

# Non-fermi liquid metals: Some facts, ideas, and speculations

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## Zeroth order fact

Breakdown of fermi liquid increasingly commonly seen in many metals, typically with ``active'' electrons from atomic d or f orbitals.

Eg: ``Bad metals'' (Kivelson talk)

Far too many examples to list.....

How can we make progress in describing them?

# A crucial question

## Is More different?

Are the growing number of non-fermi liquid examples all the same basic phenomenon?

Is there a common explanation for all ``bad metal'' phenomena?

Probably not.....

# Some strategies for progress

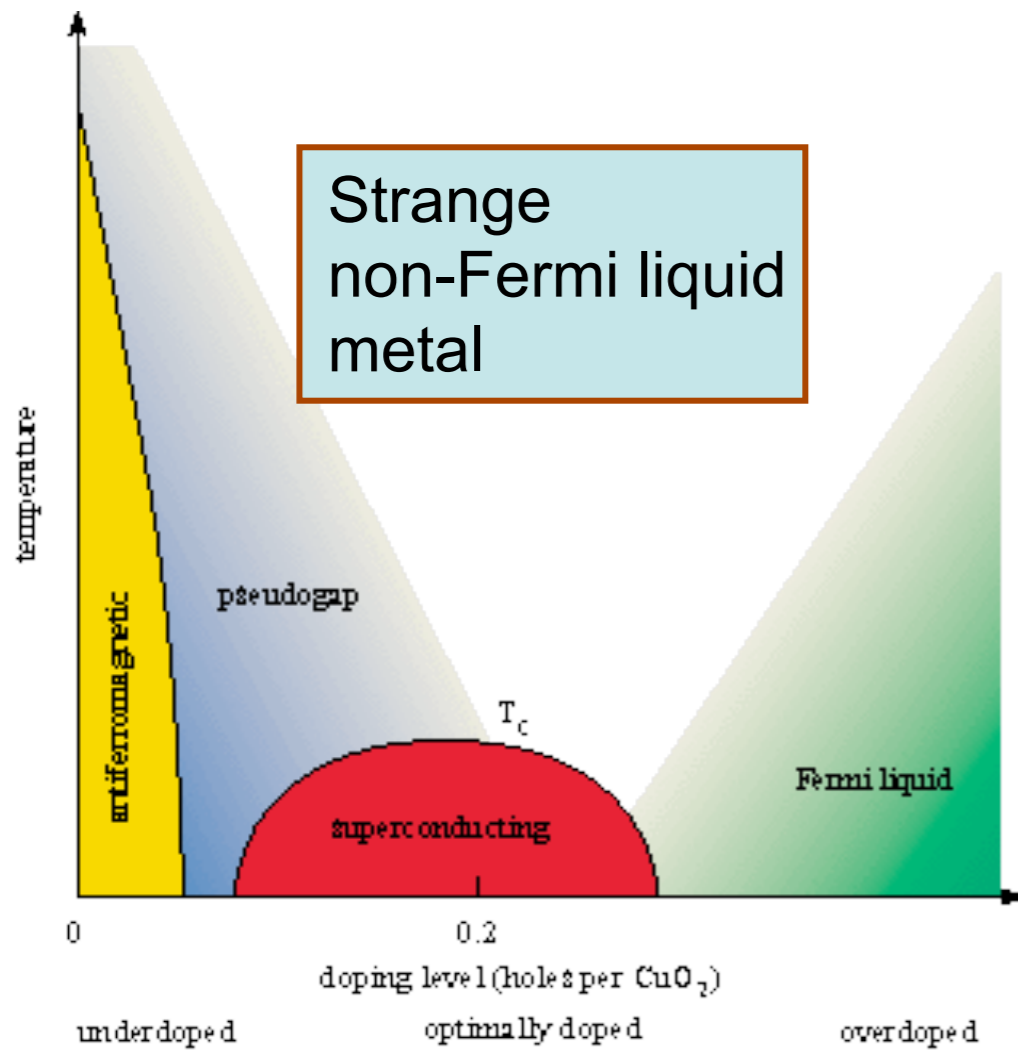
Focus on few best characterized examples

Look for systems for which

1. Much more is known than just the electrical resistivity.
2. Non-fermi liquid extends over wide range of energy scales
3. Microscopic physics is reasonably simple

Fortunately a few such systems exist.

# Example 1: high temperature superconductors



Strange metal regime:

Power laws in many physical quantities distinct from that expected in a Fermi liquid.

Fermi surface but no Landau quasiparticles.

## Example 2: Magnetic ordering in certain rare earth alloys

$\text{CePd}_2\text{Si}_2$ ,  $\text{CeCu}_{6-x}\text{Au}_x$ ,  $\text{YbRh}_2\text{Si}_2, \dots$

Partially filled f-orbitals form localized magnetic moments

## Example 2: Magnetic ordering in certain rare earth alloys

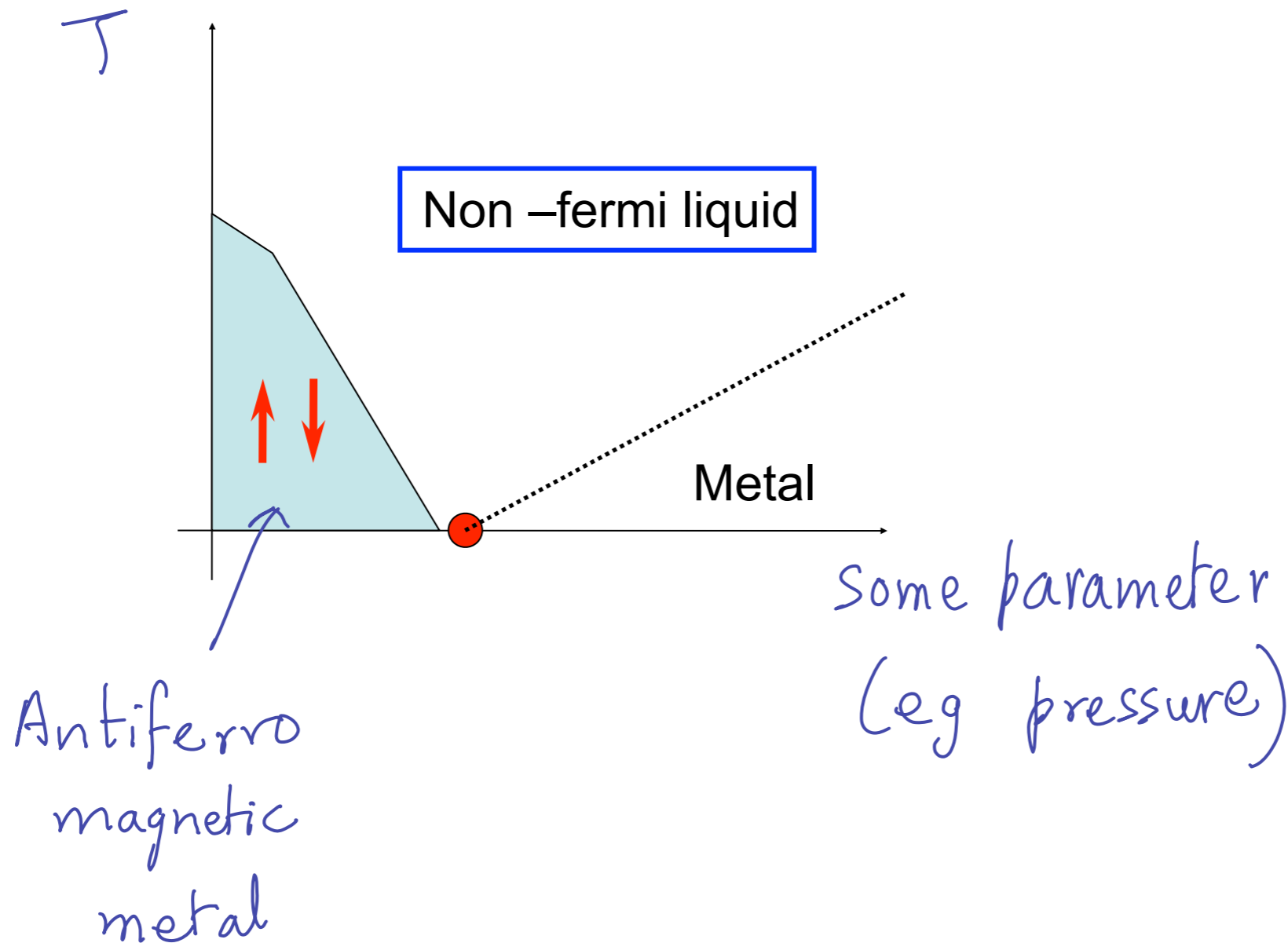
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Model as lattice of localized magnetic moments  
coupled to conduction electrons by spin exchange  
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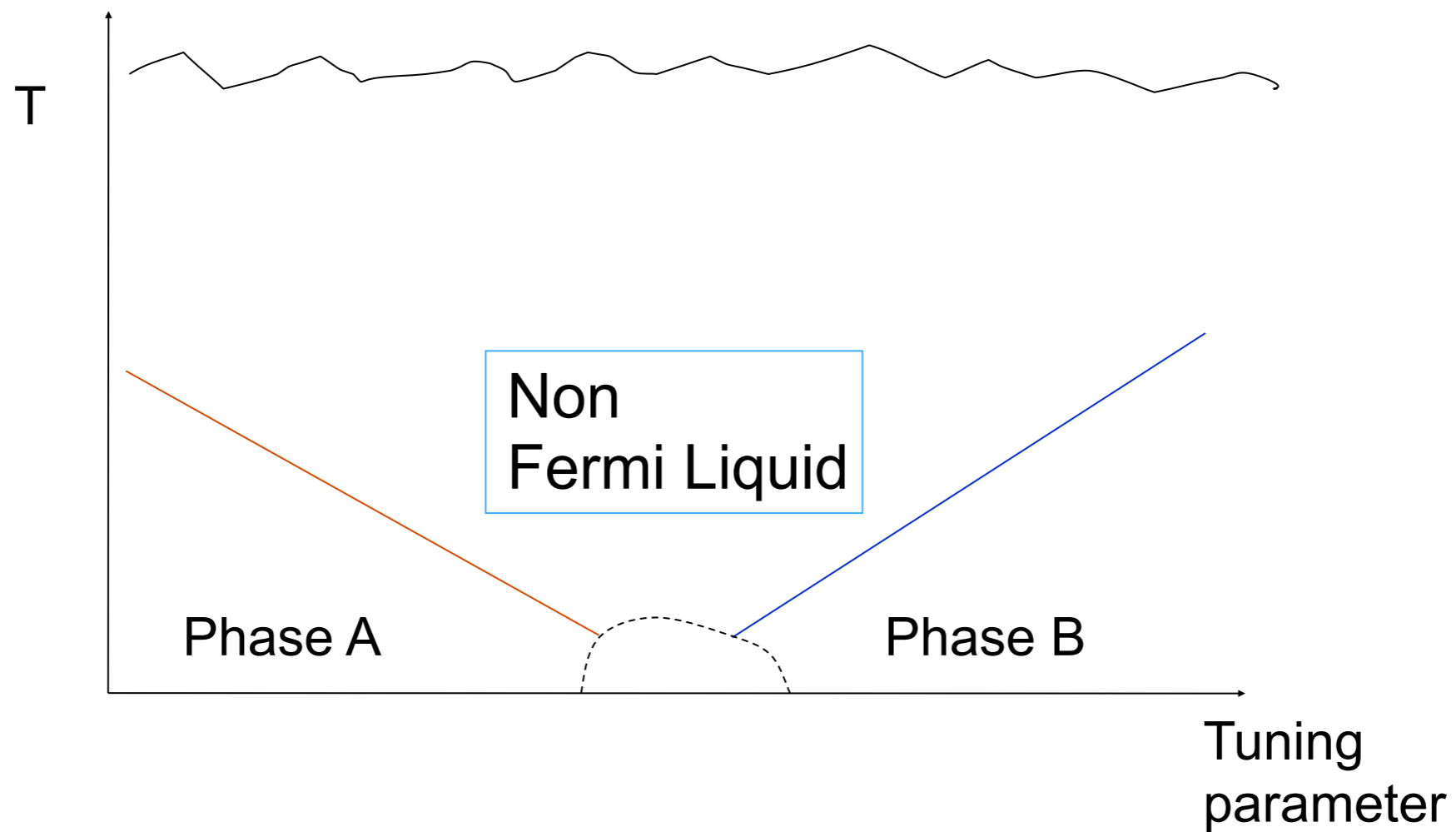
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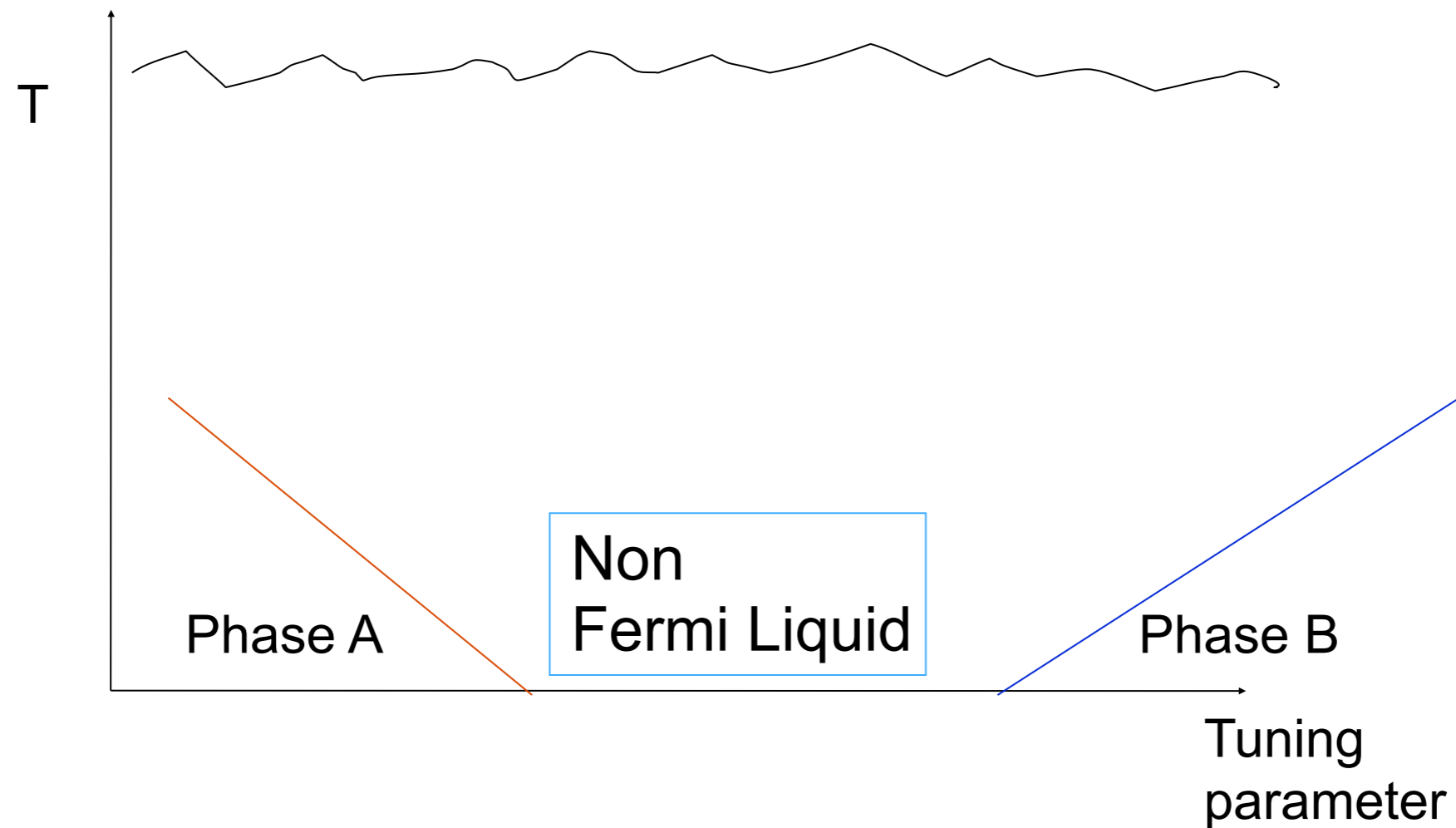


# Roughly common phase diagram



Roughly similar phase diagrams for many other non-fermi liquid metals.

# A less common phase diagram



Reasonably stable non-fermi liquid phases at  $T = 0$  seem to occur only rarely.

Examples: heavy fermion metals  $\beta$ -YbAlB<sub>4</sub>, YbRh<sub>2-x</sub>Ir<sub>x</sub>Si<sub>2</sub> (?), may be also d-electron metal MnSi (?).

?? Physics or humankind history ??

# General question

How to think about unstable intermediate energy scale non-fermi liquid regimes?

# Understanding unstable Non-Fermi Liquids (NFL)

Two kinds of intermediate energy states of quantum matter

## 1. "Semi-quantum" NFL regimes

- NFL which is necessarily unstable at low-T.

=> does not correspond to ground state behavior with any finite amount of tuning.

Theories of such NFL need "IR-completion".

## 2. Quantum $T = 0$ NFL fixed points

NFL in intermediate energy regime that is controlled by approach to finitely unstable  $T = 0$  quantum fixed point.

# ``Semi-Quantum'' non-fermi liquids

Several examples

1. Metals at  $T_{\text{Debye}} \ll T \ll E_F$

Phonons are classical but electrons are quantum.  
Resistivity linear in  $T$ , etc.....

2. Other similar: quantum electrons coupled to some other ``classical'' fluctuation (eg: antiferromagnetic spin fluctuations, gauge fields, etc)

3. ``Spin-incoherent'' metals

Electron spin in its high temperature incoherent regime but charge motion quantum.

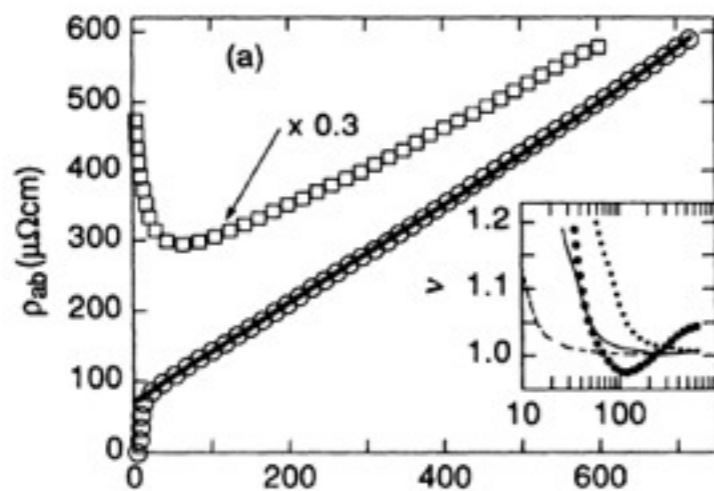
Best studied in 1d: spin-incoherent Luttinger liquids  
(Matveev 03, Cheienov et al 04, Fiete, Balents 04)

# Are cuprate/heavy fermion strange metals “semi-quantum” liquids?

Experiment: Does NFL extend to energy scales low compared to microscopic scales?

Cuprates: Microscopic energies  $J \approx 1500$  K,  $t \approx 3J$ .

Example: Linear resistivity in cuprates

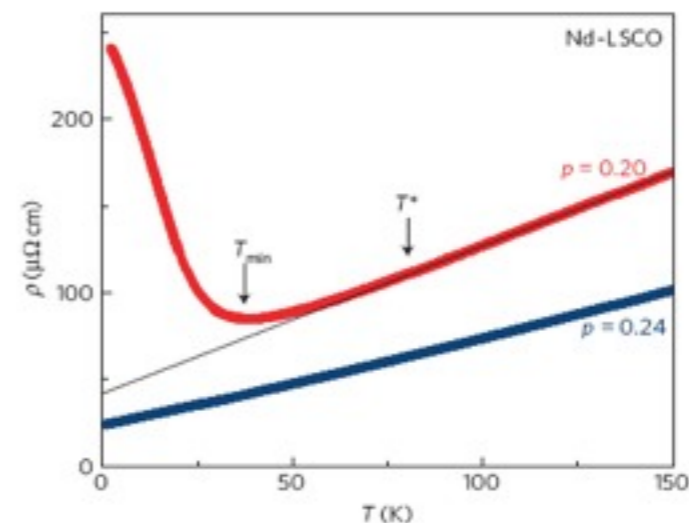


Bi-2201: Martin et al, PR B, 1990

Linear resistivity down to 10 K

Many aspects of observed NFL unlikely to be “semi-quantum”.

Similar for many heavy fermion NFLs.....



Nd-LSCO, Daou, ..., Taillefer, Nat. Phys., 2008

# Understanding unstable Non-Fermi Liquids (NFL)

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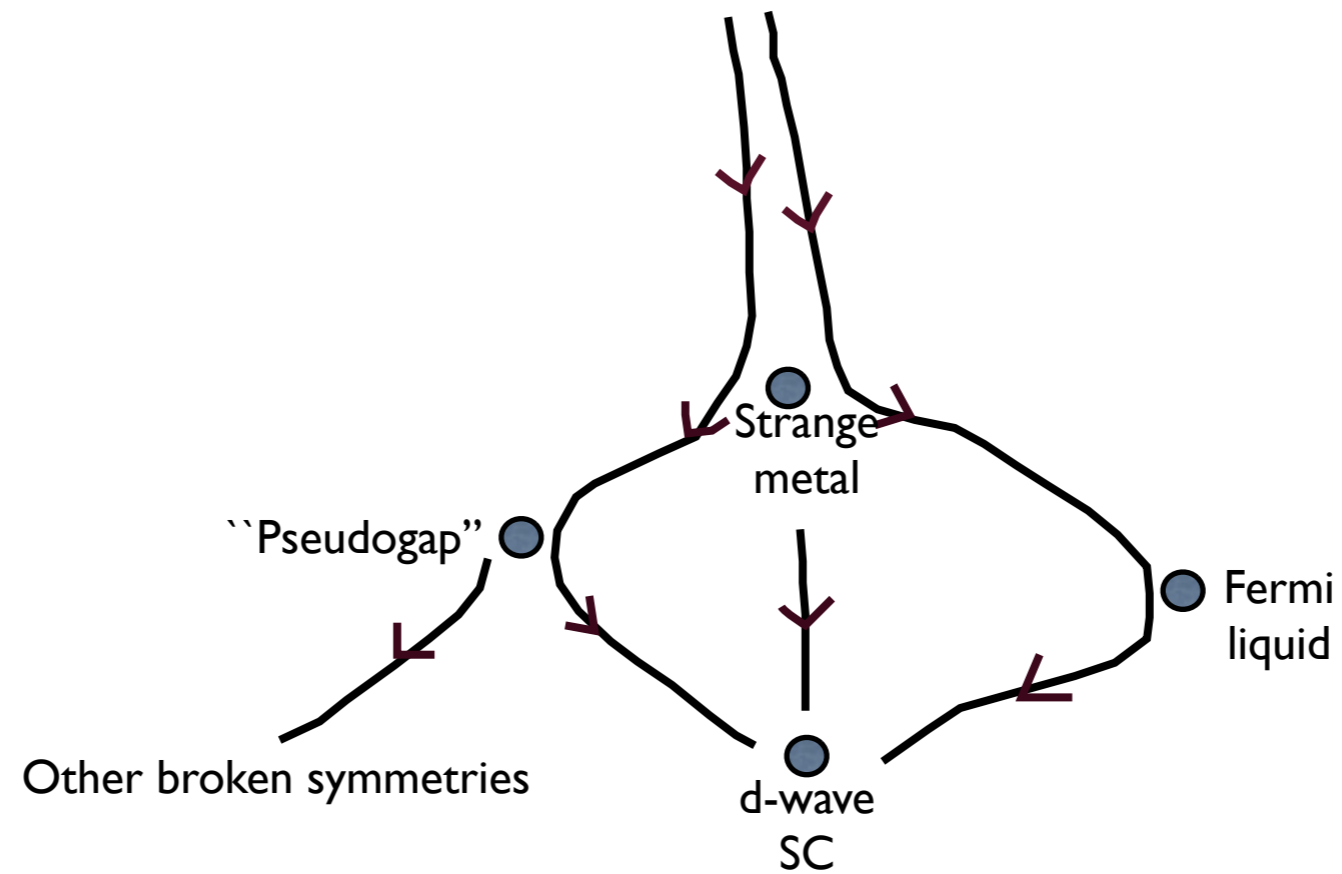
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# Unstable quantum non-fermi liquids

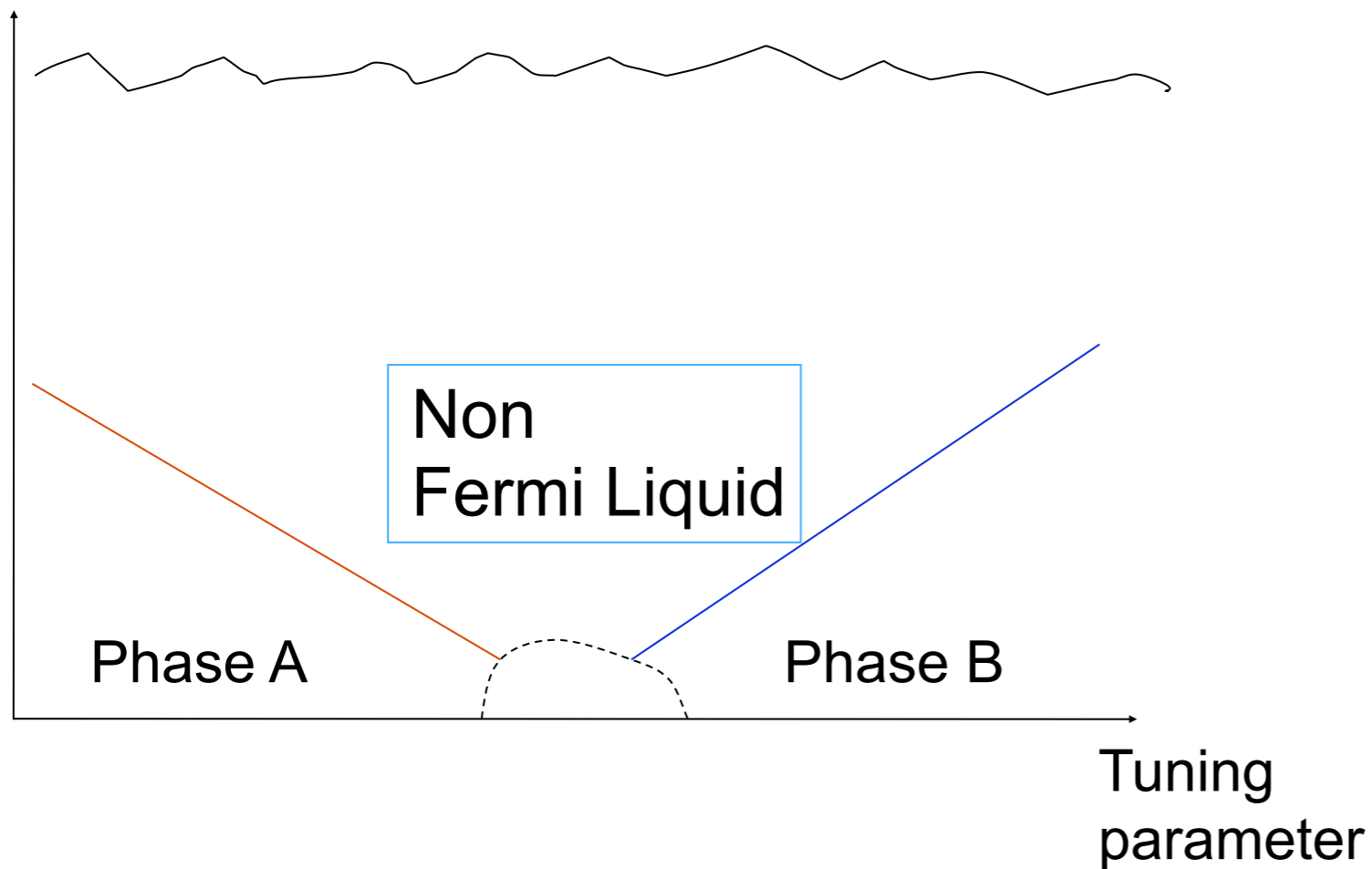
RG point of view of, eg, cuprate strange metal



Strange metal physics controlled by close approach to a quantum non-fermi liquid fixed point with small number of relevant perturbations.



# Unstable quantum fixed points and quantum criticality



Such an unstable quantum fixed point clearly also controls putative quantum phase transition between Phases A and B.

May be can turn it around:

Whatever causes phase transition between A and B also responsible for NFL physics ??

# ``Natural'' assumptions

1. Non Fermi Liquid: Universal physics associated with (quantum) critical point between phases A and B.
2. Landau: Universal critical singularities  $\sim$  fluctuations of order parameter for transition between phases A and B.

Try to play Landau versus Landau.

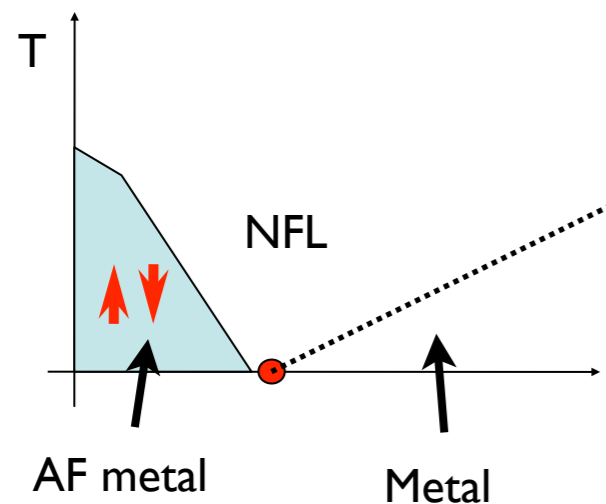
Attempt to describe NFL as Fermi surface + X.

X = gapless order parameter fluctuations

# Quantum criticality: the savior or a distraction?

Despite its apparent appeal, a naive invocation of quantum criticality to explain NFL has serious problems.

Example: heavy fermion metals - the best case?



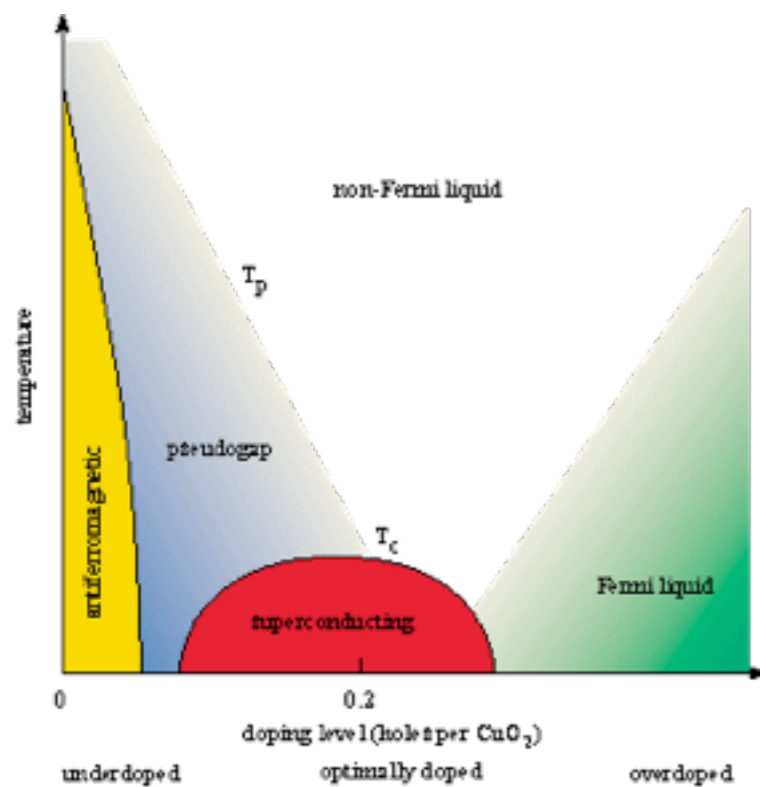
Theory of critical antiferromagnetic spin fluctuations coupled to Fermi surface well developed in 3d;

Weak departures from Fermi liquid spectacularly inconsistent with experiment.

(Eg: no linear resistivity, no scaling in spin fluctuation spectrum, etc)

# What about cuprates?

Despite its apparent appeal, a naive invocation of quantum criticality to explain NFL has serious problems.



Overdoped - no broken symmetry above SC  $T_c$

Underdoped: Several broken symmetry orders (apart from SC)

1. Antiferromagnetism

2. Broken translation (stripes)

3. Nematic

4. Broken T-reversal

5. Loop currents (break T-reversal, rotation but preserve combination, translation invariant)

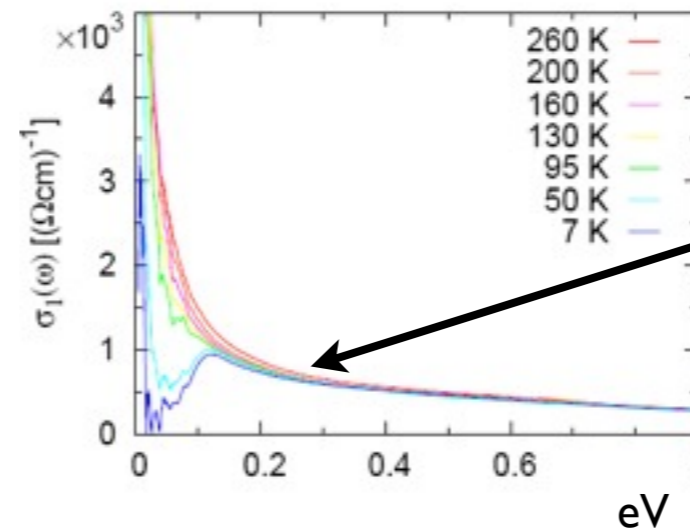
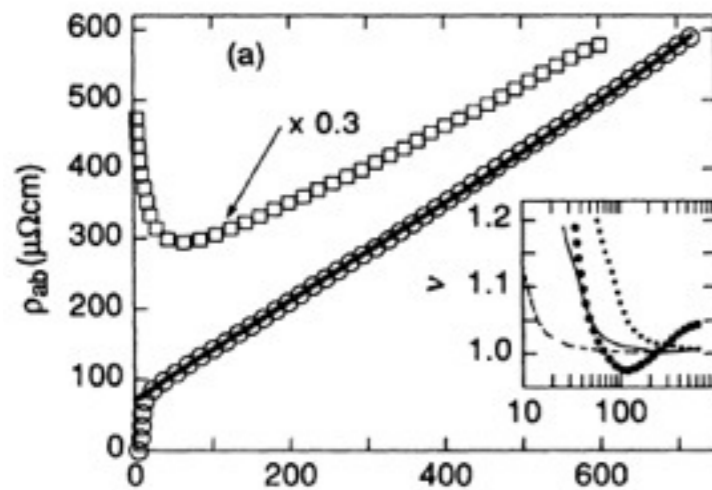
Could critical fluctuations of any of these + Fermi surface be strange metal?

# Quantum criticality: the savior or a distraction?

Despite its apparent appeal, a naive invocation of quantum criticality to explain NFL has serious problems.

In cuprates, some NFL properties extend to very high energy scales.

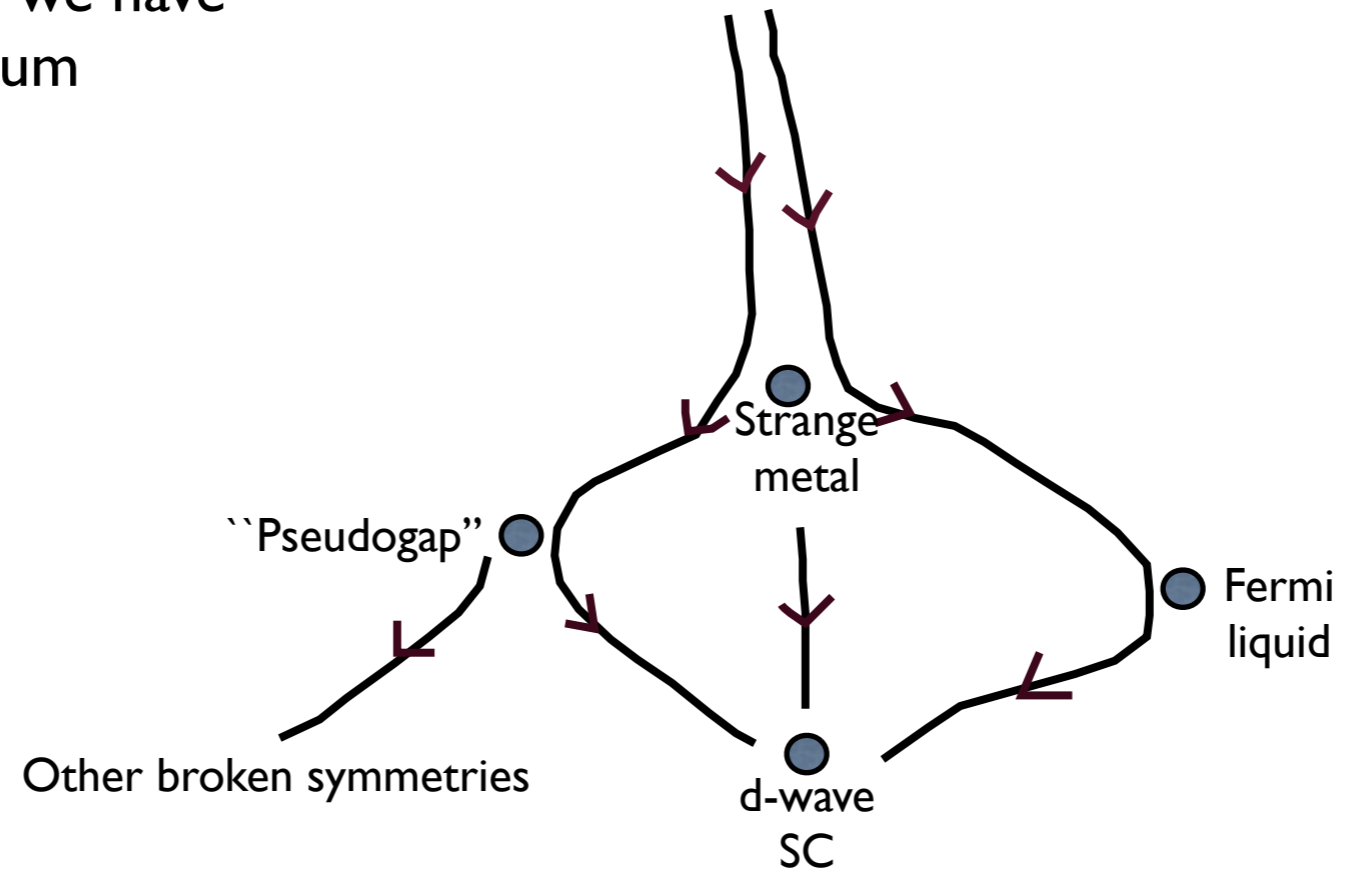
Broken symmetry order parameters are emergent low energy variables => difficulty with UV scale.



Theory of onset of some of these orders not well understood (see Metlitski talk).

# Naive quantum critical point of view problematic

Within RG perspective we have wrong picture of quantum critical point.....



So possibly.....

(i) Failure of textbook theory of metals

AND

(ii) Failure of textbook theory of phase transitions

Important clue from experiments



# Killing a Fermi surface

At certain such  $T = 0$  phase transitions in metals, an entire Fermi surface may **disappear**.

Eg: (i) Onset of magnetism in rare earth alloys

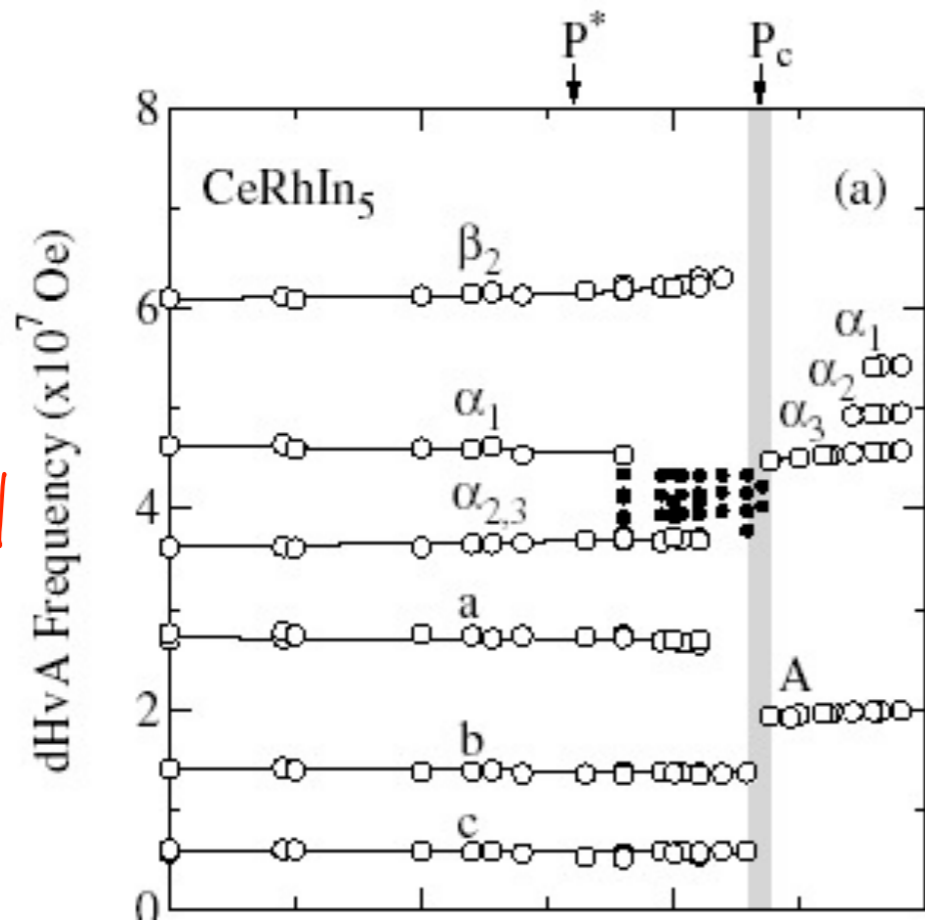
(ii) Transition from metal to (Mott) insulator

(iii) High- $T_c$  cuprates as function of doping?

IF second order, non-fermi liquid very natural!

# Example: Evolution of Fermi surface across the magnetic phase transition in CeRhIn<sub>5</sub>

Area of extremal Fermi surface orbits in some direction



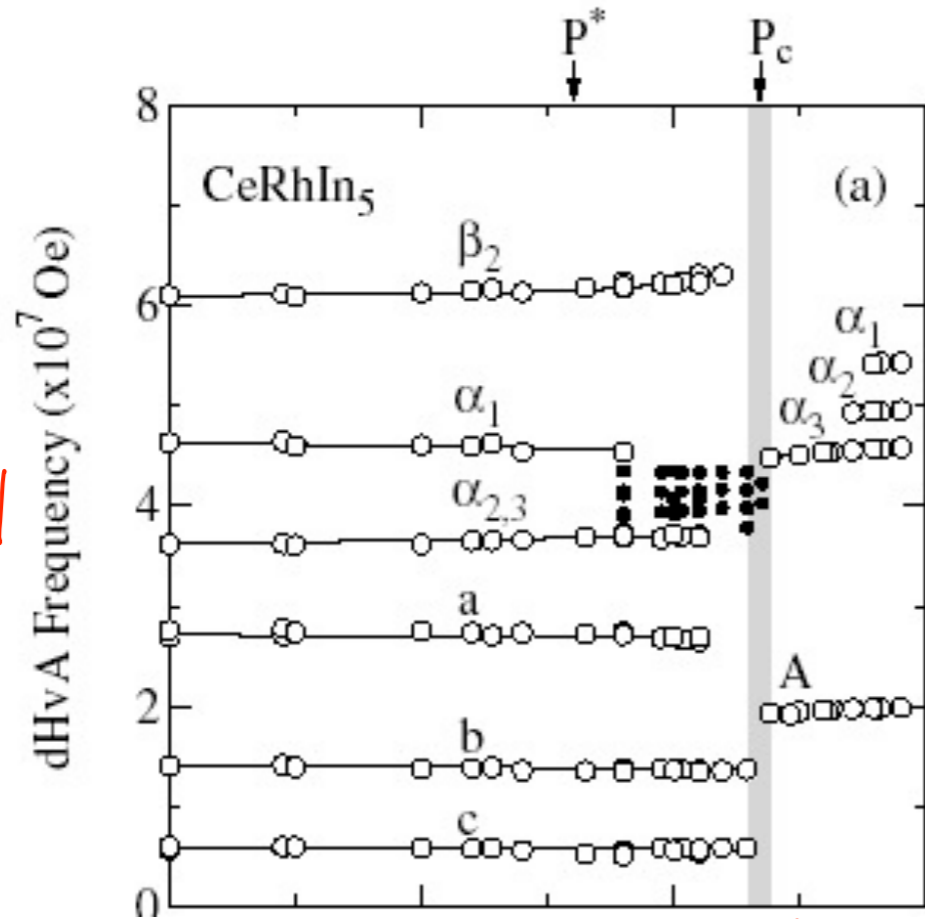
Magnetic phase transition

*H. Shishido, R. Settai, H. Harima, & Y. Onuki, JPSJ 74, 1103 (2005)*

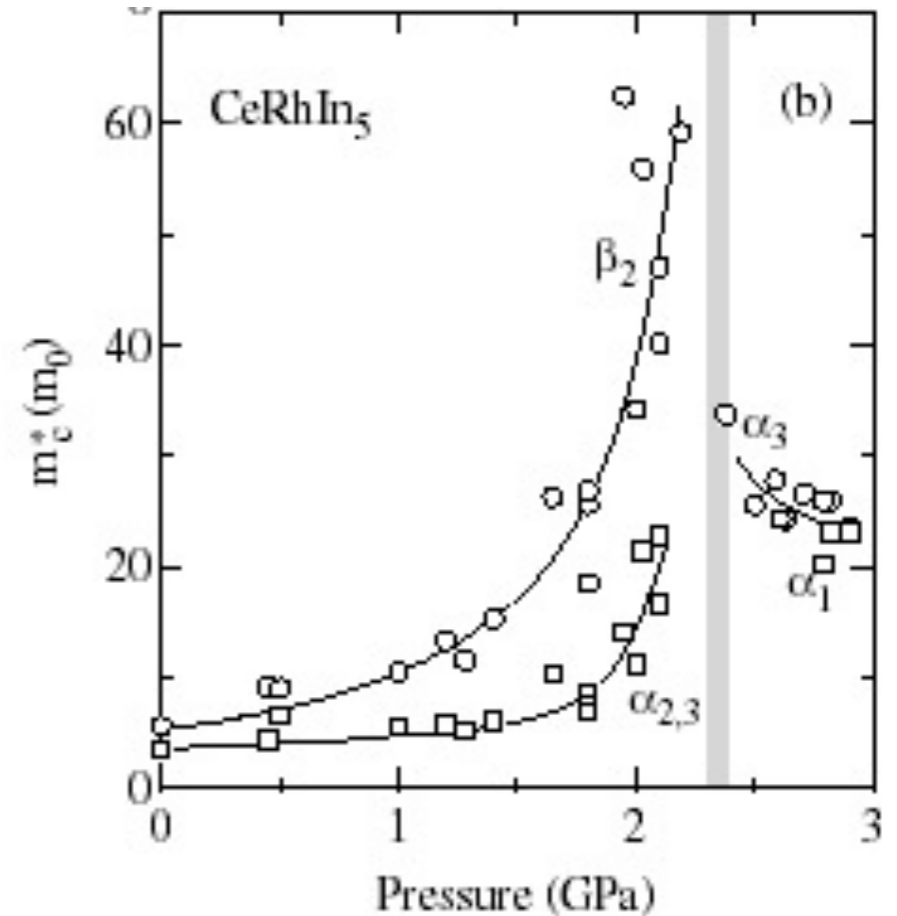
Entire sheets of Fermi surface disappear at the phase transition.

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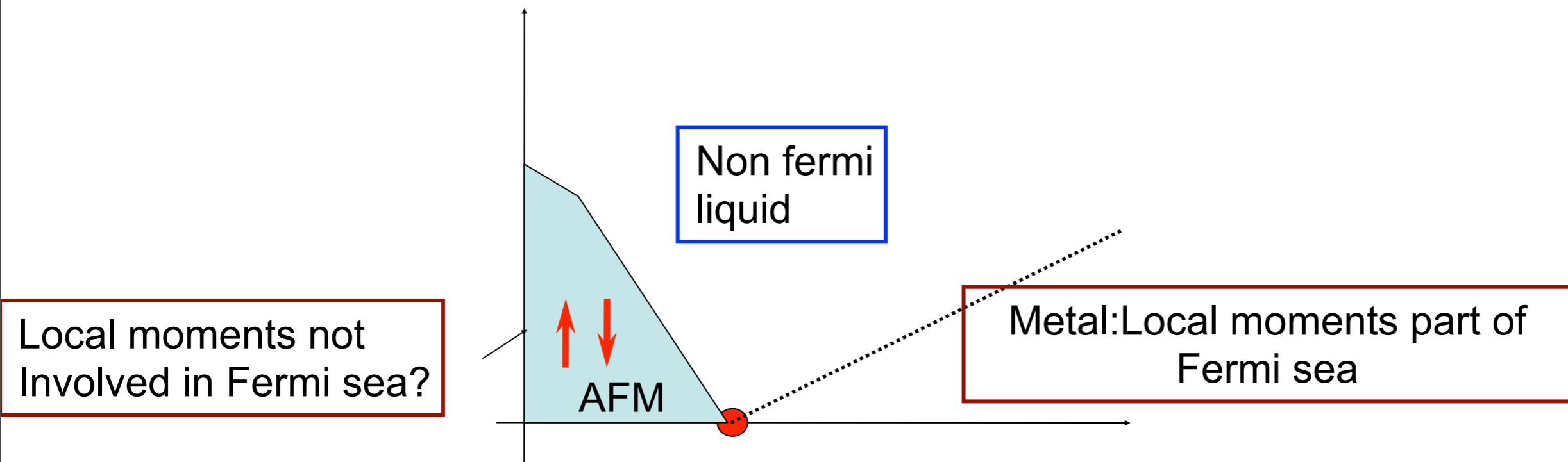
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# Non fermi liquid due to fluctuations different from natural order parameter?



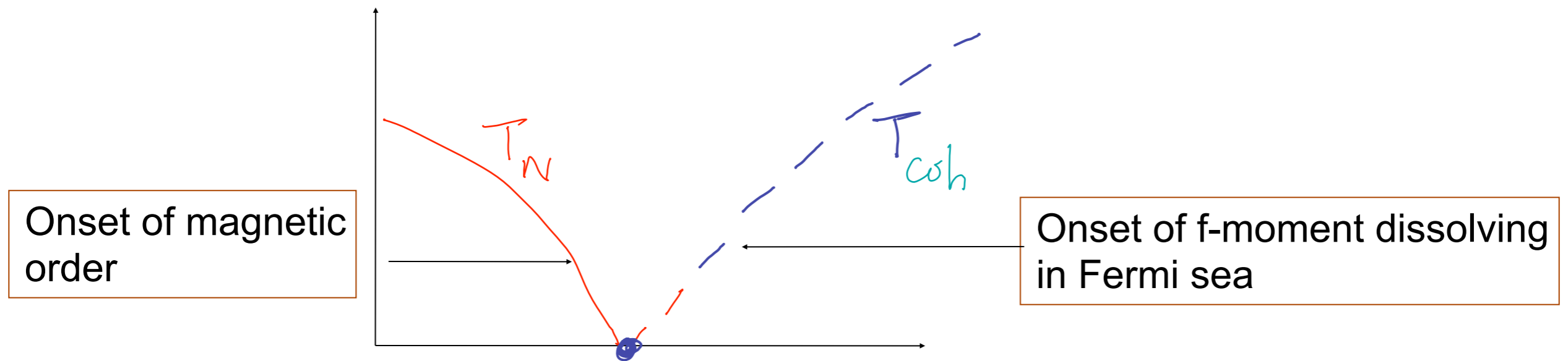
Local moments not Involved in Fermi sea?

Non fermi liquid

Metal:Local moments part of Fermi sea

Non-fermi liquid physics: fluctuations of loss of local moments from Fermi sea? (Si, Coleman, Pepin, TS, Vojta, Sachdev.....)  
Magnetic order – a distraction?? (TS, Vojta, Sachdev)

# Questions



1. Is such a second order transition generically possible?  
(Loss of magnetic order happens at same point as f-moments dissolving in Fermi sea)
2. Theoretical description?
3. Will it reproduce observed non-fermi liquid behaviour?

Answers not known!!

# Interesting possibility

- ``Underlying`` transition: loss of participation of the f-electrons in forming the fermi liquid. (View as a Mott ``metal-insulator`` transition of f-band).
- Magnetic order: ``secondary`` effect – a low energy complication once f-moments drop out of Fermi surface.
- Non-fermi liquid due to fluctuations associated with change of electronic structure rather than those of magnetic order parameter

## The promise

1. Changes of electronic structure involve fundamental electronic scales  
=> effects may extend to higher energy scales than for emergent order parameter fluctuations
2. Discontinuous change of Fermi surface at a continuous phase transition: Non-fermi liquid guaranteed.

# A simpler example: the metal- insulator transition

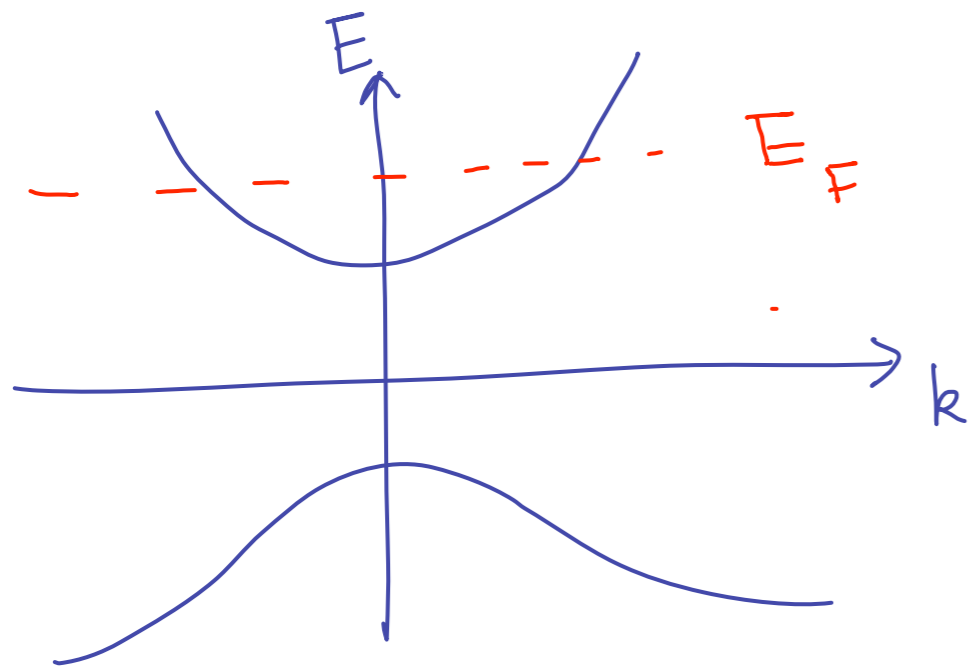
Insulators do not have Fermi surfaces!

When a metal evolves into an insulator (eg by carrier doping, or by tuning pressure), it must lose its Fermi surface.

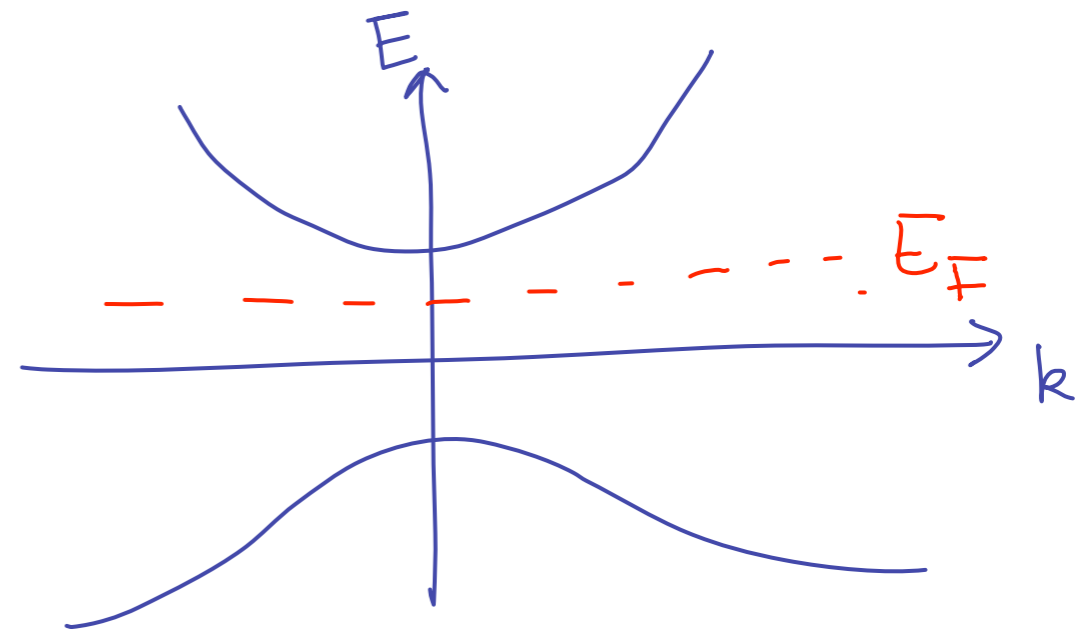


How does Fermi surface die when a metal evolves into an insulator?

Simplest possibility: Fermi surface shrinks in size and disappears.



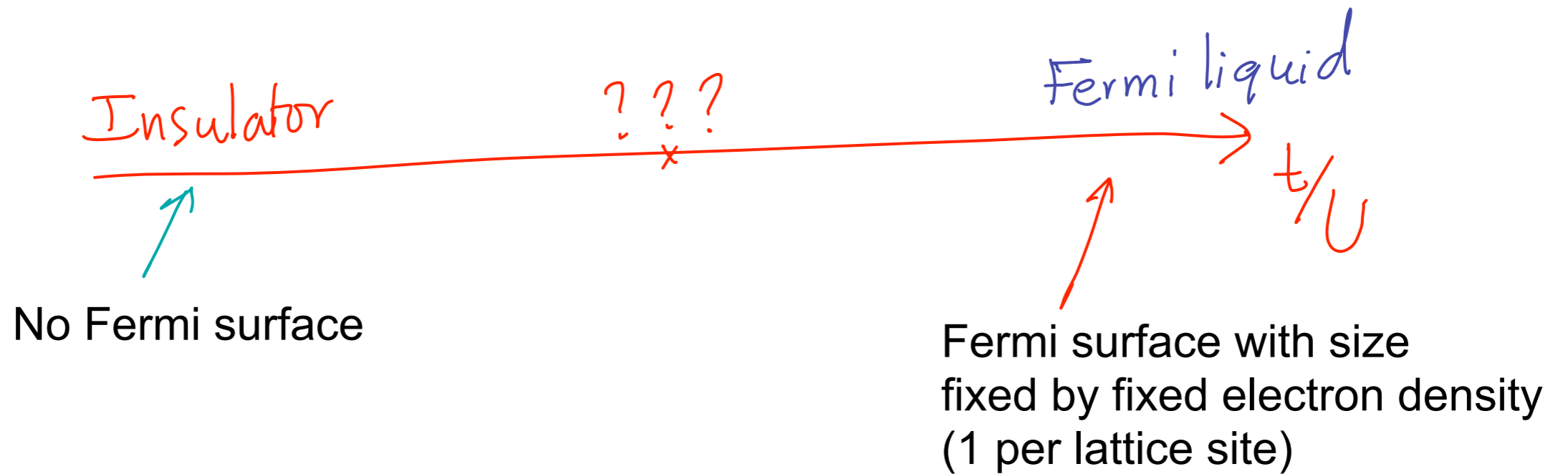
Metal



Band insulator

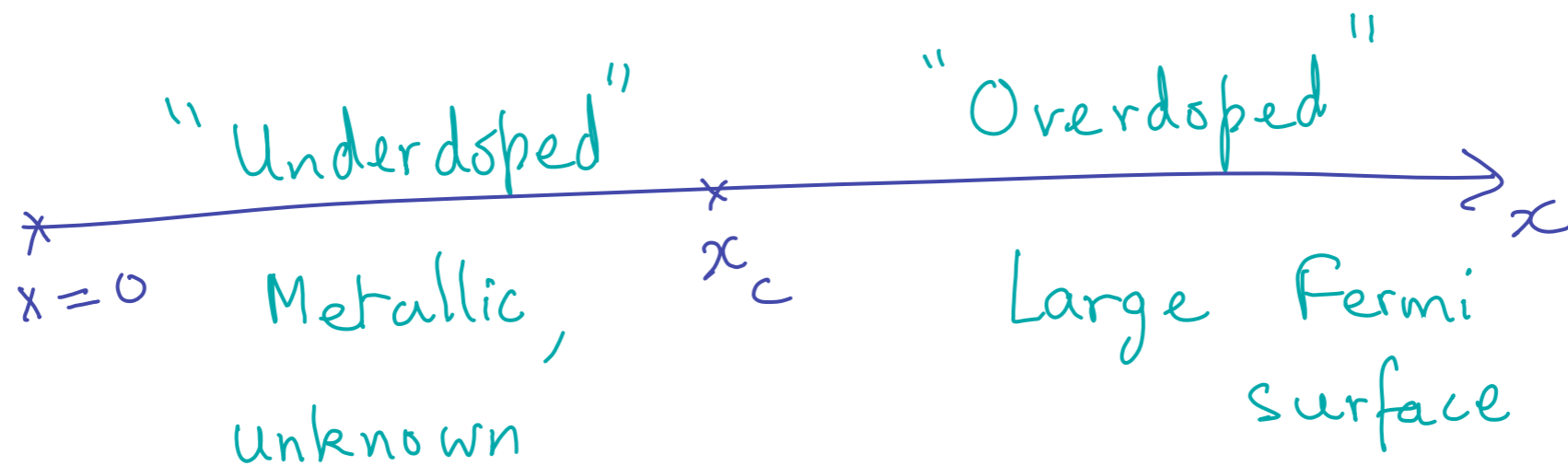
Question more interesting if insulation is due to Coulomb repulsion, i.e, a 'Mott' insulator

# Evolution from metal to insulator



In evolving from metal to insulator, entire Fermi surface of metal needs to disappear.

"Underlying" normal state (i.e. state in absence of superconductivity)



Large Fermi surface disappears for  $x < x_c$ .

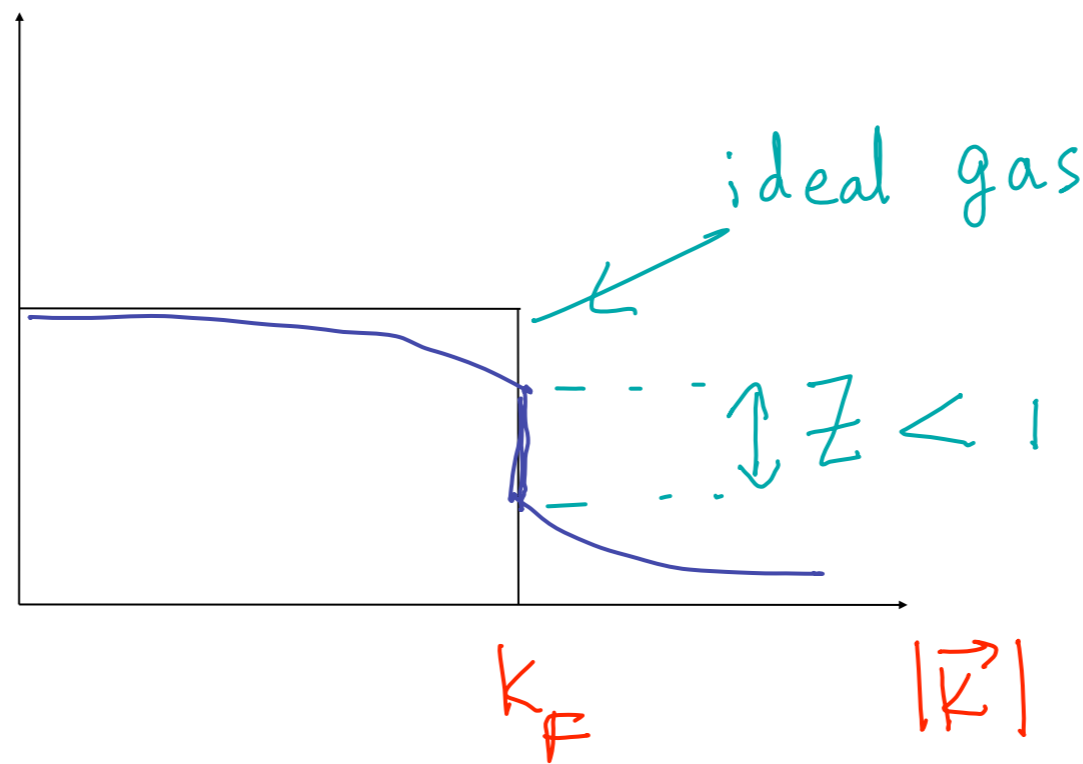
# A basic question for theory

How can an entire Fermi surface disappear continuously?

# Even more basic: What is the Fermi surface?

Interacting Landau Fermi liquid

Momentum occupation  $n(\vec{k}) = \langle c_{\vec{k}}^{\dagger} c_{\vec{k}} \rangle$



Sharp jump discontinuity  $Z$  in  $n(\vec{k})$  at  $k_F$ .

$Z$  = extent to which  $e^-$  overlaps with Landau quasiparticle

# How can a Fermi surface die continuously?

Continuous disappearance of Fermi surface if quasiparticle weight  $Z$  vanishes continuously everywhere on the Fermi surface (Brinkman, Rice, 1970).

Concrete examples: Hubbard model in infinite  $d$  (Vollhardt, Metzner, Kotliar, Georges 1990s), slave particle theories in  $d = 2, d = 3$  (TS, Vojta, Sachdev 2003, TS 2008)

# Electronic structure at a continuous Mott transition

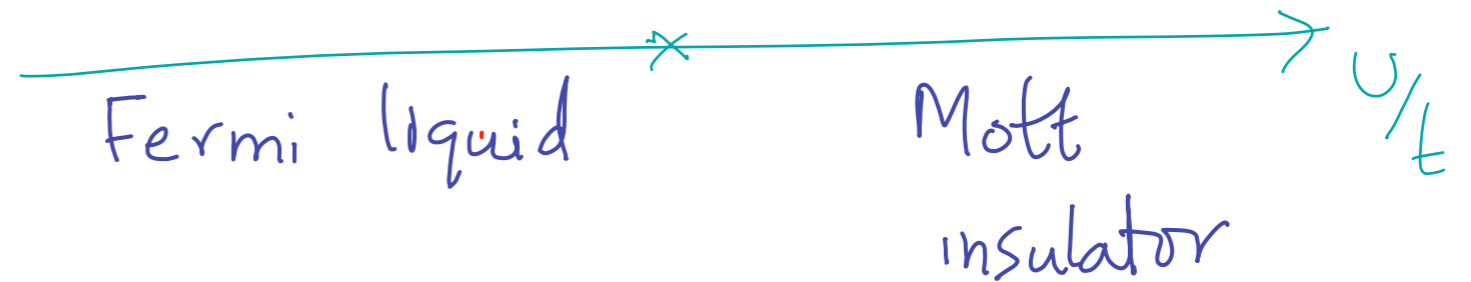
Crucial question: Electronic excitation structure right at Mott critical point when  $Z$  has just gone to zero?

Claim: At critical point, Fermi surface remains sharply defined even though there is no Landau quasiparticle (TS, 2008)

“Critical Fermi surface”

# Why a critical Fermi surface?

Mott transition



What is gap  $\Delta(\vec{k})$  to add an electron at momentum  $\vec{k}$ ?

Fermi liquid:  $\Delta(\vec{k} \in FS) = 0$

Mott insulator: Sharp gap  $\Delta(\vec{k}) \neq 0$  for all  $\vec{k}$



# Evolution of single particle gap

Approach from Mott

2<sup>nd</sup> order transition to metal  $\Rightarrow$  expect Mott gap

$\Delta(\vec{k})$  will close continuously

To match to Fermi surface in metal,  $\Delta(\vec{k}) \rightarrow 0$   
for all  $\vec{k} \in FS$ .

$\Rightarrow$  Fermi surface sharp at critical point.

But as  $Z = 0$  no sharp quasiparticle

$\Rightarrow$  Non-Fermi liquid with sharp "critical" Fermi surface!

# Killing the Fermi surface

Disappearance of Fermi surface through a continuous phase transition requires at critical fixed point

1.  $Z = 0$

2. Fermi surface sharp

# Comments

Destruction of Fermi surface related to competition between Mott physics and electron delocalization.

Many of the interesting low energy phenomena happen when neither overwhelmingly dominant.

Natural that these phenomena emerge from a universal ``mother'' fixed point where this basic competition critically destroys the Fermi surface.

Hard problem - concrete results on simplified models will be extremely useful.

# Computational framework

Only currently available framework: Slave particle methods

View electron as composite of `slave' particles with fractional quantum numbers

Reformulate electron model in terms of slave particles interacting through gauge forces.

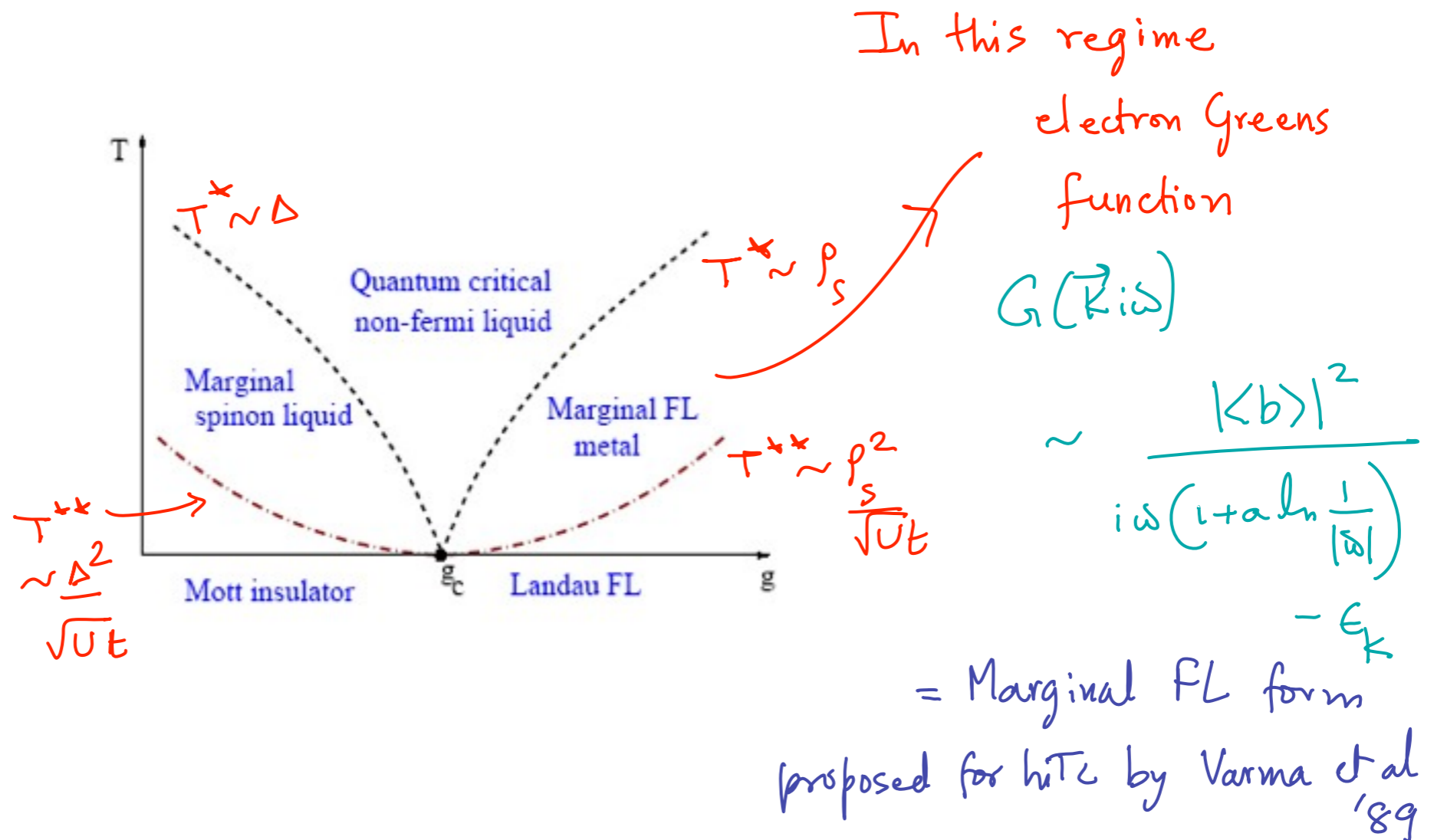
Provides concrete examples of phase transitions where an entire Fermi surface disappears continuously.

Successes: Demonstrate critical Fermi surface, emergence of non-fermi liquids (TS, 2008)

Important as proof of principle, application to experiment with caution.

# An example

Continuous Mott transition between a metal and a spin liquid Mott insulator in 2d (TS, 2008)



Cannot be described as Fermi surface + X

# Other results

Similar models for destruction of Fermi surfaces can be studied in variety of contexts.

1. Chemical potential tuned version (TS, 08; TS and P.A. Lee, 09)
2. Continuous Mott transitions in 3d to a spin liquid  
(Podolsky, Paremakanti, Kim, TS, 09)
3. Kondo breakdown transitions between heavy Fermi liquids and  
“fractionalized” Fermi liquids where f-band becomes spin liquid  
(TS, Vojta, Sachdev, 03; Paul, Pepin, Norman, 07; Pepin, 08)

# Comments/summary

All examples involve a `ghost' Fermi surface of fractionalized slave particles to survive when the electron Fermi surface is destroyed.

This is the only state we can naturally access within the slave particle framework.

Probably need to go beyond this framework to describe situations where such ghost Fermi surfaces do not survive.

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**Need help!**