Holographic approach to the Quantum Hall Effect

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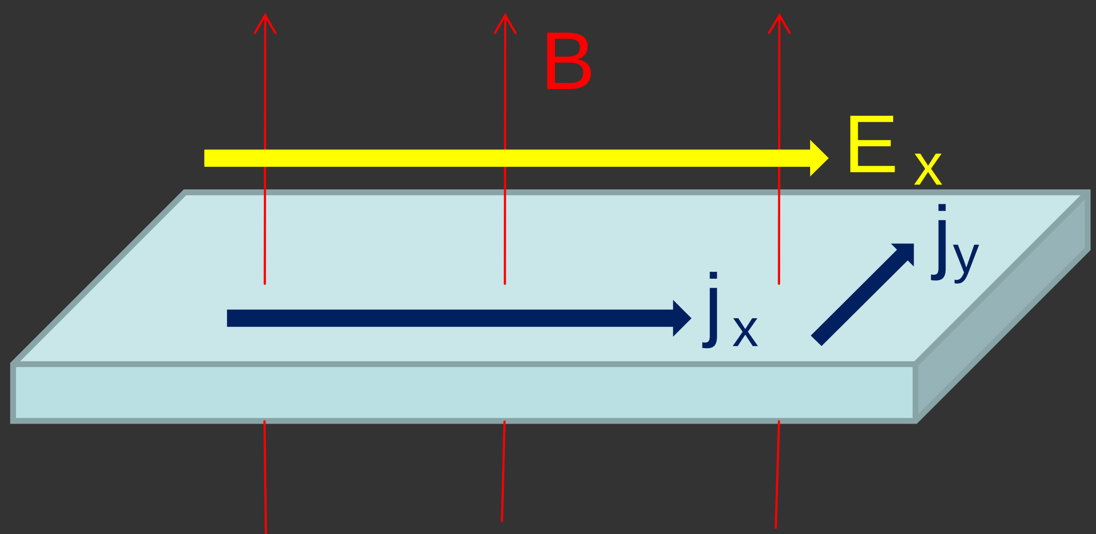
Mainly based on: 0809.1876 (Davis, P.K., Shah)
0905.438 (Alanen, Keski-Vakkuri, P.K, Suur-Uski) .

Introduction

- AdS/CFT ideas are being profitably applied to problems of interest in nuclear and condensed matter physics.
- New approach to tackling strongly coupled field theories.
- Integer and fractional QHE intensely studied since early 80s. Still open questions, some involving strong coupling.
- Can we learn something from gauge/gravity duality?

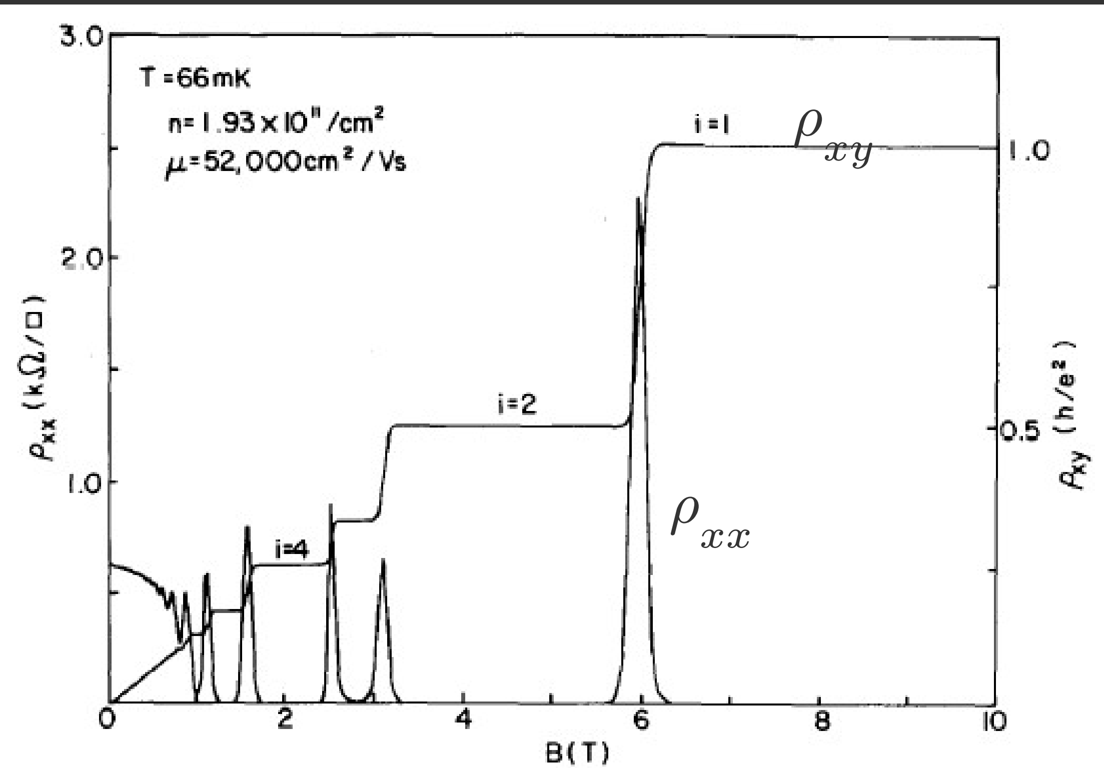
QHE primer

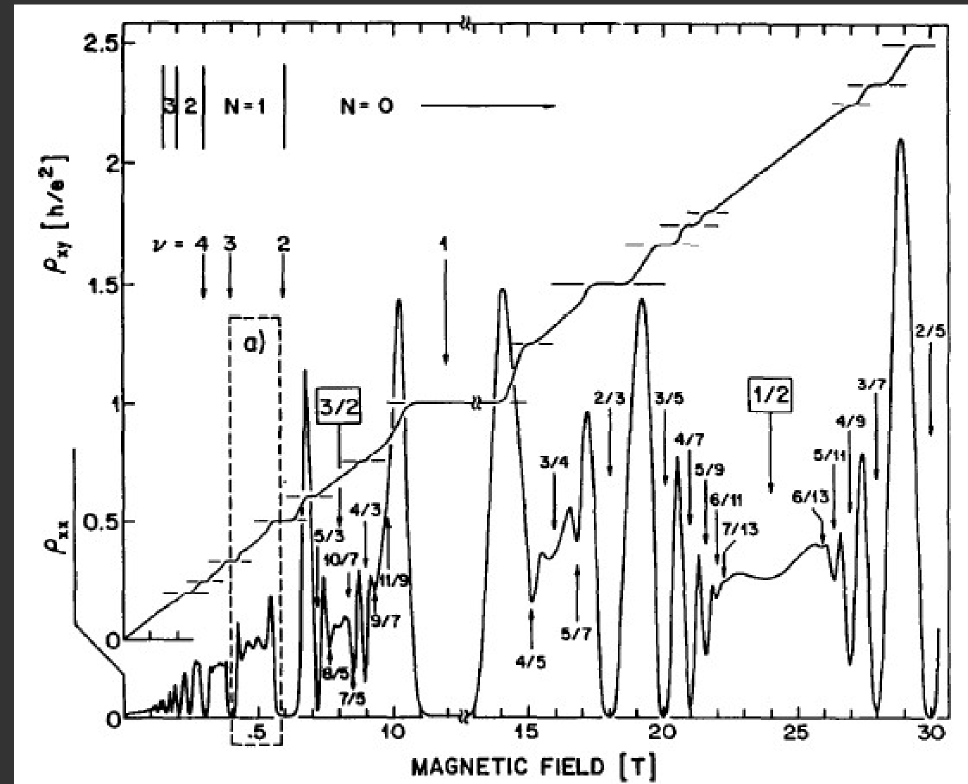
- Standard setup: D=2+1 system of electrons at large B and small T



- Measure longitudinal and transverse conductivities as function of B

$$j_i = \sigma_{ij} E_j \qquad \rho_{ij} = \sigma_{ij}^{-1}$$





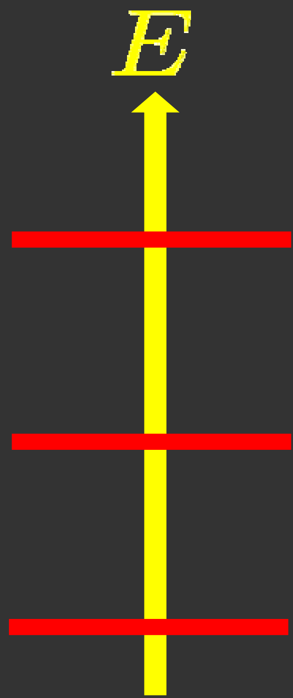
(Tsui, 1990)

Along plateaus: $\rho_{xy} = \left(\frac{h}{e^2}\right)\frac{1}{\nu}$ $\rho_{xx} = 0$

Transitions occur when: $\nu = \frac{nhc}{eB}$

ρ_{xy} jumps to a new value and ρ_{xx} becomes nonzero

In a pure system electrons inhabit Landau levels:

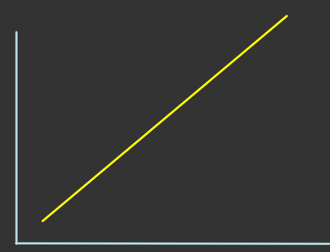


$$E_n = (n + \tfrac{1}{2})\frac{B}{m}$$

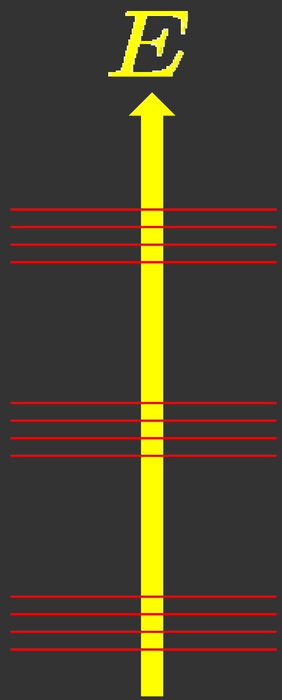
$$\mathrm{degeneracy} = \frac{BA}{2\pi}$$

elementary computation: $\sigma_{xy} = \frac{\rho}{B}$ $\quad \sigma_{xx} = 0$

no plateaus:



real system has disorder and interactions:



degeneracy lifted

discrete energy levels

How does this mess give rise to quantized plateaus?

Compute effective action for EM field

- gapped electron spectrum ⇨ local action
- organize in derivative expansion
- DC response controlled by single derivative part

unique term: $S_{CS} = \frac{k}{4\pi} \int d^3x \ \epsilon^{\alpha\beta\gamma} \partial_\alpha A_\beta A_\gamma$

$$\Longrightarrow \quad j_i = \frac{k}{2\pi} \epsilon_{ij} E_j$$

$$\Longrightarrow \quad \sigma_{xy} = \frac{k}{2\pi} \qquad \sigma_{xx} = 0$$

- k arbitrary at this stage
- nonzero k requires broken parity (e.g. from B)

Key point: k unchanged by any continuous deformation of theory that preserves gap

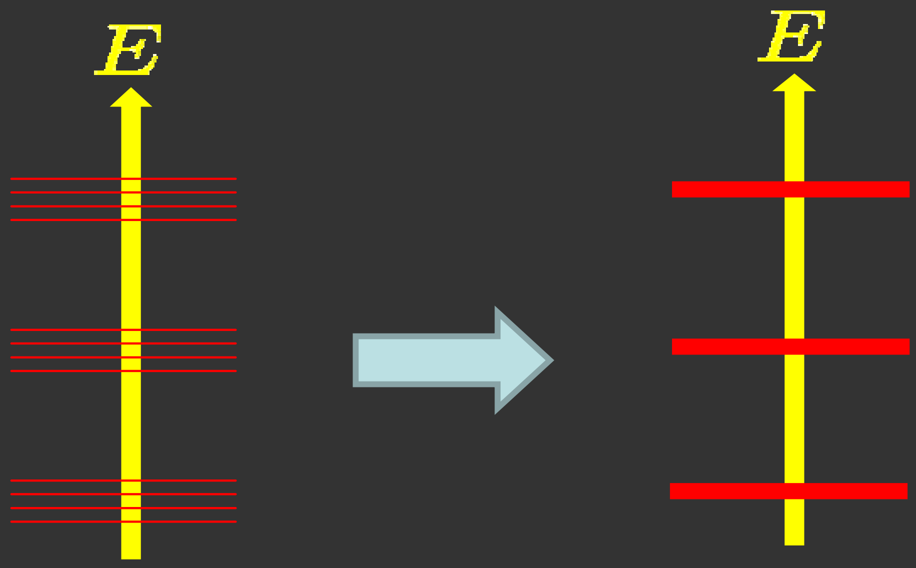
Thinking of changing B as one such deformation, this immediately explains existence of plateaus

B independent

$$\sigma_{xy} = \frac{k}{2\pi} \qquad \sigma_{xx} = 0$$

How do we compute k?

Consider case in which we can smoothly turn off disorder and interactions while maintaining gap



Presence of gap implies n filled Landau levels

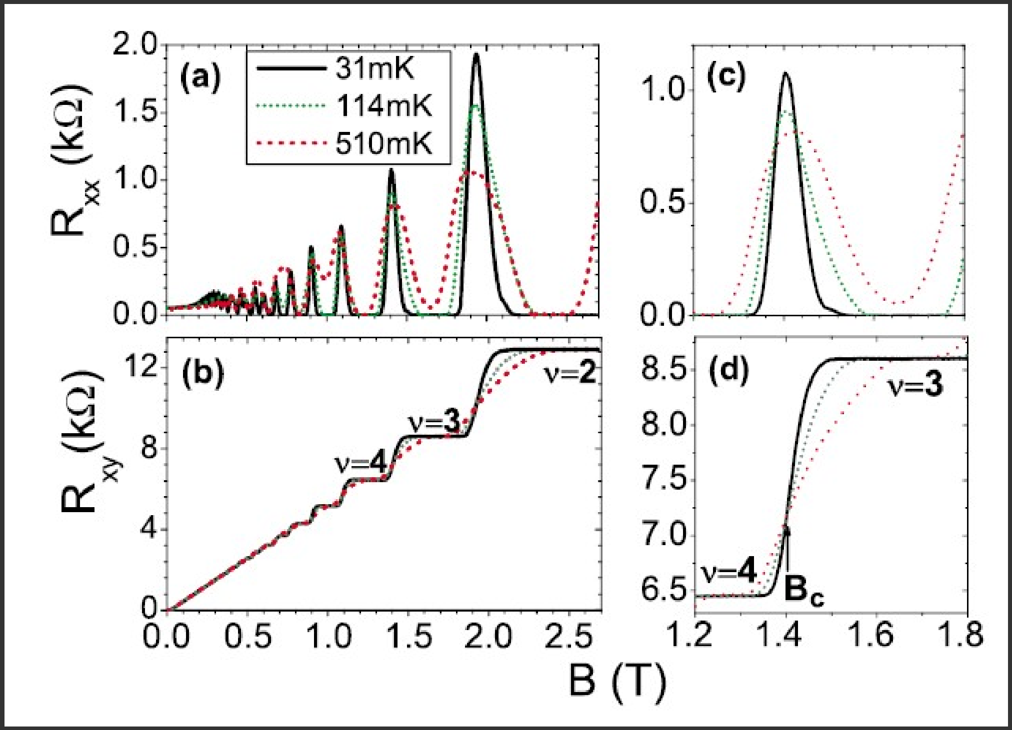
$$k = \frac{2\pi\rho}{B} = n$$ integer QHE

Fractional QHE occurs when we can't smoothly turn off disorder and interactions

- nonzero k requires some delocalized states
- k jumps when Fermi energy crosses these levels

plateau transition

- Temperature smooths out transitions:



(Tsui et. al.)

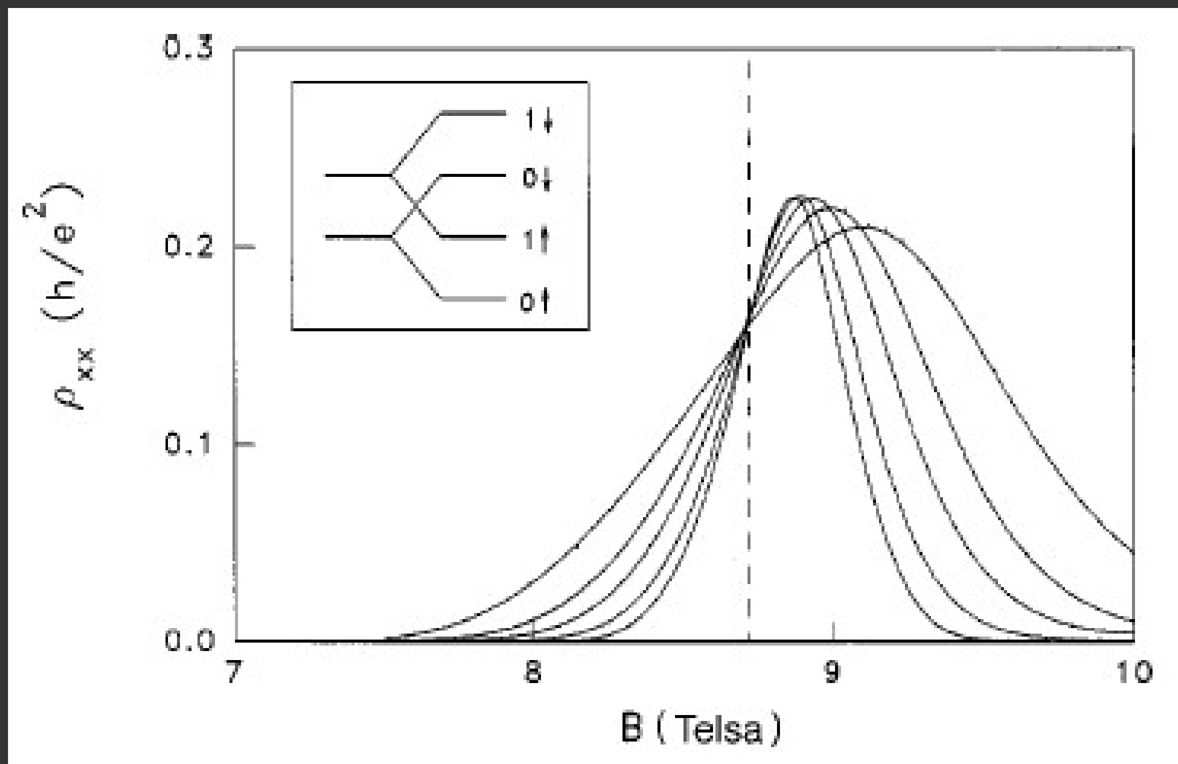
Slope at transition:

$$\frac{\partial \rho_{xy}}{\partial T} \sim \frac{1}{T^{3/7}}$$

Same exponent measured in variety of integer and fractional transitions

Universal conductivity

- At critical B, evidence of universal conductivities, independent of T:



(Coleridge)

$$\sigma^c_{xy} = (n - \tfrac{1}{2})\frac{e^2}{h} \qquad \sigma_{xx} = \frac{e^2}{2h}$$

Quantum critical point

- Behavior viewed as evidence for scale invariant theory at the transition.
- Dynamical exponent $z=1$
- For integer effect, understanding in terms of delocalization transition. Numerical computation of 3/7 exponent and critical conductivities.
- Fractional case requires simultaneous treatment of interactions and disorder. Hard.

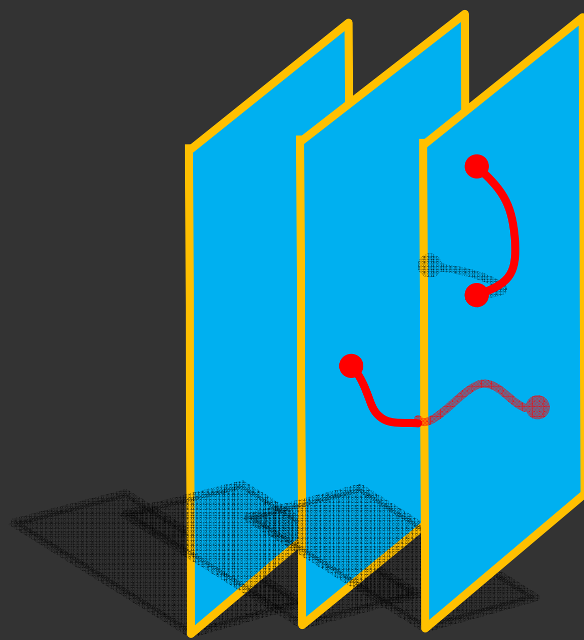
- Lesson: to generate QHE transitions we need a system with a gap and broken parity, and a knob that lets us tune the gap to zero

- Simple realization: charged fermions where knob = fermion mass

- Realize in string theory via intersecting branes

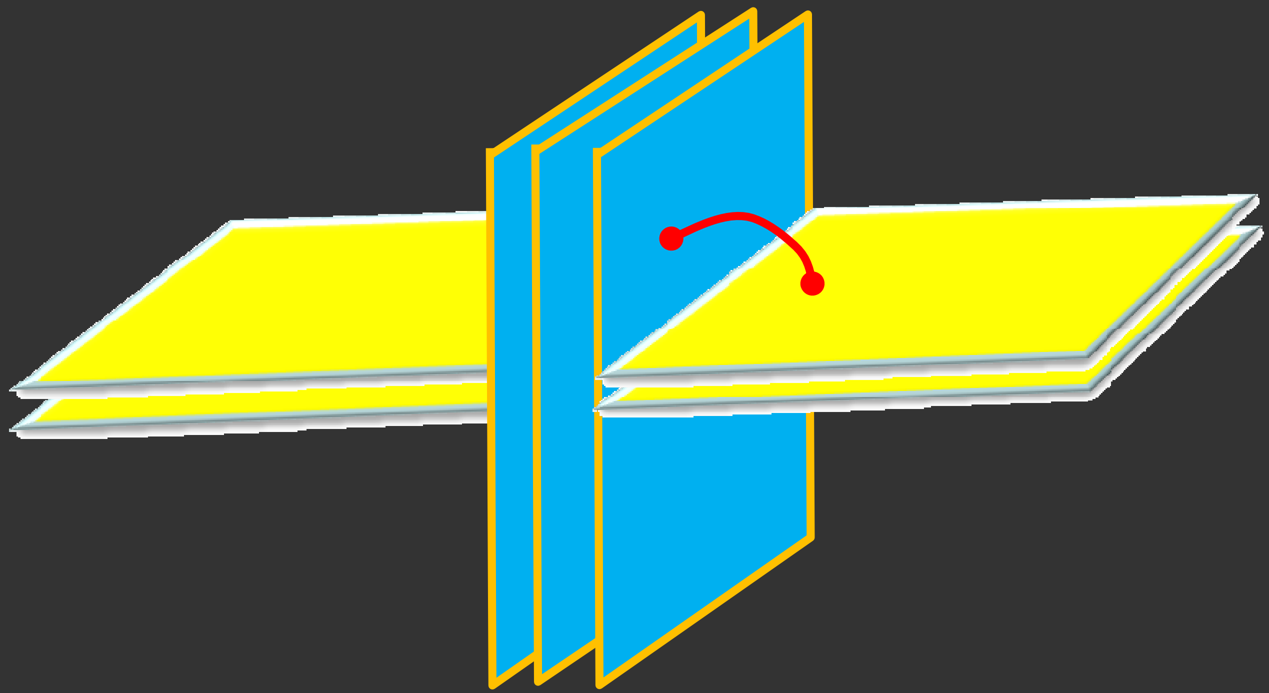
Intersecting branes

- On a stack of N Dp-branes lives a p spatial dimensional U(N) gauge theory with with matter in the adjoint representation



Intersecting branes

- On two intersecting stacks there is additional matter confined to the intersection and transforming in the bifundamental representation

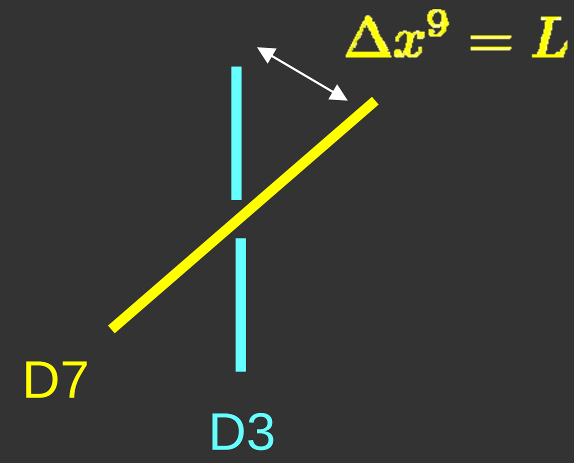


- Think of photon as U(1) gauge field on yellow stack

D-brane construction

- D=2+1 fermions from D3-D7 intersection

	0	1	2	3	4	5	6	7	8	9
$D3:$	×	×	×	×						
$D7:$	×	×	×		×	×	×	×	×	



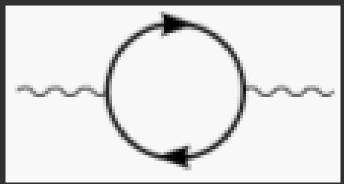
#ND=6: non-susy, but no tachyons or massless scalars

$$m_{fermion} = \frac{L}{2\pi\alpha'}$$

Transition corresponds to varying L through 0: $m\bar{\psi}\psi \to -m\bar{\psi}\psi$

EM response

- Turn on D7 gauge field: $\mathcal{L} = \overline{\psi}^i (i\partial\!\!\!/ + m - A\!\!\!/)\psi_i$



$$S_{eff}(A) = \int \frac{d^3p}{(2\pi)^3} \Pi^{\mu\nu}(p) A_\mu(p) A_\nu(-p)$$

$$\Pi^{\mu\nu} = (p^\mu p^\nu - p^2 g^{\mu\nu})\Pi_{even} + \epsilon^{\mu\nu\lambda} p_\lambda \Pi_{odd}$$

$$\Pi_{even} = -\frac{N}{2\pi}\frac{1}{\sqrt{p^2}}\left[\frac{1}{2}\frac{|m|}{\sqrt{p^2}} - (\frac{m^2}{p^2} - \frac{1}{4})\arctan\left(\frac{\sqrt{p^2}}{2|m|}\right)\right]$$

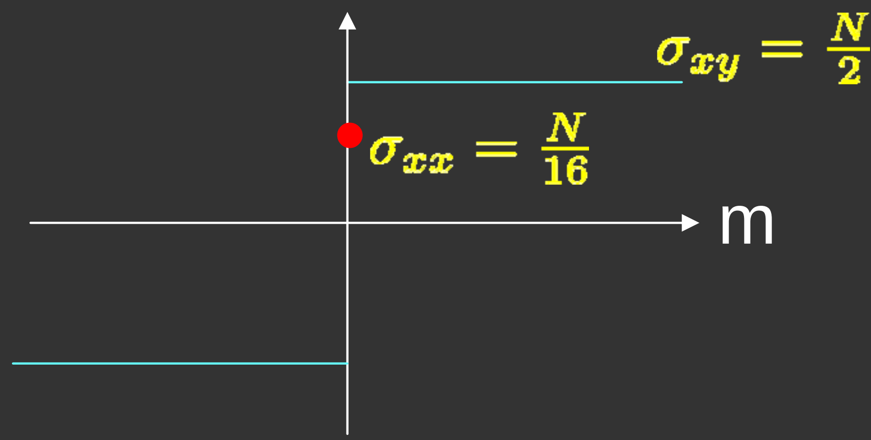
$$\Pi_{odd} = -\frac{N}{2\pi}\frac{m}{\sqrt{p^2}}\arctan\left(\frac{\sqrt{p^2}}{2|m|}\right)$$

Conductivities:

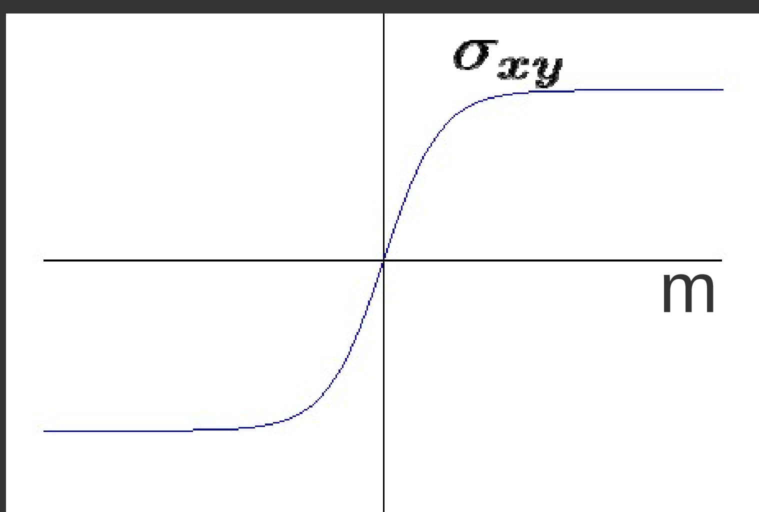
$$\sigma^T_{ij} = \frac{k}{2\pi}\epsilon_{ij} \qquad k = \begin{cases} \frac{N}{2} sgn(m) & m \neq 0 \\ 0 & m = 0 \end{cases}$$

$$\sigma^L_{ij} = \mathrm{Im}\frac{1}{\omega}\Pi^{ij}_{even} = \begin{cases} 0 & m \neq 0 \\ \frac{N}{16}\delta_{ij} & m = 0 \end{cases}$$

Weak coupling transition



- Transition is smoothed out at finite T



$$k(T) = N \tanh\left(\frac{|m|}{2T}\right) \mathrm{sgn}(m)$$

$$\frac{\partial \sigma_{xy}}{\partial m} \sim \frac{1}{T} \qquad \mathrm{expt} : \sim \frac{1}{T^{.42}}$$

Including interactions

- At finite string coupling fermions have various short and long range interactions.
- Consider 4-fermi interactions:

$$L = \bar{\psi} i \partial\!\!\!/ \psi + m \bar{\psi}\psi + \frac{g^2}{2N} (\bar{\psi}\psi)^2$$

- For small g situation same as before.
- At strong coupling situation can change due to spontaneous parity breaking:

$$\langle \bar{\psi}\psi \rangle |_{m=0} \neq 0$$

Can be studied using standard large N methods

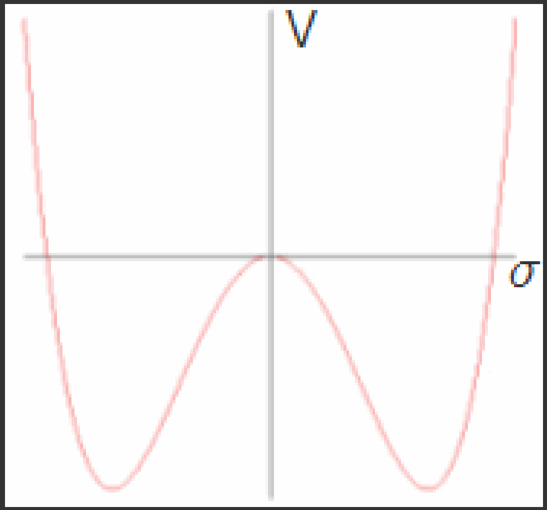
- Effective potential for $\sigma = -\frac{2g^2}{N}\langle\overline{\psi}\psi\rangle$

$$\frac{1}{N}\frac{dV_{eff}(\sigma)}{d\sigma} = \frac{\sigma}{g^2} - \frac{\sigma}{\pi^2}\left[\Lambda - \sigma\arctan\left(\frac{\Lambda}{\sigma}\right)\right]$$

UV cutoff

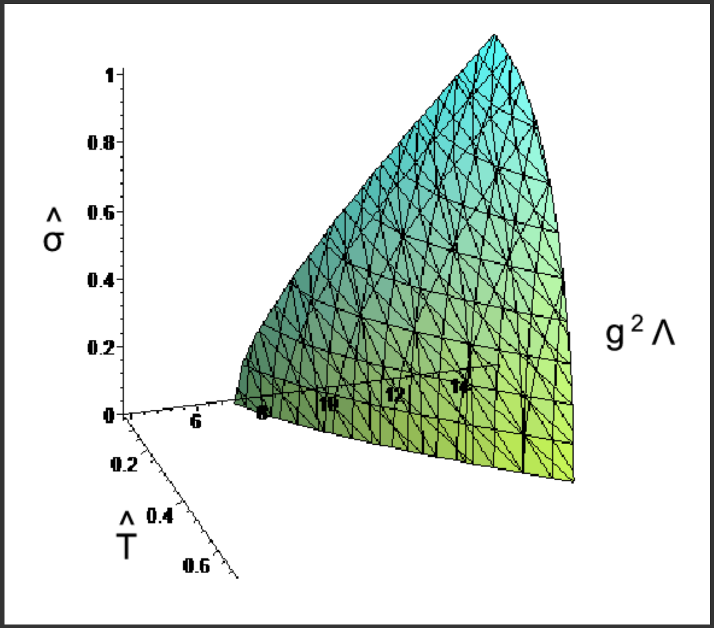
- Nontrivial solutions (hence SSB) for

$$g > g_c = \pi/\sqrt{\Lambda}$$



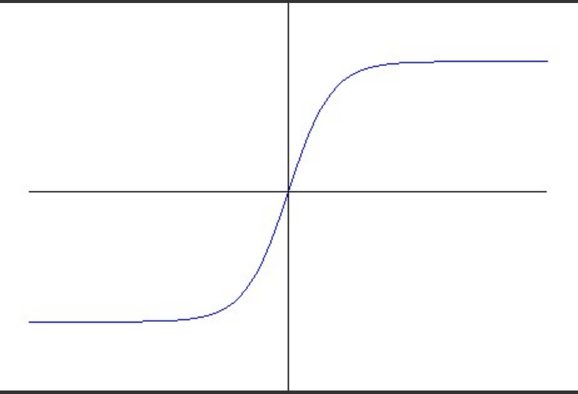
At low temperature, as m passes through zero we jump between minima, and hence transition remains discontinuous.

- parity restored for $T > T_c$



second order phase transition

- transition is smoothed out above critical temperature



$$\frac{\partial \sigma^T}{\partial m} \sim \frac{1}{(T - T_c)^{1/\nu z}}$$

$$\nu z = 1 + O(\tfrac{1}{N})$$

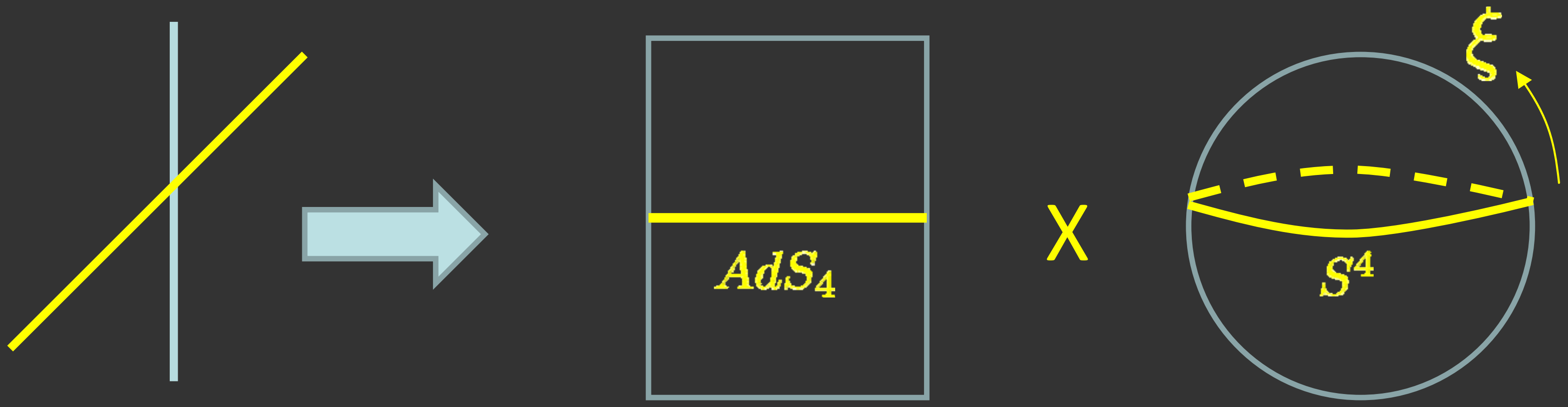
Gravity side

- Probe approximation:

$$N_3 \gg N_7 \qquad N_3, g_s N_3 \gg 1$$

D3-branes ⇨ $AdS_5 \times S^5$

D7-branes ⇨ surfaces embedded in $AdS_5 \times S^5$

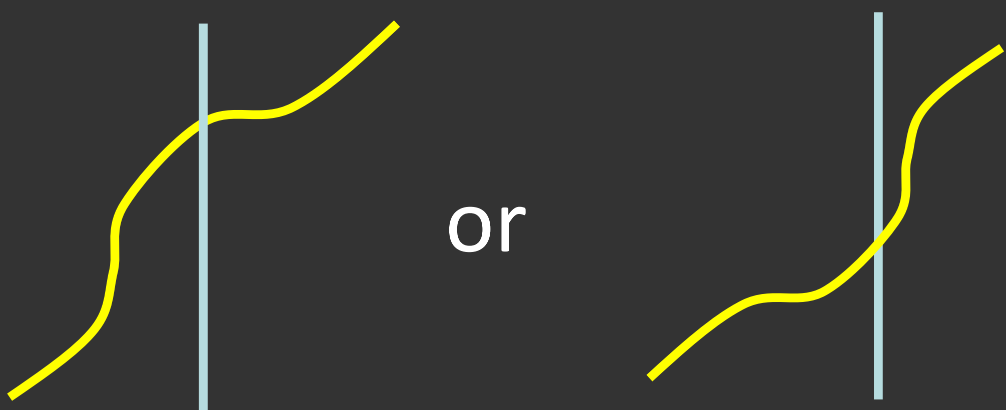


- brane potentially unstable to slipping off equator

- ξ fluctuations yield an AdS_4 scalar of mass

$$m^2 R^2 = -4 < -\frac{9}{4}$$ unstable

- Stable end points break parity:



branes repel

- corresponds to $\langle \overline{\psi}\psi \rangle|_{m=0} = \pm v$
- fermions are massive in stable vacua

Induced CS term

- CS term for D7-brane gauge field comes from interaction with D3-brane flux

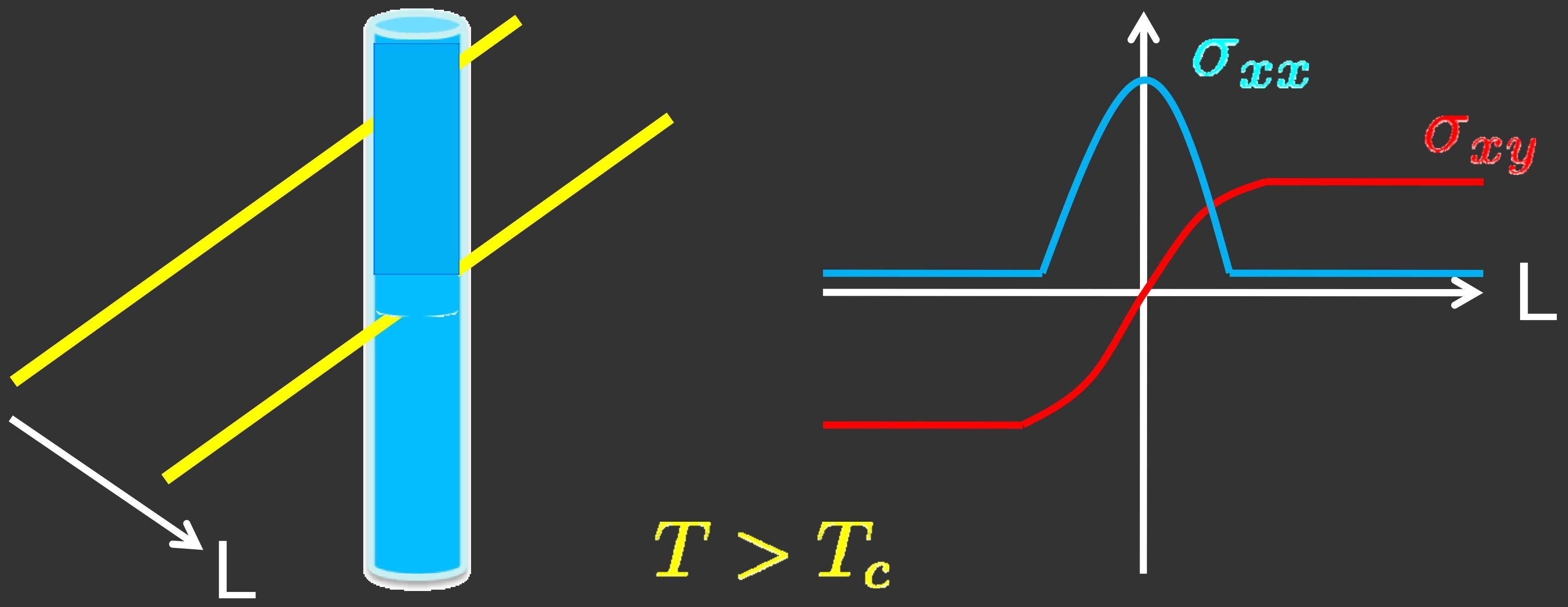
$$S_{WZ} = -\frac{1}{2}(2\pi\alpha')^2 N_7 \int A \wedge F \wedge G_5 = \frac{k}{4\pi}\int A \wedge F$$

$$k = \pm\frac{1}{2} N_3 N_7$$

since D7 sees half of D3 flux

- agrees with weak coupling result

- Finite temp: D3-brane becomes black brane



- Hall conductivity proportional to fraction of D3-flux passing through D7
- σ_{xx} nonzero only for black hole embeddings
- multiple D7s give multiple steps (or insulator)

Generalizations

- Many other possible setups

  e.g. D7-branes on $AdS_5 \times X$

  (some of these are hopefully stable…)

- Study transport in general version of this setup

- After integrating over compact space, probe action has two parts:

$$S_{BI} = \int d^4x \, \tau(\xi(r))\sqrt{-\det(g+F)}$$

Born-Infeld

↑ volume of compact space at fixed ξ

induced metric: $ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + V(r)dx^i dx^i$

Generalizations

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- Study transport in general version of this setup

- After integrating over compact space, probe action has two parts:

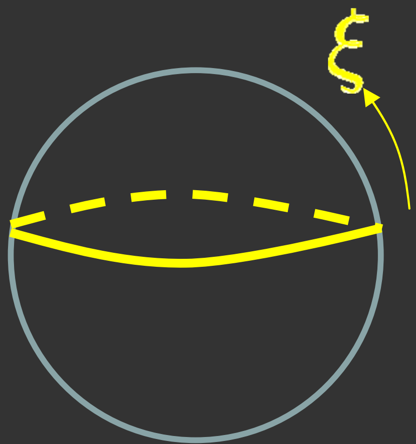
$$S_{CS} = \frac{1}{4\pi}\int dk(r) \wedge A \wedge F$$

Chern-Simons

$$k = k(\infty) - k(r_+)$$

flux through probe

Critical Exponent



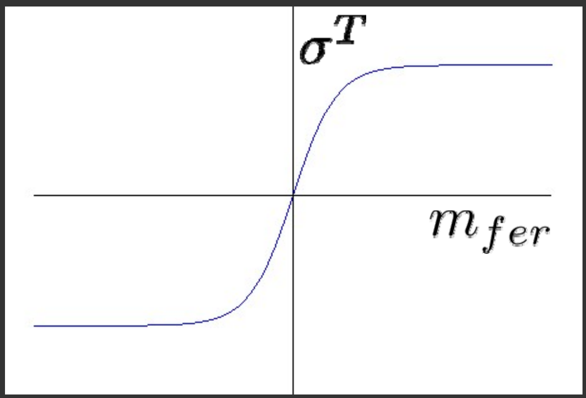
- slipping mode becomes AdS_4 scalar

$$S_\xi = \frac{1}{2}\int d^4x\sqrt{-g}\left((\partial\xi)^2 - m_\xi^2\xi^2\right)$$

- corresponds to operator of dimension

$$\Delta = \frac{3}{2} + \sqrt{\frac{9}{4} + m_\xi^2}$$

- fermion mass related to ξ falloff: $\delta\xi \sim m_{fer} r^{\Delta-3}$



$$\text{slope} \sim \frac{1}{T^{3-\Delta}}$$

DC conductivity

- Natural to turn on B and ρ

$$\sigma_{xx} = \frac{\sqrt{\rho(r_+)^2/\tau(r_+)^2 + B^2 + V(r_+)^2}}{B^2 + V(r_+)^2} V(r_+)\tau(r_+)$$

$$\sigma_{xy} = \frac{\rho(r_+)B}{B^2 + V(r_+)^2} + \frac{k(\xi_0)}{2\pi}$$

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + V(r)dx^i dx^i$$

$$r_+ \sim T \qquad \rho(r_+) = \rho - \frac{kB}{2\pi}$$

- all dependence on ambient metric drops out, except for entropy density $S \sim V(r_+)^{\frac{d-1}{2}}$
- only other parameters are tension and flux as measured at the horizon

- D3-D7 example with $m_{fer} = B = \rho = 0$:

$$\sigma_{xx} = \tau = \frac{N_3 N_7}{3\pi^2}$$

- at weak coupling had: $\sigma_{xx} = \frac{N_3 N_7}{16}$

conductivity decreases by 1.85 in going from weak to strong coupling

AC conductivity at critical point

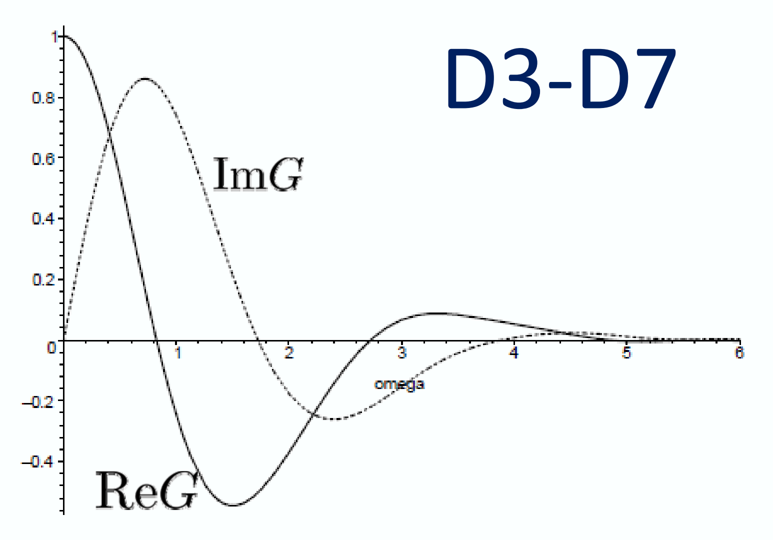
- no frequency dependence when $B=\rho=0$

$$\sigma_{xx}(\omega)+i\sigma_{xy}(\omega)=\tau$$

- Expansion for small (B,ρ) :

$$\sigma_{xx}(\omega)+i\sigma_{xy}(\omega)=\tau-\frac{\tau}{2}(B\pm i\rho/\tau)^2\,G(\omega)$$

$$G(\omega)=\frac{1}{V(r_+)^2}+2i\omega\int_{r_+}^{\infty}\frac{dx}{U(x)}\left(\frac{1}{V(r_+)^2}-\frac{1}{V(x)^2}\right)e^{2i\omega\int_x^{\infty}\frac{dy}{U(y)}}$$

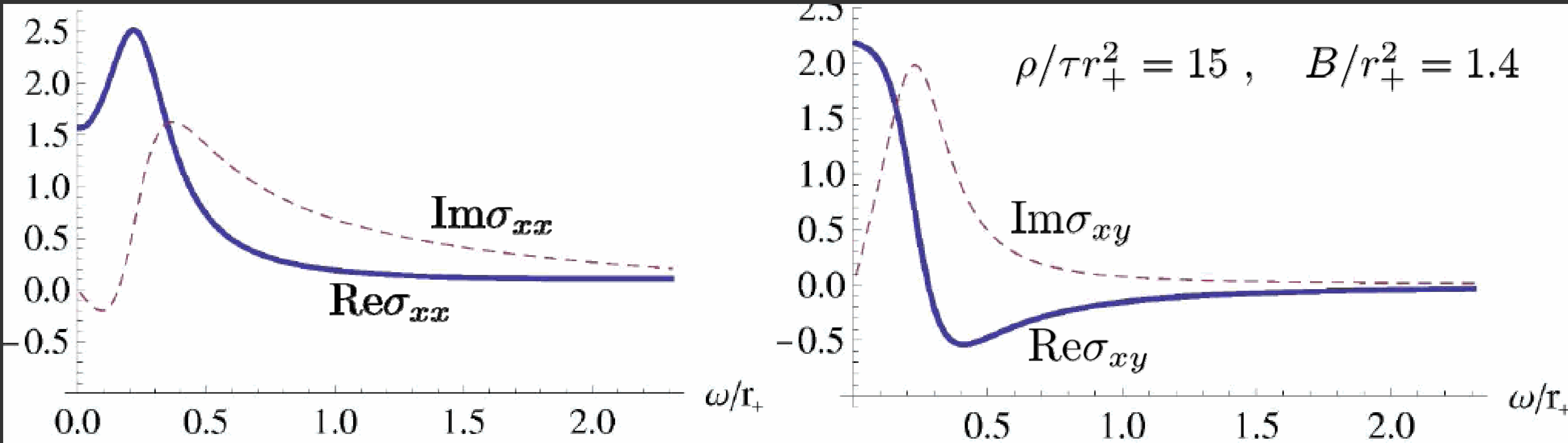


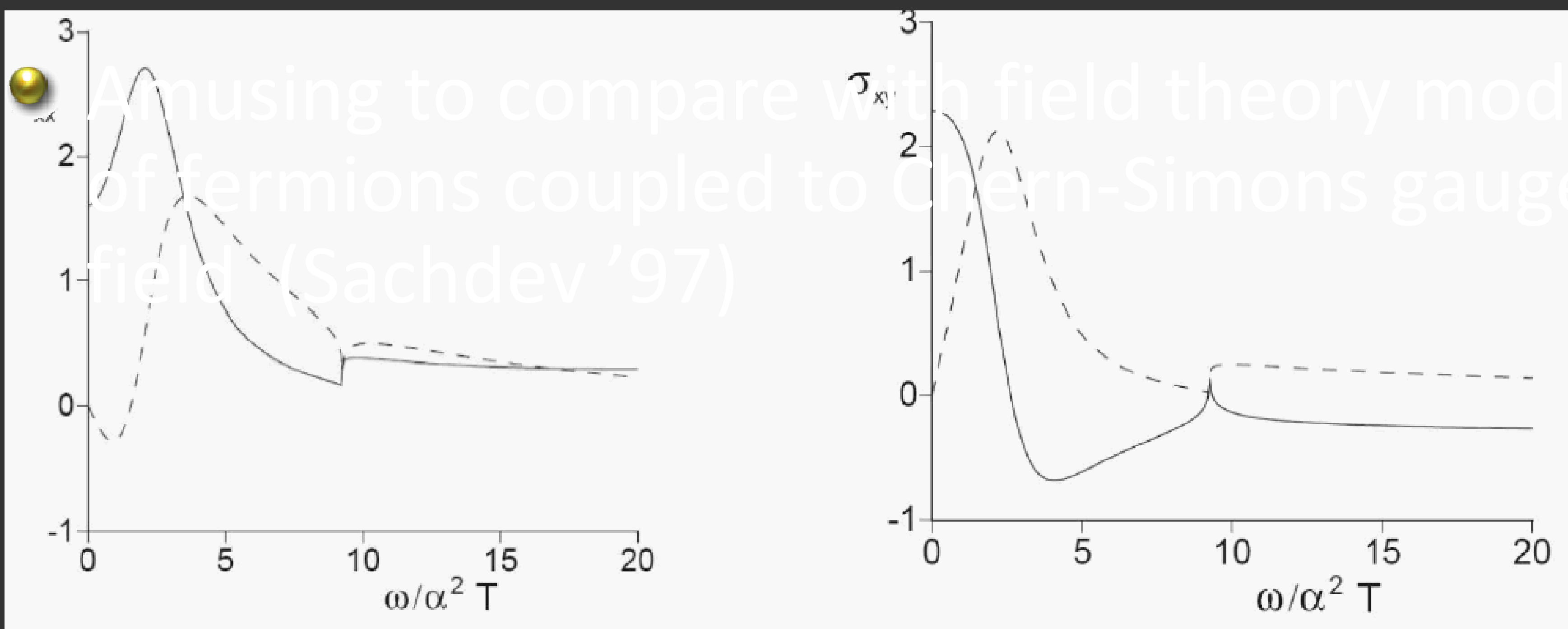
- high frequency asymptotics can also be computed in the general case

$$\sigma_{xx} + i\sigma_{xy} = \left(1 - \frac{3}{4}\frac{(B \pm i\rho/\tau)^2}{\omega^4} + \cdots\right)\tau$$

- result is completely universal up to overall normalization

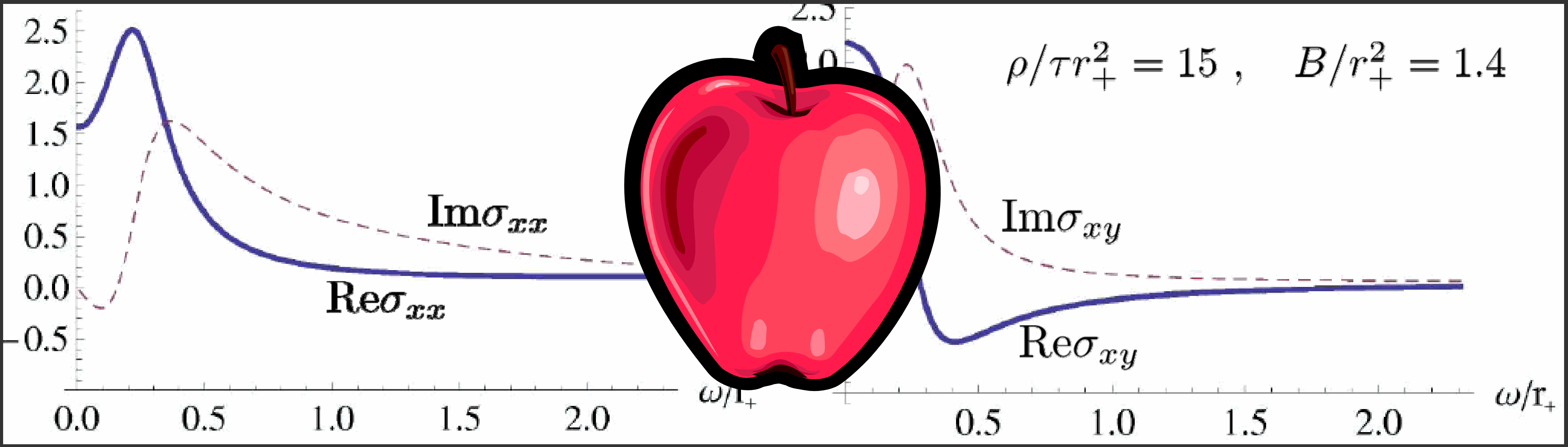
Numerical results

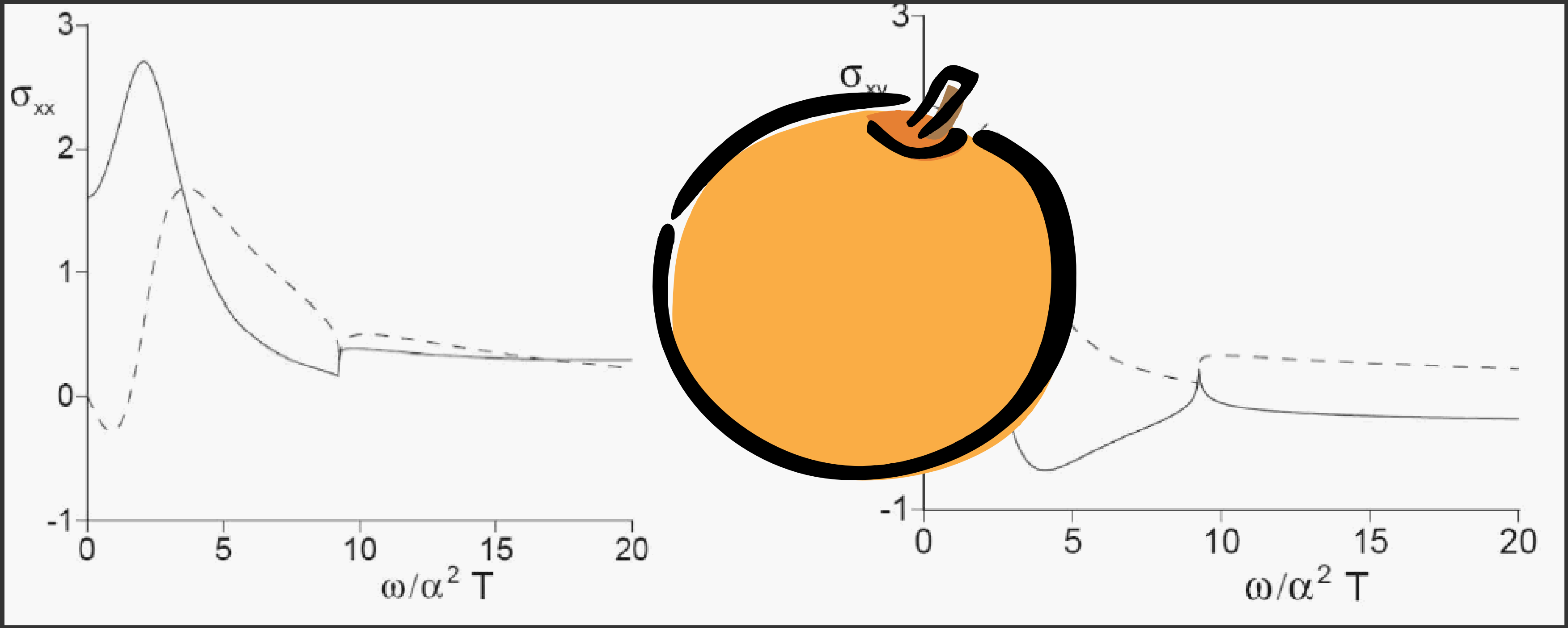




- Amusing to compare with field theory model of fermions coupled to Chern-Simons gauge field (Sachdev '97)

Numerical results





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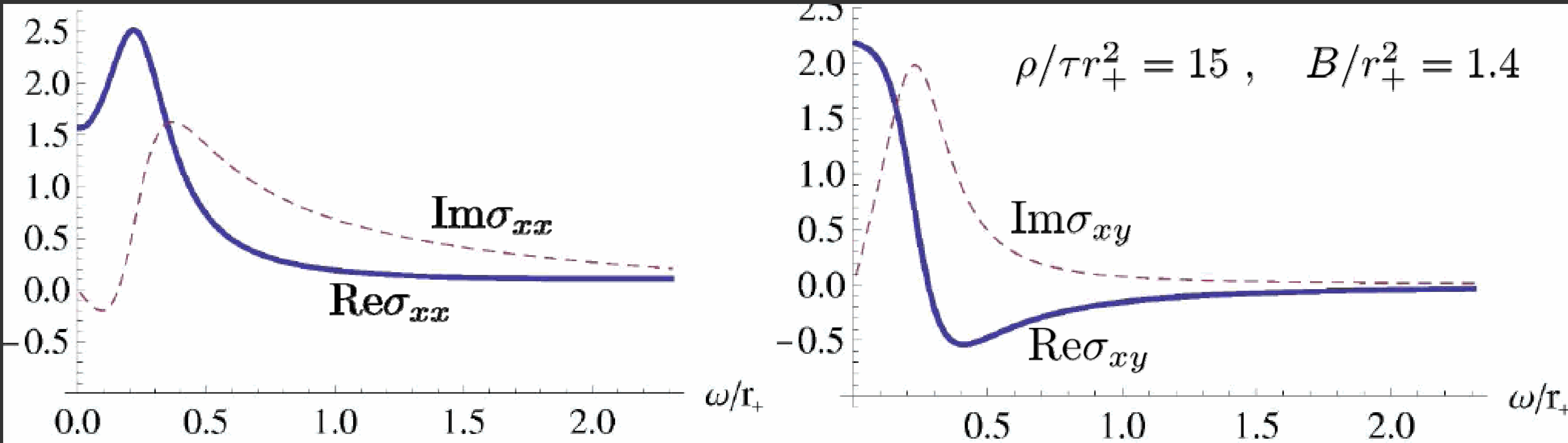

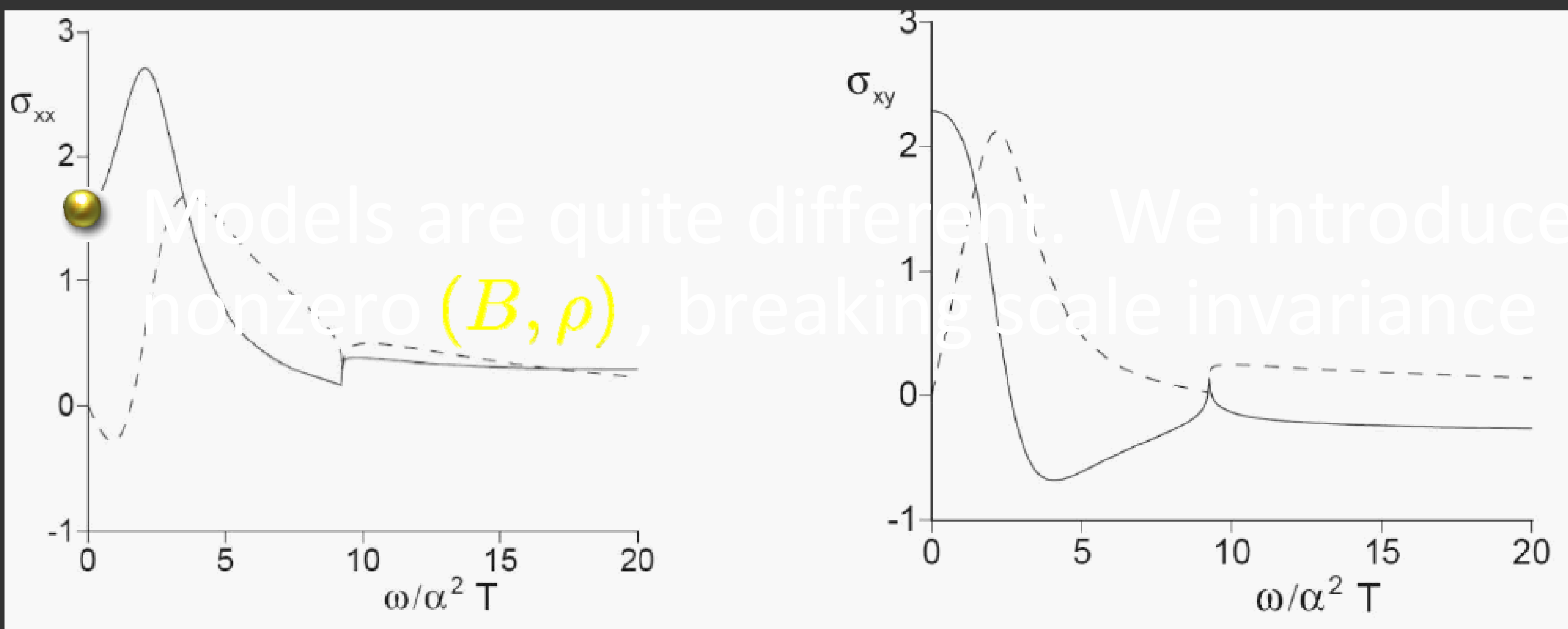

- Models are quite different. We introduced nonzero (B, ρ) , breaking scale invariance

Conclusion

- Stringy quantum Hall transitions can be studied using probe branes and gauge/gravity duality
- Would be interesting to model delocalization transitions, FQHE transitions, …