The Emergent Fermi Liquid

Fermions, Fermions, Fermions...

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arxiv/0904.1993

Lee: arxiv/0809.3402
Liu, McGreevy, Vegh: arxiv/0903.2477
Faulkner, Liu, McGreevy, Vegh: arxiv/0907.xxxx
Cubrovic, Sadri, Schalm, Zaanen: arxiv/090x.xxx

AdS/CMT July 2009 KITP UCSB

- Electrons in the real world/in the "mundane" [Hartnoll (Monday)]

Experiment	**Theory**
Fermi Liquid	adiabatic continuation Fermi-gas Landau quasi-particles
BCS superconductor	BCS Pairing Spontaneous symmetry breaking

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Non Fermi Liquids - high T_c cuprates - heavy fermions - ...	**?**

- The non-Fermi liquid: two important characteristics
  - Physics controlled by a QCP [conjecture]
  - Adjacent phase is regular Fermi Liquid

- Fundamental problem:

Fermion-sign-problem

[original]

~~[Computer]~~

[**not** weakly interacting electrons:
interacting CFT?
strongly coupled?]

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Fermion-sign-problem [here]

How can a Fermi Liquid (defined by E_F, k_F)
***disappear* into a quantum critical state?**

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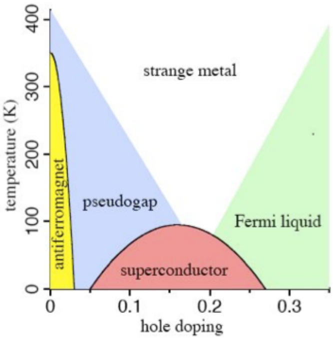
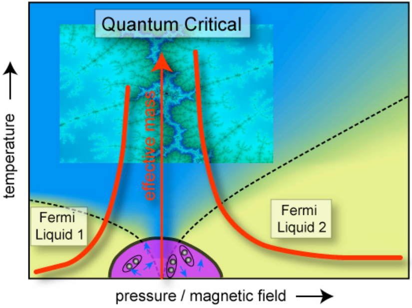
How can a Fermi Liquid (defined by E_F, k_F)
***emerge* from a quantum critical state?**

[**not** weakly interacting electrons:
interacting CFT?
strongly coupled?]

$\Rightarrow$difficult for standard QFT...
..."natural fit" for AdS dual

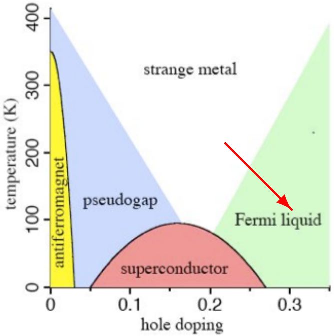




High T_c superconductors

Heavy fermion systems

The fermion-sign-problem II



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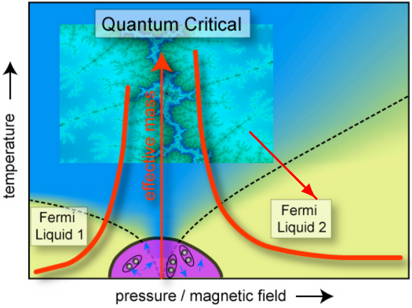

Heavy fermion systems

- Essential: Single *fermionic* quasi-particle spectrum *encodes* Fermi Liquid ground state

$$A(\omega, k) = -\frac{1}{\pi}\mathrm{Im}G_R(\omega, k)$$

AdS-to-ARPES

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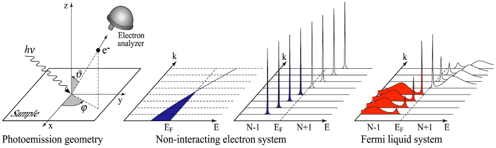
$$A(\omega, k) = -\frac{1}{\pi}\mathrm{Im}G_R(\omega, k)$$

- Experimental probe: ARPES [Source: A. Damascelli CIAR 2003]

Angle-Resolved Photoemission Spectroscopy



Photoemission intensity: $I(\mathbf{k},\omega) = I_0\,|M(\mathbf{k},\omega)|^2 f(\omega)\,A(\mathbf{k},\omega)$

Single-particle spectral function

$$A(\mathbf{k},\omega) = -\frac{1}{\pi}\frac{\Sigma''(\mathbf{k},\omega)}{[\omega-\epsilon_{\mathbf{k}}-\Sigma'(\mathbf{k},\omega)]^2+[\Sigma''(\mathbf{k},\omega)]^2}$$

$\Sigma(\mathbf{k},\omega)$: the "self-energy" - captures the effects of interactions

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$$A(\omega,k)=-\frac{1}{\pi}\mathrm{Im}G_R(\omega,k)$$

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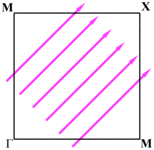
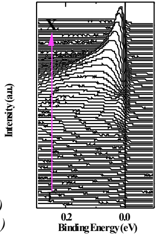
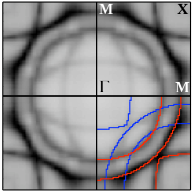
ARPES : present day



*A. Damascelli et al., PRL **85**, 5194 (2000)*
*K.M. Shen et al., PRB **64**, 180502R (2001)*





- Spectral density from charged AdS BH [e.g. McGreevy's lectures]
  - $G_R^{\Psi\Psi}(\omega,k)$: infalling b.c. at BH Horizon $\Leftrightarrow T>0$
  - $G_R^{\Psi\Psi}(\omega,k)$: finite μ_F? $\Leftrightarrow \mu_{U(1)}$

$\Rightarrow$ Charged AdS BH in 3+1 dim dual to 2+1 dim relativistic CFT:

$$\begin{aligned} ds^2 &= \frac{\alpha^2}{z^2}\left(-f(z)dt^2+dx_1^2+dx_2^2\right)+\frac{1}{f(z)}\frac{dz}{z^2} \\ A_0 &= 2q\alpha(z-1) \\ f(z) &= (1-z)(z^2+z+1-q^2z^3) \end{aligned}$$

$$4\pi T=\alpha(3-q^2)\ ,\quad \mu_0=-2q\alpha$$

Is $\mu_{U(1)} = \mu_F$ *?*

Caveat: dynamical instability in full theory

- AdS Phenomenology:
  - Do not know exact quantum theory (CFT or AdS) i.e. do not know constituents of the BH. There could be many $U(1)$ charged particles (fermions and bosons)

    It is not a priori guaranteed that $\mu_{U(1)}$ will act as μ_F

- Empirical approach:
  - QFT: In an interacting system μ_F renormalizes: $\mu_F^{(IR)}$ is empirically determined by the pole in G_R
  - AdS: We will follow the same approach. Expectation $\mu_{U(1)}^{(UV)} \equiv \mu_0$ induces a $\mu_F \equiv E_F$, whose value we read off from the spectrum. A priori $\mu_F \neq \mu_{U(1)}^{(UV)}$

- Action: Einstein YM + charged fermions

$$S = \int \sqrt{-g} \left[R + 6 - \frac{1}{4} F^2 - \bar{\Psi} e_A^M \Gamma^A \left(D_M + igA_M \right) \Psi - m \bar{\Psi} \Psi \right] + S_{bnd}$$

- Fermions:

[Henningson,Sfetsos Mueck,Viswanathan Henneaux Contino,Pomarol]

[1.] constrained (first order) system:

⇒ reduce to physical d.o.f.

$$\Gamma_z \Psi_\pm = \pm \Psi_\pm$$

[2.] action vanishes on shell.

⇒ add boundary term

$$\boxed{S_{bnd} = \int \sqrt{-h} \bar{\Psi}_+ \Psi_-}$$

⇒ variation $\frac{\partial S}{\partial \Psi_-}$ well-defined

[$L = \kappa = 1$; also set $g = 1$ from hereon]

- Standard AdS/CFT: .... with some subtleties
  - to solve Dirac Equation...

$$(\partial_z + \mathcal{A}_z)\Psi_\pm = \mp \not{\mathcal{T}} \Psi_\pm$$

  - ... write as second order equation

$$(\partial_z^2 + P\partial_z + Q)\Psi_+ = 0$$

  - ... compute on-shell action
    (bdy term only; bdy Ψ^0 determined via EOM)

$$\Psi_\pm = F_\pm(z)F_+^{-1}(z_0)\Psi_+^0 \Leftrightarrow \Psi_-^0 = F_-(z)F_+^{-1}(z_0)\Psi_+^0$$

  - Green's function

$$\boxed{G(z_0) = F_-(z_0)F_+^{-1}(z_0)}$$

- Unitarity bound Fermion mass [Contino,Pomarol;CZS;Iqbal,Liu]
  - 2nd order equation $(\partial_z^2 + P\partial_z + Q)\Psi_+ = 0$ has asymptotic solutions

$$\Psi_+ = z^{\frac{d+1}{2} - |m+\frac{1}{2}|}(A + \ldots) + z^{\frac{d+1}{2} + |m+\frac{1}{2}|}(B + \ldots)$$

  - CFT Green's function ($T = 0$) has a pole unless

$$-\frac{1}{2} < m \qquad (S_{bnd} \text{ breaks } m \Leftrightarrow -m)$$

$\Rightarrow$ Unitarity bound on CFT fermionic operator dimension

$$\langle \mathcal{O}_\Psi(-k)\mathcal{O}_\Psi(k)\rangle = k^{2\Delta_\Psi - d} \qquad \Delta_\Psi = \frac{d-1}{2} + |m + \frac{1}{2}|$$

$$\boxed{\frac{d-1}{2} < \Delta}$$

Which Δ/m?

- IR emergence of a new state
  - Recall from holographic superconductors [Gubser;Hartnoll,Herzog,Horowitz]

    Unitarity bound

    relevant perturbations = scalars with $\frac{-d^2}{4} < m_\phi^2 < 0$

  $\Rightarrow$ For fermions: choose mass near unitarity bound: $-\frac{1}{2} < m_\Psi$

  - Upper bound on mass?

    Green's function marginal at: $m_\Psi = 0, \ \Delta_\Psi = \frac{d}{2}$

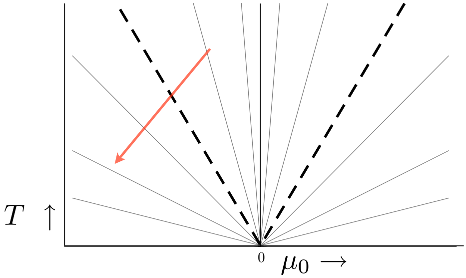
  $\Rightarrow$ Interesting range to obtain new physics

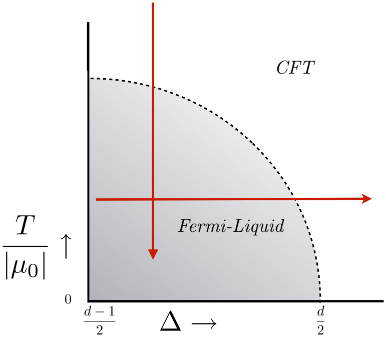
$$-\frac{1}{2} < m_\Psi < 0 \qquad -\frac{d-1}{2} < \Delta < \frac{d}{2}$$

[$m_\Psi = 0$: Lee:0809.3402; Liu, McGreevy, Vegh:0903.2477]

[Qualitative argument ($g = \kappa = 1$): quantitative cf. Denef,Hartnoll]

- Our expectation:





- Boundary AdS Green's function invariant under

$$\omega \to \lambda\omega \; , \; k \to \lambda k \; , \; \alpha \to \lambda\alpha$$

  - (recall that $4\pi T = \alpha(3 - q^2) \; , \;\; \mu_0 = -2q\alpha$)

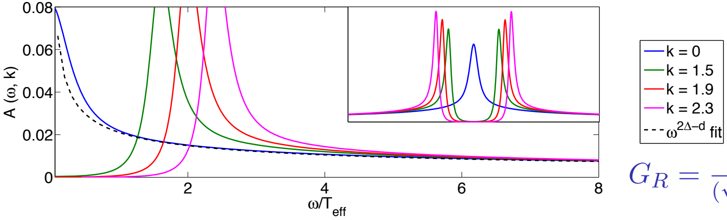
- in terms of CFT quantities

$$T_{eff}(\mu_0) = T\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{\mu_0^2}{T^2}}\right)$$

$$G_{\bar{\Psi}\Psi}^{\mu_0/T}(\omega, k) = T^{2\Delta_\Psi - d} f\left(\frac{\omega}{T_{eff}}, \frac{k}{T_{eff}}; \frac{\mu_0}{T}\right)$$

Spectral functions and a Quasiparticle peak

$$A(\omega, k) = -\frac{1}{\pi}\mathrm{Im}\mathrm{Tr}\left(i\gamma_0 G_R(\omega, k)\right)$$



$$\frac{\mu_0}{T} \to 0$$

$$G_R = \frac{1}{\left(\sqrt{-\omega^2+k^2}\right)^{d-2\Delta_\Psi}}$$

$$\Delta = 1.25$$

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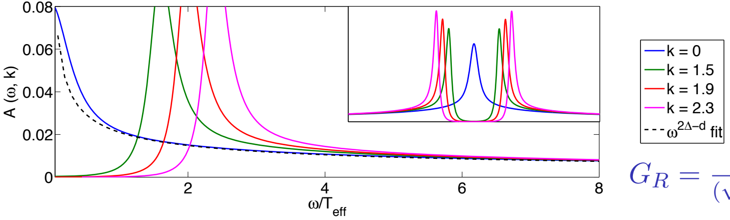
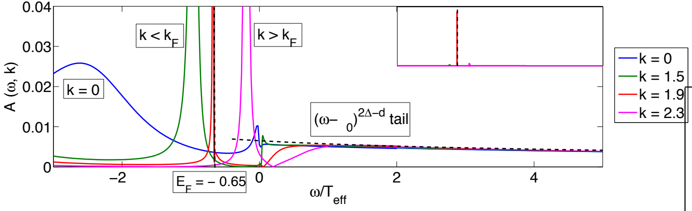


$$\frac{\mu_0}{T} \to 0$$

$$G_R = \frac{1}{(\sqrt{-\omega^2 + k^2})^{d-2\Delta_\Psi}}$$

$$\frac{\mu_0}{T} = 30.9$$



$$\frac{\mu_0}{T} > \left(\frac{\mu_0}{T}\right)_c$$

QP Peaks

$$\Delta = 1.25$$

- Single particle Green's functions encodes the ground-state

$$
\begin{aligned}
G(\omega,k) &= \frac{1}{\omega-\mu_0-\frac{k^2}{2m}+\Sigma(\omega,k)} \\
&\equiv \frac{Z}{(\omega-E_F)-v_F(k-k_F)+\ldots}
\end{aligned}
$$

  – Quasiparticle peak (QP) $\omega = E_F$, $k = k_F$ and a sharp Fermi surface $|k| = k_F$

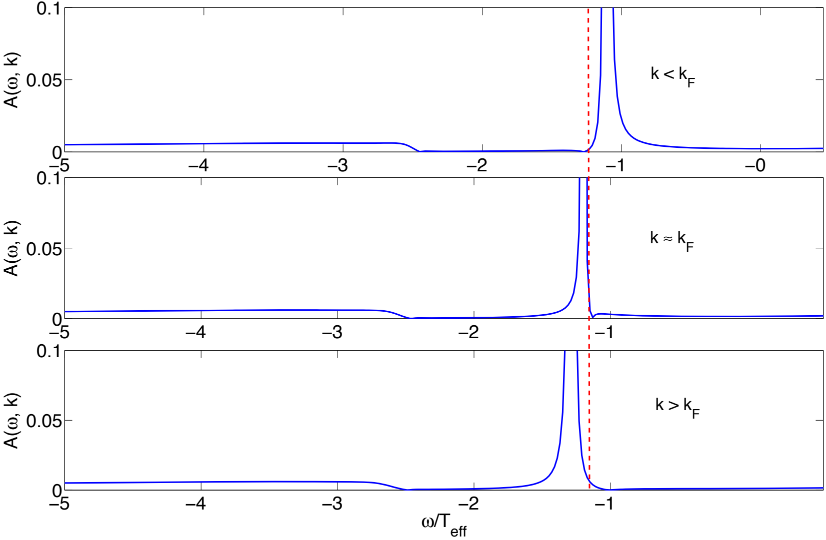
- Spectral function

$$
A(\omega,k) = \frac{\mathrm{Im}\Sigma(\omega,k)}{|\omega+\mu_0+\frac{(k-k_F)^2}{2m}+\mathrm{Re}\Sigma(\omega,k)|^2+|\mathrm{Im}\Sigma(\omega,k)|^2}
$$

List of Fermi Liquid tests:

- Quasiparticle peaks
- Zero density of states at the Fermi surface $A(E_F, k) = 0$
- Quadratic scaling of the QP-widths with temperature
- Analytical structure of the self-energy $\Sigma(\omega, k)$
- Linear dispersion of QP-excitations

Identifying $E_F,\ k_F$



$$\boxed{T=0:}\ A(E_F,k)=Z\delta(k-k_F) \Leftrightarrow \mathrm{Im}\Sigma(\omega,k)=\frac{1}{2}(\omega-E_F)^2\left.\frac{\partial^2\mathrm{Im}\Sigma}{\partial\omega^2}\right|_{\omega=E_F}$$

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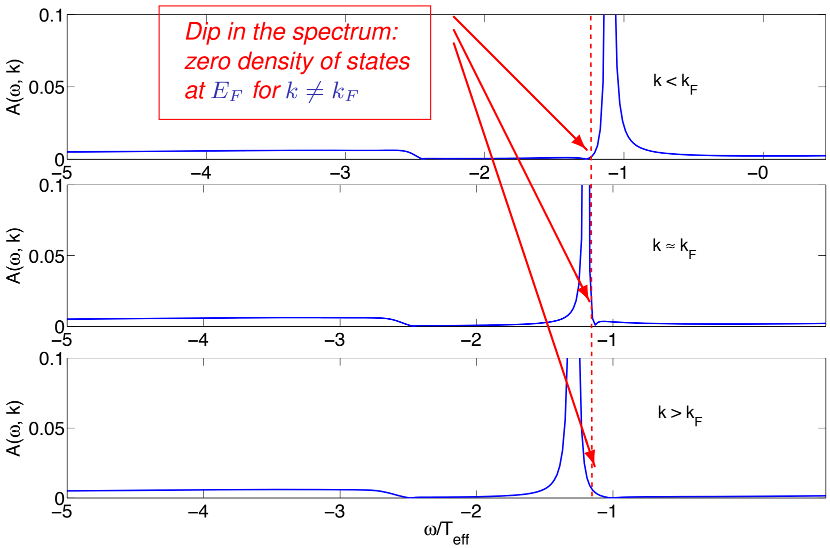

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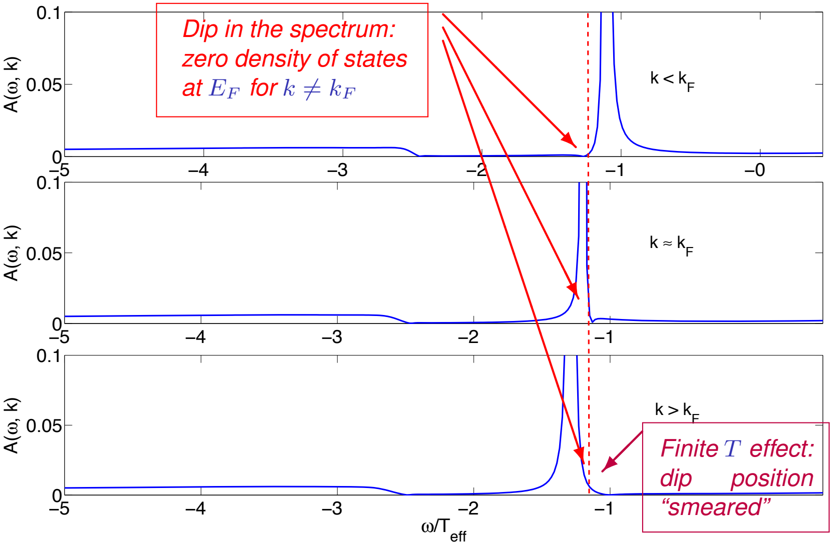

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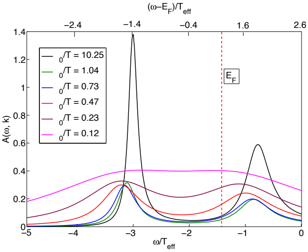
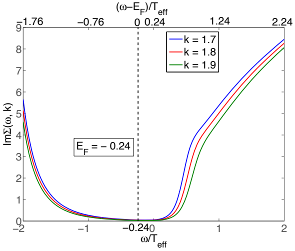
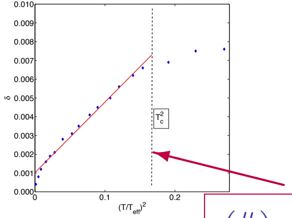
Temperature Dependence/Self-energy structure



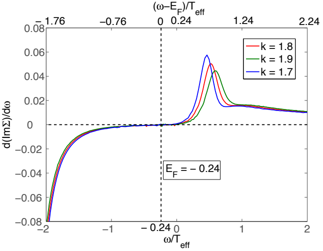


No constant term in $\partial \mathrm{Im}\Sigma/\partial\omega$

Peak width δ *vs.* T*:* $\delta \sim T^2$



$$\left(\frac{\mu}{T}\right)_c \simeq 4$$

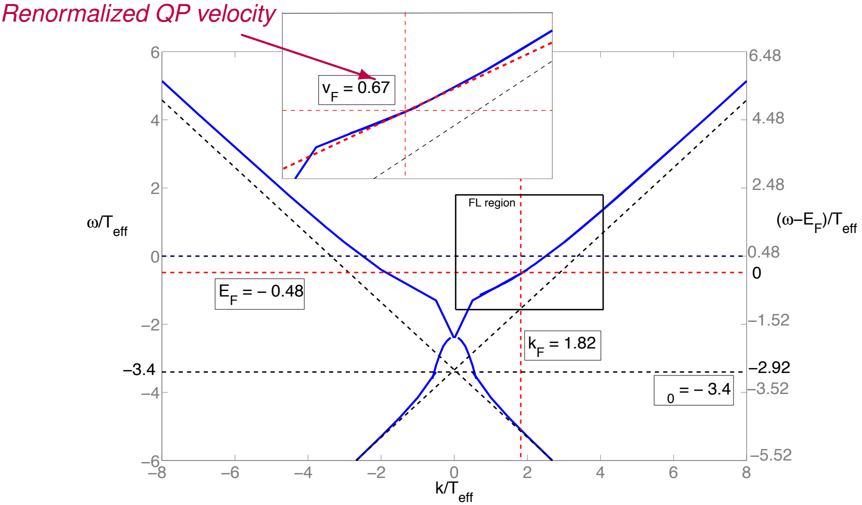


$$\Delta_\Psi = 1.05$$

$$\Delta_\Psi = 1.40,\ \mu_0/T = 30.9$$

Dispersion relation

Quasiparticle dispersion: $\omega - E_F = v_F(k - k_F) + \mathcal{O}((k - k_F)^2)$



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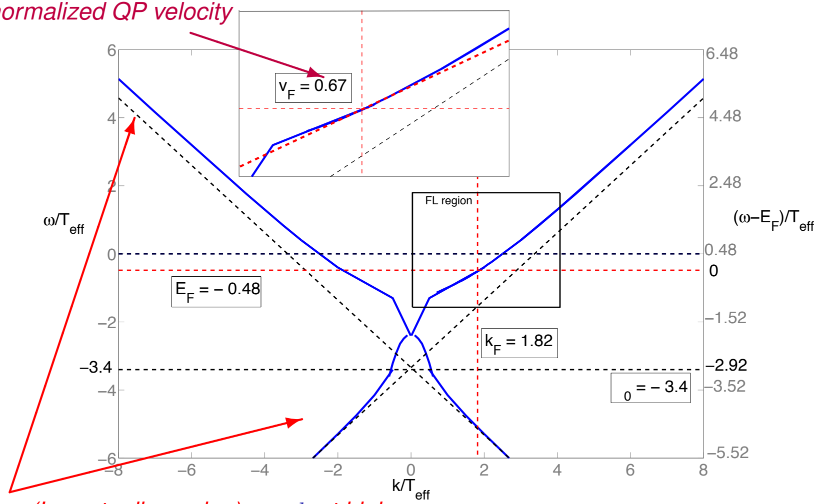
Renormalized QP velocity



Bare (Lorentz dispersion) $\omega = k$ *at high* ω

Dispersion relation

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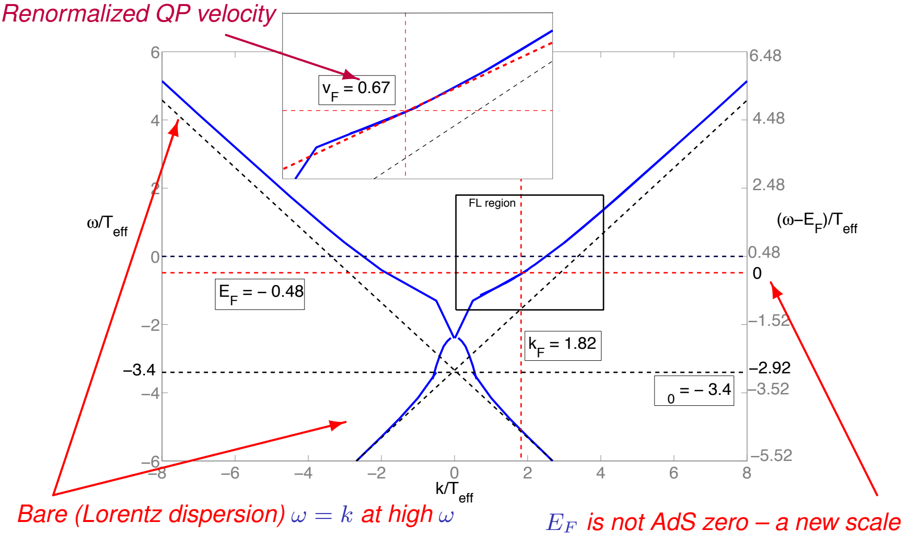

List of Fermi Liquid tests:

- Quasiparticle peaks ✓
- Zero density of states at the Fermi surface $A(E_F, k_F) = 0$ ✓
- Quadratic scaling of the QP-widths with temperature ✓
- Analytical structure of the self-energy $\Sigma(\omega, k)$ ✓
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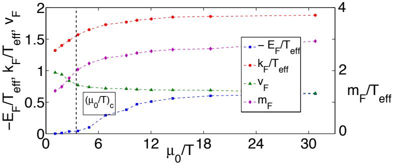
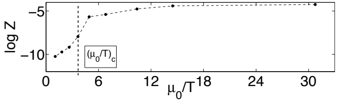
Charge density dependence





Fermi Liquid

$Z = \frac{m_{bare} v_F}{k_F}$

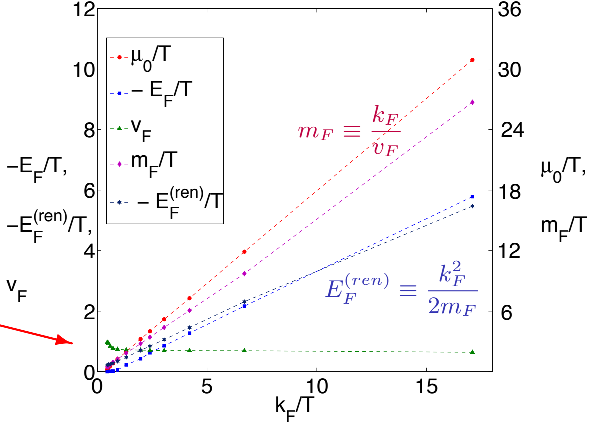
$v_F = Z(1 + \partial_k \mathrm{Re}\Sigma|_{k_F}) + \ldots$

$Z = 1/(1 - \partial_\omega \mathrm{Re}\Sigma|_{E_F}) + \ldots$

restored Lorentz invariance

[cf. Randeria et al cond-mat/0307217]

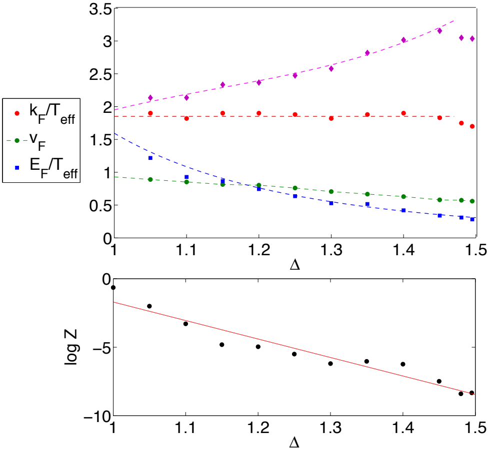
$\partial_k \mathrm{Re}\Sigma - \partial_\omega \mathrm{Re}\Sigma \sim 0$



$\Delta_\Psi = 1.25$

Coupling strength dependence

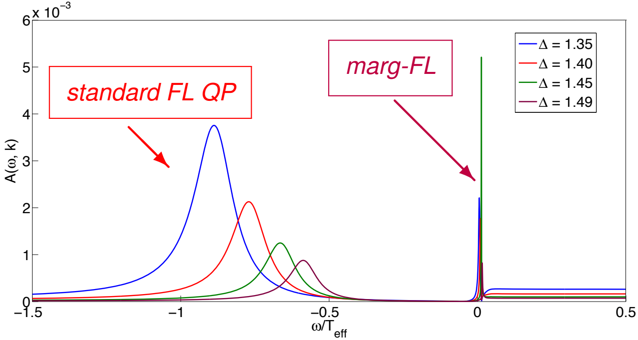
Scaling dimension Δ_Ψ proxy for coupling strength



$\mu_0/T = 30.9$

- $k_F \simeq$ constant for all Δ $\Rightarrow$ *Luttinger theorem!?*
- Pole strength Z drops exponentially as $\Delta \to 1.5$... [cf. Lawler et al cond-mat/0508747]
- ...but v_F, m_F stay finite $\Rightarrow$ emergence of Lorentz invariance [cf. Randeria et al cond-mat/0307217]

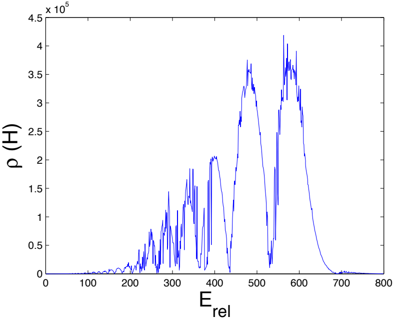
- Always two peaks (are there more?) :
  - standard FL,marginal FL [Faulkner,Liu,McGreevy,Vegh] ,. . .
  - CFT pole = BH quasinormal mode [Kovtun,Starinets]
    AdS BH: No a priori reason ∃ only one Dirac quasinormal mode.
  - What is the true IR? "Band structure"?



$$\frac{\mu_0}{T} = 30.9$$

Landau levels in a magnetic field

[Preliminary....]



$$\rho(\omega) = \frac{dN}{d\omega} \quad , \quad N(\omega) = \int d^2\vec{k}\, n_{FD}(\omega, k - k_F; E_F) A(\omega, k)$$

[Cubrovic,KS,Zaanen,Sadri]

- **Theoretical Calculation of an emergent Fermi Liquid**
  (CFT) **=** (AdS)
  probing a charged black hole with electrons

- **What is k_F on the gravity side?**
  - in string theory geometry depends on the probe...
- **Transition to BCS superconductor**
- Physical interpretation of the $\Delta = 3/2$ transition?
- Interpretation of/co-existence with $\omega = 0$ pole?
  [Faulkner,Liu,McGreevy,Vegh]
- Inclusion magnetic field/.../(better understanding of the emergence)

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- ... **direct connection with experimental data?!** ...

- **Theoretical Calculation of an emergent Fermi Liquid**
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Thank you.

- **...** **direct connection with experimental data?!** **...**

Boundary behavior of $\Psi_\pm$

- Asymptotic behavior near $z = 0$:

$$\begin{aligned} \Psi_+(z) &= z^{\frac{d+1}{2}-|m+\frac{1}{2}|}(\psi_+ + \ldots) + z^{\frac{d+1}{2}+|m+\frac{1}{2}|}(A_+ + \ldots) \\ \Psi_-(z) &= z^{\frac{d+1}{2}-|m-\frac{1}{2}|}(\psi_- + \ldots) + z^{\frac{d+1}{2}+|m-\frac{1}{2}|}(A_- + \ldots) \end{aligned}$$

- The boundary value of Ψ_- is not independent.

$$(\partial_z - \frac{d/2 - m}{z})\Psi_+ = -\not{T}|_{z=0}\Psi_- + \ldots$$

Thus $\psi_- \propto \psi_+$ and $A_- \propto A_+$.

- The scaling behavior of $G(\omega, k)$:

$$G(\omega,k) = \frac{1}{\mathcal{N}} F_- F_+^{-1} \sim \frac{z^{\frac{d+1}{2}-|m-\frac{1}{2}|}(\psi_- + \ldots) + z^{\frac{d+1}{2}+|m-\frac{1}{2}|}(A_- + \ldots)}{z^{\frac{d+1}{2}-|m+\frac{1}{2}|}(\psi_+ + \ldots) + z^{\frac{d+1}{2}+|m+\frac{1}{2}|}(A_+ + \ldots)} \, .$$

- Three different regimes:

$$G(\omega,k) \sim \begin{cases} z\left(\frac{\psi_-}{\psi_+} + \ldots\right) + z^{2m}\left(\frac{A_-}{\psi_+} + \ldots\right) & m > \frac{1}{2} \, , \\ z^{2m}\left(\frac{\psi_-}{\psi_+} + \ldots\right) + z\left(\frac{A_-}{\psi_+} + \ldots\right) & \frac{1}{2} > m > -\frac{1}{2} \, , \\ \frac{1}{z}\left(\frac{\psi_-}{\psi_+} + \ldots\right) + \frac{1}{z^{2m}}\left(\frac{A_-}{\psi_+} + \ldots\right) & -\frac{1}{2} > m \, . \end{cases}$$

Boundary behavior of $\Psi_\pm$

- Asymptotic behavior near $z=0$:

$$\begin{aligned}\Psi_+(z) &= z^{\frac{d+1}{2}-|m+\frac{1}{2}|}(\psi_+ + \ldots) + z^{\frac{d+1}{2}+|m+\frac{1}{2}|}(A_+ + \ldots)\\ \Psi_-(z) &= z^{\frac{d+1}{2}-|m-\frac{1}{2}|}(\psi_- + \ldots) + z^{\frac{d+1}{2}+|m-\frac{1}{2}|}(A_- + \ldots)\end{aligned}$$

- The boundary value of Ψ_- is not independent.

$$(\partial_z - \frac{d/2-m}{z})\Psi_+ = -\not{T}|_{z=0}\Psi_- + \ldots$$

Thus $\psi_- \propto \psi_+$ and $A_- \propto A_+$.

- The scaling behavior of $G(\omega,k)$:

$$G(\omega,k) = \frac{1}{\mathcal{N}}F_-F_+^{-1} \sim \frac{z^{\frac{d+1}{2}-|m-\frac{1}{2}|}(\psi_- + \ldots) + z^{\frac{d+1}{2}+|m-\frac{1}{2}|}(A_- + \ldots)}{z^{\frac{d+1}{2}-|m+\frac{1}{2}|}(\psi_+ + \ldots) + z^{\frac{d+1}{2}+|m+\frac{1}{2}|}(A_+ + \ldots)}\ .$$

- Three different regimes:

$$G(\omega,k) \sim \begin{cases} z\left(\frac{\psi_-}{\psi_+} + \ldots\right) + z^{2m}\left(\frac{A_-}{\psi_+} + \ldots\right) & m > \frac{1}{2}\ ,\\ z^{2m}\left(\frac{\psi_-}{\psi_+} + \ldots\right) + z\left(\frac{A_-}{\psi_+} + \ldots\right) & \frac{1}{2} > m > -\frac{1}{2}\ ,\\ \frac{1}{z}\left(\frac{\psi_-}{\psi_+} + \ldots\right) + \frac{1}{z^{2m}}\left(\frac{A_-}{\psi_+} + \ldots\right) & -\frac{1}{2} > m\ .\end{cases}$$