Toward an AdS/cold atom correspondence

D. T. Son

Institute for Nuclear Theory, University of Washington

Plan

- History/motivation
  - BCS/BEC crossover
  - Unitarity regime
- Schrödinger symmetry: nonrelativistic conformal invariance
- Geometric realization of Schrödinger symmetry
- Green's functions in vacuum
- Conclusion

Collaborators: Y. Nishida, M. Rangamani, S. Ross, E. Thompson

Refs.: Y. Nishida, DTS, 0706.3746 (PRD)
Balasubramanian and McGreevy, 0804.4053 (PRL 2008)
DTS, 0804.3972 (PRD 2008)
Rangamani, Ross, DTS, Thompson, arXiv:0811.2049

Will not talk about

- Finite temperature and density
- "Lifshitz geometry"

BCS mechanism

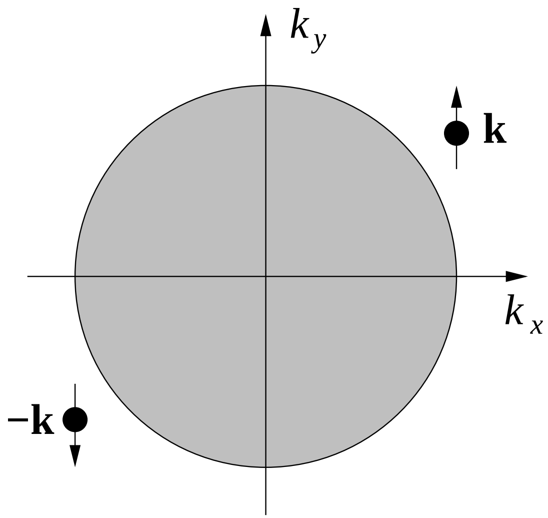
Explains superconductivity of metals

Consider a gas of spin-1/2 fermions ψ_a, $a = \uparrow, \downarrow$

No interactions: Fermi sphere: states with $k < k_{\rm F}$ filled

Cooper phenomenon: any attractive interaction leads to condensation

$$\langle \psi_\uparrow \psi_\downarrow \rangle \neq 0$$



BCS/BEC crossover

Critical temperature:

$$T_c \sim \epsilon_{\mathrm{F}} e^{-1/g}, \qquad g \ll 1$$

Leggett (1980): If one increases the interaction strength g, how large can one make $T_c/\epsilon_{\mathrm{F}}$?

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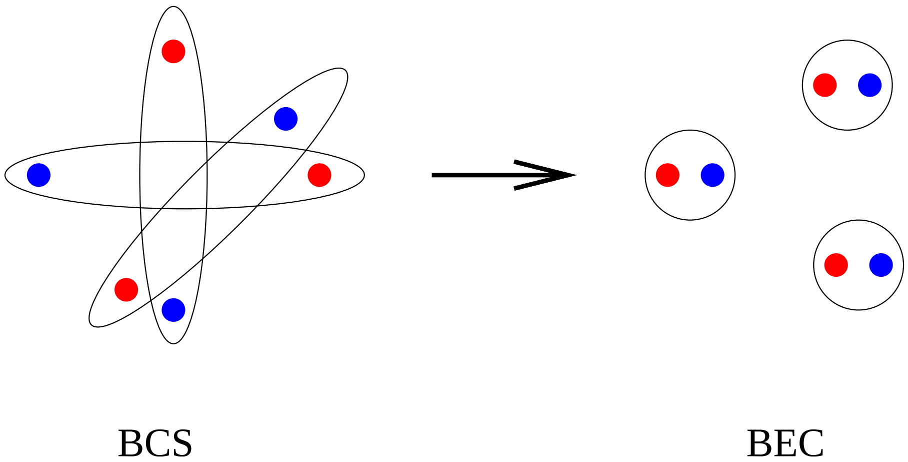
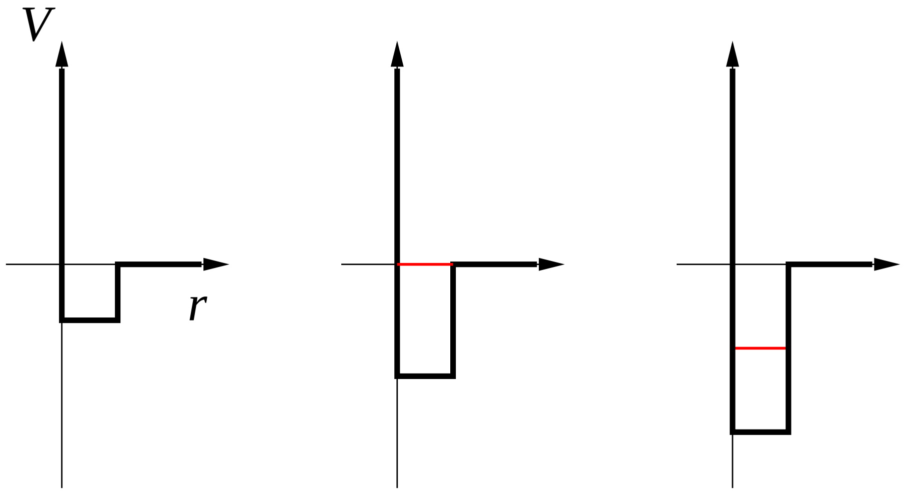


Strong attraction: leads to bound states, which form a dilute Bose gas
$T_c =$ temperature of Bose-Einstein condensation $\approx 0.22\epsilon_{\mathrm{F}}$.

Unitarity regime

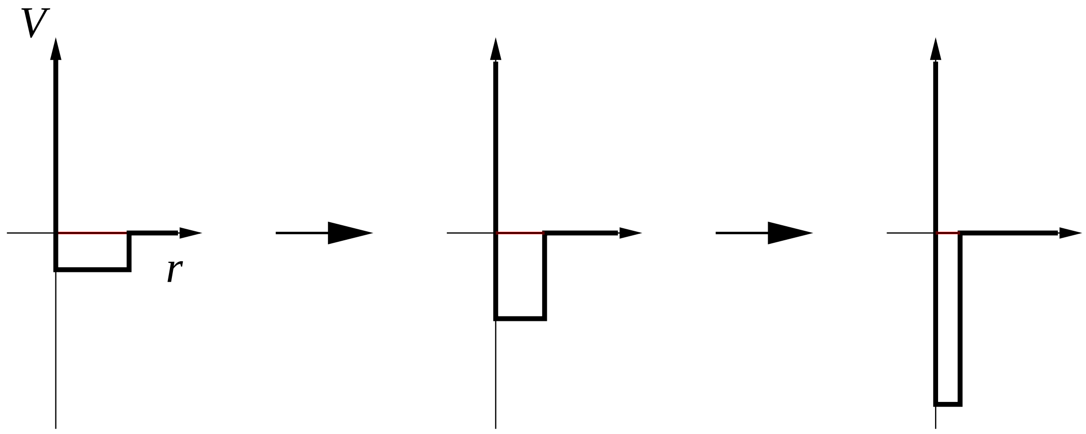
Consider in more detail the process of going from BCS to BEC
Assume a square-well potential, with fixed but small range r_0.

- $V_0 < 1/mr_0^2$: no bound state
- $V_0 = 1/mr_0^2$: one bound state appears, at first with zero energy
- $V_0 > 1/mr_0^2$: at least one bound state



Unitarity regime (II)

Unitarity regime: take $r_0 \to 0$, keeping one bound state at zero energy.



In this limit: no intrinsic scale associated with the potential

In the language of scattering theory: infinite scattering length $a \to \infty$

s-wave scattering cross section saturates unitarity

Boundary condition interpretation

Unitarity: taking Hamiltonian to be free:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m}$$

but imposing nontrivial boundary condition on the wavefunction:

$$\Psi(\underbrace{\mathbf{x}_1, \mathbf{x}_2, \ldots}_{\text{spin-up}}, \underbrace{\mathbf{y}_1, \mathbf{y}_2, \ldots}_{\text{spin=down}})$$

When $|\mathbf{x}_i - \mathbf{y}_j| \to 0$:

$$\Psi \to \frac{C}{|\mathbf{x}_i - \mathbf{y}_j|} + 0 \times |\mathbf{x}_i - \mathbf{y}_j|^0 + O(|\mathbf{x}_i - y_j|)$$

Free gas corresponds to

$$\Psi \to \frac{0}{|\mathbf{x}_i - \mathbf{y}_j|} + C + O(|\mathbf{x}_i - y_j|)$$

Field theory interpretation

Consider the following model

Sachdev, Nikolic; Nishida, DTS

$$S = \int dt\, d^d x \, \left(i\psi^\dagger \partial_t \psi - \frac{1}{2m} |\nabla\psi|^2 - c_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \right)$$

Dimensional analysis:

$$[t] = -2, \quad [x] = -1, \quad [\psi] = \frac{d}{2}, \quad [c_0] = 2 - d$$

Contact interaction is irrelevant at $d > 2$

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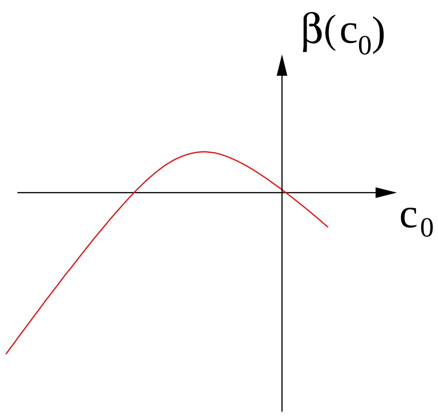
Contact interaction is irrelevant at $d > 2$
RG equation in $d = 2 + \epsilon$:

$$\frac{\partial c_0}{\partial s} = -\epsilon c_0 - \frac{c_0^2}{2\pi}$$

Two fixed points:

- $c_0 = 0$: trivial, noninteracting
- $c_0 = -2\pi\epsilon$: unitarity regime



Field theory in $d = 4 - \epsilon$ dimensions

Sachdev, Nikolic; Nishida, DTS; Nussinov and Nussinov

$$S = \int dt\, d^d x \left(i\psi^\dagger \partial_t \psi - \frac{1}{2m}|\nabla\psi|^2 - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger - g\phi^*\psi_\downarrow\psi_\uparrow \right.$$
$$\left. + i\phi^*\partial_t\phi - \frac{1}{4m}|\nabla\phi|^2 + C\phi^*\phi \right)$$

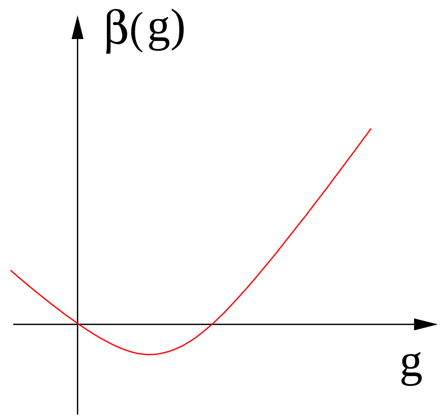
C fined tuned to criticality

Dimensions: $[g] = \frac{1}{2}(4-d) = \frac{1}{2}\epsilon$

RG equation for g:

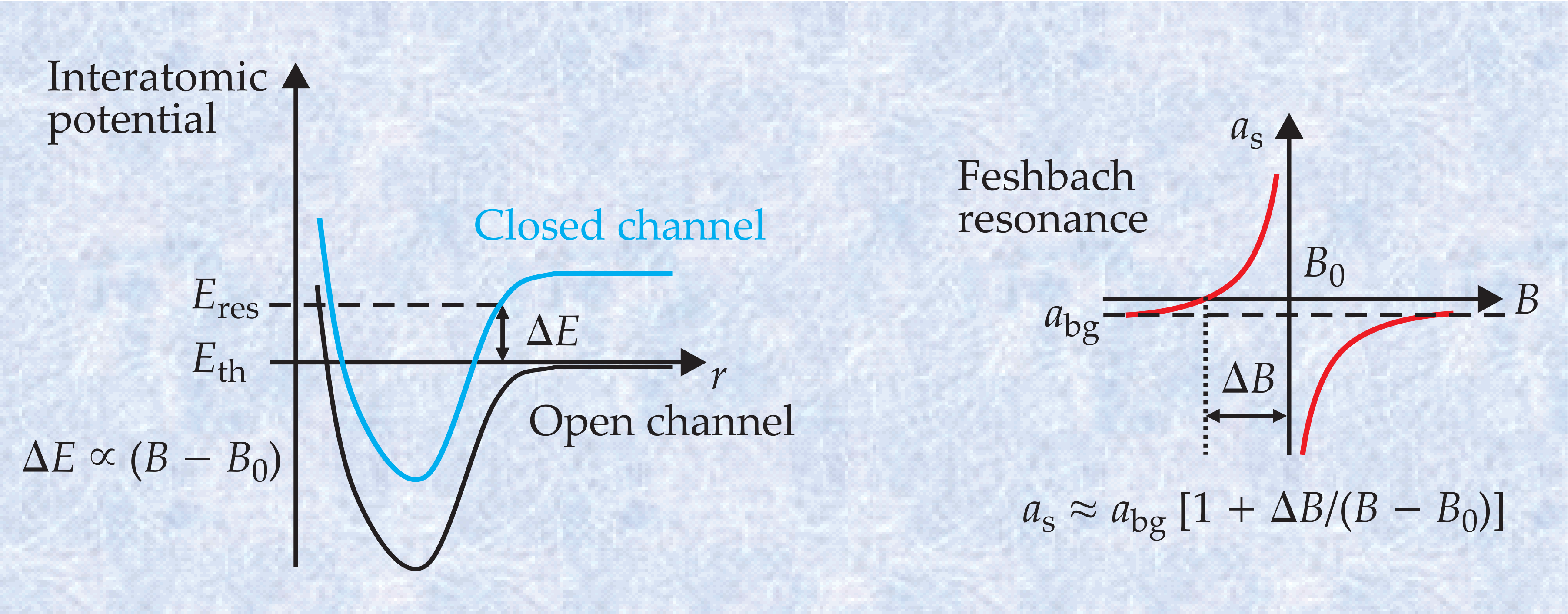
$$\frac{\partial g}{\partial \ln \mu} = -\frac{\epsilon}{2} g + \frac{g^3}{16\pi^2}$$

Fixed point at $g^2 = 8\pi^2\epsilon$



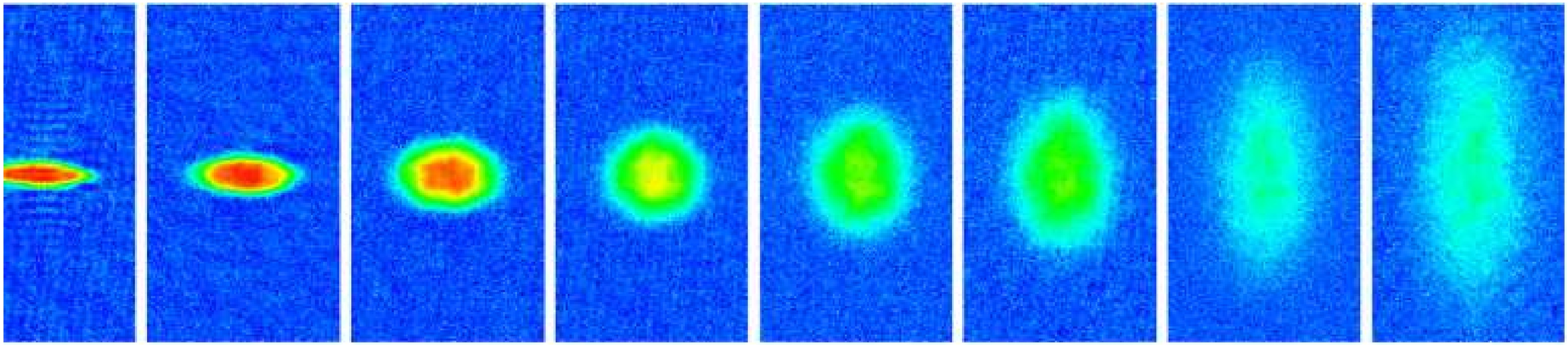
Examples of systems near unitarity

- Neutrons: $a = -20$ fm, $|a| \gg 1$ fm
- Trapped atom gases, with scattering length a controlled by magnetic field

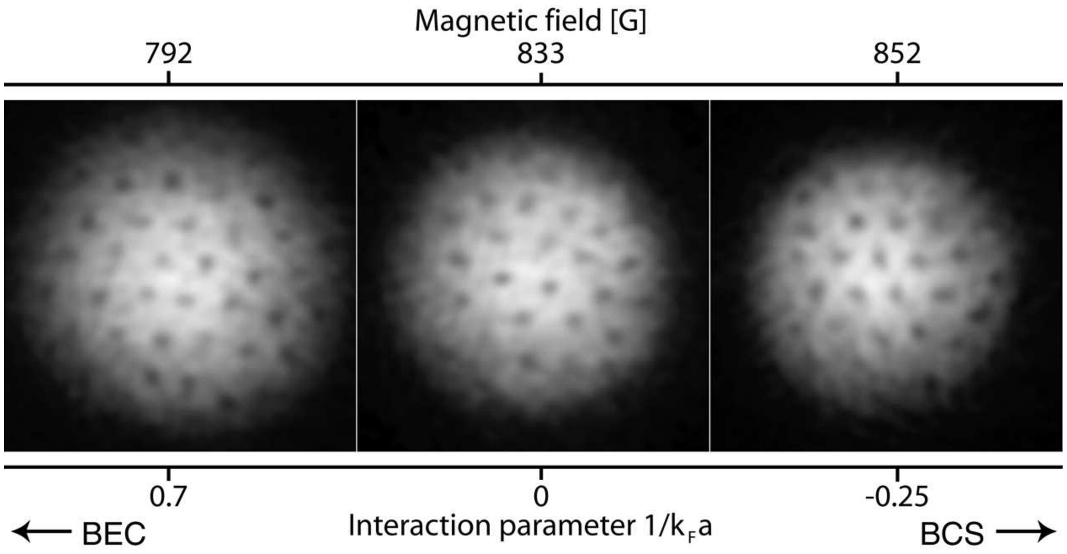


Experiments

A cloud of gas is released from the trap:



Vortices indicating superfluidity



Ground state energy

George Bertsch asked the question:

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Dimensional analysis: no intrinsic length/energy scale, only density n. So:

$$\epsilon \equiv \frac{E}{V} = \# \frac{n^{5/3}}{m}$$

The same parametric dependence as the energy of a free gas

$$\epsilon(n) = \xi \epsilon_{\text{free}}(n)$$

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Since then, ξ is called the Bertsch parameter.

Analogy with AdS/CFT

$$\epsilon(n) = \xi \epsilon_{\text{free}}(n)$$

Current estimate: $\xi \approx 0.4$

Similar to pressure in $\mathcal{N} = 4$ SYM theory

$$P(T)|_{\lambda\to\infty} = \frac{3}{4} P(T)|_{\lambda\to 0}$$

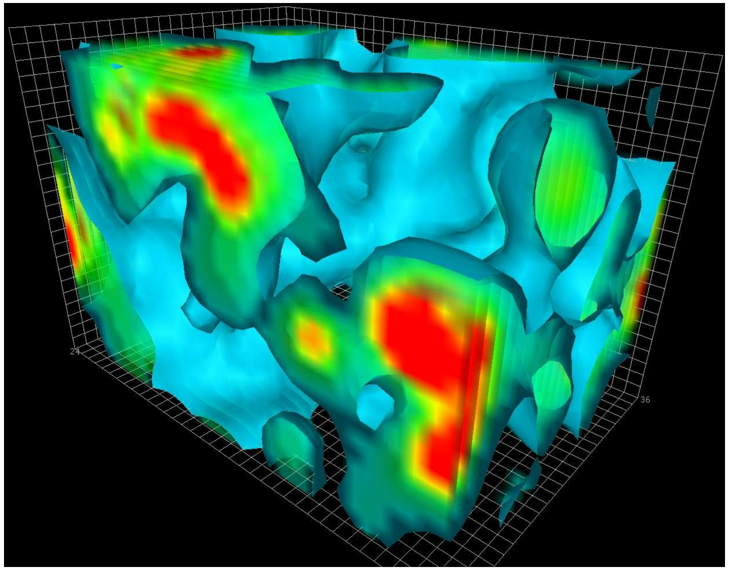
Is there an useful AdS/CFT-type duality for unitarity Fermi gas?

As in $\mathcal{N} = 4$ SYM, perhaps we should start with the vacuum (zero temperture and density)

- More symmetry
- Temperature and chemical potential can (hopefully) be accomodated later

Vacuum

Relativistic vacuum



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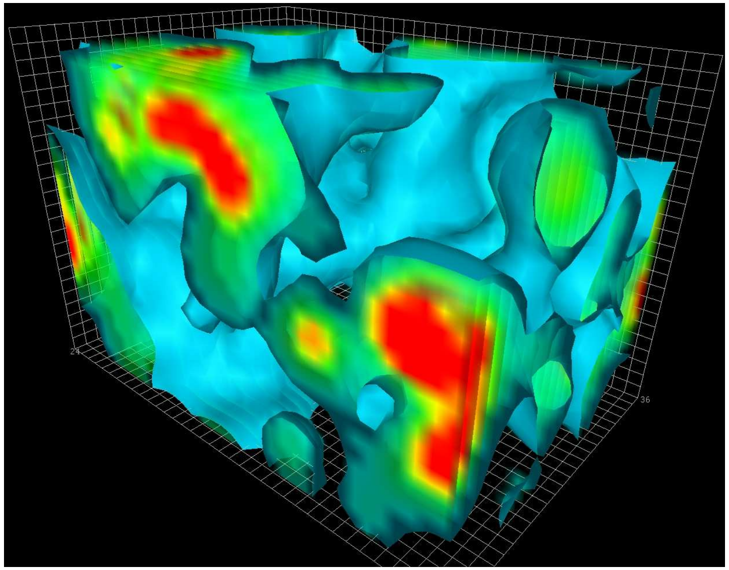

Nonrelativistic vacuum

Vacuum

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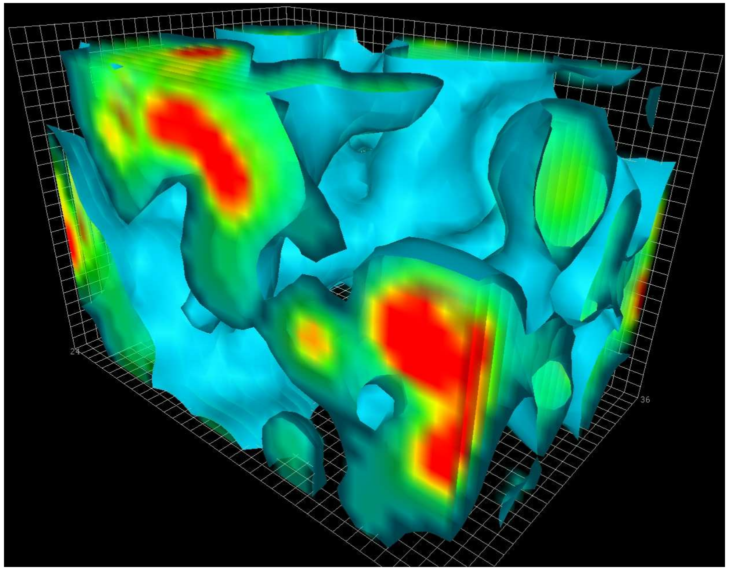


Nonrelativistic vacuum

Nonrelativistic vacuum is much simpler (no particle-hole pair creation)

Still: nontrivial conformal dimensions and correlation functions.

AdS/CFT correspondence

$\mathcal{N} = 4$ super-Yang-Mills theory $\Leftrightarrow$ type IIB string theory on AdS$_5 \times$ S^5.

First evidence: matching of symmetries SO$(4,2) \times$ SO(6).

- SO$(4,2)$: conformal symetry of 4-dim theories (CFT$_4$), isometry of AdS$_5$
- SO(6): $\sim$ SU(4) is the R-symmetry of $N = 4$ SYM, isometry of S^5.

Conformal algebra: P^μ, $M^{\mu\nu}$, K^μ, D

$$[D,\, P^\mu] = -iP^\mu, \qquad [D,\, K^\mu] = iK^\mu$$

$$[P^\mu,\, K^\nu] = -2i(g^{\mu\nu}D + M^{\mu\nu})$$

Schrödinger algebra

Nonrelativistic field theory is invariant under:

- Phase rotation M: $\psi \to \psi e^{i\alpha}$
- Time and space translations, H and P^i
- Rotations M^{ij}
- Galilean boosts K^i
- Dilatation D: $\mathbf{x} \to \lambda \mathbf{x}$, $t \to \lambda^2 t$
- Conformal transformation C:

$$\mathbf{x} \to \frac{\mathbf{x}}{1-\lambda t}, \qquad t \to \frac{t}{1-\lambda t}$$

$$[D,\, P^i] = -iP^i, \qquad [D,\, K^i] = iK^i, \qquad [P^i,\, K^j] = -\delta^{ij} M$$

$$[D,\, H] = -2iH, \qquad [D,\, C] = 2iC, \qquad [H,\, C] = iD$$

D, H, C form a SO(2,1)

Chemical potential $\mu\psi^\dagger\psi$ breaks the symmetry.

Generators

$$M = \int d\mathbf{x}\, n(\mathbf{x}), \qquad P_i = \int d\mathbf{x}\, j_i(\mathbf{x})$$

$$K_i = \int d\mathbf{x}\, x_i n(\mathbf{x}), \qquad C = \frac{1}{2}\int d\mathbf{x}\, x^2 n(x), \qquad D = -\int d\mathbf{x}\, x_i j_i(\mathbf{x})$$

$H \to H + \omega^2 C$: putting the system in an external potential $V(\mathbf{x}) = \frac{1}{2}\omega^2 x^2$.

Operator $\mathcal{O}$ has dimension Δ if $[D,\, \mathcal{O}(0)] = -i\Delta\mathcal{O}(0)$

$[D,\, K_i] = iK_i$: K_i lowers dimension by 1

$[D,\, C] = 2iC$: C lowers dimension by 2

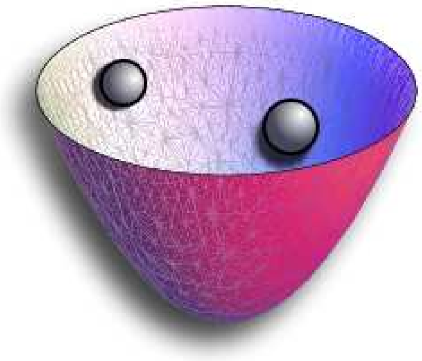
Define primary operators: $[K_i,\ \mathcal{O}(0)] = [C,\ \mathcal{O}(0)] = 0$

Operator-state correspondence

Nishida, DTS

Primary operator with dimension Δ $\Longleftrightarrow$ eigenstate in harmonic potential with energy $\Delta \times \hbar\omega$

$\psi_\uparrow \psi_\downarrow$ $\Longleftrightarrow$



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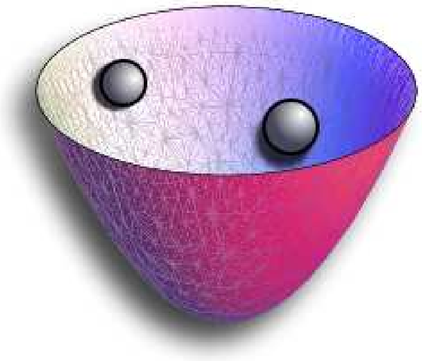
$\psi_\uparrow \psi_\downarrow$ $\Longleftrightarrow$ 

Proof: let $\omega = 1$, $H_{\rm osc} = H + C$; Define $|\Psi_{\mathcal{O}}\rangle = e^{-H}\mathcal{O}^\dagger(0)|0\rangle$

From Schrödinger algebra one finds $e^H H_{\rm osc} e^{-H} = C + iD$

from $[C, \mathcal{O}^\dagger(0)] = 0$, $[D, \mathcal{O}^\dagger(0)] = -i\Delta_{\mathcal{O}}$, and $C|0\rangle = D|0\rangle = 0$:

$$H_{\rm osc}|\Psi_{\mathcal{O}}\rangle = \Delta_{\mathcal{O}}|\Psi_{\mathcal{O}}\rangle$$

Example

Two particles in harmonic potential: ground state with unitarity boundary condition can be found exactly

$$\Psi(\mathbf{x}, \mathbf{y}) = \frac{e^{-(x^2+y^2)/2}}{|\mathbf{x} - \mathbf{y}|}, \qquad E_0 = 2\hbar\omega$$

$\longrightarrow$ Dimension of operator $O_2 = \psi_\uparrow \psi_\downarrow$ is 2 (naively 3)

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Can be see explicitely: two-particle wavefunctions behave as

$$\Psi(\mathbf{x},\mathbf{y}) \sim \frac{1}{|\mathbf{x}-\mathbf{y}|}, \qquad x \to y$$

the operator O_2 has to be regularized as

$$O_2(\mathbf{x}) = \lim_{\mathbf{x}\to\mathbf{y}} |\mathbf{x}-\mathbf{y}|\psi_\uparrow(\mathbf{x})\psi_\downarrow(\mathbf{y})$$

so that $\langle 0|O_2|\Psi(\mathbf{x},\mathbf{y})\rangle$ is finite.

Lowest 3-body operators: $\Delta_{l=1} = 4.27272...$, $\Delta_{l=0} = 4.66622...$

Embedding the Schrödinger algebra

- Sch(d): is the symmetry of the Schrödinger equation

$$i\frac{\partial\psi}{\partial t} + \frac{\nabla^2}{2m}\psi = 0$$

- CFT$_{d+2}$: is the symmetry of the Klein-Gordon equation

$$\partial_\mu^2\phi = 0, \qquad \mu = 0, 1, \cdots, d+1$$

In light-cone coordinates $x^\pm = x^0 \pm x^{d+1}$ the Klein-Gordon equation becomes

$$-2\frac{\partial}{\partial x^+}\frac{\partial}{\partial x^-}\phi + \partial_i\partial_i\phi = 0, \qquad i = 1, \cdots, d$$

Restricting ϕ to $\phi = e^{-imx^-}\psi(x^+, \mathbf{x})$: Klein-Gordon eq. $\Rightarrow$ Schrödinger eq.:

$$2im\frac{\partial}{\partial x^+}\psi + \nabla^2\psi = 0, \qquad \nabla^2 = \sum_{i=1}^{d}\partial_i^2$$

This means Sch$(d) \subset$ CFT$_{d+2}$

Embedding (2)

Sch(d) is the subgroup of CFT$_{d+2}$ containing group elements which does not change the ansatz

$$\phi = e^{imx^-}\psi(x^+, x^i)$$

Algebra: sch(d) is the subalgebra of the conformal algebra, containing elements that commute with P^+

$$[P^+, O] = 0$$

One can identify the Schrödinger generators:

$$M = P^+, \quad H = P^-, \quad K^i = M^{i-},$$

$$D_{\rm nonrel} = D_{\rm rel} + M^{+-}, \quad C = \frac{1}{2}K^+$$

Nonrelativistic dilatation

$$\begin{array}{ccccc} & D_{\rm rel} & & M^{+-} & \\ x^+ & \longrightarrow & \lambda x^+ & \longrightarrow & \lambda^2 x^+ \\ x^- & \longrightarrow & \lambda x^- & \longrightarrow & x^- \\ x^i & \longrightarrow & \lambda x^i & \longrightarrow & \lambda x^i \end{array}$$

Geometric realization of Schrödinger algebra

Start from AdS$_{d+3}$ space:

$$ds^2 = \frac{-2dx^+dx^- + dx^i dx^i + dz^2}{z^2}$$

Invariant under the whole conformal group, in particular with respect to relativistic scaling

$$x^\mu \to \lambda x^\mu, \qquad z \to \lambda z$$

and boost along the x^{d+1} direction:

$$x^+ \to \tilde{\lambda} x^+, \qquad x^- \to \tilde{\lambda}^{-1} x^-$$

Break the symmetry down to Sch(d):

$$ds^2 = \frac{-2dx^+dx^- + dx^i dx^i + dz^2}{z^2} - \frac{2(dx^+)^2}{z^4}$$

The additional term is invariant only under a combination of relativistic dilation and boost:

$$x^+ \to \lambda^2 x^+, \quad x^- \to x^-, \quad x^i \to \lambda x^i, \qquad z \to \lambda z$$

Model

$$ds^2 = \frac{-2dx^+dx^- + dx^i dx^i + dz^2}{z^2} - \frac{2(dx^+)^2}{z^4}$$

Is there a model where this is a solution to the Einstein equation?

The additional term gives rise to a change in $R_{++} \sim z^{-4}$: we need matter that provides $T_{++} \sim z^{-4}$.

Can be provided by A_μ with $A_+ \sim 1/z^2$: has to be a massive gauge field.

$$S = \int d^{d+3}x \sqrt{-g} \left(\frac{1}{2} R - \Lambda - \frac{1}{4} F_{\mu\nu}^2 - \frac{m^2}{2} A_\mu^2 \right)$$

Can be realized in string theory ($d = 2$)
Herzog, Rangamani, Ross;
Maldacena, Martelli, Tachikawa;
Adam, Balasubramanian, McGreevy (2008)

Black-hole solutions also constructed: allow studying hydrodynamics

Two-point function

Following standard prescription

$$S = -\int d^{d+3}x\,\sqrt{-g}(g^{\mu\nu}\partial_\mu\phi^*\partial_\mu\phi + m_0^2\phi^*\phi)$$

Consider $\phi \sim e^{iMx^-}$

$$S = \int d^{d+2}x\,dz\,\frac{1}{z^{d+3}}\left(2iMz^2\phi^*\partial_t\phi - z^2\partial_i\phi^*\partial_i\phi - m^2\phi^*\phi\right), \quad m^2 = m_0^2 + 2M^2$$

$$\langle O(\tau,\mathbf{x})O(0,0)\rangle \sim \frac{\theta(\tau)}{\tau^\Delta}\exp\left(-\frac{Mx^2}{2\tau}\right)$$

where

$$\Delta = \frac{d+2}{2} + \nu, \qquad \nu = \sqrt{m^2 + \frac{(d+2)^2}{4}}$$

Form dictated by Schrödinger symmetry.

Three-point function

$$\langle O_1(t_1, x_1) O_2(t_2, x_2) O_3^\dagger(0,0)\rangle = \frac{\theta(t_1)\theta(t_2)}{t_1^{\Delta_{13,2}} t_2^{\Delta_{23,1}} (t_1 - t_2)^{\Delta_{12,3}}} \exp\left[-\frac{M_1 x_1^2}{2t_1} - \frac{M_2 x_2^2}{2t_2}\right] \Psi(y)$$

$\Delta_{ij,k} = \Delta_i + \Delta_j - \Delta_k$, where y is a Schrödinger invariant

$$y = \frac{x_1 t_2 - x_2 t_1}{(t_1 - t_2) t_1 t_2}$$

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$$\Psi(y)\sim y^{1-d/2}\gamma\left(\frac{d}{2}-1,y\right)\longleftarrow \text{incomplete beta function}$$

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Holography: computing Witten diagram Fuertes, Moroz

$$\psi(y) \sim \int dv\, dv'\, e^{-iM_1v-iM_2v'}(v-v'+iy)^{-\Delta_{12,3}/2}(v')^{-\Delta_{23,1}}v^{-\Delta_{13,2}}$$

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Qualitative understanding that we gain

- $m^2 < -(d+2)^2/4$: complex dimension

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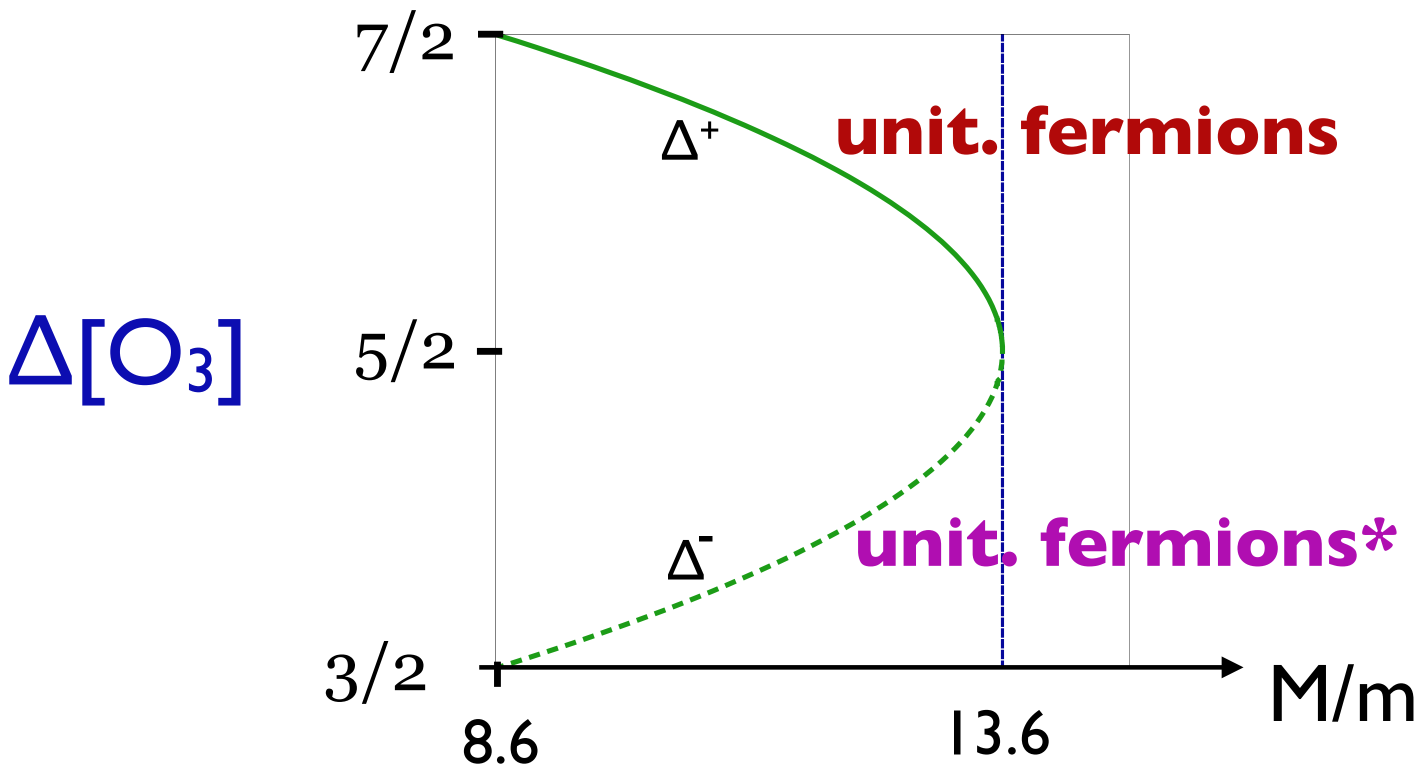
- Unitarity fermions with different masses for 2 flavors $M/m \neq 1$
  - One three-body operator $\Psi \partial_i \Psi \psi$: dimension between $7/2$ and $5/2$ when M/m varies from 8.6 to 13.6
  - There exists another scale-invariant theory with two and three-body resonances in this range of mass ratio Nishida, DTS, Shina Tan

Unitarity fermions*



Things we don't understand

- Holographic renormalization
- Role of large N? (Sp($2N$) model?)
- Hierarchical organization in NR field theories
  - to understand n-body sector we don't need to know solution to $n+1$, $n+2$ etc. body problem
  - In gravity: throwing away fields with mass $> n$ should be a consistent truncation!
  - Not a feature of current string constructions of Schrödinger background

Conclusion

- Unitarity fermions have Schrödinger symmetry, a nonrelativistic conformal symmetry
- Universal properties, studied in experiments
- There is a metric with Schrödinger symmetry
- Starting point for dual phenomenology of unitarity fermions?
- Deeper connection between few- and many-body physics?