Horizon formation and far-from-equilibrium dynamics in strongly coupled plasma

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based on work with Paul Chesler: arXiv:0906.4426, arXiv:0906.4426

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Thermal plasma physics from AdS/CFT

- Equilibrium ($\mathcal{N} = 4$ SYM) (static, Euclidean signature)
  - equation of state
  - correlation lengths, screening
  - flavor physics
  - finite volume
    - confinement/deconfinement
    - chemical potentials
    - rotation

SUGRA mode	$J^{CR_t}_{R_y}$	SYM operator	mass/πT
G_{00}	0^{++}_{+}	T_{00}	2.3361
a	0^{+-}_{-}	$\mathrm{tr}\, E \cdot B$	3.4041
G_{ij}	2^{++}	T_{ij}	3.4041
ϕ	0^{++}_{+}	$\mathcal{L}$	3.4041
G_{i0}	1^{+-}	T_{i0}	4.3217
B_{ij}	0^{-+}_{-}	$\mathcal{O}_{ij}$	5.1085
C_{ij}	0^{--}_{+}	$\mathcal{O}_{30}$	5.1085
B_{i0}	1^{--}	$\mathcal{O}_{i0}$	6.6537
C_{i0}	1^{-+}	$\mathcal{O}_{3j}$	6.6537
G^{a}_{a}	0^{++}_{+}	$\mathrm{tr}\, F^4$	7.4116

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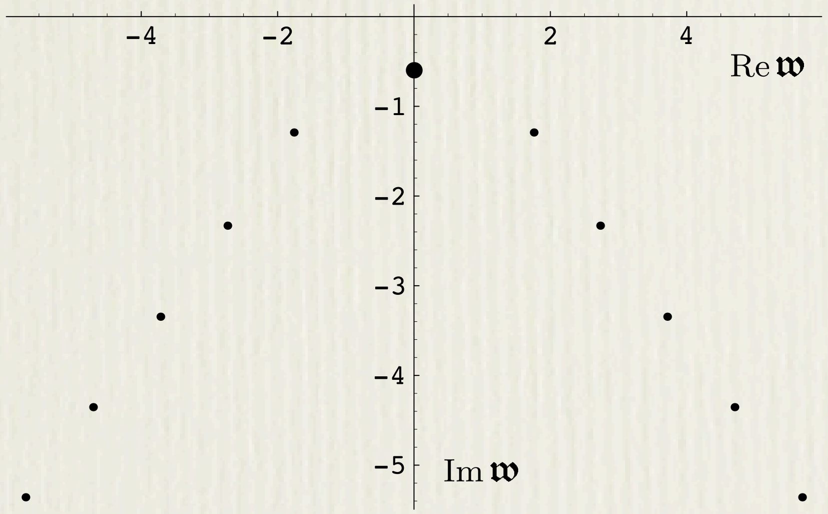
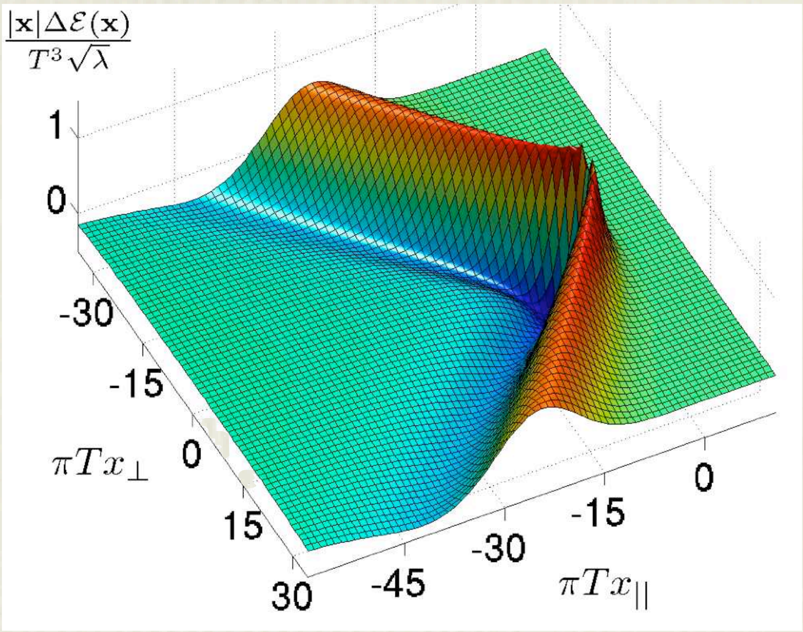
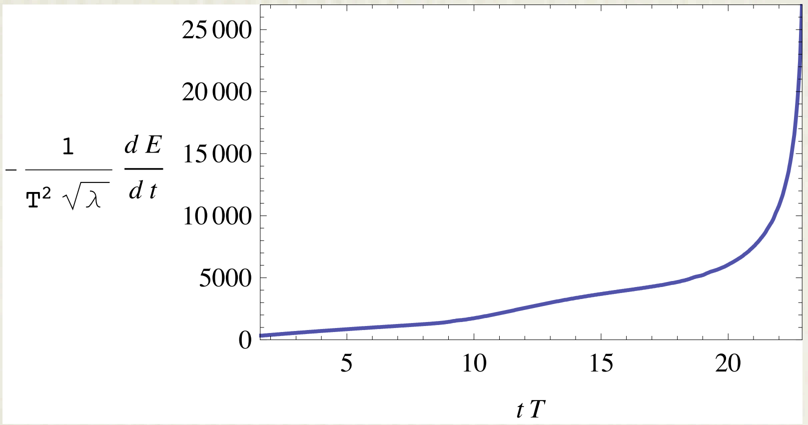
- Near-equilibrium (real-time response, Minkowski signature)
  - viscosity, diffusion
  - quasi-normal modes, late time expansions
  - photo-emission
  - second-order transport coefficients
  - non-linear conductivity



Thermal plasma physics from AdS/CFT

- Probe dynamics (classical string dynamics)
  - heavy quark drag
  - wakes, Brownian motion
  - heavy meson stability, dispersion
  - light quark jets





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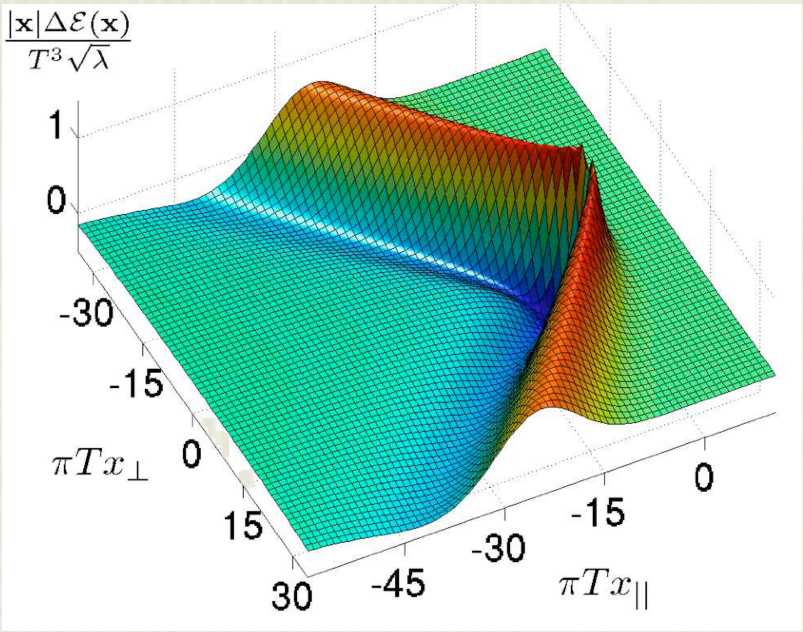
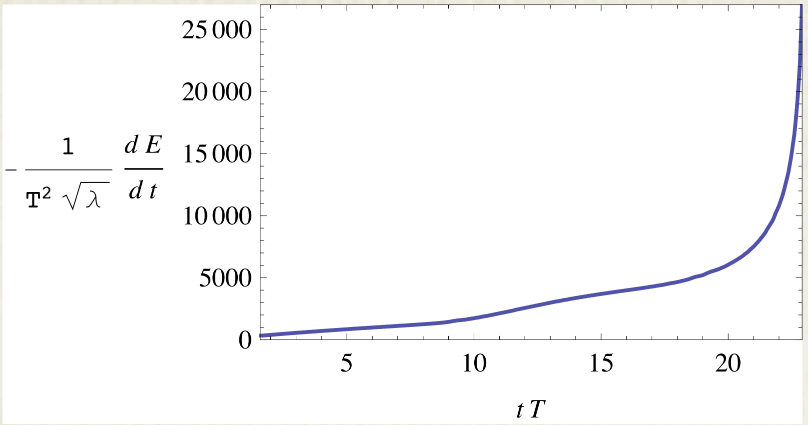


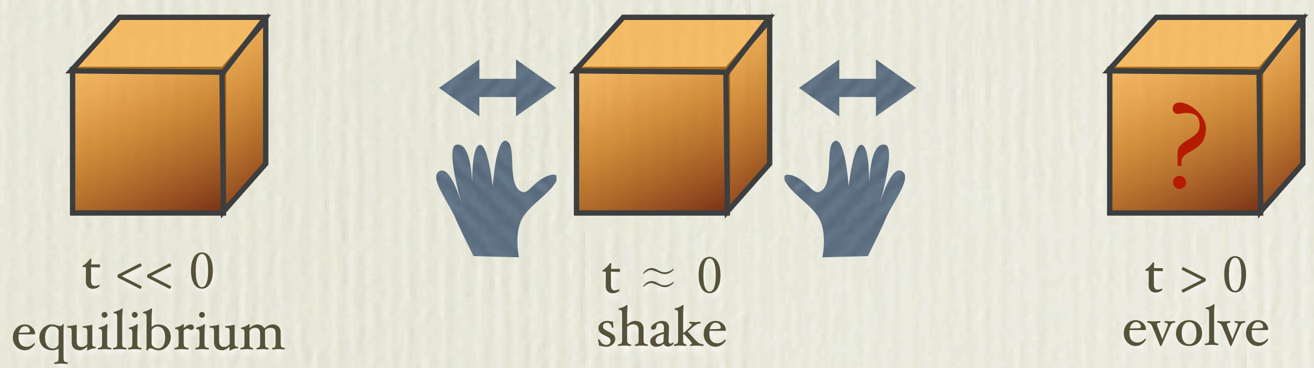


- Far-from-equilibrium dynamics ???
  - plasma formation
  - early thermalization
  - turbulence

} initial value problems with non-trivial time-dependent bulk geometry

Non-equilibrium initial states

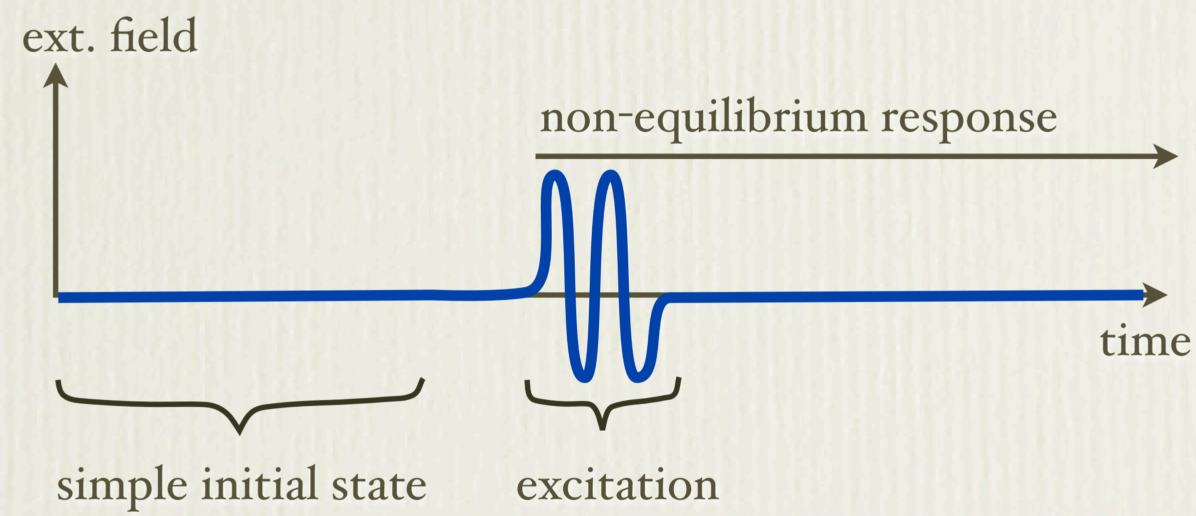
- Specify complete density matrix $\boldsymbol{\rho}$? Ugh!
- Pick geometry on initial Cauchy surface ? Ugh!
- Want “operational” description:



∴ Specify time-dependent external fields

➡ time-dependent dynamics

➡ external work done on system



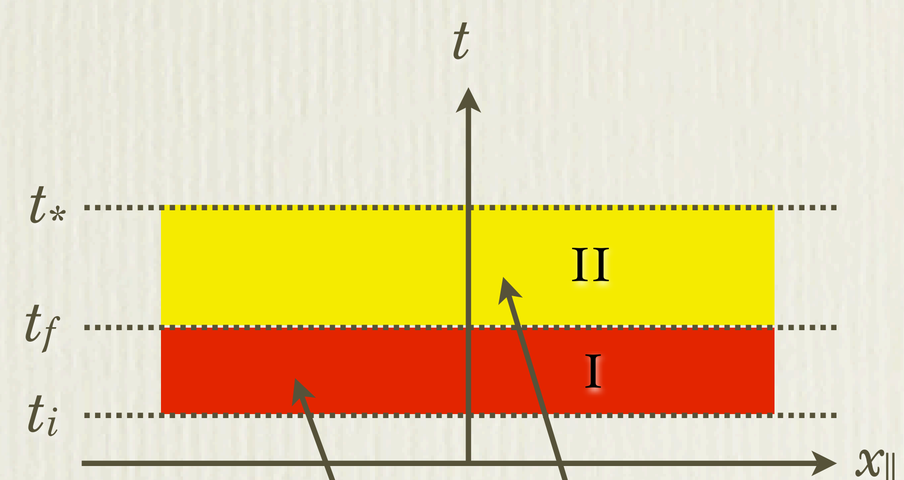
Anisotropy dynamics

- Metric $g^{\mu\nu}$ = external field coupling to stress-energy $T^{\mu\nu}$

  $\therefore$ time-dependent geometry ➡ non-equilibrium $\langle T^{\mu\nu} \rangle$

- Case I: perfect spatial homogeneity, arbitrary anisotropy

$$ds^2 = -dt^2 + e^{f(t)}(dx^2 + dy^2) + e^{-2f(t)}\, dz^2$$

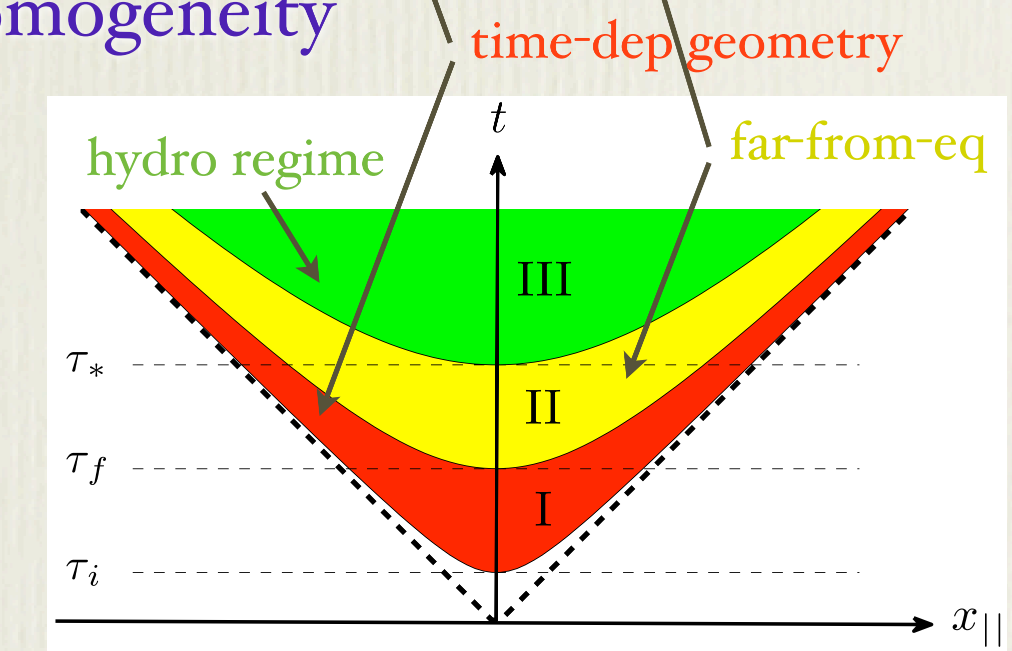


- Case II: boost invariance & transverse homogeneity

$$ds^2 = -d\tau^2 + e^{\gamma(\tau)}\, d\boldsymbol{x}_\perp^2 + \tau^2\, e^{-2\gamma(\tau)}\, dy^2$$

proper time (↑ τ) — rapidity (↑ y)

$$\langle T^{\mu\nu}(t, \mathbf{x}) \rangle = \begin{bmatrix} \varepsilon(t) & & & \\ & p_\perp(t) & & \\ & & p_\perp(t) & \\ & & & p_{||}(t) \end{bmatrix}$$



Gravitational description

- Solve 5-*d* Einstein equations with time-dependent boundary condition $G^{AB} \to g^{\mu\nu}$ and simple initial condition (AdS or AdS-BH)

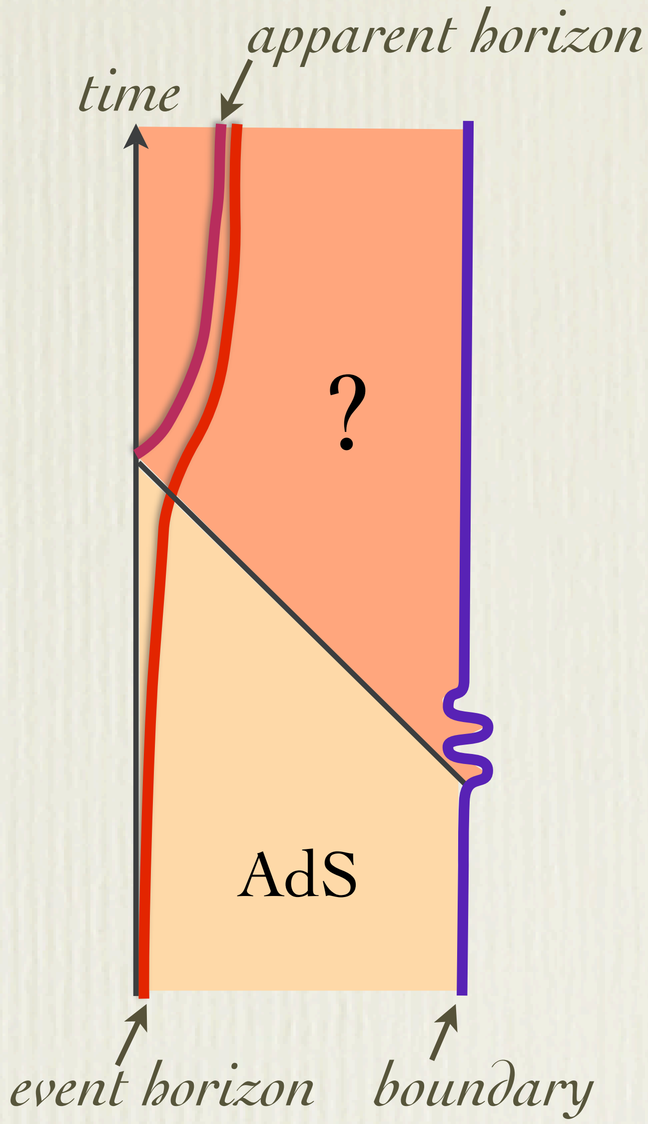
- Extract $\langle T^{\mu\nu} \rangle$ from sub-leading near-boundary asymptotics

- Note:

  - time-dependent boundary conditions produce dynamic event horizon

  - "Teleological" event horizon growth occurs outside causal future of boundary time dependence

  ➡ event horizon area (pulled back to boundary) *cannot* represent entropy in non-equilibrium setting



Practical issues (I)

- Coordinate choice:
  - ✘ Bad: Fefferman-Graham or similar $(r, t, \boldsymbol{x})$
  - ✓ Good: Incoming Eddington-Finkelstein

$$ds^2 = -A(v,r)\,dv^2 + 2\,dv\,dr + \Sigma(v,r)^2 \left[e^{B(v,r)}(dx^2 + dy^2) + e^{-2B(v,r)}dz^2\right]$$

  - v = const. on incoming (radial) null geodesics
  - $dv/dr = \frac{1}{2}A$ on outgoing (radial) null geodesics
    - $g' \equiv \partial_r g$ = directional derivative along incoming null geodesics,
    - $\dot{g} \equiv \partial_v g + \frac{1}{2}A\,\partial_r g$ = directional derivative along outgoing null geodesics

- Boundary conditions as $r \to \infty$:
  - Case I: $A \to r^2, \quad \Sigma \to r, \quad B \to f(v)$
  - Case II: $A \to r^2, \quad \Sigma \to r\,\tau^{1/3}, \quad B \to -\frac{2}{3}\ln\tau + \gamma(\tau)$

Einstein equations

- $R_{MN} - \frac{1}{2} G_{MN}(R + 2\Lambda) = 0$

- Non-trivial components: *vv, rr, vr, zz, xx+yy*

  ➡ *5 equations, 3 unknown functions* (A,B,Σ)

- Need to separate dynamics from constraints

  ➡
$$
\begin{aligned}
0 &= \Sigma\,(\dot{\Sigma})' + 2\Sigma'\,\dot{\Sigma} - 2\Sigma^2\,, \\
0 &= \Sigma\,(\dot{B})' + \tfrac{3}{2}\left(\Sigma'\dot{B} + B'\,\dot{\Sigma}\right), \\
0 &= A'' + 3B'\dot{B} - 12\Sigma'\,\dot{\Sigma}/\Sigma^2 + 4\,, \\
0 &= \ddot{\Sigma} + \tfrac{1}{2}\left(\dot{B}^2\,\Sigma - A'\,\dot{\Sigma}\right), \quad \leftarrow \text{boundary value constraint} \\
0 &= \Sigma'' + \tfrac{1}{2}B'^2\,\Sigma\,, \quad \leftarrow \text{initial value constraint}
\end{aligned}
$$

  - N.B.: A = non-dynamical auxillary field

Practical issues (II)

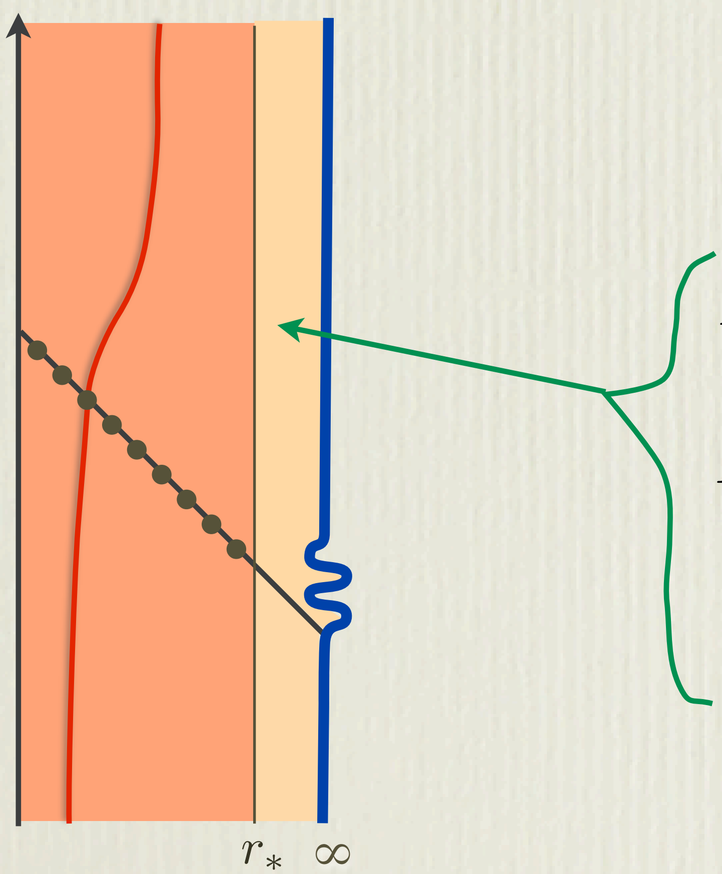
- Need to solve for "velocities," $\partial_v B$, $\partial_v \Sigma$, and auxillary field A

$$\dot{\Sigma}(r,v) = -\frac{2}{\Sigma(r,v)^2}\int_r dw\, \Sigma(w,v)^3$$

$$\dot{B}(r,v) = -\frac{3}{\Sigma(r,v)^{3/2}}\int_r dw\, \frac{B'(w,v)}{\Sigma(w,v)^{3/2}}\int_w d\bar{w}\, \Sigma(\bar{w},v)^3$$

- Discretize $r \to \infty$ system of coupled ODEs

- Must treat near-boundary behavior accurately
  - ➡ match discretized numerics to large r asymptotics



$$A(r,v) = \sum_{n=0}[a_n(v) + \alpha_n(v)\log r]\, r^{2-n},$$

$$B(r,v) = \sum_{n=0}[b_n(v) + \beta_n(v)\log r]\, r^{-n},$$

$$\Sigma(r,v) = \sum_{n=0}[s_n(v) + \sigma_n(v)\log r]\, r^{1-n}.$$

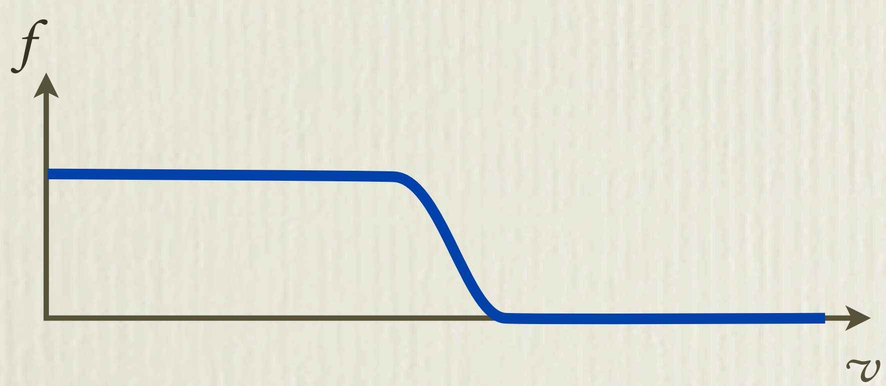
Practical issues (III)

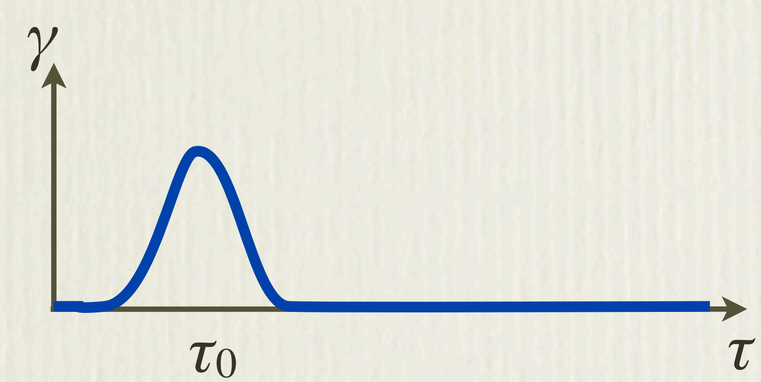
- Must remove residual reparameterize freedom: $r \to r + \alpha(v)$
  - ✘ Bad: fix coordinate location of event horizon
  - ✓ Good: fix $a_1 = 0$
- Must excise region surrounding singularity: $r < r_{\min}(v) < r_{\text{horizon}}(v)$
- Must choose specific boundary time dependence

Ex: $f(v) = \frac{1}{2} c\,[1 - \tanh(v/\tau)]$

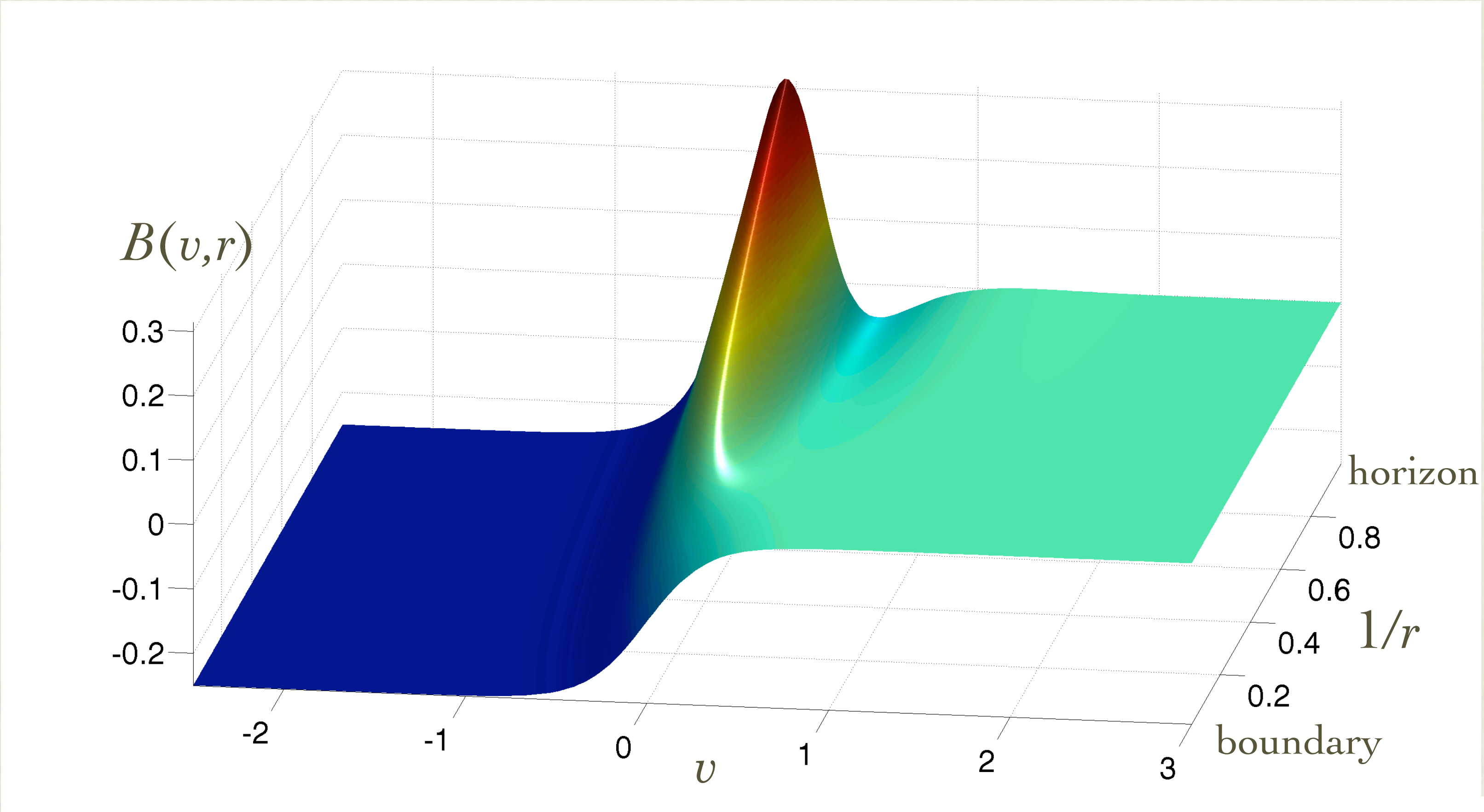
$$\gamma(\tau) = c\,h(\tau - \tau_0)^6\, e^{-1/h(\tau-\tau_0)}$$

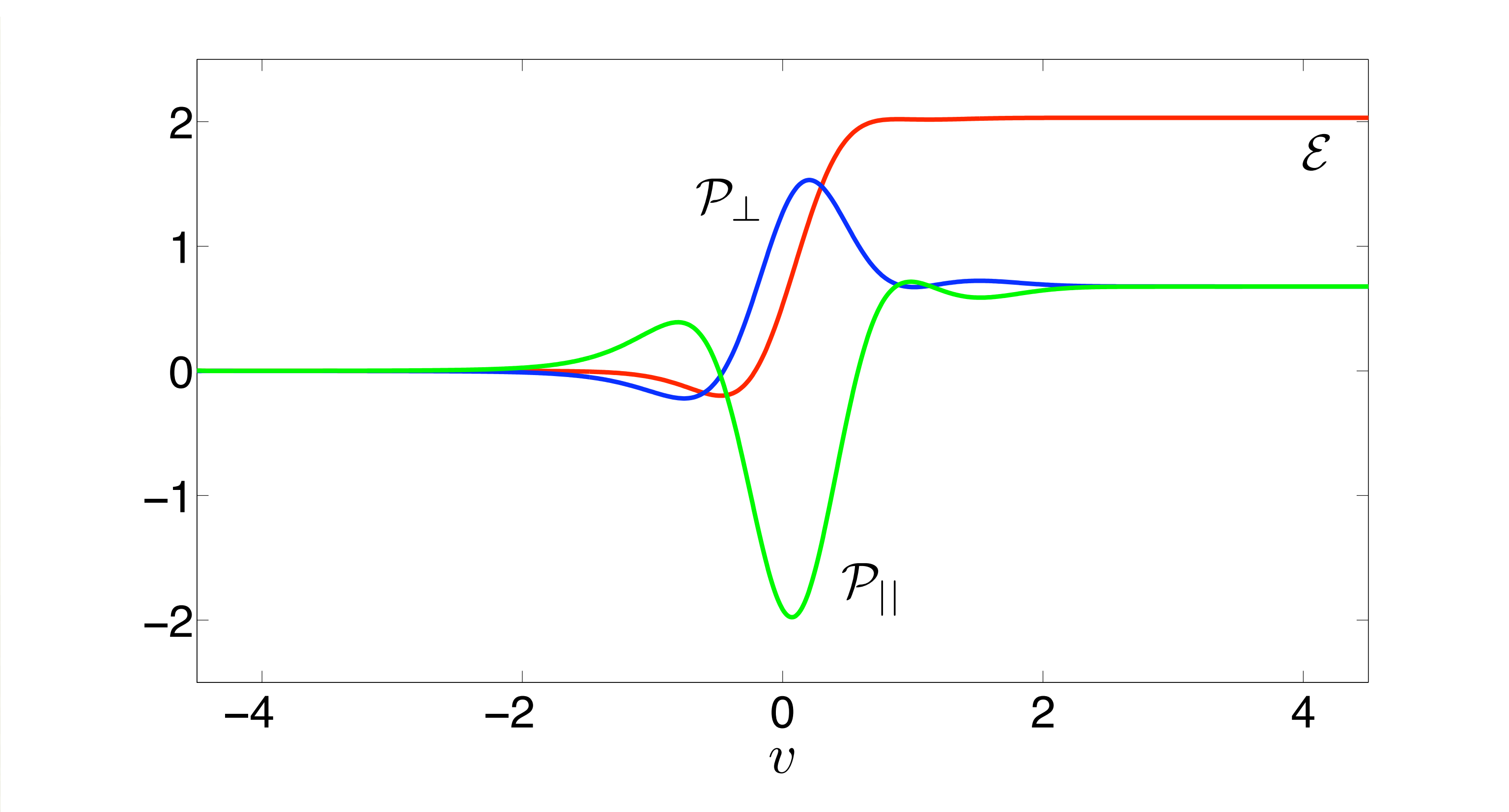
$$h(\delta\tau) = 1 - (\delta\tau)^2/\Delta^2$$



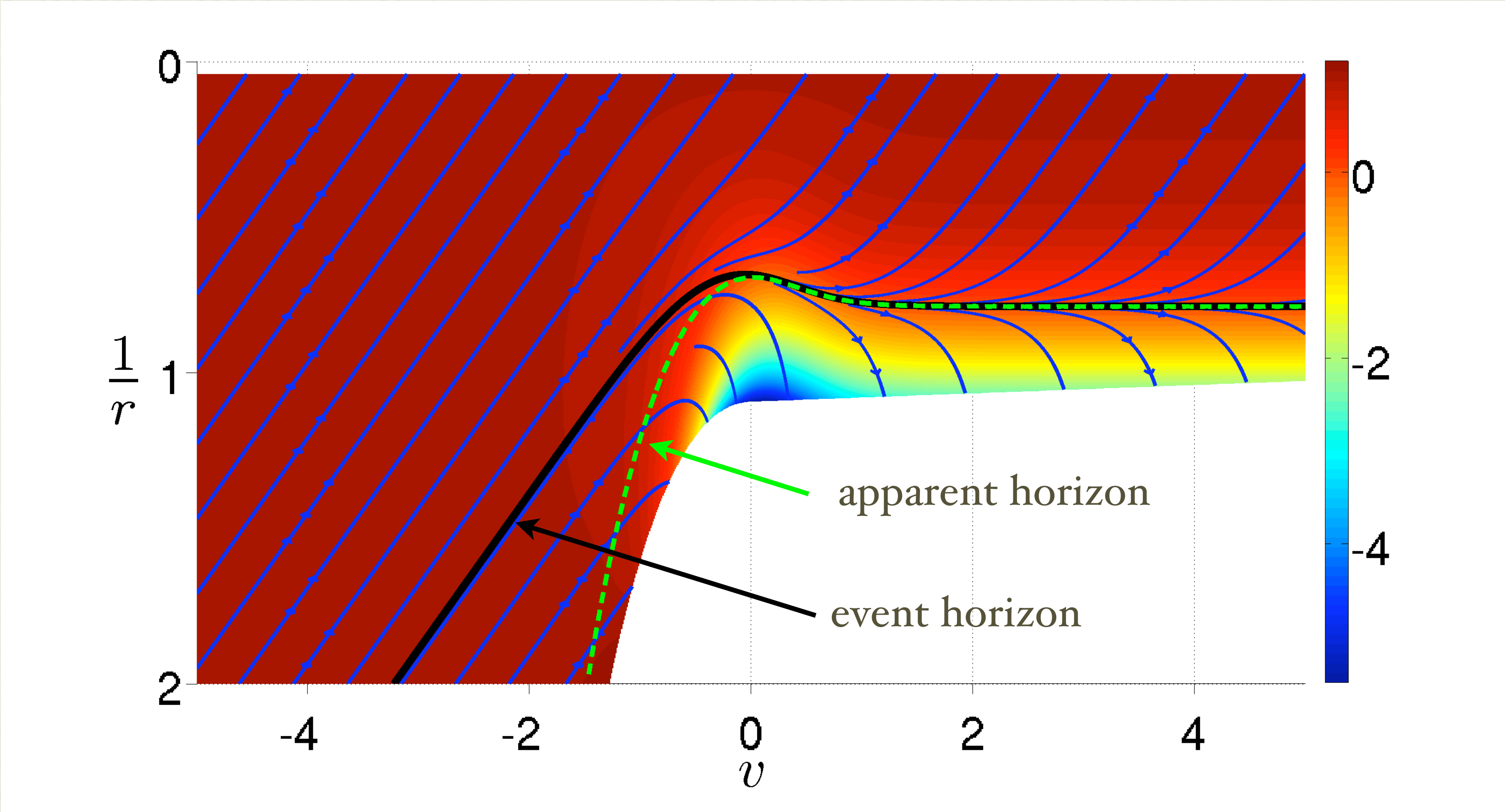


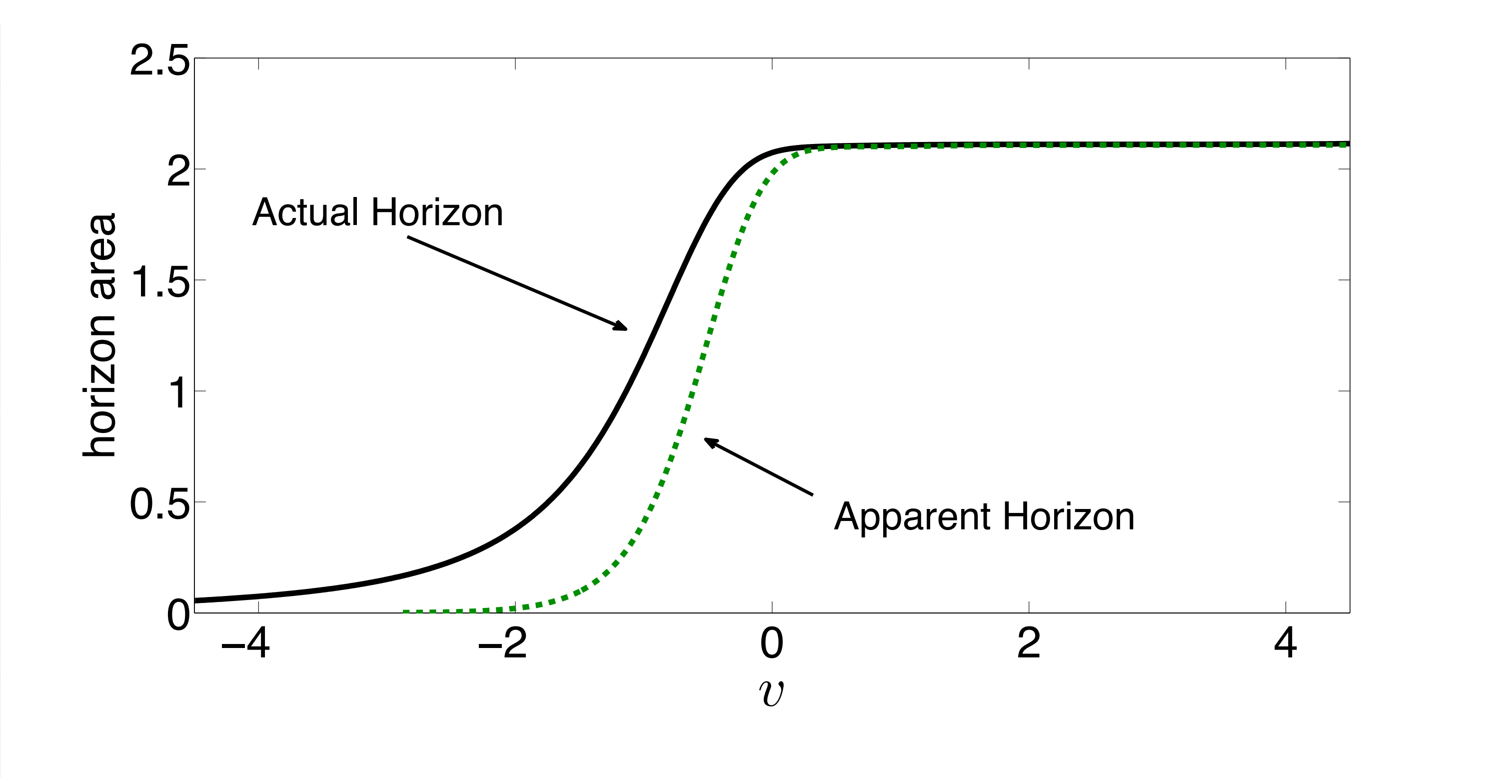
Case I: Results





Case I: Horizon area





Case I: Isotropization time

$\|c\|$	1	1.5	2	2.5	3	3.5	4
τT	0.23	0.31	0.41	0.52	0.65	0.79	0.94
$\tau_{\rm iso} T$	0.67	0.68	0.71	0.92	1.2	1.5	1.8
$\tau_{\rm iso}/\tau$	3.0	2.2	1.7	1.8	1.8	1.9	1.9

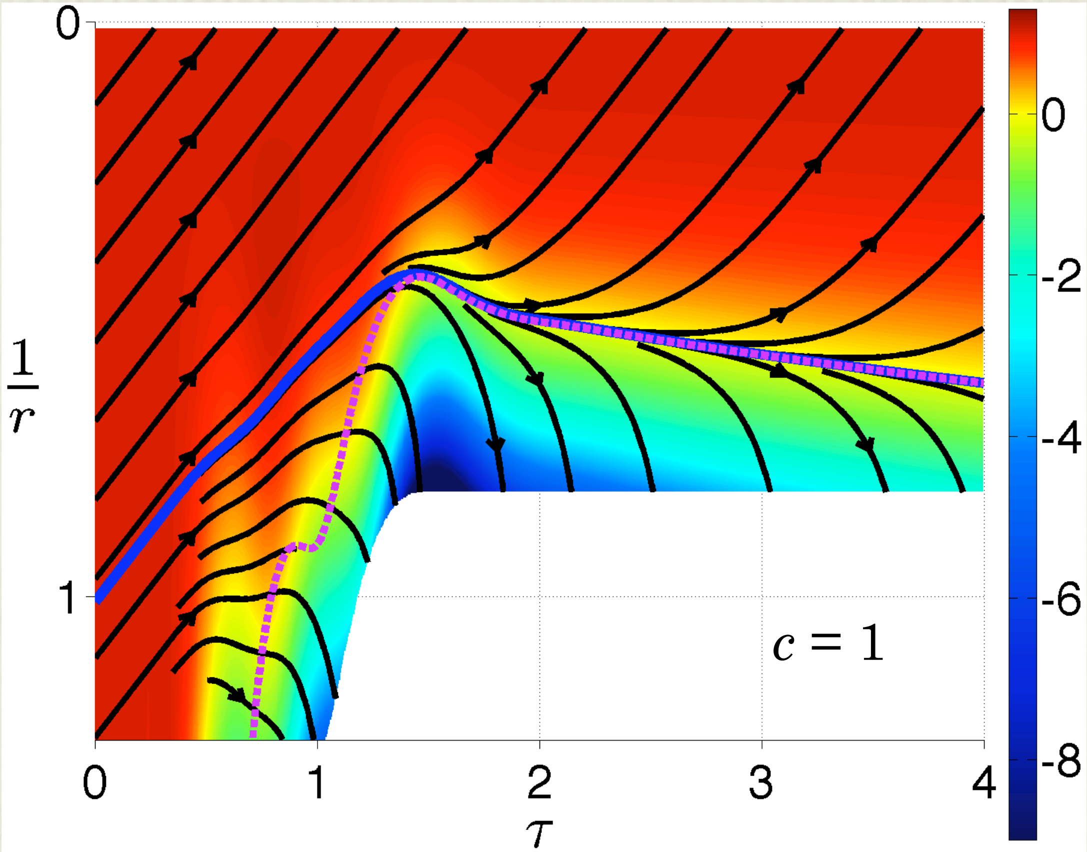
T = final equilibrium temperature

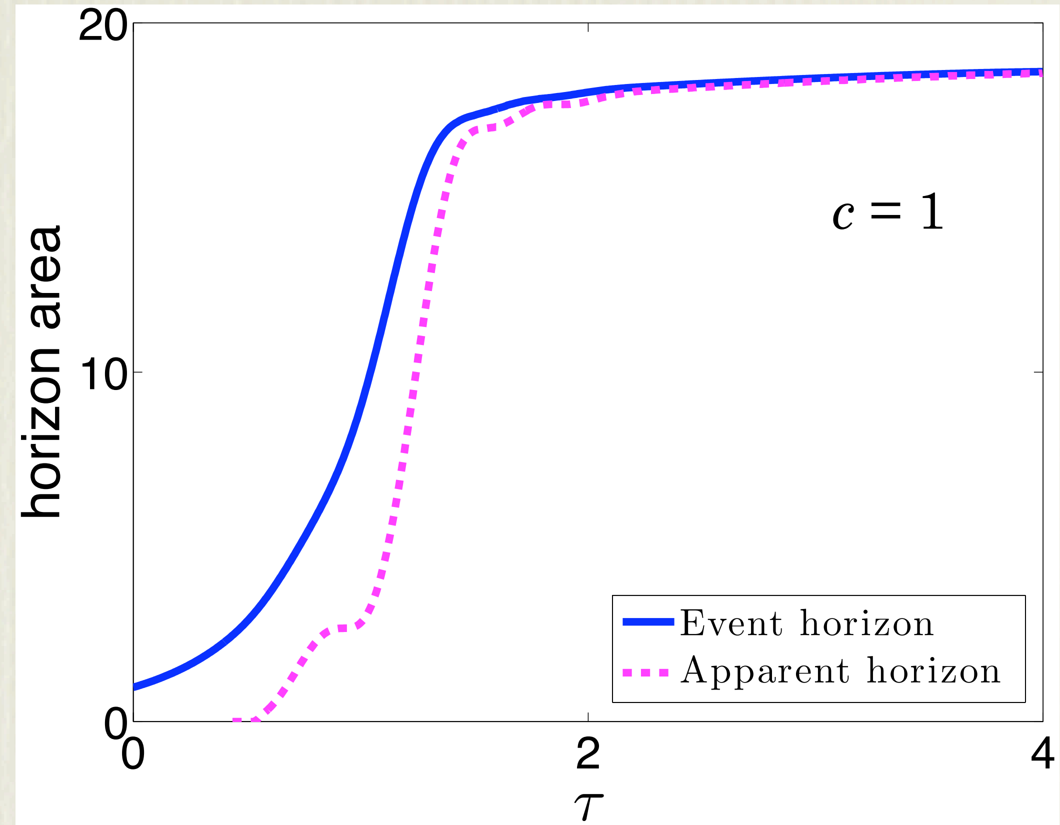
$\tau_{\rm iso}$ = isotropization time

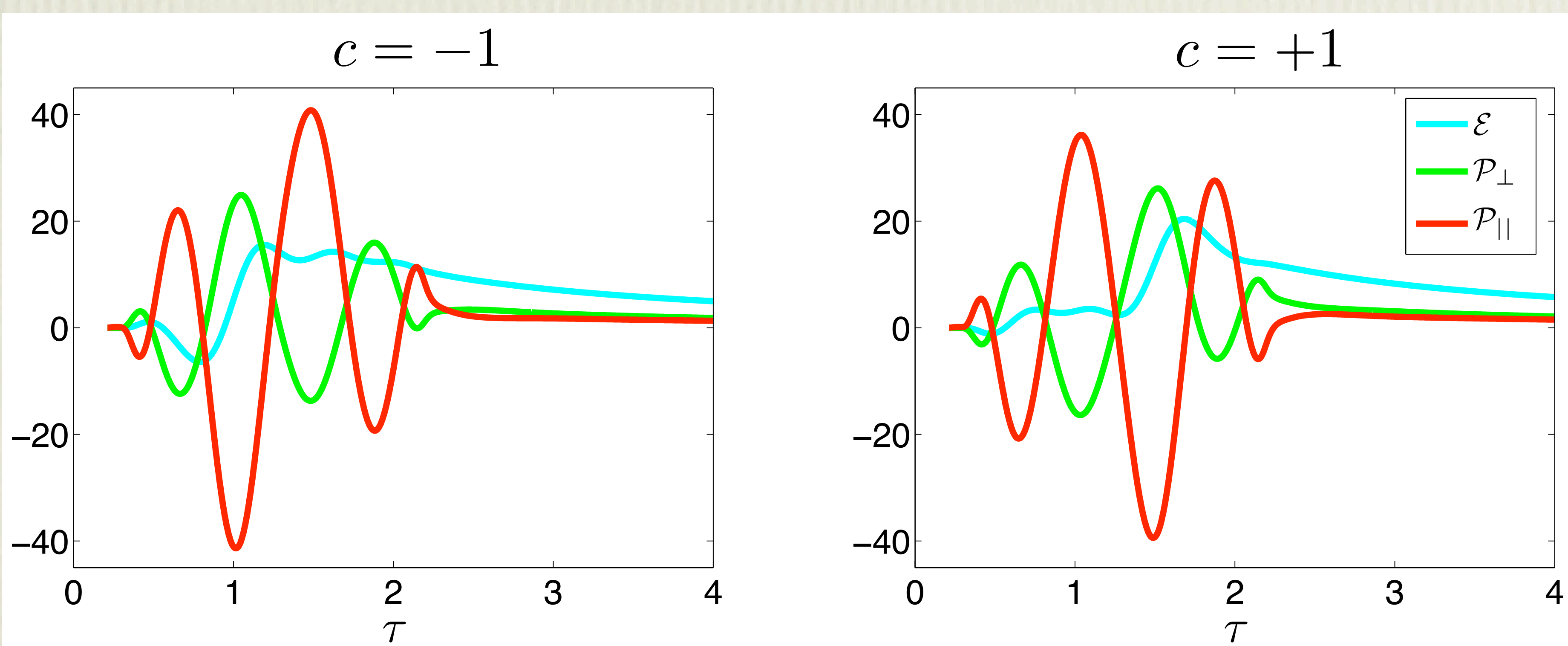
τ = plasma creation time scale

$\tau_{\rm iso} \approx 0.7/T$ ➡ $\tau_{\rm iso} \approx 0.5$ fm/c at $T \approx 350$ MeV --- relevant at RHIC???

Case II: Results



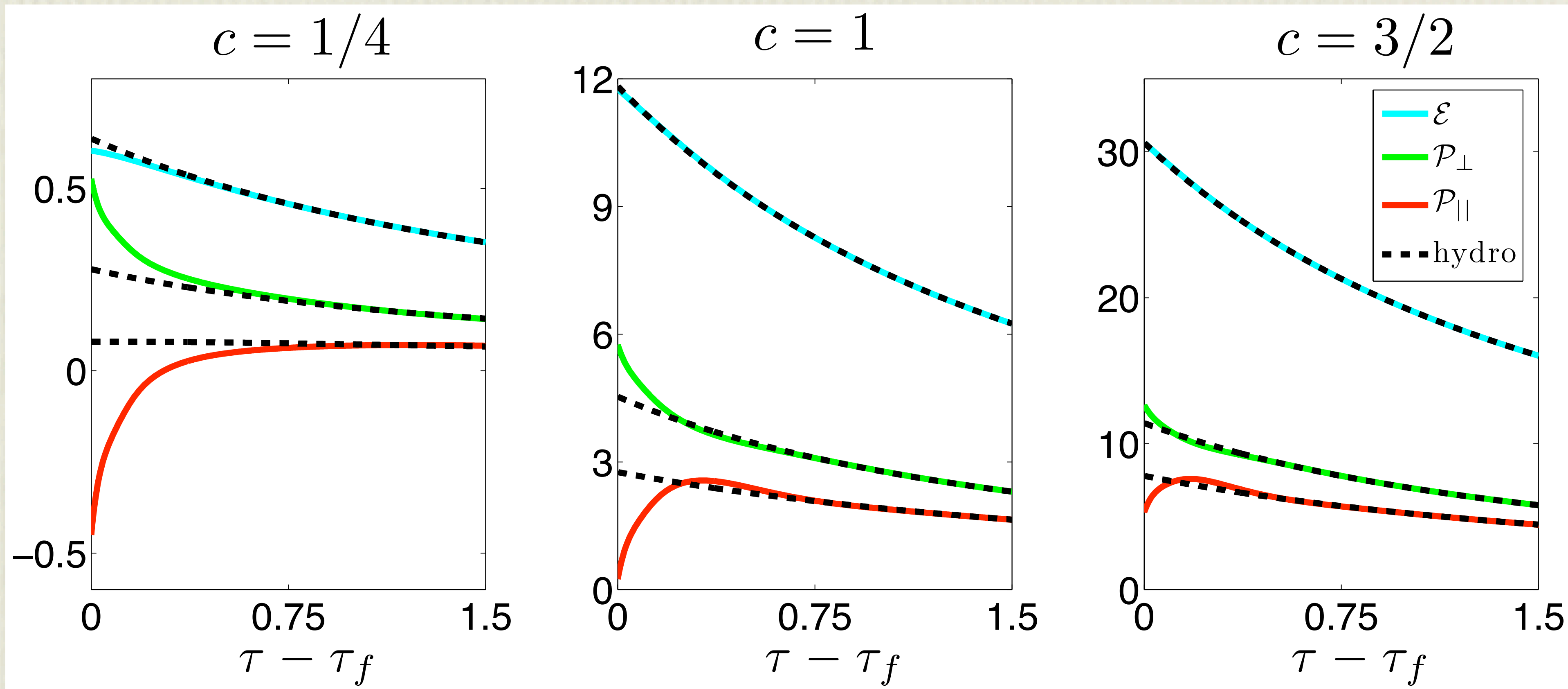


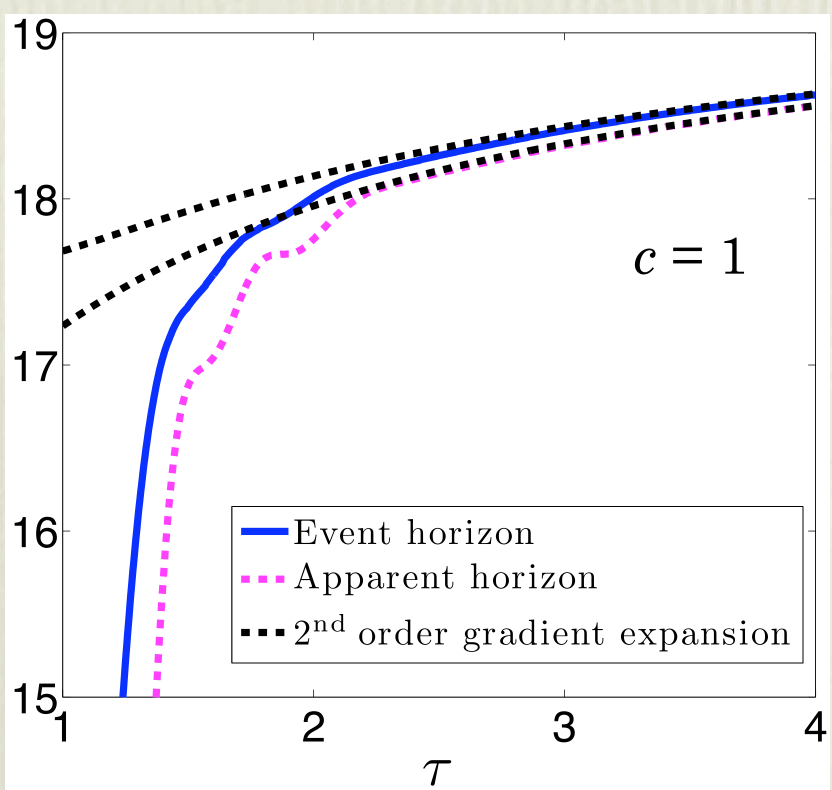


$\tau_i = 0.25$

$\tau_f = 2.25$

Case II: Hydro comparison





$$\mathcal{E} = \frac{3\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}}\left[1 - \frac{2C_1}{(\Lambda\tau)^{2/3}} + \frac{C_2}{(\Lambda\tau)^{4/3}}\right], \qquad C_1 = \frac{1}{3\pi}$$

$$\mathcal{P}_\perp = \frac{\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}}\left[1 - \frac{C_2}{3(\Lambda\tau)^{4/3}}\right], \qquad C_2 = \frac{2+\ln 2}{18\pi^2}$$

$$\mathcal{P}_{||} = \frac{\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}}\left[1 - \frac{2C_1}{(\Lambda\tau)^{2/3}} + \frac{5C_2}{3(\Lambda\tau)^{4/3}}\right]$$

$$A_{\mathrm{EH}} = \pi^3\Lambda^2\left[1 - \frac{1}{2\pi(\Lambda\tau)^{2/3}} + \frac{6+\pi+6\ln 2}{24\pi^2(\Lambda\tau)^{4/3}}\right]$$

$$A_{\mathrm{AH}} = \pi^3\Lambda^2\left[1 - \frac{1}{2\pi(\Lambda\tau)^{2/3}} + \frac{2+\pi+\ln 2}{24\pi^2(\Lambda\tau)^{4/3}}\right],$$

Janik, Peschanski; Kinoshita, Mukohyama, Nakamura; Booth, Heller, Spalinski

Case II: Relaxation time

c	-2	$-3/2$	-1	$-1/2$	$-1/4$	$1/4$	$1/2$	1	$3/2$	2
τ_*	2.2	2.3	2.4	2.7	3.1	3.1	2.7	2.4	2.3	2.2
T_*	0.93	0.77	0.60	0.40	0.27	0.27	0.41	0.62	0.80	0.97
$\Lambda\tau_*$	3.1	2.5	1.9	1.2	0.87	0.89	1.3	1.9	2.6	3.3
$(\tau_* - \tau_i)\,T_*$	2.0	1.7	1.4	1.1	0.84	0.85	1.1	1.5	1.8	2.1
$(\tau_* - \tau_f)\,T_*$	0.00	0.05	0.11	0.19	0.24	0.24	0.20	0.11	0.04	0.00
$\frac{\mathcal{P}_\perp(\tau_f) - P_{\|\|}(\tau_f)}{\mathcal{E}(\tau_f)}$	0.06	-0.03	-0.22	-0.56	-1.1	1.6	0.91	0.47	0.24	0.13

τ_* = hydro onset time

T_* = initial hydro temperature

$c \to \infty$: $\tau_* \to \tau_f$, $T_* \to \infty$, "instantaneous" relaxation to local equilibrium

$c \to 0$: $\tau_* \sim \Delta/\sqrt{c}$, $\tau_* T_* \sim O(1)$

$\Lambda\tau_* \geq 0.9$ always ➡ limit of validity of hydro controlled by relaxation of non-hydro modes, not by growth of higher-order viscous terms

Open questions

- Sensitivity to choice of boundary time dependence?
    - wider range of amplitudes
    - periodic forcing
- Precise connection between entropy & apparent horizon area?
    - ambiguities in definition of non-equilibrium entropy
    - foliation dependence of apparent horizon area
- Feasibility of evolving anisotropic & inhomogeneous geometries?
    - non-boost invariant colliding shocks
    - finite expanding fluids
    - turbulent driven systems
- Relevance for heavy ion collisions?