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MHD-PIC Simulations of the Cosmic-ray Streaming Instability

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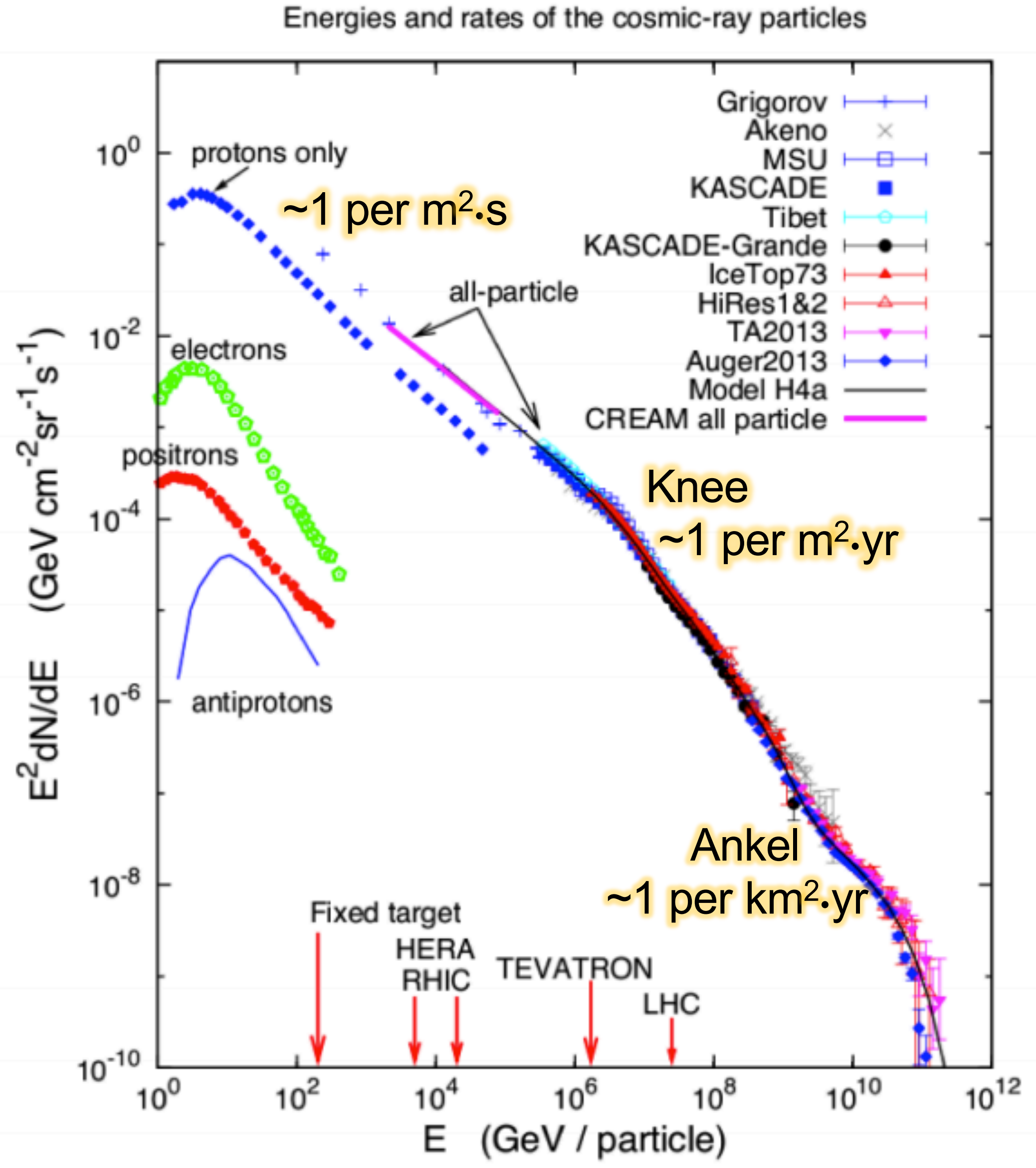
Outline

- Motivation and MHD-PIC method
- CR streaming instability (CRSI): basic physics
- Simulating CRSI: method
- Simulating CRSI: results
- Summary

Cosmic-rays in the ISM



(Blasi 2013)

Energy density ~1 eV cm^{-3}, comparable to thermal/magnetic/turbulent energy density in the ISM

Most CR pressure from ~GeV particles (protons).

Typical CR gyroradii are tiny:

$$r_L = 1.08 \times 10^{-6}\text{pc}\left(\frac{E}{\text{GeV}}\right)\left(\frac{B}{\mu\text{G}}\right)^{-1}$$

~sub-AU

How do CRs interact with a thermal plasma?

- CRs are collisionless and diffuse by scattering off MHD waves/turbulence:

  Galactic CRs' residence time: ~10s Myr in total.

  Diffusion coefficient: $\kappa \sim R^2/T \sim$ a few x 10^{28} cm^{2}s^{-1}. (e.g., Ginzburg & Syrovatskii 64)

  Passive "test particles"

- CRs provide pressure support:

$$F = -\nabla_\perp P_{\rm CR} = -\frac{\boldsymbol{J}_{\rm CR} \times \boldsymbol{B}}{c}$$

  Formally, it is perpendicular to B due to a magnetization current.

- When the bulk CRs drift through background plasma faster than the Alfven speed $v_{D,CR} > v_A$, they will drive *streaming instabilities*.

  (e.g., Kulsrud & Pearce 1969, Bell 2004)

  CRs transfer energy and momentum to gas via Alfven waves.

  Active CR feedback !

Consequences of CR feedback: (See E. Zweibel's talk)

CR self-confinement:

- CR streaming creates Alfven waves, which scatter and isotropize the CRs (in wave frame), reducing CR streaming.
- CRs are trapped by waves they create by themselves.

CR-driven outflows:

- CR can drive outflow by its pressure gradient.
- CR streaming leads to energy/momentum exchanges with ISM gas.
- CR streaming can lead to significant heating.

Important feedback mechanism to galaxy formation and evolution.

Fluid treatment at macroscopic scales

(see C. Pfrommer, K. Yang's talks)

The CR energy equation (taking moments of the CR transport Eq.):

$$\frac{\partial \varepsilon_{\rm cr}}{\partial t} + \nabla \cdot \boldsymbol{F}_{\varepsilon} = (\boldsymbol{v} + \boldsymbol{v}_{\rm st}) \cdot \nabla P_{\rm cr} + \Gamma_{\rm acc} + \bar{Q}_{\varepsilon}$$

CR heating

$$\boldsymbol{F}_{\varepsilon} = (\boldsymbol{v} + \boldsymbol{v}_{\rm st})(P_{\rm cr} + \varepsilon_{\rm cr}) - \kappa_{\varepsilon} \boldsymbol{b} \, (\boldsymbol{b} \cdot \nabla \varepsilon_{\rm cr})$$

CR advection and streaming

CR diffusion

Underlying assumptions:

- We are at scales >> CR mfp >> CR gyro radii. (no kinetic physics)
- CRs relaxes to an isotropic distribution in the "streaming" frame on short timescales. (likely true, but not without caveats)

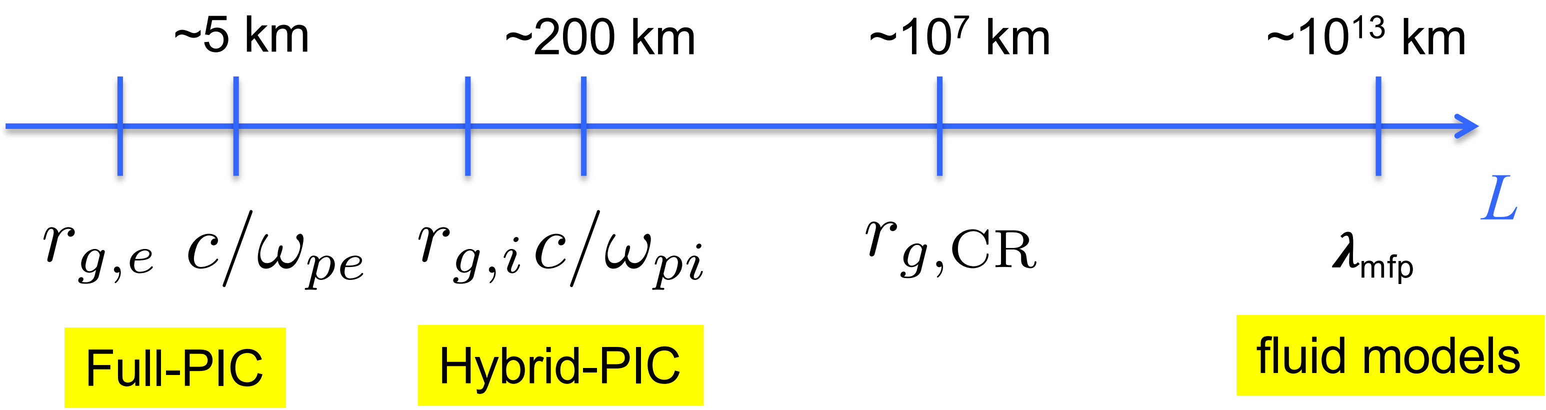
What is the diffusion coefficient? What is the streaming speed?

Need kinetic physics (for subgrid models)!

Simulating CR physics at kinetic level

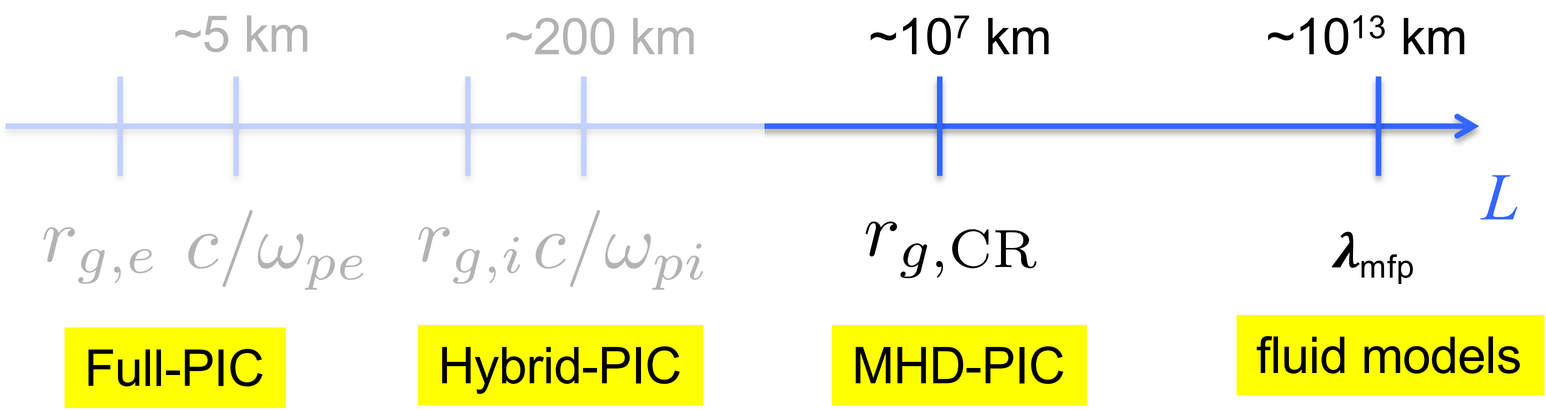
- Minimum requirement: resolve CR gyro-radii.
- Huge scale separation involved, challenging for conventional PIC codes:

  - *Full-PIC*: treat all (background+CR) particles as kinetic particles
  - Hybrid-PIC: all ions (background+CR) are kinetic, electrons as massless fluid



Simulating CR physics at kinetic level

- Minimum requirement: resolve CR gyro-radii.
- Overcome this scale separation: skipping over the kinetic scales of the background plasma.
  - MHD-PIC: treat background plasma by MHD, while CRs are kinetic



Similar approaches have been proposed/implemented, e.g., Zachary & Cohen (1986), Lucek & Bell (2000), Reville & Bell (2012).

MHD-PIC: formulation and implementation

Equations for the (relativistic) CR particles:

$$\frac{d(\gamma_j \boldsymbol{u}_j)}{dt} = \frac{q_j}{m_j}\left(\boldsymbol{E} + \frac{\boldsymbol{u}_j}{c} \times \boldsymbol{B}\right)$$

Specify the numerical speed of light c >> any velocities in MHD.

Full equations for the gas:

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \mathbf{P}^*) =$$ - Lorentz force on the CRs

$$\frac{\partial E}{\partial t} + \nabla \cdot [(E + P^*)\boldsymbol{v} - \boldsymbol{B}(\boldsymbol{B} \cdot \boldsymbol{v})] =$$ - energy change rate on the CRs

Implemented in the Athena MHD code (Bai, Caprioli, Sironi & Spitkovsky 2015).

(See also van Marle, Casse & Marcowith 2018, Mignone et al. 2018)

Applicability of MHD-PIC

Using MHD to describe background plasmas.

Collisional background plasmas?

Missing ion and electron kinetic-scale physics (e.g., linear/non-linear Landau)

"Magneto-immutability": even if background plasma is collisionless, MHD can still be a reasonable approximation. (J. Squire's talk)

Separation between background plasma and high-energy CRs

Need a clear division between non-thermal and thermal particles. Treating the former using PIC, and treating the latter as fluid.

This may not always hold, creating ambiguities.

Additional requirement:

Numerical peed of light >> MHD velocities.

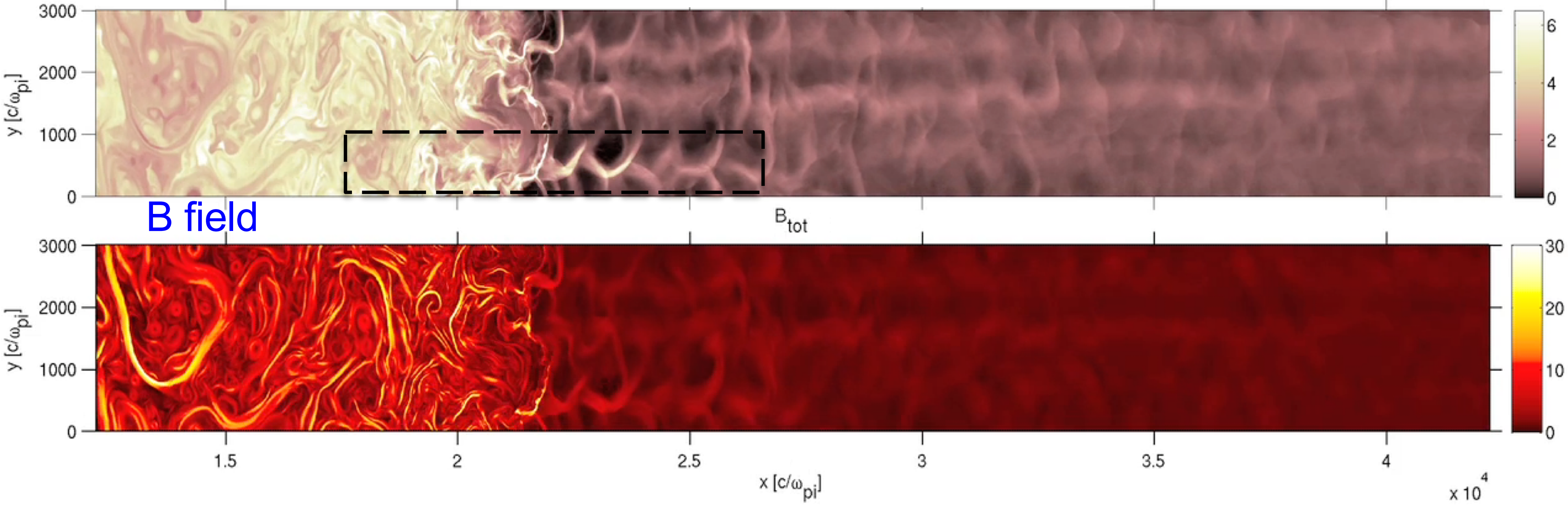
CR number density << background gas.

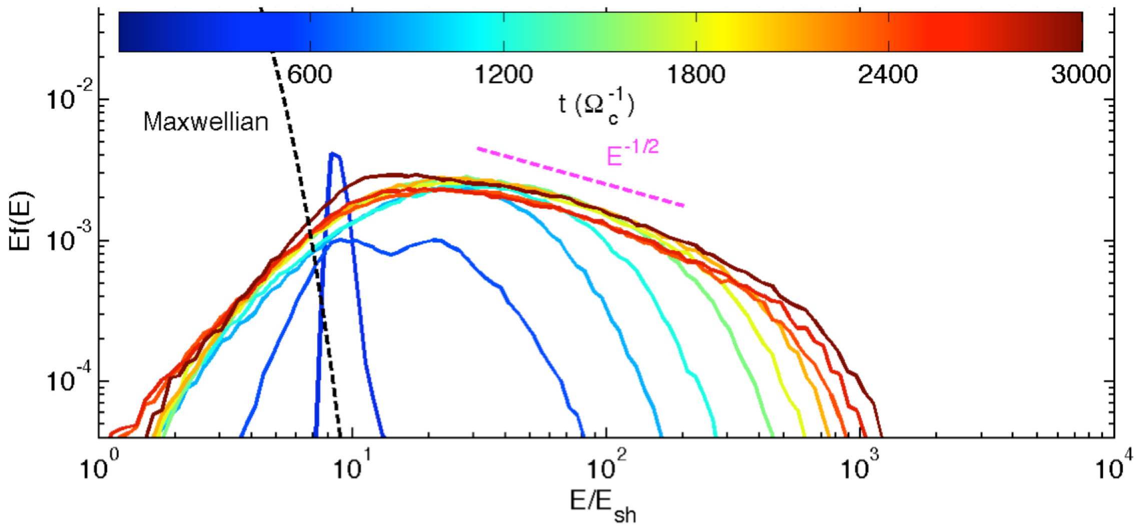
(easy to satisfy)

Demo: simulating non-relativistic shocks

Bai+ 2015

Density M_A~30, parallel shock θ=0.





12 ion skin depths per cell

(v.s. 0.5 in hybrid-PIC)

$f(E)\sim E^{-3/2}$ (non-relativistic)

Efficiency: ξ~13%

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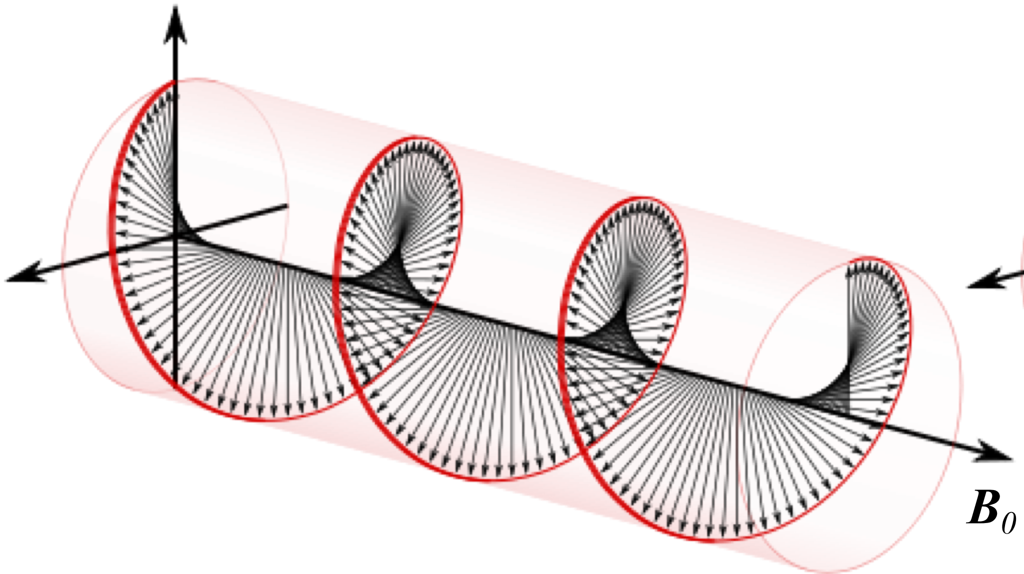
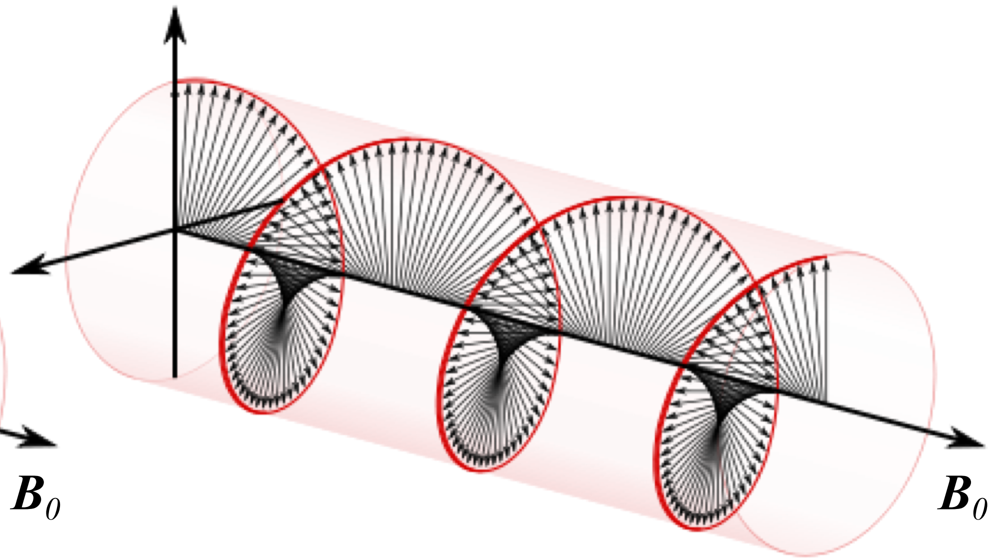
Resonant interactions with Alfvén waves

Right polarization:



Resonant with backward-traveling ions.

Left polarization:



Resonant with forward-traveling ions.

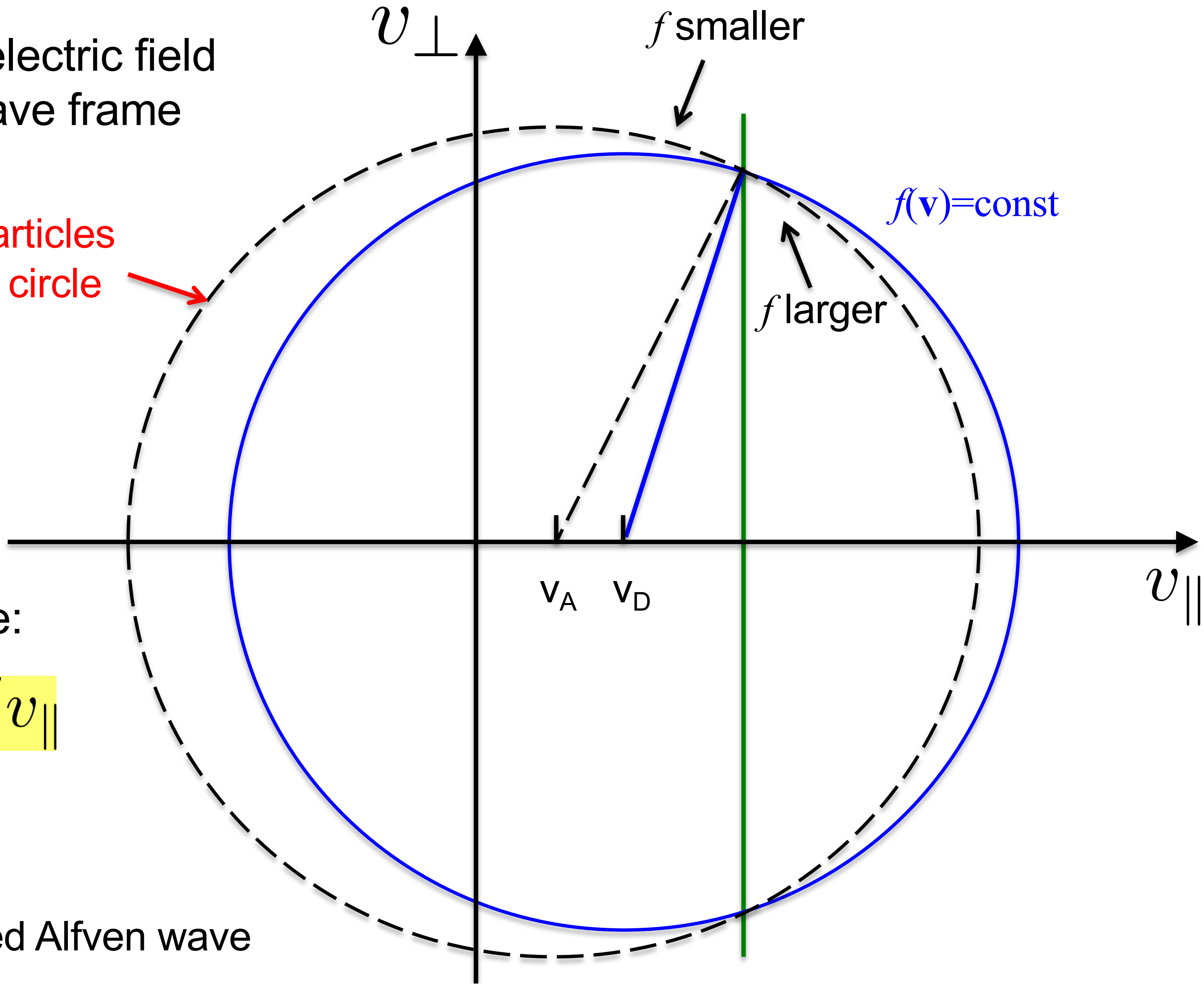
Gyro resonance: $$\omega - k v_{\parallel} = \pm\Omega$$

In general, $\omega << \Omega$: $$k = \pm\Omega / v_{\parallel}$$

CR streaming instability: basic physics

Alfven wave: electric field vanishes in wave frame

Individual CR particles move along this circle



Gyro resonance:

$$k = \pm\Omega/v_\parallel$$

Excite left-polarized Alfven wave

Resonant CR streaming instability

When CR drift velocity v_D exceeds v_A:

- Forward-traveling CRs resonantly excite (right) polarized, forward-propagating Alfven waves.
- Backward-traveling CRs resonantly excite (left) polarized, forward propagating Alfven waves.
- Backward-propagating Alfven waves are suppressed.

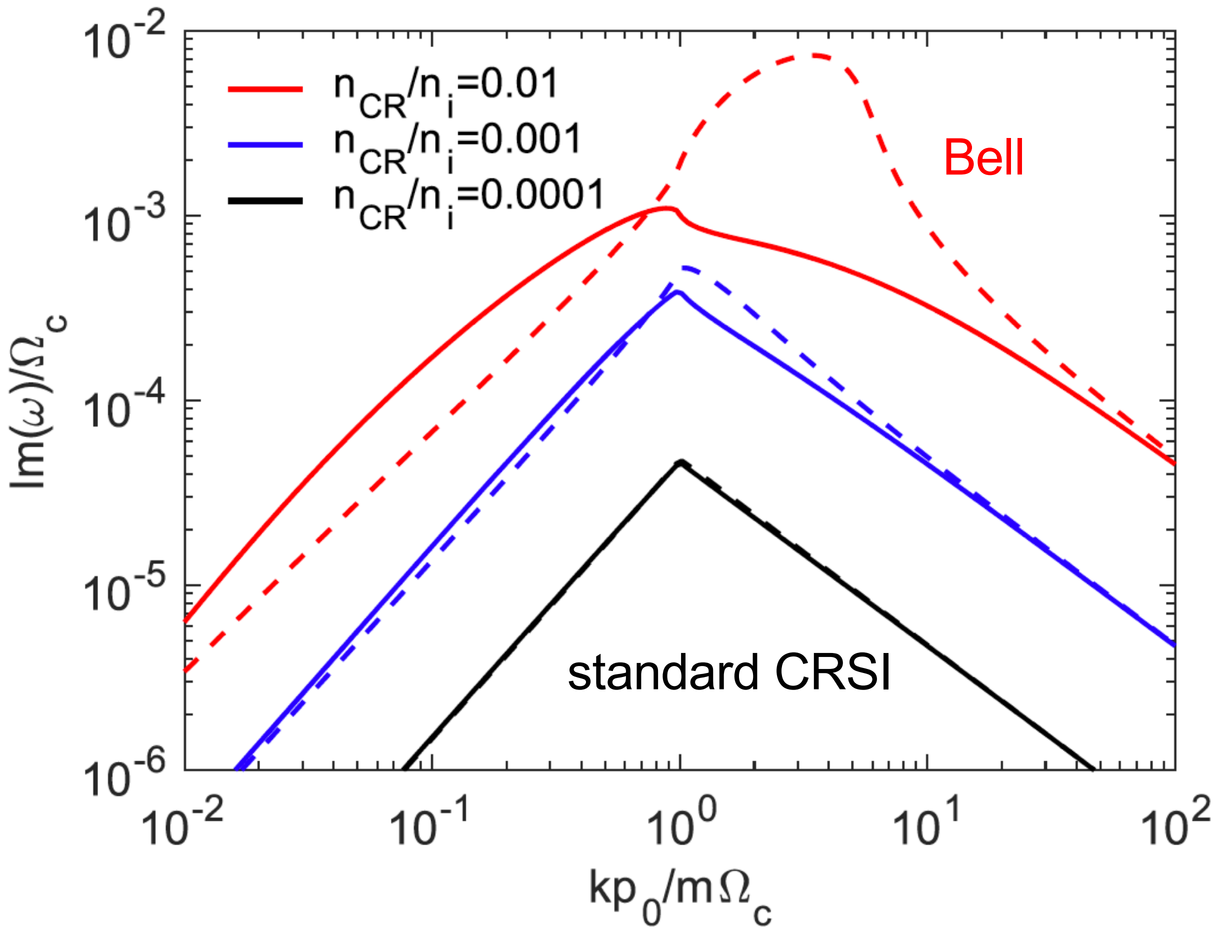
(e.g., Kulsrud & Pearce 69, Skilling 75)

Characteristic growth rate:

$$\Gamma(k) \approx \Omega_c \frac{N_{\rm CR}(p > p_{\rm res}(k))}{n_i} \frac{v_D - v_A}{v_A}$$

Driven primarily by low-energy CRs (i.e. the dominant CR population by number)

Dispersion relation



$$f(p) = p^{-\alpha}$$

$$(p_0 < p < p_{\max})$$

Instability is dominant by the non-resonant (Bell) mode in the case of strong streaming.

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Computational challenges

- Huge scale separation:

Microscopic plasma scale ion skin depth: $\delta_i = \dfrac{c}{\omega_{pi}} = \dfrac{v_A}{\Omega_c}$

CR resonant wavelengths are much longer: $\lambda \approx \dfrac{p_{\rm CR}}{m\Omega_c}$

$$\frac{\lambda}{\delta_i} \sim \frac{c}{v_A}$$

Conventional PIC:

Must resolve skin depth, very expensive!

- Solution:

Use MHD-PIC, which mainly needs to resolve resonant scale.

But see work by Holcomb & Spitkovsky (2019, in revision) in full-PIC in more extreme regimes, and talks by C. Haggerty and A. Beresnyak.

Computational challenges

- Resonant interaction: only a tiny fraction of particles involved

Level of anisotropy ~v_D/c <<1 => Need huge # of particles per cell

- Solution:

When distribution function f does not deviate much from some initial equilibrium f_0, one can implement the δf method, based on Liouville's theorem (particle moves along characteristics of constant f).

=> Dramatically reduced Poisson noise.

Caveat: need a smooth distribution for f_0 => use a κ distribution.

Computational challenges

- Pitch-angle scattering of particles (quasi-linear diffusion):

Particles should traverse independent wave packets over wave growth time:

$$L > L_{\min} \equiv \frac{\mathbb{C}\mu}{\Gamma_{\max}} \sim \frac{n_i}{n_{\rm CR}} \frac{k_{\rm res}^{-1}}{\gamma}.$$

=> Need excessively long simulation box.

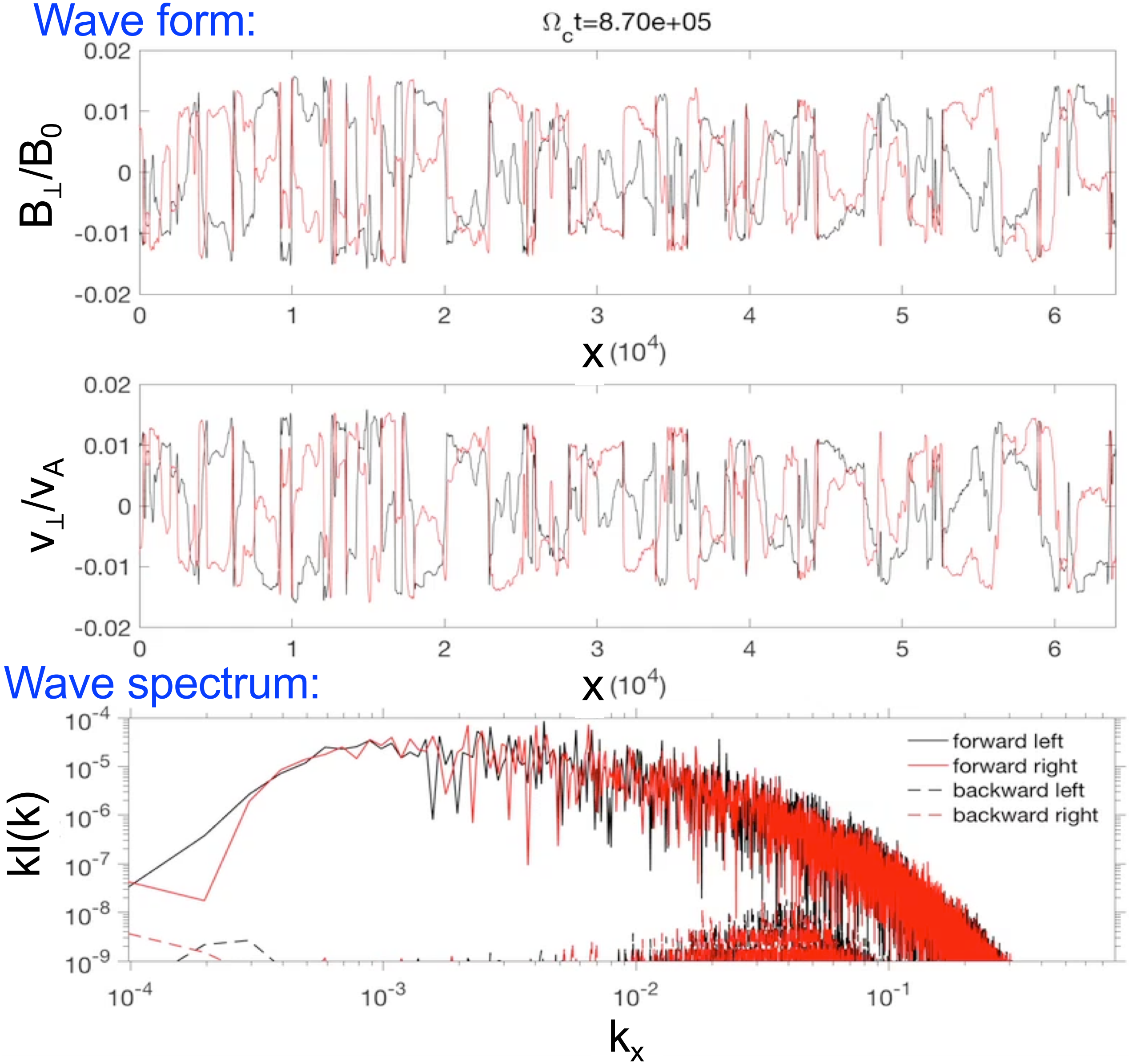
- Solution:

Randomize particle phases when they cross periodic boundaries.

1D simulation: growth and saturation

Wave form:

Wave spectrum:



Bai, Ostriker, Plotnikov & Stone 2019

Simulation performed in the rest frame of the CRs.

Periodic BC.

Gas travels to the left at v_D.

Fiducial parameters:

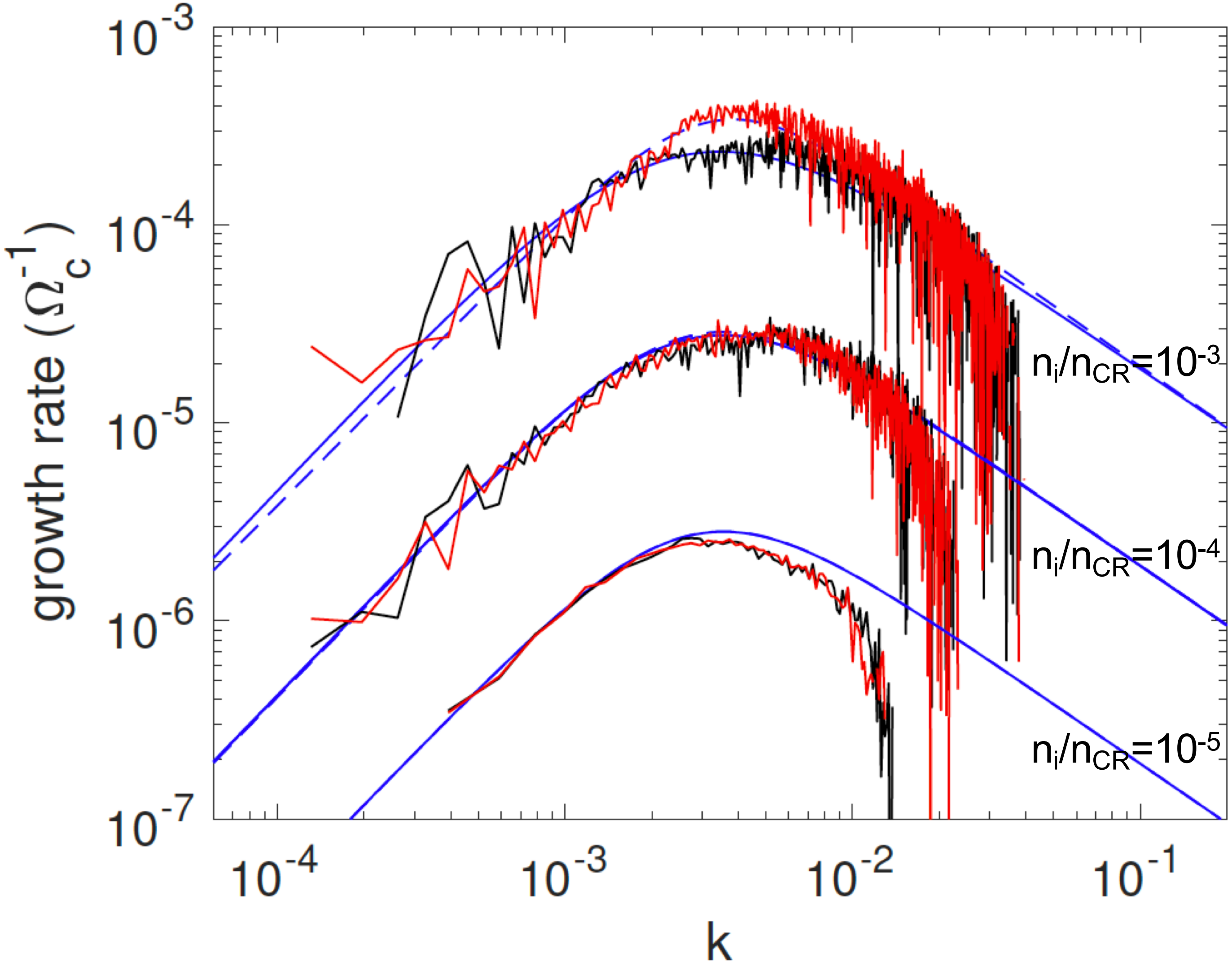
$v_D=2v_A$

$N_{CR}/n_0=10^{-4}$

$p_0/m=300v_A$

2048 ppc, Lx~50 most unstable wavelength.

Matching analytical dispersion relation



Accurately reproduce the linear growth rate over broad spectrum.

Bai, Ostriker, Plotnikov & Stone 2019

Towards saturation: quasi-linear diffusion

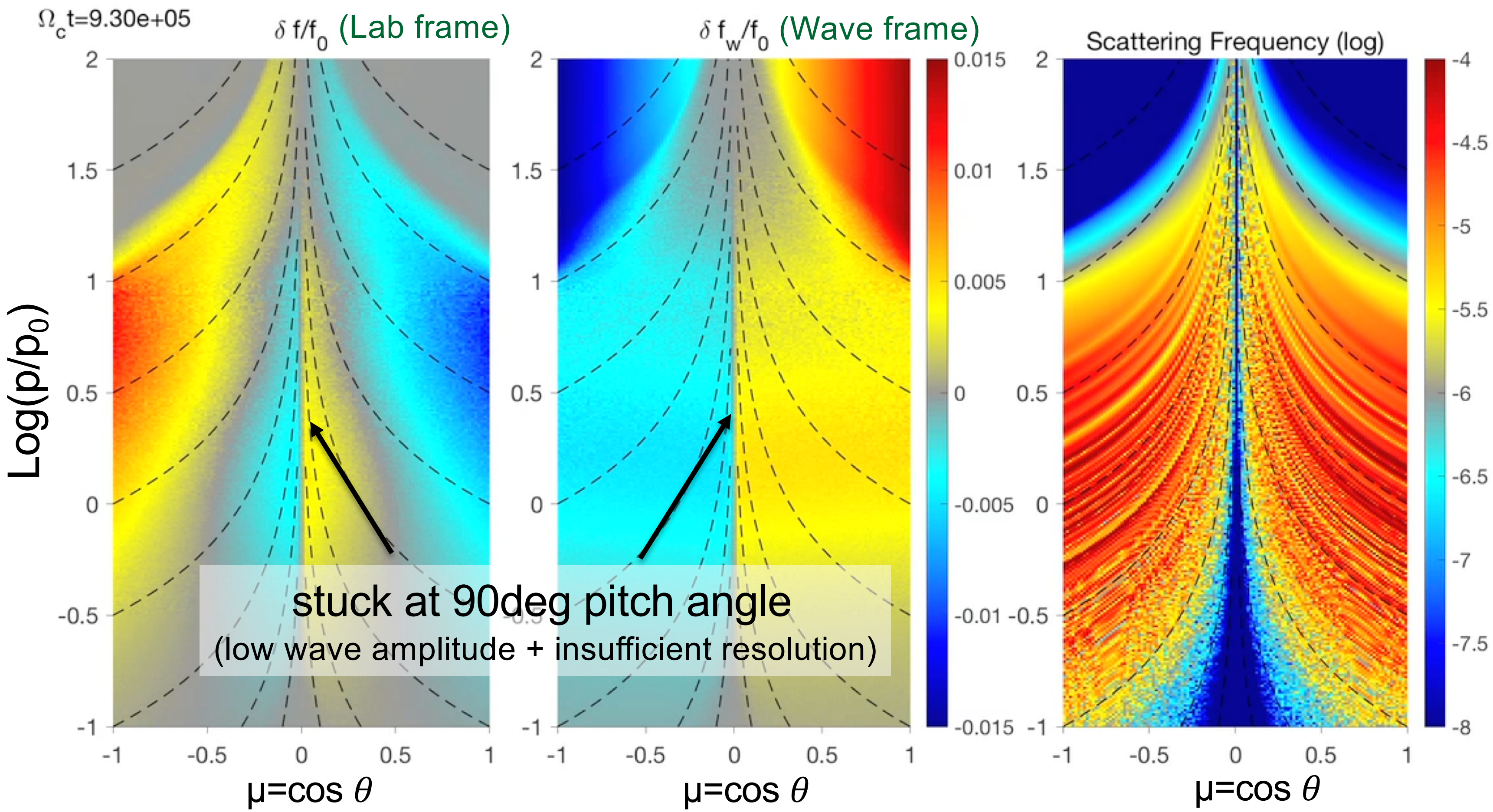
$$\frac{\partial f_w}{\partial t} = \frac{\partial}{\partial \mu_w}\left[\frac{1-\mu_w^2}{2}\nu(\mu_w)\frac{\partial f_w}{\partial \mu_w}\right]$$ + reflection

Scattering frequency:

$$\nu(\mu_w) = \pi\Omega k_{\rm res} I(k_{\rm res})$$

wave intensity

Parameters: $v_D=2v_A$; $N_{CR}/n_0=10^{-4}$



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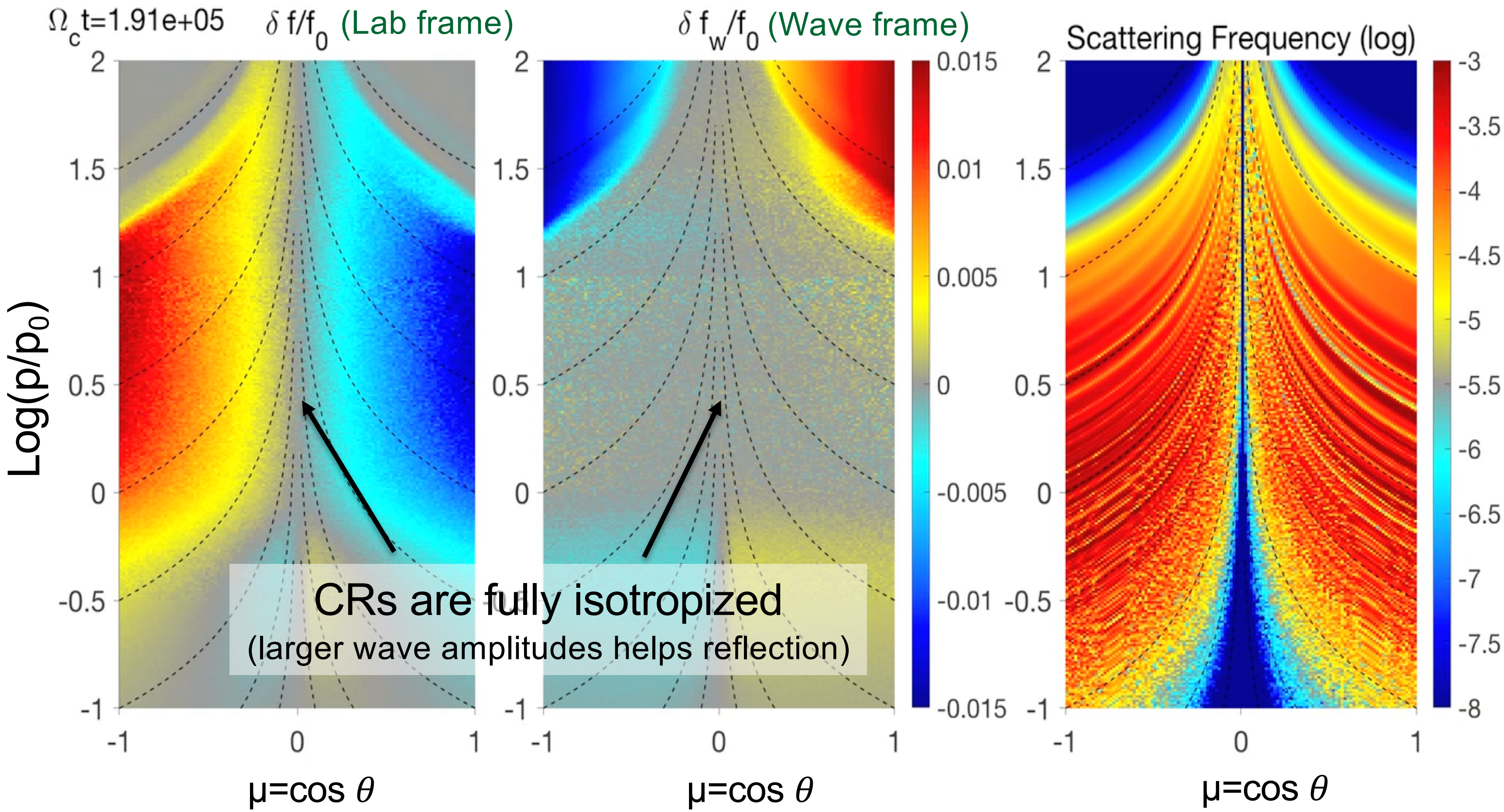
Scattering frequency:

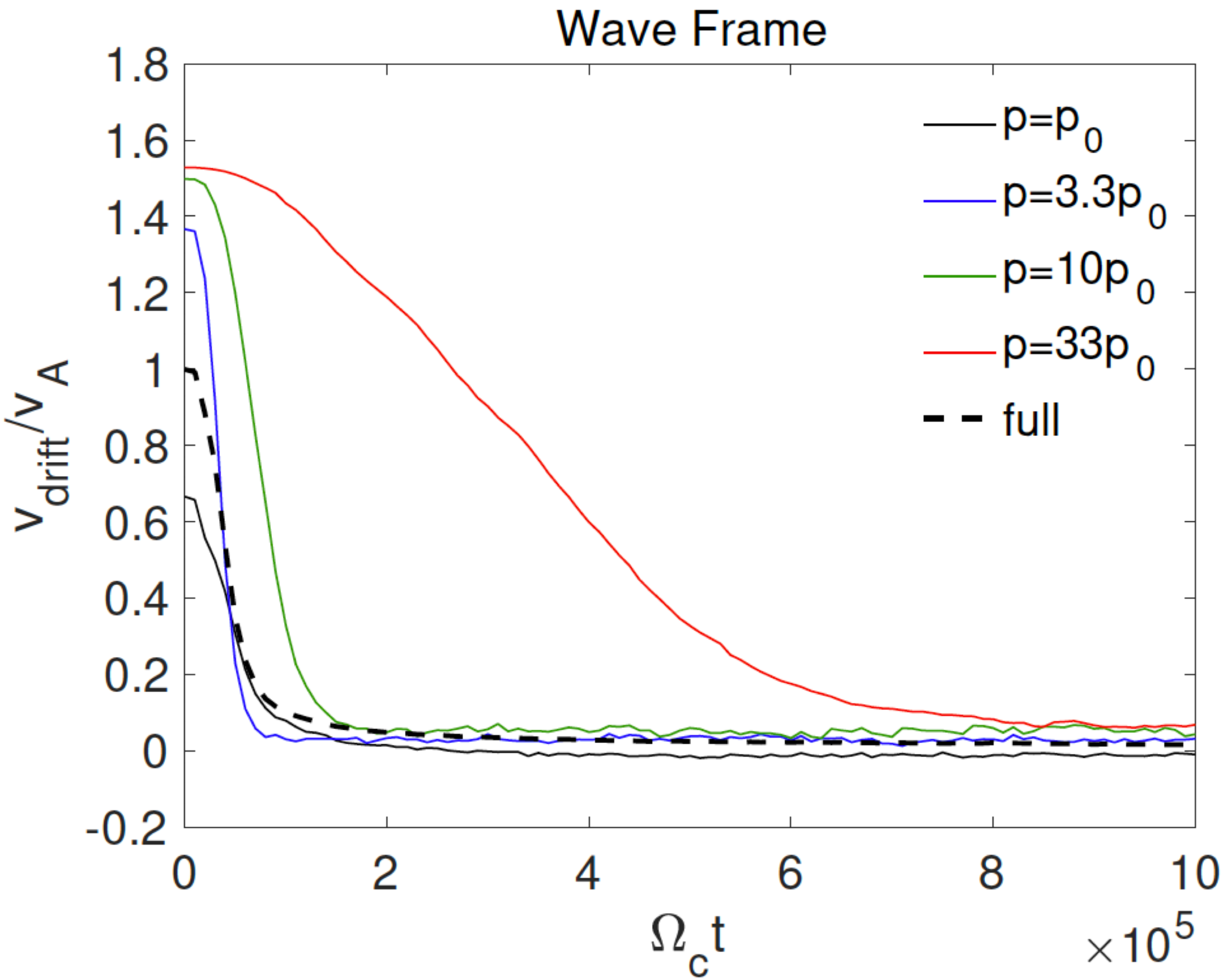
$$\nu(\mu_w) = \pi \Omega k_{\rm res} I(k_{\rm res})$$

wave intensity

Parameters: $v_D=2v_A$; $N_{CR}/n_0=10^{-3}$



Reduction of CR drift speeds



$v_D=2v_A$

$N_{CR}/n_0=10^{-3}$

Bai, Ostriker, Plotnikov & Stone. 2019

Beyond quasi-linear theory

- Scattering across 90° pitch angle:

$$k = \pm\Omega / v_{\|}$$

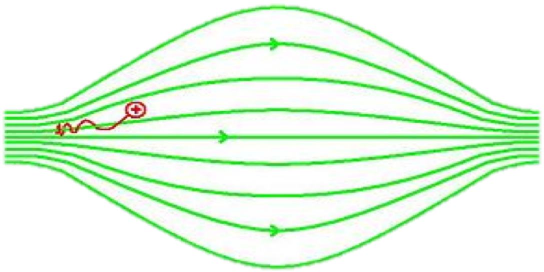
⇒ Resonant wavelength approaches 0...

⇒ Need infinite resolution?

- Solution 1: Reflection by magnetic mirroring (Felice & Kulsrud 02).

Due to conservation of magnetic moment in slow-varying field (adiabatic invariant):

$$M \equiv p_{\perp}^2 / 2B$$



Beyond quasi-linear theory

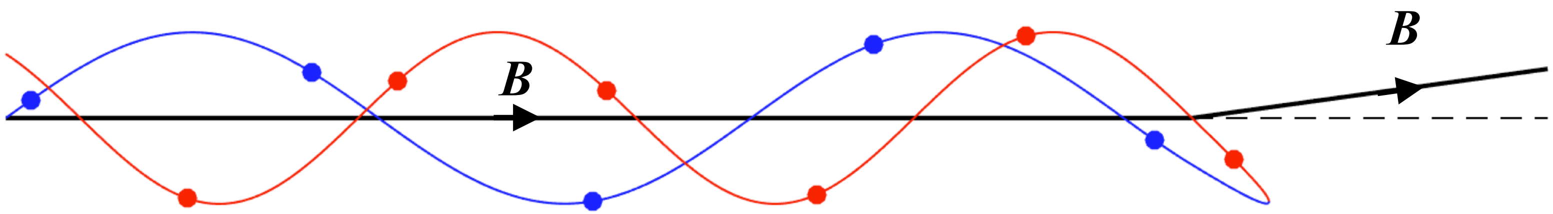
- Scattering across 90° pitch angle:

$$k = \pm\Omega/v_{\|}$$

⇒ Resonant wavelength approaches 0...

⇒ Need infinite resolution?

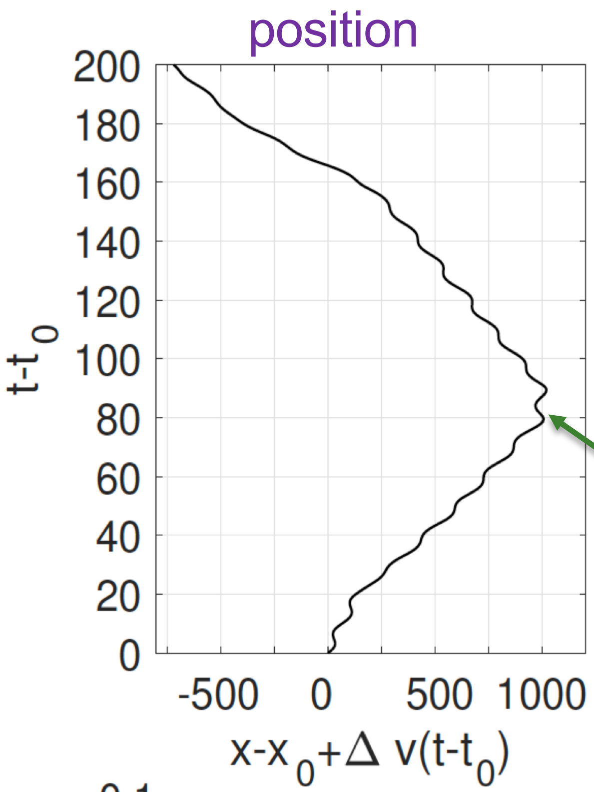
- Solution 2: Non-linear wave-particle interaction (e.g., Volk 73).



Reflection by more abrupt changes in field configuration.

Which mechanism is responsible?

A representative reflection event



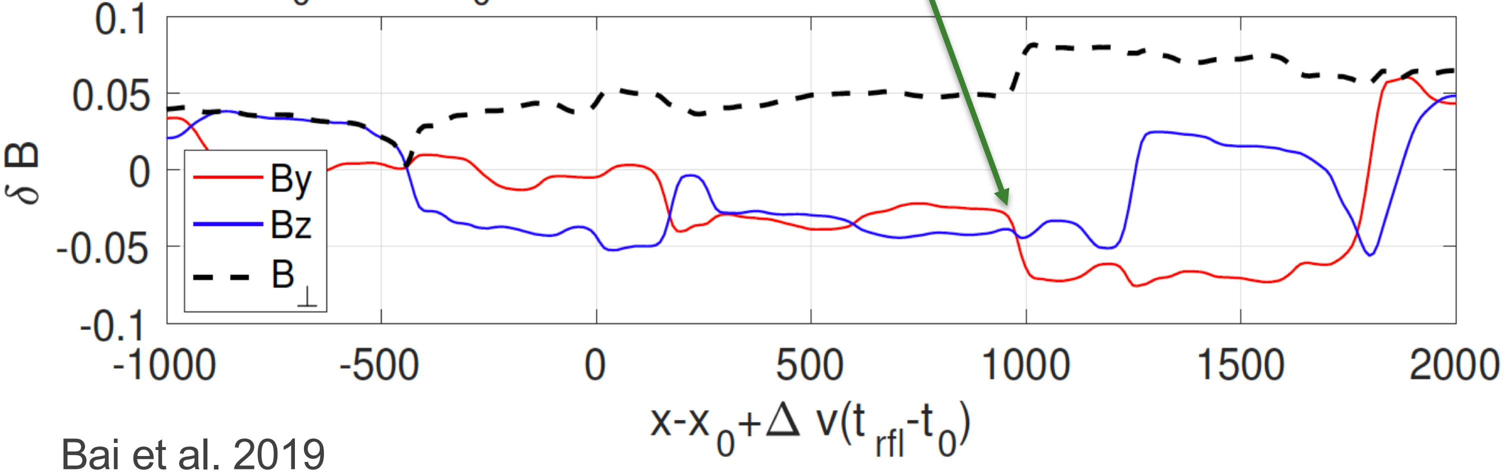
During reflection, magnetic moment is not conserved:

Not mirror reflection.

Abrupt changes in By during reflection:

Consistent with NL wave-particle interaction.

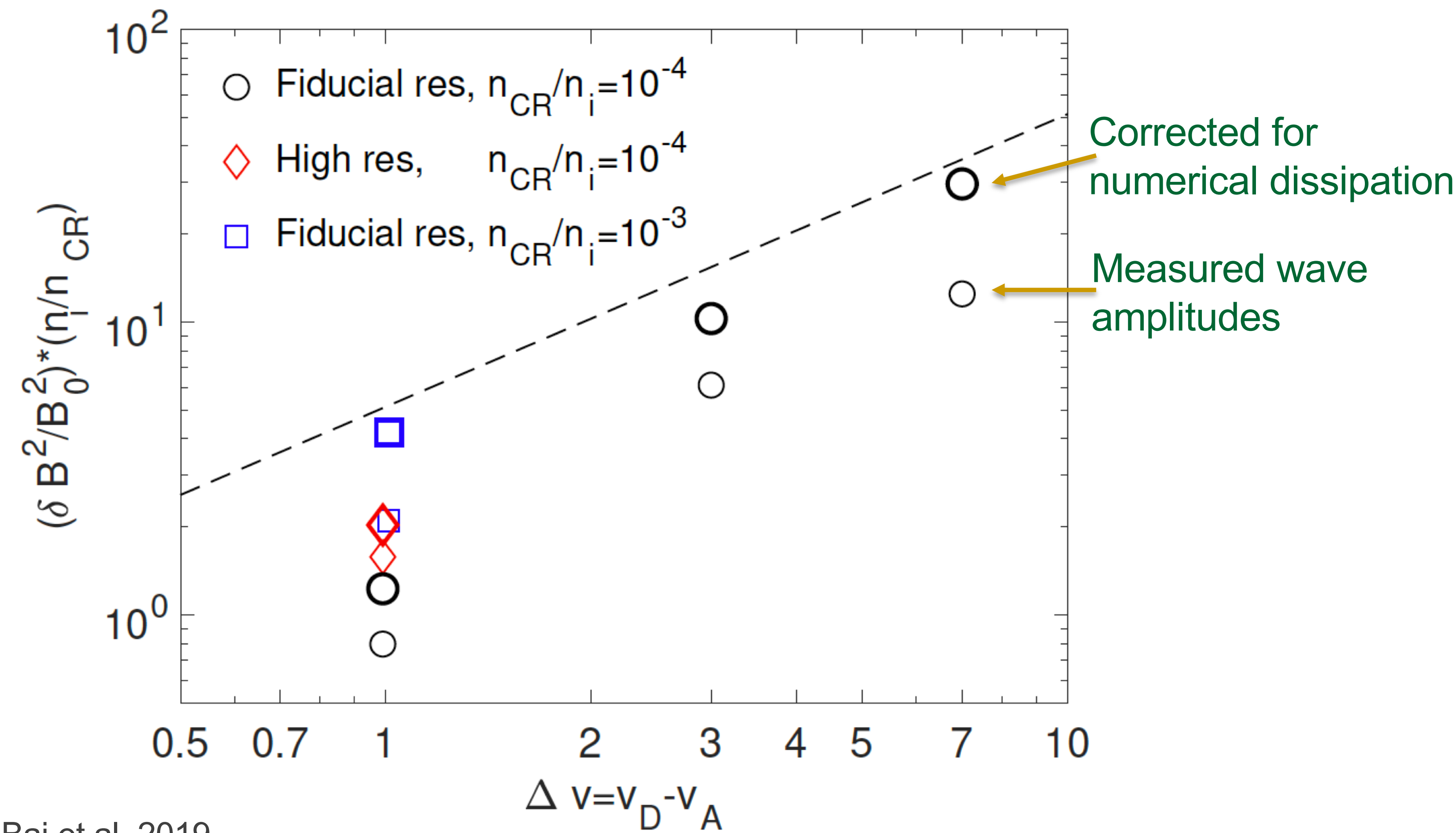
when reflection occurs



Bai et al. 2019

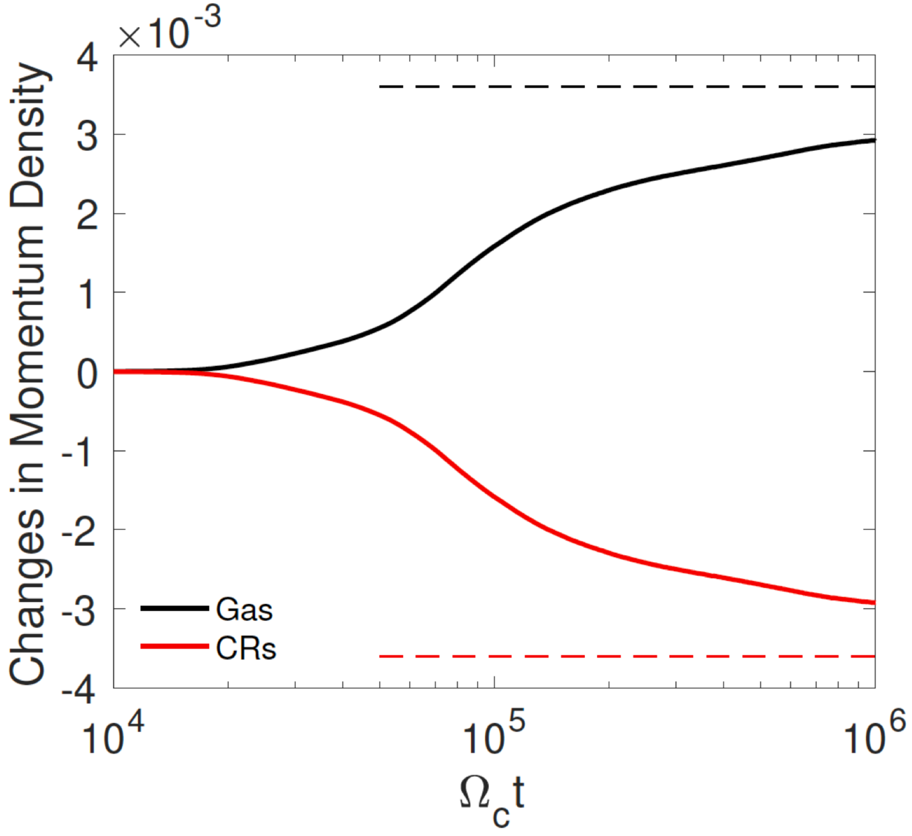
Saturation level

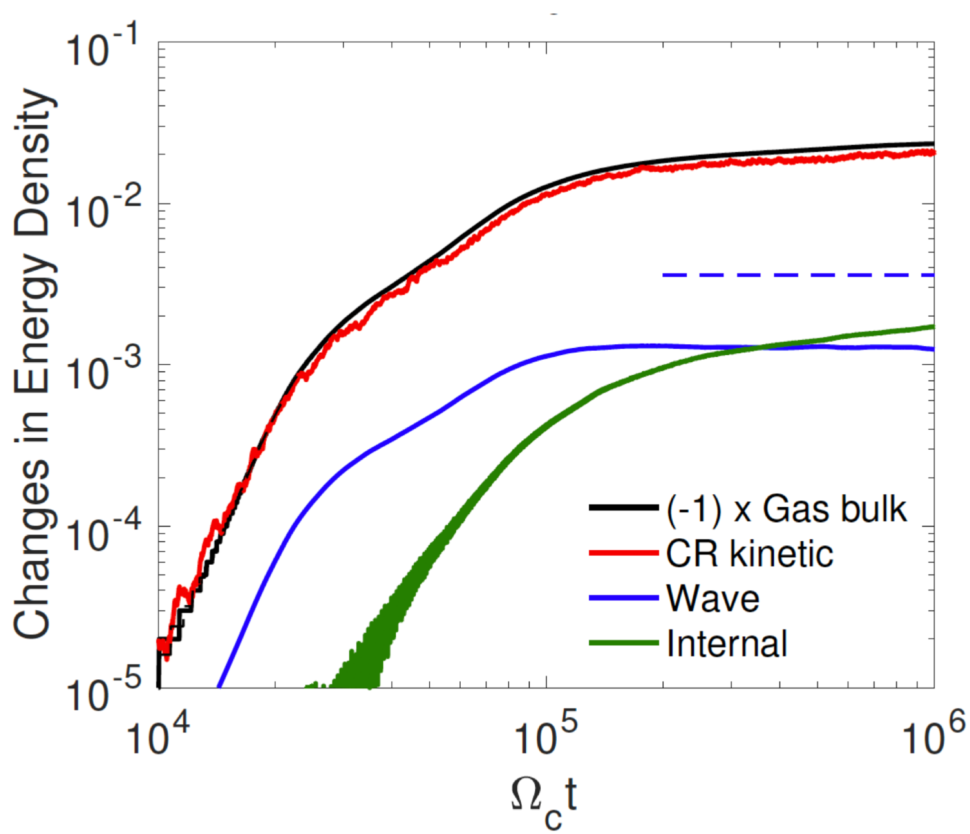
Condition for saturation: Reduction of CR momentum = wave momentum



Bai et al. 2019

CR feedback





CR momentum goes to the gas.

CRs do not directly heat the gas:
CRs excite waves, wave damping leads to heating.

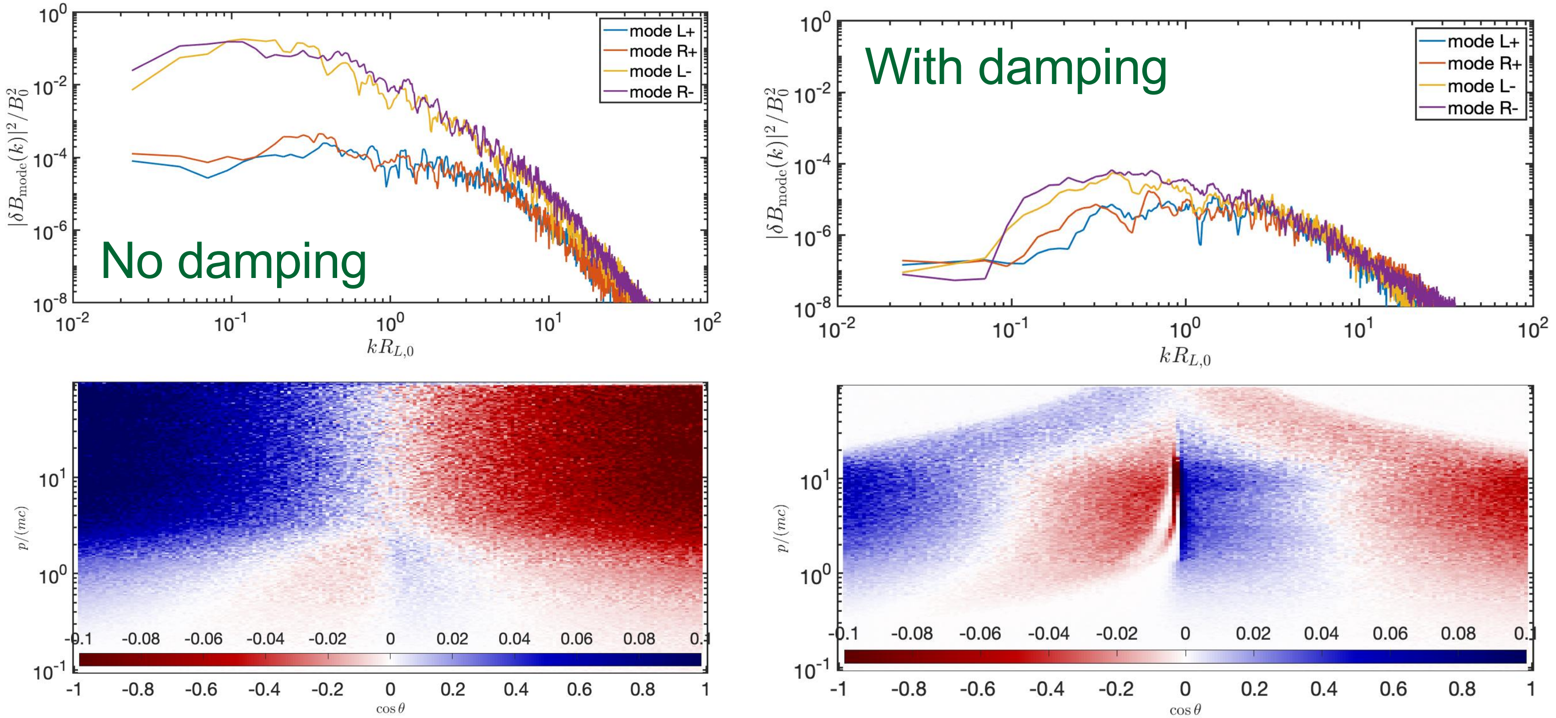
Bai et al. 2019

Wave damping

Ion-neutral damping (Kulsrud & Pearce 69)
Non-linear Landau damping (Lee & Volk 73)
Turbulent damping (Goldreich & Farmer 04)

=> When effective, CRs no longer well isotropized, weaken CR feedback.

We start from ion-neutral damping:



Crossing 90deg is more demanding

(Plotnikov+, in prep)

Future directions

- More realistic damping
- Towards multi-dimensions
- Other gyro-resonant instabilities
- Non-periodic BC / source problem
- Applications to multi-phase ISM
- Prescriptions for sub-grid model

Summary

- Coupling of CRs with background plasma
  - Collisionless charged particles scattered by waves
  - Dynamically important and feedback to background gas.
- Development of the MHD-PIC method/code
  - Valid on scales > ion skin depth, fully conservative, well tested.
- MHD-PIC simulations of resonant CR streaming instability
  - Overcome the challenges: need δf method and phase randomization.
  - First numerical study: confirmation of linear growth rates, and can follow CR quasi-linear evolution.
  - Crossing 90deg pitch angle: dominated by non-linear wave-particle interaction.
- Future: more realistic microphysics, provide subgrid model