Continuum (Approximate) Models of Star Cluster Dynamics

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Overview

- 1. Introduction: Plasma, Fokker-Planck, core collapse
- 2. Rotation: Fokker-Planck models, no black hole
- 3. Central Star Accreting Black Hole: Gas and Fokker-Planck

Extra Topics:

- From Newton to Einstein…
- GRAPE and GRACE Hardware

People: Pau Amaro-Seoane (now AEI), Eunhyeuk Kim (now Harvard), Jose Fiestas, Ilsang Yoon, Hyung Mok Lee, Koji Takahashi (Univs. of Heidelberg, Seoul, Tokyo)

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Introduction: Plasma, Fokker-Planck, core collapse

Star Cluster:

$$\mathbf{a}_i = -\sum_j G m_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

Plasma:

$$\mathbf{a}_i = \frac{q_i}{m_i} \sum_j \frac{q_j}{4\pi\varepsilon_0} \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$

Binding Energy of Binaries $E = Gm^2/(2a)$ (analogous Rydberg atoms)

…seems no problem in principle to implement in codes…
even with moving ions …

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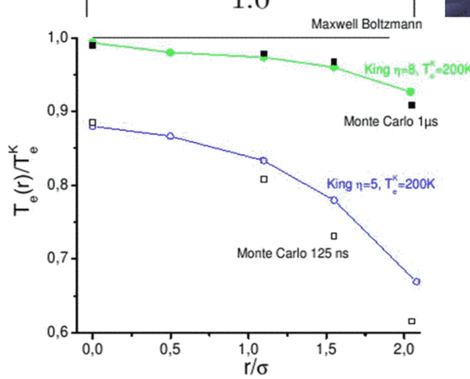
Introduction: Plasma, Fokker-Planck, core collapse

	Star Cluster	Ultracold Plasma
Number of particles	$5 \cdot 10^5$	10^6
Peak Density	$10^6 \mathrm{M}_\odot \mathrm{pc}^{-3}$	$10^{10} \mathrm{cm}^{-3}$
Size/Core Radius r	$1 \text{ pc} \approx 3 \cdot 10^{18}$ cm	$200\ \mu\mathrm{m} = 2 \cdot 10^{-2}$ cm
d/r	0.01	$10^{-5.33}$
mass range M/m	100 (1000)	1800 (m_i/m_e)
Γ_e (e-e)	0.01	0.1
Γ_i (i-i)	$(M/m)\Gamma_e$	$(m_i/m_e)\Gamma_e$
t_{e-e}/t_{dyn}	100	0.01 A
t_{e_i}/t_{dyn}	1.0	1.0



How do they compare?
(Table inspired by
Comparat et al. 2004)

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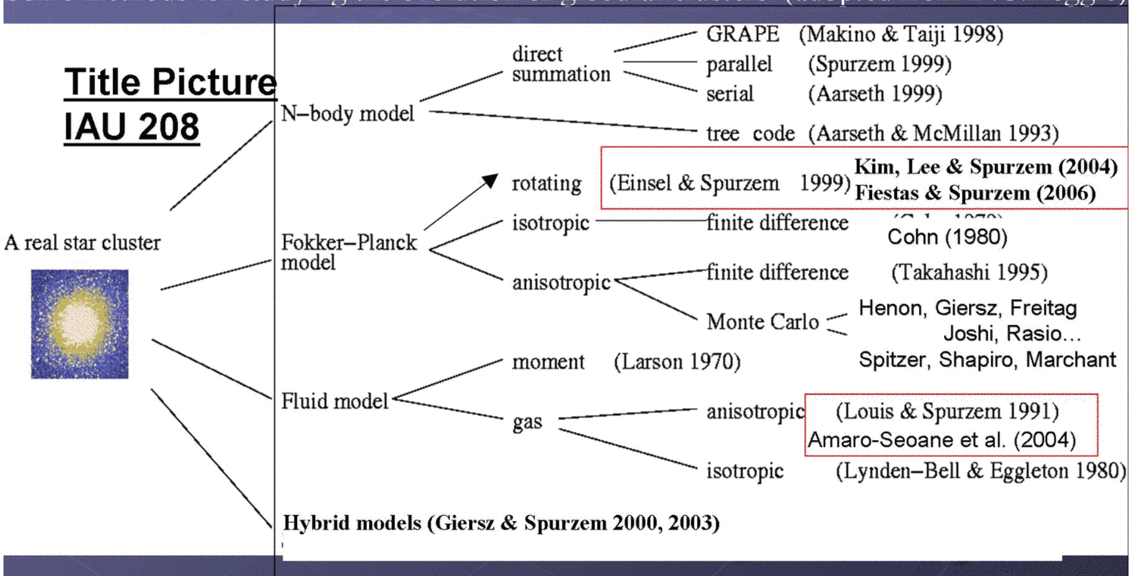
Introduction: Plasma, Fokker-Planck, core collapse

Some methods for studying the evolution of globular clusters (adopted from D.C.Heggie)



Introduction: Plasma, Fokker-Planck, core collapse

$$\left(\frac{\delta f}{\delta t}\right)_{\rm enc} = -\sum_{i=1}^{3}\left[\frac{\partial}{\partial x_i}\Big(f(\mathbf{x},\mathbf{v})D(\Delta x_i)\Big) + \frac{\partial}{\partial v_i}\Big(f(\mathbf{x},\mathbf{v})D(\Delta v_i)\Big)\right]$$
$$+\frac{1}{2}\sum_{i,j=1}^{3}\left[\frac{\partial^2}{\partial x_i\partial x_j}\Big(f(\mathbf{x},\mathbf{v})D(\Delta x_i\Delta x_j)\Big) + \frac{\partial^2}{\partial v_i\partial v_j}\Big(f(\mathbf{x},\mathbf{v})D(\Delta v_i\Delta v_j)\Big)\right.$$
$$\left. + \frac{\partial^2}{\partial x_i\partial v_j}\Big(f(\mathbf{x},\mathbf{v})D(\Delta x_i\Delta v_j)\Big) + \frac{\partial^2}{\partial v_i\partial x_j}\Big(f(\mathbf{x},\mathbf{v})D(\Delta v_i\Delta x_j)\Big)\right].$$

Full Fokker-Planck Equation, use Liouville's Theorem and Rosenbluth Potentials (local equation):

$$\left(\frac{\delta f}{\delta t}\right)_{\rm enc} = -4\pi G^2 m_f \ln\Lambda\left[\sum_{i=1}^{3}\frac{\partial}{\partial v_i}\left(f(\mathbf{v})\frac{\partial h}{\partial v_i}\right) + \frac{1}{2}\sum_{i,j=1}^{3}\frac{\partial^2}{\partial v_i\partial v_j}\left(f(\mathbf{v})\frac{\partial^2 g}{\partial v_i\partial v_j}\right)\right].$$

(Rosenbluth, McDonald, Judd 1956)

Introduction: Plasma, Fokker-Planck, core collapse

$$h(\mathbf{v}) = (m + m_f) \int \frac{f(\mathbf{v}_f)}{|\mathbf{v} - \mathbf{v}_f|} d^3\mathbf{v}_f$$

$$g(\mathbf{v}) = m_f \int f(\mathbf{v}_f) |\mathbf{v} - \mathbf{v}_f| d^3\mathbf{v}_f \ .$$

Rosenbluth Potentials (Rosenbluth, McDonald & Judd 1956)

$$D(\Delta v_i) = 4\pi G^2 m_f \ln\Lambda \frac{\partial}{\partial v_i} h(\mathbf{v})$$

$$D(\Delta v_i v_j) = 4\pi G^2 m_f \ln\Lambda \frac{\partial^2}{\partial v_i v_j} g(\mathbf{v})$$

Diffusion Coefficients (Local)

$$f(v) = \sum_{l=0}^{\infty} A_l(v) P_l(\mu)$$

Gas Models:
up to l=2 for background f
(in principle higher possible)

Introduction: Plasma, Fokker-Planck, core collapse

Spurzem & Takahashi, 1995

784 *R. Spurzem and K. Takahashi*

$$g(v,\mu) = 4\pi m \left\{ \left[v I_2 + \frac{1}{3v} I_4 + \frac{v^2}{3} K_1 + K_3 \right] + \frac{1}{5}\frac{c_2}{\sigma^2} P_2(\mu) \left[-\frac{1}{3v} I_6 + \frac{1}{7v^3} I_8 + \frac{v^4}{7} K_1 - \frac{v^2}{3} K_3 \right] \right\}.$$

$$h(v,\mu) = 4\pi(m + m_\mathrm{p}) \left\{ \left[\frac{1}{v} I_2 + K_1 \right] + \frac{1}{5} P_2(\mu) \frac{c_2}{\sigma^2} \left[\frac{1}{v^3} I_6 + v^2 K_1 \right] \right\}$$

$$I_n = \int_0^v v_\mathrm{p}^n g_\mathrm{p}(v_\mathrm{p}) \mathrm{d}v_\mathrm{p}$$

$$K_n = \int_v^\infty v_\mathrm{p}^n g_\mathrm{p}(v_\mathrm{p}) \mathrm{d}v_\mathrm{p} \ .$$

Fully analytic, get coll. Terms,
Fully self-consistent.

Exists for arbitrary order but not
yet implemented.

Introduction: Plasma, Fokker-Planck, core collapse

Spurzem & Takahashi, 1995

$$Dp_r = \left(\frac{\delta p_r}{\delta t}\right)_c = \frac{2\pi\rho}{\lambda_{eq}} \int_0^\infty v^2 dv \int_{-1}^{+1} d\mu v^2 \mu^2 (FP)$$

$$Dp_t = \left(\frac{\delta p_t}{\delta t}\right)_c = \frac{2\pi\rho}{\lambda_{eq}} \int_0^\infty v^2 dv \int_{-1}^{+1} d\mu v^2 \frac{1}{2}(1-\mu^2)(FP)$$

$$De = \frac{1}{2}\left(Dp_r + 2Dp_t\right) = \left(\frac{3}{2}C_1 + \frac{1}{2}C_4 + C_4'\right)/(T\lambda_{eq})$$

$$Dp_a = Dp_r - Dp_t = \left(\frac{1}{2}(C_2+C_3) + C_4 - C_4'\right)/(T\lambda_{eq}).$$

$$C_1 = -\frac{3}{4}\rho'\left(\sigma^2 - \frac{m'}{m}\sigma'^2\right)$$

$$C_2 = -\frac{1}{5}\rho'\sigma_a{}^2\left(1 + \frac{3}{2}\frac{m'}{m}\right) + \frac{1}{5}\rho'\sigma_a'^2\left(\frac{3}{2} + \frac{m'}{m}\right)$$

$$C_3 = -\frac{3}{10}\rho'\left(\sigma_a{}^2 + \sigma_a'^2\right)\cdot\left(1 + \frac{m'}{m}\right)\Big/\left(1 + \frac{\sigma^2}{\sigma'^2}\right)$$

$$C_4 = -\frac{27}{140}\rho'\sigma_a\sigma_a'^2\left(\sigma^2 + \sigma'^2\right)^{-2} \times \left[\sigma^2\left(1 + \frac{16}{27}\frac{m'}{m}\right) - \frac{28}{27}\sigma'^2\left(1 + \frac{39}{28}\frac{m'}{m}\right)\right]$$

C_1: classical equipartition term
C_2, C_3: first order anisotropy interactions
C_4: second order anisotropy interactions

$$T = \frac{9}{16\sqrt{\pi}} \frac{\sigma^3}{G^2 m\rho \log(\gamma N)}$$

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Introduction: Plasma, Fokker-Planck, core collapse

Louis & Spurzem, 1991, Amaro-Seoane Freitag & Spurzem 2004

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u\rho\right) = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{GM_r}{r^2} + \frac{1}{\rho}\frac{\partial p_r}{\partial r} + 2\frac{p_r - p_t}{\rho r} = 0$$

$$\frac{\partial p_r}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u p_r\right) + 2p_r\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) - \frac{2F_t}{r} = \left(\frac{\delta p_r}{\delta t}\right)_{\rm enc} + \left(\frac{\delta p_r}{\delta t}\right)_{\rm bin3}$$

$$\frac{\partial p_t}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u p_t\right) + 2\frac{p_t u}{r} + \frac{1}{2}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_t\right) + \frac{F_t}{r} = \left(\frac{\delta p_t}{\delta t}\right)_{\rm enc} + \left(\frac{\delta p_t}{\delta t}\right)_{\rm bin3}.$$

$$F = -\kappa\frac{\partial T}{\partial r} = -\Lambda\frac{\partial \sigma^2}{\partial r}.$$

$$v_r - u + \frac{\lambda}{4\pi G\rho t_{\rm rx}}\frac{\partial \sigma^2}{\partial r} = 0$$

$$v_r = v_t .$$

$$\dot{n}_{3b} = \tilde{C}\, n_{2b}\, n\, \sigma A = \tilde{C} n^3 \sigma \left(\frac{Gm}{\sigma^2}\right)^5 .$$

$$\left(\frac{\delta e}{\delta t}\right)_{\rm bin3} = E_{\rm heat}\cdot\dot{n}_{3b} = \frac{15x}{2}\tilde{C} m n^3 \sigma^3 \left(\frac{Gm}{\sigma^2}\right)^5 .$$

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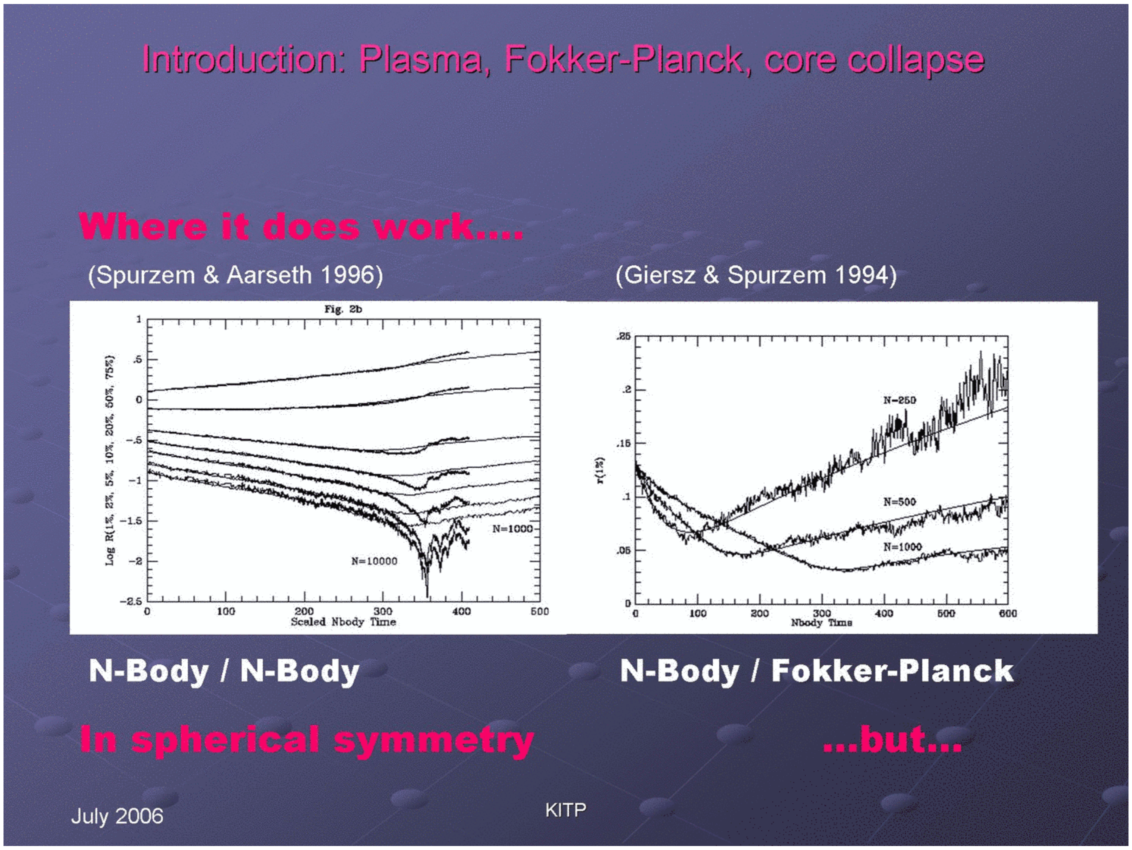
Introduction: Plasma, Fokker-Planck, core collapse
Where it does work....
(Spurzem & Aarseth 1996)
(Giersz & Spurzem 1994)
Fig. 2b
Log R(1%, 2%, 5%, 10%, 20%, 50%, 75%)
Scaled Nbody Time
N=1000
N=10000
r(1%)
Nbody Time
N=250
N=500
N=1000
N-Body / N-Body
N-Body / Fokker-Planck
In spherical symmetry
...but...
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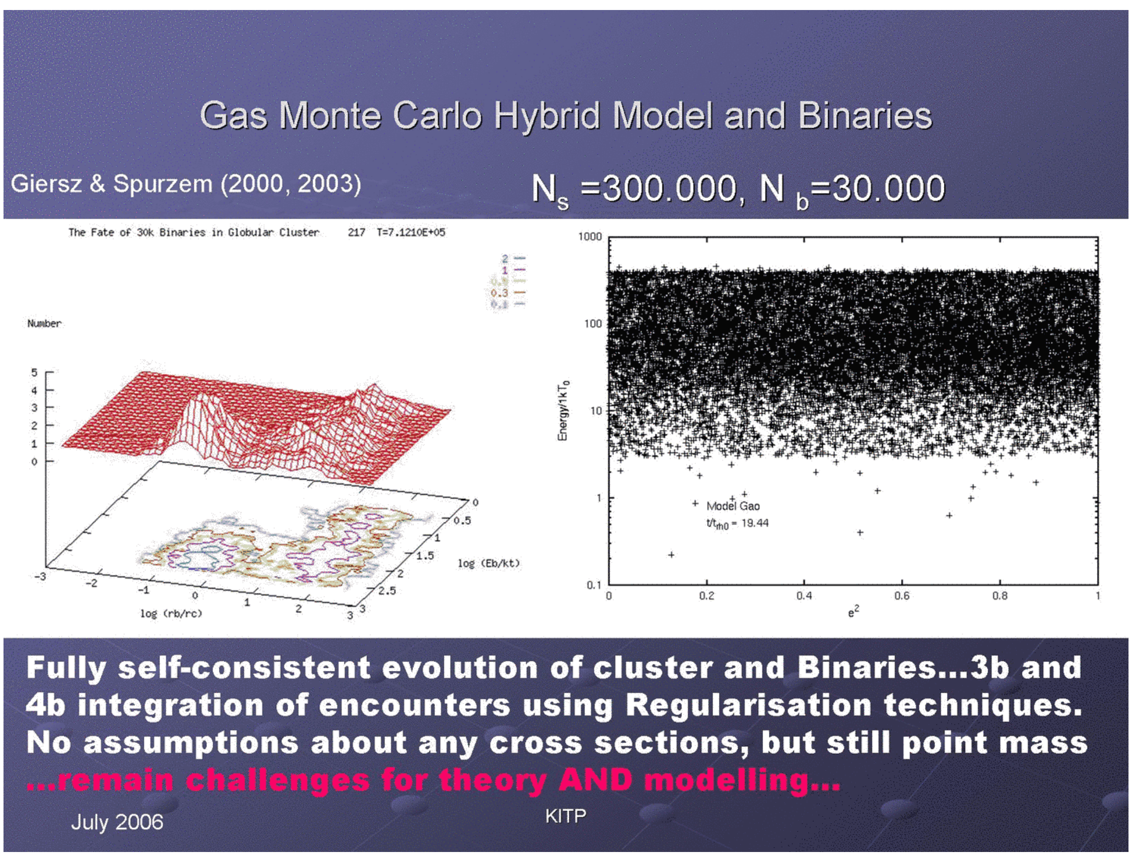
Gas Monte Carlo Hybrid Model and Binaries
Giersz & Spurzem (2000, 2003)
N s =300.000, N b=30.000
The Fate of 30k Binaries in Globular Cluster 217 T=7.1210E+05
Number
log (rb/rc)
log (Eb/kt)
Energy/1kT0
e2
Model Gao
t/trh0 = 19.44
Fully self-consistent evolution of cluster and Binaries...3b and
4b integration of encounters using Regularisation techniques.
No assumptions about any cross sections, but still point mass
...remain challenges for theory AND modelling...
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Rotation: Fokker-Planck models, no black hole

- Agekian (1959)

* biased escape from rotating Maxwellian

- Hachisu (1979)

* gravo-gyro catastrophe

- Goodman (1983) (unpublished Phd thesis)

* first Fokker-Planck modeling of rotating stellar systems

- Einsel & Spurzem (1999)

* higher numerical accuracy, only pre-collapse

- Kim, Einsel, Lee, Spurzem, Lee (2002, 2004)

* post-collapse evolution, multi-mass

- Fiestas, Spurzem (2006, in prep.)

* with star-accreting black hole

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Rotation: Fokker-Planck models, no black hole

$$\frac{\partial f}{\partial t} + \frac{\partial \phi}{\partial t}\frac{\partial f}{\partial E} = \left(\frac{\partial f}{\partial t}\right)_{\rm enc},$$

with the potential ϕ advanced according to the **Poisson** equa

$$\nabla^2 \phi = 4\pi G n,$$

and the collisional term on the right-hand side of equatic expressed under the **Fokker–Planck** assumption of small scatterings:

$$\left(\frac{\partial f}{\partial t}\right)_{\rm enc} = \frac{1}{V}\left[-\frac{\partial}{\partial E}(<\Delta E> fV) - \frac{\partial}{\partial J_z}(<\Delta J_z> fV) + \frac{1}{2}\frac{\partial^2}{\partial E^2}(<(\Delta E)^2> fV) + \frac{\partial^2}{\partial E \partial J_z}(<\Delta E \Delta J_z> fV) + \frac{1}{2}\frac{\partial^2}{\partial J_z^2}(<(\Delta J_z)^2> fV)\right],$$

Orbit averaged Fokker-Planck Equation

(here in the 2D form for axisymm. systems, Einsel & Spurzem 1999)

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Rotation: Fokker-Planck models, no black hole

The Fokker–Planck equation is transformed into flux conservation form in order to improve conservation of several quantities:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{1}{p}\left(-\frac{\partial F_E}{\partial E} - \frac{\partial F_{J_z}}{\partial J_z}\right), \tag{10}$$

with particle flux components

$$F_E = -D_{EE}\frac{\partial f}{\partial E} - D_{EJ_z}\frac{\partial f}{\partial J_z} - D_E f, \tag{11}$$

$$F_{J_z} = -D_{J_zJ_z}\frac{\partial f}{\partial J_z} - D_{J_zE}\frac{\partial f}{\partial E} - D_{J_z} f\,. \tag{12}$$

Flux Conserving Form of Fokker-Planck Equation
(Einsel & Spurzem 1999), Chang-Cooper Scheme (1970)

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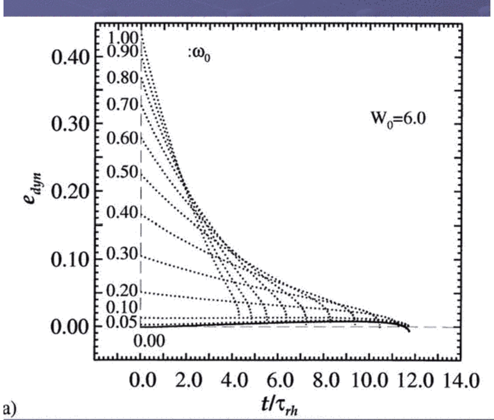
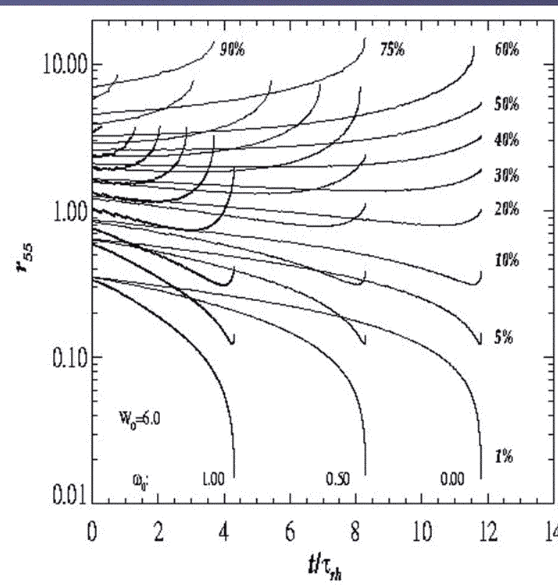
Rotation: Fokker-Planck models, no black hole

Rotation accelerates core collapse significantly and takes away flattening



a)



(Einsel & Spurzem 1999)
(Kim, Einsel, Lee, Spurzem & Lee 2002)

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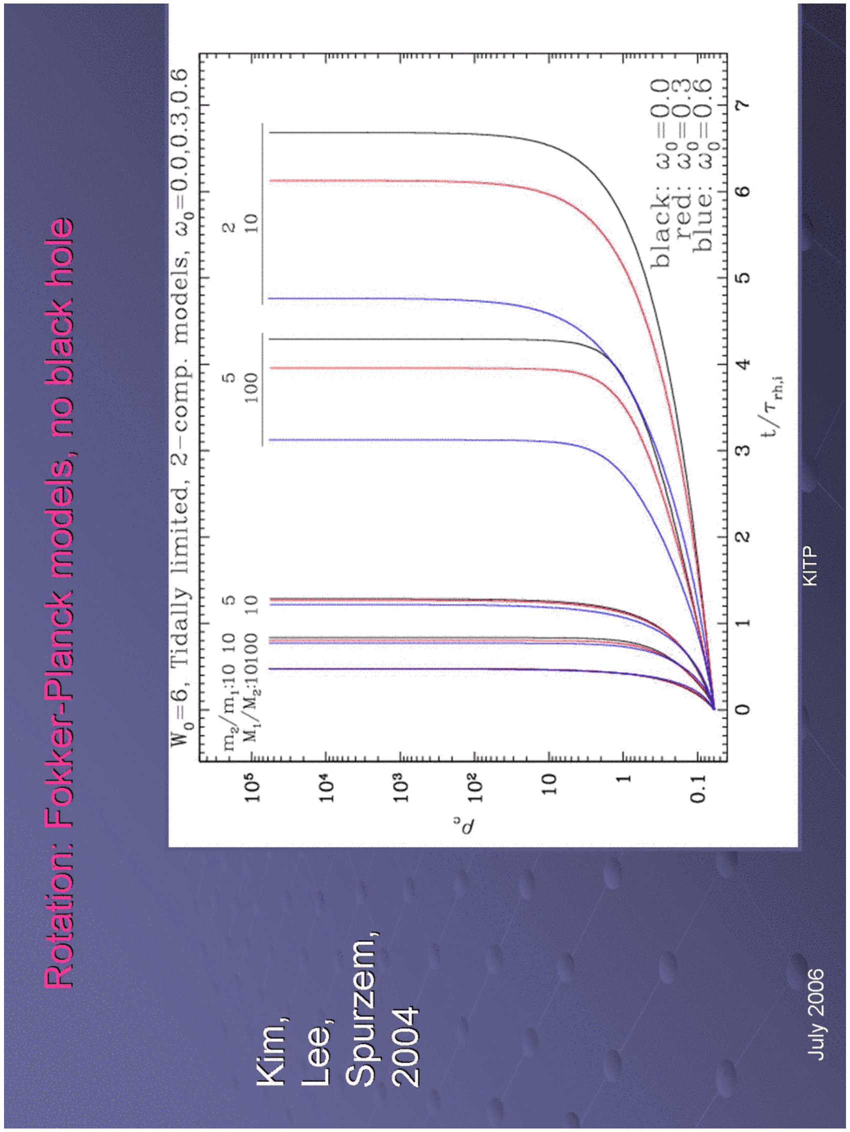
Rotation: Fokker-Planck models, no black hole
Kim, Lee, Spurzem, 2004
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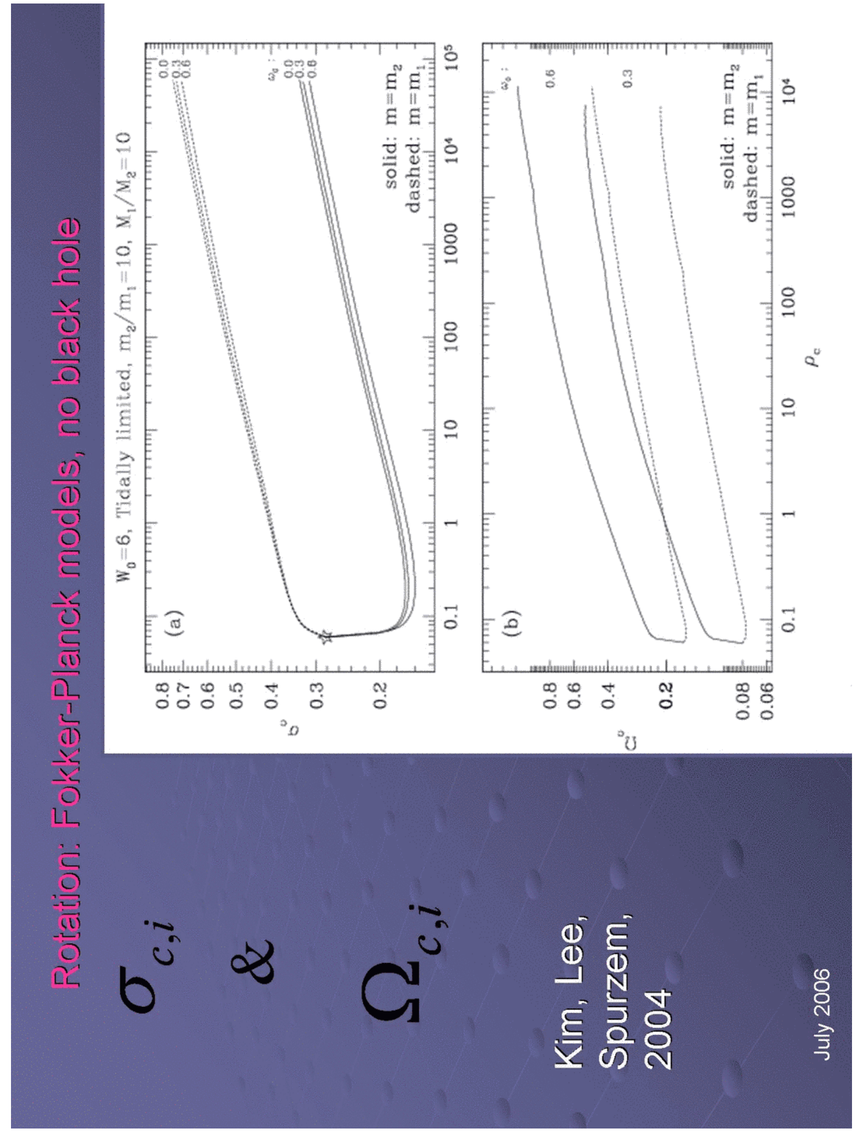
Rotation: Fokker-Planck models, no black hole
$\sigma_{c,i}$ & $\Omega_{c,i}$
Kim, Lee, Spurzem, 2004
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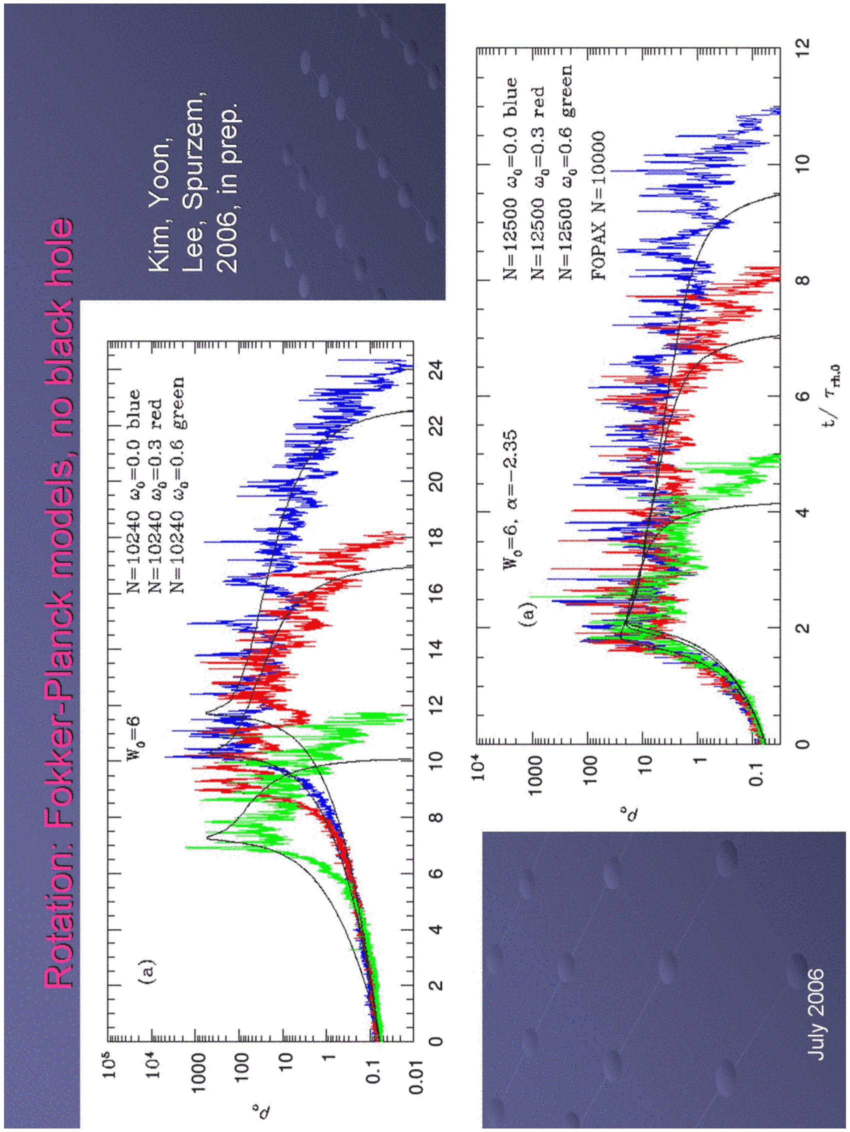
Rotation: Fokker-Planck models, no black hole
Kim, Yoon, Lee, Spurzem, 2006, in prep.
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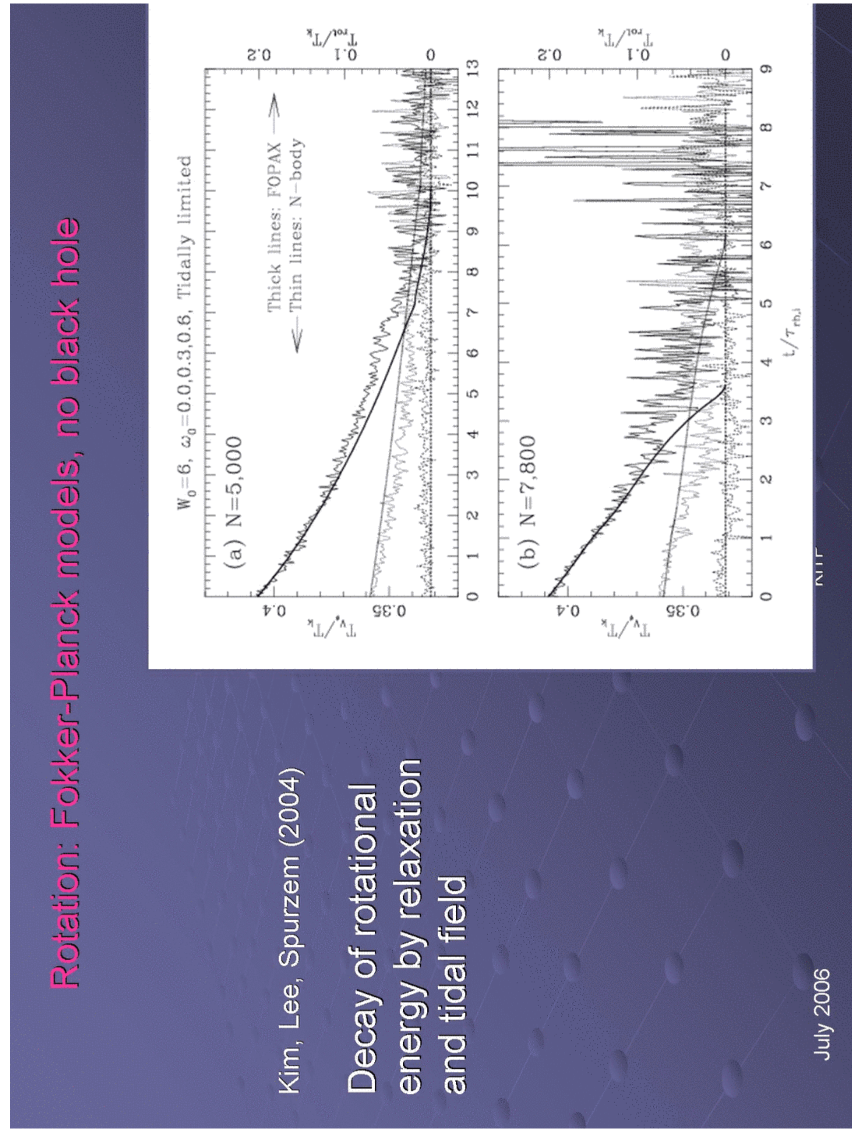
Rotation: Fokker-Planck models, no black hole
Kim, Lee, Spurzem (2004)
Decay of rotational energy by relaxation and tidal field
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Gas Model and Black Hole

- Time-Dep. Loss-Cone Diffusion (Milos. & Merritt, 2003) in orbit averaged Fokker-Planck model
- Anisotropic gaseous model (Louis & Spurzem 1991, Spurzem 1994) plus local simplified diffusion equation (Amaro-Seoane, Freitag, Spurzem, in prep.)
- Using gaseous model: http://www.ari.uni-heidelberg.de/gaseous-model (P. Amaro-Seoane)

$$\frac{d\rho_{\rm lc}}{dt} = -\frac{\rho_{\rm lc} P_{\rm lc}}{t_{\rm out}} + \frac{(\rho\Omega - \rho_{\rm lc})}{t_{\rm in}}. \quad P_{\rm lc} = \frac{1}{1+(t_{\rm out}/t_{\rm in})}$$

$$k = \rho_{\rm lc}/(\rho\Omega)$$

$$k(t) = k_0 \exp\left(-\frac{P_{\rm lc} K (t-t_0)}{t_{\rm out}}\right) + \frac{t_{\rm out}}{P_{\rm lc} t_{\rm in} K}\cdot\left(1 - \exp\left(-\frac{P_{\rm lc} K (t-t_0)}{t_{\rm out}}\right)\right)$$

$$k_\infty := \lim_{t\to\infty} k(t) = \frac{2}{3}.$$

$$K := 1 + (t_{\rm out}/P_{\rm lc} t_{\rm in})$$

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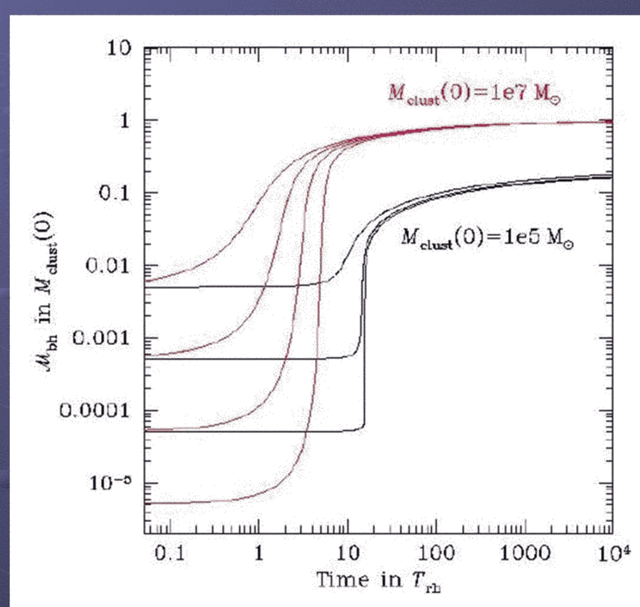
Central Star Accreting Black Hole: Gas and Fokker-Planck

Amaro-Seoane, Spurzem (2001)

Amaro-Seoane, Freitag, Spurzem (2004) in prep.

Black Hole Growth Self-Regulation Gas Model single mass



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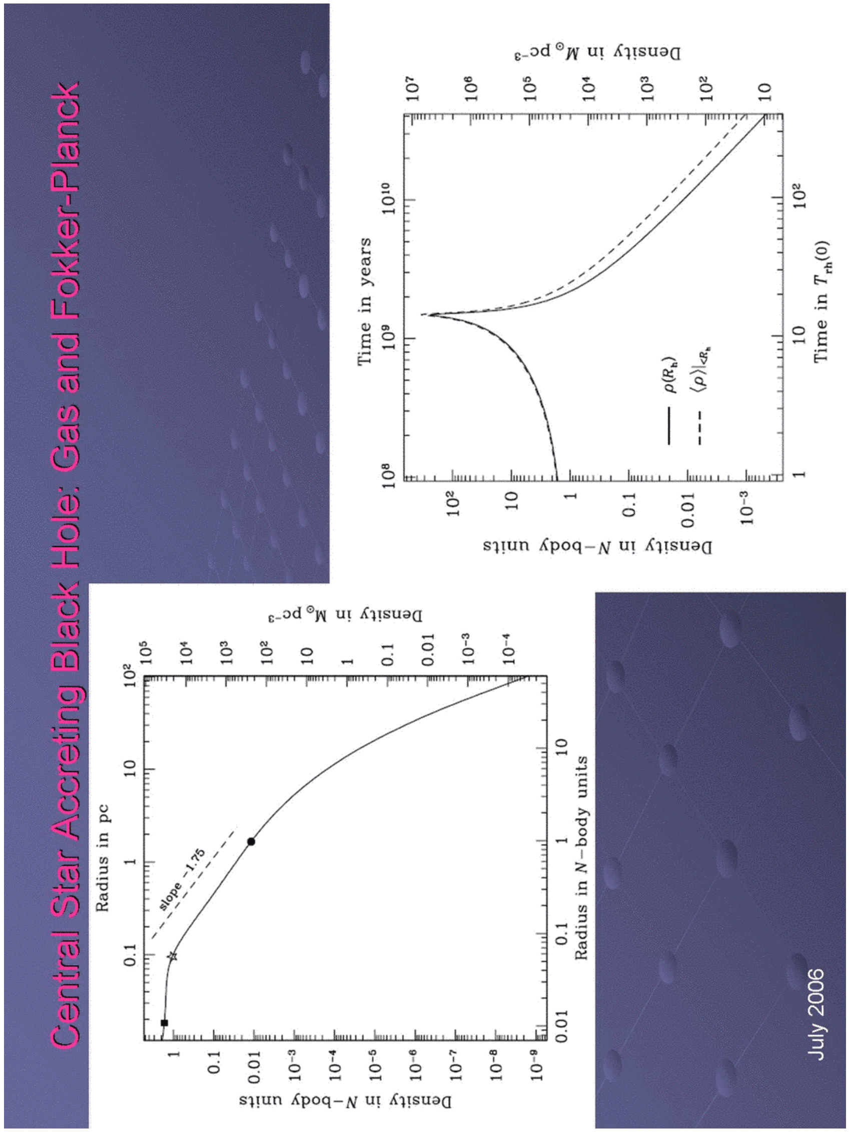
Central Star Accreting Black Hole: Gas and Fokker-Planck
Radius in pc
Density in N−body units
Radius in N−body units
Density in M☉ pc⁻³
slope −1.75
Time in years
Time in T_rh(0)
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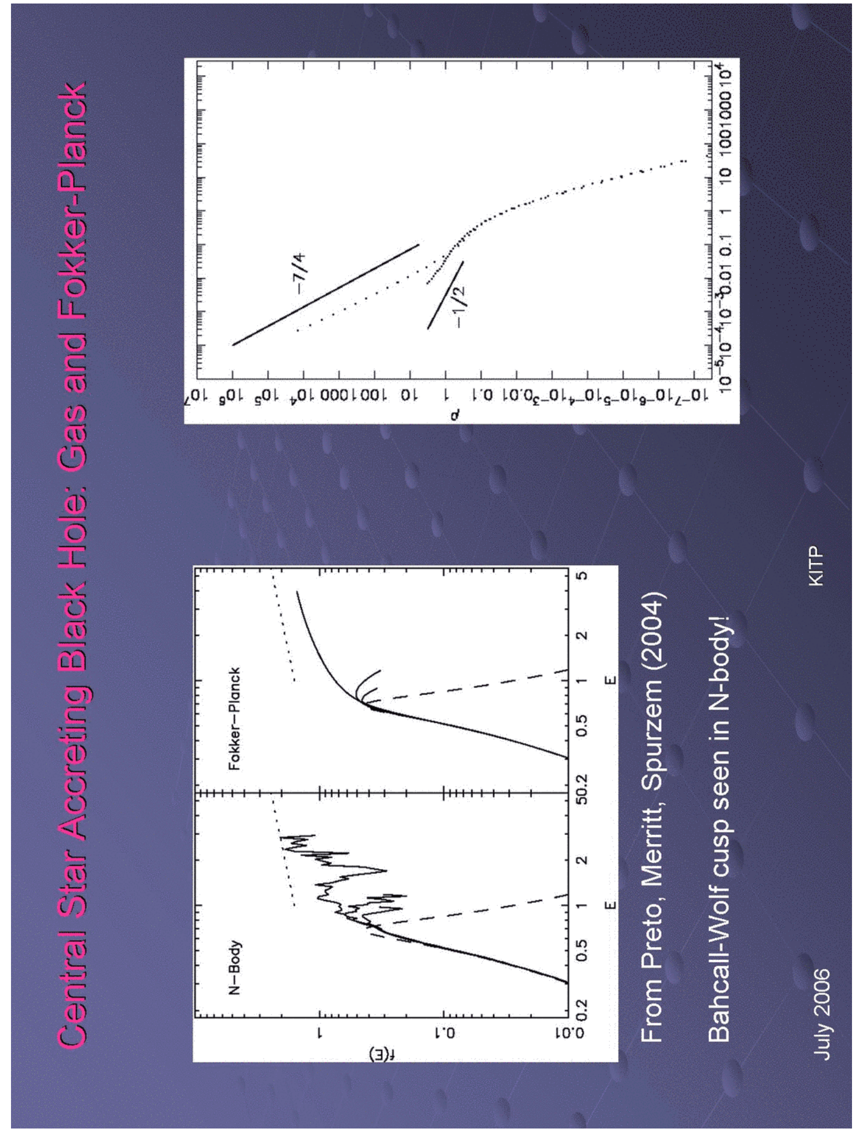
Central Star Accreting Black Hole: Gas and Fokker-Planck
N−Body
Fokker−Planck
f(E)
E
−7/4
−1/2
From Preto, Merritt, Spurzem (2004)
Bahcall-Wolf cusp seen in N-body!
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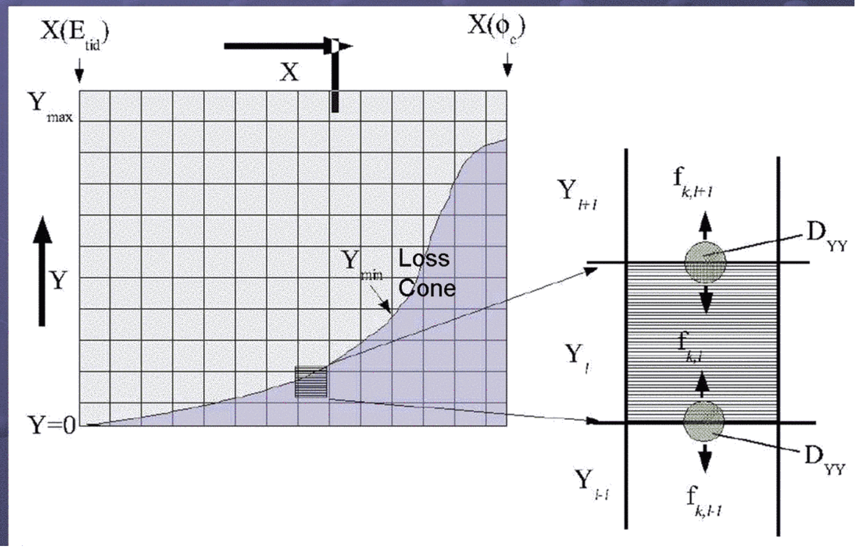
Central Star Accreting Black Hole: Gas and Fokker-Planck

The „loss-cone" in axisymmetric systems, use normalized angular momentum Y (0<Y<1) and energy X, adaptive mesh in X.

Fiestas & Spurzem 2006 (to be subm.)



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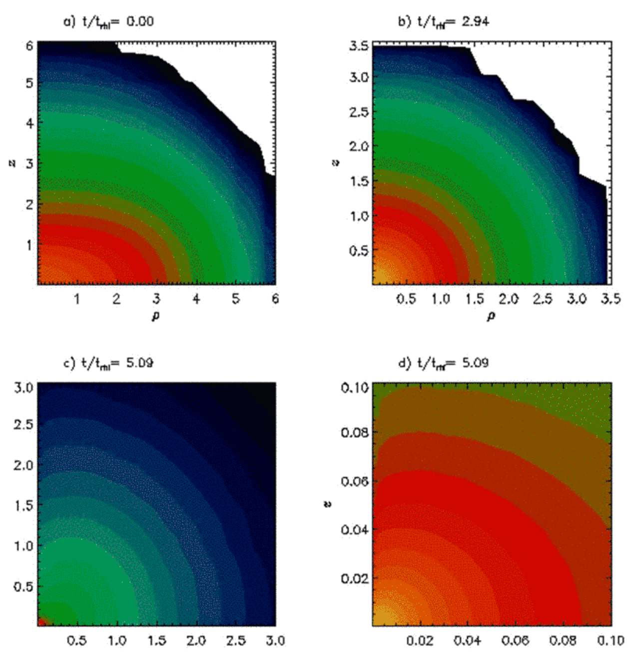
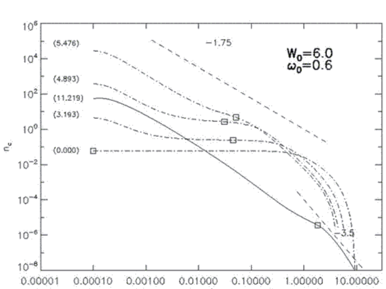
Central Star Accreting Black Hole: Gas and Fokker-Planck

Figure 4. Evolution of density distribution in the meridional plane for a model B4. Cylindrical coordinates (ϖ, z) are used. Lighter zones represent higher isodensity contours. Note that scales are different in each plot and Fig. 4d shows a zoom of the central parts of Fig. 4c (insider r_a). The time is given in units of initial half-mass relaxation tim



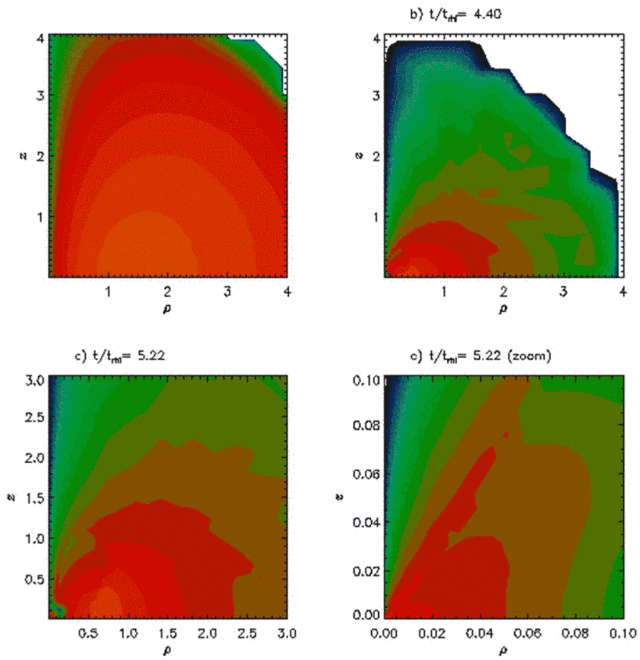
Density Evolution of Rotating Star Cluster with central star-accreting black hole.

(Fiestas & Spurzem 2006, to be submitted)



Central Star Accreting Black Hole: Gas and Fokker-Planck

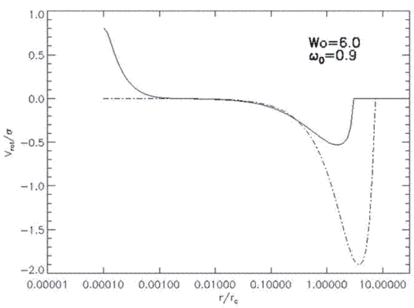
Figure 12. 2D rotational velocity distribution in the meridional plane for a model B4 $(6.0, 0.9, 5 \cdot 10^{-6})$. Note the formation of a maxima of rotation in the central region. Note that scales are different in each plot and Fig. 12d shows a zoom of the central parts of Fig. 12c (insider r_a). Time is given in units of initial half-mass relaxation time (t_{rhi}).



Evolution of rotation in Star Cluster with central star-accreting black hole.

(Fiestas & Spurzem 2006, to be submitted)

Counter Rotation near SMBH



Status of Continuum Models

☼ yellow sun: accomplished 🔑 green key: in progress or straightforward

st.dyn. st. evol.	FP 1D f(E) sph	FP 2D f(E,J) sph	FP 2D f(E,J$_z$) rot	Gas f(E,J) sph.	M-Carlo f(E,J) sph.
multi-mass post-collapse	☼	☼	☼	☼	☼
stell. evolution	☼	☼	🔑	☼	☼
tidal cut/ mass loss	☼	☼	☼	☼	☼
tidal shock	☼	🔑		🔑	
many hard binaries	(☼) approx.			☼ eq.m. (hybr. MC)	☼
binary stell. evolution					🔑

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Status of Continuum Models

☼ yellow sun: accomplished 🔑 green key: in progress or straightforward

more	FP 1D f(E) sph	FP 2D f(E,J) sph	FP 2D f(E,J_z) rot	Gas f(E,J) sph.	M-Carlo f(E,J) sph.
cent. bl. hole star accretion	☼	☼	🔑	☼	☼
stell. collisions	(☼)			🔑	☼
gas dynamics (single comp.)	(☼) hyd.-stat.			🔑	
star-gas interactions	(☼)			🔑	
star formation					
full chemo-dynamics					

Conclusions Approximate Models

- Continuum Models still very fast even in times of GRAPE-6,8,...,NBODY6++,...
- Continuum Models help understanding physics
- Continuum Models are far from being exploited to their limits – much to do still!
- Comparisons with NBODY and Monte-Carlo should be continued

From Newton to Einstein…

What happens? Use Post-Newtonian approximation…

$$\mathcal{H}(\mathbf{r},\hat{\mathbf{p}}) = \mathcal{H}_0(\mathbf{r},\hat{\mathbf{p}}) + \frac{1}{c^2}\mathcal{H}_1(\mathbf{r},\hat{\mathbf{p}}) + \frac{1}{c^4}\mathcal{H}_2(\mathbf{r},\hat{\mathbf{p}}) + \frac{1}{c^6}\mathcal{H}_3(\mathbf{r},\hat{\mathbf{p}}), \quad + c^{-5} H_{2.5} + c^{-7} H_{3.5}$$

- Non-Dissipative Terms, so-called PN 1,2,3.... (Perihel Shifts)
- Dissipative Terms PN 2.5, 3.5
  (Emission of Energy in gravitational waves, emission of linear momentum!)
- Spin-Spin, Spin-Orbit-Couplings PN 3.5!

Schäfer, Gauge Theor. Grav. 36, 2223 (2004)
Memmesheimer, Gopakumar, Schäfer, Phys. Rev.D 70, 104011 (2004)

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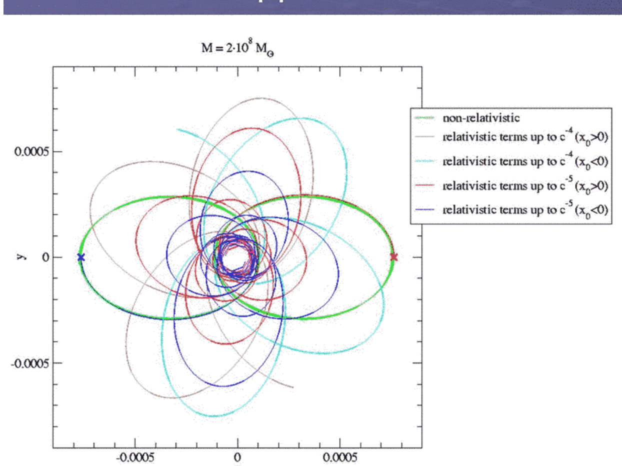
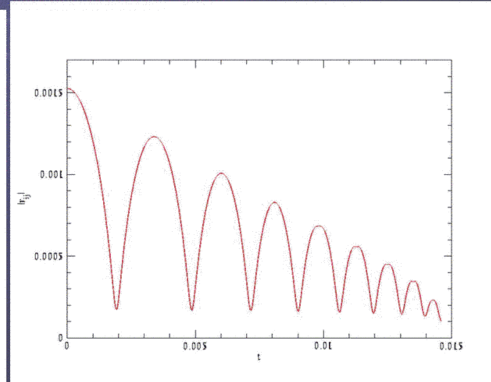
From Newton to Einstein…

What happens afterwards? Post-Newton Order „2.5"…





Kupi & Spurzem 2004

<u>Final Ring-Down and Merger ⇒ grav. wave signal!</u>

$t_{GW} = 1.1 \cdot 10^6 \text{ a } M_8^{-5/3} \quad (P/a)^{8/3} \quad (1+q)^2/q$ (Jaffe & Backer 2003)

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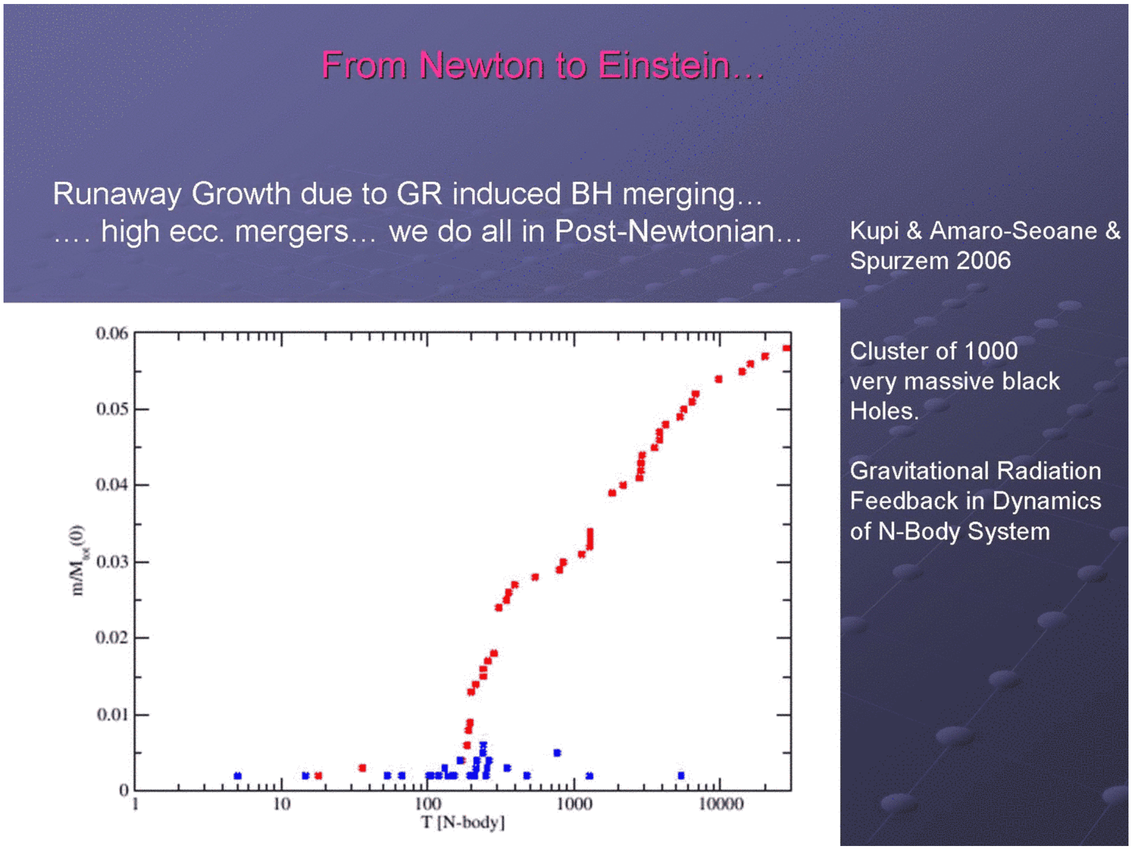
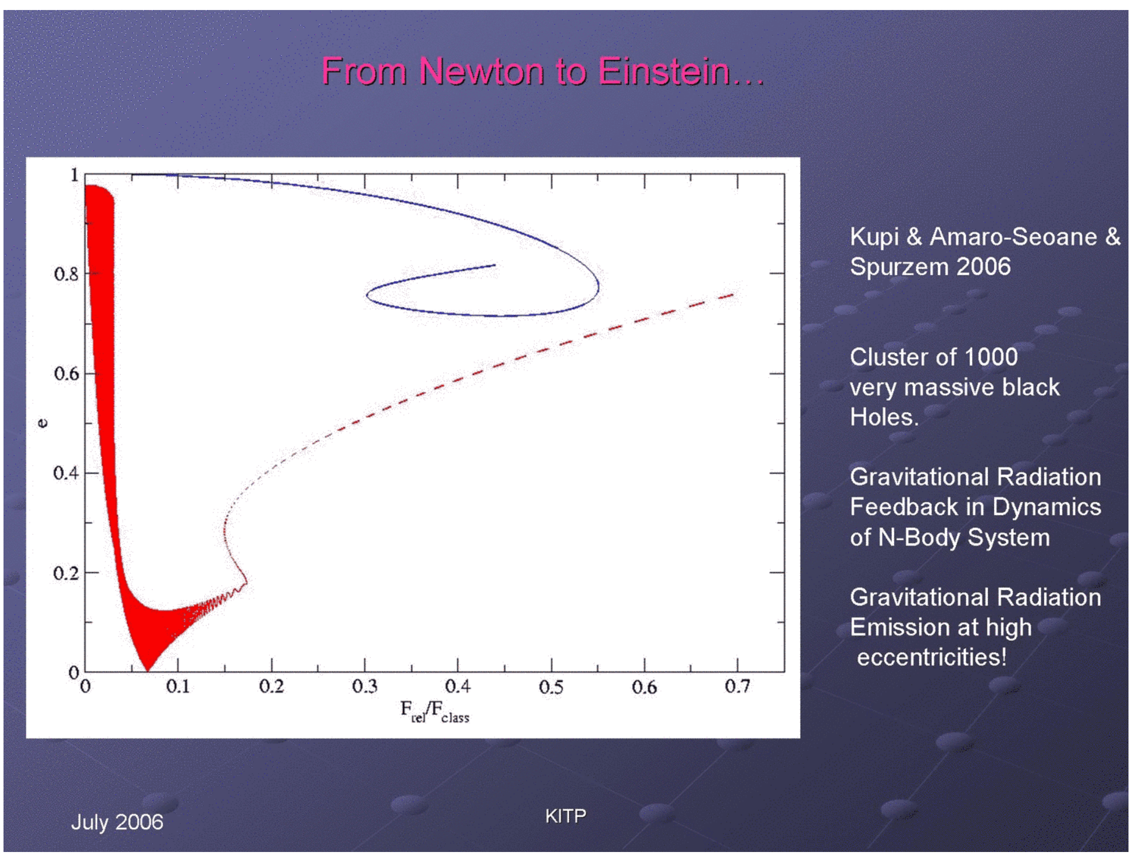

From Newton to Einstein…
Runaway Growth due to GR induced BH merging…
…. high ecc. mergers… we do all in Post-Newtonian…
Kupi & Amaro-Seoane & Spurzem 2006
Cluster of 1000 very massive black Holes.
Gravitational Radiation Feedback in Dynamics of N-Body System
m/M_tot(0)
T [N-body]
0.06
0.05
0.04
0.03
0.02
0.01
0
1
10
100
1000
10000

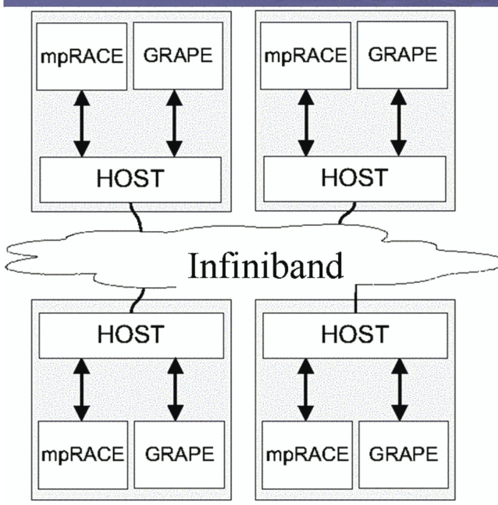

From Newton to Einstein…
Kupi & Amaro-Seoane & Spurzem 2006
Cluster of 1000 very massive black Holes.
Gravitational Radiation Feedback in Dynamics of N-Body System
Gravitational Radiation Emission at high eccentricities!
e
F_rel/F_class
1
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
July 2006
KITP

VolkswagenStiftung

MWK Baden-Württembg.

The GRACE architecture
(GRAPE+MPRACE)

Univ. Heidelberg (ARI) Univ. Mannheim (LIV)
Univ. Munich (USM) RIKEN Institute Tokyo



4 Tflops, 128 CPUs (64 P4 Xeon, 32 GRAPE,
32 Xilinx FPGA-MPRACE)

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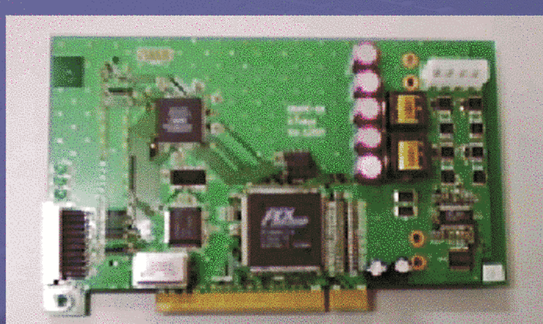
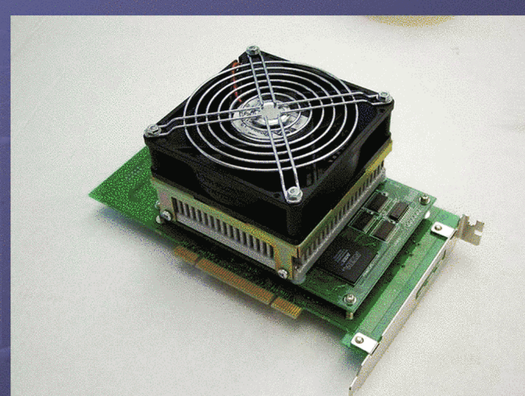
GRAPE and GRACE Hardware





GRAPE6a PCI board

GRAPE6a - PCI Board for PC-Clusters, recent
development of the University of Tokyo

~128 Gflops for a price ~5K USD
Memory for N, up to 128K particles

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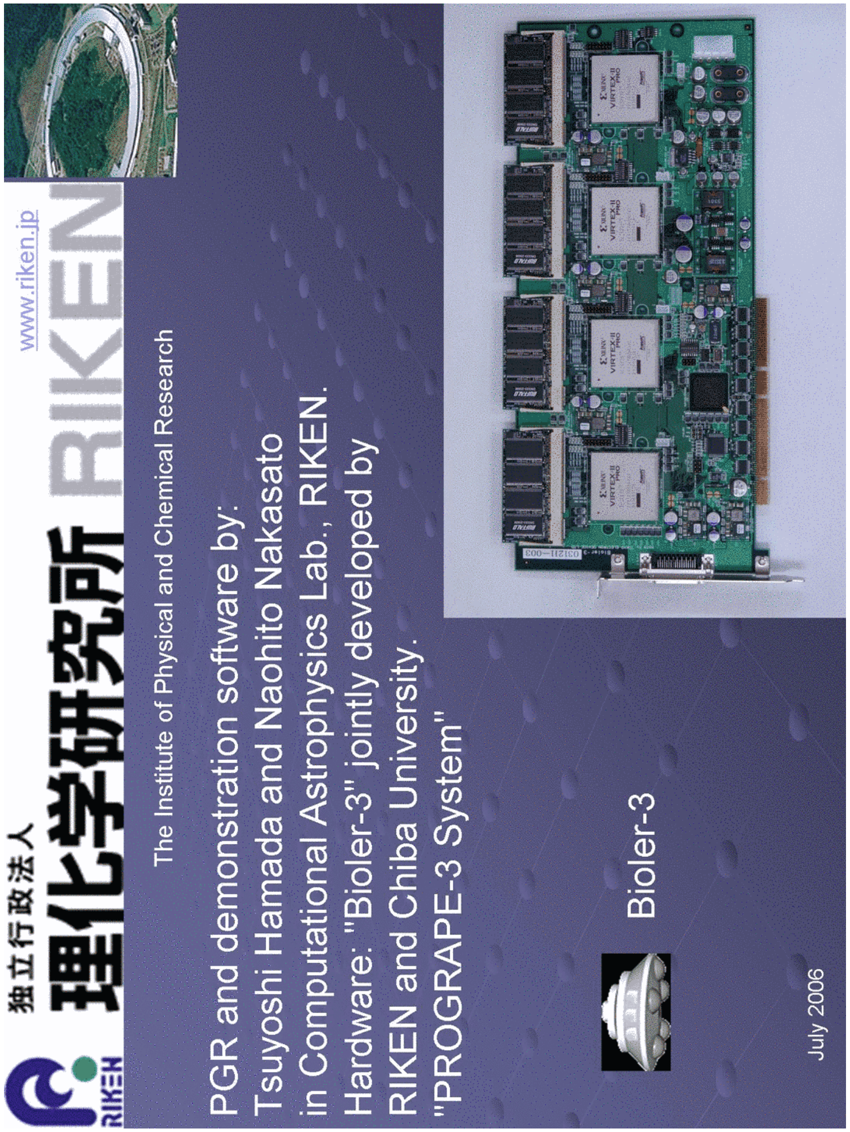
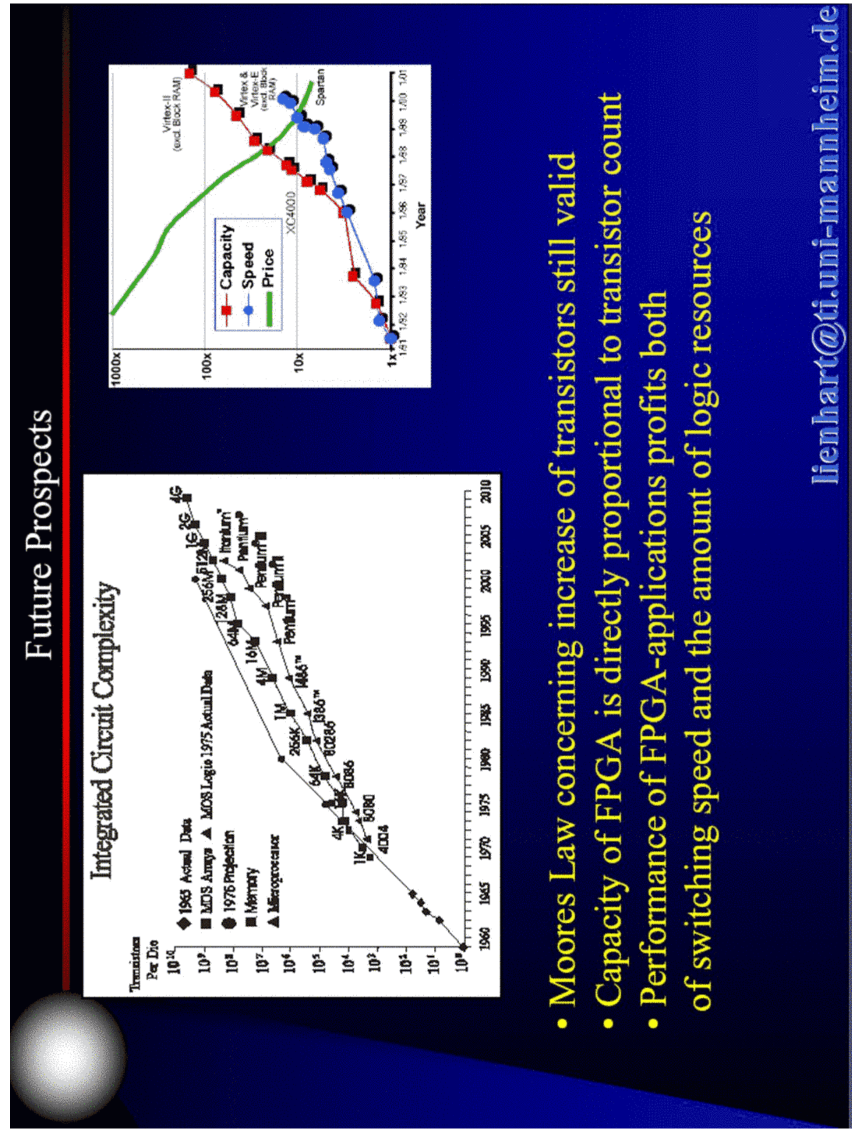
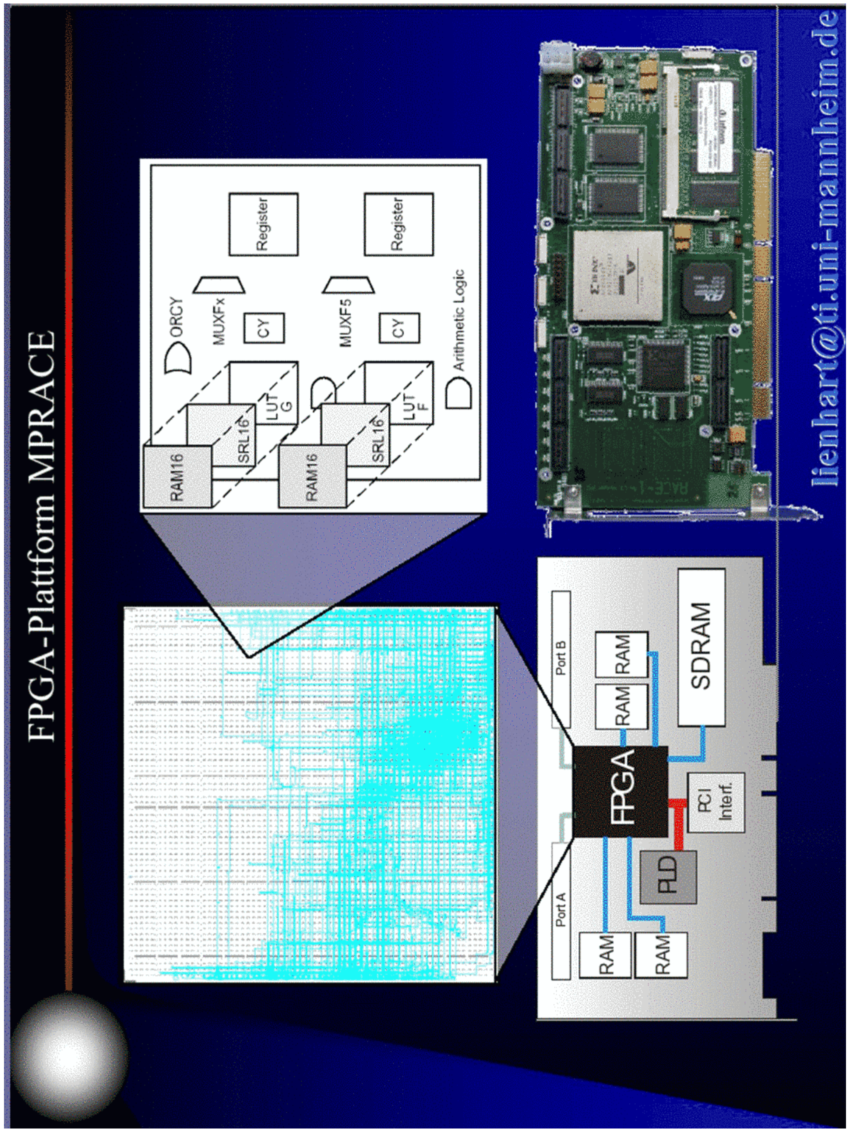

独立行政法人
理化学研究所
RIKEN
www.riken.jp
The Institute of Physical and Chemical Research
PGR and demonstration software by:
Tsuyoshi Hamada and Naohito Nakasato
in Computational Astrophysics Lab., RIKEN.
Hardware: "Bioler-3" jointly developed by
RIKEN and Chiba University.
"PROGRAPE-3 System"
Bioler-3
July 2006


Future Prospects
Integrated Circuit Complexity
Capacity
Speed
Price
Year
XC4000
Spartan
• Moores Law concerning increase of transistors still valid
• Capacity of FPGA is directly proportional to transistor count
• Performance of FPGA-applications profits both of switching speed and the amount of logic resources
lienhart@ti.uni-mannheim.de


FPGA-Plattform MPRACE
lienhart@ti.uni-mannheim.de
RAM16
SRL16
LUT G
LUT F
ORCY
MUXFx
MUXF5
CY
Register
Arithmetic Logic
Port A
Port B
RAM
SDRAM
FPGA
PCI Interf.
PLD