Sub-Larmor Turbulence in Low-Beta Collisionless Plasma

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Connecting Micro and Macro Scales: Acceleration, Reconnection, and Dissipation in Astrophysical Plasmas

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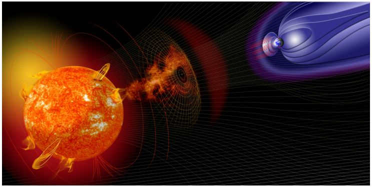
Motivations: Solar wind turbulence observations

In a weakly collisional and $\beta \sim 1$ plasma like the solar wind, compressive fluctuations are ought to be subject to strong kinetic damping.

Why does spacecraft measurements of compressive fluctuations in the solar-wind turbulence show healthy Kolmogorov-like power-law spectra?



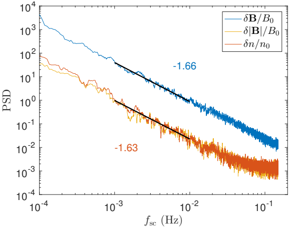


Figure: C. H. K. Chen JPP 2016

Phase space turbulent cascade

In the inertial range, energy injected into perturbations can be thermalised (produce entropy) by:

- **Phase mixing** (Landau damping), producing fine scales in $v_{\|}$.
- **Turbulent mixing**, producing fine scale in real space.

Which thermalisation route does the system favour?

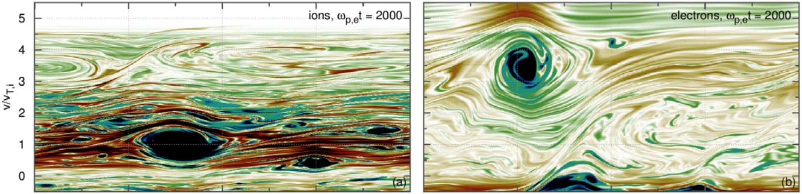


Figure: Lesur et al. PPCF 2014

Theoritical framework: Kinetic reduced MHD

Ordering

- Plasma near Maxwellian equilibrium: $f = F_0 + \delta f$
- Strong (uniform) magnetic field: $\omega \ll \Omega_i, k_\parallel \ll k_\perp$
- Long wavelength: $k_\perp \ll \rho_i$

KRMHD = hybrid fluid-kinetic description of magnetized weakly collisional plasma.

Fluid part:

$$\left(\frac{\partial}{\partial t} \mp v_A \frac{\partial}{\partial z}\right)\omega^{\pm} = -\left[\xi^{\pm}, \omega^{\mp}\right] - \left[\partial_j \xi^{\mp}, \partial_j \xi^{\pm}\right]$$

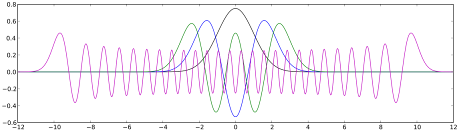
Kinetic part:

$$\frac{dg^{(i)}}{dt} + v_\parallel \nabla_\parallel g^{(i)} + v_\parallel F_0 \nabla_\parallel \phi^i = 0$$

$$\phi^i = \alpha^i \int dv_\parallel g^i(v_\parallel)$$

Hermite Space Formulation

Hermite formulation enables a spectral representation of velocity space. It is natural way to separate the 'fluid' part of the problem from the 'kinetic' one.



$$H_m(x) = (-1)^m e^{x^2} \frac{d^m}{dx^m} e^{-x^2} \Rightarrow$$

$$g(v_\parallel) = \sum_{m=0}^{\infty} \frac{H_m(v_\parallel / v_{th}) F_0}{\sqrt{2^m m!}} g_m$$

$$\begin{cases} H_0 = 1 \Rightarrow g_0 = \dfrac{\delta n}{n} \\ H_1 = 2x \Rightarrow g_1 = \sqrt{2}\dfrac{u_\parallel}{v_{th}} \\ H_2 = 4\left(x^2 - \dfrac{1}{2}\right) \Rightarrow g_2 = \dfrac{1}{\sqrt{2}} \dfrac{\delta T_\parallel}{T} \end{cases}$$

$m = 0, 1, 2$ corespond to 'fluid' moments.

Hermite Space Formulation

- makes the numerical scheme spectrally accurate in the $v_{\|}$ coordinate.
- provides an elegant analytical framework to study phase mixing.
- the integro-differential kinetic equations becomes a fluid-like hierarchy of equations.

$$\frac{dg_0^i}{dt} + v_{th}\nabla_{\|}\frac{g_1^i}{\sqrt{2}} = 0,$$

$$\frac{dg_1^i}{dt} + v_{th}\nabla_{\|}\left(g_2^i + \frac{(1-1/\Lambda^i)}{\sqrt{2}}g_0^i\right) = 0,$$

$$\frac{dg_m^i}{dt} + v_{th}\nabla_{\|}\left(\sqrt{\frac{m+1}{2}}g_{m+1}^i + \sqrt{\frac{m}{2}}g_{m-1}^i\right) = C[g_m^i], \quad m \geq 2.$$

Dynamics in Phase space

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_\perp \cdot \nabla\right) g^i_m + v_{th}\nabla_\parallel \left(\sqrt{\frac{m+1}{2}}g^i_{m+1} + \sqrt{\frac{m}{2}}g^i_{m-1}\right) = -\eta m g^i_m$$

all moments are advected by the same velocity

higher moments couple to lower ones ⇒cascade in hermite space

at large enough m, free energy is removed by collisions

- **Landau damping/phase mixing** is the transfer of free energy from low moments $(\delta n, u_\parallel, \delta T_\parallel)$ into higer one $(g^i_m \geq 3)$.
- **Turbulence** is the mixing of $(\delta n, u_\parallel, \delta T_\parallel)$ by $\mathbf{u}_\perp$ and $\mathbf{b}_\perp$ transferring their energy to small scales.

Hermite "Cascade"

How transfer of free energy to high m's occurs <u>linearly</u>?

$$\left(\frac{\partial}{\partial t} + \cancel{\mathbf{u}_\perp \cdot \nabla_\perp}\right) g_m + v_{th} \nabla_\| \left(\sqrt{\frac{m+1}{2}} g_{m+1} + \sqrt{\frac{m}{2}} g_{m-1}\right) = -\nu m g_m$$

Let Fourier transform in the z direction $(\frac{\partial}{\partial z} \rightarrow ik_\|)$ and introduce:

$$\tilde{g}_m(k_\|) \equiv \left(i \mathrm{sgn} k_\|\right)^m g_m(k_\|)$$

$$\frac{\partial \tilde{g}_m}{\partial t} + \frac{|k_\||v_{th}}{\sqrt{2}} \left(\sqrt{m+1}\tilde{g}_{m+1} - \sqrt{m}\tilde{g}_{m-1}\right) = -\nu m \tilde{g}_m$$

$$\frac{\partial \tilde{g}_m}{\partial t} + \frac{|k_\||v_{th}}{\sqrt{2}} m^{1/4} \frac{\partial}{\partial m} m^{1/4} \tilde{g}_m = -\nu m \tilde{g}_m$$

$$\boxed{\tilde{g}_m = \tilde{g}_m^+ + (-1)^m \tilde{g}_m^-}$$

$$\frac{\partial \tilde{g}_m^\pm}{\partial t} \pm \sqrt{2}|k_\parallel| v_{th} m^{1/4} \frac{\partial}{\partial m} m^{1/4} \tilde{g}_m^\pm = -\nu m \tilde{g}_m^\pm$$

Phase Mixing

$\tilde{g}_m^+ = \dfrac{\tilde{g}_m + \tilde{g}_{m+1}}{2}$, propagate from low to high m.

Anti-Phase-Mixing

$\tilde{g}_m^- = (-1)^m \dfrac{\tilde{g}_m - \tilde{g}_{m+1}}{2}$, propagate from high to low m.

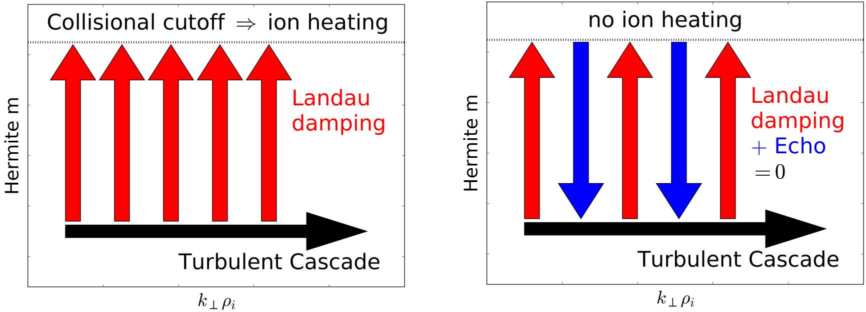
In energy terms: $C_m = C_m^+ + C_m^-$ satisfies:

$$\frac{\partial C_m}{\partial t} + \frac{\partial}{\partial m} |k_\parallel| v_{th} \sqrt{2m} \left(C_m^+ - C_m^- \right) = -2\nu m C_m$$

Hermite flux to high m can be cancelled by the ‘$-$’ modes

$$\left(\frac{\partial \tilde{g}_m^{s\pm}}{\partial t}\right)_{nl} = -\sum_{p_\parallel+q_\parallel=k_\parallel} \mathbf{u}_\perp(p_\parallel)\cdot\nabla_\perp\left[\delta^{+}_{k_\parallel,q_\parallel}\tilde{g}_m^{s\pm}(q_\parallel)+\delta^{-}_{k_\parallel,q_\parallel}\tilde{g}_m^{s\mp}(q_\parallel)\right],$$

Hermite flux can be cancelled by the anti-phase-mixing modes



Asterix

KRMHD + KREHM (Zocco et al. 2013)

A Fourier-Hermite pseudo-spectral code for strongly magnetised fluid-kinetic plasma dynamics

- Extension of an incompressible MHD Open Source solver (http://aqua.ulb.ac.be/turbo)
- FFTW
- Runge-Kutta RK4
- Parallelized in one direction using MPI
- Shift dealiasing

Parallel cascade of compressible fluctuations

If $k_{||}$ remains small, compressive fluctuations are not strongly damped.

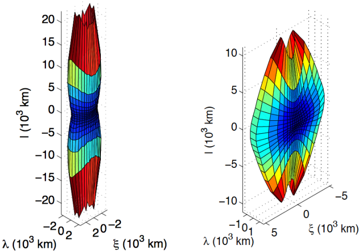


Figure: C. H. K. Chen et al. APJ 2012

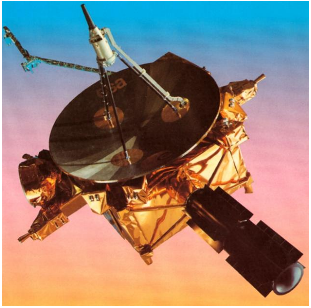
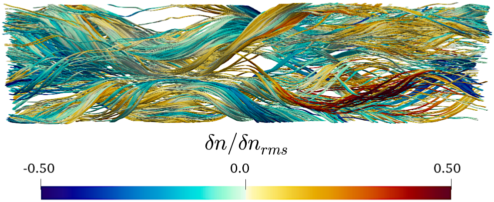
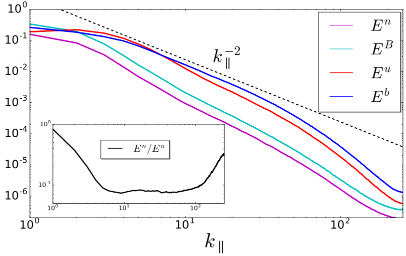


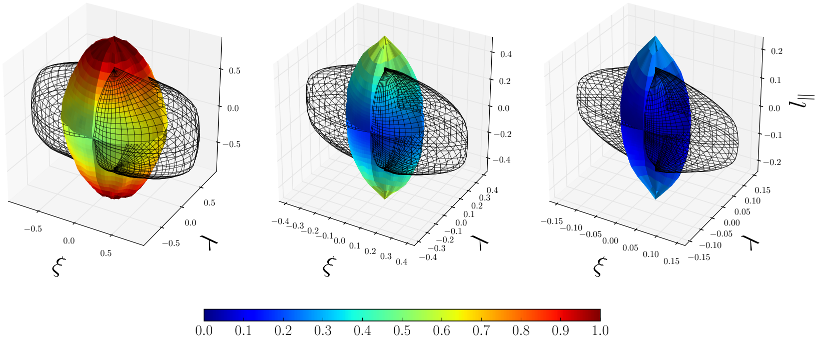
Figure: Ulysses space-craft

$$\mid k_{\parallel} \mid v_{th} \sim \mid k_{\parallel} \mid v_A \sim \tau_{nl}^{-1}$$



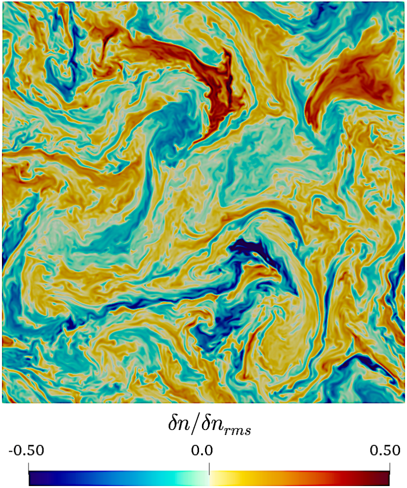


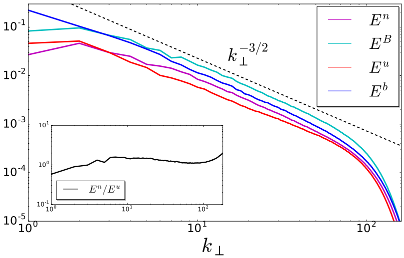
Contours of 3D correlation functions or "statitistical eddy shapes" of Alfvénic (black netting) and compressive fluctuations:



Perpendicular cascade

Meyrand et al. PNAS 2019

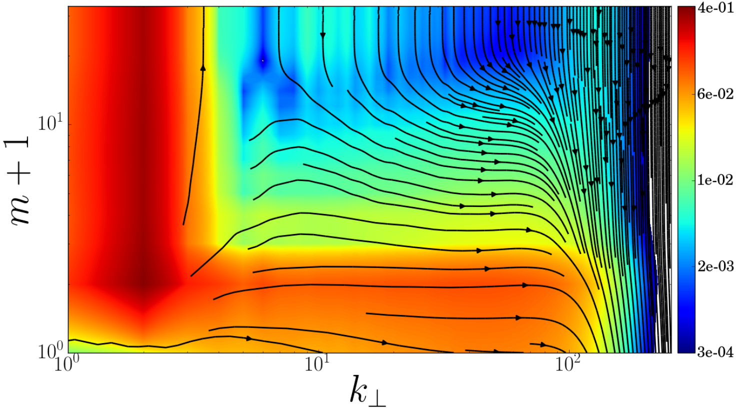




The compressive fluctuations' spectra track the Alfvénic velocity spectrum, as one might expect an undamped passive scalar to do.

Hermite-Fourier flux
Meyrand et al. PNAS 2019

$$\frac{\partial |g_m(k_\perp)|^2}{\partial t} = -\frac{\partial \Pi(k_\perp, m)}{\partial k_\perp} - \frac{\partial \Gamma(k_\perp, m)}{\partial m}$$



Sub-Larmor Low Beta Turbulence

KREHM equations

Asymptotic expansion of gyrokinetic under $\beta_e \sim \dfrac{Zm_e}{m_i} \ll 1$ ordering (Zocco et al. 2011):

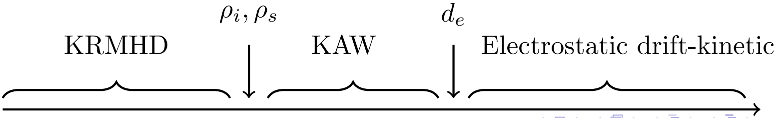
$$\frac{d}{dt}\frac{\delta n}{n} + v_{th}\nabla_{\parallel} de^2 \nabla_{\perp}^2 A = 0,$$

$$\frac{d}{dt}\left(A - de^2\nabla_{\perp}^2 A\right) = -v_{th}\left[\frac{\partial \varphi}{\partial z} - \nabla_{\parallel}\left(\frac{\delta n}{n} + \frac{\delta T_{\parallel}}{T}\right)\right],$$

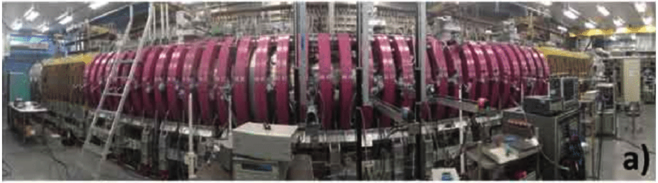
$$\frac{d}{dt}\frac{\delta T_{\parallel}}{T} + v_{th}\nabla_{\parallel}\left(\sqrt{3}\hat{g}_3 + 2de^2\nabla_{\perp}^2 A\right) = 0,$$

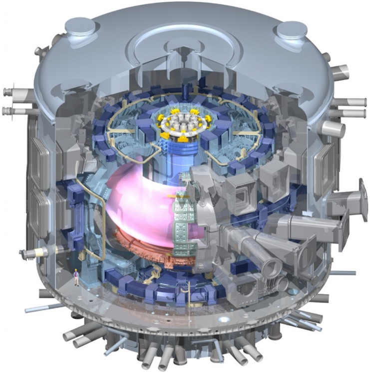
$$\frac{d\hat{g}_m}{dt} + \frac{v_{th}}{\sqrt{2}}\nabla_{\parallel}\left(\sqrt{m+1}\hat{g}_{m+1} + \sqrt{m}\hat{g}_{m-1}\right) + \nu_{ei} m\hat{g}_m = 0, m \geqslant 3,$$

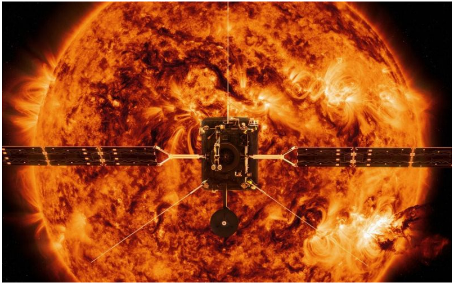
$$\frac{\delta n}{n} = -\frac{Z}{\tau}\left(1 - \hat{\Gamma}_0\right)\varphi, \quad \Gamma_0 = I_0(k_{\perp}^2\rho_i^2/2)e^{-k_{\perp}^2\rho_i^2/2}$$



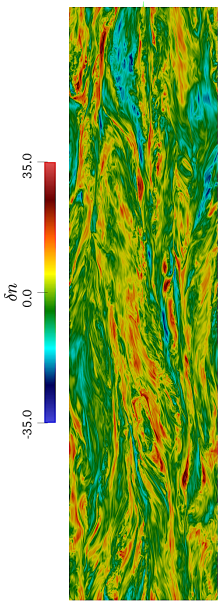
Sub-Larmor Low Beta Turbulence

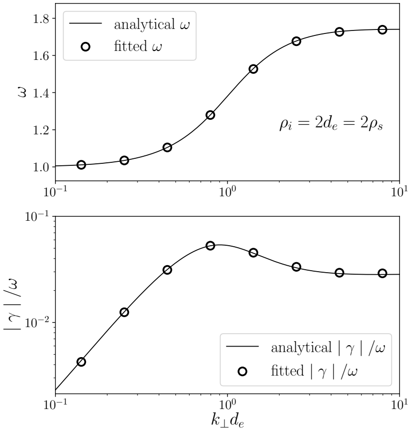






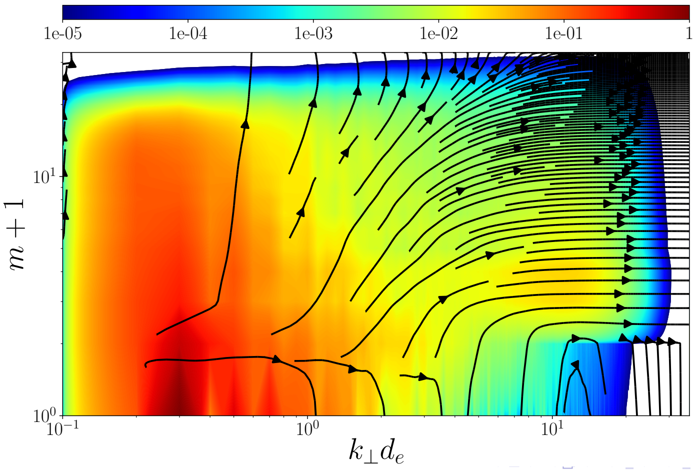
Sub-Larmor Low Beta Turbulence





Sub-Larmor Low Beta Turbulence

$$W = \frac{1}{2}\varphi\frac{Z}{\tau}\left(1-\hat{\Gamma}_0\right)\varphi + \frac{1}{2}\frac{\delta n^2}{n^2} + |\, de^2\nabla_\perp A\,|^2 + \frac{u_\parallel^2}{v_{th}^2} + \frac{1}{4}\frac{\delta T_\parallel^2}{T^2} + \sum_{m\geqslant 3}\hat{g}_m^2$$

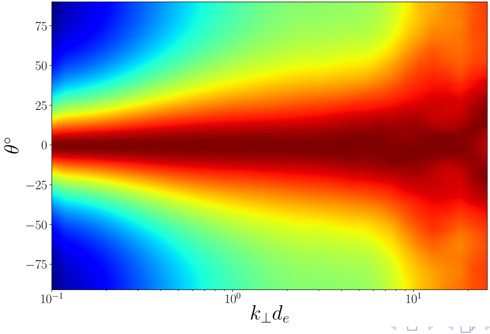


Sub-Larmor Low Beta Turbulence

Polarization alignment:

- $$[f, g] = \hat{\mathbf{z}} \cdot (\nabla_\perp f \times \nabla_\perp g)$$
- Gradients are perpendicular to contour lines.
- Solutions with either circularly symmetric or 1D perpendicular profiles are exact nonlinear solutions (Daniel Groselj et al. PRX 2019).

$$\theta = \overline{sin^{-1}\left(\mathbf{v}_E \times (\hat{\mathbf{z}} \times \nabla_\perp \hat{g}_m)\right) / \mid \mathbf{v}_E \mid\mid \hat{\mathbf{z}} \times \nabla_\perp \hat{g}_m \mid}$$



Conclusions

Inertial range turbulence

- Nonlinear cascade scatters energy in the phase space so as to generate a stochastic version of the plasma echo.
- Stochastic echo impede Landau damping by reducing the net flux to small velocity space scales.
- **Collisionless plasma turbulence in the solar wind behave in a more “fluid-like” fashion than expected.**

Sub-Larmor turbulence

- Nonlinear cascade scatters energy in the phase space so as to impede plasma echo at $k_\perp d_e \sim 1$.
- Landau damping probably plays a key role around $k_\perp d_e \sim 1$.