Holography and Dynamical Critical Phenomena

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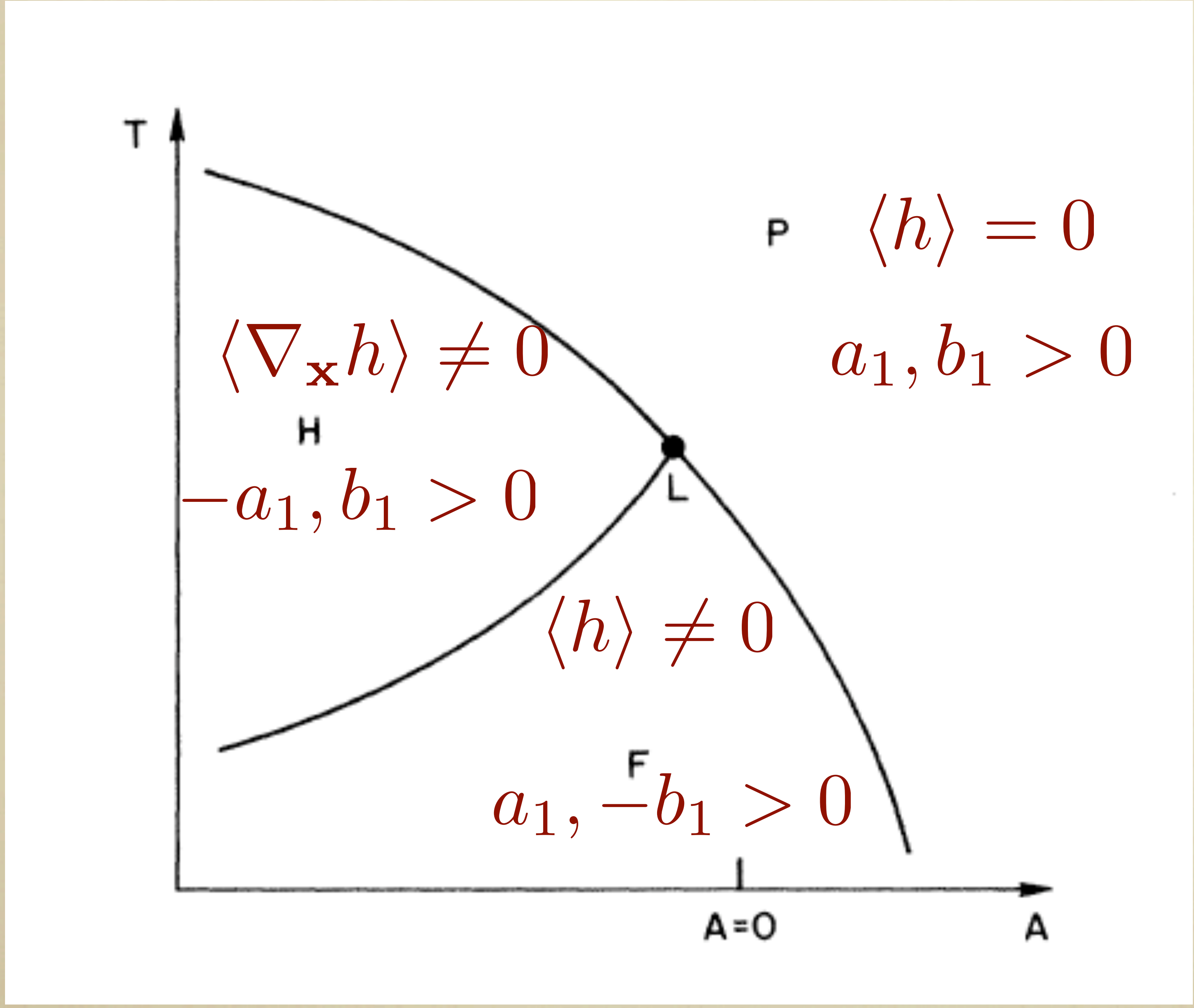
Shamit Kachru, Xiao Liu, and MM, hep-th/0808.1725.

Consider the following phenomenological free energy and associated phase diagram obtained by varying T and A which control the parameters a_1, a_2, b_1, b_2.

$$F = \int d^d\mathbf{x}\,\left[a_1(\nabla_{\mathbf{x}}h)^2 + a_2(\nabla^2_{\mathbf{x}}h)^2 + b_1 h^2 + b_2 h^4 + ...\right]$$

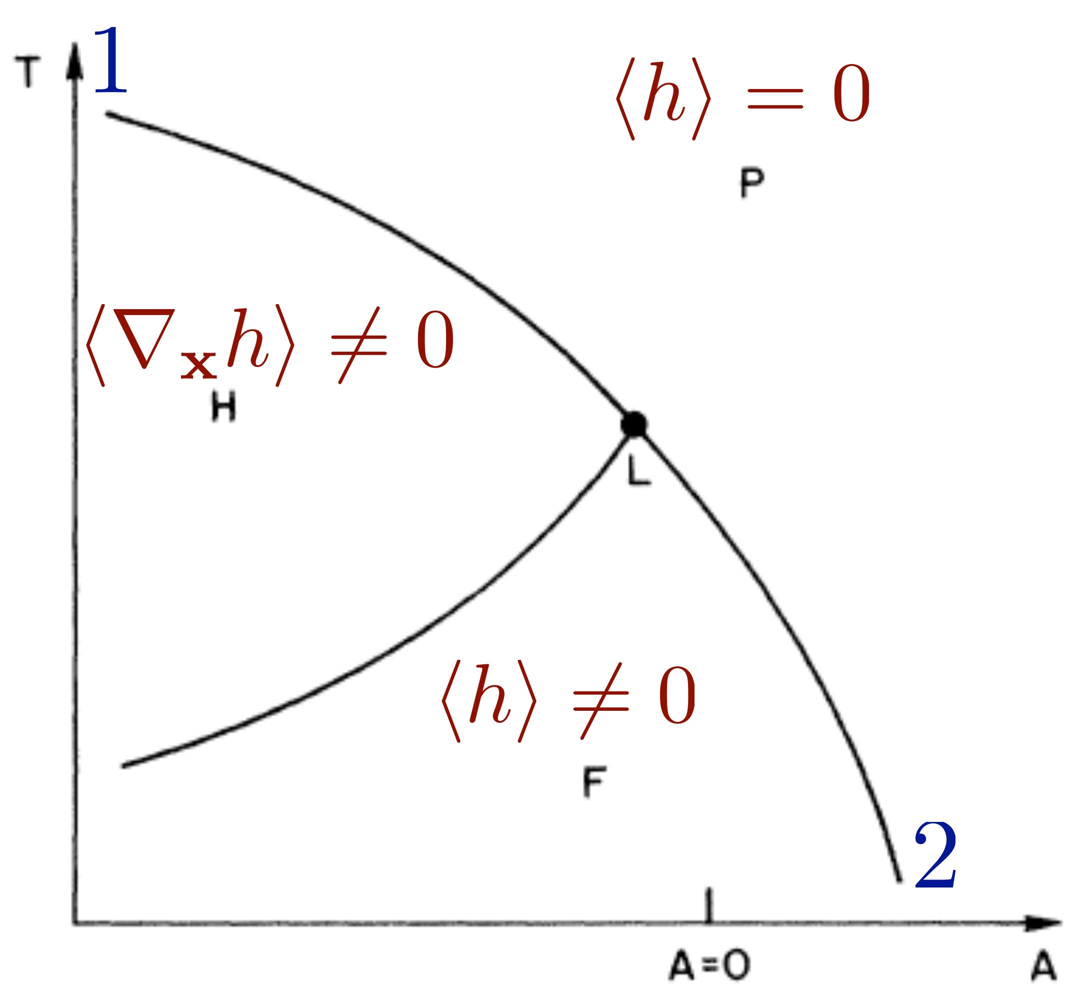


$d > 2$

$a_2, b_2 > 0$

- The line $1-2$ is a line a continuous transitions.
- $L-2$ is controlled by an h^4 - theory fixed point.
- $1-L$ is controlled by the "Lifshitz field theory:"

$$L=\int dt d^d\mathbf{x}\left[(\partial_t h)^2-\kappa^2(\nabla^2_{\mathbf{x}}h)^2\right]$$



Invariant under,

$$t\to\lambda^2 t,\mathbf{x}\to\mathbf{x};$$

$$[h]=\frac{d-2}{2}$$

Now, there are a number of condensed matter systems that contain anisotropic critical points.

Instead of invariance under the usual scaling

$$t \to \lambda t, \mathbf{x} \to \lambda \mathbf{x},$$

they are invariant under the modified scaling

$$t \to \lambda^z t, \mathbf{x} \to \lambda \mathbf{x}.$$

Such systems are said to exhibit dynamical critical phenomena with dynamical critical exponent, z .

Hohenberg and Halperin '77

Example $z \neq 1$ fixed points:

- BEC-BCS crossover in cold atomic systems

Bloch et al. 2007

- itinerant fermions in one dimension; ferro-transitions "$.7(2e^2/h)$ anomaly" in conductivity?

Hertz '76, Sachdev & Senthil '96, K. Yang 2004

- liquid crystals: nematic, smectic-A,C transitions

Grinstein '81, Chaikin & Lubensky's book, (Wikipedia)

- VBS transitions in coupled bilayer spin-systems

Vishwanath, Balents, & Senthil '03

- generally at any tricritical point obtained by dialing two or more parameters, one of the parameters being the coefficient of the term quadratic in spatial derivatives.

Honreich et al. '75

Now I'd like to provide an extended example where the Lifshitz theory arises.
I focus on the Lifshitz theory for two reasons:

1. It arises in descriptions of a variety of unrelated systems in various dimensions.
2. It provides useful intuition for interacting theories with the same symmetries.

In the example below, we will find that the Lifshitz theory arises as a certain approximation to a particular phase of strongly correlated electrons. Moving away from this approximation necessarily involves strongly coupled physics.

An example of where the Lifshitz theory arises is provided by the quantum dimer model at the **Rokhsar-Kivelson point**.
(This model is motivated by the study of the high temperature superconducting cuprates.)

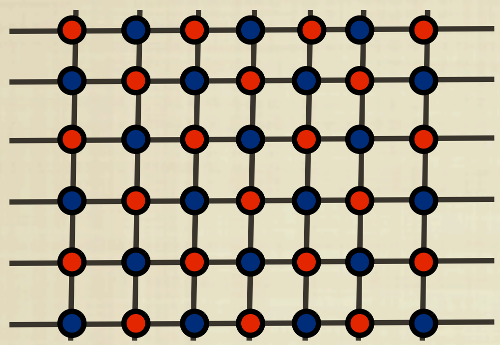
To see this, begin with the quantum $SU(2)$ Heisenberg antiferromagnet in 2+1 dimensions (on a square lattice)

$$H = + \sum_{\langle i,j \rangle} |J_{ij}| \sigma_{\mathbf{i}} \cdot \sigma_{\mathbf{j}}.$$

It is energetically favorable for neighboring spins to point in the opposite direction.

Restricting to nearest-neighbor interactions, the ground state possesses so-called **Neel order**.

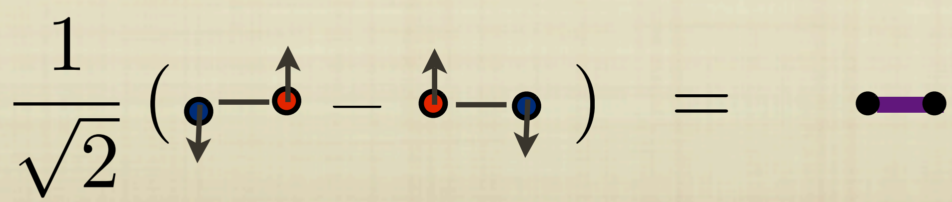
(working at zero temperature)



$$\langle M_z \rangle \sim \cos(\pi x_1 + \pi x_2)$$

By adding in, for example, next-nearest neighbor antiferromagnetic interactions, we can "frustrate" the Neel order, and thereby achieve other phases.

A **dimer phase** occurs when each spin forms a spin singlet with a *single* neighboring spin.



Parenthetical high T_c comment.

(Spectroscopic studies of the normal state of the high temperature superconductors lead some to believe that there is separation of **charge** and **spin** degrees of freedom.

Orgad et al.

Dimer models provide simple toy models where the charge gap has been taken to infinity (and, therefore, effectively decoupled), while retaining the spin degrees of freedom.

In such a model, one then tries to find ground states of model Hamiltonians where the spin degrees of freedom (spinons) are deconfined.)

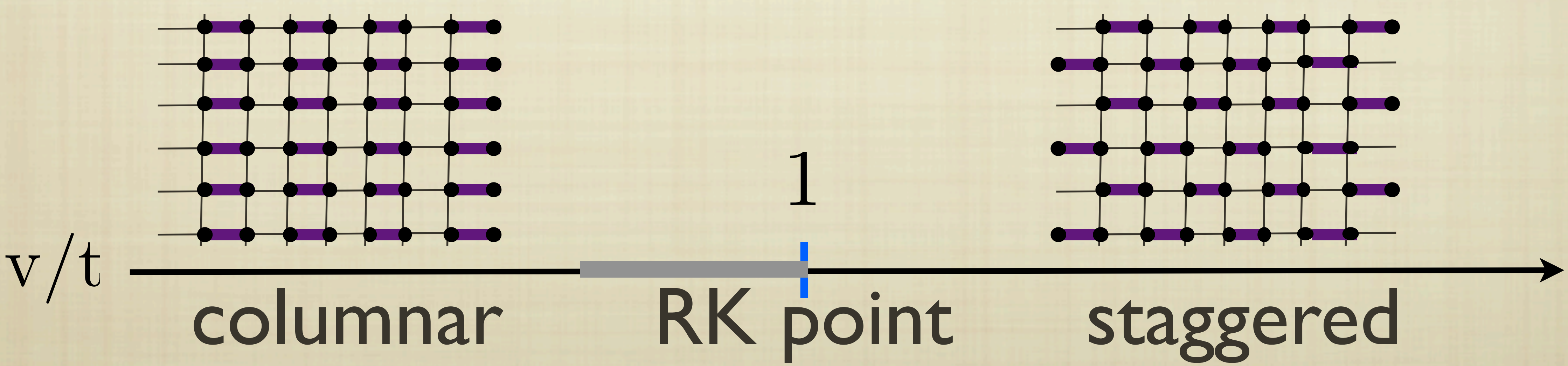
The **Rokhsar-Kivelson** (RK) Hamiltonian describes the dynamics of such a dimer phase,

Note: In writing down this Hamiltonian, we are vastly reducing the dimension of the Hilbert space: restricting only to dimer configurations.

$$H = \sum_{\square} -t(|\,\|\,\rangle\langle\,=\,| + \text{h.c.}) + \text{v}(|\,=\,\rangle\langle\,=\,| + |\,\|\,\rangle\langle\,\|\,|).$$

The kinetic (t) and potential (v) terms act either by flipping or as the identity on "flippable plaquettes", respectively. Otherwise they give eigenvalue zero.

As a function of v/t the phase diagram (roughly) looks like:



The phase existing in the gray area, , depends upon the lattice: its dimensionality, and whether or not it's bipartite.

For example:

- d=2 bipartite: plaquette ...
- d=2 non-bipartite (triangular): Z_2 RVB liquid ...

RVB = resonating valence bond

Moessner and Sondhi

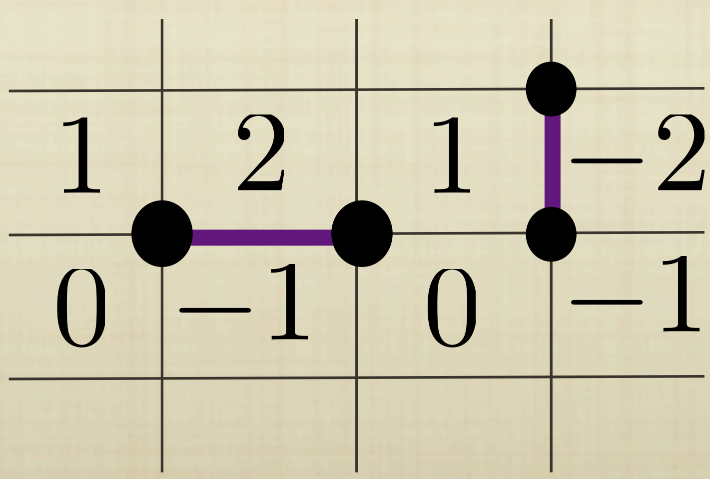
(There are a number of more subtle phases around the RK point that I don't mention.)

general references: Fradkin's book '91, reviews by Sachdev, Moessner and Raman '08

The continuum description of this model is the Lifshitz field theory. Here's the mapping:

Given any dimer configuration, we can associate a number, $h(x)$, or height to each plaquette:

1. choose a zero plaquette with height 0.
2. moving around an *even* vertex counter-clockwise, beginning at the zero plaquette (to the lower right of the vertex, for definiteness)
   (a) **increase** the height by 1 if **no** dimer is crossed
   (b) **decrease** the height by -3 if a dimer is crossed
3. moving around an *odd* vertex, change $1 \leftrightarrow -1$ and $-3 \leftrightarrow 3$ in 2. above.



Henley '97 discovered the long wavelength limit of this dimer/height model is described by the following (deformed) Lifshitz field theory.

$$L = \int d\tau d^2\mathbf{x} \left[(\partial_\tau h)^2 - (1 - \frac{v}{t})(\nabla_{\mathbf{x}} h)^2 - (\nabla_{\mathbf{x}}^2 h)^2 \right]$$

The Lifshitz field theory has the following properties:

- non-relativistic
- parity and time-reversal invariance
- spatial $SO(2)$ invariance

At the RK point, $v = t$

- $\tau \to \lambda^z \tau, \mathbf{x} \to \lambda \mathbf{x}$ scale invariance with dynamical critical exponent $z = 2$.
- $h \to h + f(z) + \bar{f}(\bar{z})$ "Kahler" invariance.

This Lifshitz theory captures a few gross properties of the RK model, e.g.:

1. critical for $v = t$

2. columnar phase: $v < t$, $\langle \nabla_{\mathbf{x}} h \rangle = 0$

3. staggered phase: $v > t$, $\langle \nabla_{\mathbf{x}} h \rangle \neq 0$

Further, the RK point is an example of so-called deconfined criticality where new degrees of freedom emerge or deconfine at the critical point.

Here, the deconfined degrees of freedom are **spinons**.

$$\int_{\partial A} \nabla h = \int_A \sigma \quad \Rightarrow \quad h(r) = \int_{r'} \sigma(r') \log(r - r')$$

where σ represents the vortex density.

Plugging this solution into energy functional, we find the vortices are free. (Compare this to the logarithmic interaction obtained with linear dispersion, $(\nabla h)^2$.)

$$E = \int (\nabla^2 h)^2 \sim \int_r \int_{r'} \sigma(r)\sigma(r')\delta(r - r')$$

Now the RK theory is supposed to be a toy model describing a certain phase of strongly correlated electrons.

And the derivation by Henley of the Lifshitz field theory is mean field theoretical in nature.

It is, therefore, natural to ask if there exist other weakly coupled descriptions of (strongly interacting) P and T preserving $z \neq 1$ theories.

Further, there exists a strongly interacting $SU(2)$ generalization of the Lifshitz field theory studied by Freedman, Nayak, and Shtengel.

In the remainder of the talk, I will introduce a candidate weakly curved gravitational description of strongly coupled anisotropic fixed points.

Because I will mostly focus on the case, $z = 2$, I will sometimes refer to these fixed points as Lifshitz-like. By this, I mean a $z = 2$, parity and time-reversal invariant field theory dual to our bulk gravitational system. In fact, our gravitational system will actually be valid for any $z \geq 1$.

Note: I do NOT mean that I am describing a dual to the Lifshitz field theory itself! Only strongly coupled versions with the same symmetries.

In the rest of (the bulk of) the talk I'll do the following.

1. Guess a metric
2. Provide some support for the guess
   a. 2-pt function
   b. RG flow

Note: I will focus on 3+1D bulk theories dual to 2+1D "boundary" theories.

(see M. Taylor's '08 paper on the generalization to other dimensions)

Geometry.

In the usual AdS_4/CFT_3 correspondence the $SO(3,2)$ conformal group of the field theory is geometrized by the AdS_4 space,

$$ds^2_{\mathrm{AdS}_4} = \frac{L^2(-dt^2 + dx^2 + dy^2 + du^2)}{u^2}.$$

Therefore, we guess the metric

$$ds^2_{\mathrm{LiF}} = L^2\left(-\frac{dt^2}{u^{2z}} + \frac{dx^2 + dy^2 + du^2}{u^2}\right).$$

It is invariant, e.g., under the anisotropic scaling,

$$t \to \lambda^z t, \mathbf{x} \to \lambda \mathbf{x}, u \to \lambda u.$$

In the well-studied AdS_5 example: $L^4 = 4\pi g_s N \alpha'^2$.

The classical gravity description is good when:

1. $g_s \ll 1$ (weakly coupled string theory) and
2. $N \gg 1$ (low curvature), where N^2 (roughly) corresponds to the number of degrees of freedom in the holographically dual field theory.

Also, the (dual)'t Hooft coupling, $\lambda = g_s N$. The above limit is taken while keeping λ large and fixed.

The u coordinate can be usefully thought to geometrize the RG scale of the holographic dual.

Note the following.

$$ds^2_{\rm LiF} = L^2(-\frac{dt^2}{u^{2z}} + \frac{dx^2 + dy^2 + du^2}{u^2}).$$

•From now on, the overall length scale in the geometry, L , will be set to unity.

•All curvature invariants are finite.

•There are, however, diverging tidal forces as one approaches the horizon $u \longrightarrow \infty$.

(see Horowitz and Ross '97 for a discussion about BHs with such properties)

•Any internal symmetries of the field theory are matched by the compact part of the metric.

I will not discuss any compact part. Instead, I'll focus on a truncation to 3+1 dimensions.

Sourcing the Geometry.

Consider the action:

$$S = \int d^4x\sqrt{g}(R - 2\Lambda) - \frac{1}{2}\int (F_{(2)} \wedge *F_{(2)} - F_{(3)} \wedge *F_{(3)}) - c\int B_{(2)} \wedge F_{(2)}.$$

The Lifshitz metric is a solution to the field equations derived from this action. The field equations also determine the fluxes, C.C. and the parameter c :

$$\begin{aligned}
\Lambda &= -\frac{z^2+z+4}{2}, \\
c^2 &= 2z, \\
F_{(2)} &= A\frac{du \wedge dt}{u^{z+1}}, \text{ with } A^2 = 2z(z-1), \\
F_{(3)} &= B\frac{du \wedge dx \wedge dy}{u^3}, \text{ with } B^2 = 4(z-1).
\end{aligned}$$

Alternatively, we can source the Lifshitz metric with a massive gauge field of mass, $m^2 = 2z$, by dualizing the 3-form flux and letting the scalar field Higgs the vector. The 2-form flux and C.C. remain the same.

I discuss the solution on the previous page in the remainder because of an holographic renormalization group flow that exists using the 2- and 3-form fluxes in an essential way.

From now on, I specialize to the case $z = 2$.

2-point Correlator.

Let's test our guess: couple a real scalar to the background metric. It obeys the equation,

$$\partial_u^2\phi - \frac{3}{u}\partial_u\phi + u^2\partial_\tau^2\phi + (\partial_x^2 + \partial_y^2)\phi - \frac{m^2}{u^2}\phi = 0.$$

Near the boundary, $u = 0$, ϕ takes the form

$$\phi \sim c_1 u^{\Delta_+}\phi_+(\tau, x, y) + c_2 u^{\Delta_-}\phi_-(\tau, x, y),$$

where $\Delta_\pm$, $\Delta_+ \geq \Delta_-$ satisfy

$$\Delta(\Delta - 4) = m^2.$$

m^2 is not allowed to take arbitrary values. A generalized Breitenlohner-Freedman bound requires $m^2 > -4$. In the remainder we set $m^2 = 0$.

Setting $m^2 = 0$ we see that ϕ should be seen as the fluctuation dual to a marginal operator $\mathcal{O}_\phi$ since marginal operators have dimension $z + d$ where d is the number of spatial dimensions.

Recall:

$$\int dt d^d \mathbf{x} \; \mathcal{O}(x) \rightarrow \lambda^{z+d-[\mathcal{O}(x)]} \int dt d^d \mathbf{x} \; \mathcal{O}(x)$$

under the anisotropic scale transformation.

Naively use the usual AdS/CFT prescription for calculating two-point correlation functions (at zero temperature):

1. solve the scalar E.O.M with boundary conditions
a. $\phi(0) = 1$ and b. $\lim_{u \to \infty} \phi(u) = 0$
2. plug this solution back into the action
3. differentiate this generating functional twice w.r.t. ϕ.

(The full power of holographic renormalization is necessary to get consistent normalizations of all the n-point functions. However, the "naive" procedure described above will be good enough for us.)

The result is:

$$\langle \mathcal{O}_\phi(-\mathbf{k},\omega)\mathcal{O}_\phi(\mathbf{k},\omega)\rangle = -\tfrac{1}{2}\mathbf{k}^2|\omega| - \tfrac{1}{8}(4\omega^2 - \mathbf{k}^4)\log(|\omega|) - \tfrac{1}{8}(4\omega^2 - \mathbf{k}^4)\psi(\tfrac{3}{2} + \tfrac{\mathbf{k}^2}{4|\omega|}).$$

Note:

•the correlator corresponds to a dimension four operator and for fixed time decays as $1/|\mathbf{x}|^8$.

•only the third term gives rise (upon Fourier transformation) to correlations between points with both spatial and temporal separation

•the first two terms contribute terms localized in space with long-range correlations in time. Such terms may be related to the ultra-local behavior discussed by Ghaemi, Vishwanath, and Senthil.

An RG Flow.

We found an holographic renormalization group flow between the Lifshitz-like spacetime and AdS_4 .

This is analogous to the flow generated by $(\nabla_{\mathbf{x}} h)^2$ in the Lifshitz field theory:

$$L = \int dt d^2\mathbf{x}[(\partial_t h)^2 - (\nabla^2_{\mathbf{x}} h)^2].$$

Perturbed by this operator, the Lifshitz field theory would flow towards the free Gaussian fixed point (and presumably hit the Wilson-Fisher one).

The flow is achieved by making the ansatz (after switching to r=1/u coordinates):

$$ds^2 = -r^4 f(r)^2 dt^2 + r^2(dx^2 + dy^2) + g(r)^2 dr^2/r^2$$

$$F_{(2)} \quad = \quad 2h(r)f(r)g(r)rdr \wedge dt, \quad \Lambda = -5,$$

$$F_{(3)} = 2j(r)g(r)rdr \wedge dx \wedge dy.$$

Plugging this ansatz into the EOMs, we are left with four independent equations. The first determines $f(r)$ in terms of $g(r), h(r), j(r)$. The three remaining equations form a closed set. We concentrate on these three equations in the remainder.

1. $$2rf'/f = (5 - h^2 + j^2)g^2 - 5$$

2. $$rg' = \frac{1}{2}g^3(h^2 + j^2 - 5) + \frac{3}{2}$$

3. $$rh' = 2gj - 2h$$

4. $$rj' = 2gh + \frac{1}{2}j + \frac{1}{2}jg^2(h^2 - j^2 - 5)$$

Before we (numerically) solve these equations, let's investigate the stability of the Lifshitz fixed point by performing a linearized fluctuation analysis (within the above ansatz).

The resulting linear equations take the general form:

$$\frac{d\mathbf{v}}{d\log(r)} = \underline{M}\mathbf{v}, \quad \mathbf{v}^T = \begin{pmatrix} g(r) & h(r) & j(r) \end{pmatrix}.$$

The eigenvalues of the matrix $\underline{M}$ determine the relevancy of the operators dual to the fluctuations represented by $g(r), h(r), j(r)$.

It was found that the Lifshitz fixed point has two **relevant** directions and one **marginal** direction.

Now imagine perturbing the Lifshitz fixed point by the relevant operators found in the previous slide.

Using the intuition we mentioned from the free field theory example, we can consider a flow to an AdS_4 spacetime.

Indeed, by numerically solving the nonlinear ODEs, we can find such an holographic RG flow.

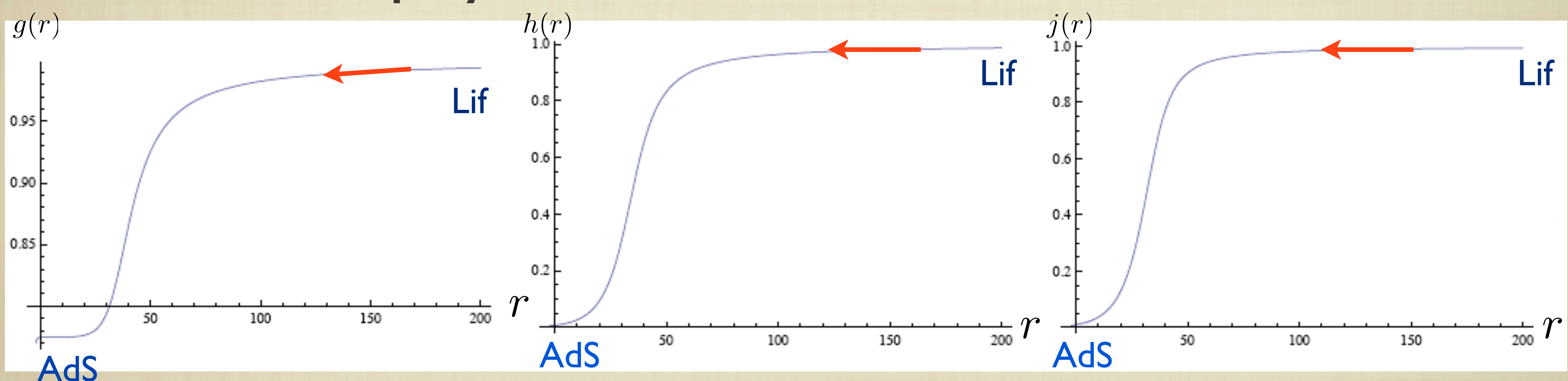
The Lifshitz-like spacetime corresponds to

$$f(r) = g(r) = h(r) = j(r) = 1,$$

while AdS_4 corresponds to

$$f(r) \sim 1/r; \quad g(r) = \text{nonzero constants};$$
$$h(r) = j(r) = 0.$$

Below I display the RG flow of $g(r), h(r), j(r)$.



$f(r)$ flows correctly to $1/r$ as well (but I don't show this).

Summary (so far):

- P,T invariant critical points with $z \neq 1$ occur in many condensed matter systems.
- We provided candidate weakly coupled duals of strongly interacting theories with such symmetries.
- Specializing to the $z = 2$ case, we tested (did consistency checks on) our guess by computing a two-point correlation function and finding a RG flow to AdS_4 space.

There are a number of directions for future work.

First, one could put the theory at finite temperature.

Second, one could study other field theoretic models with $z \neq 1$.

Third, it's interesting to ask how "(meta-)observables" like the entanglement entropy change when one considers field theories with anisotropic scale invariance.

Fourth, it's important to find microscopic realizations of the bulk gravity solutions.

I would like to briefly describe work in these directions.

1.Finite Temperature Solutions.

Ghaemi, Vishwanath, and Senthil studied the Lifshitz field theory at finite temperature, T. They found that two-point correlators of the local operators of the theory $e^{2\pi i h(x)}$ displayed a certain type of ultra-local behavior at equal times.

$$\langle e^{2\pi i h(0,\mathbf{x})} e^{-2\pi i h(0,\mathbf{0})} \rangle \sim \delta(\mathbf{x})$$

in the thermodynamic limit. Long-ranged correlations are restored by moving away from the scaling limit by including the leading irrelevant operators. It would be interesting to find such behavior on the gravity side. Finite T tranport properties would also be interesting to study.

Recently, Danielsson and Thorlacius, Mann, and Peet et al. made progress towards understanding gravity duals of field theories with $z = 2$ scaling.

Among many things, DT numerically found a black hole with Lifshitz-like asymptotics and took its temperature. Mann generalized the work of DT to black holes with horizon topologies of higher genera. Peet et al. studied solutions with $z \neq 1$.

It would be nice to find an analytic solution (for *flat* horizon topologies). And it would also be interesting to see if any ultra-locality existed in any correlators.

2. An $SU(N)$ Generalization of the Abelian Lifshitz Field Theory.

see also Horava '08, and Das & Murthy '09 for related work

By dualizing the scalar field into a vector in 2+1 D, the abelian Lifshitz theory takes the form,

$$H = \int d\tau d^2\mathbf{x}[(\partial_i E_j)^2 + B^2].$$

Freedman, Nayak, and Shtengel (FNS) studied the following $z = 2$ $SU(2)$ generalization:

$$H = \frac{1}{g^2}\int d\tau d^2\mathbf{x}[E_i\partial_\tau A_i + A_0 D_i E_i - \frac{1}{2}(D_i E_j)^2 + \frac{1}{2}B^2 + ...]$$

where the ellipses represent quartic E^4 and $E_i E_j F_{ij}$ terms. This action is invariant under,

$$\mathbf{x} \to \lambda\mathbf{x}, \quad \tau \to \lambda^2\tau, \quad E_i \to \lambda^{-1}E_i, \quad A_i \to \lambda^{-1}A_i, \quad A_0 \to \lambda^{-2}A_0.$$

There are a number of reasons to be interested in this model.

A. Study the properties of a SU(N) generalization.

B. Is this model critical as is? If not, we could consider looking for the analogue of Banks-Zaks fixed points by adding in appropriate matter fields

C. Notice that a Chern-Simons term, $A \wedge dA + 2/3 A^3$, is also **marginal** in this theory. This means that the Chern-Simons term and magnetic term B^2 can compete. This is to be contrasted with what usually happens in pure 3D gauge theories.

D.Further, study of this theory could be useful in understanding the holographic duals to the gravitational systems described in this talk.

There is a more general reason to be interested in the FNS + CS theory.

Topologically ordered systems are described by a topological field theory in the deep IR.

The leading irrelevant operators describe the departure away from the scaling limit.

The ‘regular’ CS theory is a good example of the above situation; the gauge field kinetic term is the leading irrelevant operator.

Because the gauge kinetic term is marginal in a 3d theory with modified scaling, it is interesting to ask how the above picture changes.

$$H = \int d\tau d^2\mathbf{x}\mathrm{Tr}[\frac{1}{g_1^2}(E_i\partial_\tau A_i + A_0 D_i E_i) - \frac{1}{2g_2^2}(D_i E_j)^2 + \frac{1}{2g_3^2}B^2]$$

Let me summarize our current understanding of this theory. To one-loop, using background field gauge, we find the following beta functions

$$\partial_t g_1 \sim -\frac{g_1^3 g_3}{g_2} \qquad \partial_t g_2 \sim \pm g_1^2 g_3 \qquad \partial_t g_3 \sim \frac{g_1^2 g_3^2}{g_2}$$

where $\partial_t := \dfrac{\partial}{\partial \log(\ell)}$.

So the first coupling is IR free, imposing the Gauss' law constraint during the flow to the IR, while the third coupling is UV free. The flow of the second coupling is work in progress.

3.Entanglement Entropy

Entanglement entropy has been used in quantum information studies. This quantity measures the non-local correlations of the ground state wavefunction. (Besides black holes) it has been applied to quantum critical systems and systems in a topological phase. The entanglement entropy can be thought of as a quantum number partially characterizing the phase of the system.

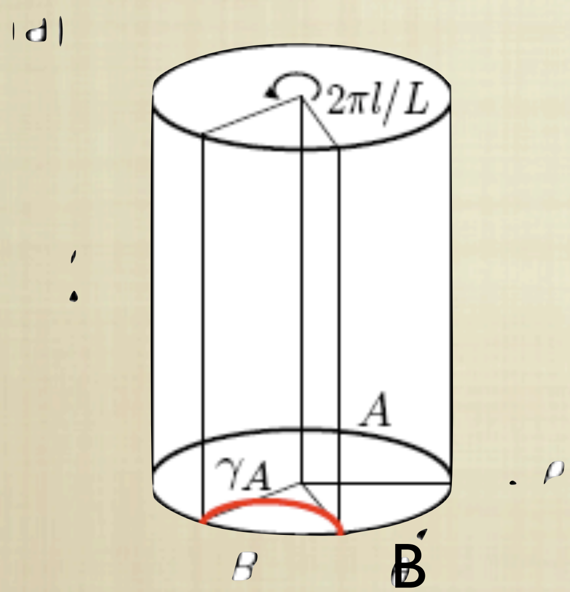
Ryu and Takayanagi (RT) have given a prescription for calculating the entanglement entropy for theories with a classical gravitational dual.

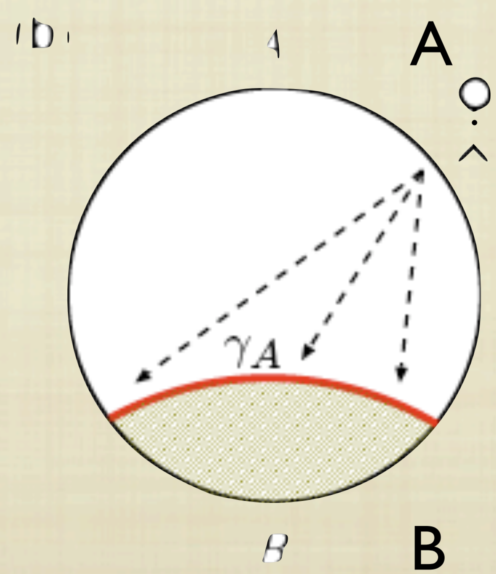
In quantum field theory, the entanglement entropy is calculated in the following way.

- Take the system to be in the pure state $|\Psi\rangle$.
- Form the density matrix $|\Psi\rangle\langle\Psi|$.
- Partition the system into two subsystems A, B.
- Form the reduced density matrix $\rho_A = \mathrm{Tr}_B|\Psi\rangle\langle\Psi|$ by performing a partial trace over the degrees of freedom in region B.
- The entanglement entropy is now identified by the von Neumann entropy $\mathrm{Tr}_A\left[\rho_A \log(\rho_A)\right]$ associated to the mixed state described by the reduced density matrix ρ_A.

(Luckily, we don't need to know/remember any of the above to actually use the RT holographic method.)

We wish to calculate the entanglement between two regions A,B on a constant time surface in the boundary. The prescription of RT is that the entanglement entropy is simply the area divided by $4G_N$ of the minimal surface extending into the bulk which ends on the boundary between A and B.

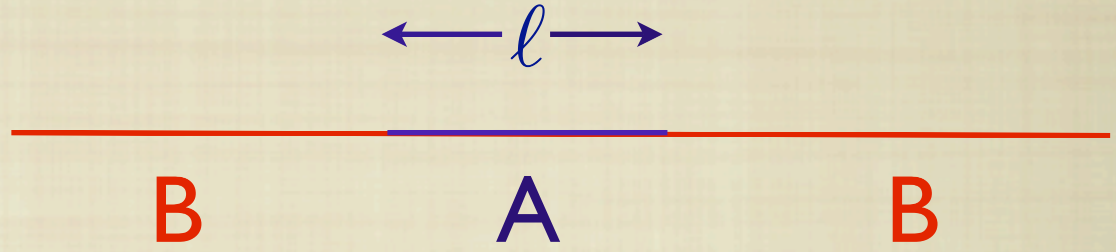




$$\text{Area} = \int \sqrt{\text{Det}(g_{\gamma_A})}$$

The scaling of the entanglement entropy is best understood in 1+1 D CFTs:

$$S_{\text{entanglement}} = \frac{c}{3}\log(\frac{\ell}{a}),$$



where ℓ is the size of region A, a is an UV cutoff, and $c\ (= 3L/2G_N)$ is the central charge. This result is consistent with the gravity calculations.

Using the prescription given by RT for the dual to a 2+1 D spacetime with dispersion $\omega \sim k^z$, the coefficient of the log is modified in a simple way consistent with the anisotropy,

$$S_{\text{anisotropic entanglement}} = \frac{Lz}{2G_N}\log(\frac{\ell}{a}).$$

4. UV Embedding.

There has been some recent progress in finding anisotropic spacetimes from string theoretic UV completions.

Azeyanagi, Li, and Takayanagi (ALT) '09

ALT found the following solution from IIB SUGRA:

$$ds^2 = \tilde{R}^2(r^3(-dt^2 + dx^2 + dy^2) + r^2 dw^2 + \frac{dr^2}{r^2}) + R^2 ds^2_{X_5}$$

$$\tilde{R}^2 = \frac{11}{12} R^2$$

X_5 - Einstein 5-manifold

This geometry is sourced by a:

1. Cosmological Constant

and the two fields,

2. Axion
3. Dilaton.

A microscopic D3-D7 system motivates the above solution.

The idea is that this solution is dual to a fixed point with $z = 3/2$ invariance in 3+1 dimensions.

However, the dilaton and axion break the scaling symmetry.

The dilaton $e^{\phi} \sim r^{2/3}$ shifts by an arbitrary constant in any scale transformation. Approaching the boundary $r \to \infty$ any shift becomes unimportant. In the regime of small r, where string theoretic corrections can be neglected, it is the arbitrarily large N which protects this scaling symmetry.

The objection is more serious for the axion $\chi \sim w$. This explicitly violates the scaling symmetry.

Nevertheless, ALT's construction is instructive and it would be interesting to learn how the scale non-invariance of the dilaton and axion fields manifests itself in correlators or other observables of the dual field theory.

For example, one could imagine the following relation in the boundary theory between the trace of the stress tensor and operators dual to fluctuations of the dilaton and axion fields:

$$T^{\mu}_{\mu} = \sum_{a} \beta_a \mathcal{O}_a .$$

Other future directions.

•We studied a scalar probe of the Lifshitz spacetime. One can imagine studying other types of fluctuations. Look for possible phase transitions.

•"Holographically understand" the fluctuations about the Lifshitz-like spacetime. Such fluctuations can be mapped to operators in the holographic dual.

•Study transport properties by "phenomenologically" incorporating a bulk gauge field into the spacetime, and computing correlators.

•Provide a string theoretic embedding of an/the anisotropic spacetime.