Fermions at Criticality;
heavy fermion and critical spin liquid

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Why interesting?

- Fermi surface is the most generic state of fermionic matter
- Infinitely many gapless modes : very different from relativistic QFT
- Critical states serve as unifying platforms to understand many other `ordered' phases
- New state of matter : non-Fermi liquid
- Many experiments yet to be understood

Outline

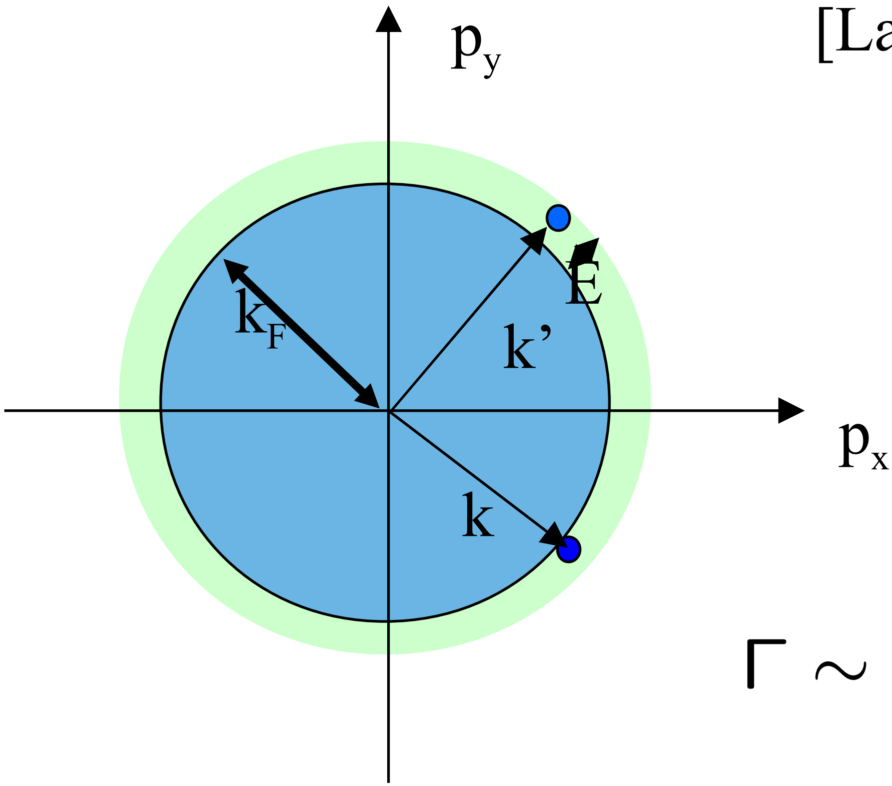
- Fermi Liquid Theory : the Standard Theory of Metal
- Non Fermi Liquid at critical point : Heavy Fermion
  - Single impurity model
  - Kondo lattice model
  - Heavy Fermi Liquid
  - Two Magnetic Phases
  - Phase Transitions from Fermi Liquid to Magnetic Phases
- Non Fermi Liquid Phase
  - Fermi Surface Coupled with Gauge Field
- Open Theoretical Problems

General References

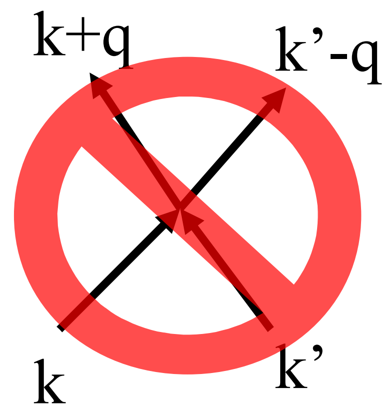
- Heavy fermion :
  - P. Coleman, arXiv:cond-mat/0612006
  - Lohneysen et al., Rev. Mod. Phys. **79**, 1015 (2007)
  - T. Senthil, cond-mat/0604250

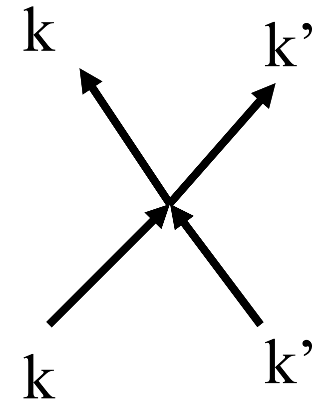
- Spin liquid :
  - P. Lee, N. Nagaosa and Xiao-Gang Wen, Rev. Mod. Phys. **78**, 17 (2006)

Fermi Liquid



[Landau] [Shankar,Polchinski]





$$\Gamma \sim E^2$$

At low energies, only forward scatterings are important (generically)

$\rightarrow$ Quasiparticles have long life time

$\rightarrow$ electron has a finite over-lap with quasiparticle $Z > 0$

adiabatically connected to non-interacting Fermi gas

Emergent symmetry in Fermi Liquid

[Haldane]

$$H = \sum_k \epsilon_k c_k^\dagger c_k + \sum_{k,k',q} V_q c_{k+q}^\dagger c_{k'-q}^\dagger c_{k'} c_k$$

Low energy theory :

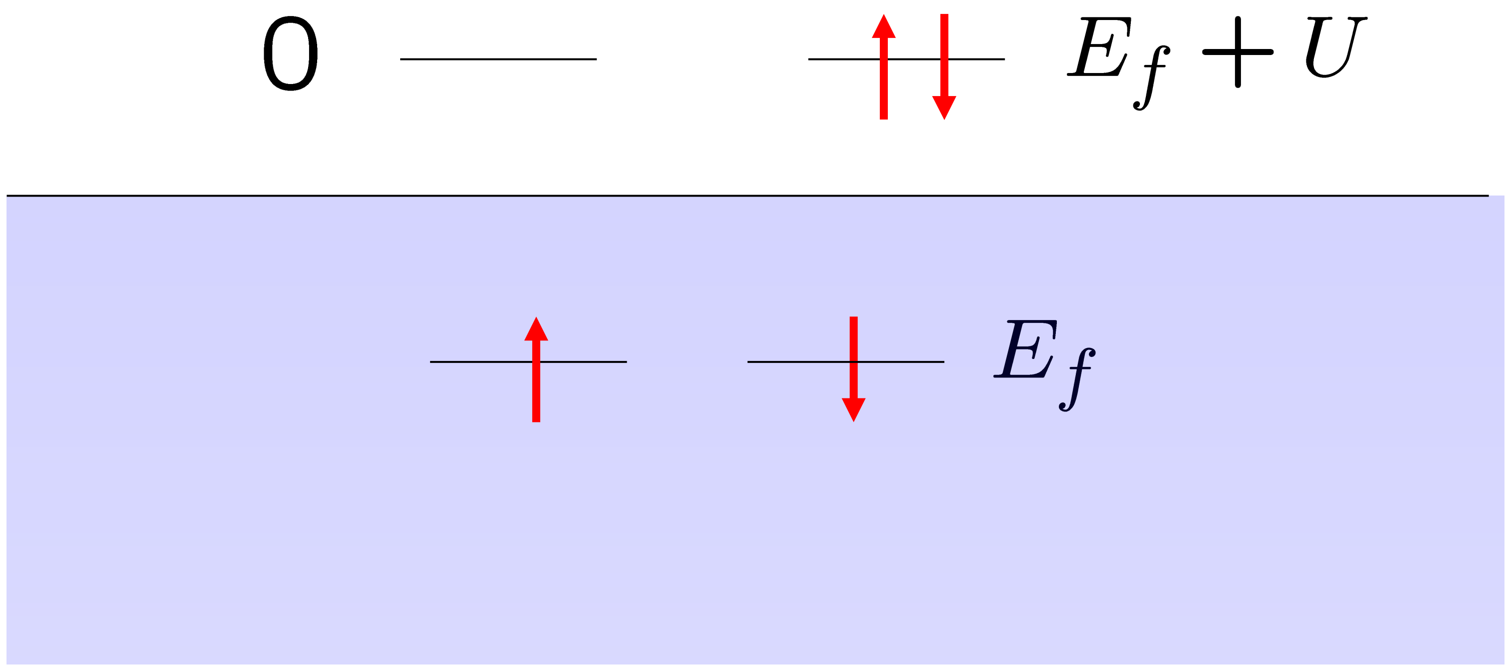
$$H_{eff} = \sum_k \epsilon_k c_k^\dagger c_k + \sum_{k,k'} f(k,k')(c_k^\dagger c_k)(c_{k'}^\dagger c_{k'})$$

Infinite number of emergent U(1) symmetries : $c_k \longrightarrow e^{i\theta_k} c_k$

Single impurity in metal

Anderson model

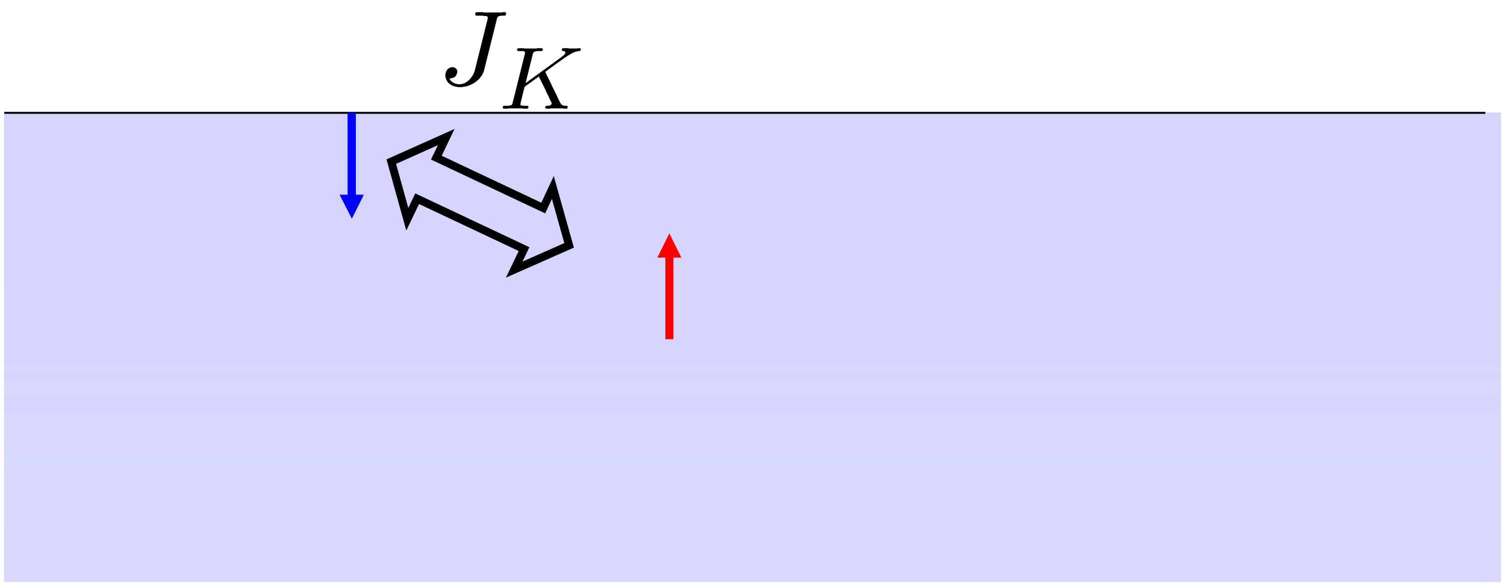
$$H = \sum_{k,\sigma} \epsilon_{k\sigma} c^{\dagger}_{k\sigma} c_k + \sum_{\sigma} V (c^{\dagger}_{0\sigma} f_{\sigma} + h.c.) + E_f n_f + U n_{f\uparrow} n_{f\downarrow}$$



Single impurity in metal

Low energy effective model : Kondo impurity model

$$H = \sum_{k,\sigma} \epsilon_{k\sigma} c^{\dagger}_{k\sigma} c_k + J_K \vec{s}_0 \cdot \vec{S}$$



Single impurity in metal

Low energy effective model : Kondo model

$$H = \sum_{k,\sigma} \epsilon_{k\sigma} c^\dagger_{k\sigma} c_k + J_K \vec{s}_0 \cdot \vec{S}$$

Kondo coupling logarithmically grows at IR :

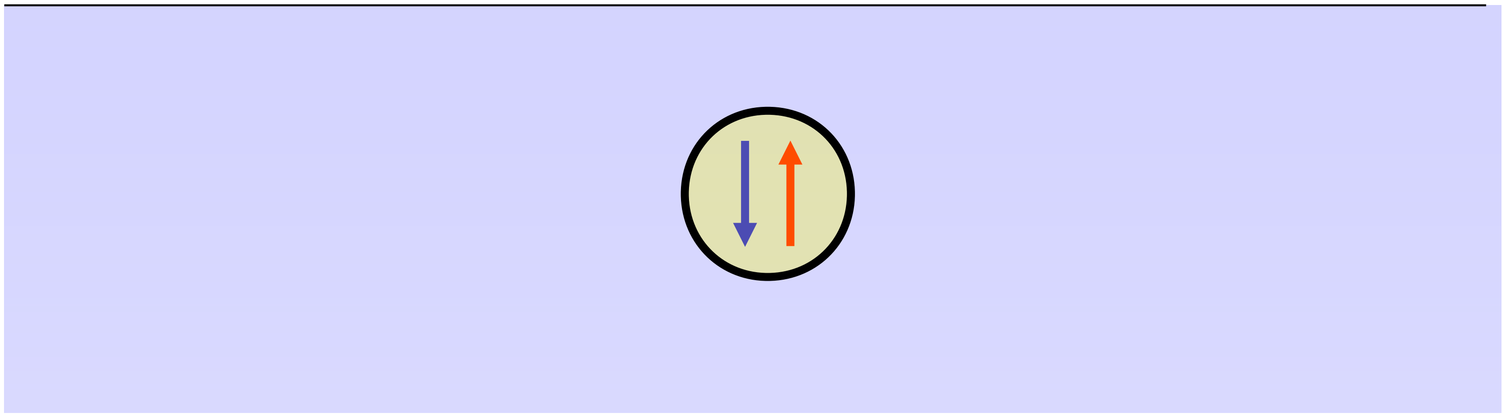
$$\frac{\partial J_K}{\partial ln\Lambda} = -2J_K^2$$

At low energy, the impurity spin is screened : Kondo singlet

Cross-over temperature :

$$T_K \sim Dexp(-\frac{1}{2J_K\rho})$$

Local Fermi liquid



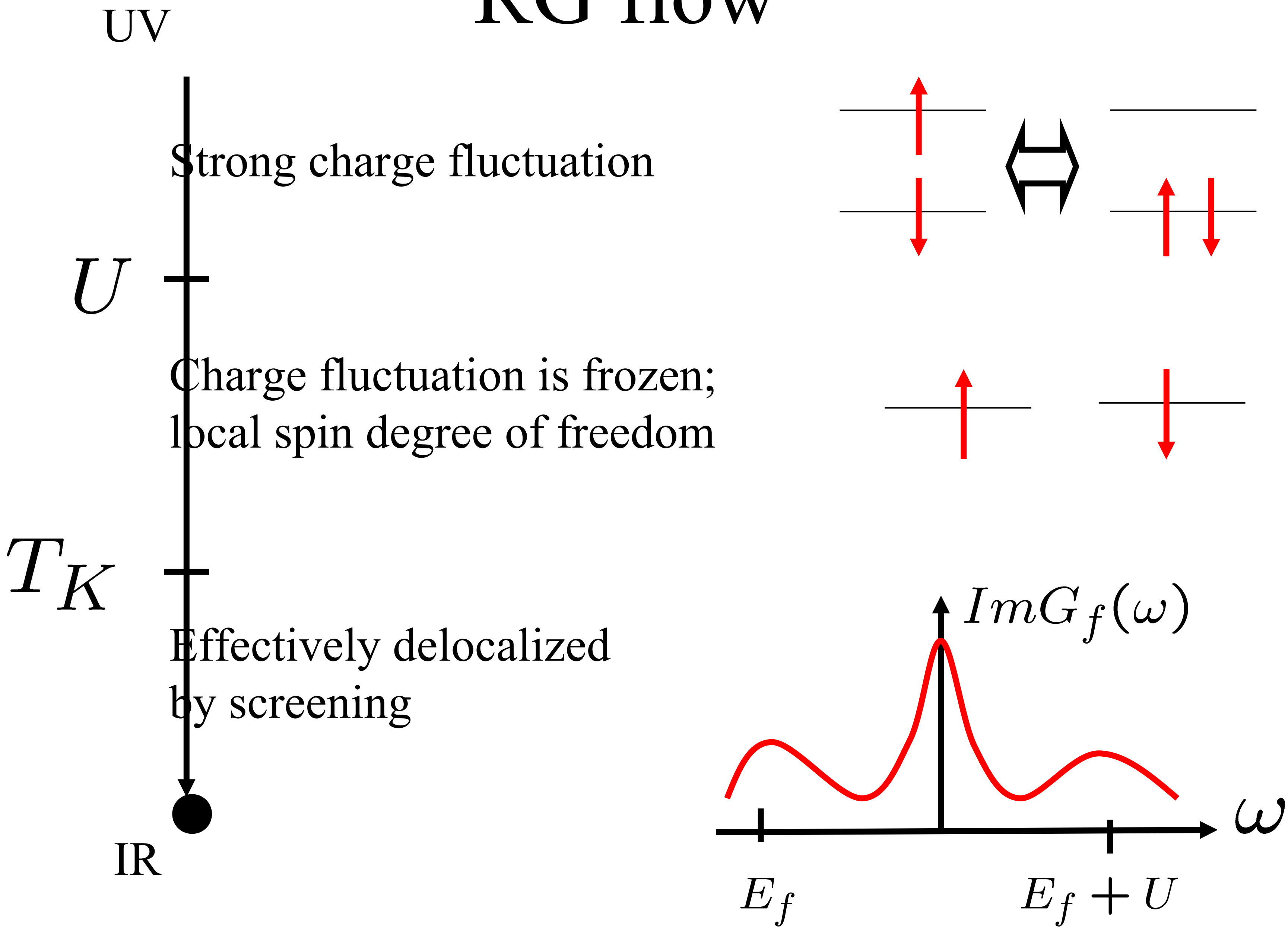
Non-magnetic scattering center

Adiabatically connected to local Fermi liquid :

$$H = \sum_{k,\sigma} \epsilon_{k\sigma} c^\dagger_{k\sigma} c_k + \sum_{\sigma} V(c^\dagger_{0\sigma} f_\sigma + h.c.)$$

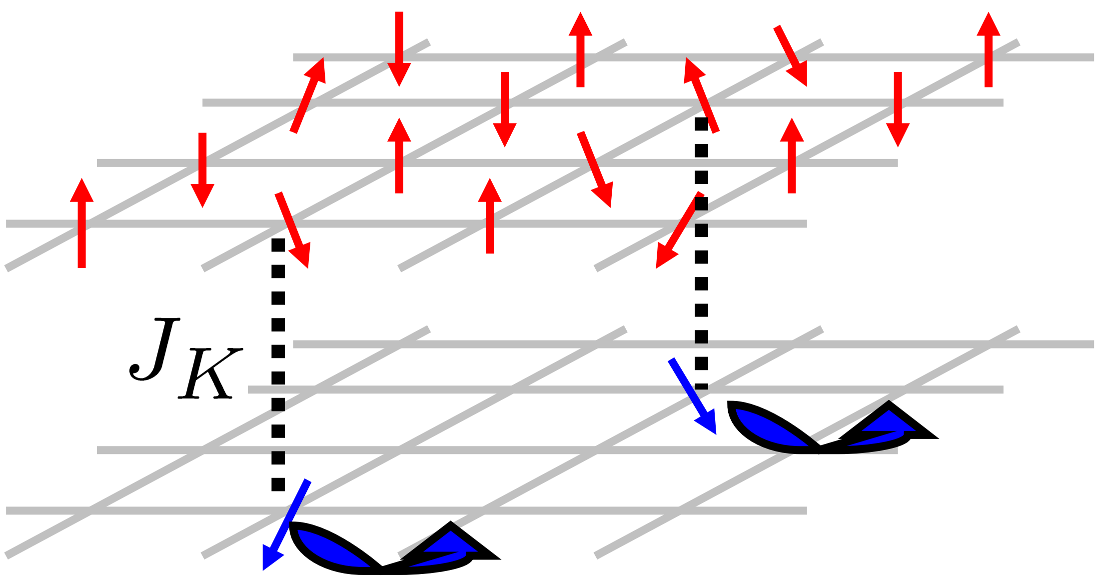
Local spin is absorbed into the conduction band and get delocalized.

RG flow


UV
Strong charge fluctuation
U
Charge fluctuation is frozen;
local spin degree of freedom
T_K
Effectively delocalized
by screening
IR
$ImG_f(\omega)$
ω
E_f
$E_f + U$

Kondo lattice model : the model for heavy fermion systems

$$H = \sum_{k,\sigma} \epsilon_{k\sigma} c^\dagger_{k\sigma} c_k + J_K \sum_r \vec{S}_r \cdot c^\dagger_{r\alpha} \vec{\sigma}_{\alpha\beta} c_{r\beta}$$

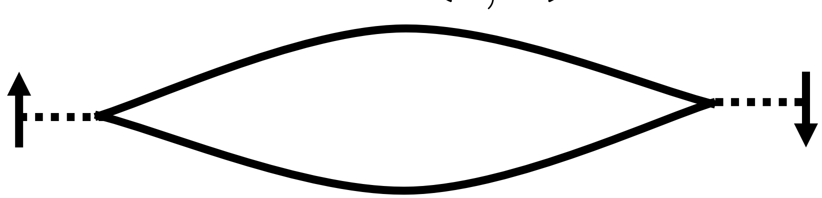


Kondo lattice model : the model for heavy fermion systems

$$H = \sum_{k,\sigma} \epsilon_{k\sigma} c^{\dagger}_{k\sigma} c_k + J_K \sum_r \vec{S}_r \cdot c^{\dagger}_{r\alpha} \vec{\sigma}_{\alpha\beta} c_{r\beta}$$

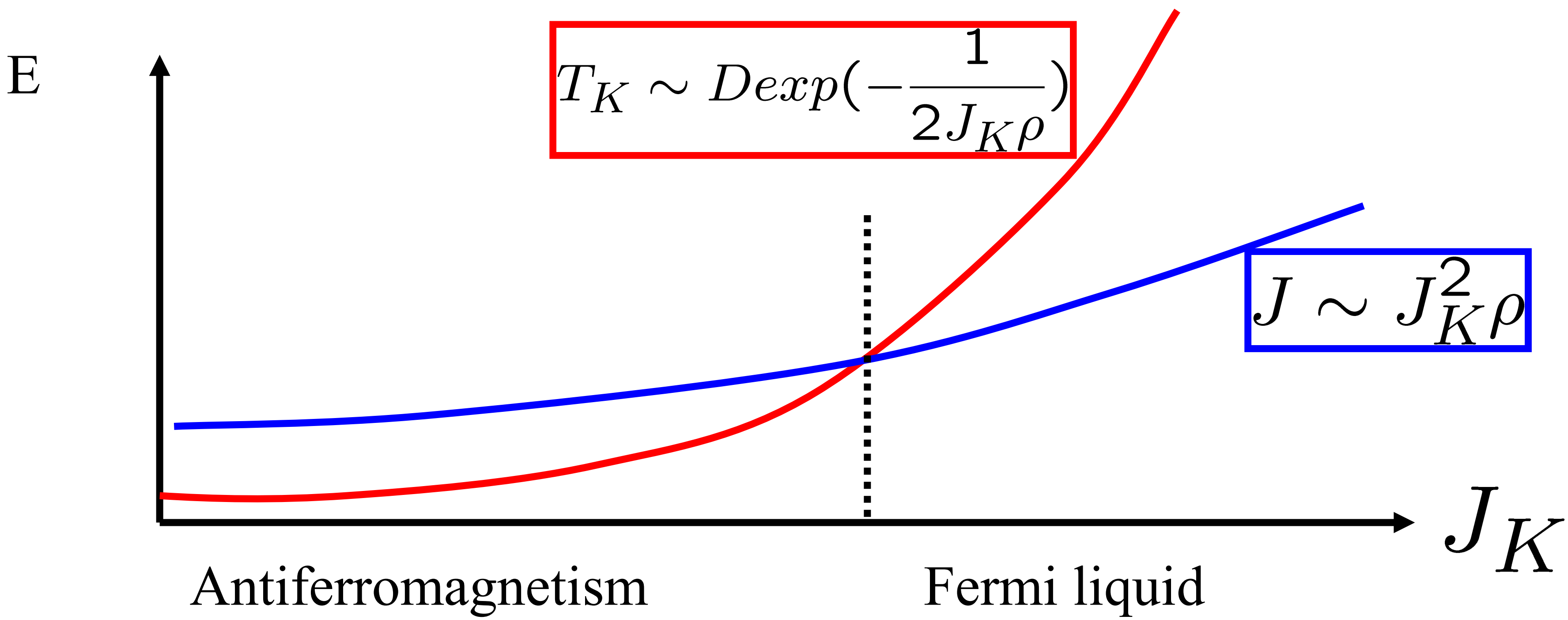
- Two competing factors
  - Kondo screening $T_K \sim Dexp(-\frac{1}{2J_K\rho})$
  - RKKY interaction : spin-spin interaction medicated by conduction electrons

$$H_{RKKY} = \sum_{<r,r'>} J(r-r')\vec{S}_r \cdot \vec{S}_{r'}$$

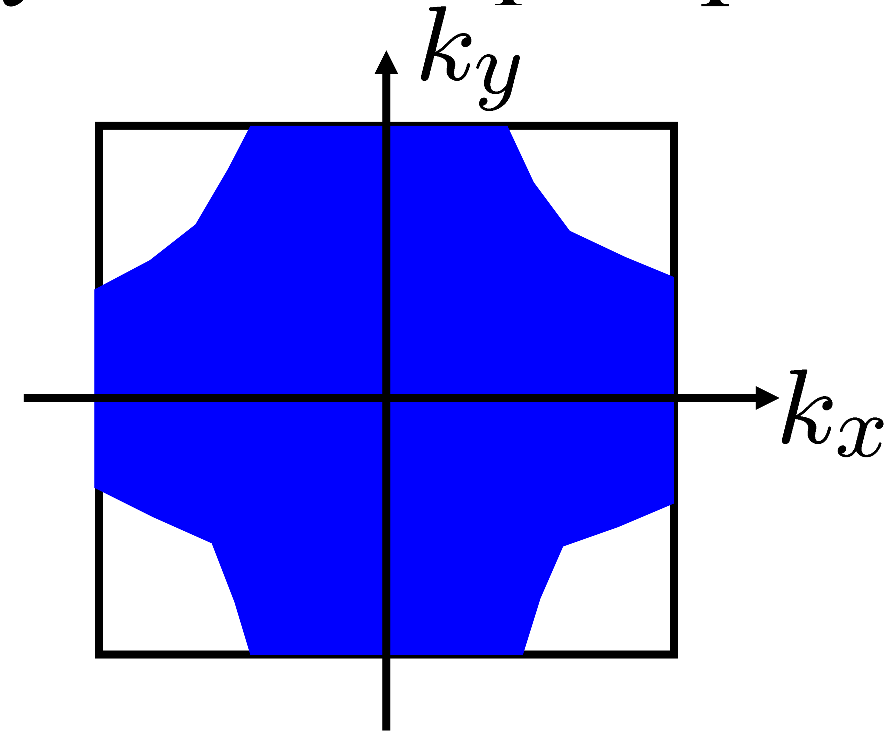


$$J \sim J_K^2 \rho$$

Doniach diagram



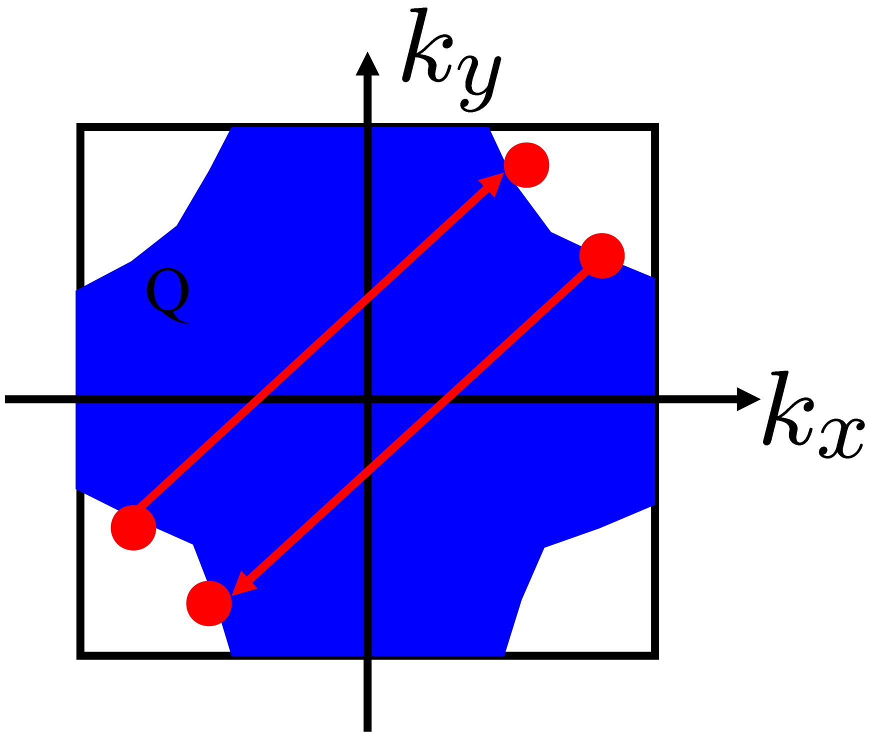
Heavy Fermi liquid phase



- Local moments are dissolved into a large FS
- Quasiparticle is mixture of conduction electron and f-electrons : it becomes heavy, but obey FL description

$$V_{FS} = n_c + n_f$$

Magnetic phase with a large FS (SDW)



- RKKY interaction mediates scatterings between quasiparticles, leading to a SDW instability
- Major part of FS remains intact

Phase transition from HFL to SDW

- Only the magnetic order undergoes a substantial change across the transition
- Integrate out fermions to obtain LGW effective action for the order parameter [Hertz, Millis]

$$s[\phi] = \int d\omega d^d k \left[\delta + k^2 + \frac{|\omega|}{\gamma(k)} \right] |\phi(k,\omega)|^2 + \ldots$$

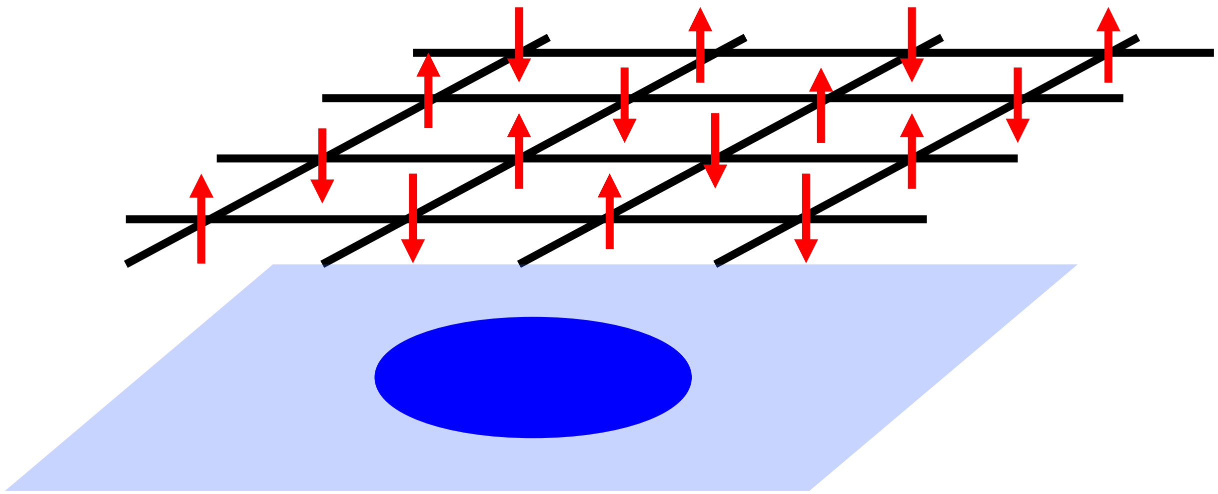
For AF, $\gamma(k) = \gamma_0 \quad \rightarrow \quad z = 2$

$$d_{eff} = d + z > d_c = 4, \text{ for } d = 3$$

Phase transition from HFL to SDW

- For d=3, the critical theory is Gaussian (mean-field-like transition)
- (Dangerously) irrelevant operator becomes important for some physical response → break down of ω/T scaling for spin susceptibility
- FS reconstruction is not drastic → continuous Hall conductivity, quantum oscillation across the phase transition

Local moment magnet (LMM)

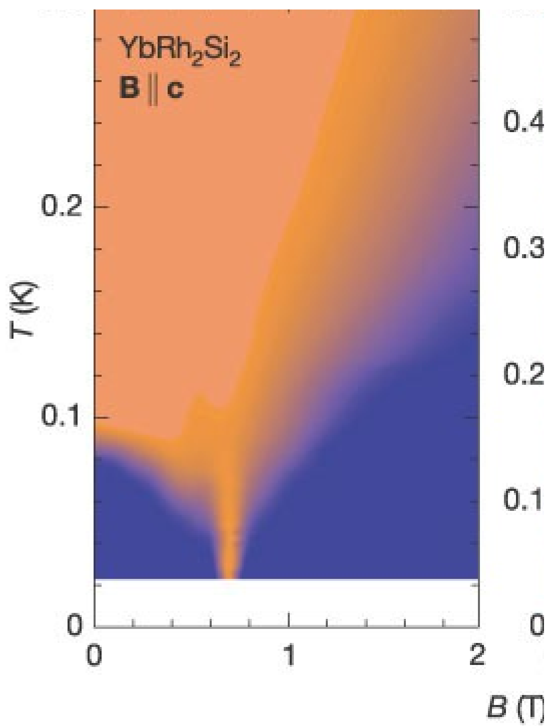


- Local moments are decoupled from conduction electrons, and form a magnetic order
- Conduction electrons form a small FS

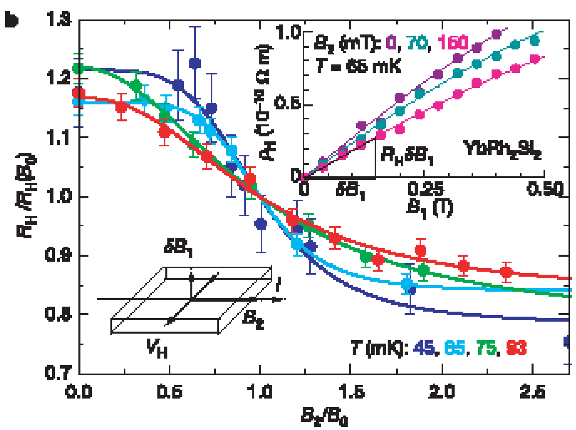
Phase transition from HFL to LMM (if there is a direct continuous transition)

- The Kondo screening should be lost concomitantly as magnetic fluctuations become critical
- Multiple diverging scales at QCP
- FS reconstruction is drastic → discontinous Hall conductivity, quantum oscillation across the phase transition (this is allowed in continuous transition if $Z \to 0$)
- NFL state at QCP
- No satisfactory theory available yet!

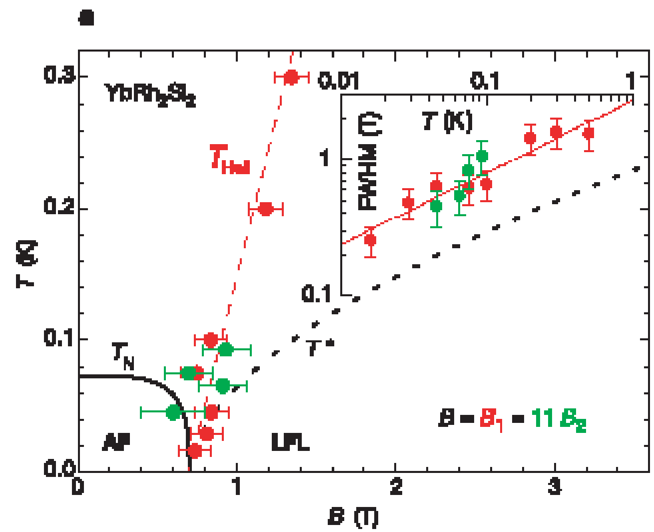
Experimental evidences for direct 2nd order transitions from HFL to LMM



$$\rho \sim T$$



Discontinuous
Hall resistivity



Crossover energy scale
associated with
FS reconstruction
Vanishes at QCP

[Custers et al.(2003); Paschen et al. (2004)]

Experimental evidences for direct 2^{nd} order transitions from HFL to LMM

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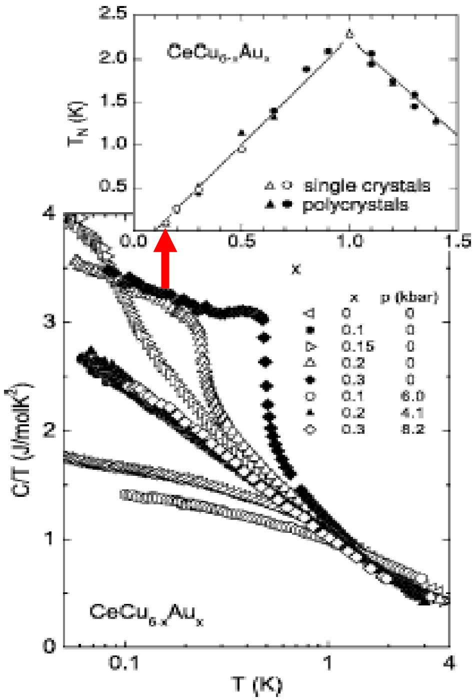

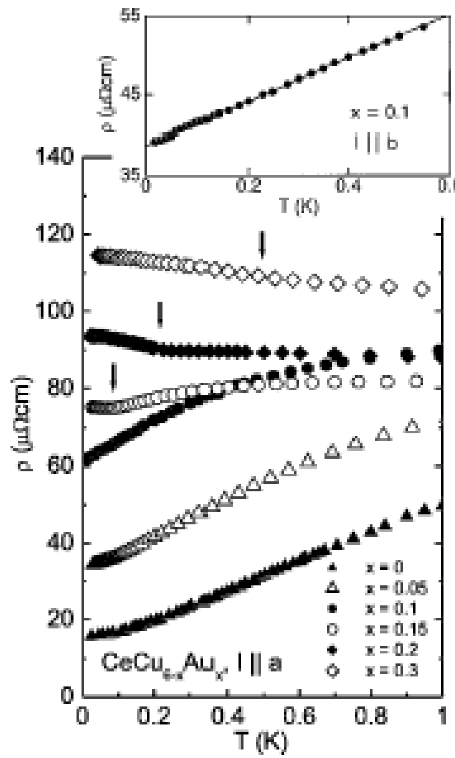

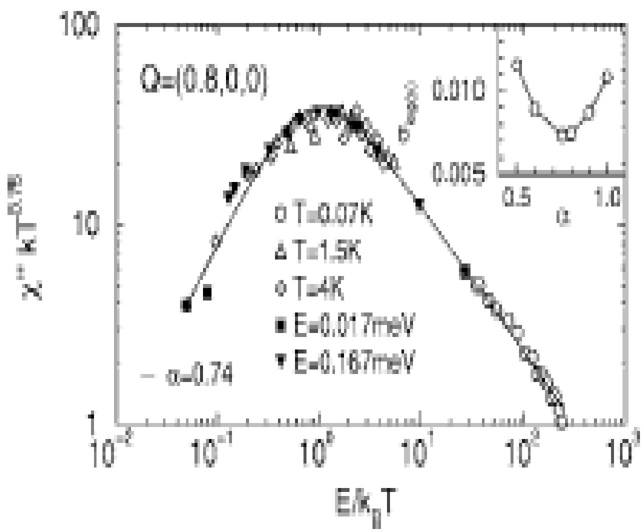

$$T^{3/4}\chi''(\omega/T)$$

$$\rho \sim T$$

$$C/T \sim ln1/T$$

[Lohneysen et al (96); Lohneysen and Mock et al(98); Schroder et al(98)]

Experimental evidences for direct 2nd order transitions from HFL to LMM

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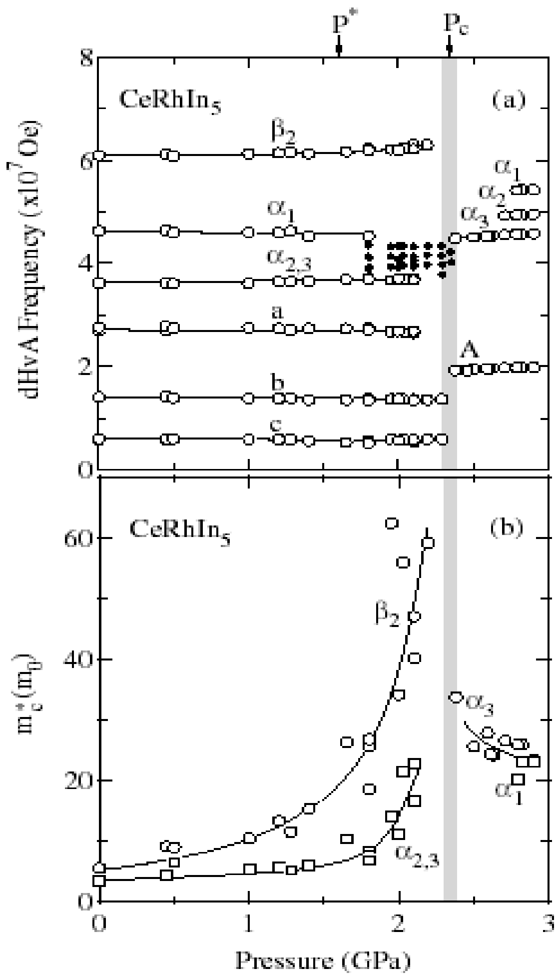


Discontinuous quantum oscillation : FS reconstruction

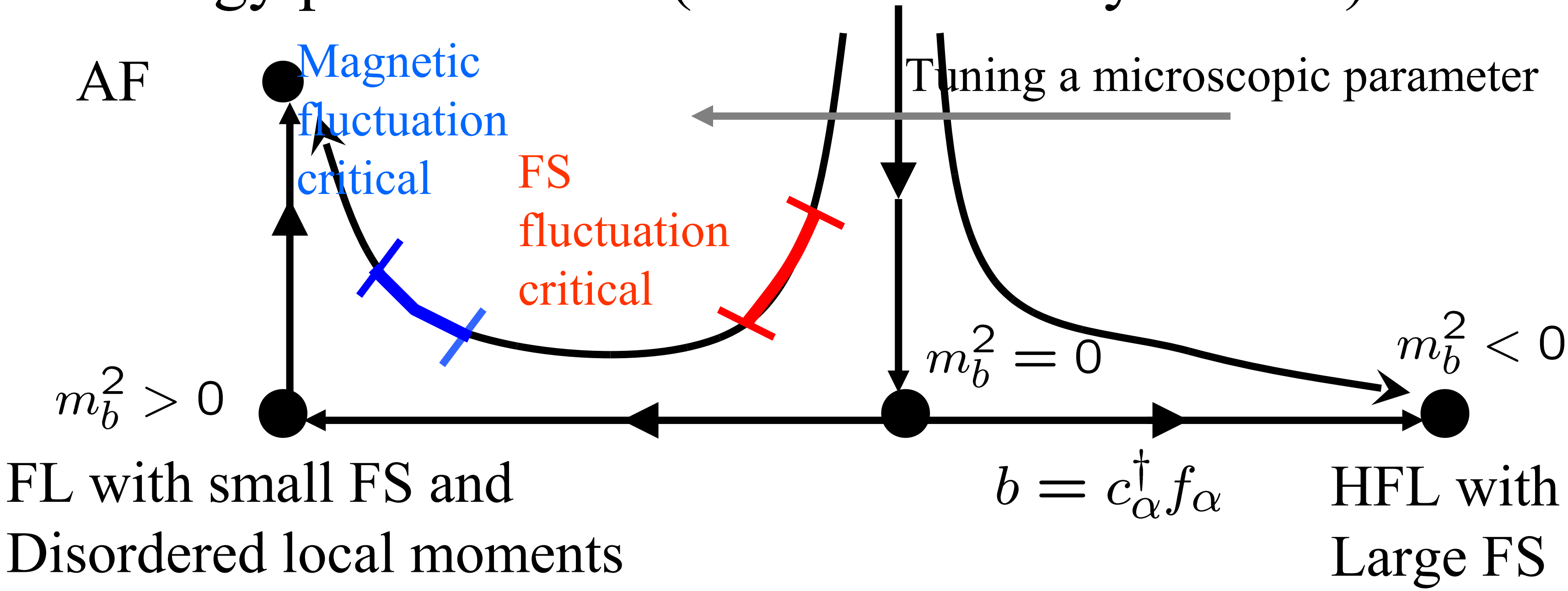
Diverging effective mass : vanishing Z at QCP

[Shishido et al. (2005)]

Two diverging time (length) scales

[Si, Robello, Ingersent, Smith(2001); Senthil, Sachdev, Vojta (2003)]

- Regard magnetic instability as a secondary & lower energy phenomenon that follows from the destruction of the Kondo screening which is the primary & high energy phenomenon (or the other way around)

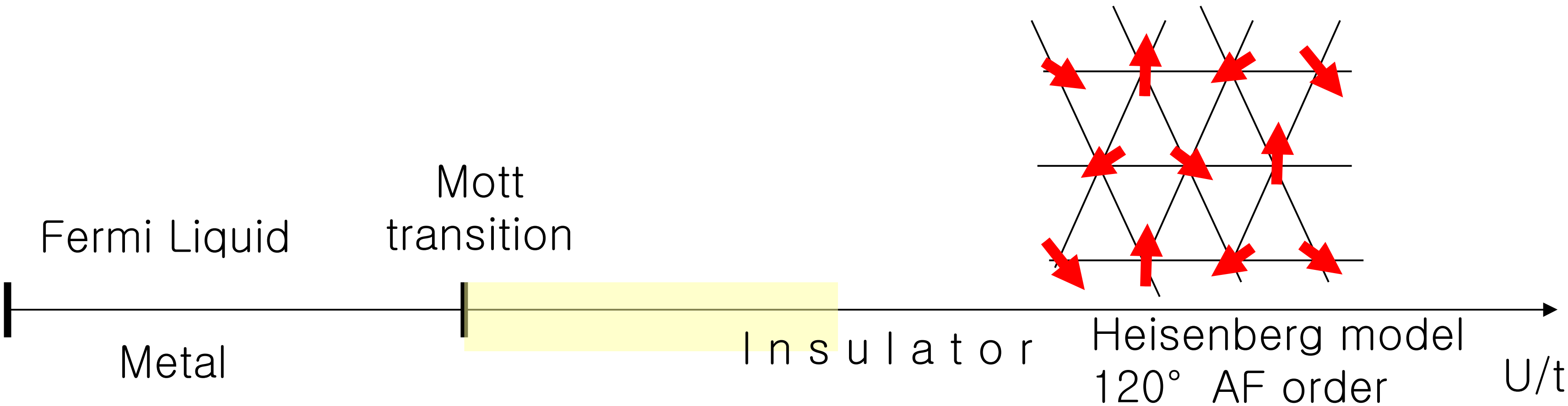


There is no concrete field theory that realizes this idea yet.

Non-Fermi liquid phase

Hubbard Model

$$H = -t \sum_{<i,j>} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i n_i$$



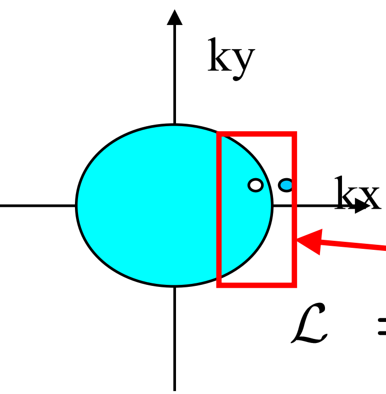
$$H_{eff} = J \sum_{<i,j>} S_i \cdot S_j + \dots.$$

Charge fluctuations / geometrical frustration may disrupt spins from ordering even at T=0 near the metal-insulator transition.

Spinon FS coupled with U(1) gauge field in 2+1D

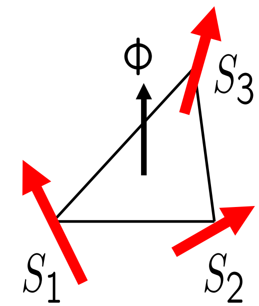
$\vec{S}_r = f^\dagger_{r\alpha}\vec{\sigma}_{\alpha\beta}f_{r\beta}$ Spinon : EM charge 0, spin 1/2

Gauge redundancy : $f_{r\alpha} \longrightarrow f_{r\alpha}e^{i\theta_r}$



Spinon
Fermi surface

Emergent U(1) gauge field



$\nabla \times a \sim \sin \Phi \sim S_1 \cdot (S_2 \times S_3)$

$$\mathcal{L} = \sum_j \psi_j^*(\partial_\tau - iv_x\partial_x - v_y\partial_y^2)\psi_j + e\sum_j a\psi_j^*\psi_j + a\left[-\partial_\tau^2 - \partial_x^2 - \partial_y^2\right]a$$

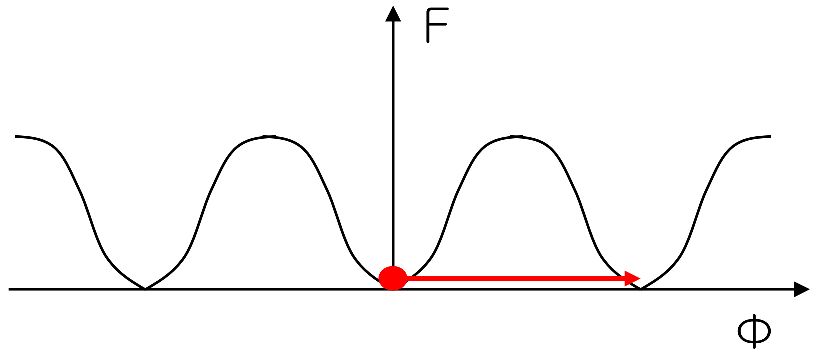
This theory arises in many strongly correlated electron systems :

spin liquid in frustrated spin systems,

half filled fractional quantum hall state

high Tc superconductor

Spinon FS coupled with U(1) gauge field in 2+1D



– Gauge group is compact : instanton (monopole) allowed
– Instanton is irrelevant for any nonzero N [SL (2008)]
(cf. in relativistic theory, one needs a large N to suppress instanton)
[Hermele et al. (2004)]

→ deconfined (Coulomb) phase is stable against confinement

Two space dimension is very special :
- Large enough to have an extended Fermi surface
- Small enough to support strong quantum fluctuations

→ non-trivial QFT underlying NFL behavior

Spinon FS coupled with U(1) gauge field in 2+1D

No dimensionless parameter except for the fermion flavor N=2

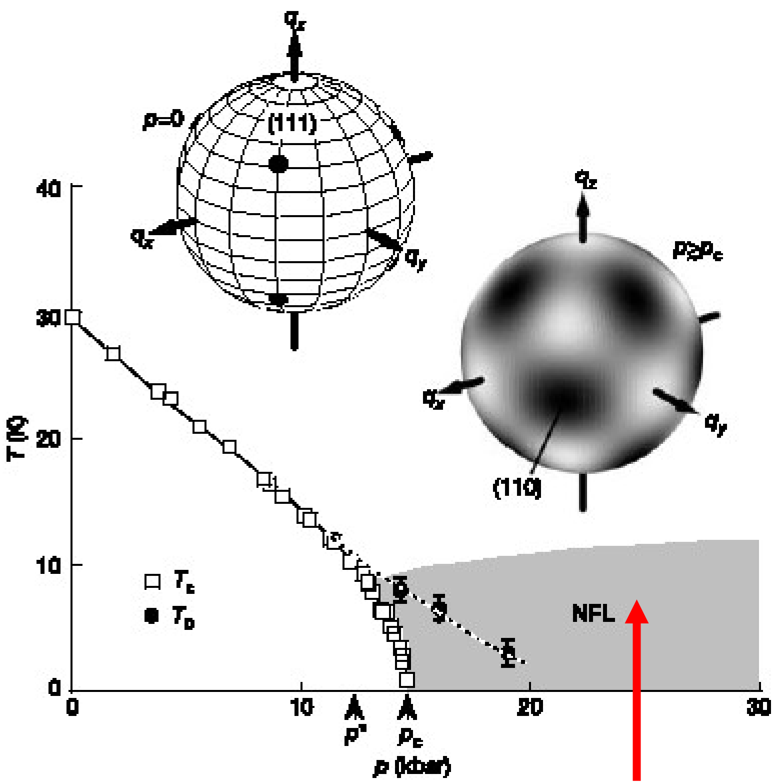
Generalize to a large N

→ **What is the nature of the theory ?**

[Polchinski(93); Althsuler,Ioffe,Millis(94); Kim, Furusaki, Wen, Lee(94); Nayak, Wilczek(94); Motrunich, Fisher (07); SL(09)]

- Low energy fermions are strongly coupled even in the large N limit
- Stable NFL phase (one-loop beta function exact)
- Genus expansion
- Weakly coupled description is absent even in the large N limit (string theory?)

A Non-Fermi liquid phase



$$\rho \sim T^{3/2}$$

- Ferromagnetic FL (with long wavelength spiral) at low pressures
- NFL behavior over a wide range of pressure at high pressures
- If the NFL state is due to a critical boson, how can a boson remain gapless with the robust critical exponent ?
- Protected by emergent symmetry (gauge boson) ?
- No satisfactory theory yet.

Some Open Problems in theories of strongly interacting fermions

- Important low energy phenomenology not understood
  - Direct continuous transition between HFL to LMM
  - NFL phase without fine tuning
- Low energy description for 2d NFL (e.g. FS coupled with U(1) gauge field)
  - Universal field theories ?
  - Classification?
- Systematic RG is not available
  - Non-local scale transformation (momentum-shell RG)