde{t}) = l^{-1}B(k, \tilde{t})$

$$U(k,t) \propto t^{-\alpha}k^{-\gamma} \qquad 2\alpha = \gamma + 1$$

$\gamma = 2$ due to the sharp magnetic reversal at current sheets





[Zhou et al 2019]

3D Analytical model

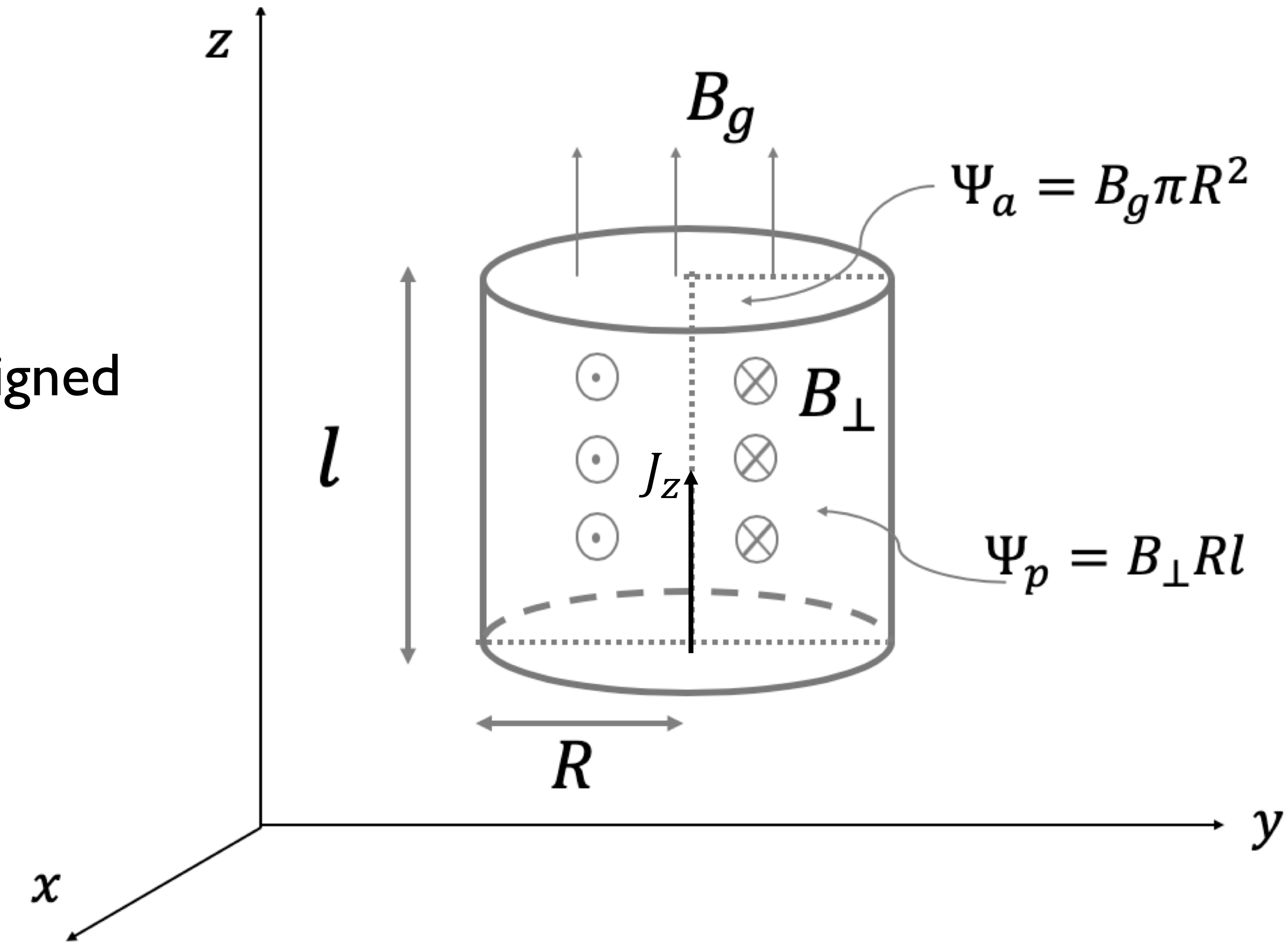
Characterization of a flux tube

consider system with:

- constant strong guide field

  (for simplicity)

- Volume-filling flux tubes aligned with the guide field, with alternating polarities



3D Analytical model

Discrete generation description

- Conservation of axial flux: $\Psi_{a,n+1} = 2\Psi_{a,n}$

  → conservation of cross-section area: $R_{n+1}^2 \pi = 2R_n^2 \pi$

- Conservation of magnetic potential $\psi_{n+1} = \psi_n \qquad (\psi = BR)$

  → conservation of Lundquist number. $S_{n+1} = S_n \qquad (S \propto RB_\perp/\eta \propto \psi/\eta)$

  → $B_{\perp,n+1} = B_{\perp,n}/\sqrt{2}$

Time scales: $\tau_\perp \sim R/B_\perp \qquad \tau_{||} \sim l/B_g$

Critical Balance: $\tau_\perp \sim \tau_{||}, \quad R/B_\perp \sim l/B_g$

[Goldreich & Sridhar 1995]

$$l_{n+1} = 2l_n$$

(No kink instability expected for tubes in RMHD)

3D Analytical model

Continuous time description

- Perpendicular dynamics: Quasi-2D merger

$$k_\perp = k_{\perp,0}\tilde{t}^{-1/2}, \quad B_\perp = B_{\perp,0}\tilde{t}^{-1/2}, \quad E_M = E_{M,0}\tilde{t}^{-1}$$

$$N_{xy} = N_{xy,0}\tilde{t}^{-1}$$

- Parallel dynamics: Alfven wave propagation

$$l = l_0\tilde{t}$$

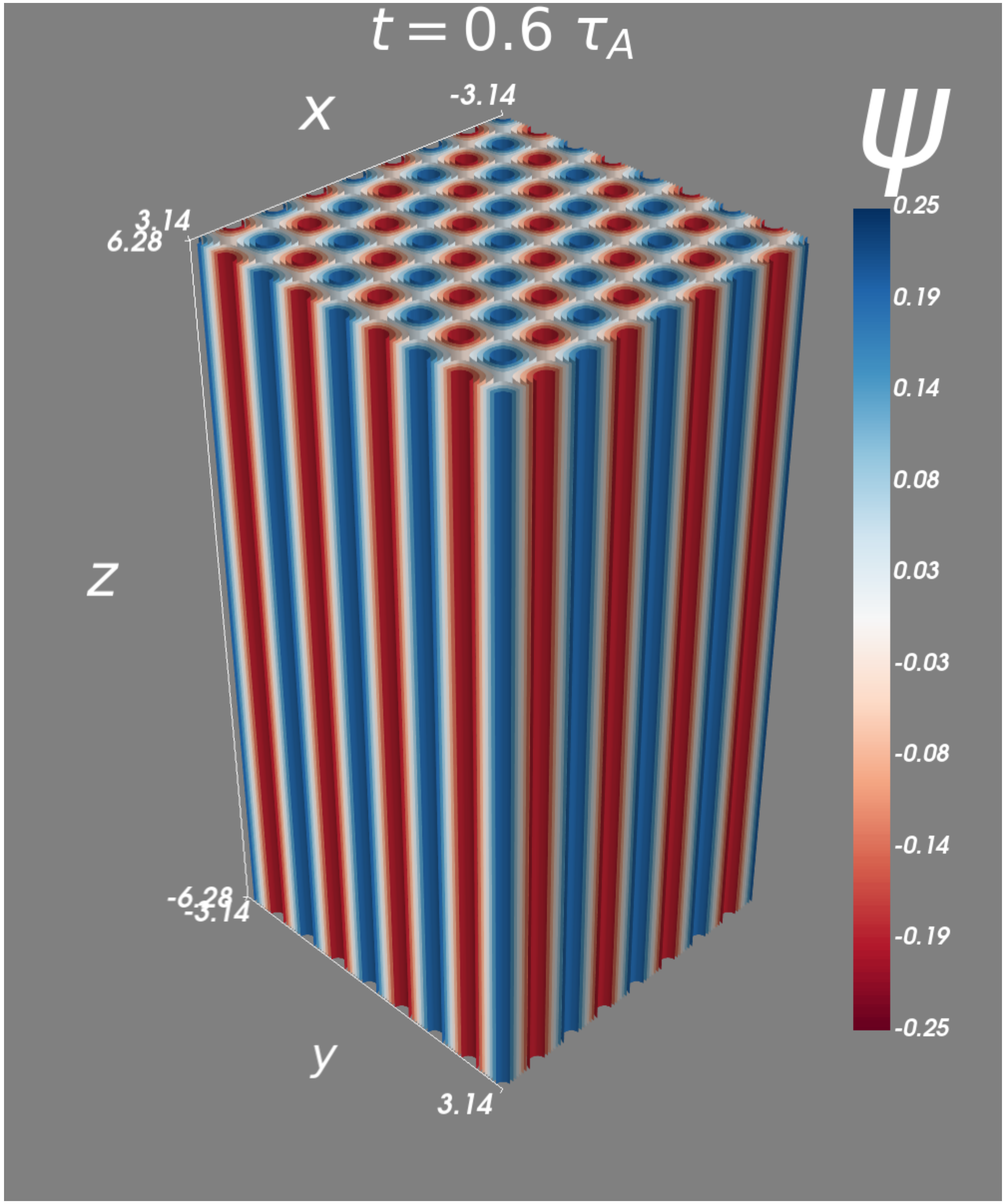
- Related through critical balance

$$\frac{l}{B_g} \sim \frac{R}{B_\perp}$$

3D Reduced-MHD simulation

$$\mathbf{B} = \mathbf{B_g} + \mathbf{B}_\perp \quad k_\perp \gg k_\parallel$$

Initial set-up of magnetic flux tubes



3D Reduced-MHD simulation

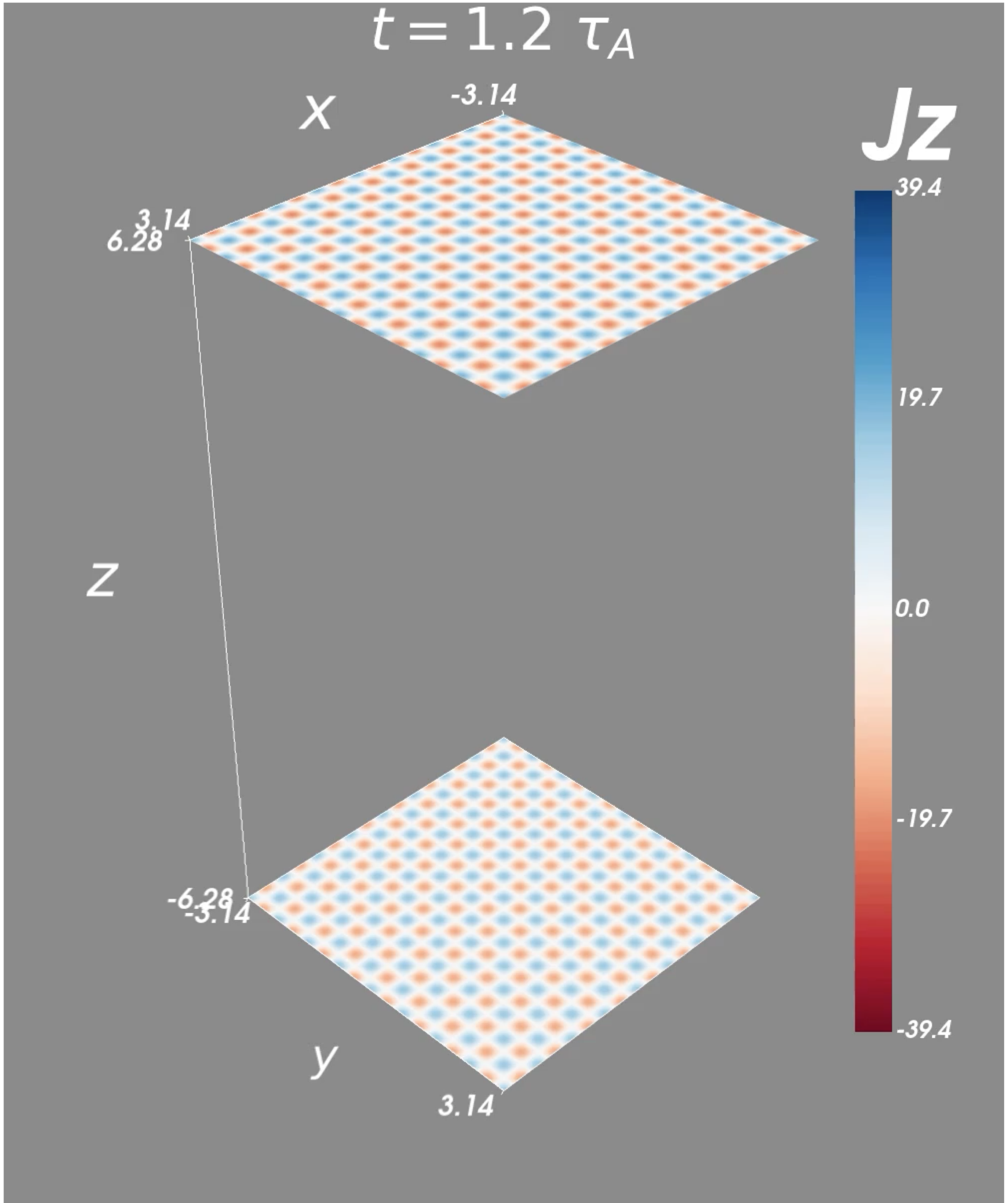
Current density at various times

$$S_0 = 1250,$$

$$L_z[L_\parallel] = 2L_x[L_\perp]$$

Current sheets

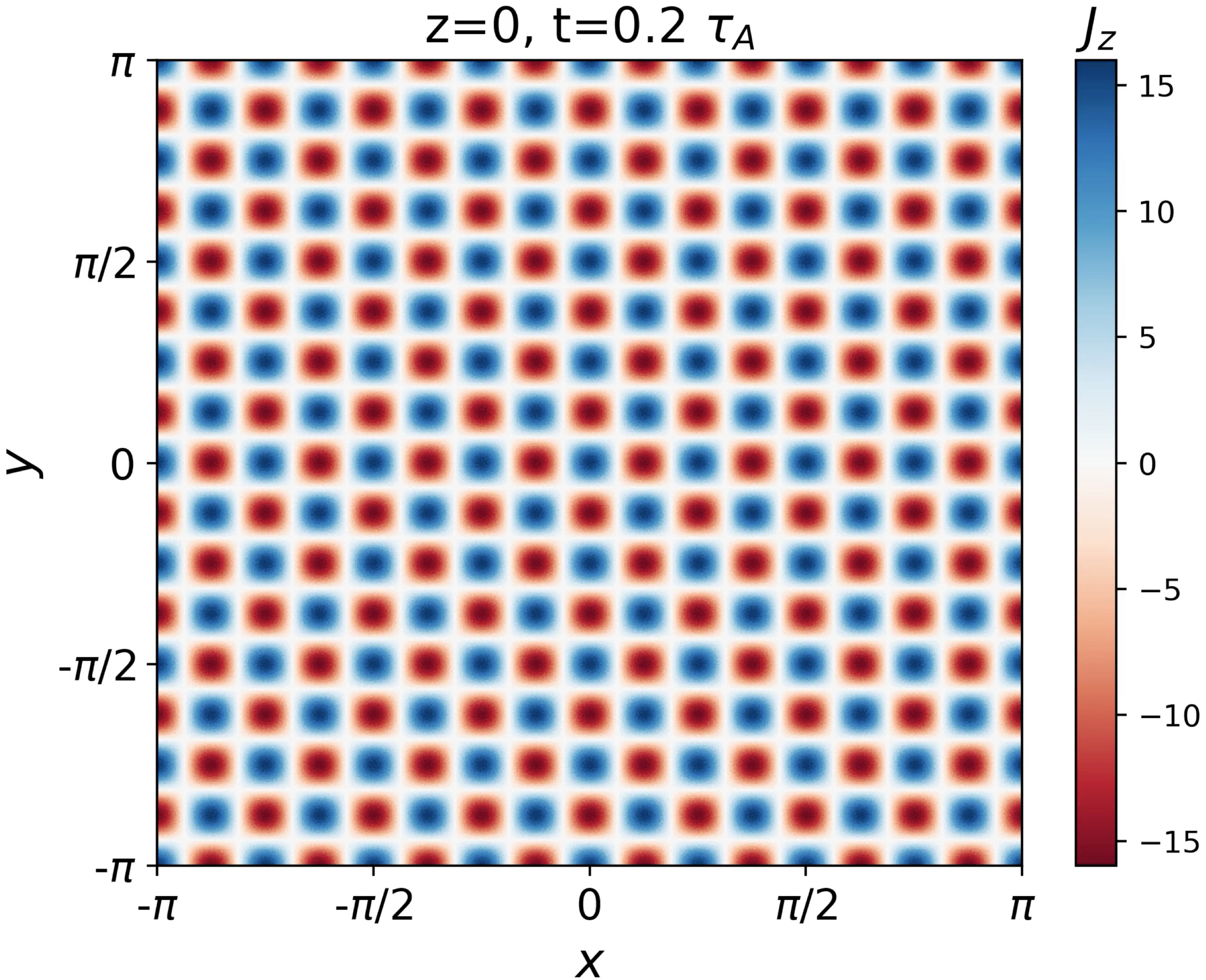
$$|\,J\,| > 3\,J_{rms}$$



3D Reduced-MHD simulation

visuals

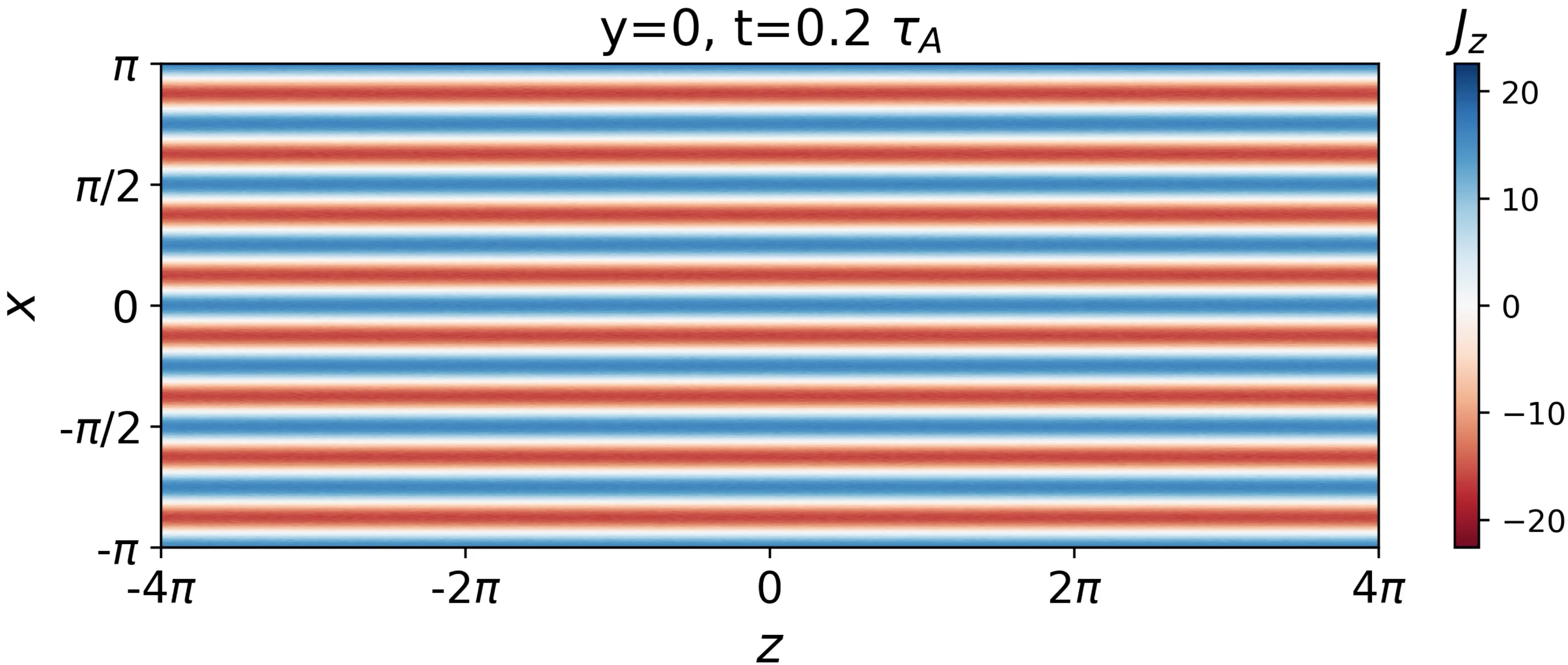
$S_0 = 1250,\ L_z[L_\parallel] = 4L_x[L_\perp]$



3D Reduced-MHD simulation

visuals

$S_0 = 1250,\ L_z[L_\parallel] = 4L_x[L_\perp]$



3D Reduced-MHD simulation

Two-stages evolution

- First stage: developing turbulence

  tubes break in parallel direction

  Increasing complexity of the system

  robust dissipation of magnetic energy

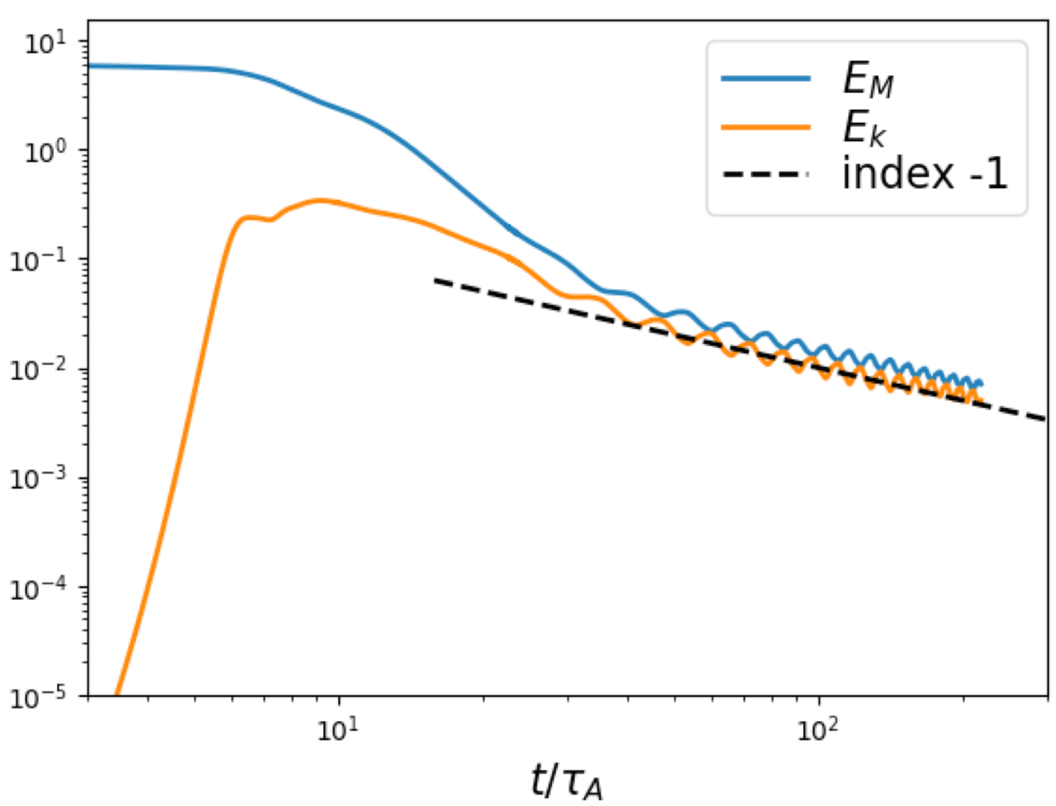
- Second stage: decaying turbulence

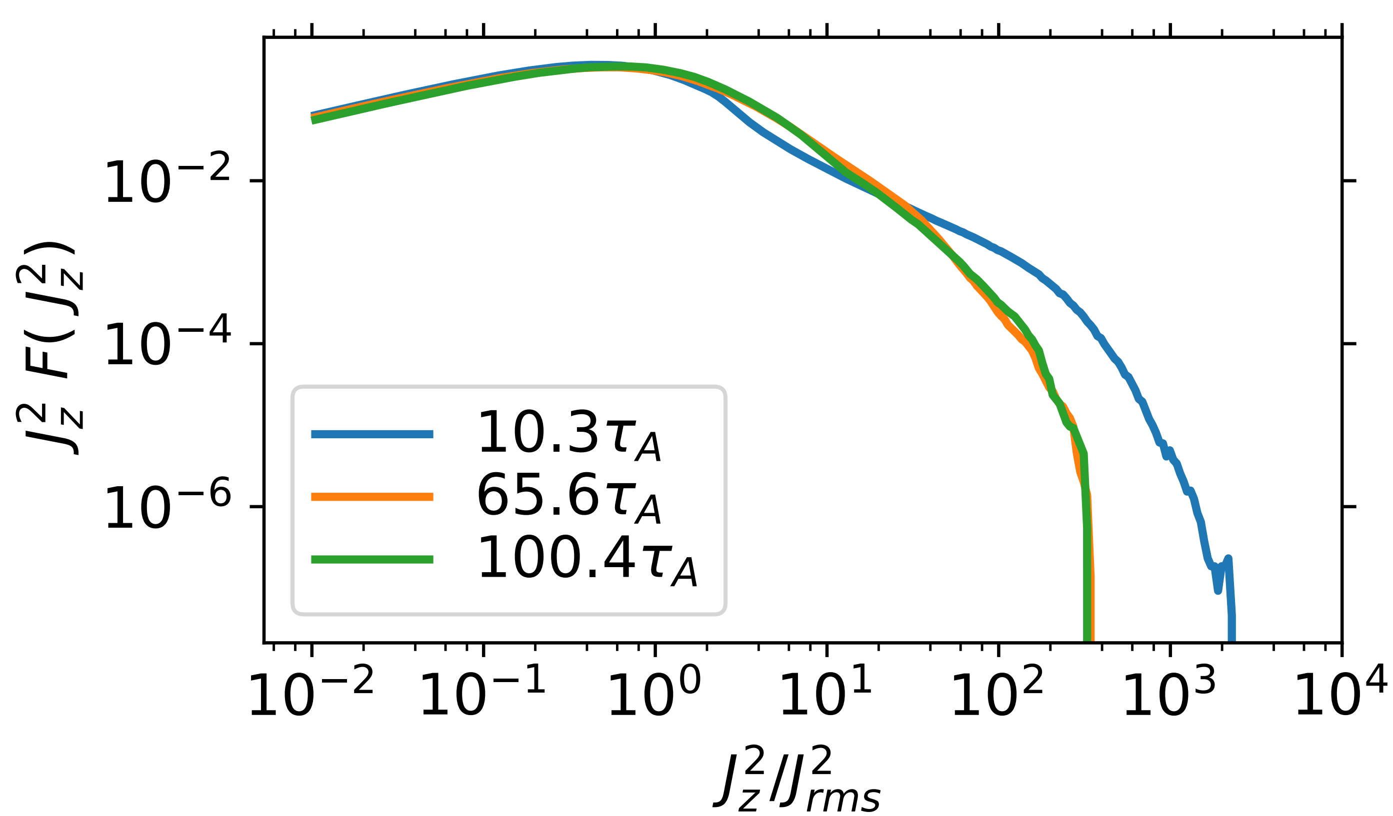
  merging of flux tubes

  self-similar evolution

  energy inverse transferred to larger scales

$$S_0 = 1250, \quad L_z\,[L_\parallel] = 4L_x\,[L_\perp]$$

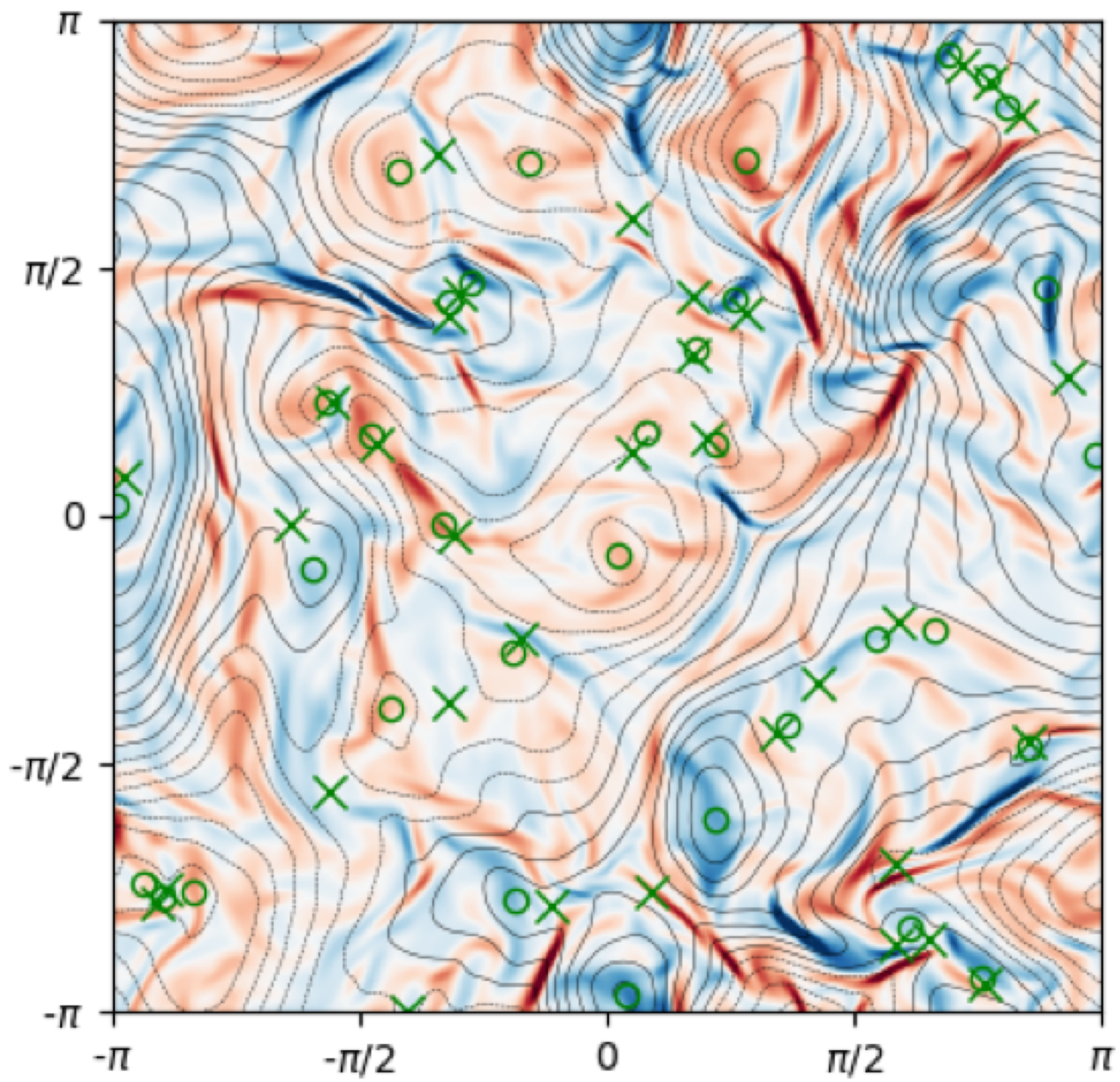


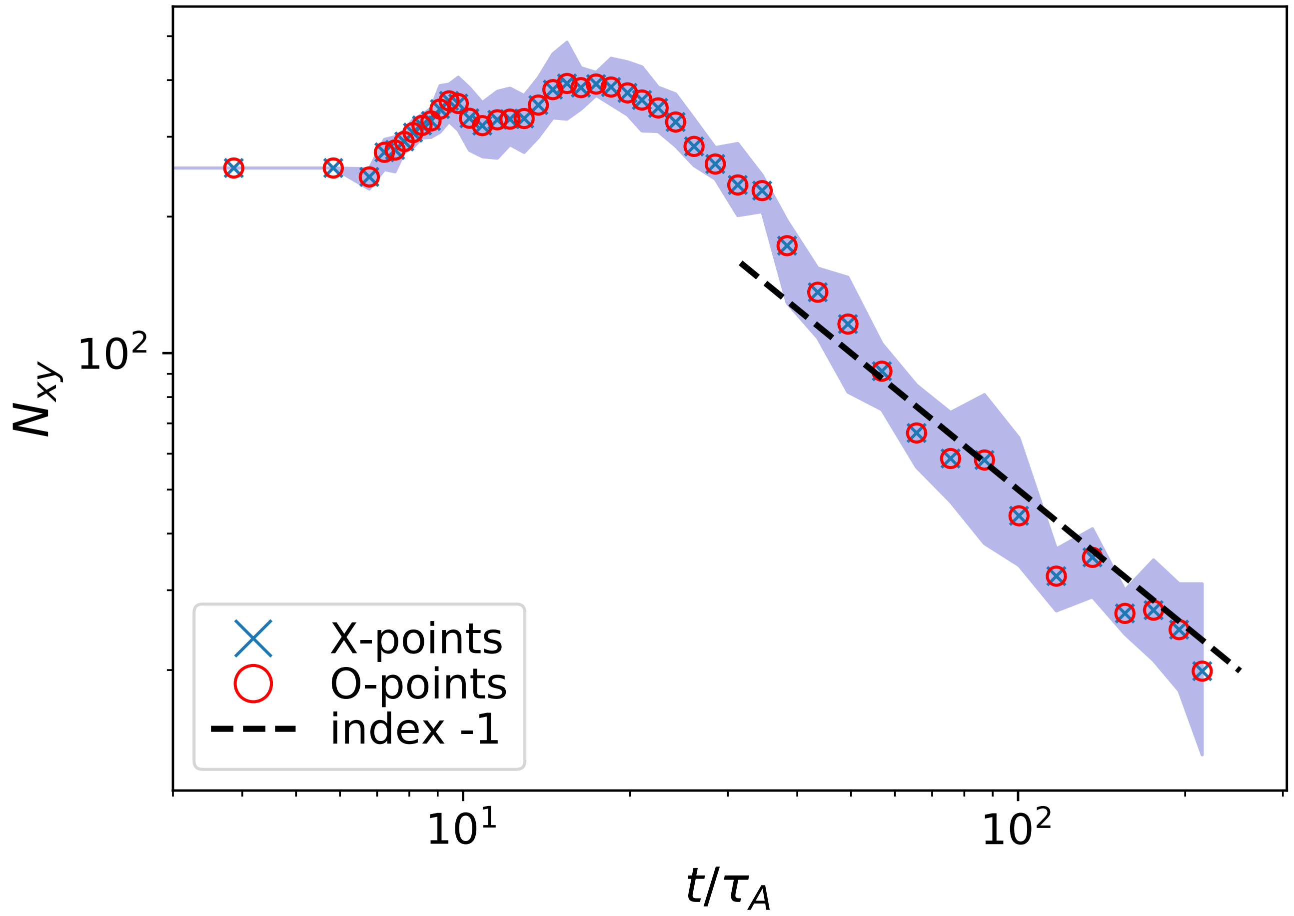


3D Reduced-MHD simulation

Number of structures

Identify the critical points of magnetic potential on x-y planes





3D Reduced-MHD simulation

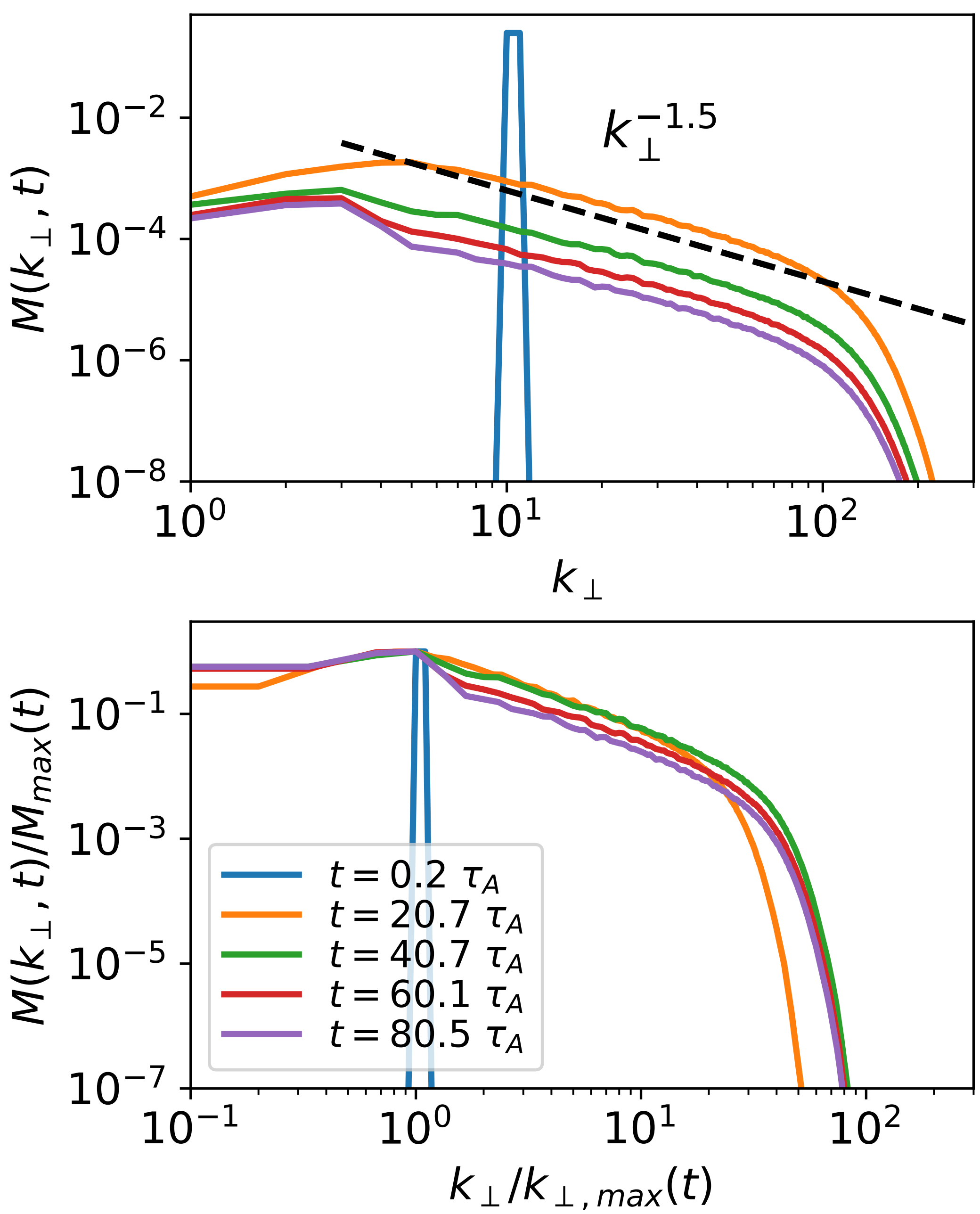
Magnetic power spectra

Hyper-resistivity, $L_z[L_\parallel] = 4L_x[L_\perp]$

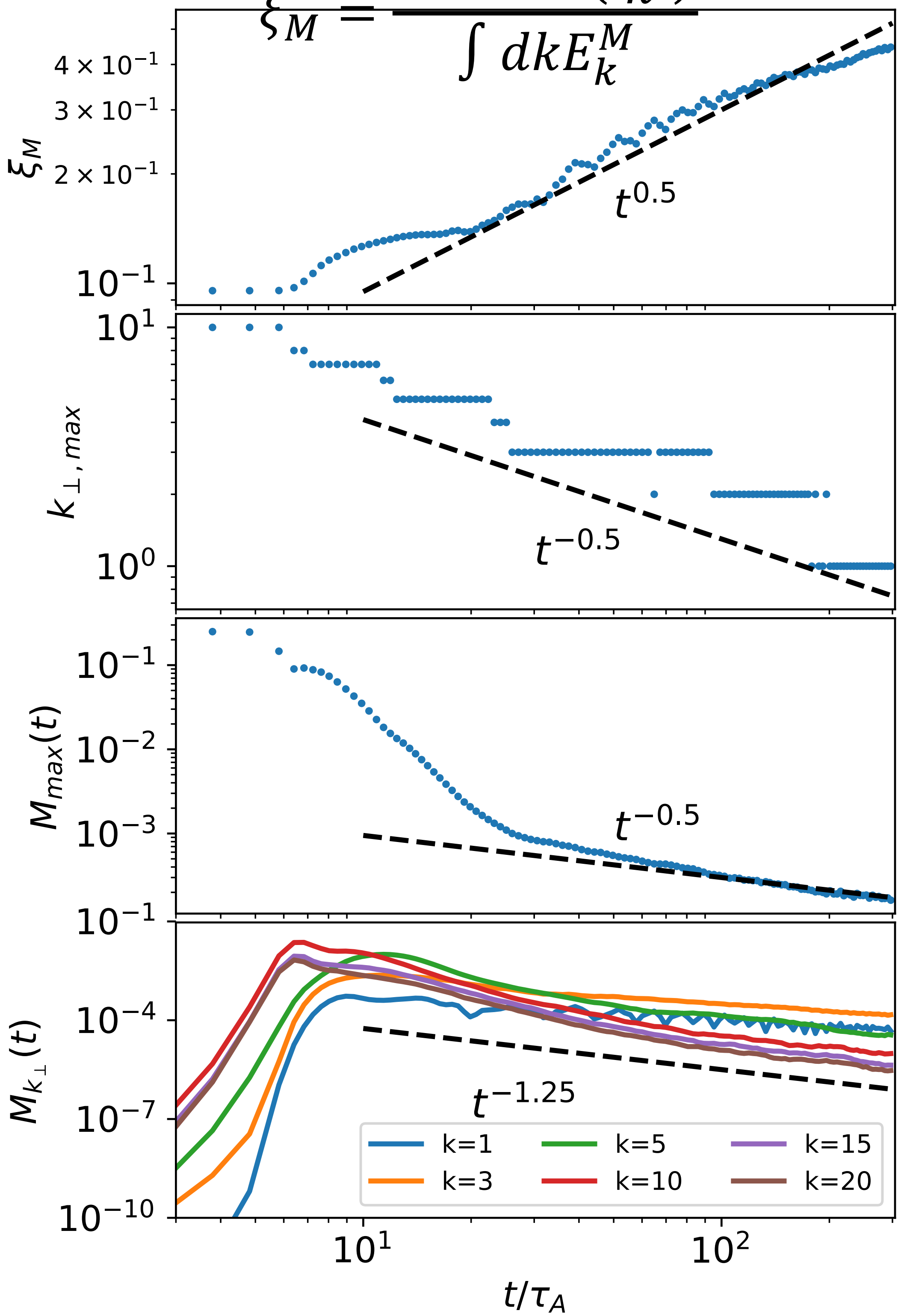
$$M(k_\perp, \tilde{t}) \propto \tilde{t}^{-\alpha} k_\perp^{-\gamma}$$

$$2\alpha = \gamma + 1$$

$$\alpha = 1.25, \gamma = 1.5$$



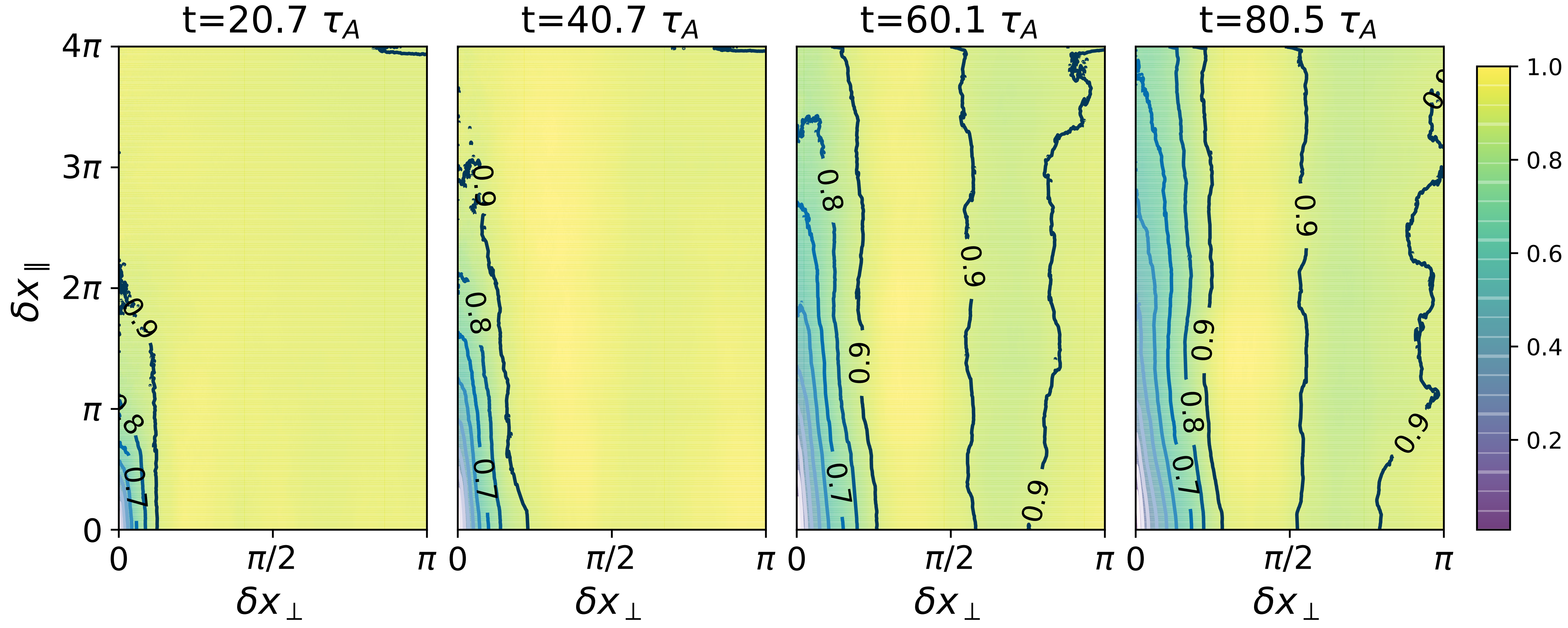
$$\xi_M = \frac{\int dk E_k^M \left(\frac{2\pi}{k}\right)}{\int dk E_k^M}$$



3D Reduced-MHD simulation

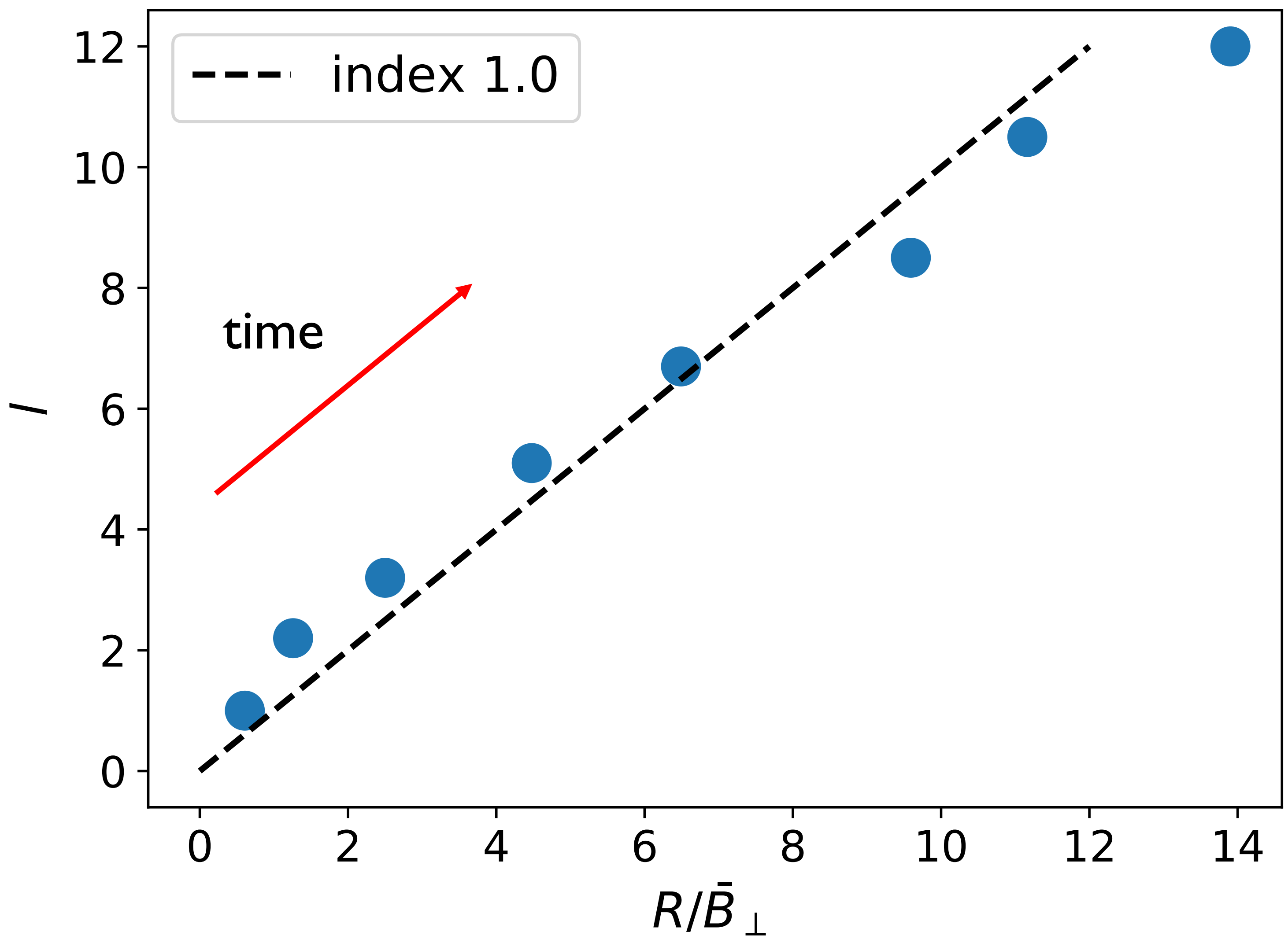
Magnetic structure function

$$S_B^2(\delta\mathbf{x}, t) = \langle|\mathbf{B}(\mathbf{x}_2, t) - \mathbf{B}(\mathbf{x}_1, t)|^2\rangle_{x_1}$$



Critical balance

time evolution of statistical flux tubes



$$\frac{l}{B_g} \sim \frac{R}{B_\perp}$$

Geometry (aspect ratio) of flux tubes is controlled by critical balance

Summary

Magnetic energy can be transferred to larger scales through magnetic reconnection

Dynamics reconnection-based model

- Hierarchical merger of magnetic structures
- Energy decay as t^{-1} and scale of magnetic field grows as $t^{1/2}$
- Reconnection regime remains the same

Numerical study

- 2D MHD---self-similar magnetically-dominated evolution
- 3D RMHD—additional parallel dynamics determined by critical balance

Acknowledgements

MIT PSFC

Center for Integrated Plasma Studies
UNIVERSITY OF COLORADO BOULDER

NSF

U.S. DEPARTMENT OF ENERGY