Holographic Superconductors (and Superfluids)

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References

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Quantum Phase Transition:

a phase transition between different quantum phases (phases of matter at $T = 0$). Quantum phase transitions can only be accessed by varying a physical parameter — such as magnetic field or pressure — at $T = 0$.

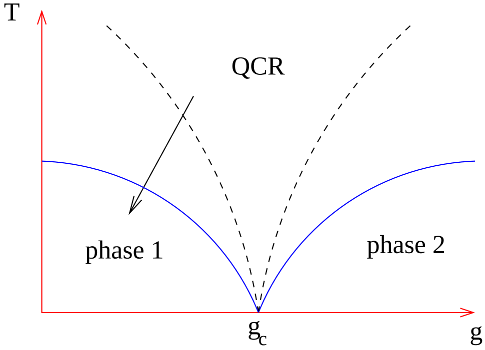


Figure: Phase diagram paradigm

The role of AdS/CFT

The AdS/CFT correspondence provides a tool to study a class of strongly interacting field theories with Lorentzian symmetry in d dimensions by mapping the field theories to classical gravity in $d+1$ dimensions.

- equation of state
- real time correlation functions
- transport properties — conductivities, diffusion constants, etc.

The ambitious program: There may be an example in this class of field theories which describes the quantum critical region of a real world material such as a high T_c superconductor.

The less ambitious program: By learning about this class of field theories, we may find universal features that could hold more generally for QCPs ($\eta/s = \hbar/4\pi k_B$).

There are a few entries in the AdS/CFT dictionary for $z > 1$ (Kachru, Mulligan, Liu, Balasubramanian, McGreevy, Son, ...), but here we consider only $z = 1$.

AdS/CFT Models

Holographic Phase Transitions

Goal: To have a simple holographic model of a (classical) phase transition where we can calculate the phase diagram and transport coefficients.

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}(R-2\Lambda) - \frac{1}{4g^2}\int d^{d+1}x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

- Einstein-Hilbert produces correlators of the stress tensor $T^{\mu\nu}$ in the boundary theory.
- Maxwell produces correlators of a global current J^{μ} in the boundary.
- To model a (classical) phase transition, we need something that will serve as an order parameter.

Two Choices of Order

- We can add a charged scalar field

$$-\int d^{d+1}x\sqrt{-g}\left(|(\partial - iqA)\Psi|^2 + V(|\Psi|)\right) \ .$$

  The order parameter is the boundary value of Ψ.
- We can promote the Abelian gauge field to an SU(2) gauge field

$$F_{\mu\nu} \to F^a_{\mu\nu} \ .$$

  We find a vector order parameter which is the boundary value of A^a_μ.

Motivating the Action

The action, in a loose sense, comes from string theory.

- For $d = 4$, there is a duality between type IIB string theory in $AdS_5 \times S^5$ and maximally supersymmetric SU(N) Yang-Mills theory in 3+1 dimensions.
- For $d = 3$, there is a duality between M-theory in $AdS_4 \times S^7$ and the maximally supersymmetric SU(N) Yang-Mills theory in 2+1 dimensions.
- The low energy limit of string theory is supergravity. The actions above are conceivably truncations of the full supergravity action.
- For conformal field theories with extended supersymmetry, there is a global R-symmetry that is dual to the gauge field $F_{\mu\nu}$ in gravity.

Dyonic Black Holes and the QCR

One solution to our scalar action with $\psi = 0$ is a dyonic black hole in AdS_4. The dyonic black hole is also a solution to the SU(2) action. Dyonic black holes have electric and magnetic charge.

- The Hawking temperature of the black hole is the temperature T of the field theory.
- The magnetic field of the black hole is the magnetic field B in the field theory.
- The electric field of the black hole becomes the charge density ρ of the field theory.

One can freely tune the temperature and charges of the black hole.

Dyonic Black Holes II

The metric or line element:

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2)$$

The electric and magnetic fields:

$$A = \rho \left(\frac{1}{r_+} - \frac{1}{r} \right) dt + B\, x\, dy$$

The warp factor:

$$g(r) = r^2 - \frac{1}{4rr_+}(4r_+^4 + \rho^2 + B^2) + \frac{1}{4r^2}(\rho^2 + B^2)$$

The temperature:

$$T = \frac{12r_+^4 - \rho^2 - B^2}{16\pi r_+^3}$$

An instability for the scalar action

Assuming $V(\Psi) = m^2|\Psi|^2$, Gubser observed an instability for the scalar to condense when ρ gets too large:

$$m_{\text{eff}}^2 = m^2 + g^{tt} A_t^2$$

where

$$g_{tt} = -g(r) \; ; \quad A_t = \frac{\rho}{r r_+}(r - r_+) \; .$$

The effective mass becomes tachyonic and the scalar condenses in a narrow region of radial coordinate r.

There is no need for a Ψ^4 term!

For the case $B = 0$, there is only one other scale in the problem, the temperature, so large ρ corresponds to small T.

The SU(2) instability

The SU(2) action has a similar instability. Let the τ^i generate $\mathfrak{s}u(2)$. For an electrically charged black hole in the τ^3 direction, there is an instability to generate a nonzero $A_x^1 = w$:

$$A = \phi \, \tau^3 \, dt + w \, \tau^1 \, dx \ .$$

The nonzero w corresponds to a nonzero current in the boundary field theory!

- The scalar action, while conceptually simple, suffers from the arbitrariness of $V(|\psi|)$. We will choose $V(|\psi|) = m^2|\psi|^2$.
- The SU(2) action is less arbitrary and in $d = 4$ can be treated analytically for $T \approx T_c$, but suffers from a phase transition that breaks rotational invariance.
- First, we will summarize the (largely numerical) results for the scalar action.
- Next, we will sketch some of the analytic results for the SU(2) action ($d = 4$).

Superfluidity and Superconductivity Results: The Scalar Case

The phase transition

Given $m^2 L^2 = -2$ (above the BF bound), we can choose a scalar in the field theory with scaling dimension one or two.

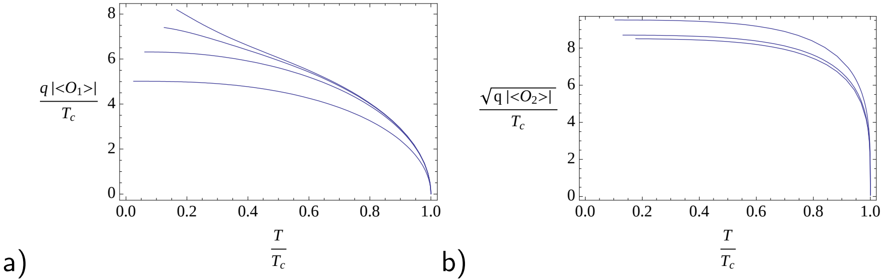


Figure: The value of the condensate as a function of temperature for the two different boundary conditions: a) from bottom to top, the various curves correspond to $q = 1$, 3, 6, and 12; b) from top to bottom, the curves correspond to $q = 3$, 6, and 12. Note that $T_c \sim \sqrt{\rho}$.

Probe limit is large q.

Mean field behavior and philosophy

For $T \lesssim T_c$,

$$\langle \mathcal{O}_i \rangle \sim (T_c - T)^{1/2} \ ,$$

the standard Landau-Ginzburg mean field result.

There is a LG interpretation on the field theory side

$$f_{LG} = \frac{1}{2m_*}|(\nabla + iqA)\varphi|^2 + a|\varphi|^2 + \frac{b}{2}|\varphi|^4 + \ldots$$

AdS/CFT gives us the full $V(\varphi)$ as a complicated function of our bulk gravitational action.

We can go away from the universal behavior at small $|\varphi|$ having assumed (or derived from string theory) a simple form for the gravitational action.

Superfluid or superconductor?

Two interpretations of the instability

- This $U(1)$ symmetry in the field theory is global, and strictly speaking we have only spontaneous symmetry breaking — a superfluid phase transition.
- We can think of the $U(1)$ as being weakly gauged, in which case we have a superconductor.

The Conductivity from BCS Theory

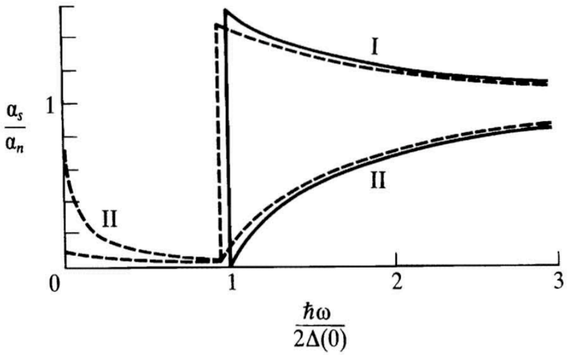


Figure: Frequency dependence of absorption processes obeying case I and II coherence factors at $T = 0$ (solid curves) and $T \approx \frac{1}{2} T_c$ (dashed curves). [Tinkham, Superconductivity, 2nd edition]

Conductivity for dimension one case, probe limit

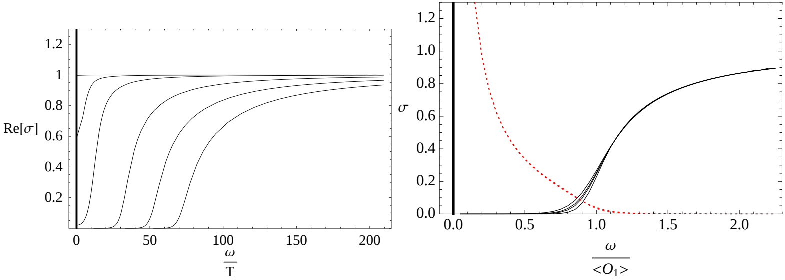


Figure: Left: Plots of the real part of the conductivity versus frequency for various temperatures. Right: Plots of the conductivity versus frequency at very low temperature. The dotted red curve is the $\mathrm{Im}(\sigma)/5$.

$\mathrm{Re}[\sigma(\omega)]$ contains a delta function $\pi n_s \delta(\omega)$ which leads to superconductivity where n_s is the superfluid density.

Conductivity for dimension two case, probe limit

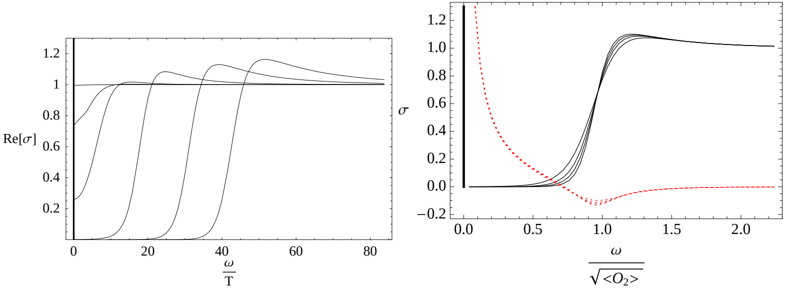


Figure: Left: Plots of the real part of the conductivity versus frequency for various temperatures. Right: Plots of the conductivity versus frequency at very low temperature. The dotted red curve is the $\text{Im}(\sigma)/5$.

One conclusion that may be drawn from these plots is that $\langle O_1 \rangle$ and $\sqrt{\langle O_2 \rangle}$ can be intepreted as twice the superconducting gap.

Second Sound (away from the probe limit)

Second sound is a collective motion in two component fluids sourced (mostly, in the incompressible limit) by temperature oscillations.

We calculate the speed of second sound from thermodynamic quantities

$$v_2^2 = \frac{\sigma^2 \rho_{\rm s}}{w} \frac{1}{(\partial \sigma / \partial T)_\mu} \ ,$$

where $w = \epsilon + P$ and $\sigma = s/\rho$.
This formula follows from a hydrodynamic analysis.

Second Sound of Helium-4 from Khalatnikov's Book

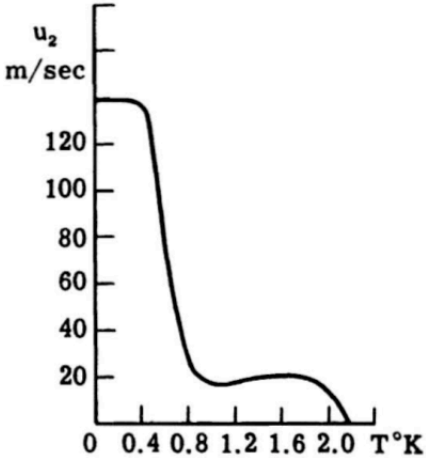
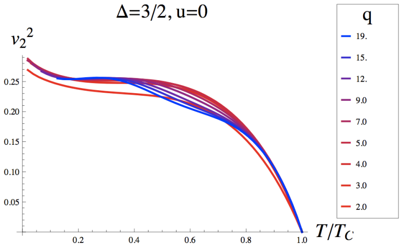
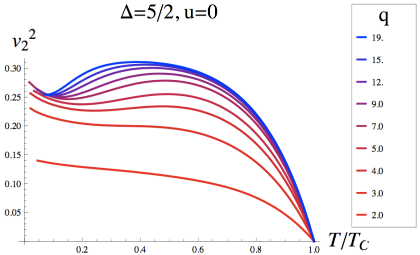


Figure 4. The temperature dependence of the velocity of second sound.

Second Sound (3+1 dimensions)



a)



b)

Figure: The speed of second sound as a function of T/T_c, computed by evaluating thermodynamic derivatives: a) $O_{3/2}$ scalar, b) $O_{5/2}$ scalar for a 3+1 dimensional field theory. The speed of second sound vanishes as $T \to T_c$. $u = 0$ means no quartic term in V.

with Amos Yarom

Universality at low T

Landau predicted that

$$\lim_{T\to 0} v_2^2 = \frac{v_1^2}{d} \ ,$$

i.e. second sound becomes a sound wave propagating in a gas of phonons. Reasonable for helium-4.

We do not find this result. We believe the reason is that the low temperature limit of this system is not a gas of phonons.

$$\lim_{T\to 0} \frac{C_\mu}{sd} \neq 1 \ , \qquad \lim_{T\to 0} \frac{sT}{sT + \mu\rho_{\rm n}} \neq v_1^2$$

What exactly is it?

Superfluidity and Superconductivity Results: The SU(2) Case

Why it's analytic

Starting with an electrically charged black hole at small μ/T where $A = \phi\,\tau^3\,dt$, there is a critical chemical potential at which an instability appears, characterized by a zero mode for A_x^1 (Basu, He, Mukherjee, Shieh, 0810.3970 [hep-th]):

$$\partial_z^2 A_x^1 + \left(\frac{f'}{f} - \frac{1}{u}\right)\partial_z A_x^1 = -\frac{\phi^2}{f^2} A_x^1$$

where $\phi = (1-z)\mu/\pi T$ and $f = 1 - z^4$:

$$A_x^1 = \epsilon\frac{z^2}{(1+z^2)^2} \qquad \text{where} \qquad \frac{\mu}{\pi T} = 4\ .$$

NB: We are working in the probe limit and neglecting the back reaction of A on the metric.

Speed of Second Sound

The phase transition breaks rotational symmetry, and there are actually two speeds of second sound

$$c_\perp^2 \approx \frac{140}{281}\left(\frac{\mu}{\pi T} - 4\right) \ ,$$
$$c_\parallel^2 \approx \frac{70}{281}\left(\frac{\mu}{\pi T} - 4\right) \ .$$

We were able to see these results in two ways:

- From the thermodynamic identity mentioned above.
- From poles in the current-current correlation functions.

NB: These results are valid only near T_c.

with Silviu Pufu

The London Equations (half the Meissner effect)

From the current-current correlation functions, we were able to verify the London equations in the small k and ω limit.

In particular, we saw that the $\omega \longrightarrow 0$ and $k \longrightarrow 0$ limits commute:

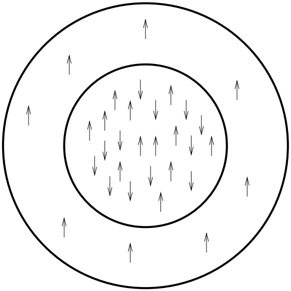
$$\lim_{k \longrightarrow 0} G^{ii}_{33}(0, k) = \lim_{\omega \longrightarrow 0} G^{ii}_{33}(\omega, 0)$$

The current is proportional to the vector potential:

$$\langle j^3_x \rangle \approx -\frac{2}{g^2}\frac{\epsilon^2}{96}\mathcal{A}^3_x \qquad \text{and} \qquad \langle j^3_y \rangle \approx -\frac{2}{g^2}\frac{\epsilon^2}{48}\mathcal{A}^3_y \ .$$

Fulde-Ferrell

- That the order parameter $\langle j_x^1 \rangle$ is a current is strange.
- Reminiscent of an idea by Fulde and Ferrell (also Larkin and Ovchinnikov) — BCS in a magnetic field



$$k_{F\uparrow} - k_{F\downarrow} > 0$$

Remarks and Plans for the Future

- Tried to convince you that AdS/CFT is a useful tool for studying strongly interacting field theories — equations of state, correlation functions, transport properties.
- The hope is that these field theories may be relevant for understanding real world condensed matter systems.
- We saw that AdS/CFT can be used to study the superconducting/superfluid phase transition.
- Embedding this model in string theory.