Nonlinear streaming instability and the inverse cascade of B in supernova precursor*

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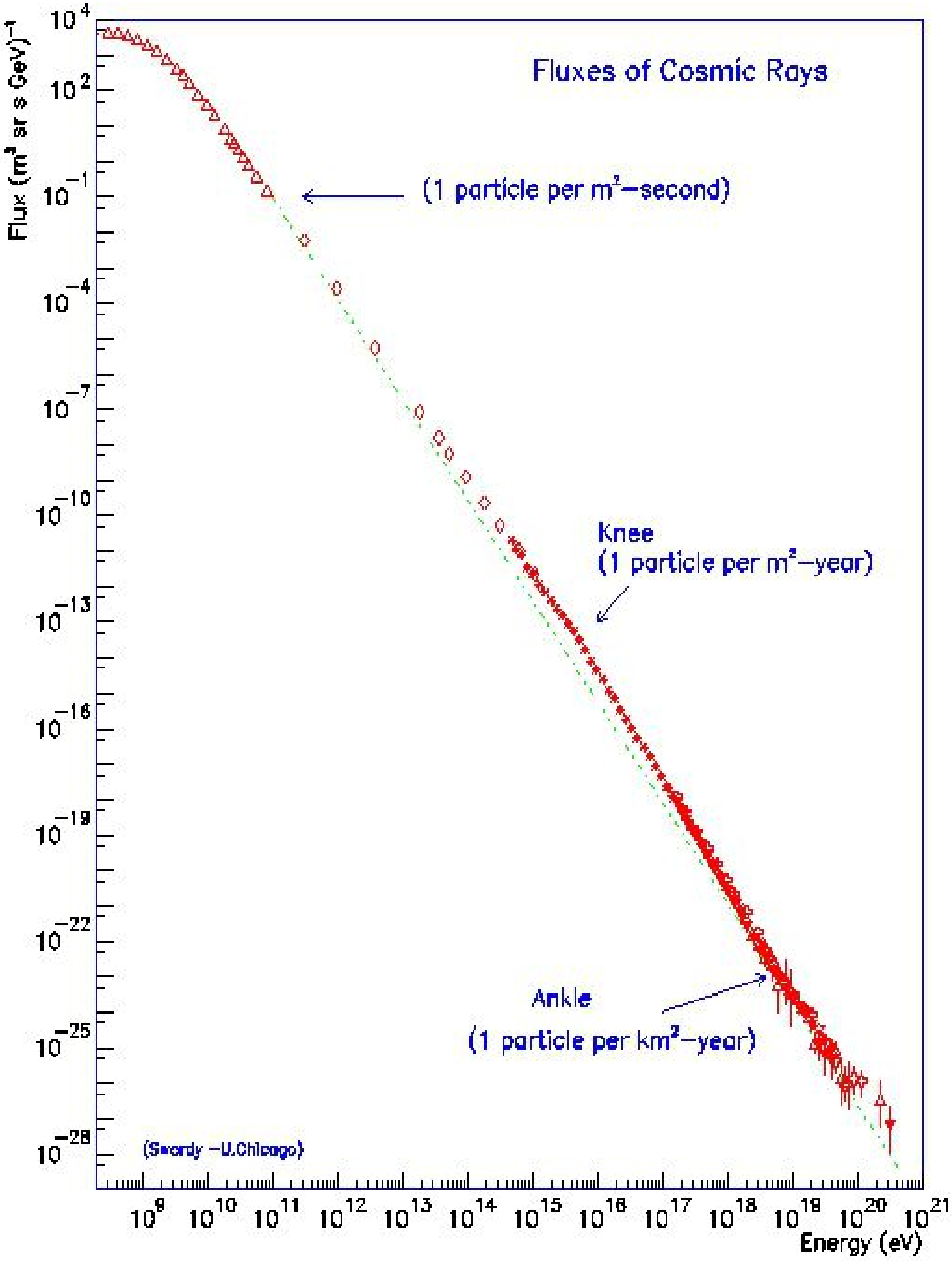
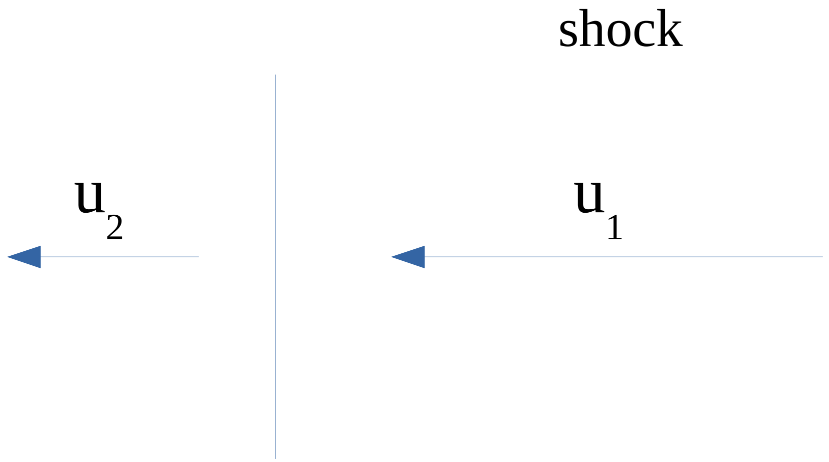
Connecting Micro and Macro Scales: Acceleration, Reconnection, and Dissipation in Astrophysical Plasmas, KITP 2019



(Swordy –U.Chicago)

How galactic cosmic rays are accelerated?



$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(D_{xx}\frac{\partial f}{\partial x}\right) + \frac{p}{3}\frac{\partial u}{\partial x}\frac{\partial f}{\partial p} .$$

Exact solution: $f \sim p^{-\alpha}$

Krymsky, 1977
Axford et al, 1977
Bell, 1978
Blandford & Ostriker 1978

How galactic cosmic rays are accelerated?

But:

1) Shock has a finite lifetime and curvature
2) Acceleration timescale:

$$\tau \sim \frac{D}{U_s^2}$$

$$D \sim 3 \cdot 10^{28} \mathrm{cm}^2/\mathrm{s}$$

$$\tau \sim 10000 \, \text{years} \gg \ \text{shock lifetime}$$

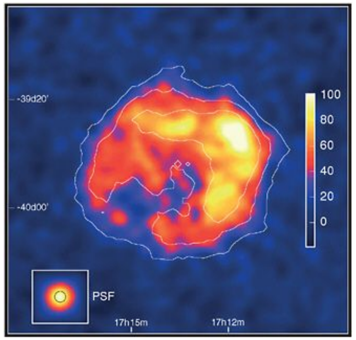
Streaming instability?

e.g, Blandford & Eichler 1987
People use QLT to obtain scattering rate close to Bohm
and acceleration timescale:

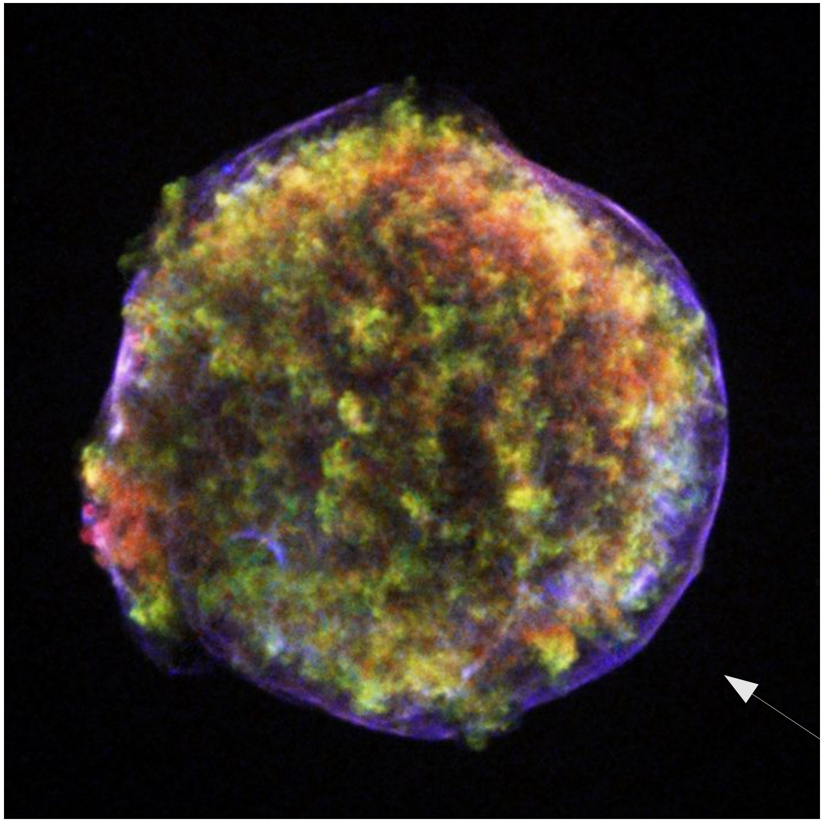
$$\tau = \frac{c^2}{U_s^2}\frac{1}{3\Omega}$$

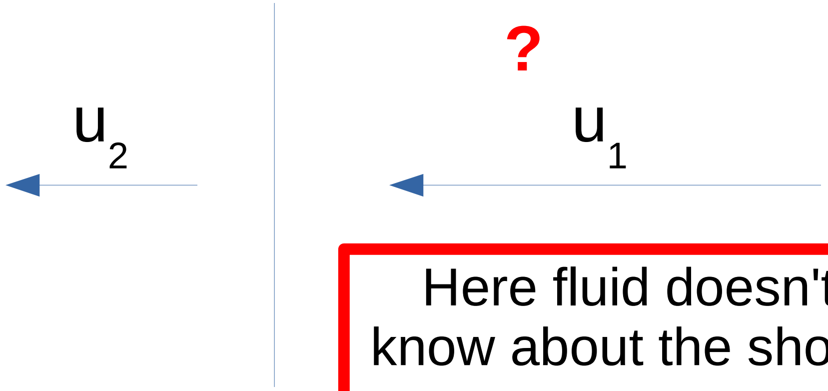
Bohm scattering in the Galactic field due to streaming
instability – still short a factor of ~100

and in contradiction with:



Field amplification?





?

u_2

u_1

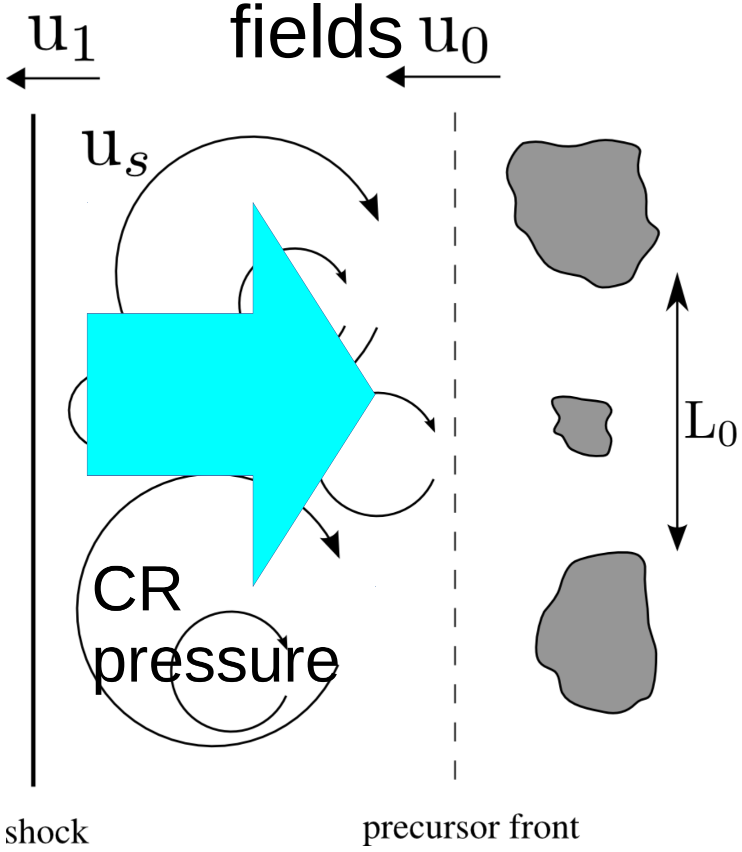
Here fluid doesn't know about the shock

Rims give:

$$100 \div 300\mu\mathrm{G}$$

shock compression would give $5 \div 20\mu\mathrm{G}$

Precursor Turbulence generates magnetic fields



Baroclinic term: $\nabla\rho_{\rm ISM} \times \nabla P_{\rm CR}$ directly produces vorticity $(\nabla \times \mathbf{v})$

$$\delta B^2_{max} \sim \delta v \epsilon_{CR}/U_s$$

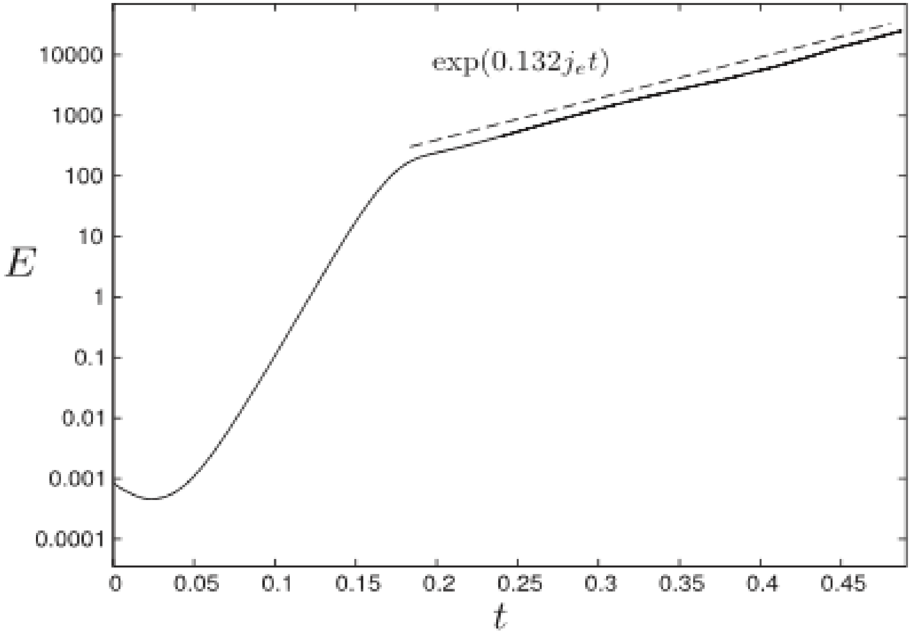
cf

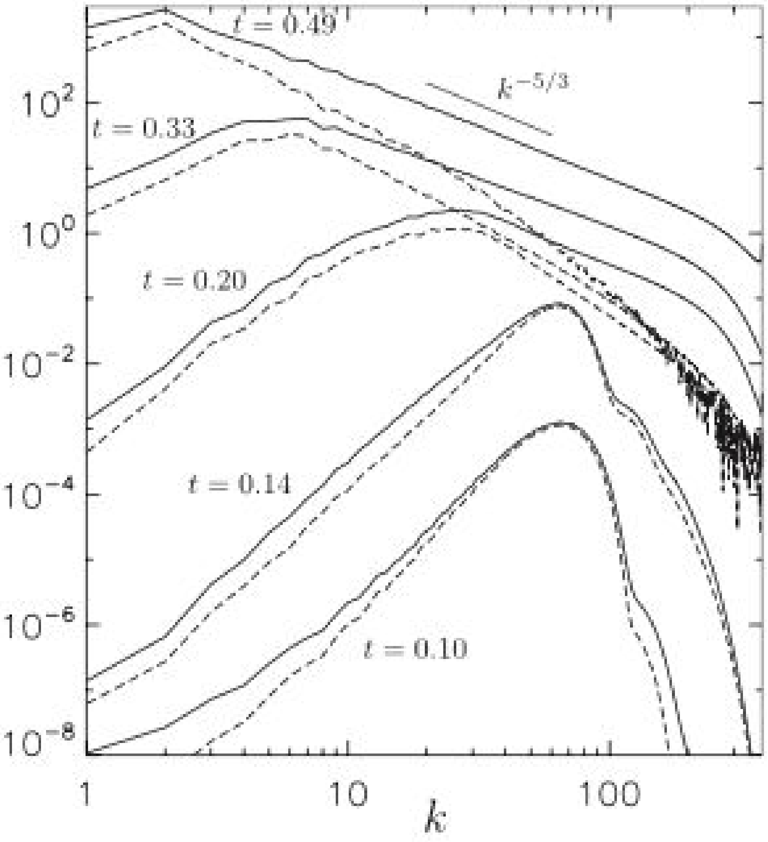
$$\delta B^2_{max} \sim U_s \epsilon_{CR}/c$$

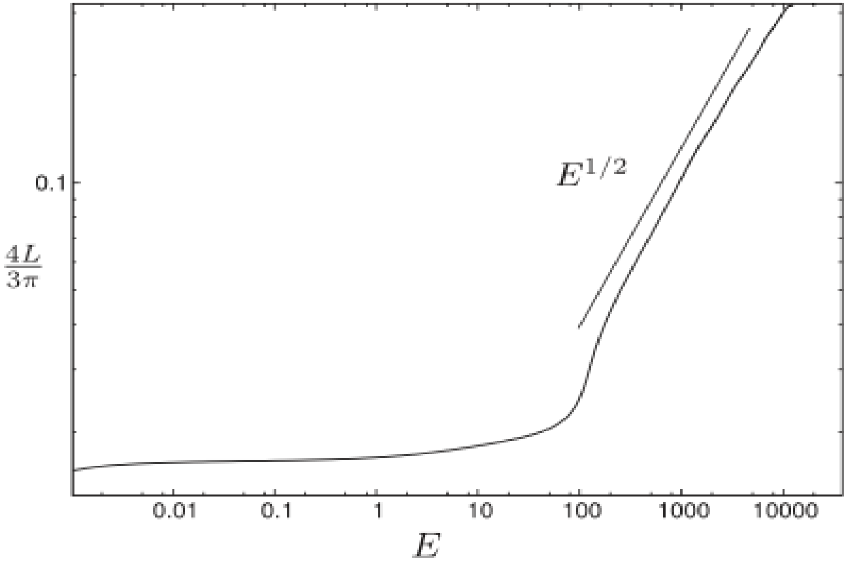
AB et al 2009

Bell’s instability?

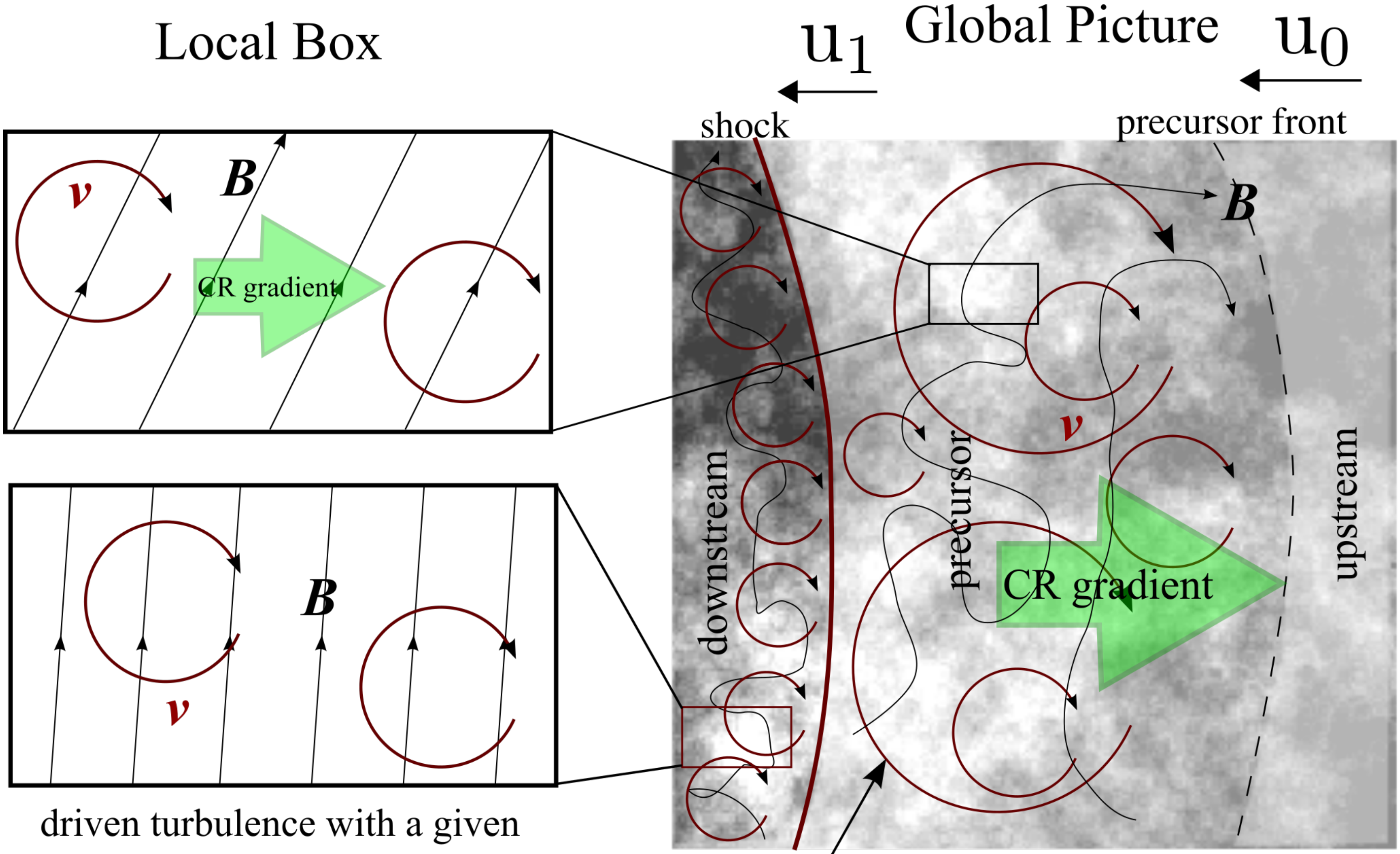
Limited on energetic grounds: *AB & Li, ApJ 2014*







Precursor Turbulence generates magnetic fields



driven turbulence with a given cascade rate ϵ

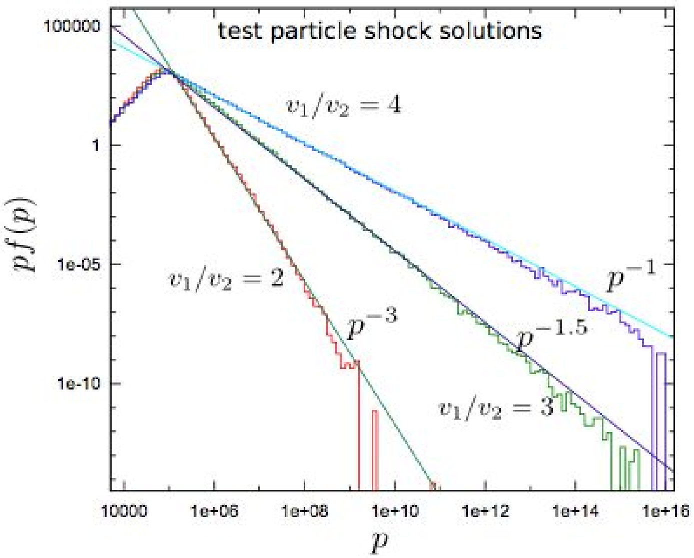
Turbulence created by $\nabla\rho_{\mathrm{ISM}} \times \nabla P_{\mathrm{CR}}$

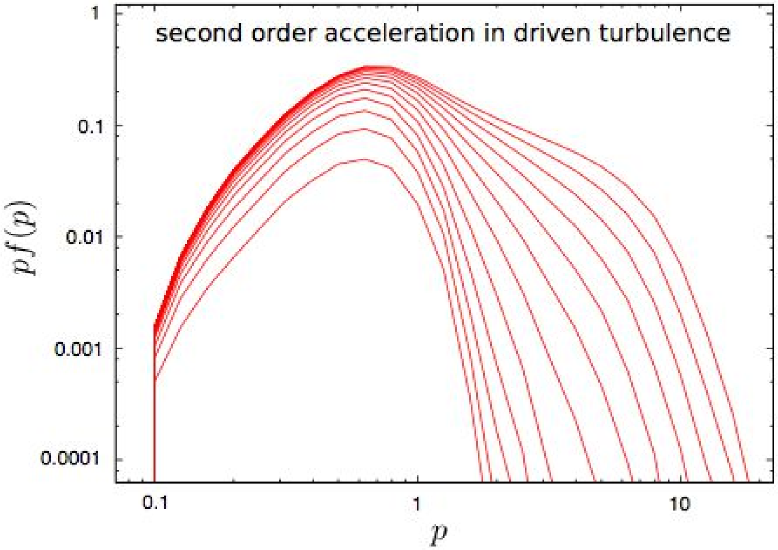
Measure D: AB et al ApJ (2011)

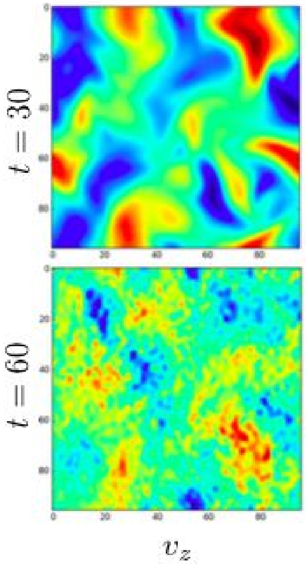
PIC-MHD code *Hephaestus*

https://sites.google.com/site/andreyberesnyak/hephaestus

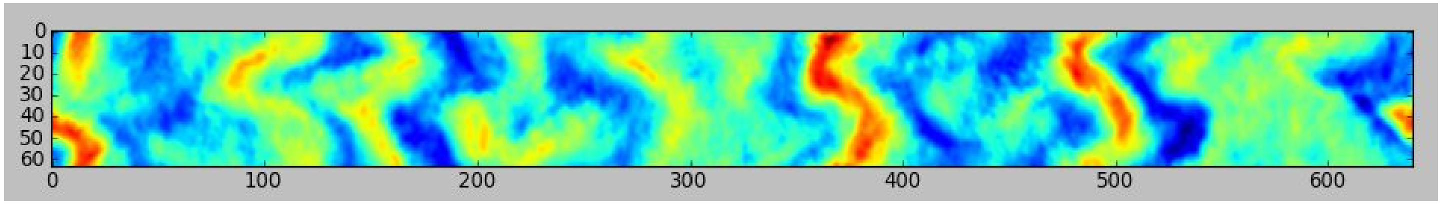
a) Lorentz force+ optional ad hoc diffusion
b) particle splitting/merging;
c) flexible precision-controlled solver







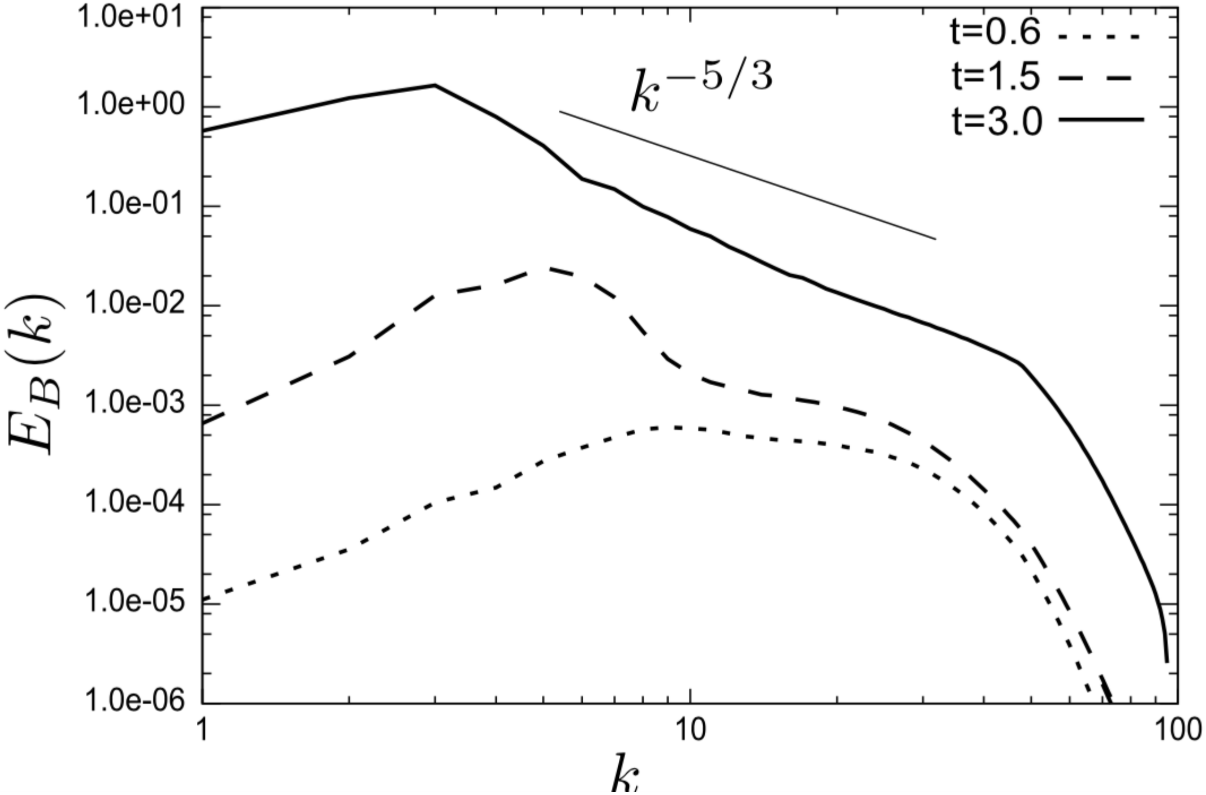
Nonlinear streaming instability



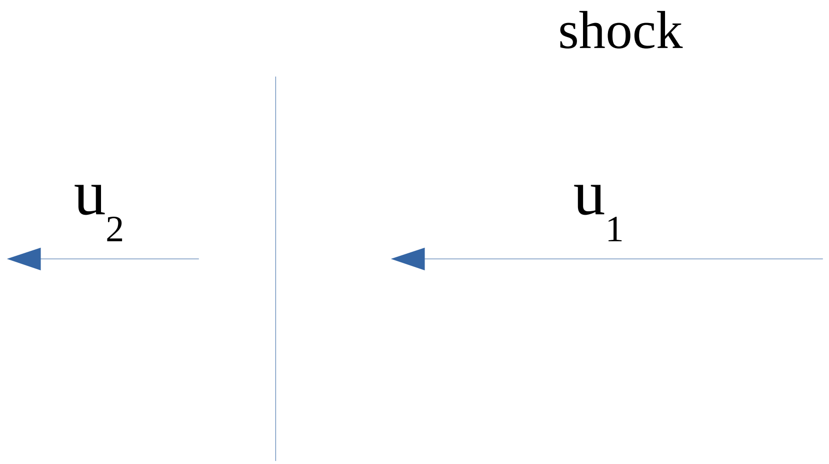
$\rho_{CR}/\rho = 6\times10^{-5} - 3\times10^{-3}$

$v_{CR}/v_A \sim 100$

$u_{CR}/(B_0^2/8\pi) = 16 - 260$



Diffusion?



?

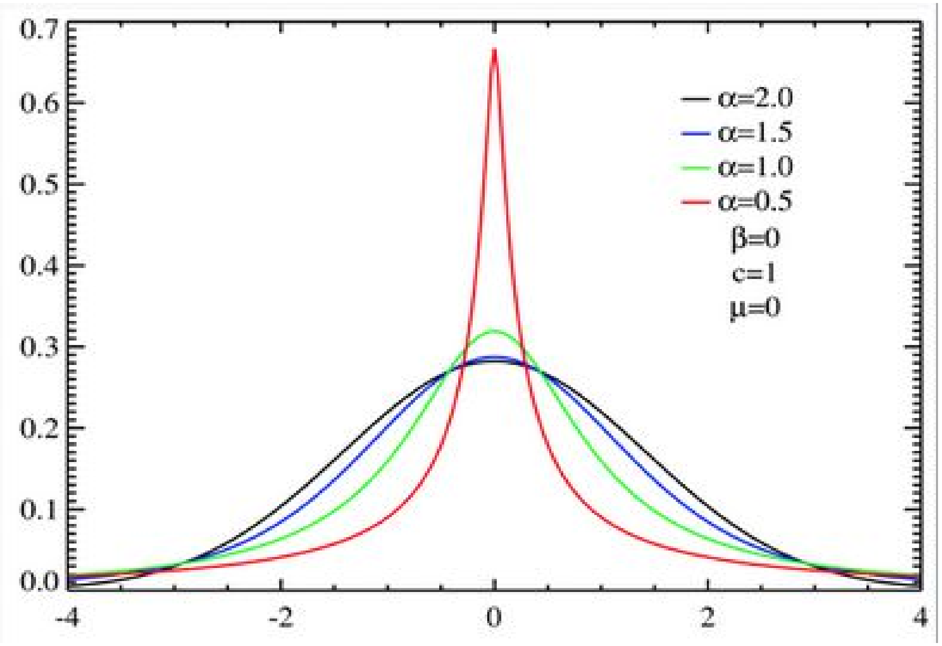
$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(D_{xx}\frac{\partial f}{\partial x}\right) + \frac{p}{3}\frac{\partial u}{\partial x}\frac{\partial f}{\partial p} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial f}{\partial p}\right)$$

Diffusion? What if dynamics is multiscale?

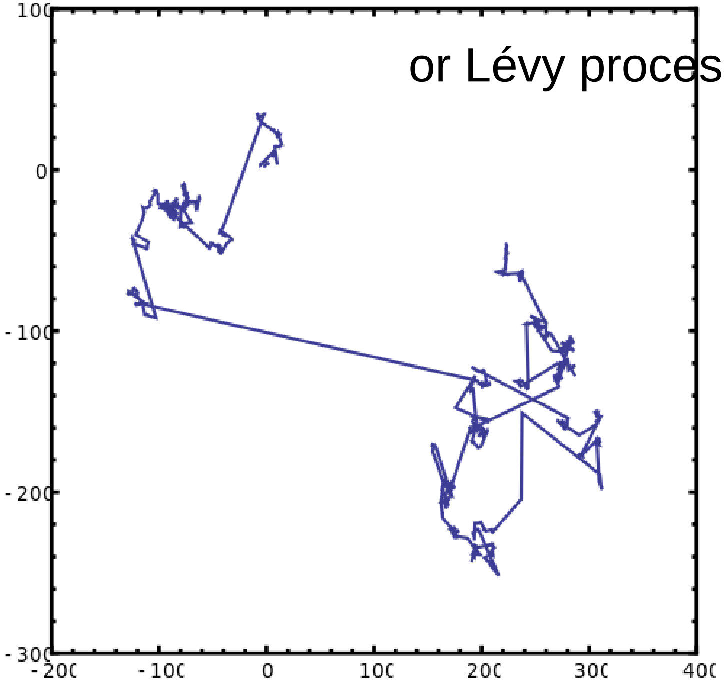
$$\frac{\partial f}{\partial t} + \mathbf{u}\cdot\frac{\partial f}{\partial \mathbf{x}} = \frac{\partial}{\partial \mathbf{x}}\left(D\frac{\partial f}{\partial \mathbf{x}}\right) \quad \Longrightarrow \quad d\mathbf{x} = \mathbf{u}dt + \sqrt{2D}d\mathbf{W}$$

Standard Wiener process?

Stable distribution

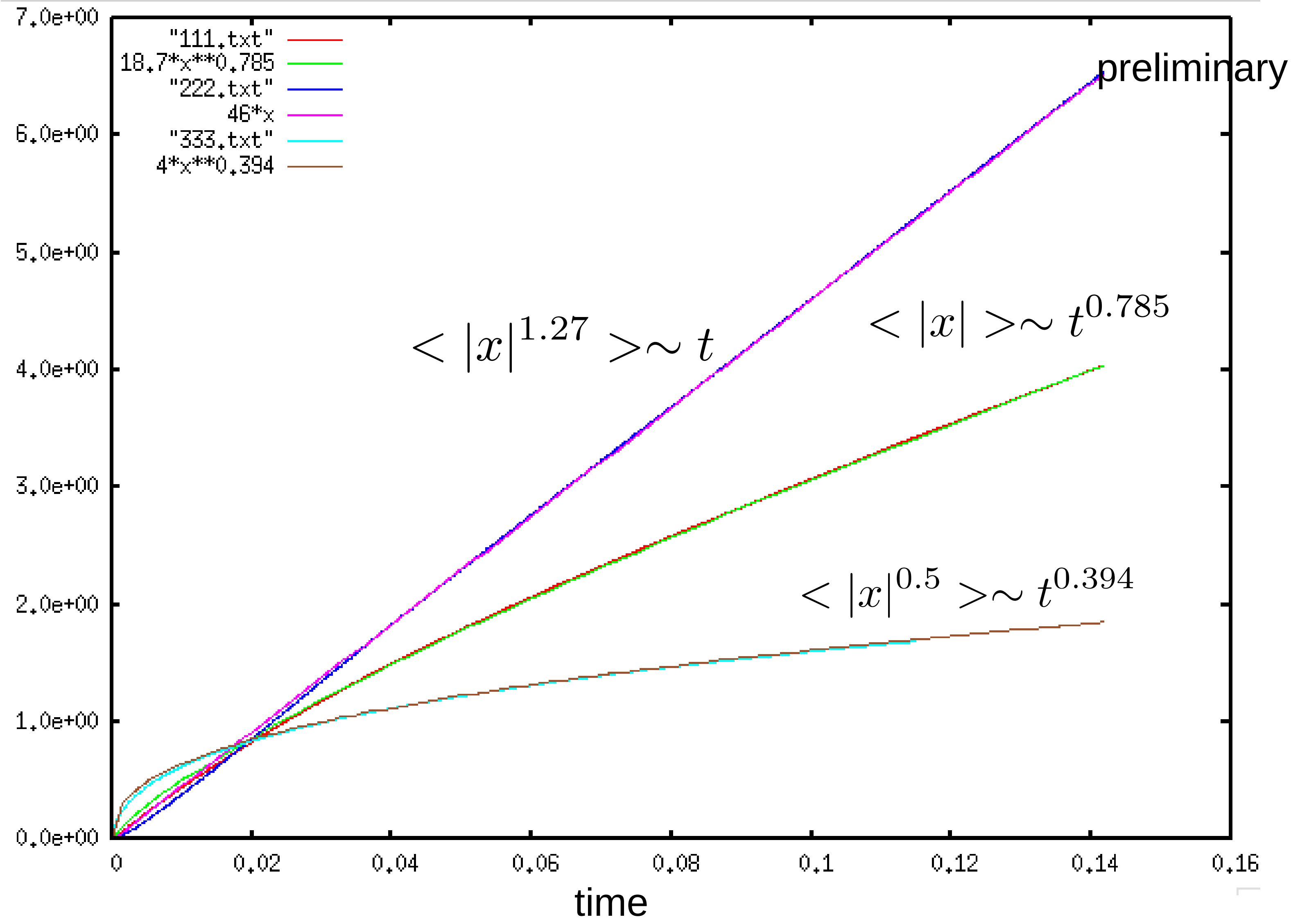


$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = D\frac{\partial^\alpha f}{\partial |x|^\alpha}$$



or Lévy process?

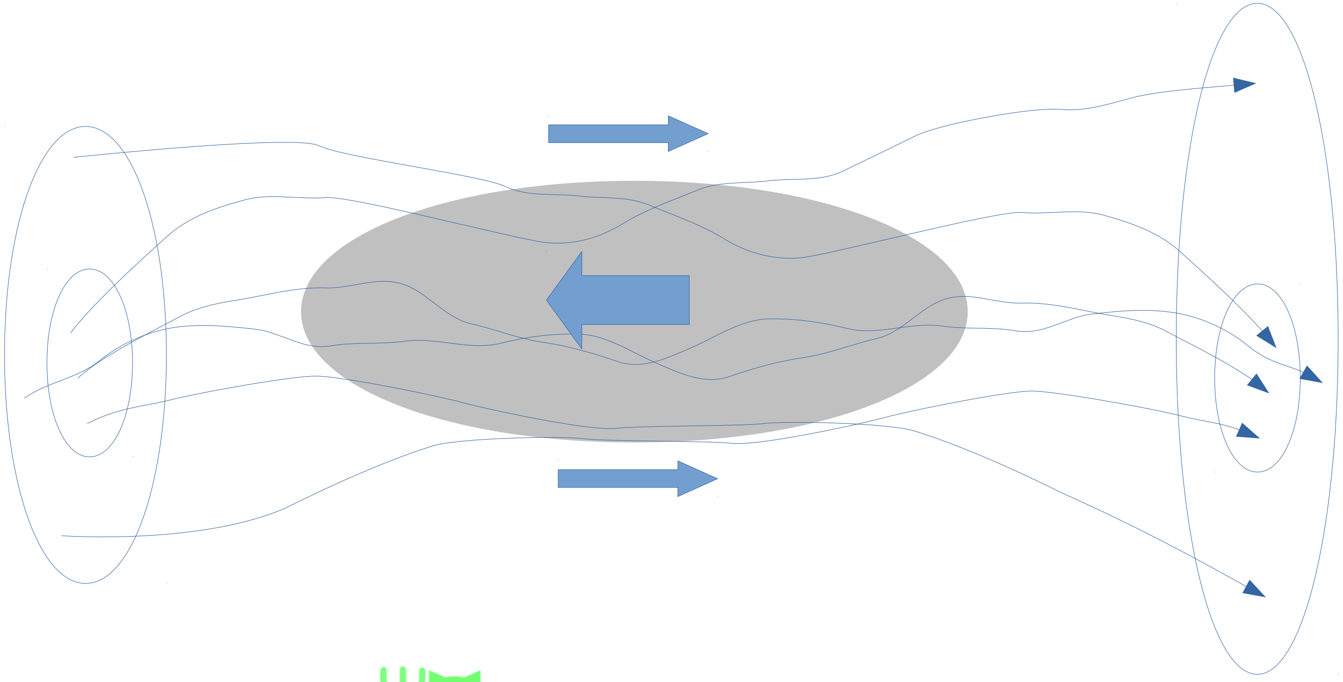
Superdiffusion in streaming instability?

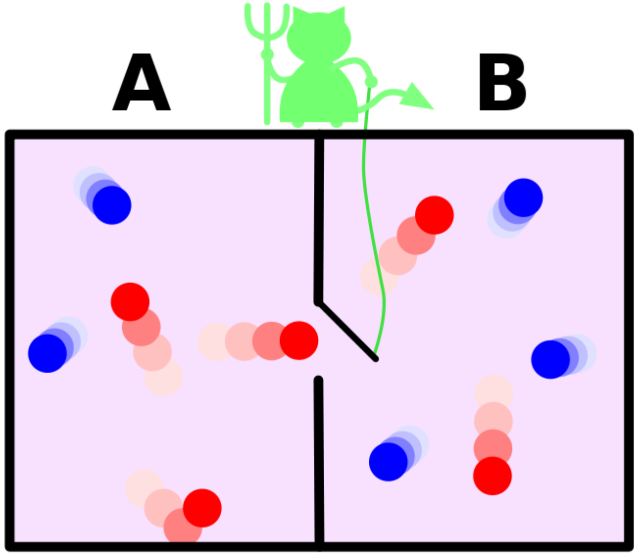


$$\alpha = 1.27$$

→ Perry & Zimbardo slope -1.6

Do you expect diffusion to be symmetric?





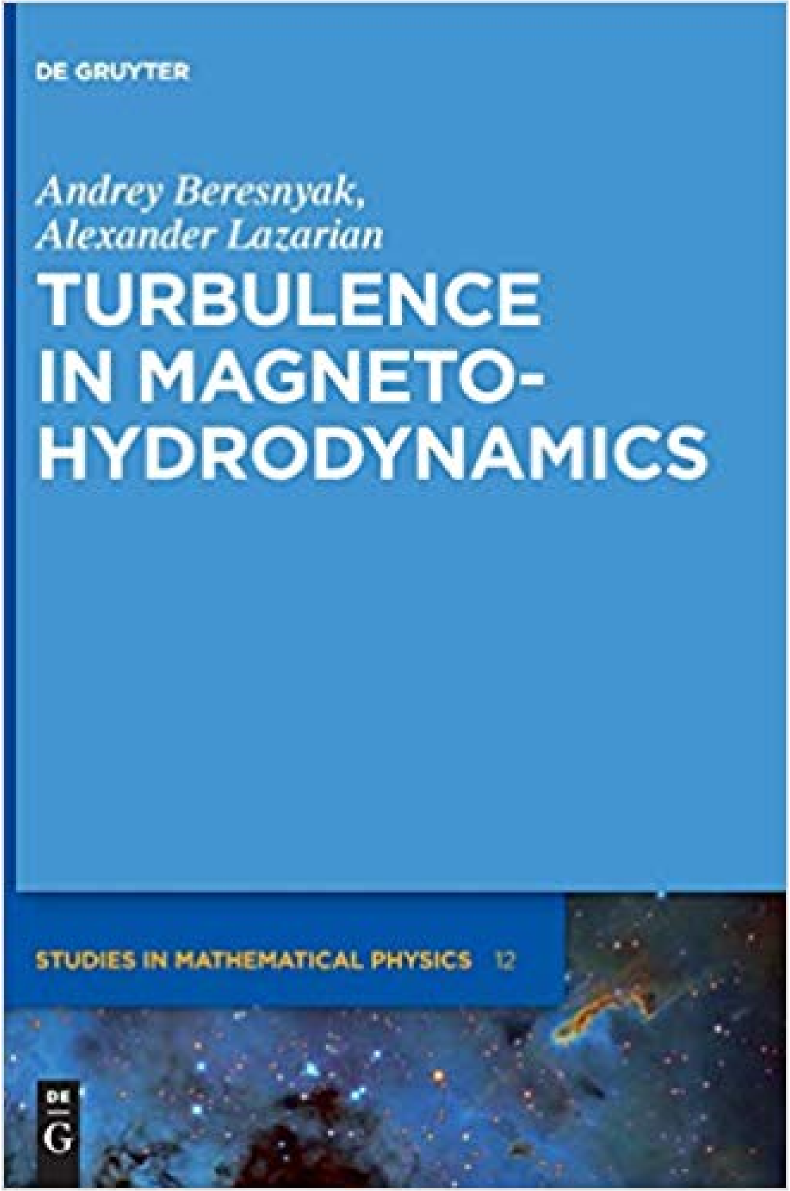
AB, ApJ 2013

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = D\frac{\partial^\alpha f}{\partial |x|^\alpha}$$

?

Summary

- Diffusive shock acceleration requires diffusion D << than D measured in the ISM. The key to understand this is turbulence created in the upstream of the shock by cosmic rays - nonlinear streaming instability and the inverse cascade of magnetic energy creating magnetic field with large correlation lengths.
- We use particle-MHD code Hephaestus that addresses the issue of particles back-reacting on the fluid.
- Our results indicate that the streaming instability proceeds into a nonlinear regime with $\delta B/B \gg 1$ without any problems, which had also been observed in earlier PIC simulations
- We also observed inverse cascade, development of perturbations with scales much larger than the Larmor radius of particles
- The particle dynamics may be different from classic 2nd order diffusion (no QLT!)



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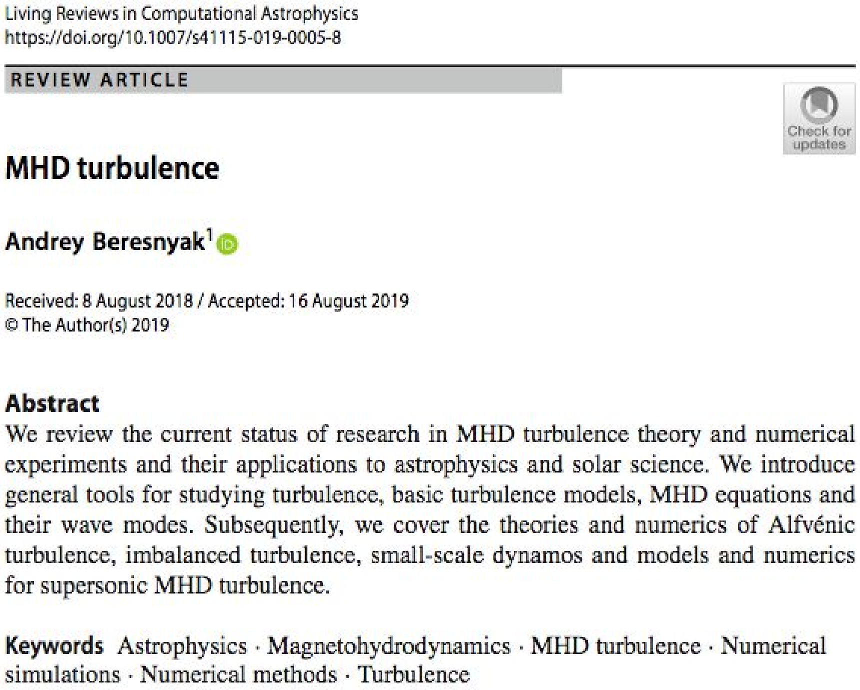

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REVIEW ARTICLE

MHD turbulence

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Abstract

We review the current status of research in MHD turbulence theory and numerical experiments and their applications to astrophysics and solar science. We introduce general tools for studying turbulence, basic turbulence models, MHD equations and their wave modes. Subsequently, we cover the theories and numerics of Alfvénic turbulence, imbalanced turbulence, small-scale dynamos and models and numerics for supersonic MHD turbulence.

Keywords Astrophysics · Magnetohydrodynamics · MHD turbulence · Numerical simulations · Numerical methods · Turbulence

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