A flavor superconductor from string theory

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Motivation

Holographic Superconductor from charged scalar in Einstein-Maxwell gravity

Gubser; Hartnoll, Herzog, Horowitz　　　　cf. Chris Herzog's talk

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p-wave superconductor (current dual to gauge field condensing)

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(AdS_4 examples)

Follow-up questions:

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Is there a holographic superconductor...

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2) within ten-dimensional type IIB supergravity?

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A holographic superconductor with field theory in 3+1 dimensions for which

1. the dual field theory is explicitly known

2. there is a qualitative ten-dimensional string theory picture of condensation

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

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'adding flavor to gauge/gravity duality' cf. Andreas Karch's talk

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Brane probes added on gravity side $\Rightarrow$ fundamental d.o.f. in the dual field theory

Additional hyperplanes within $AdS_5 \times S^5$ or deformed version thereof

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$\Rightarrow$ Rich phase structure

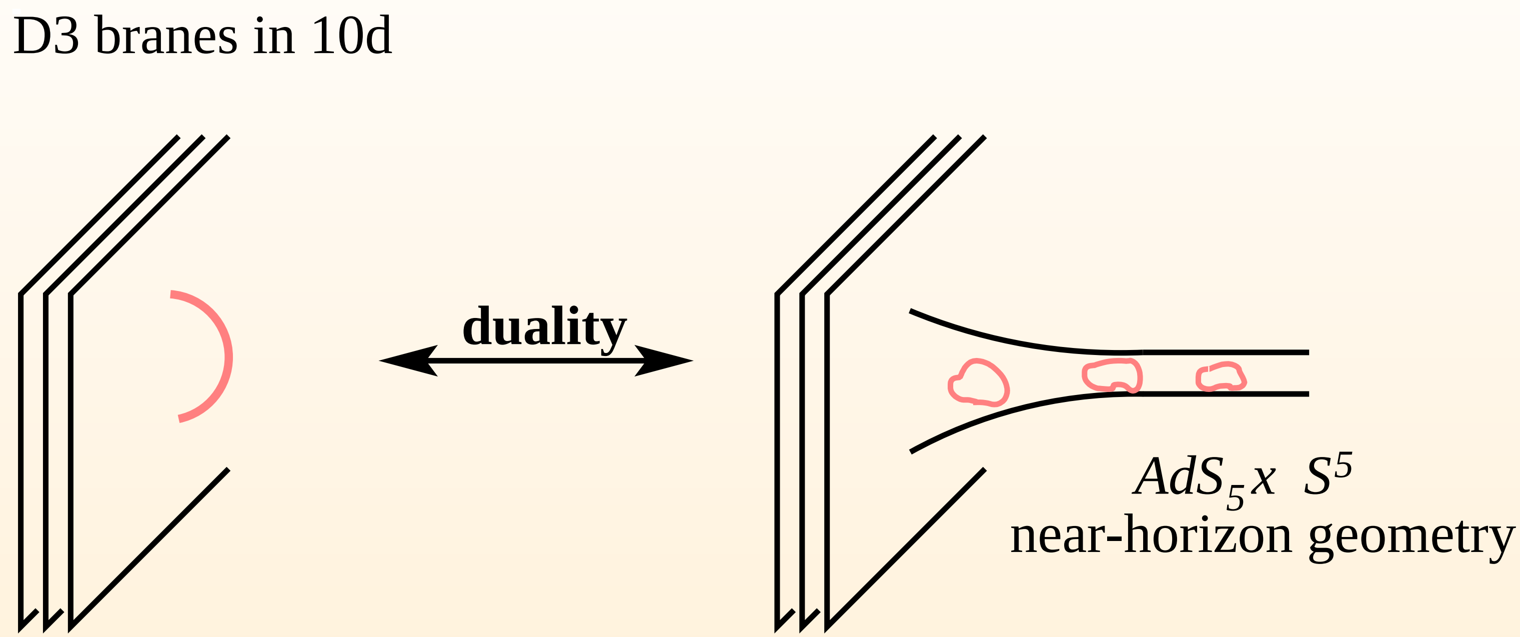
Outline

1. Adding Flavor to Gauge/Gravity Duality

2. Holographic Quarks at finite Temperature and Density

3. Superconductivity

String theory origin of AdS/CFT correspondence



$\Downarrow$ Low-energy limit

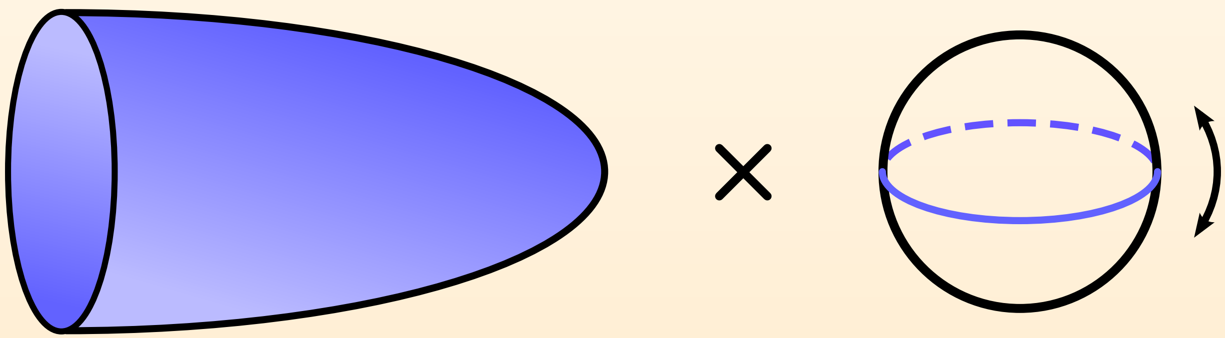
$\mathcal{N} = 4$ SUSY $SU(N)$ gauge theory in four dimensions ($N \to \infty$)

Supergravity on $AdS_5 \times S^5$

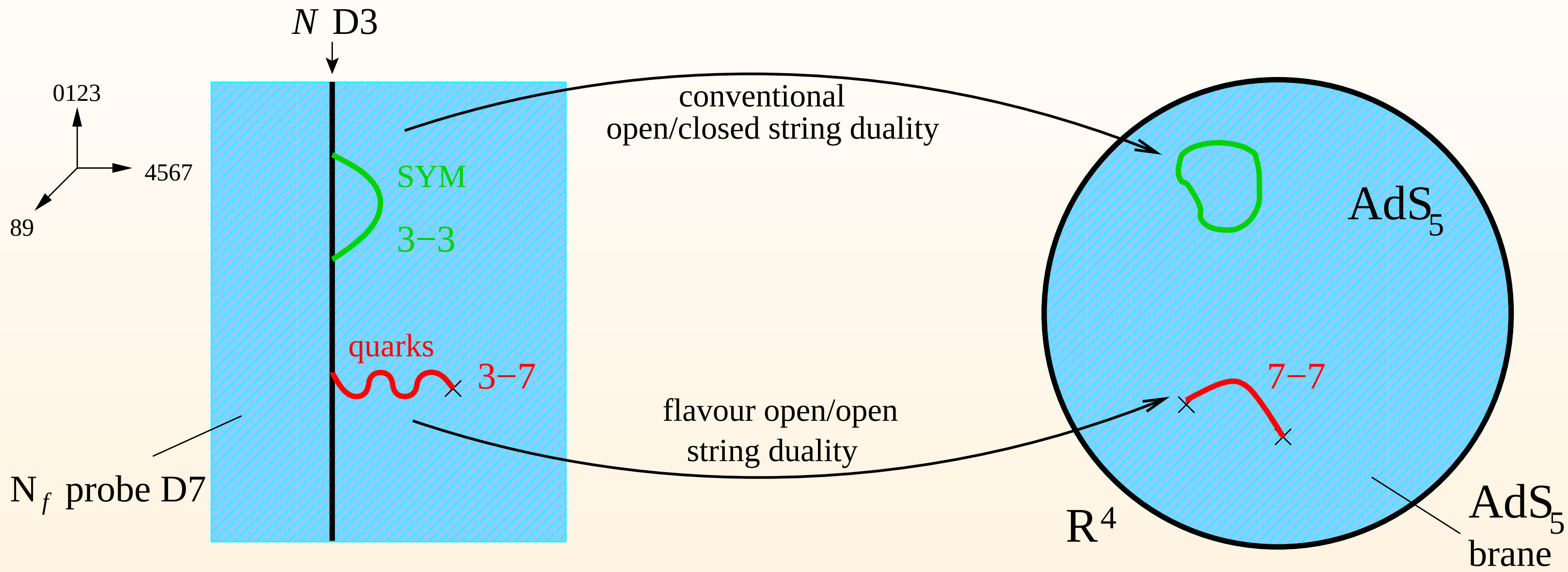
Quarks (fundamental fields) within the AdS/CFT correspondence

Adding D7 brane probe:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		



Quarks (fundamental fields) from brane probes



$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

duality acts twice:

$\mathcal{N} = 4$ SU(N) Super Yang-Mills theory coupled to $\mathcal{N} = 2$ fundamental hypermultiplet	$\longleftrightarrow$	IIB supergravity on $AdS_5 \times S^5$ + Probe brane action on $AdS_5 \times S^3$

Karch, Katz 2002

Dirac-Born-Infeld action

DBI (Dirac-Born-Infeld) action:

$$S_{DBI} = -T_7 \int d^8\xi \, \mathrm{tr}\sqrt{det(-P[G] + 2\pi\alpha' F)}$$

Contributions of order N_f/N_c

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Field theory involves fundamental fermions and scalars

Gauge/Gravity Duality at Finite Temperature

$\mathcal{N} = 4$ Super Yang-Mills theory at finite temperature is dual to AdS black hole

Witten 1998

$$ds^2 = \frac{1}{2}\left(\frac{\varrho}{R}\right)^2 \left(-\frac{f^2}{\tilde{f}}dt^2 + \tilde{f} d\vec{x}^2\right) + \left(\frac{R}{\varrho}\right)^2 (d\varrho^2 + \varrho^2 d\Omega_5^2)$$

$$f(\varrho) = 1 - \frac{\varrho_H^4}{\varrho^4}, \quad \tilde{f}(\varrho) = 1 + \frac{\varrho_H^4}{\varrho^4}$$

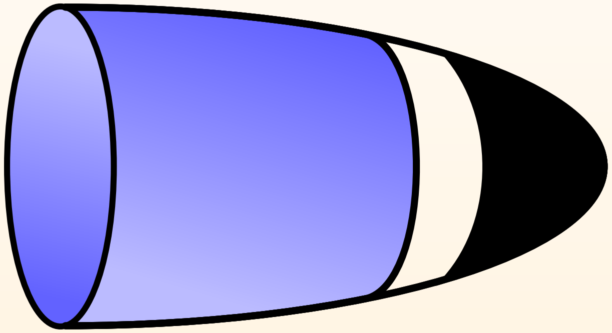
Temperature and horizon related by

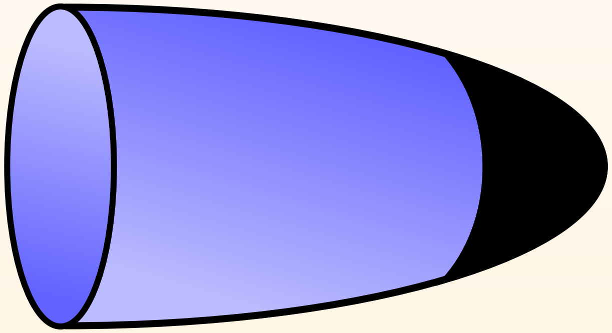
$$T = \frac{\varrho_H}{\pi R^2}$$

R: AdS radius

For $\varrho_H \to 0$, metric of $AdS_5 \times S^5$ is recovered.

D7 brane embedding in black hole background





First order phase transition

Babington, J.E., Evans, Guralnik, Kirsch
Mateos, Myers, Thomson

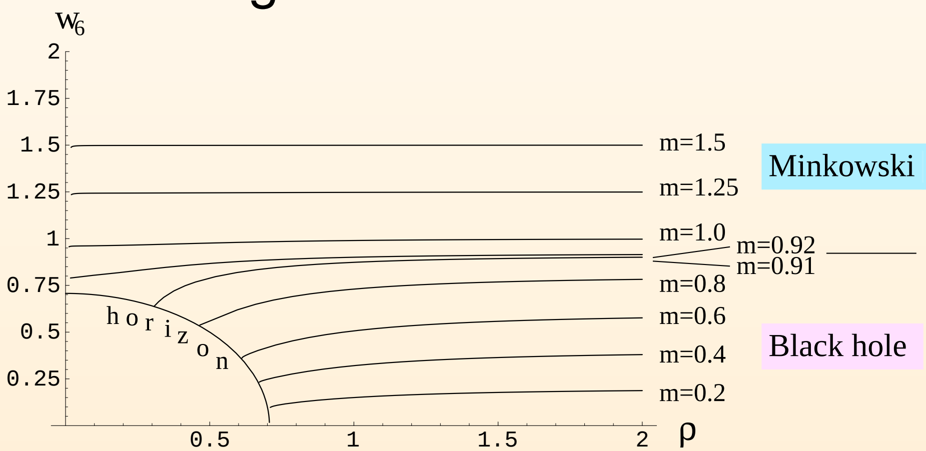
D7 brane embedding in black hole background

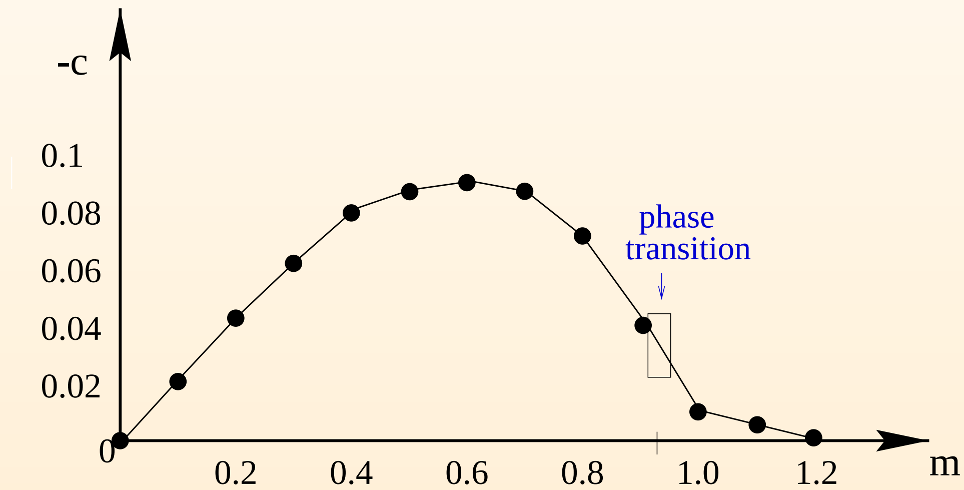
Babington, J.E., Evans, Guralnik, Kirsch 0306018

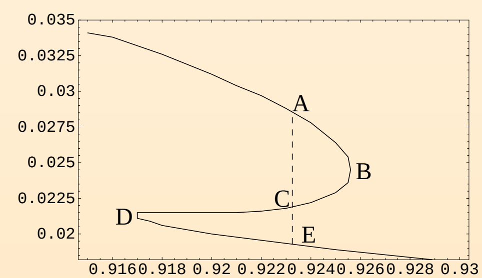
Condensate $c \equiv \langle \bar{\psi}\psi \rangle$ vs. quark mass m

m in units of T

Embeddings







Phase transition at $m_c \approx 0.92$ (1st order)

Kirsch 2004

Masses and decay widths of mesons - Spectral functions

Standard procedure in D3/D7:

Meson masses calculated from linearized fluctuations of D7 embedding

Fluctuations: $\delta w(x,\rho) = f(\rho) e^{i(\vec{k}\cdot\vec{x} - \omega t)}$, $M^2 = -k^2$

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Identify mesons with resonances in spectral function

Spectral function determined by poles of retarded Green function

Quasinormal modes

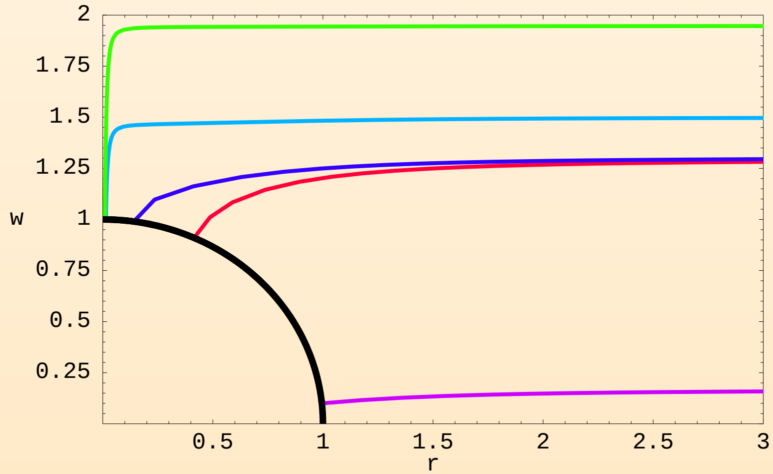
Finite $U(1)$ baryon density

Mateos, Myers, Matsuura et al

Baryon density n_B and $U(1)$ chemical potential μ
from VEV for gauge field time component:

$$\bar{A}_0(\rho) \sim \mu + \frac{\tilde{d}}{\rho^2}\,, \qquad \tilde{d} = \frac{2^{5/2}}{N_f\sqrt{\lambda}T^3}\, n_B$$

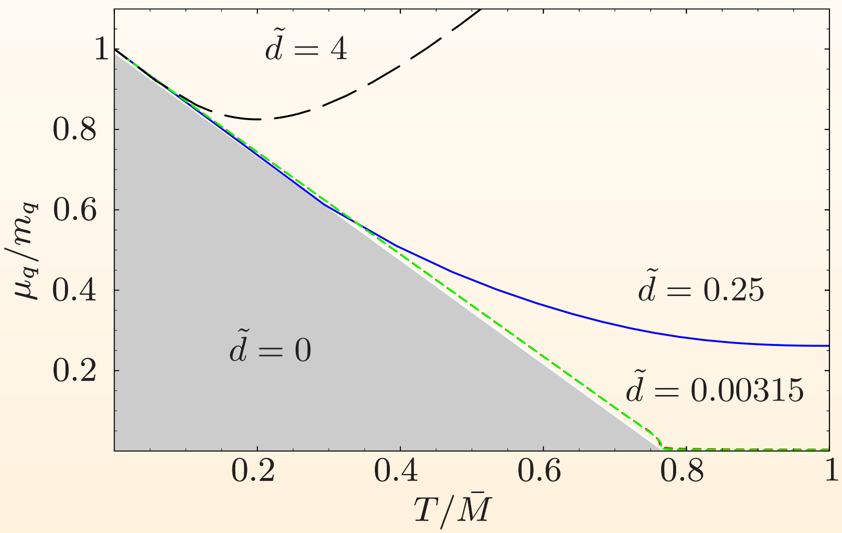
At finite baryon density, all embeddings are black hole embeddings



Phase diagram with finite $U(1)$ baryon density

Phase diagram:

grey region: $n_B = 0$
white region: $n_B \neq 0$



Sin, Yogendran et al; Mateos, Myers et al; Karch, O'Bannon; . . .

Isospin chemical potential and density

- Embed two coincident D7-branes into AdS-Schwarzschild
  gauge fields $A_\mu = A_\mu^a \sigma^a \in u(2) = u(1)_B \oplus su(2)_I$
- Dynamics of Flavour degrees is described by non-abelian DBI action
- Finite isospin density: $A_0^3 \neq 0 \Rightarrow$ Explicit breaking to $u(1)_3$

Field theory described:

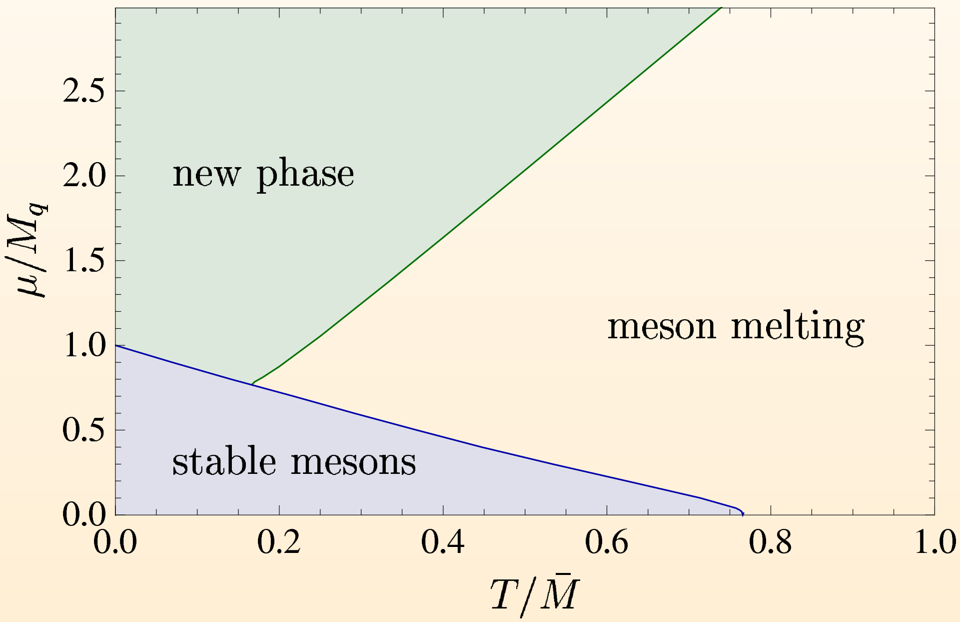
$\mathcal{N} = 4$ Super Yang-Mills plus two flavors of fundamental matter

at finite temperature and finite isospin density

ρ meson condensation

J.E., Kaminski, Kerner, Rust 0807.2663

Above a critical isospin density, a new phase forms



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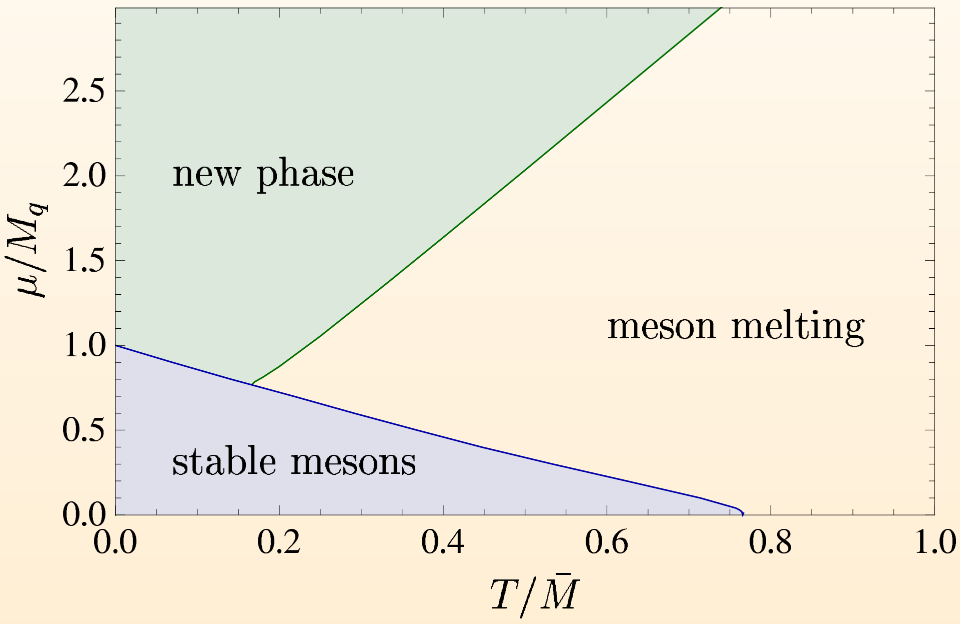
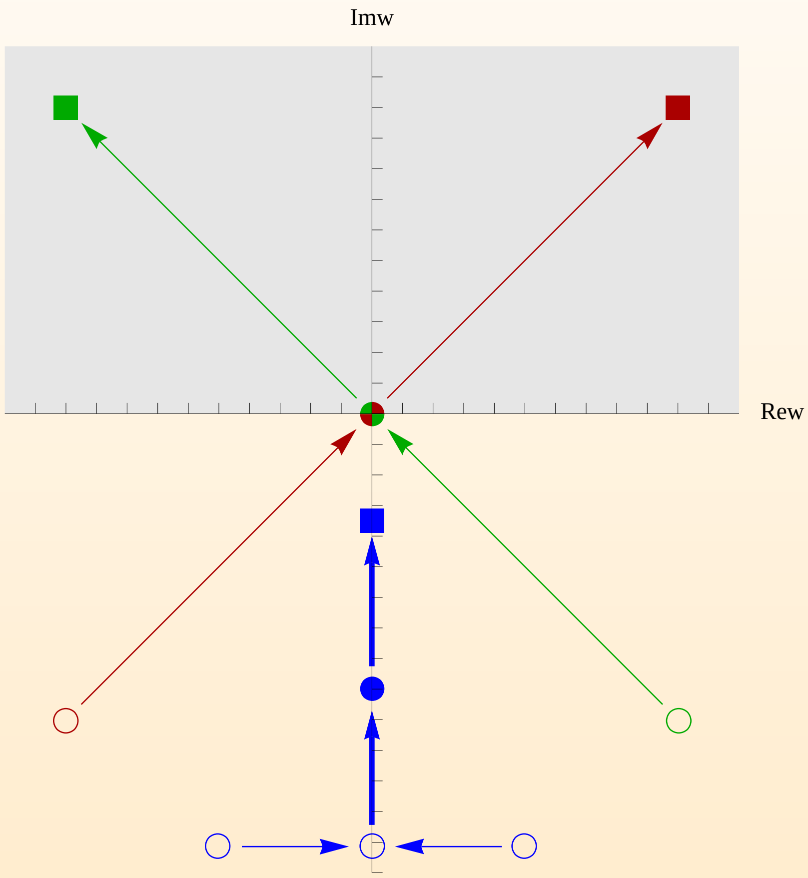


New phase is unstable

Quasinormal modes

Instability:



A new ground state forms

There is a new solution to the equations of motion

with non-zero vev for $A_3^1\sigma^1$ in addition to the non-zero $A_0^3\sigma^3$

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$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'} \frac{\rho_H}{\rho^2} + \dots , \qquad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha'} \frac{\rho_H}{\rho^2} + \dots$$

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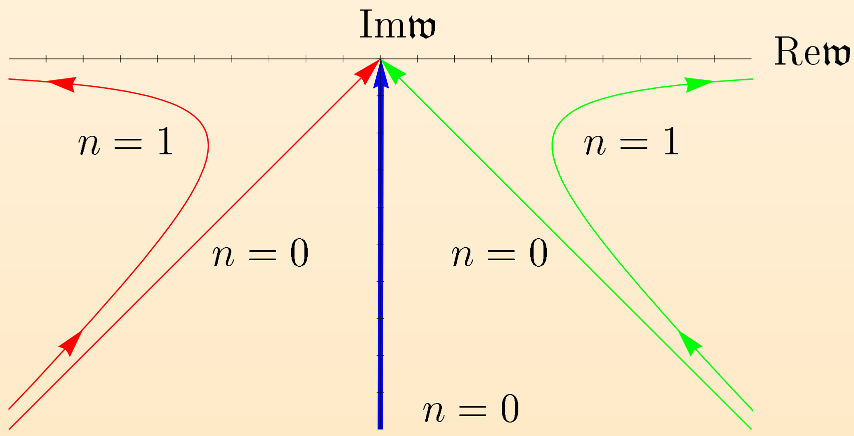
Pole structure:



Superconductivity

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

The new ground state has properties known from superconductors:

- infinite DC conductivity, gap in the AC conductivity
- second order phase transition, critical exponent of 1/2 (mean field)
- a remnant of the Meissner–Ochsenfeld effect

Superconductivity

Order parameter $\tilde{d}_3^1 \propto \langle \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \; bosons \, \rangle \neq 0$

Dual to $A_3^1 \sigma^1$ in gravity theory

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Flavor superfluid

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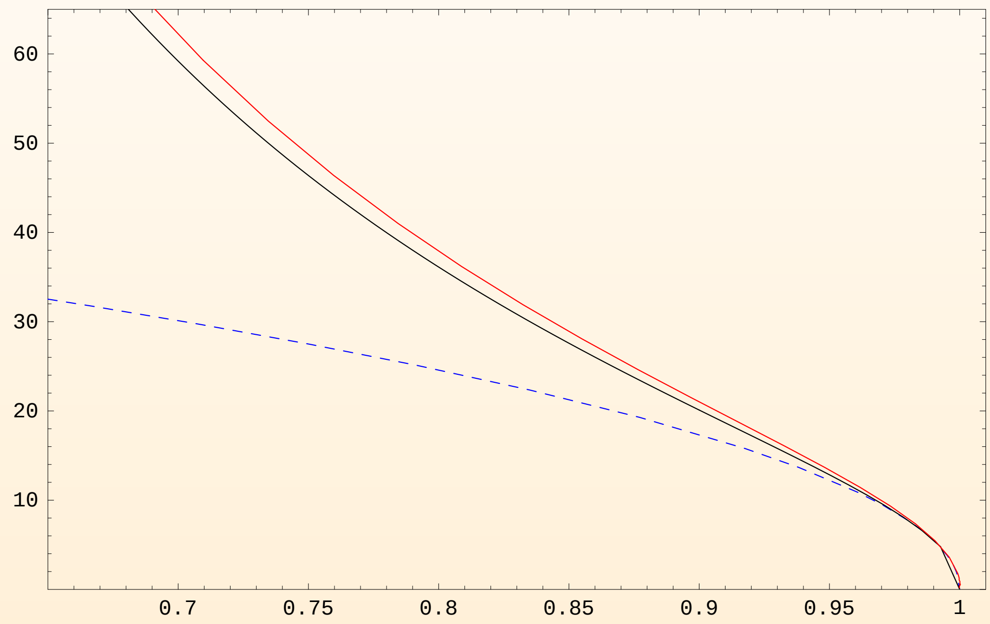
$U(1)_3$ may be weakly gauged

Order parameter: p wave condensate

$-\tilde{d}_3^1$

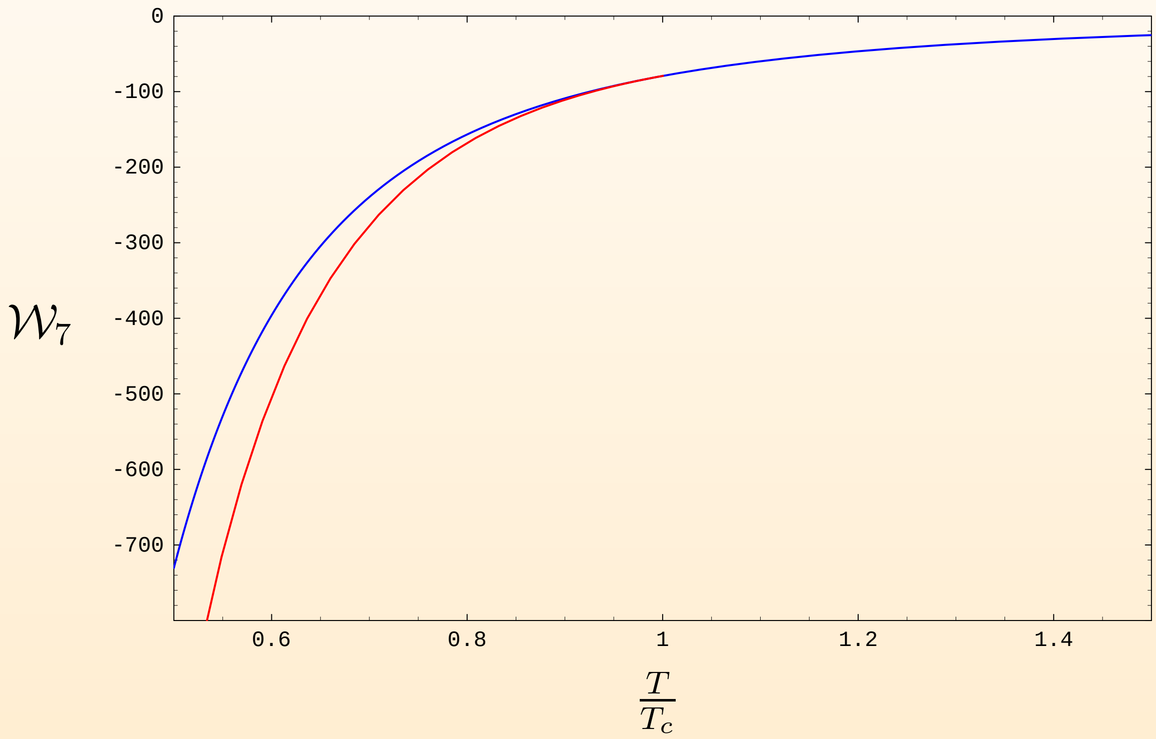


T/T_c

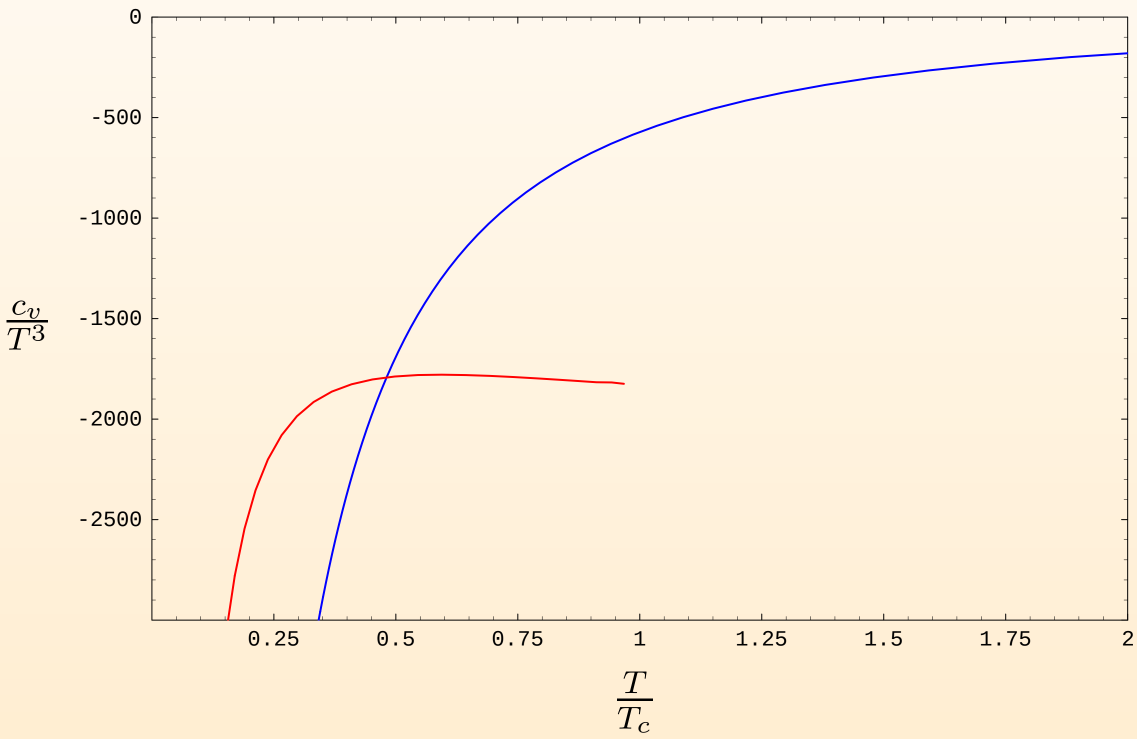
Red: Vanishing quark mass; Black: Finite quark mass, $\mu/M_q = 3$

Blue: Fit displaying critical exponent 1/2

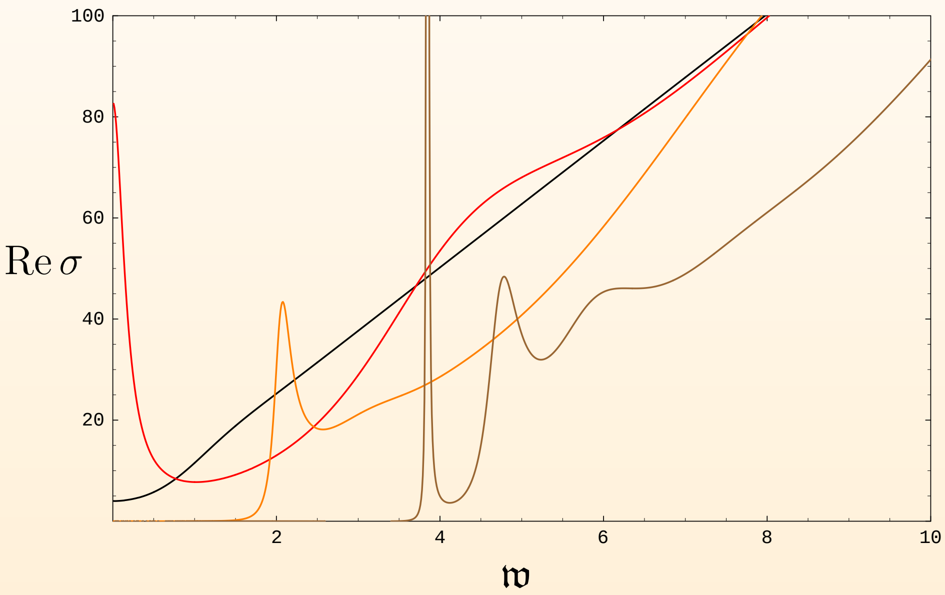
Flavor contribution to Grand potential vs. temperature



Flavor contribution to heat capacity



Frequency-dependent conductivity

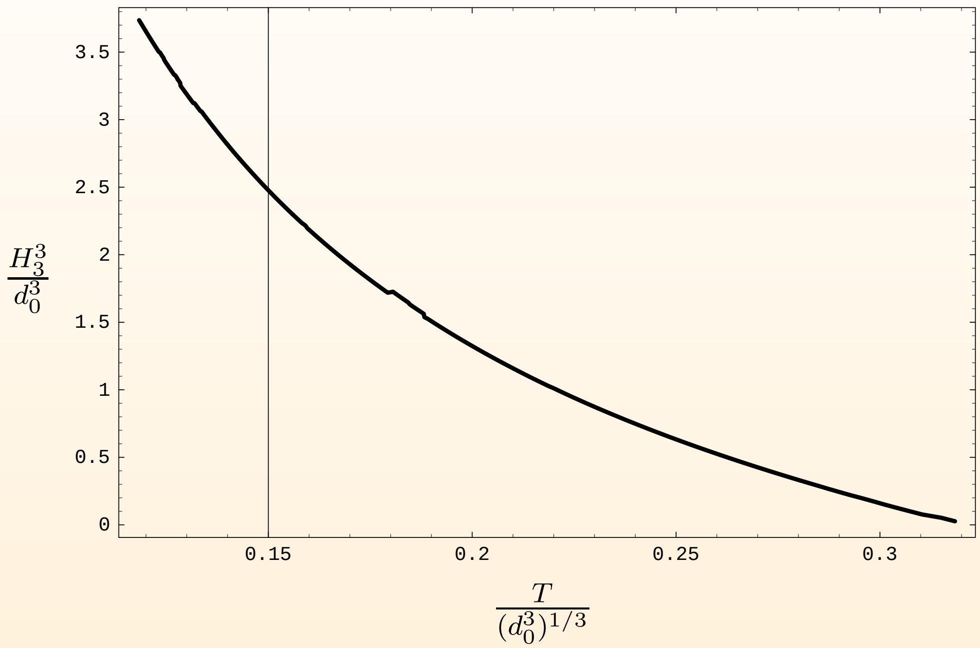


$$\mathfrak{w} = \omega/(2\pi T)$$

T/T_c: Black: ∞, Red: 1, Orange: $T = 0.5$, Brown: $T = 0.28$.

(Vanishing quark mass)

Meissner effect



Lower phase: magnetic field and condensate coexist

Upper phase: condensate vanishes

Non-abelian DBI action

Evaluation non-trivial in presence of both σ^0, σ^1

Two evaluation methods:

1) Expansion to fourth order

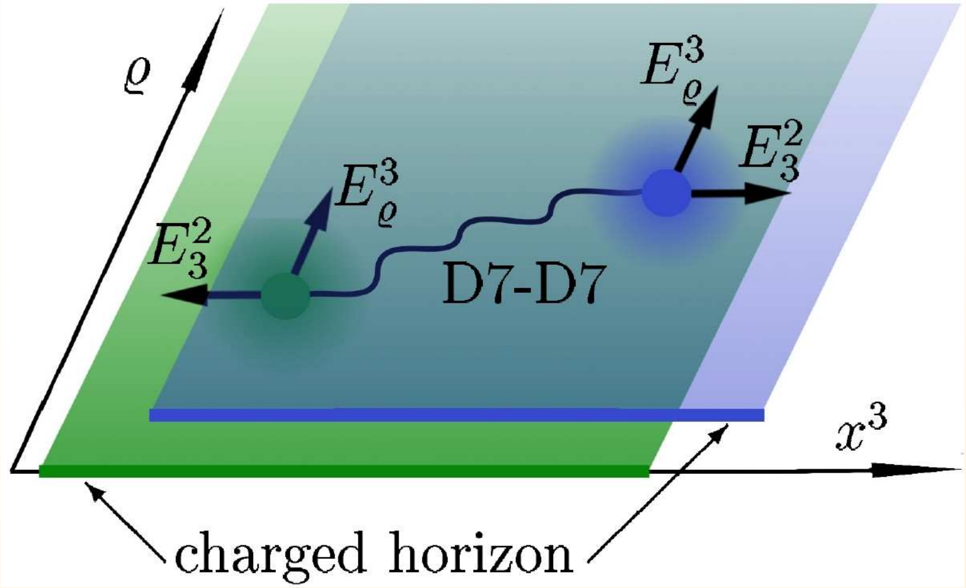
2) Simplification: Omitting commutators of Pauli matrices
Modified prescription for symmetrized trace

Allows for all-order calculation of the non-abelian DBI

Error of order $1/N_f$

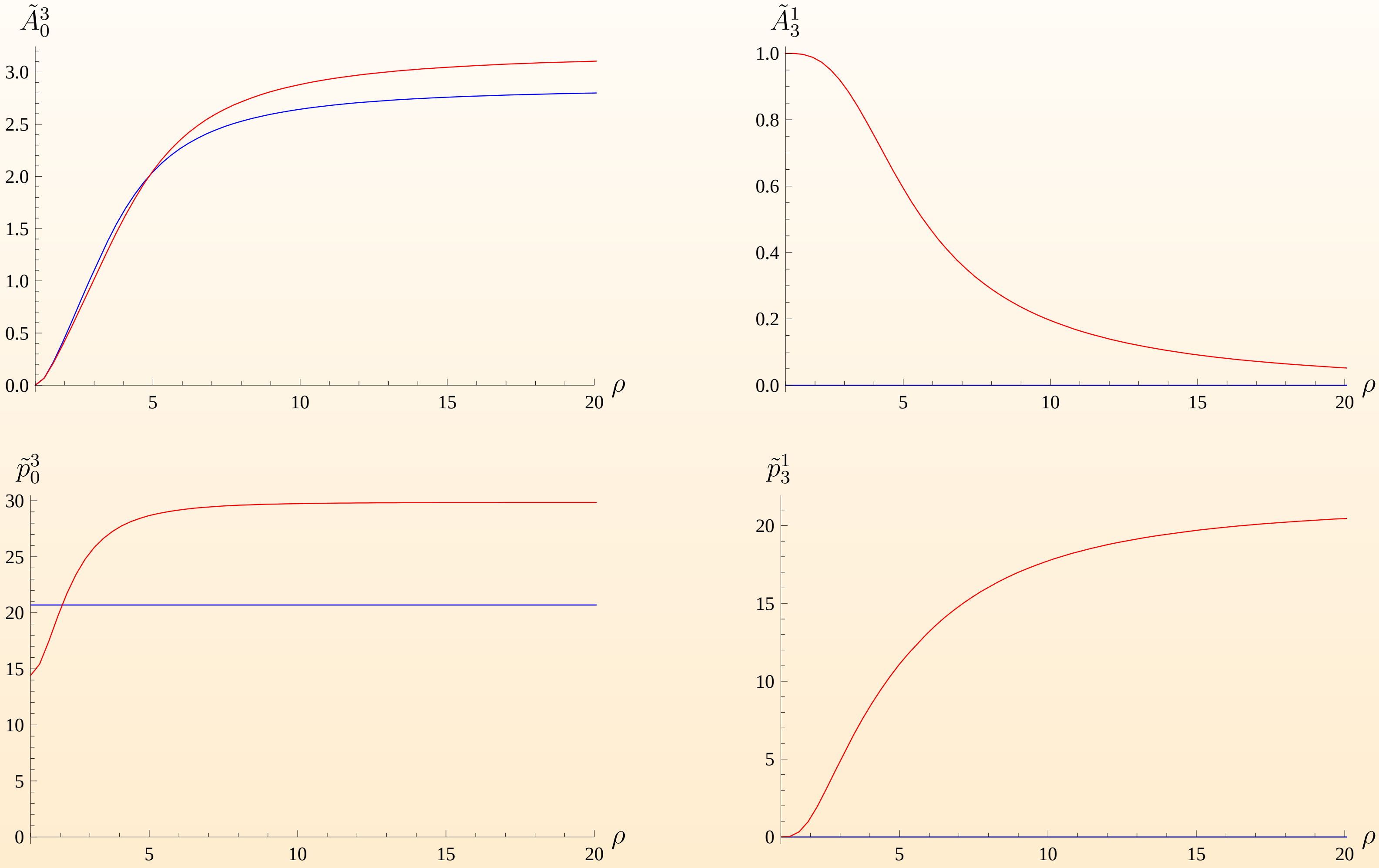
cf. Myers, Constable, Tafjord 1999

String picture



- Strings stretched between D7 branes and horizon induce a charge near the horizon
- System unstable above a critical charge density
- Horizon strings recombine to $D7 - D7$ strings
- $D7 - D7$ strings propagate into the bulk, balancing flavorelectric and gravitational forces
- $D7 - D7$ strings distribute isospin charge into the bulk $\rightarrow$ superconducting condensate

Charge distributions



Conclusion and Outlook

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- First explicit example of superconductivity (superfluidity) from 10d action
- Embedding of two coincident D7 branes $\Rightarrow$ Finite isospin density

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- Outlook: Fermions
- Outlook: Space-time dependent solutions

  Spin density waves (w. E. Caceres)