Assignment

"An intro to chirality in active matter
and how we describe it,
***both** at the micro*
and macro levels."

Activity and chirality in continuum mechanics

Vincenzo Vitelli

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Activity and chirality in continuum mechanics

"I am, in general, not a big fan of introducing continuum theories as a first step."

Activity and chirality in continuum mechanics

Tough luck,
my friend!

"I am, in general, not a big fan of introducing continuum theories as a first step."

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Activity and chirality in continuum mechanics

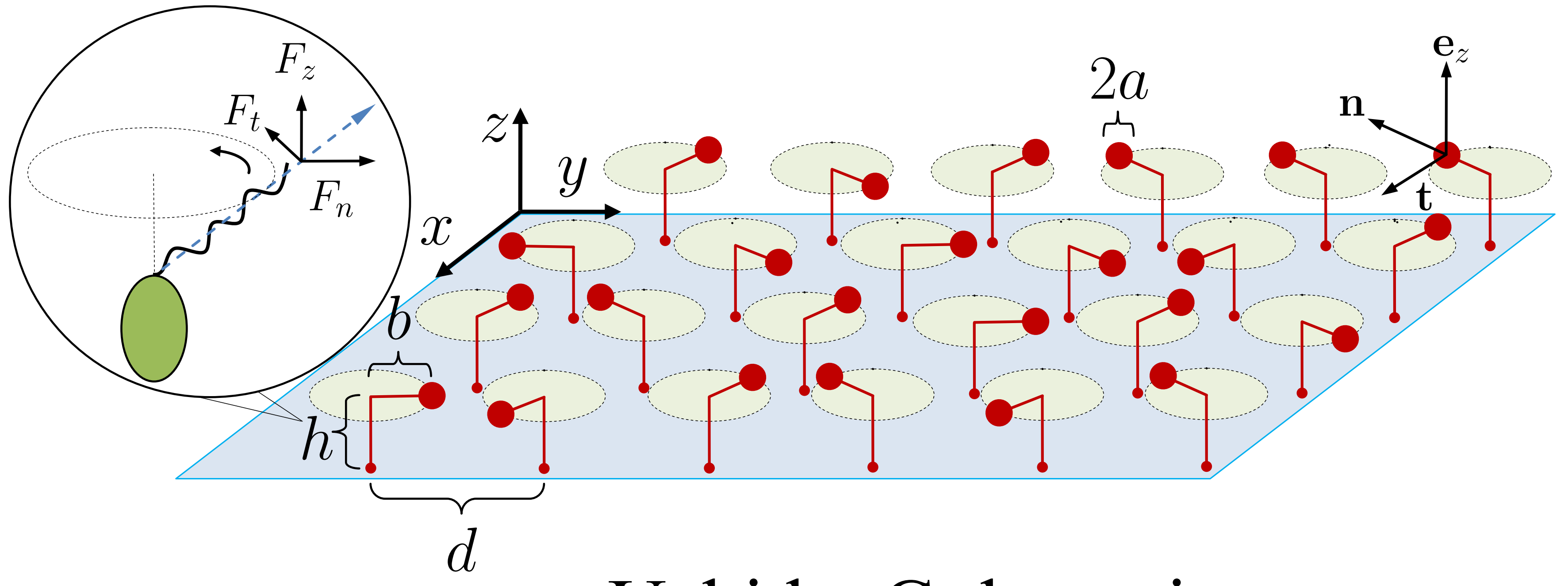
"I am, in general, not a big fan of introducing continuum theories ***as a first step****."*

Activity and chirality in continuum mechanics

Chate/Henkes
discussion

Symmetries and conservation laws

Carpet of microfluidic **rotors**, cilia



Uchida, Golestanian,
PRL 2010

coarse-
graining

continuum theories

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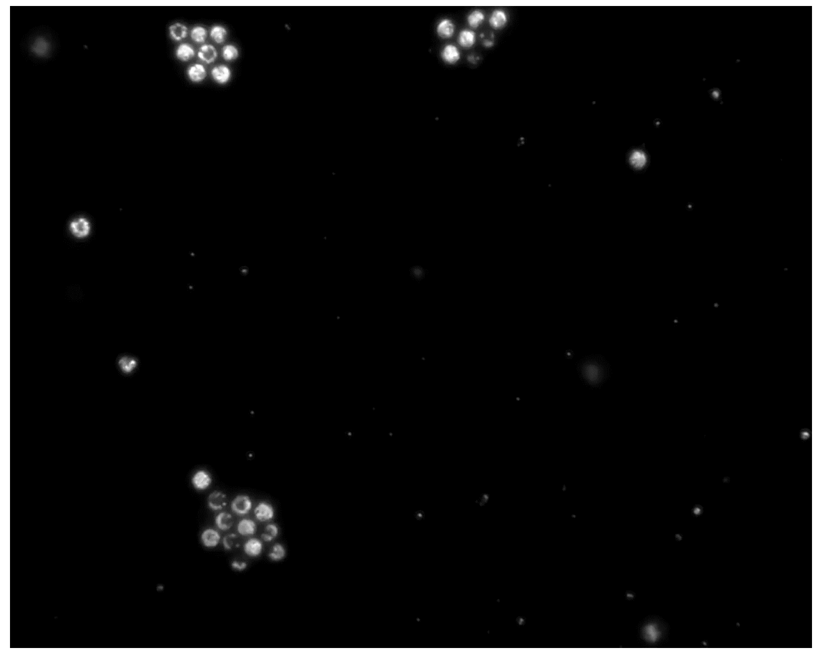
Self-spinning building blocks

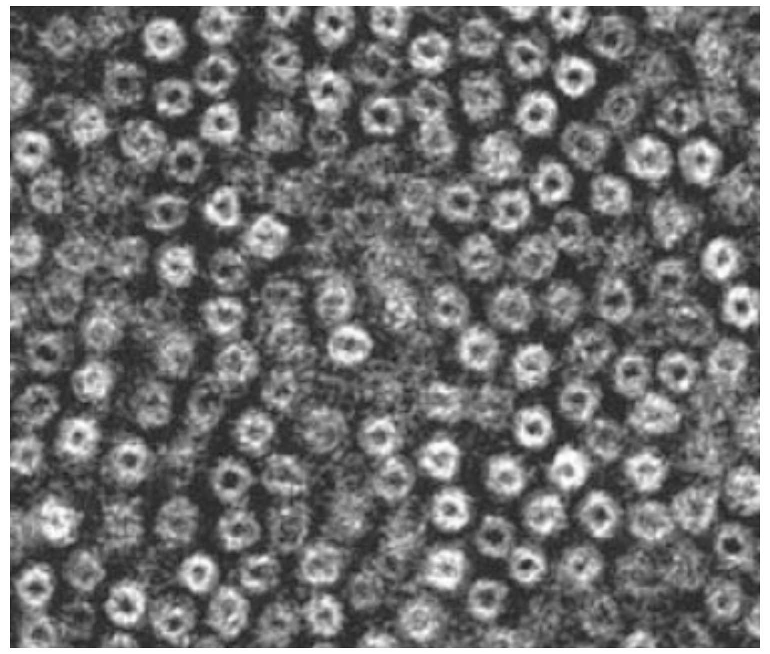
Biological:

T. Majus bacteria



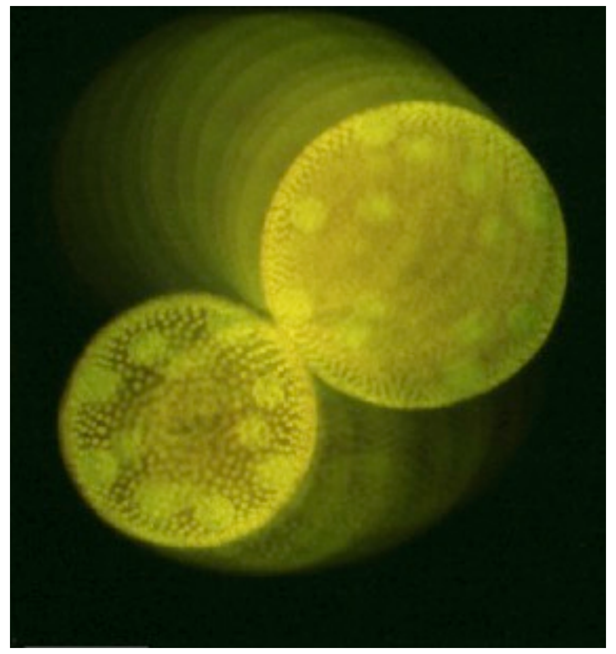
Petroff et al. *PRL* (2015)

Sperm cells

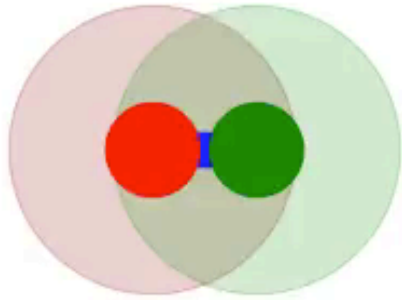


Riedel et al. *Science* (2005)

Volvox algae



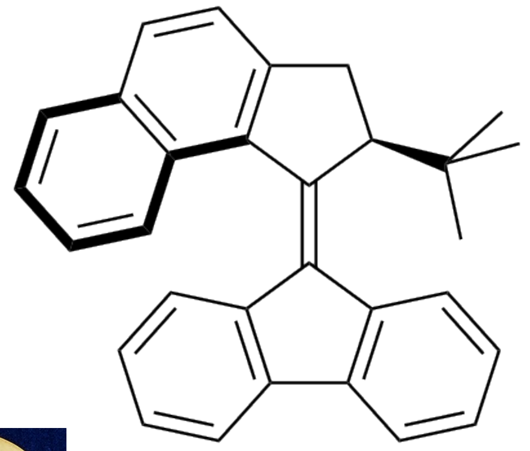
Drescher et al. *PRL* (2009)



$$\Omega = \frac{\tau}{\gamma}$$

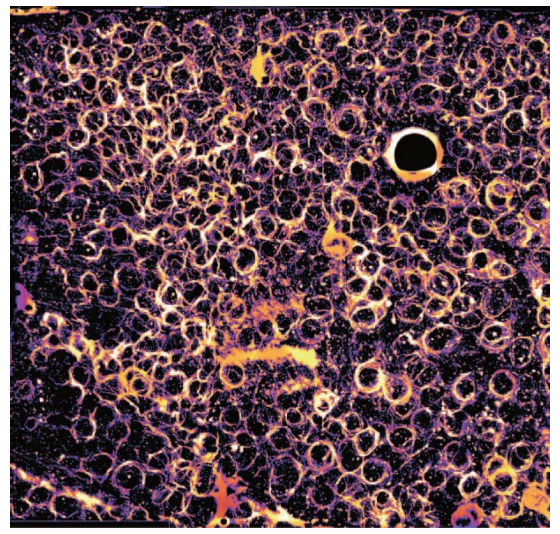
Synthetic:

Molecular motors



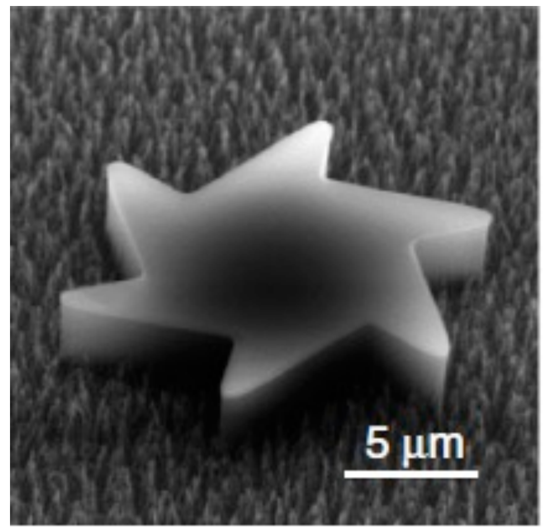
Feringa et al.
Nature (1999)

Microtubules



Sumino et al.
Nature (2012)

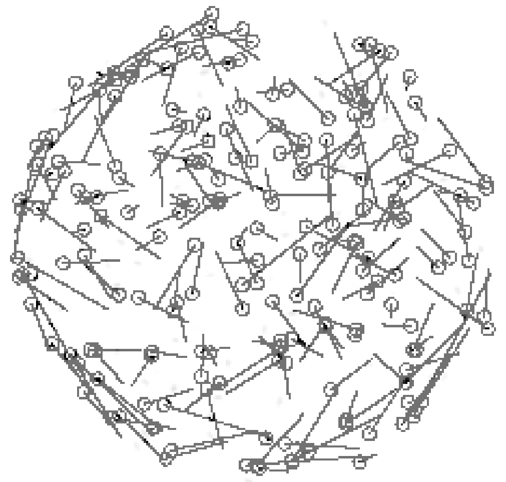
Colloids



Maggi et al.
Nat. Comm. (2015)

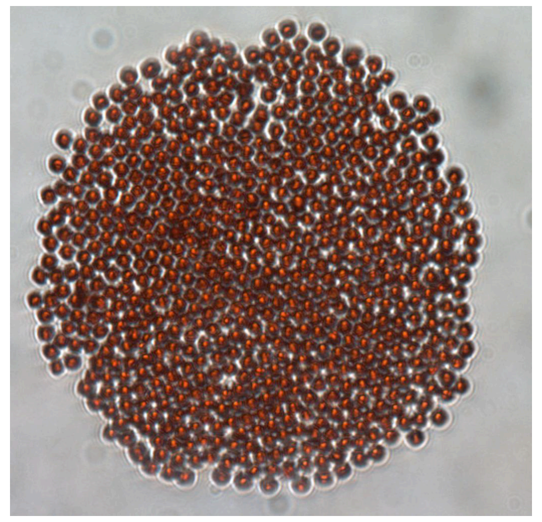
Yan J Soft Matter (2014)

Granular gas



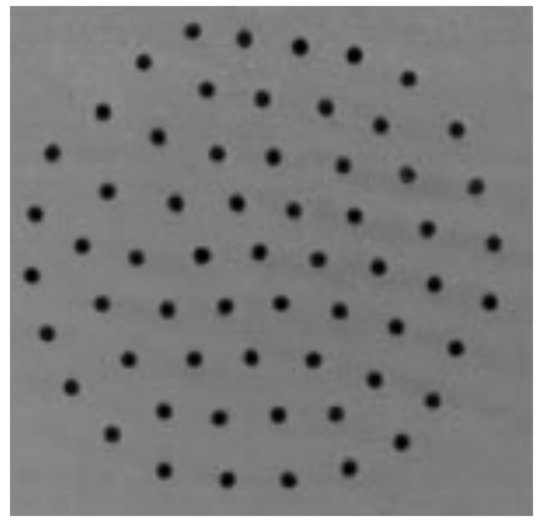
Tsai et al.
PRL (2005)

colloidal
fluids



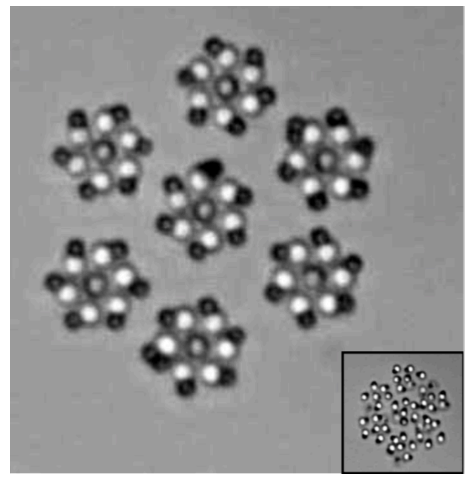
Soni et al.
Nat Phys. (2019)

magnets
at interfaces

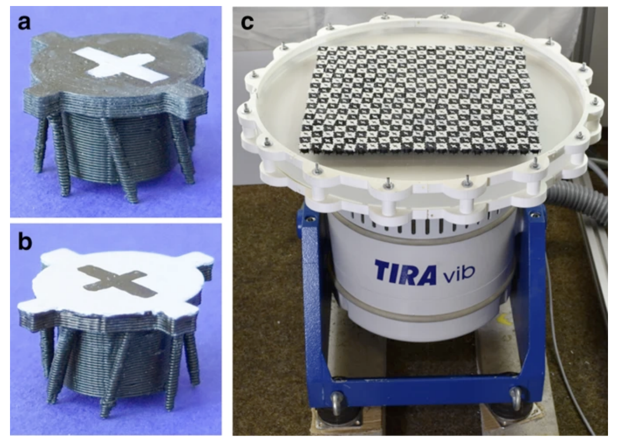


Grzybowski , Stone
and Whitesides
Nature (2000)

Microrobots
microgears,
meta materials



Aubret et al. Nat. Phys. (2018)



Scholtz et al. Nat. Comm. (2018)

Farhadi et al.
Soft Matter (2018)

Nash et al. PNAS. (2015)

Mitchell et al. Nat. Phys. (2018)

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Marchetti, M. C. et al. **Hydrodynamics of soft active matter**. *Rev. Mod. Phys.* **85**, 1143–1189 (2013).

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M. Moshe, M. J. Bowick and M. C. Marchetti, **Geometric Frustration and Solid-Solid Transitions in Model 2D Tissue**, Phys. Rev. Lett. 120, 268105 (2018).

K. Yeo, E. Lushi, P. M. Vlahovska, **Collective Dynamics in a Binary Mixture of Hydrodynamically Coupled Microrotors**, Phys. Rev. Lett. 114, (2015)

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Activity and chirality in continuum mechanics

continuum mechanics

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continuum mechanics

take out conservation of energy

Activity

continuum mechanics

take out conservation of energy

Activity ➔ **chirality in continuum mechanics**

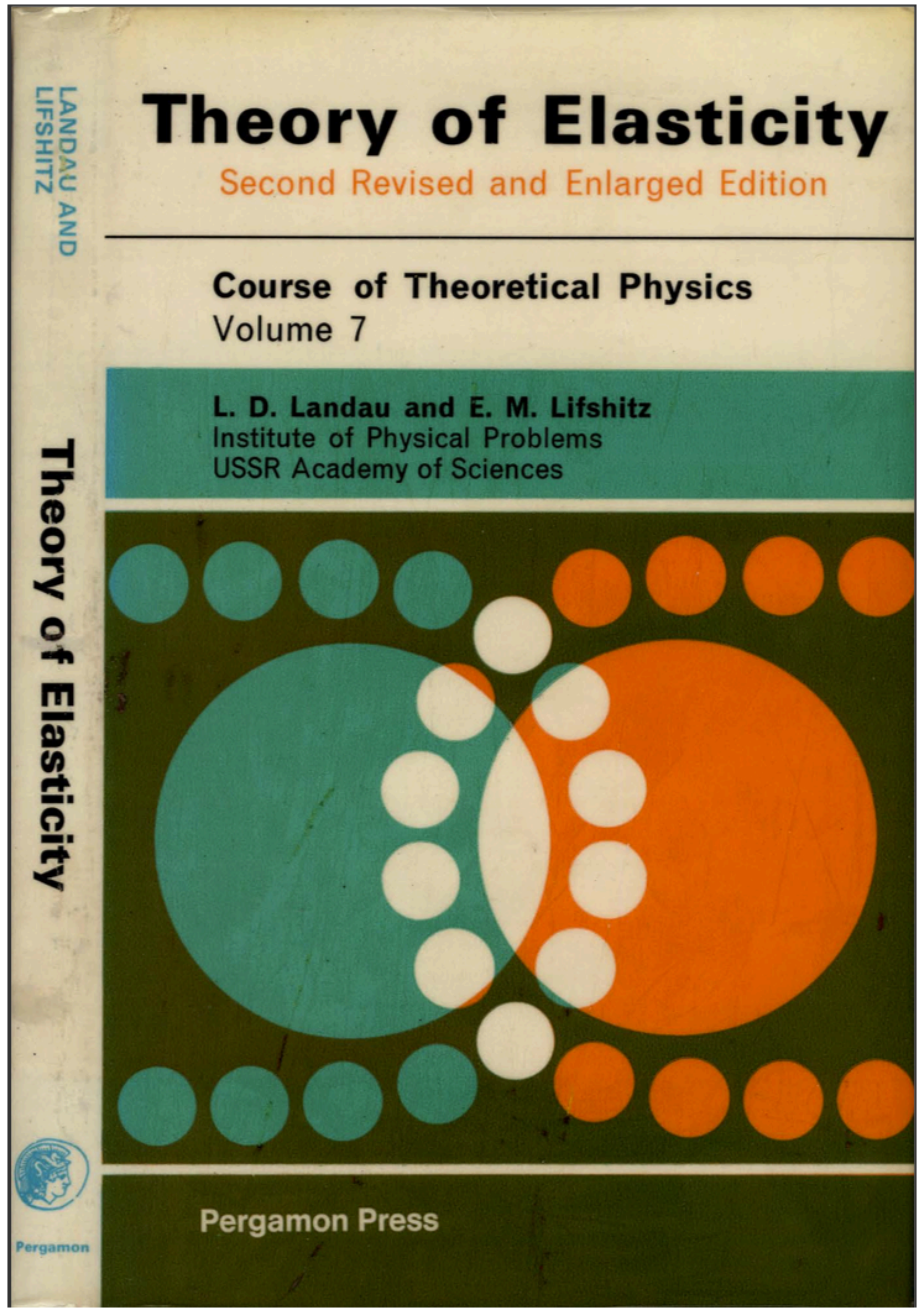
take out conservation of energy

Activity → **chirality in continuum mechanics**

take out conservation of energy

How do you understand and model “phase” transitions ?

What is elasticity ?



Linear elasticity

Stress | Stiffness Tensor | Strain

displacements

Hooke's law

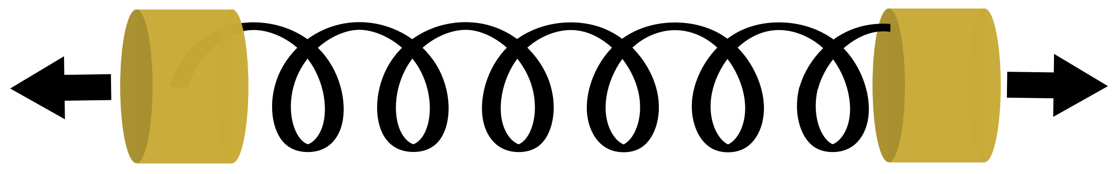
$$\sigma_{ij} = K_{ijmn} u_{mn}$$

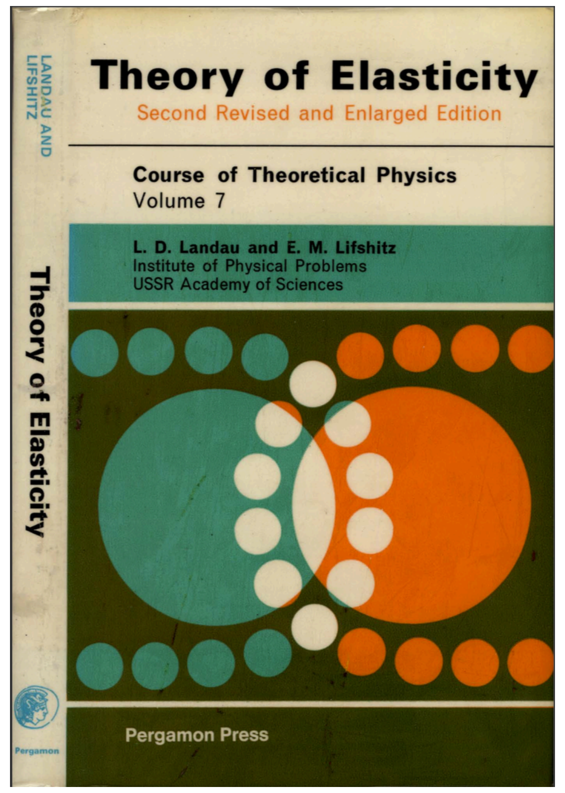
u_{xx}

gradients

$$\mathbf{F} = -(k\hat{\mathbf{r}})\delta r$$

δr





Stress is proportional to strain

Linear elasticity with internal torques

Stress | Stiffness Tensor | Strain

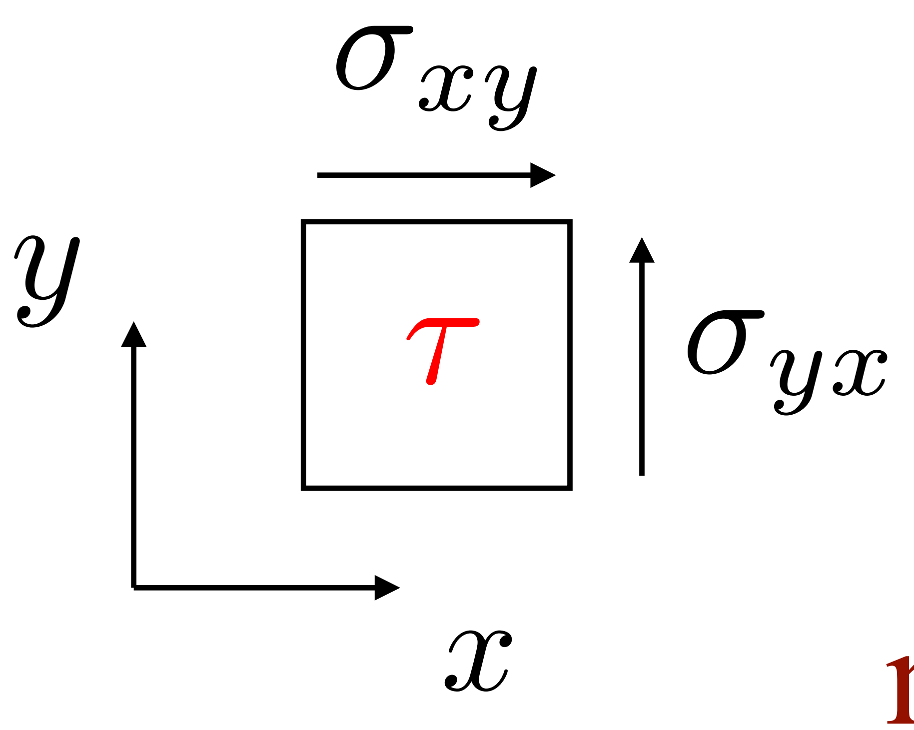
displacements

Hooke's law

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

u_{xx}

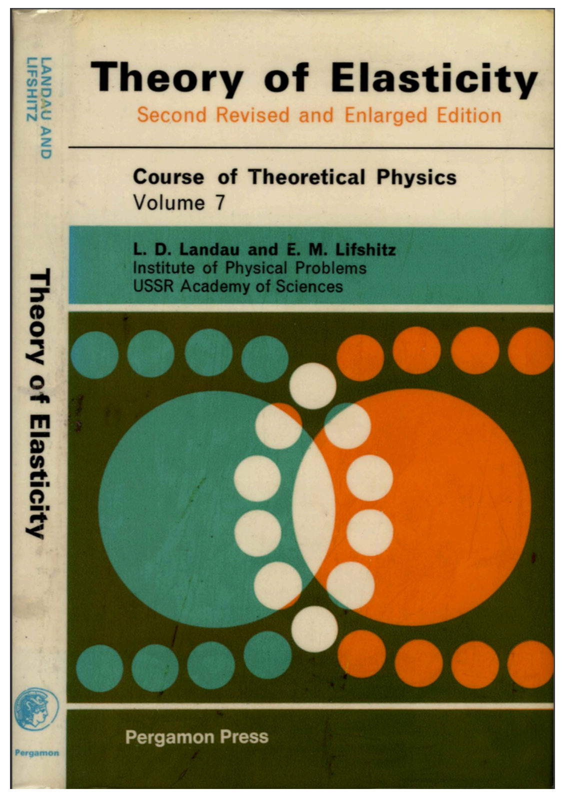
gradients



no angular momentum conservation

$\sigma_{ij} \neq \sigma_{ji}$

net torques



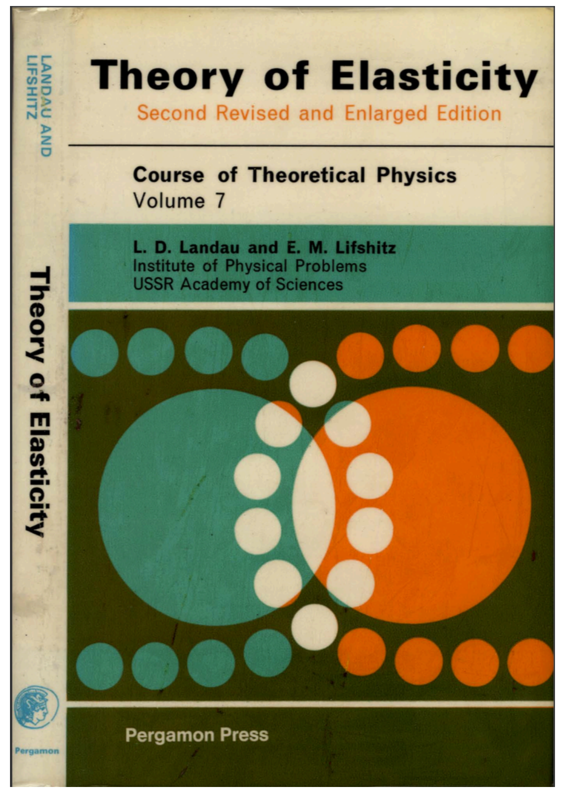
Symmetry of the stiffness tensor

Stress — Stiffness Tensor — Strain

Hooke's law $$\sigma_{ij} = K_{ijmn} u_{mn}$$

$$K_{ijmn} = K_{mnij}$$

Where does this symmetry come from?

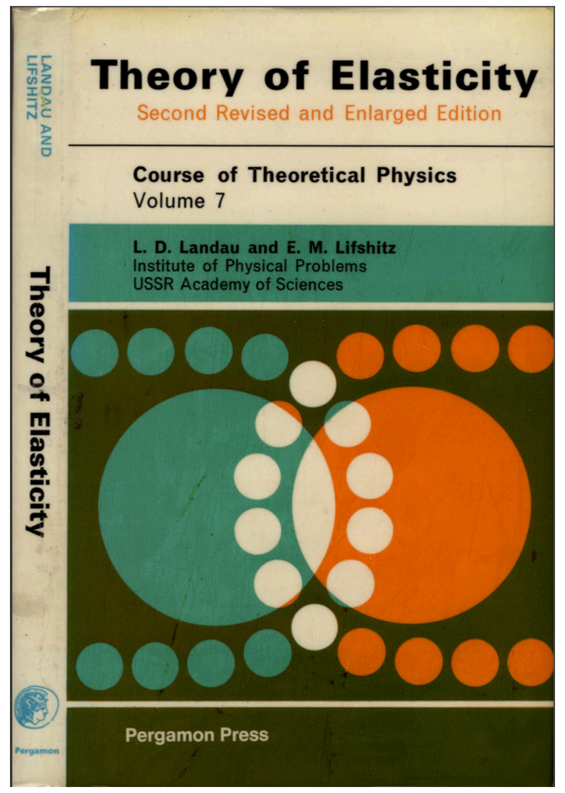


Energy conservation and other prejudices

If

$$f = \frac{1}{2} C_{ijmn} u_{ij} u_{mn}$$

Elastic Energy



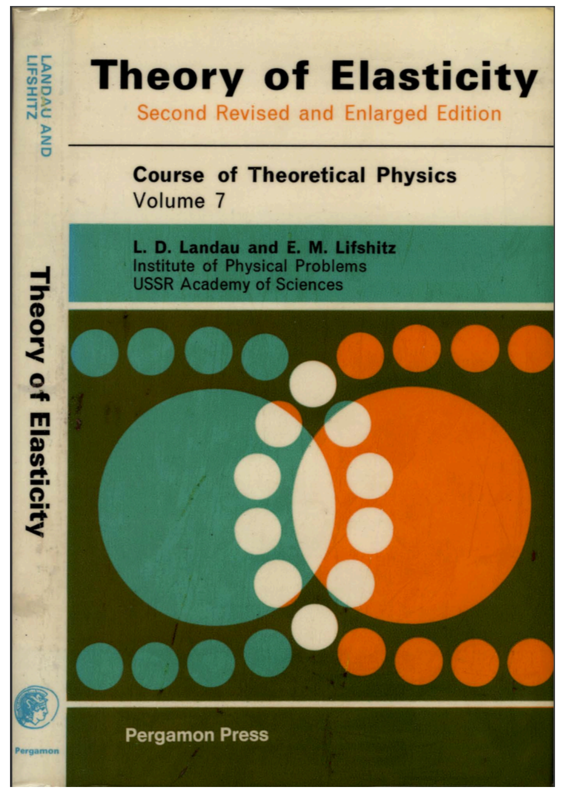
Stress from potential energy

If $$f = \frac{1}{2} C_{ijmn} u_{ij} u_{mn}$$

Elastic Energy

$$\sigma_{ij} = \frac{\partial f}{\partial u_{ij}}$$

Hooke's law $$\sigma_{ij} = \frac{1}{2}\left(C_{ijmn} + C_{mnij}\right) u_{mn}$$



Stiffness tensor must be symmetric

If
$$f = \frac{1}{2} C_{ijmn} u_{ij} u_{mn}$$

Elastic Energy

$$\sigma_{ij} = \frac{\partial f}{\partial u_{ij}}$$

Hooke's law
$$\sigma_{ij} = \underbrace{\frac{1}{2}\left(C_{ijmn} + C_{mnij}\right)}_{K_{ijmn}} u_{mn}$$

Energy conservation and other prejudices

If
$$f = \frac{1}{2} K_{ijmn} u_{ij} u_{mn}$$

Elastic Energy

$$\sigma_{ij} = \frac{\partial f}{\partial u_{ij}}$$

Hooke's law
$$\sigma_{ij} = K_{ijmn} u_{mn}$$

Then
$$K_{ijmn} = K_{mnij}$$

Symmetry is a consequence of potential elastic energy

What if energy is not conserved ?

$$f = \frac{1}{2} K_{ijmn} u_{ij} u_{mn}$$

allow for
non conservative
forces

Hooke's law

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

compatible with
linear momentum conservation

Scheibner, Souslov, Banerjee, Surowka, Irvine, Vitelli, Nat Phys 2020

Hooke's law is still valid

$$f = \frac{1}{2} K_{ijmn} u_{ij} u_{mn}$$

allow for
non conservative
forces

Hooke's law

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

compatible with
linear momentum conservation

$$K_{ijmn} = K^{e}_{ijmn} + K^{o}_{ijmn}$$

but the stiffness tensor is no longer symmetric

Scheibner, Souslov, Banerjee, Surowka, Irvine, Vitelli, Nat Phys 2020

Odd elasticity

$$f = \frac{1}{2} K_{ijmn} u_{ij} u_{mn}$$

allow for
non conservative
forces

Hooke's law

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

compatible with
linear momentum conservation

$$K_{ijmn} = K^e_{ijmn} + K^o_{ijmn}$$

$$K^e_{ijmn} = K^e_{mnij}$$

$$K^o_{ijmn} = -K^o_{mnij}$$

NEW MODULI

Scheibner, Souslov, Banerjee, Surowka, Irvine, Vitelli, Nat Phys 2020

Odd elasticity

$$f = \frac{1}{2}K_{ijmn}u_{ij}u_{mn}$$

allow for
non conservative
forces

Hooke's law

$$\sigma_{ij} = K_{ijmn}u_{mn}$$

Focus on
chirality!

$$K_{ijmn} = K^{e}_{ijmn} + K^{o}_{ijmn}$$

$$K^{e}_{ijmn} = K^{e}_{mnij}$$

$$K^{o}_{ijmn} = -K^{o}_{mnij}$$

NEW MODULI

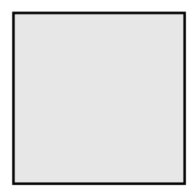
Scheibner, Souslov, Banerjee, Surowka, Irvine, Vitelli, Nat Phys 2020

Visual linear elasticity in 2D

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

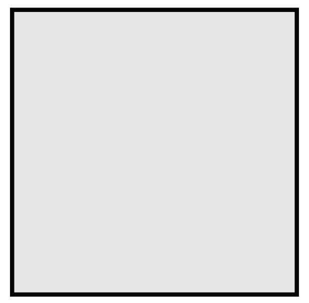
Visual representation of strain

$$\sigma_{ij} = K_{ijmn} u_{mn}$$



Visual representation of strain

$$\sigma_{ij} = K_{ijmn} u_{mn}$$



dilation

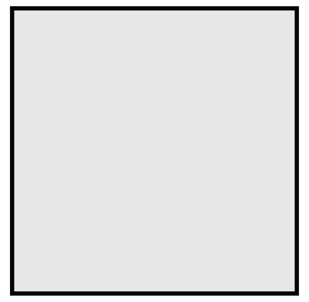
Visual representation of strain

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

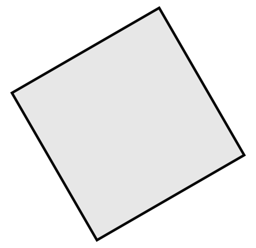
dilation

Visual representation of strain

$$\sigma_{ij} = K_{ijmn} u_{mn}$$



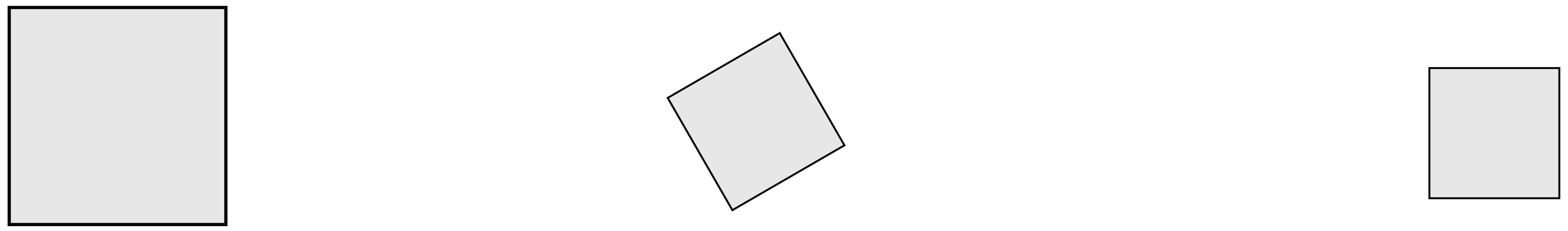
dilation



rotation

Visual representation of strain

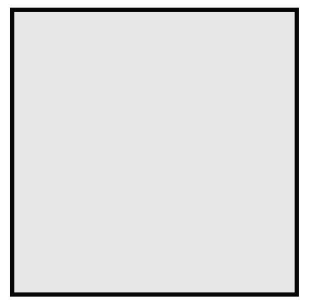
$$\sigma_{ij} = K_{ijmn} u_{mn}$$



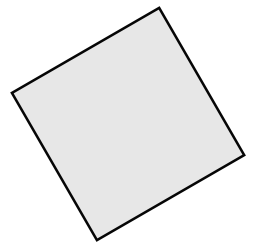
dilation rotation

Visual representation of strain

$$\sigma_{ij} = K_{ijmn} u_{mn}$$



dilation



rotation

S_1

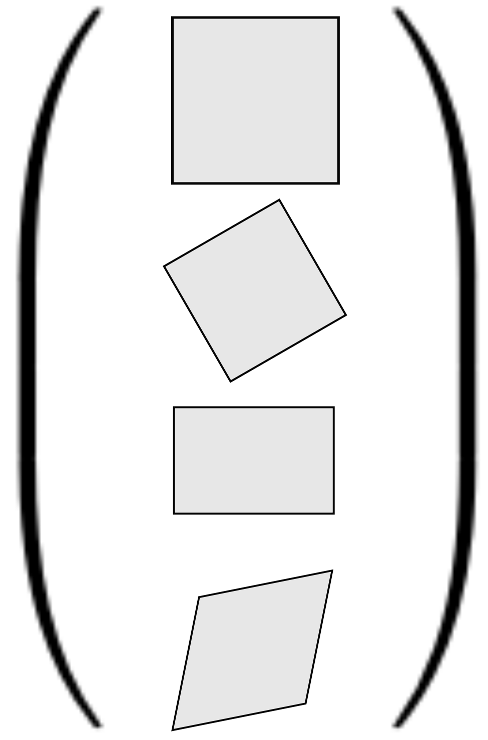
Visual representation of strain

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

dilation rotation S_1

Visual representation of strain

$$\sigma_{ij} = K_{ijmn} u_{mn}$$



Visual representation of stress

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

Let’s start from scratch

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

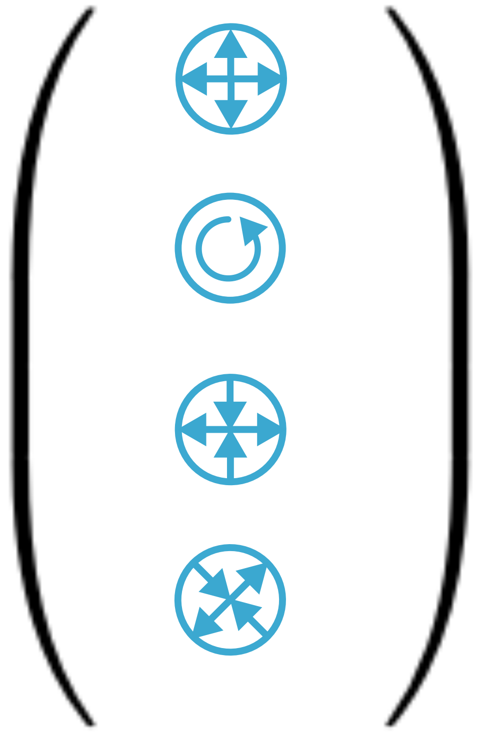
$$\sigma_a = K_{ab} u_b$$

Let’s start from scratch

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

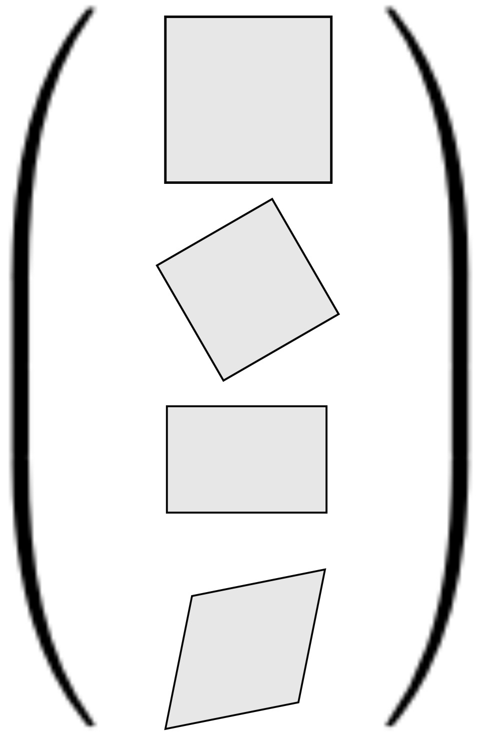
$$\sigma_a = K_{ab} u_b$$

Assumption 1: Isotropy



=

?	?	0	0
?	?	0	0
0	0	?	?
0	0	-?	?

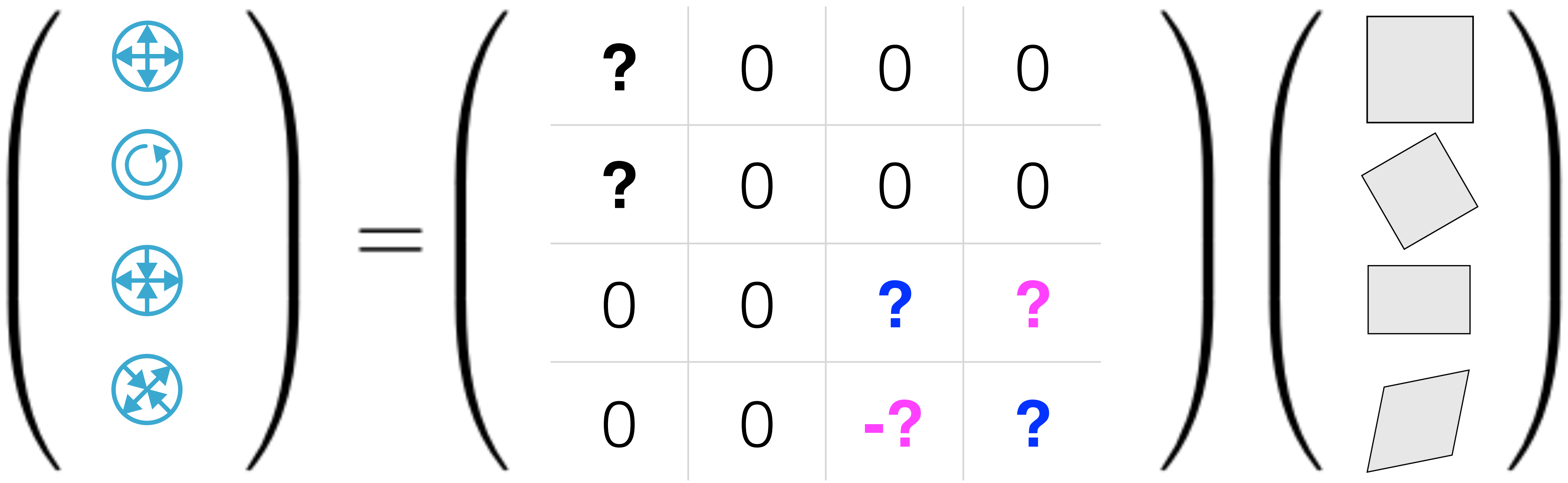


Let’s start from scratch

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

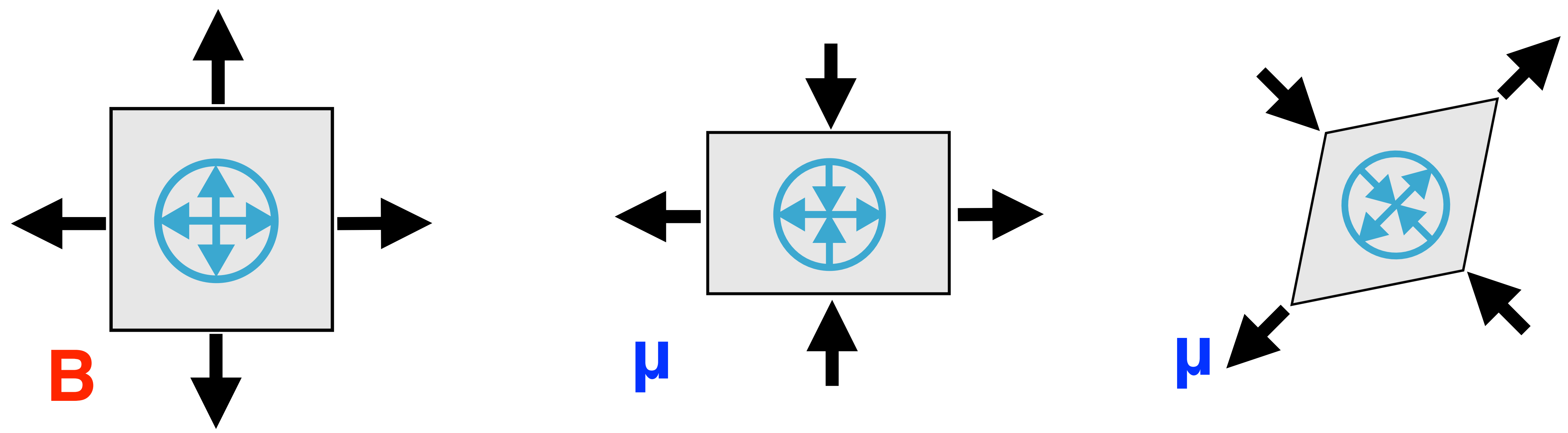
$$\sigma_a = K_{ab} \, u_b$$

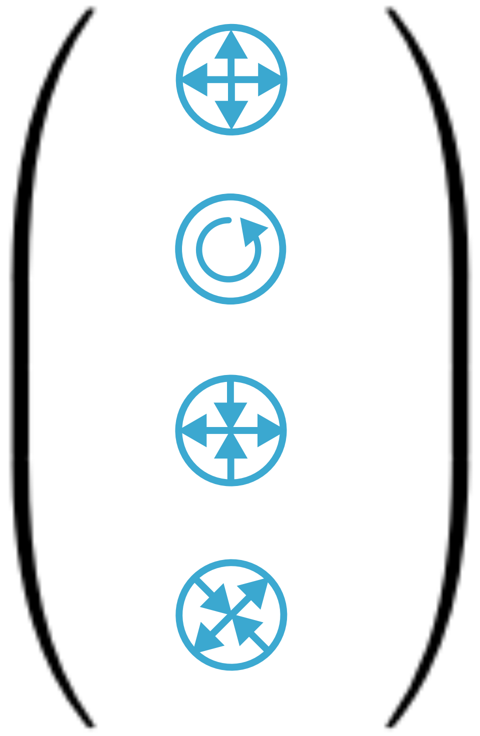
Assumption 2: Deformation Dependence



Passive elasticity of isotropic 2D solids

$$\sigma_{ij} = K_{ijmn} u_{mn}$$





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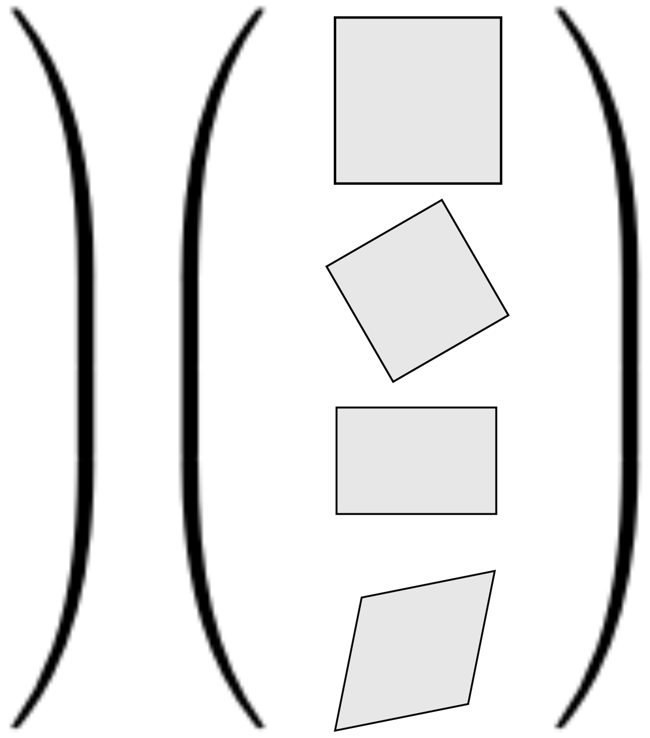
B	0	0	0
?	0	0	0
0	0	μ	?
0	0	-?	μ



Odd elasticity

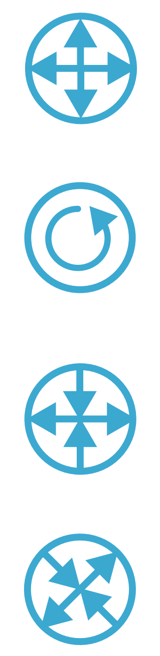
$$\sigma_{ij} = K_{ijmn} u_{mn}$$

$$\sigma_a = K_{ab} u_b$$

two additional moduli

would be zero if energy is conserved!

CHIRALITY EMERGES



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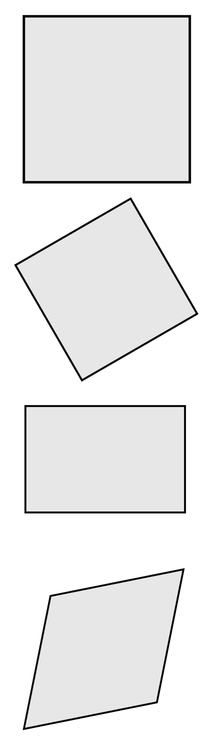
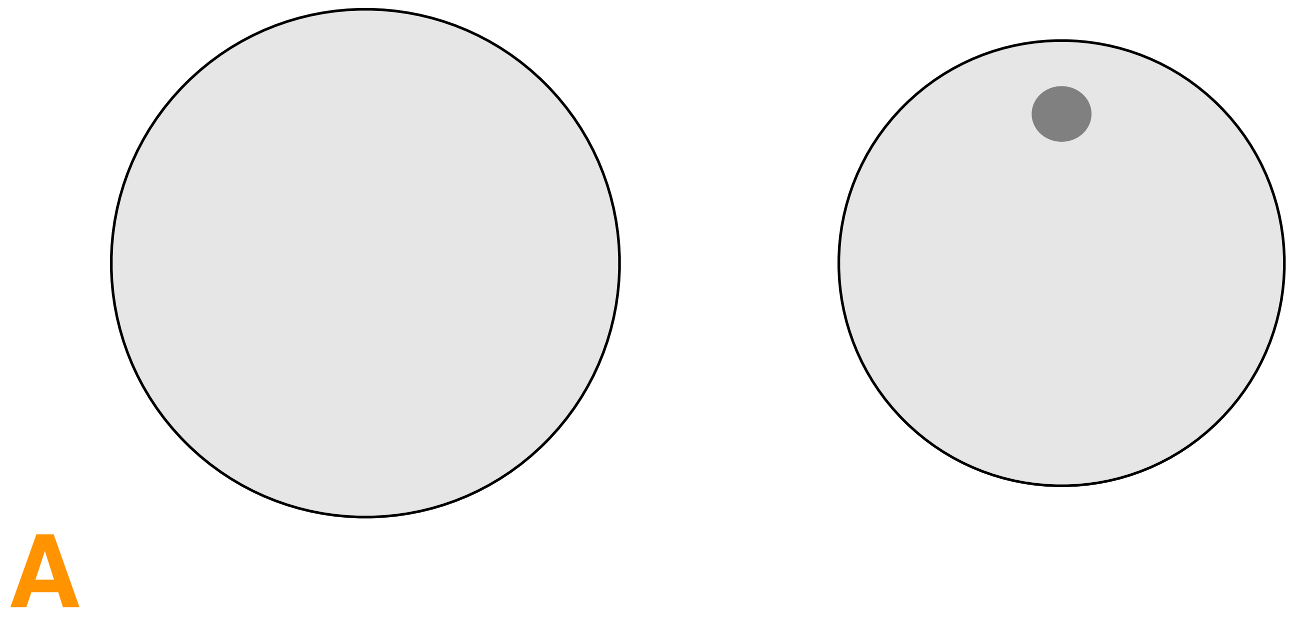
B	0	0	0
A	0	0	0
0	0	μ	K°
0	0	-K°	μ

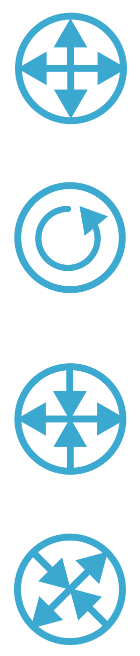


$$K^o_{ijmn} = -K^o_{mnij}$$

Odd elasticity: static response

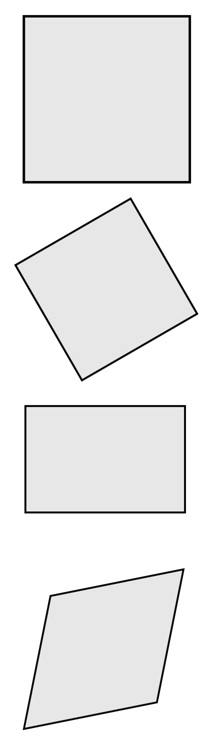
$A > 0$





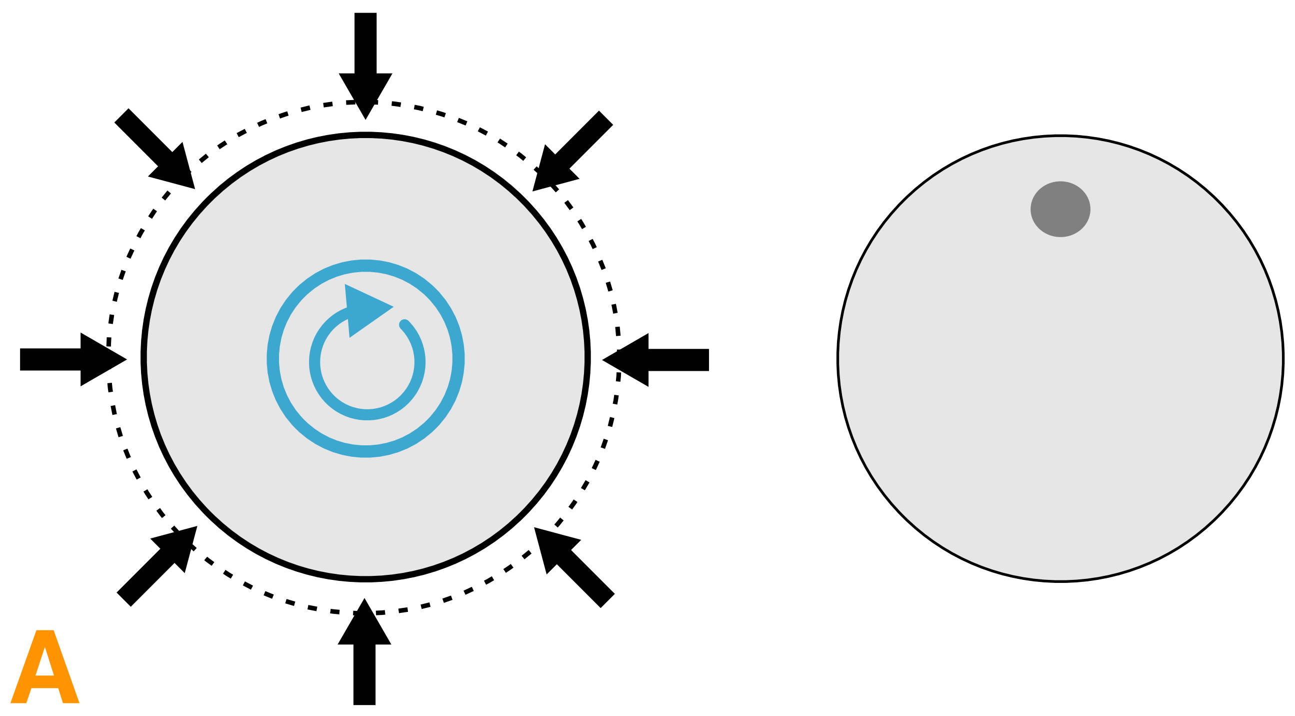
=

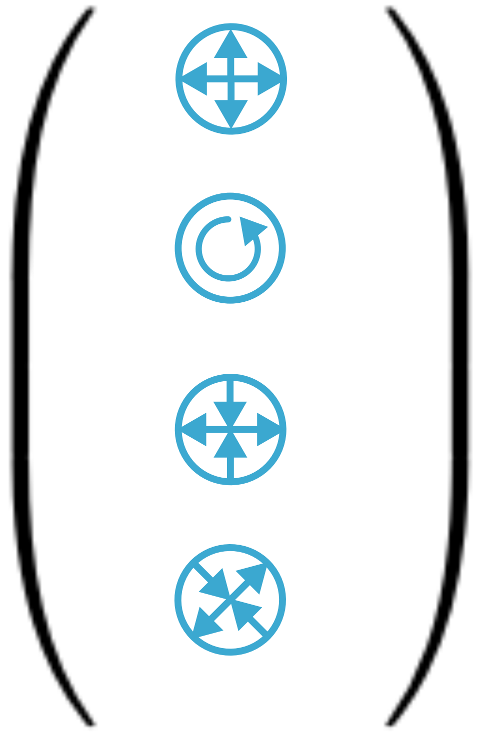
B	0	0	0
A	0	0	0
0	0	**μ**	**K°**
0	0	**-K°**	**μ**



Odd elasticity: static response

$A > 0$





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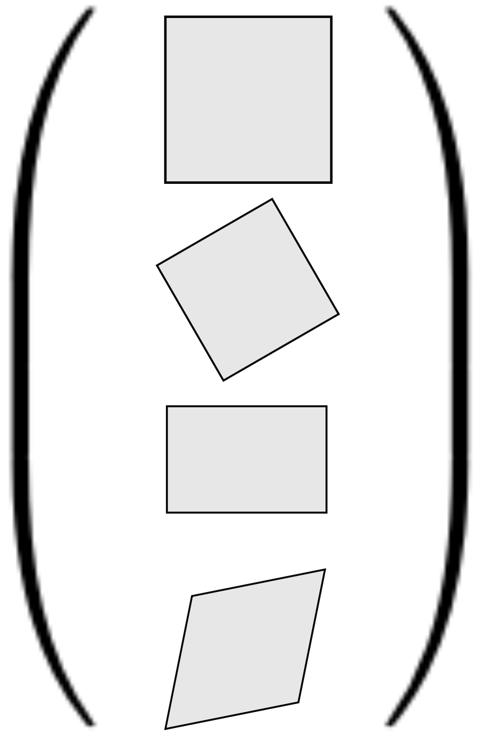
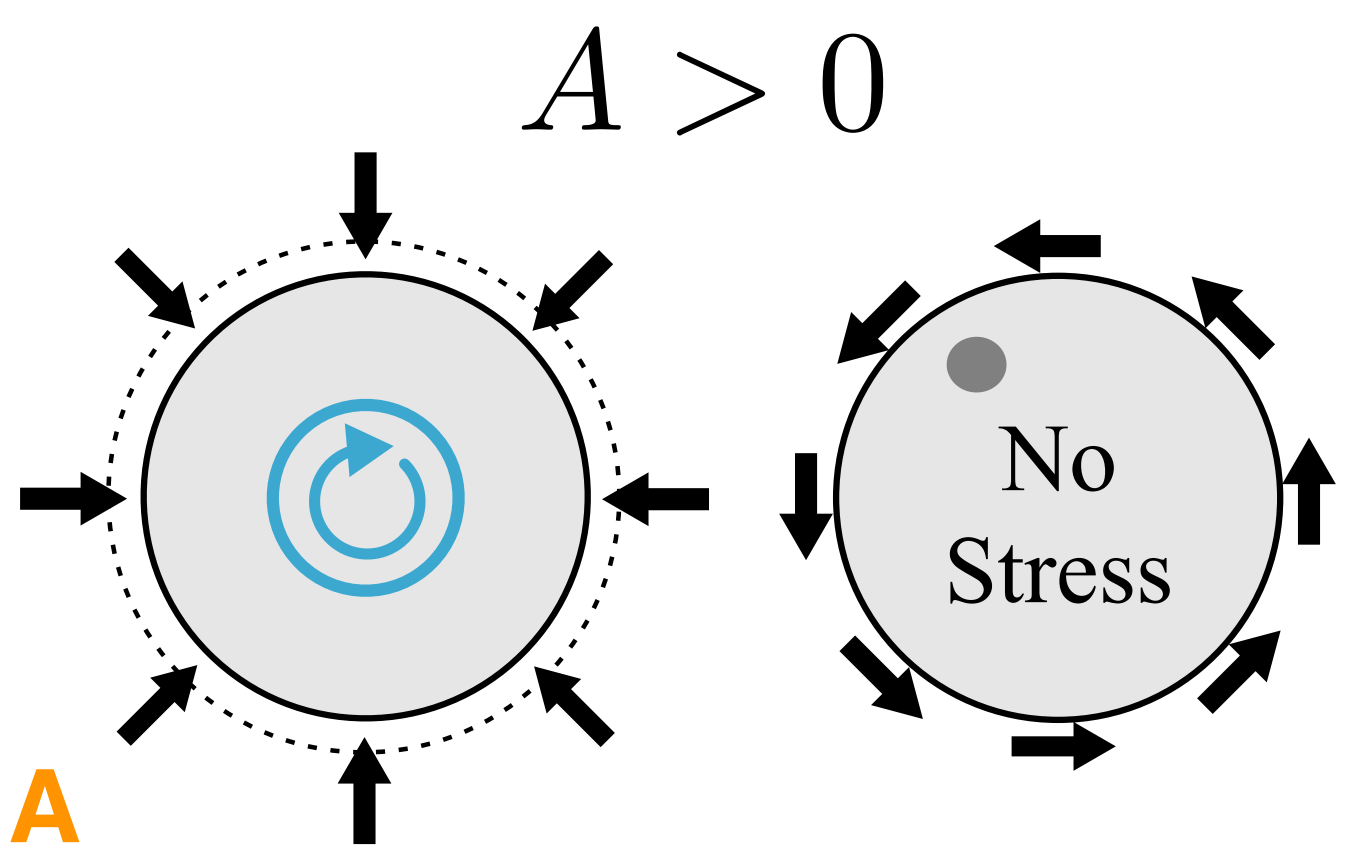
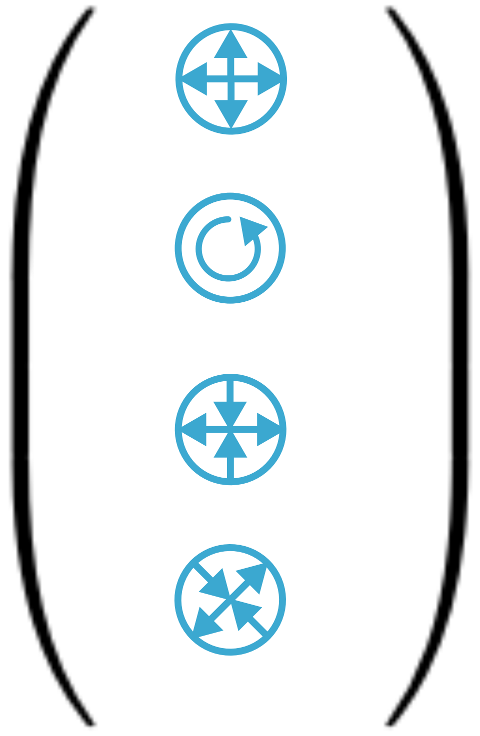
B	0	0	0
A	0	0	0
0	0	**μ**	**K°**
0	0	**-K°**	**μ**



CHIRALITY COMES FROM TORQUES

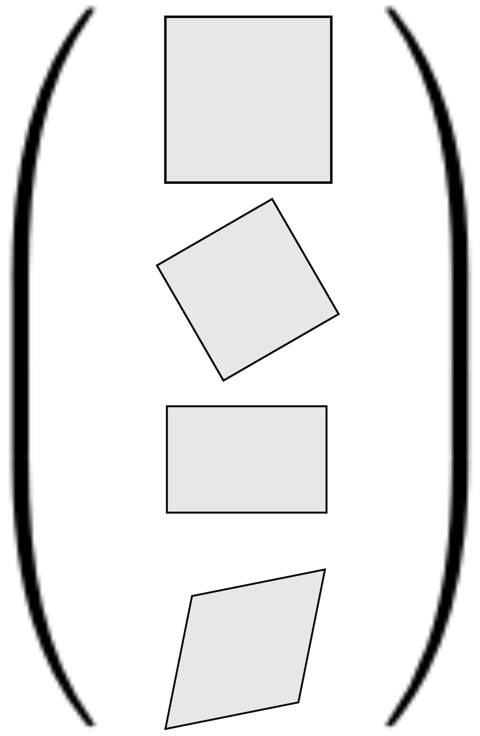
Odd elasticity: static response



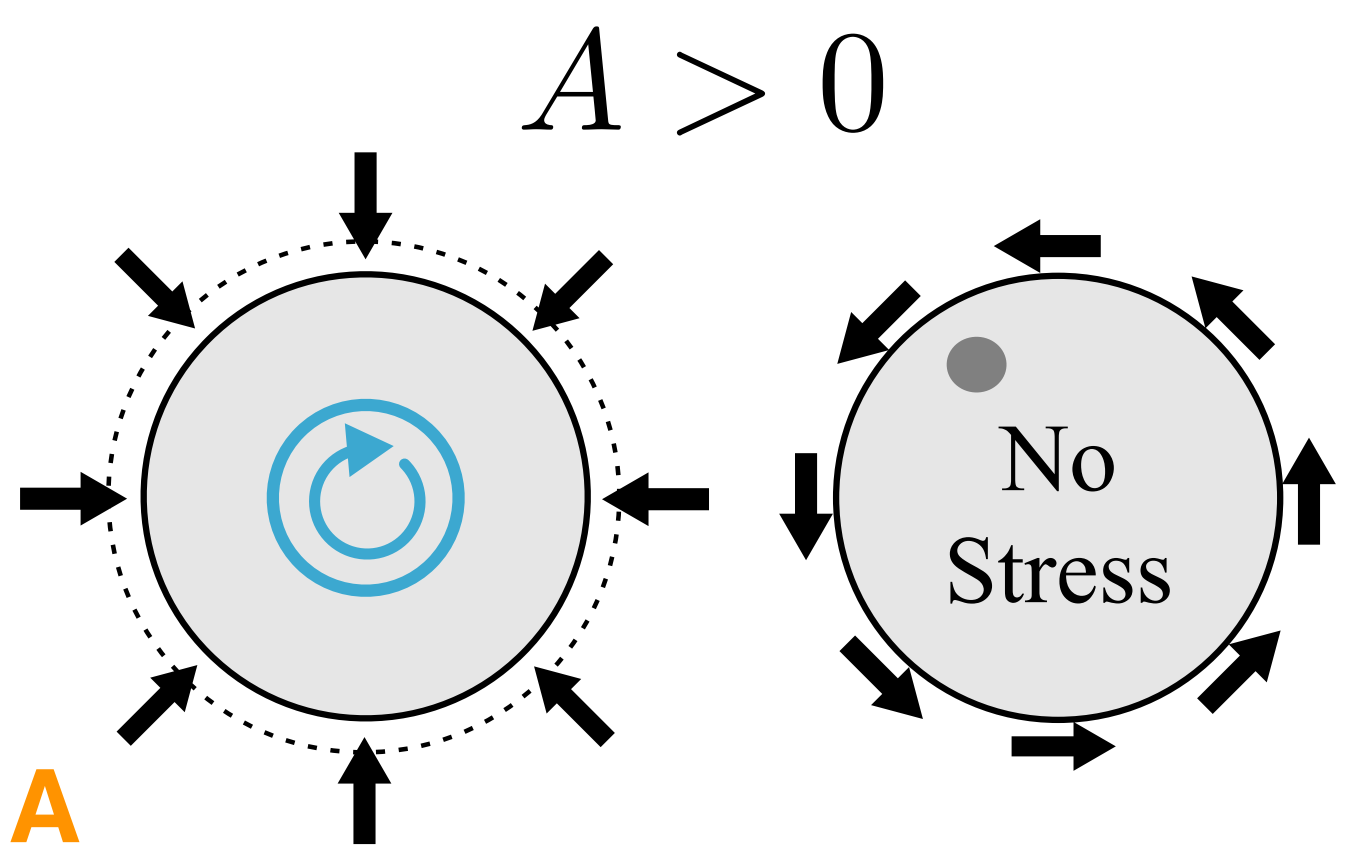


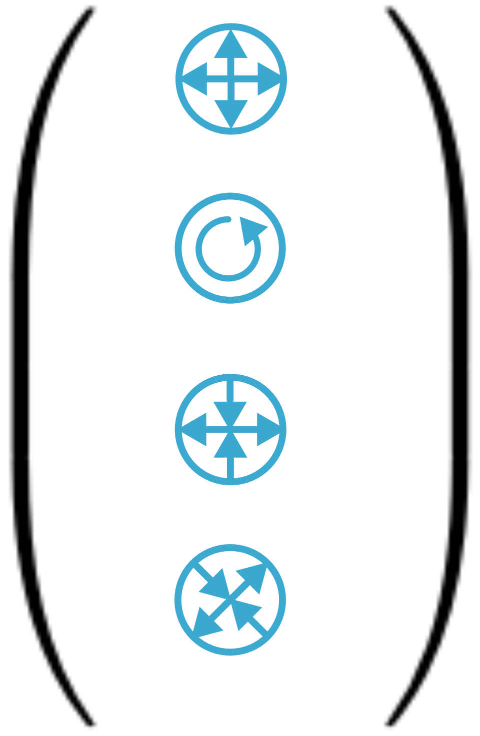
=

B	0	0	0
A	0	0	0
0	0	μ	K^o
0	0	$-K^o$	μ

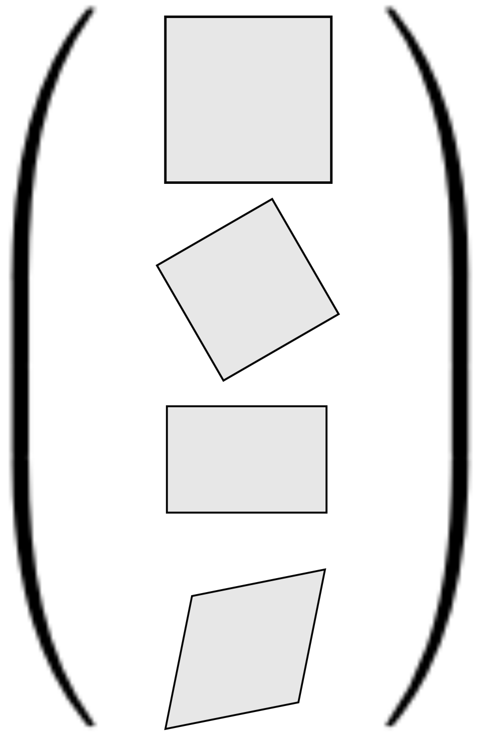


Odd elasticity: static response





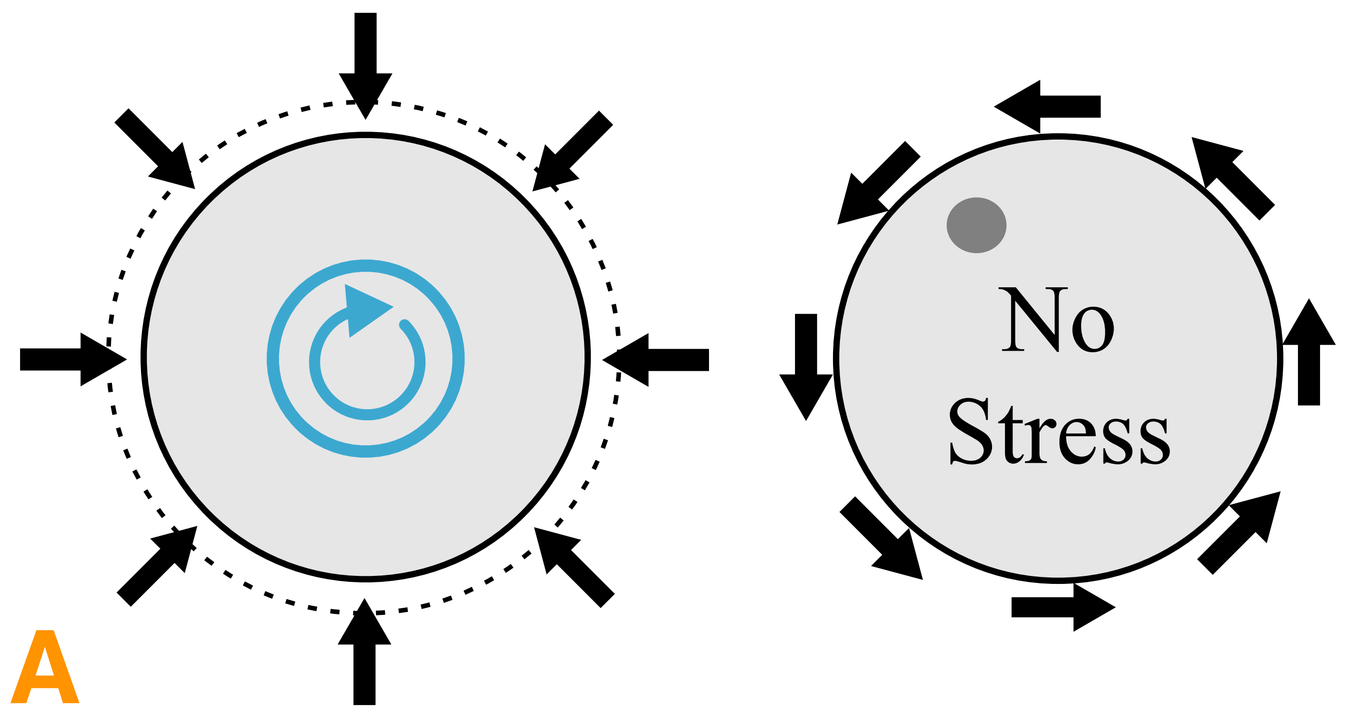
$$= \begin{pmatrix} B & 0 & 0 & 0 \\ A & 0 & 0 & 0 \\ 0 & 0 & \mu & K^o \\ 0 & 0 & -K^o & \mu \end{pmatrix}$$



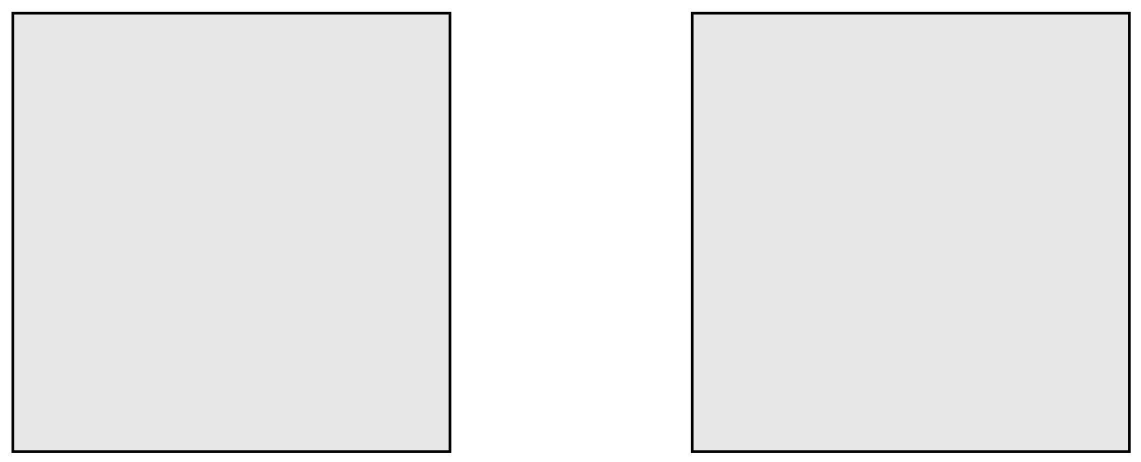
Non-reciprocity from odd coefficients

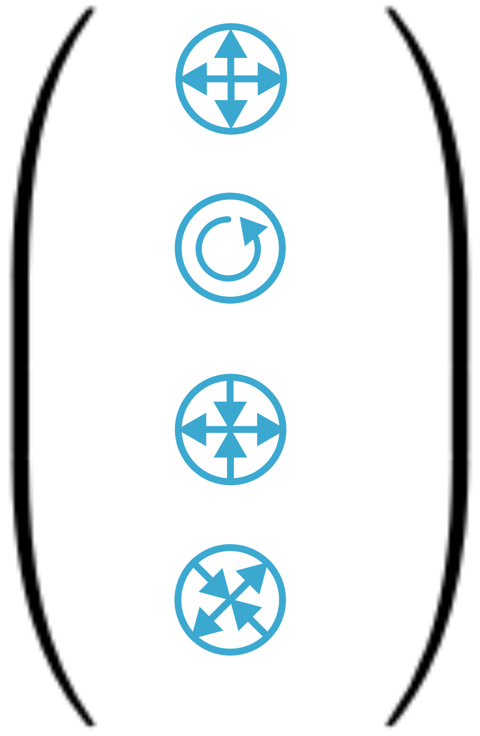
Odd elasticity: static response

$A > 0$



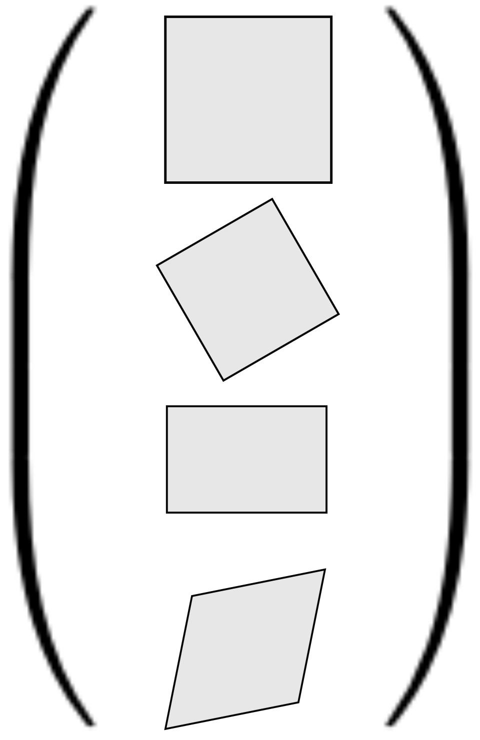
$K^o > 0$





=

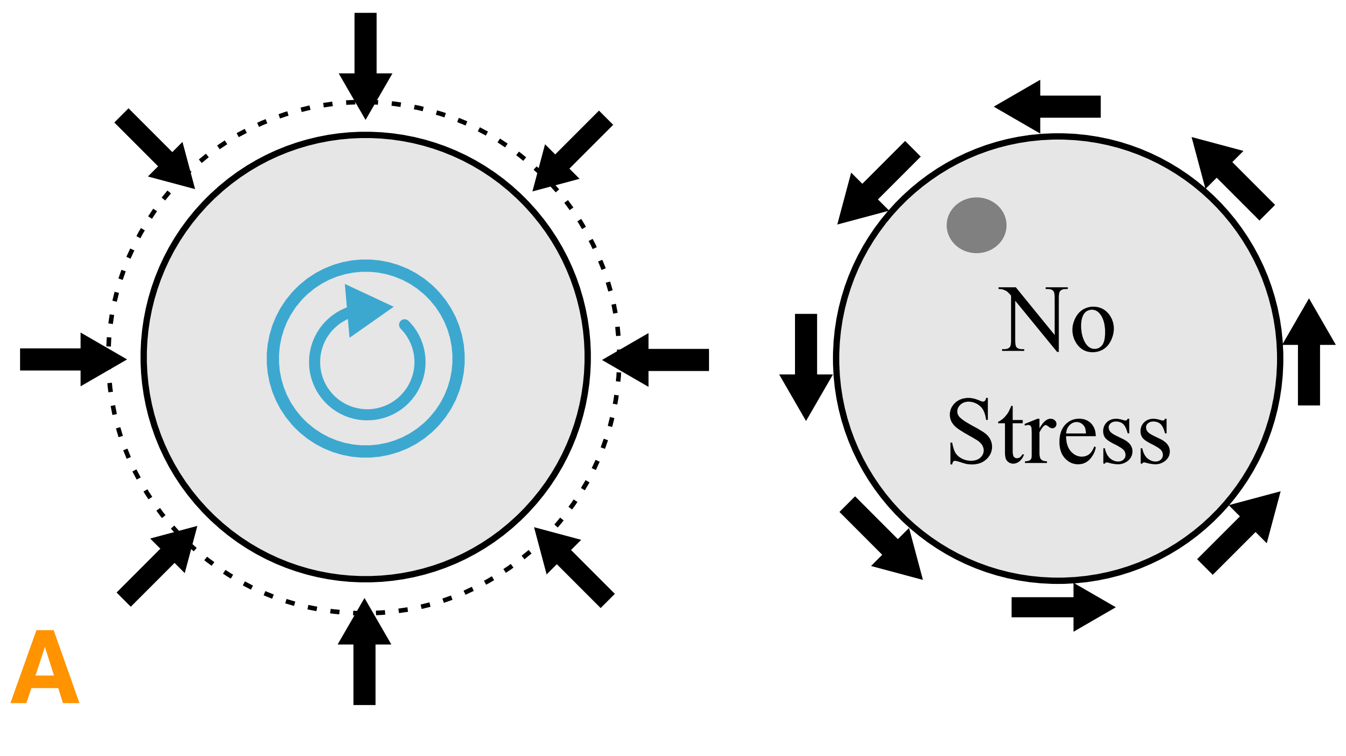
B	0	0	0
A	0	0	0
0	0	μ	Kº
0	0	-Kº	μ



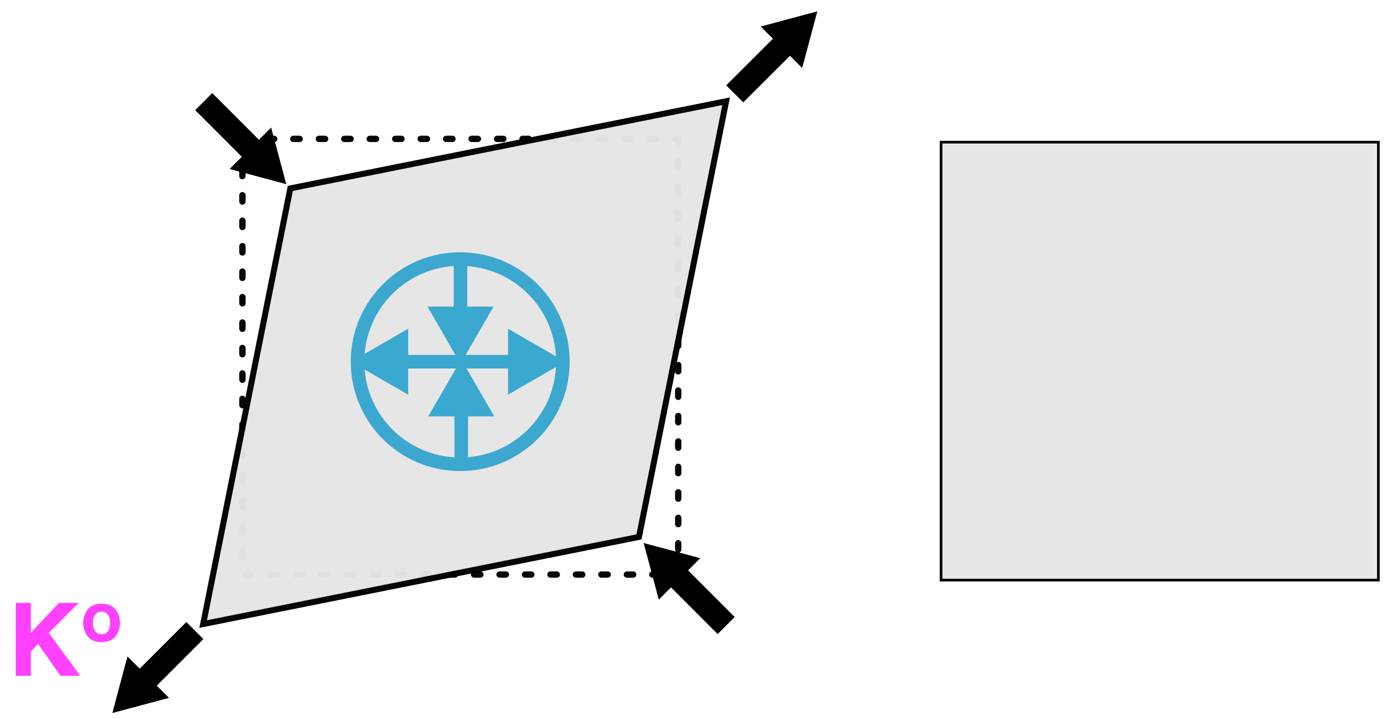
“ROTATION”
IN SHEAR
SPACE

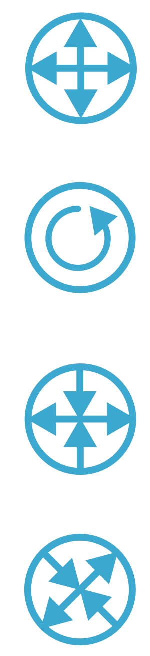
Odd elasticity: static response

$A > 0$



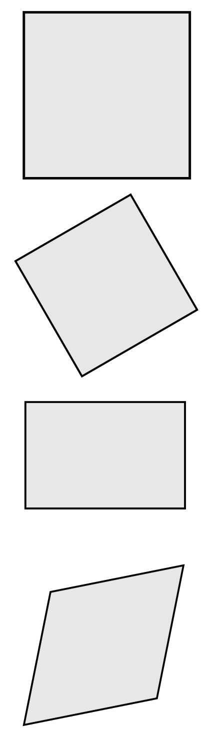
$K^o > 0$





=

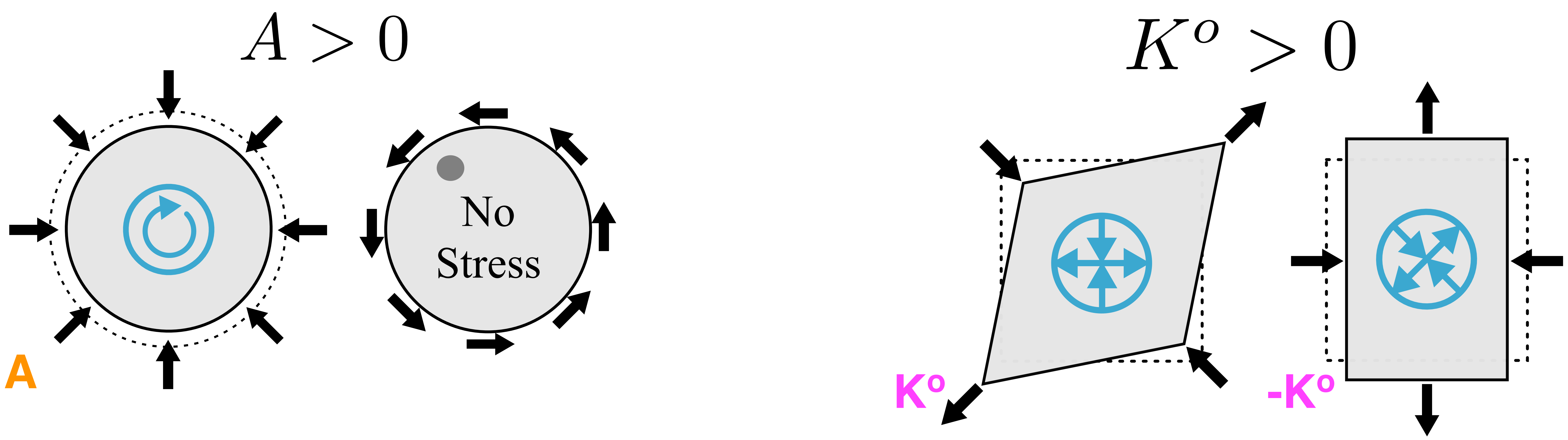
B	0	0	0
A	0	0	0
0	0	μ	Kº
0	0	-Kº	μ

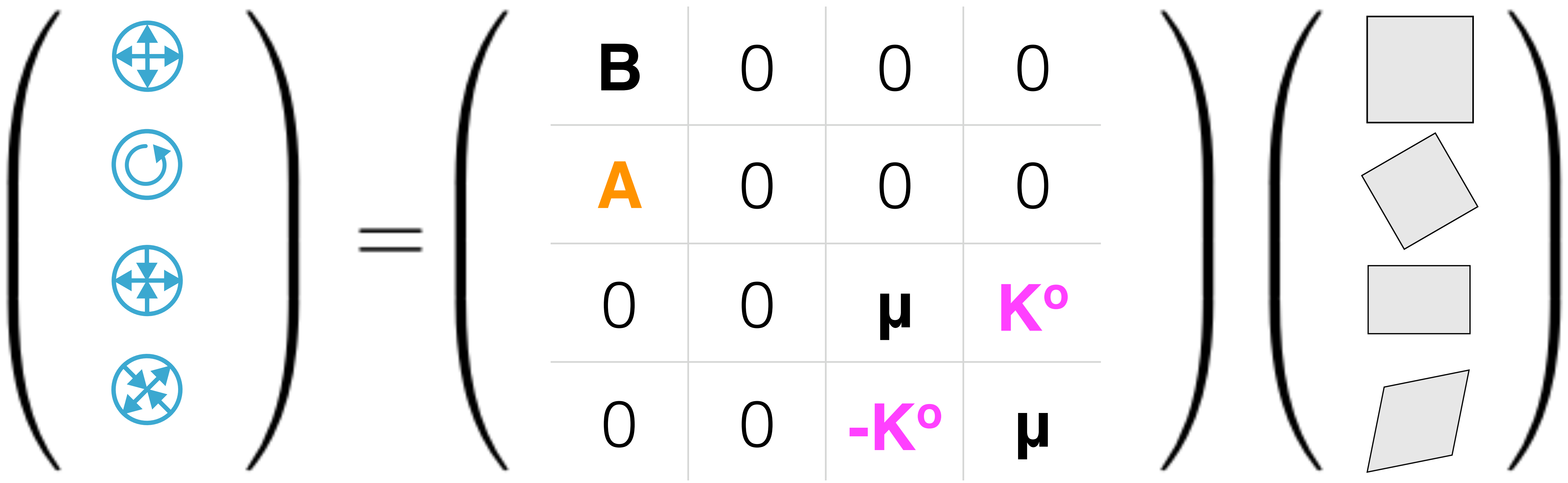


"ROTATION"
IN SHEAR
SPACE

Non-reciprocity from odd coefficients

Odd elasticity: static response





“ROTATION”
IN SHEAR
SPACE

Two odd moduli for isotropic solids

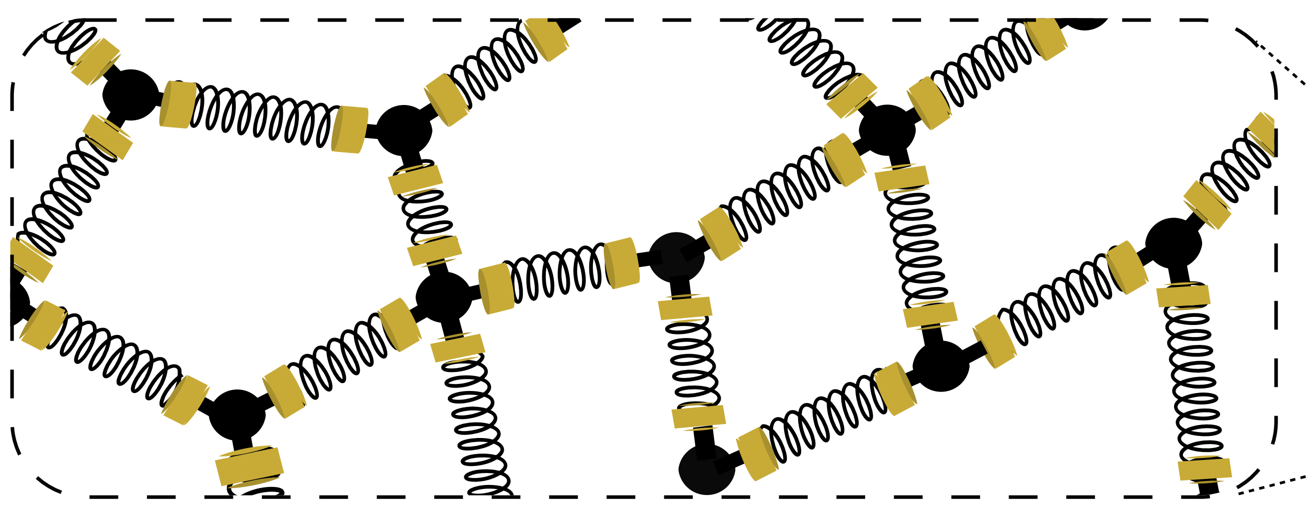
Elastic engine cycle

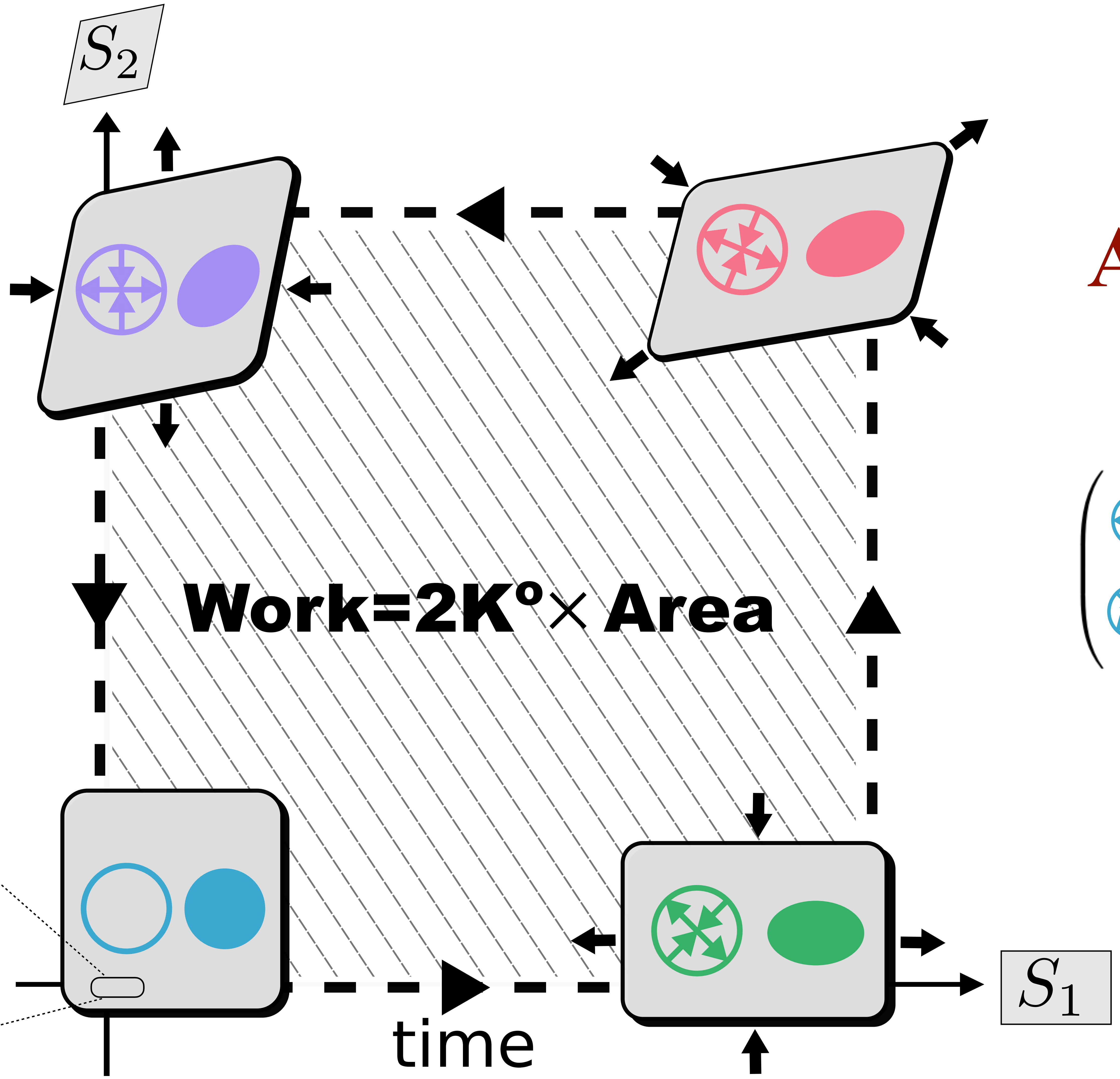
~~$\sigma_{ij} = \frac{\partial f}{\partial u_{ij}}$~~

Quasistatic

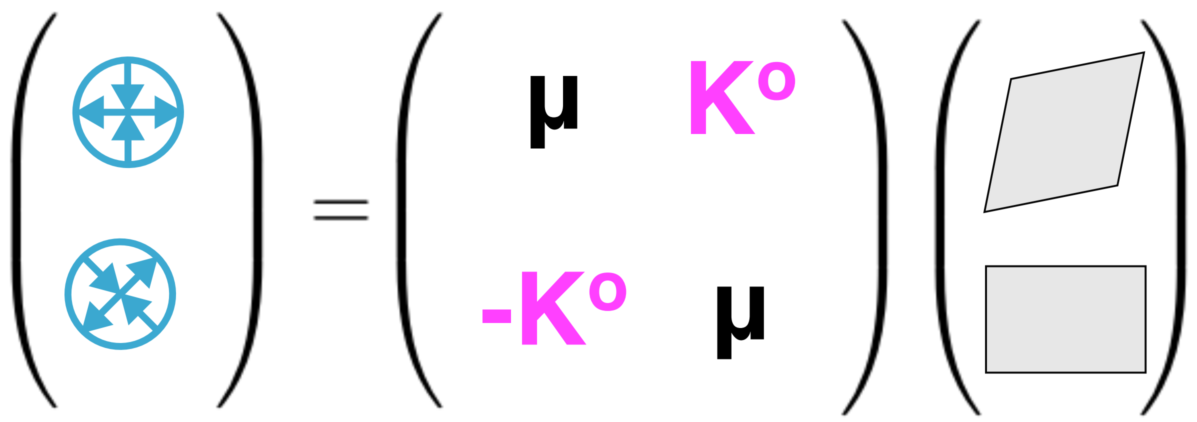
$$\mathrm{d}w = -\,\sigma_{ij}\mathrm{d}u_{ij}$$

$$w_{cycle} = -\oint K^{o}_{ijmn} u_{mn} \mathrm{d}u_{ij}$$





Active medium



"ROTATION"
IN SHEAR
SPACE

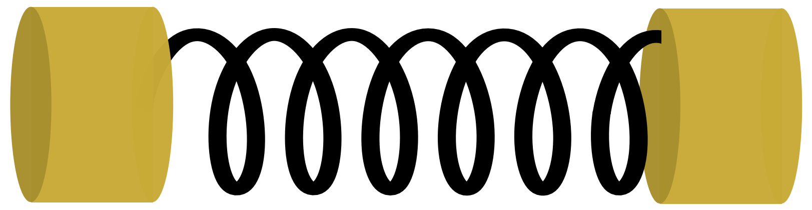
an odd elastic medium is a distributed engine

Assignment

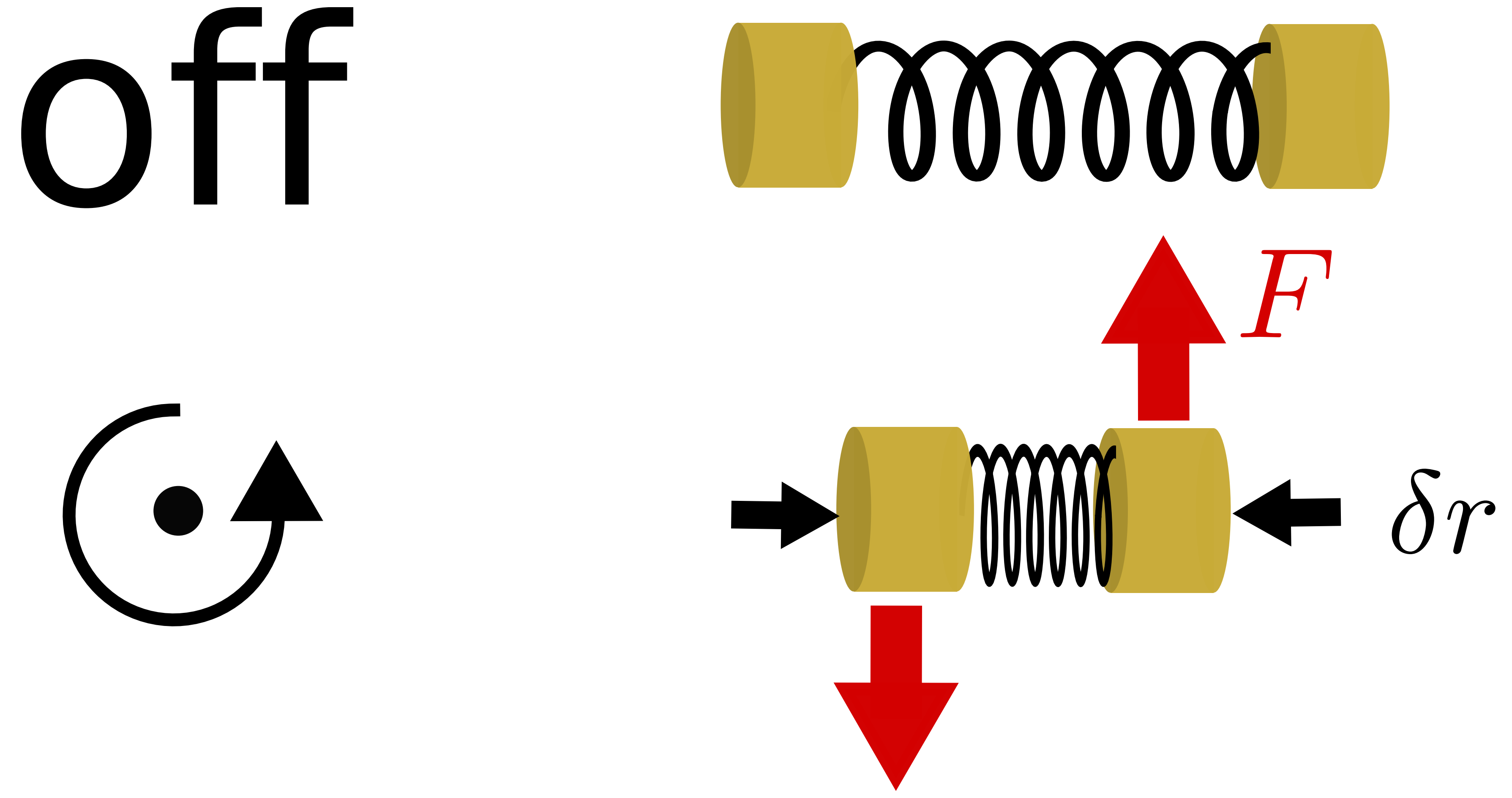
“An intro to chirality in active matter
and how we describe it,
both at the ***micro***
and macro levels.”

Microscopic model I: active bonds

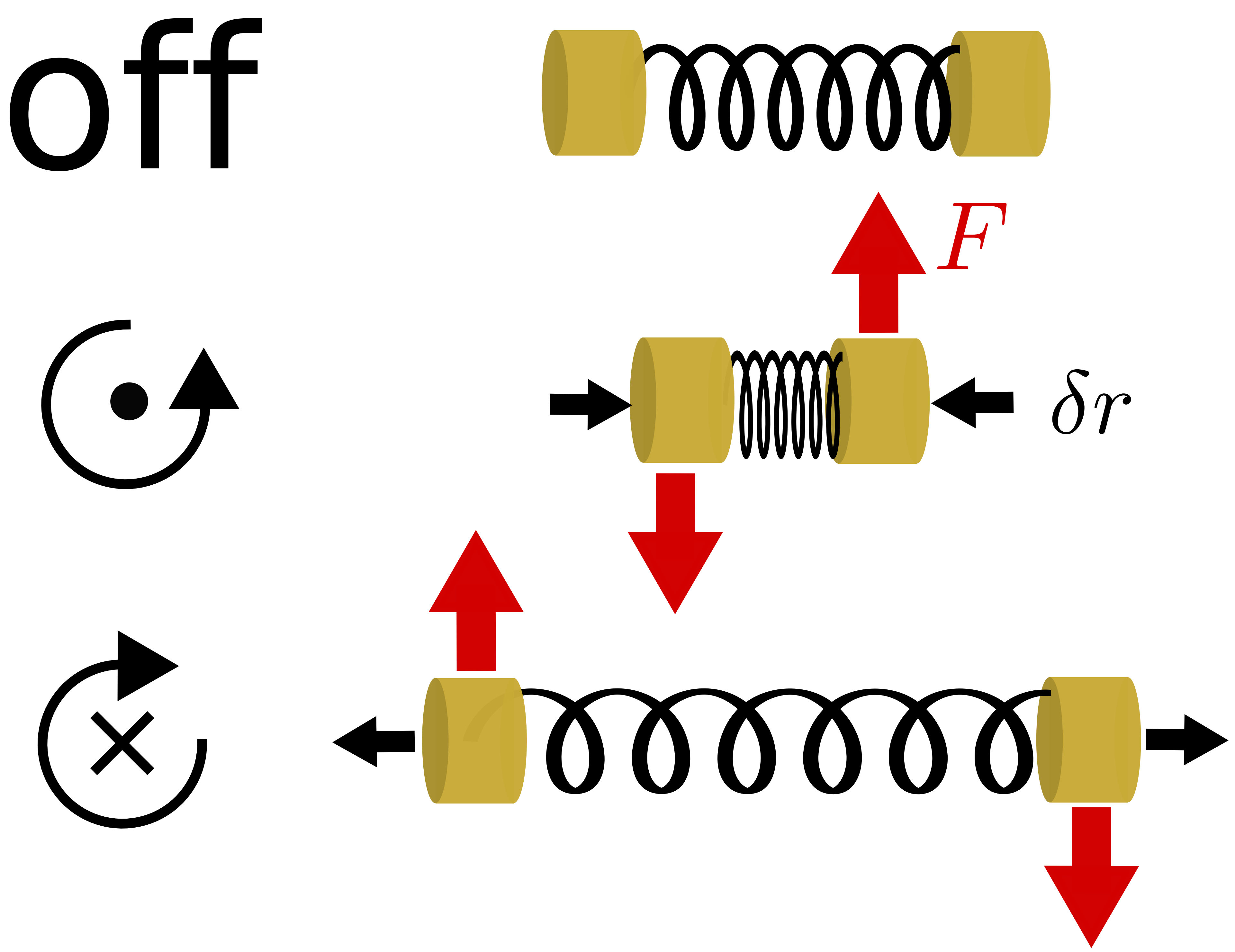
off



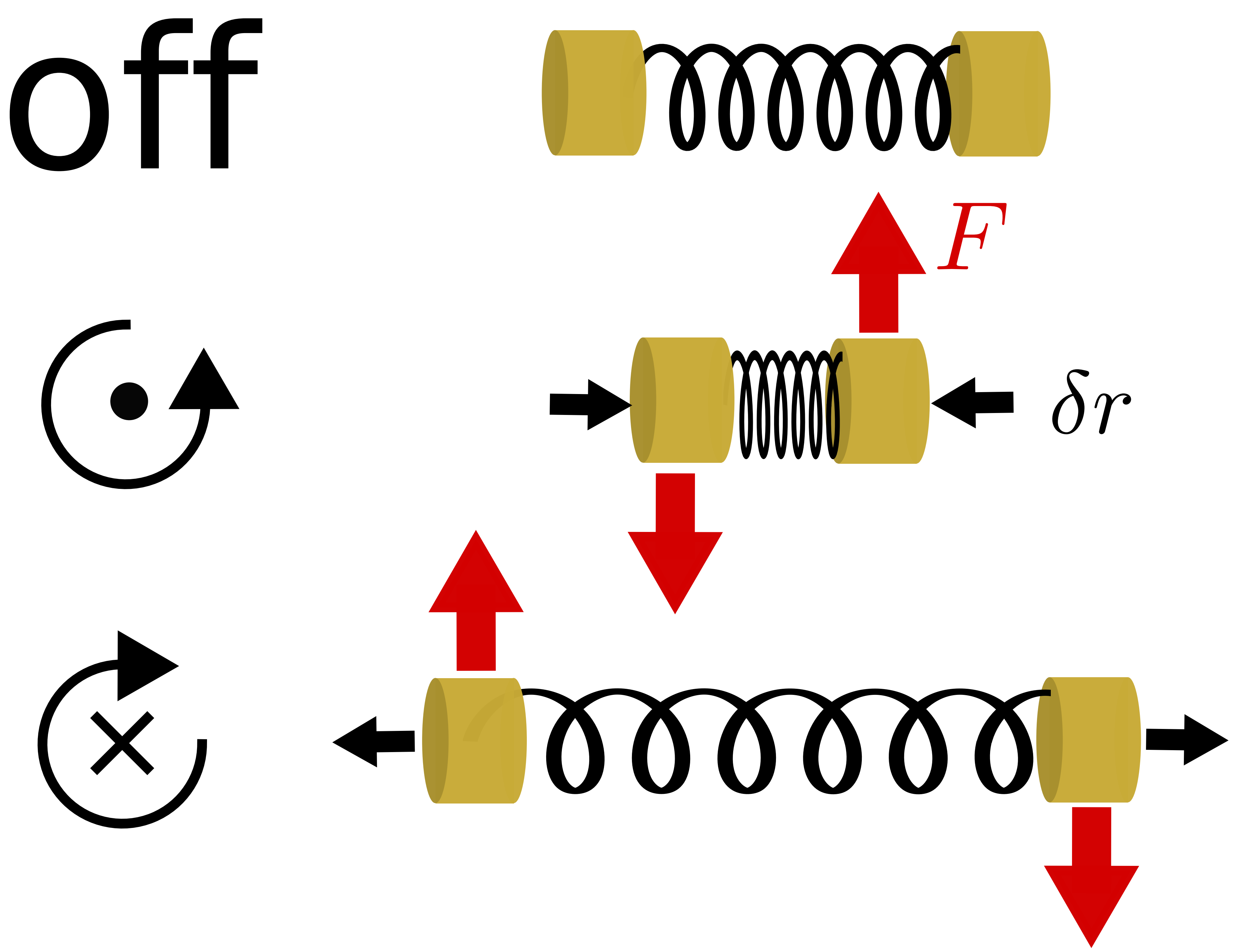
Microscopic model I: active bonds



Microscopic model I: active bonds



Microscopic model I: active bonds

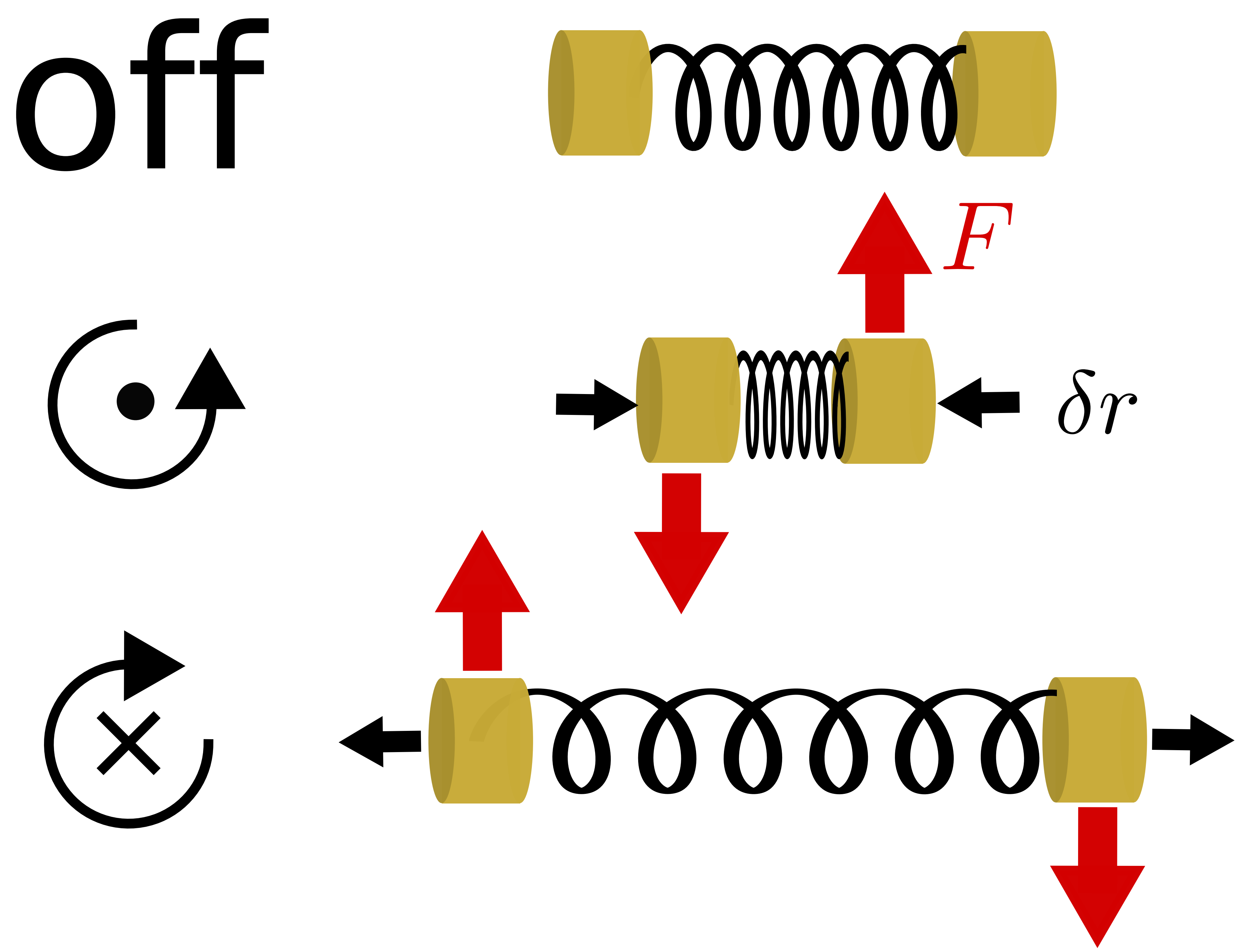


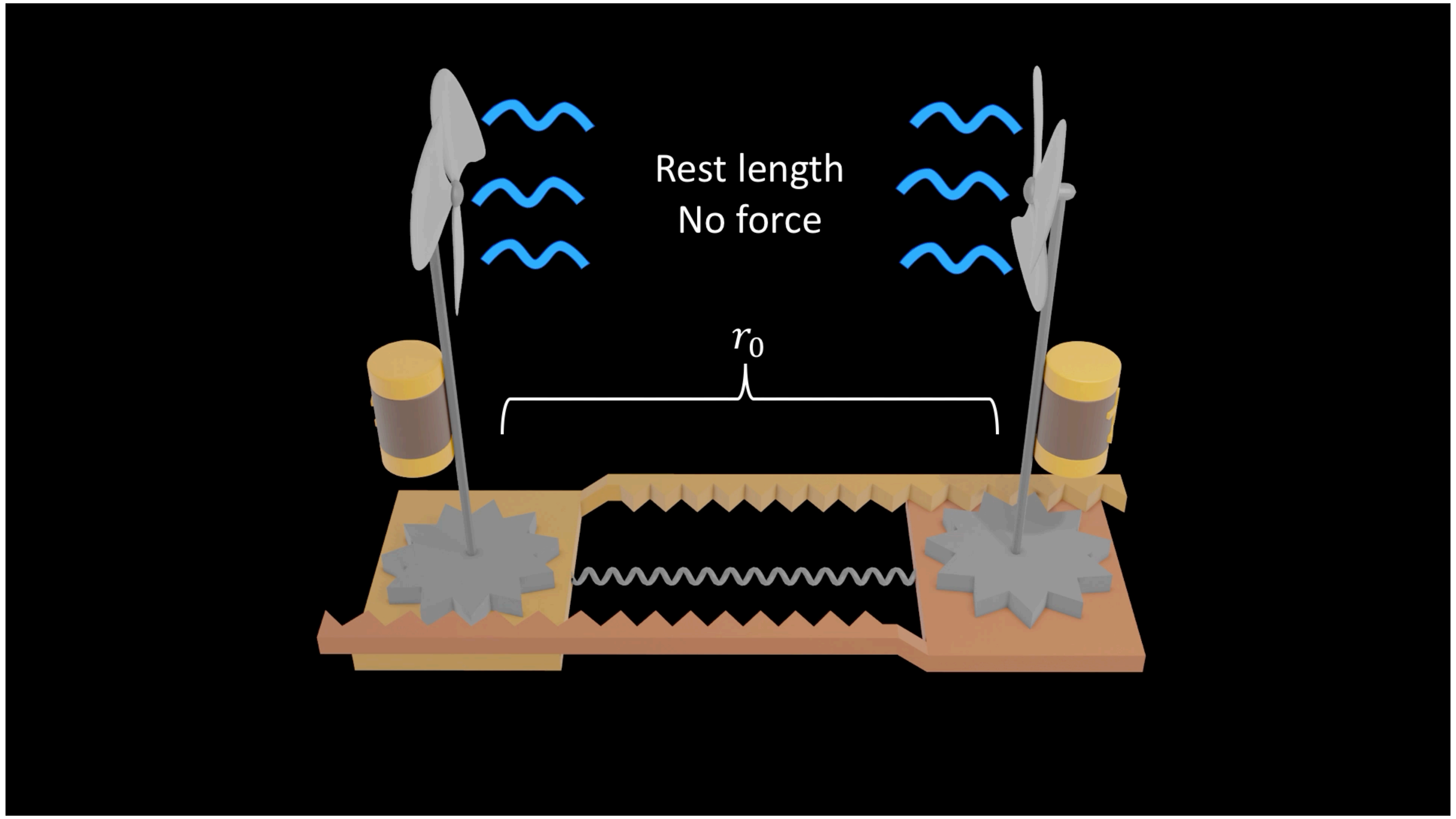
$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$

$\text{curl}\,\mathbf{F} \propto k^o$

Active

Microscopic model I: active bonds



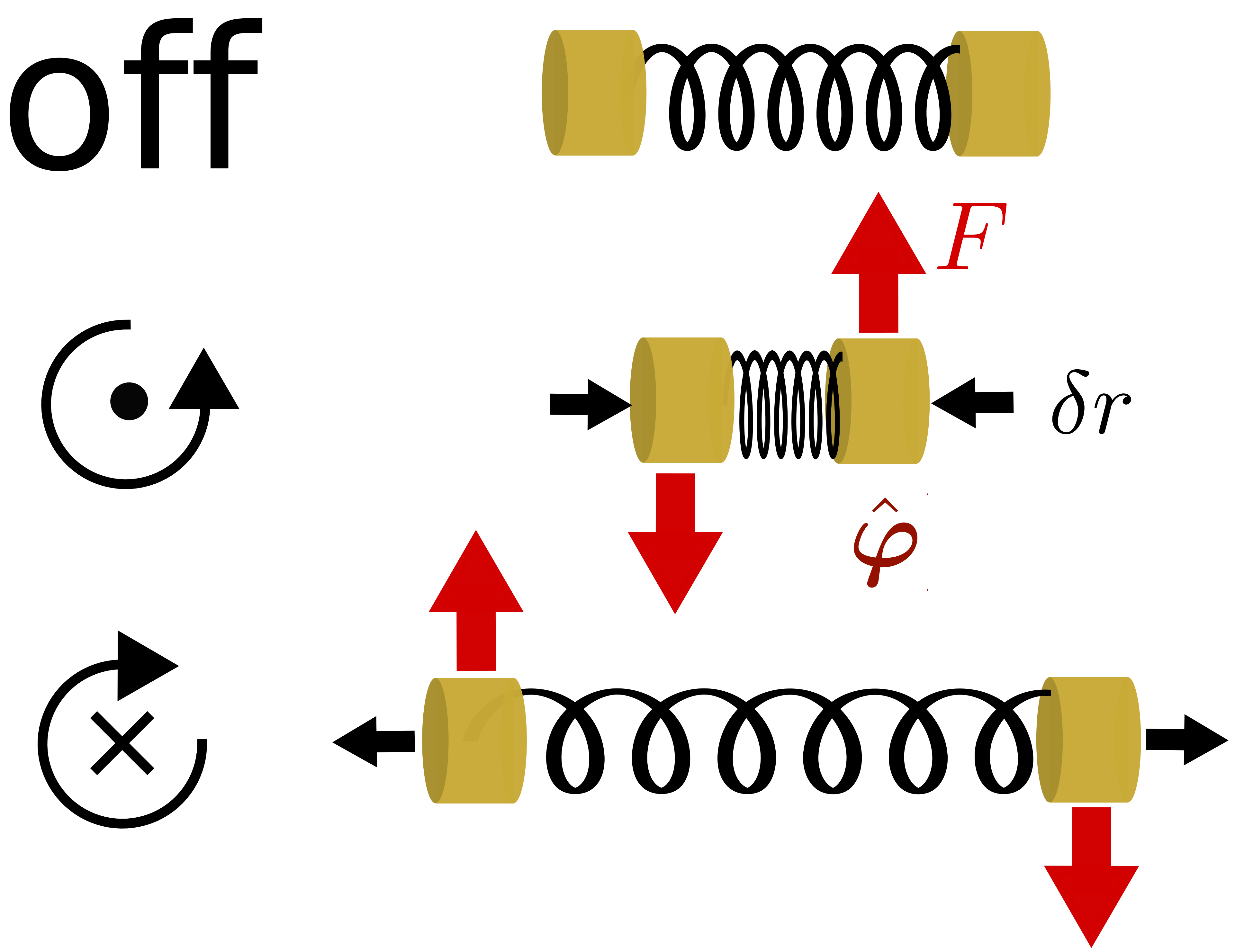


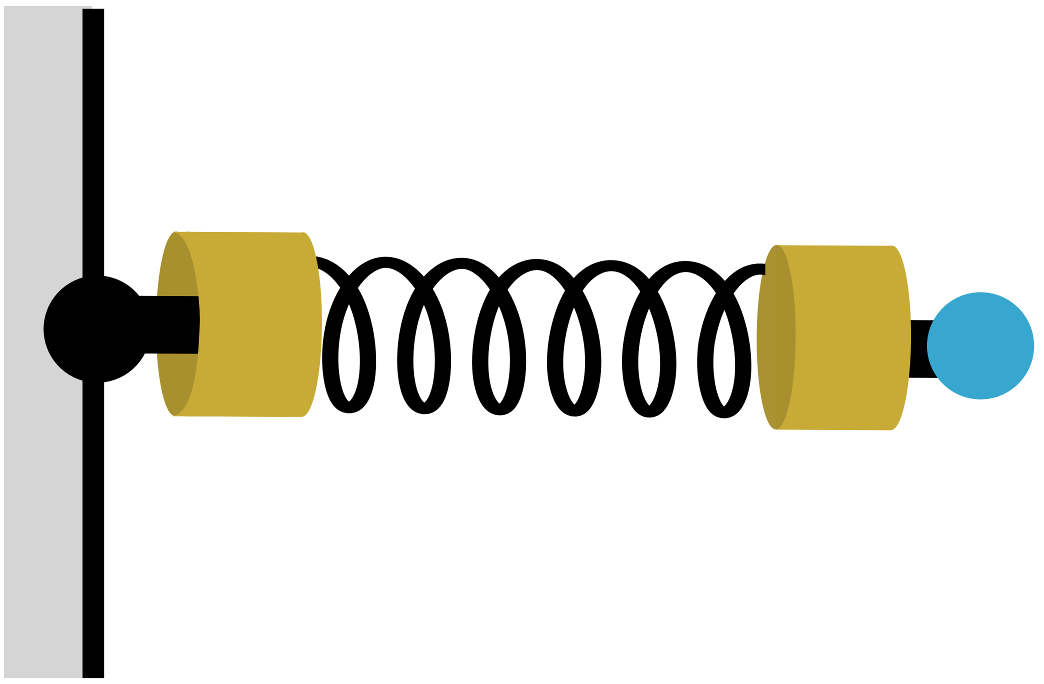
Active

$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$

$$\mathrm{curl}\,\mathbf{F} \propto k^o$$

Active bonds are microscopic engines





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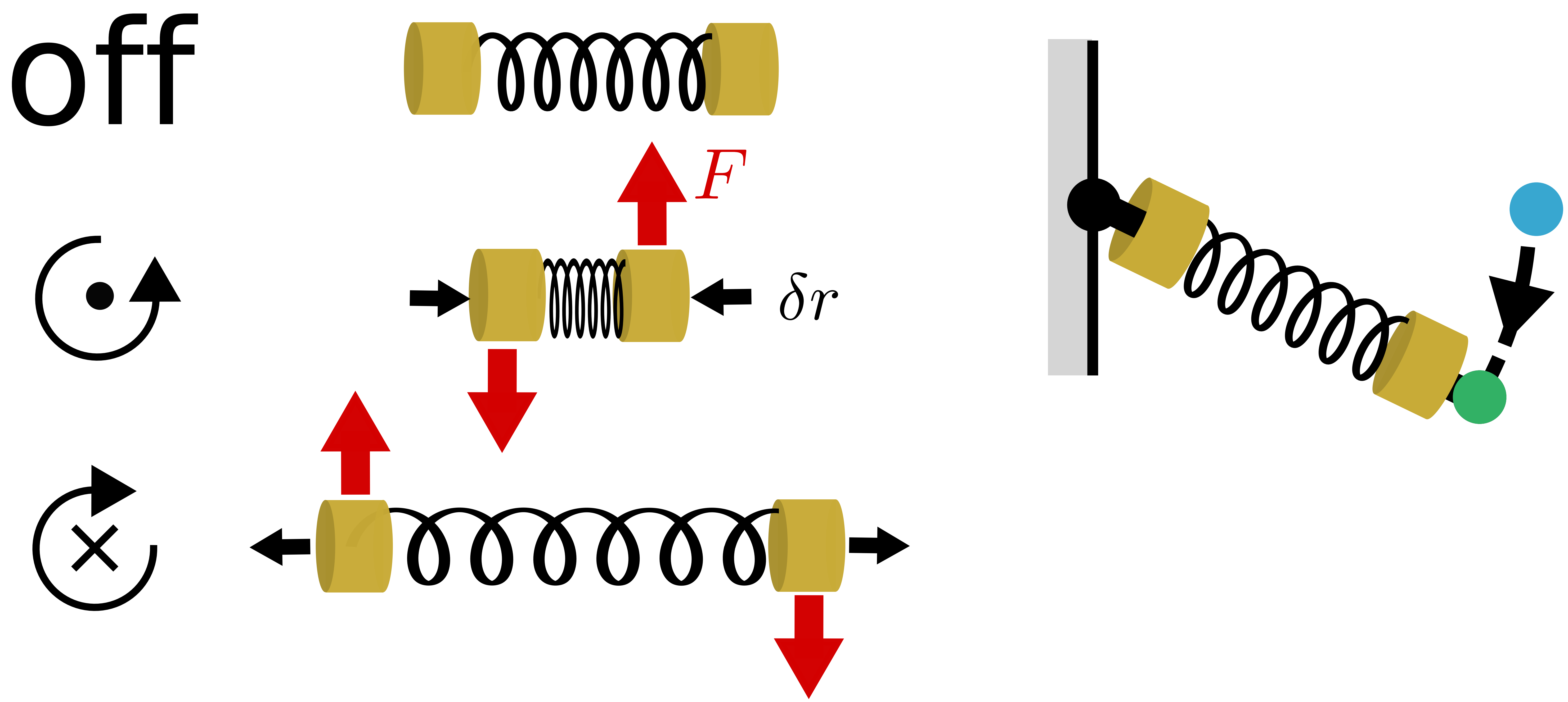
Active bonds are microscopic engines



$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$

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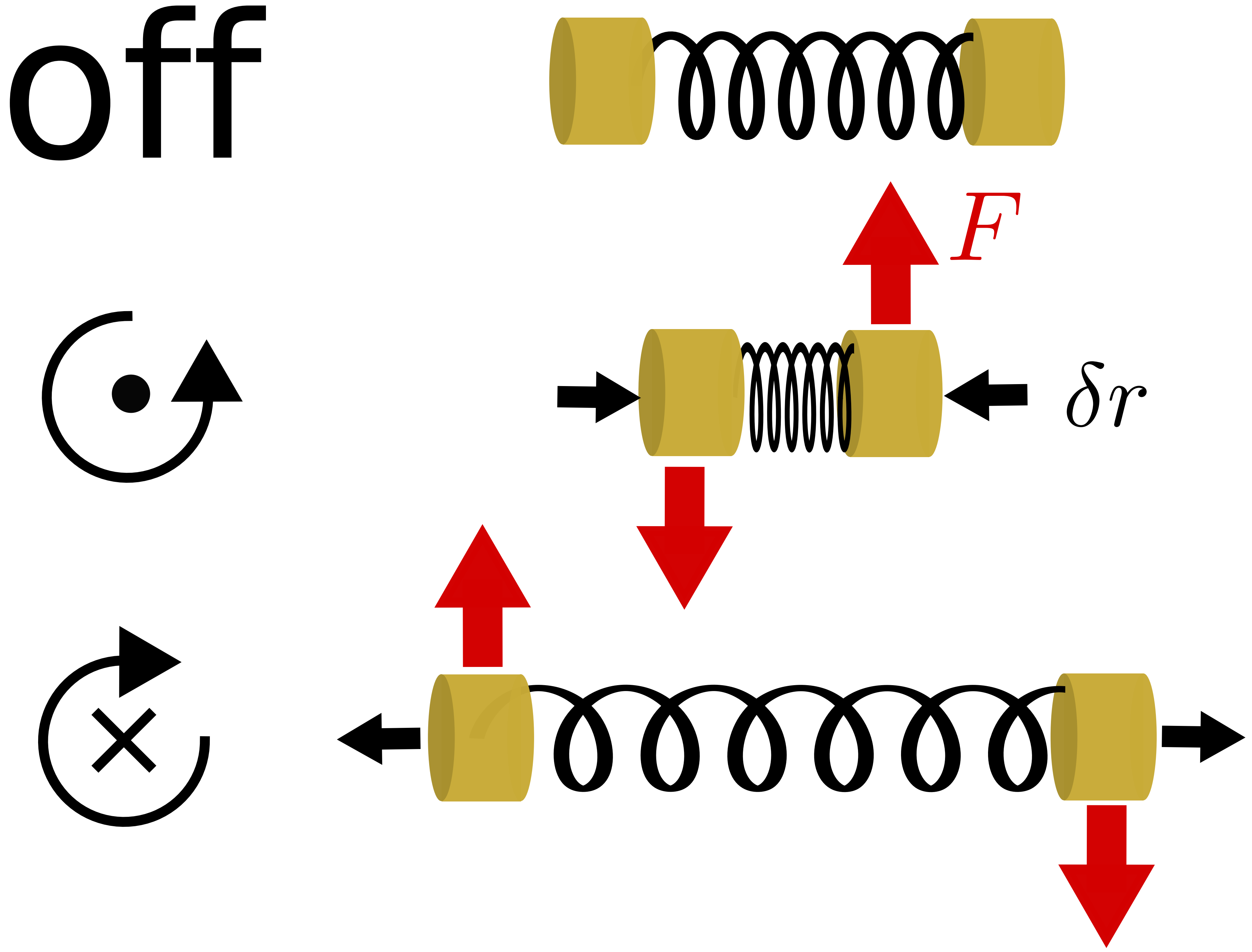
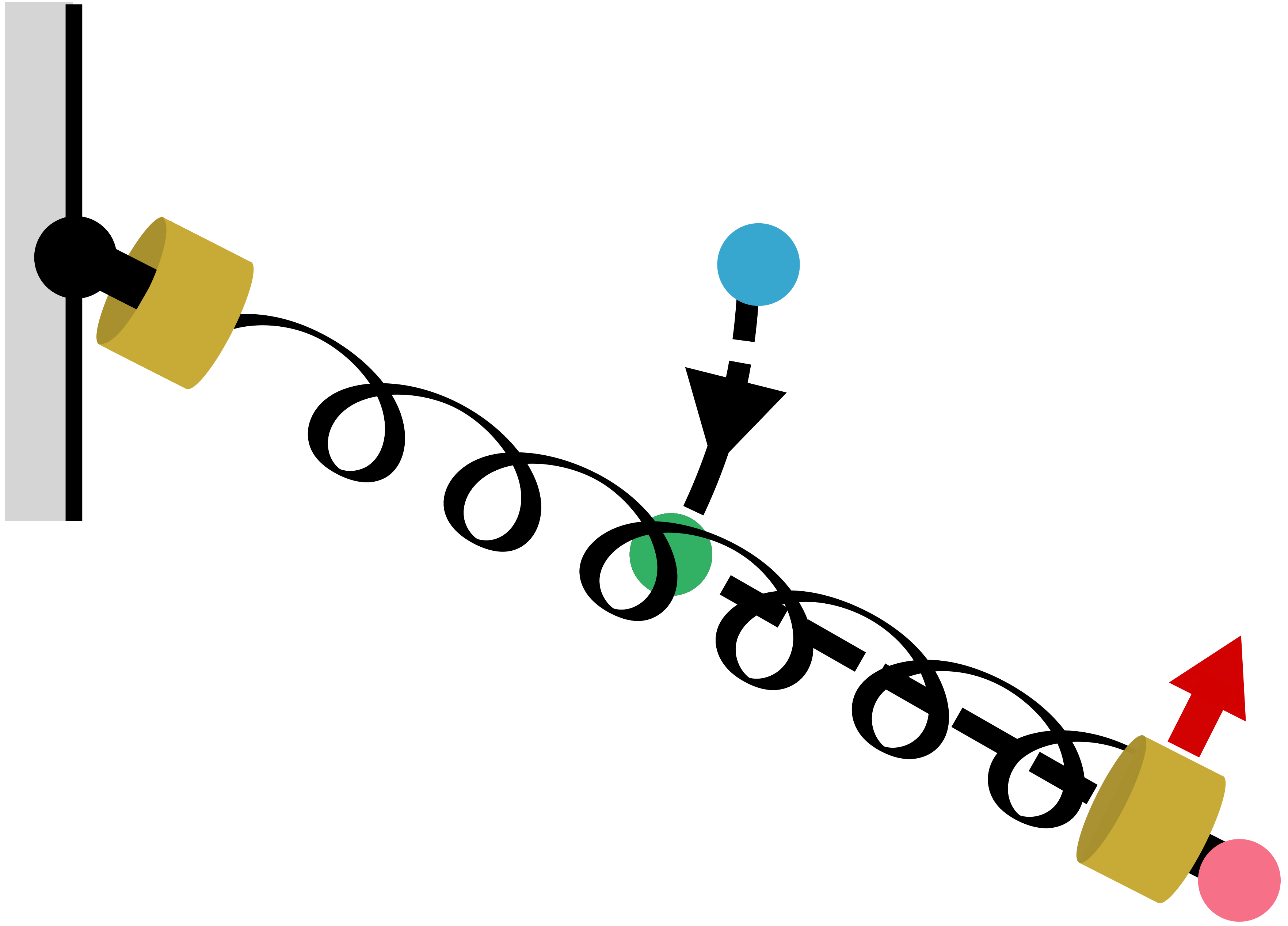
Active bonds are microscopic engines

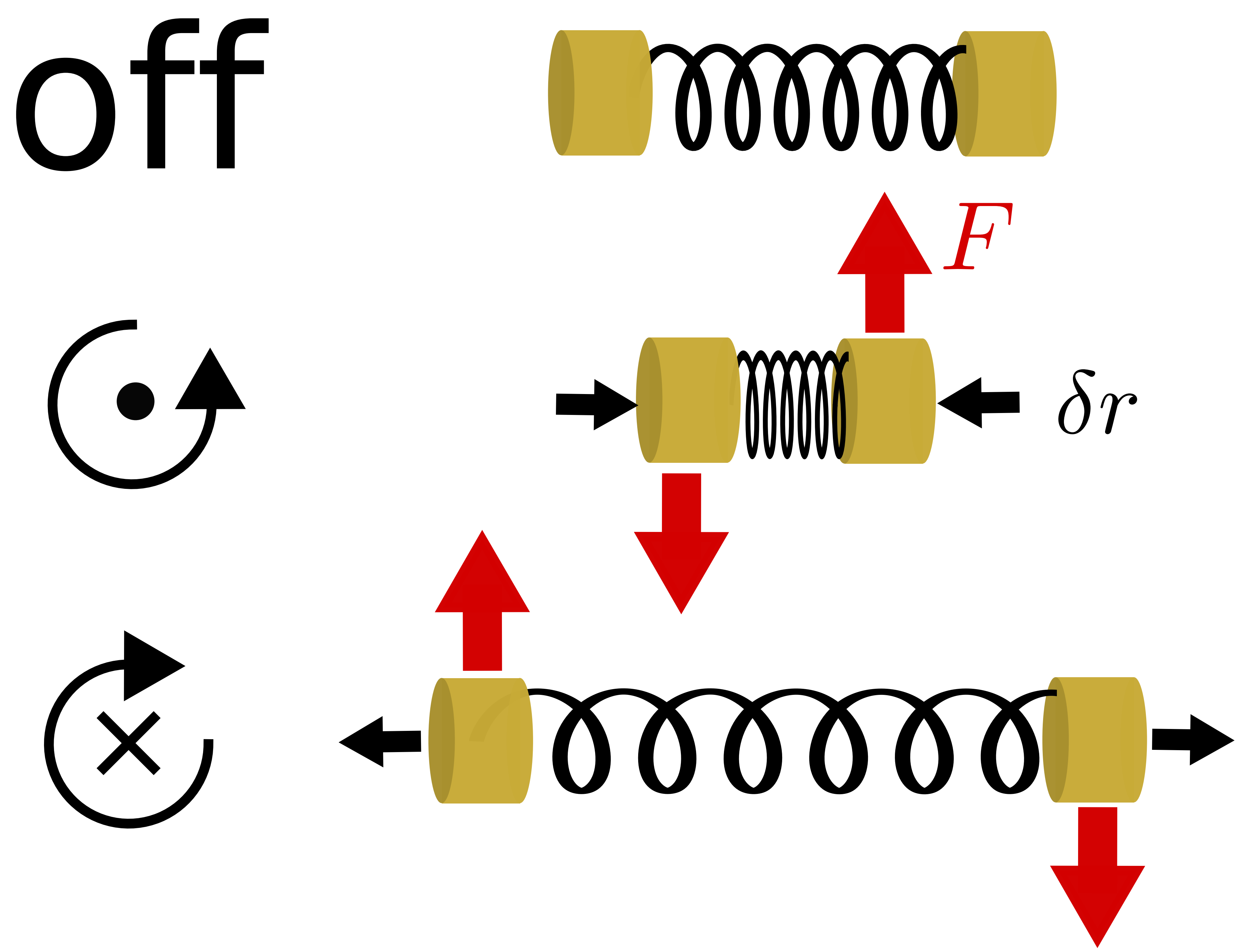


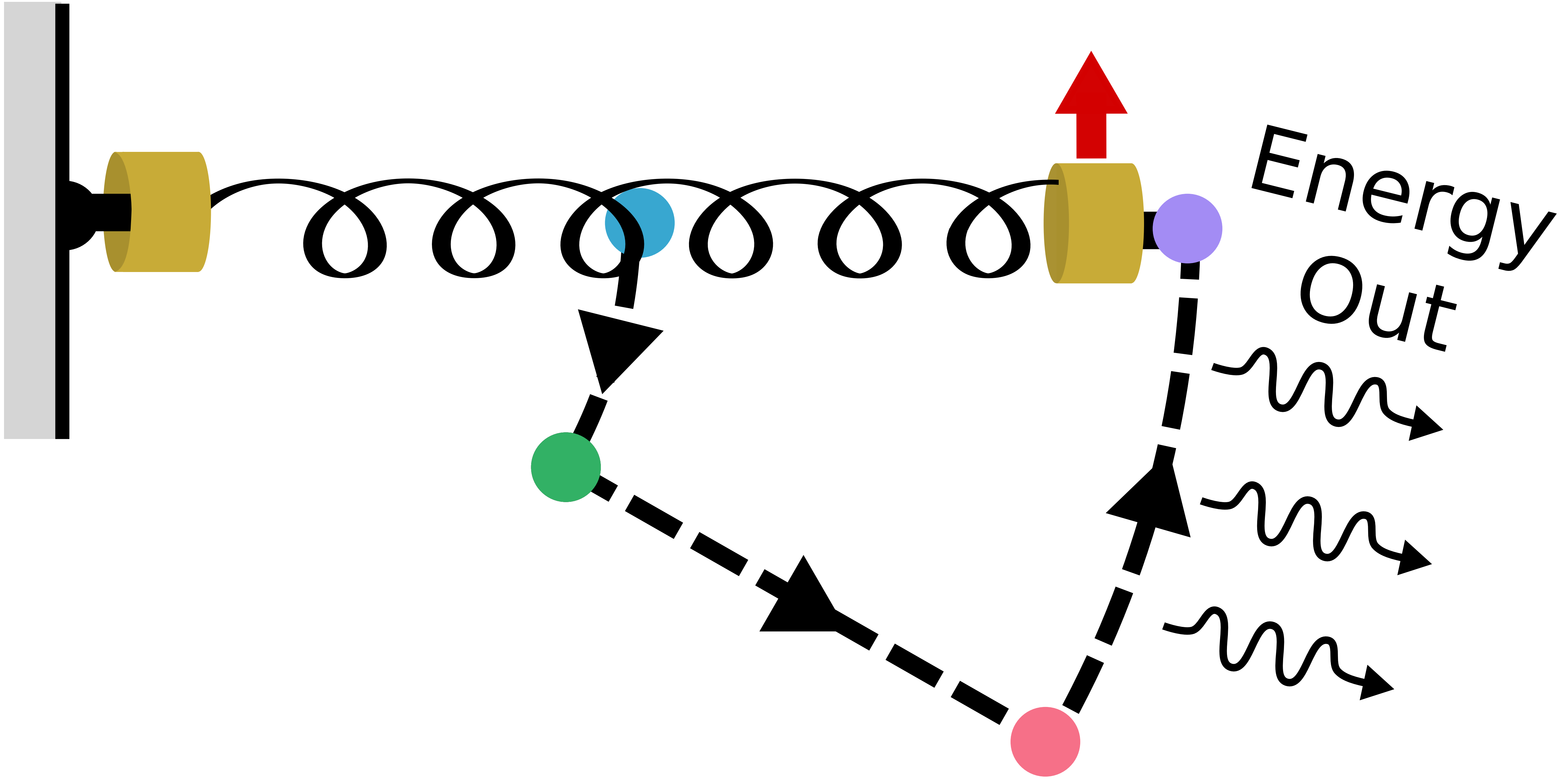
$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$

$\mathrm{curl}\,\mathbf{F} \propto k^o$



Active bonds are microscopic engines





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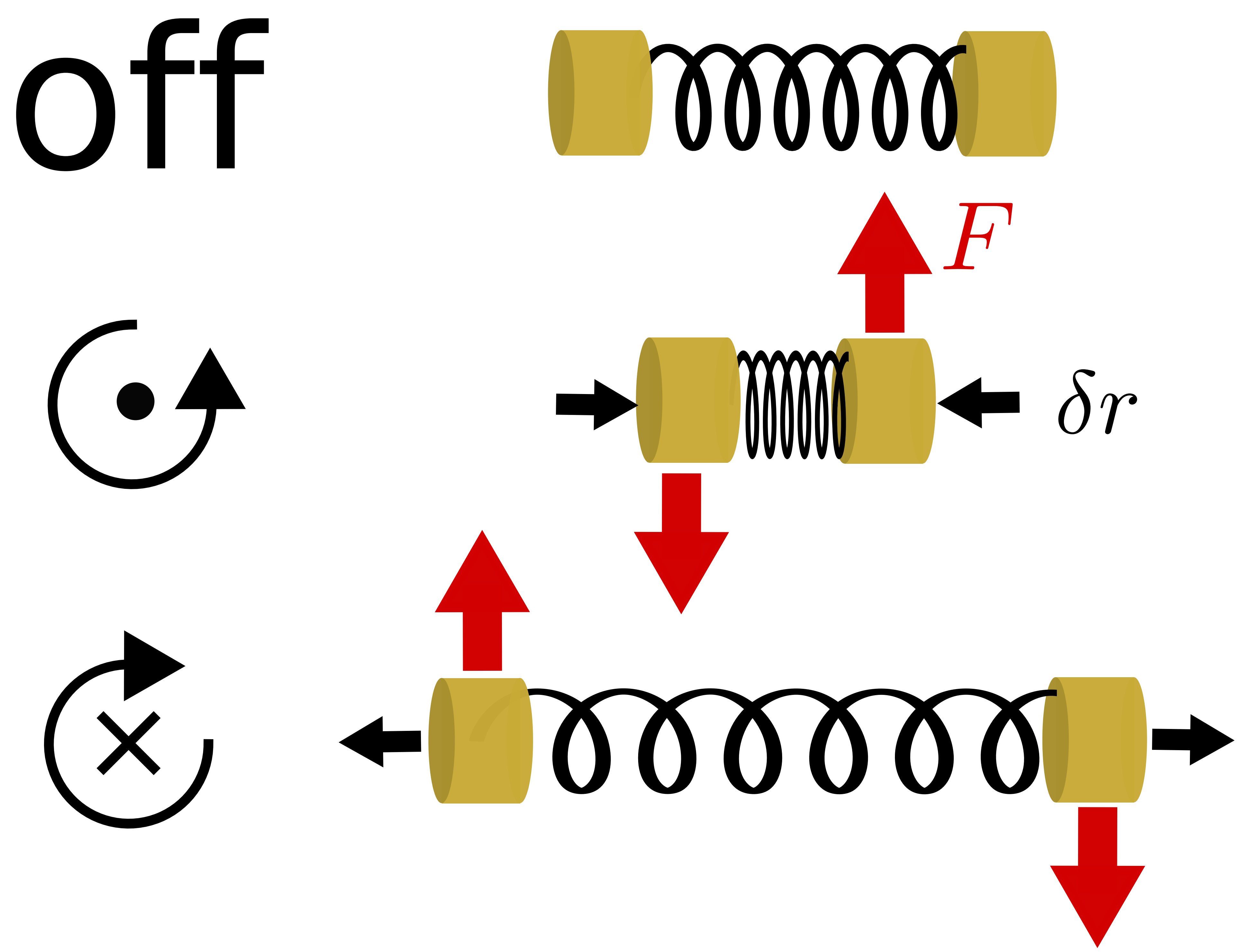
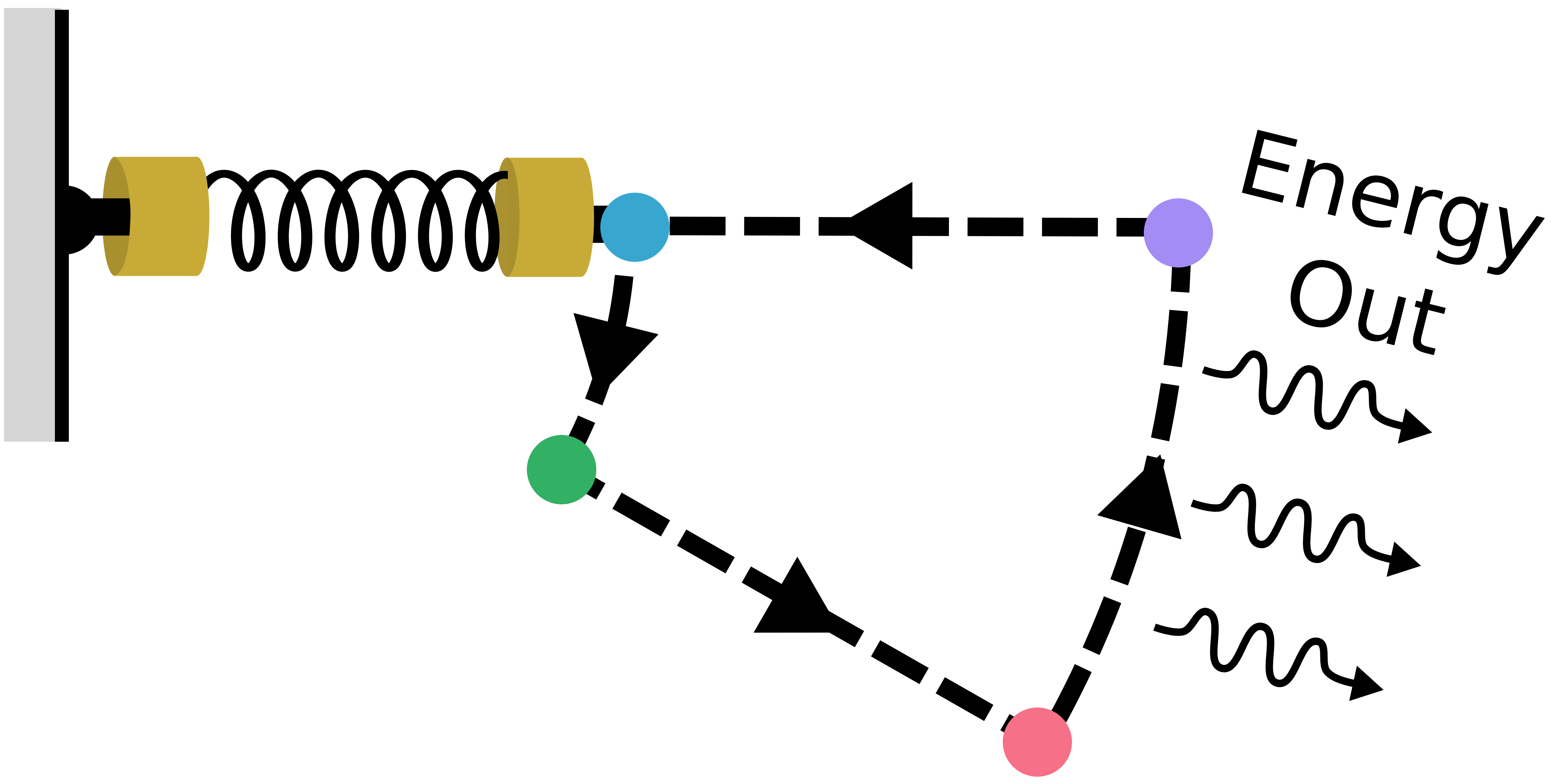
$$\operatorname{curl}\mathbf{F} \propto k^o$$

Active bonds are microscopic engines

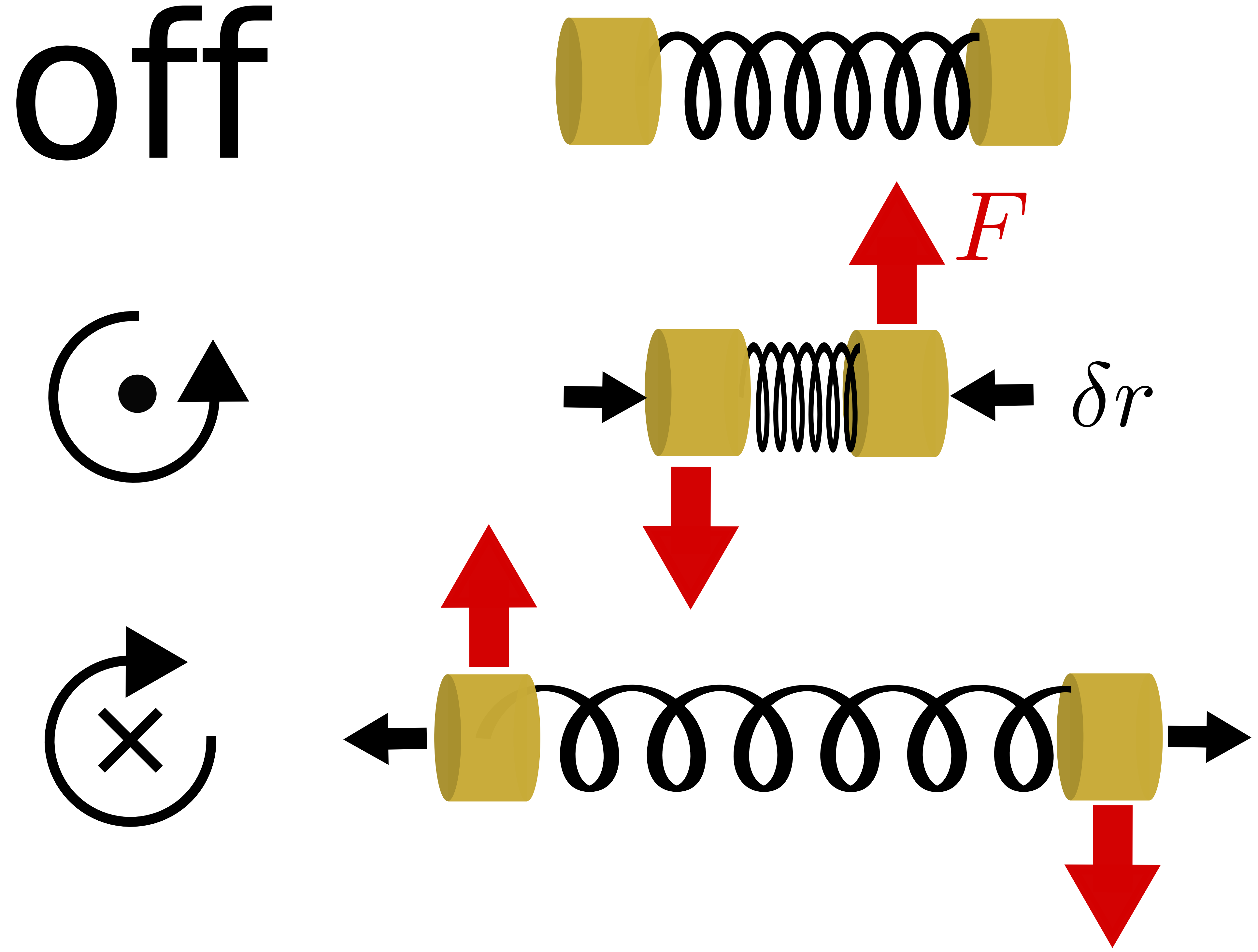




$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$

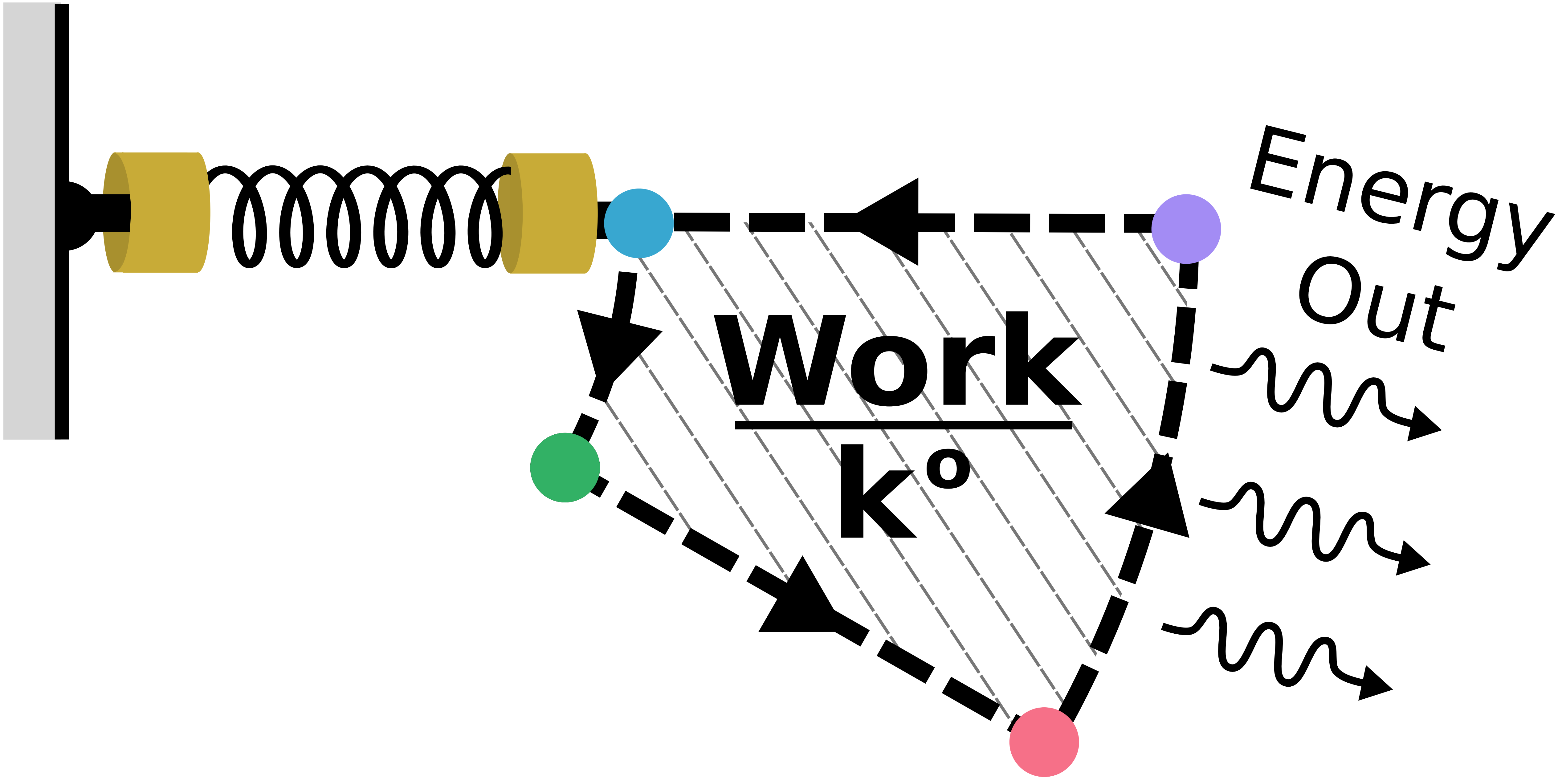
$$\mathrm{curl}\,\mathbf{F} \propto k^o$$

Active bonds are microscopic engines



$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$

$$\mathrm{curl}\,\mathbf{F} \propto k^o$$



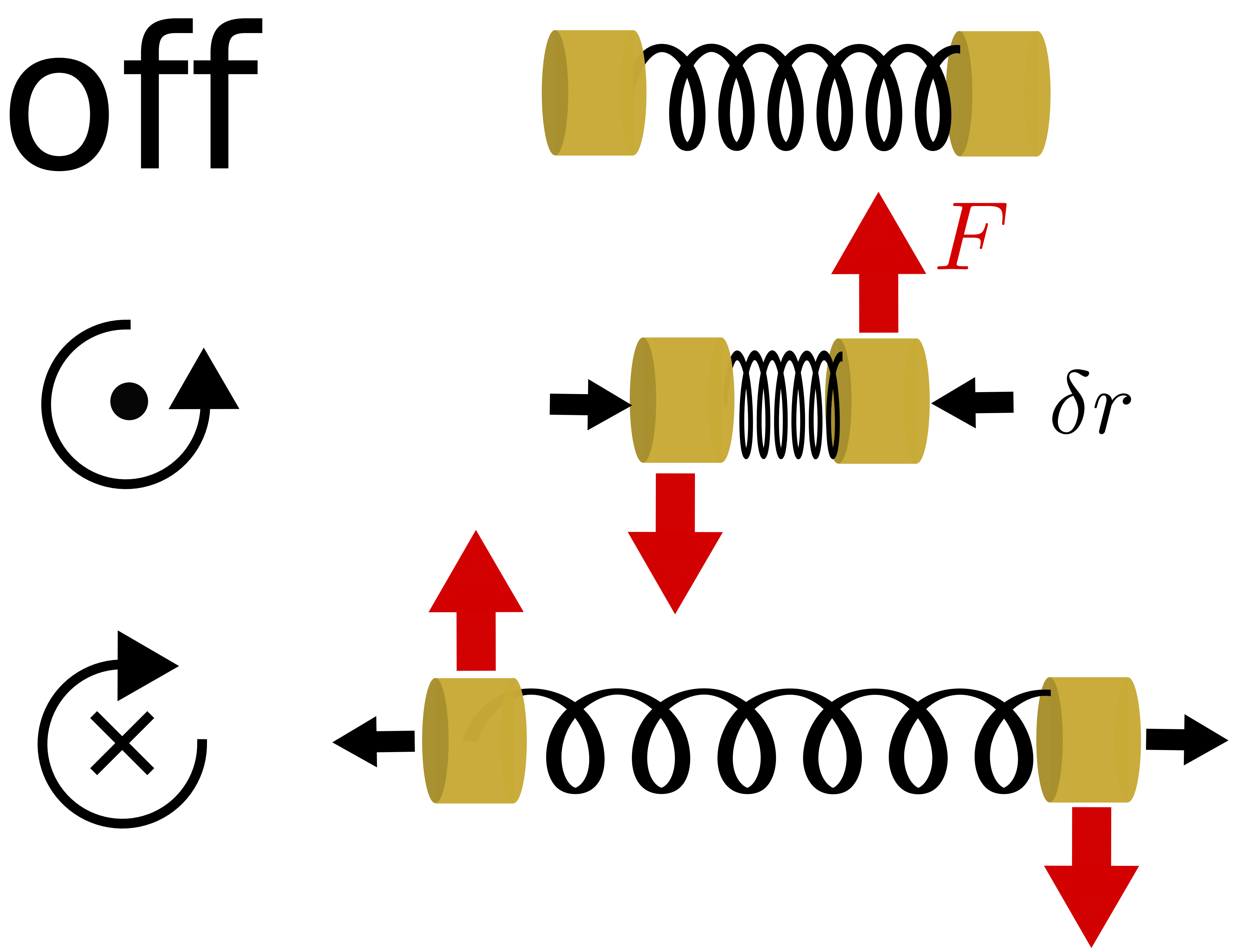
Linear Momentum conserved

Beam violates:

(1) Energy Conservation

(2) Angular momentum Conservation

Coarse-graining



$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$

$$\mathrm{curl}\,\mathbf{F} \propto k^o$$

B	0	0	0
A	0	0	0
0	0	**μ**	**K°**
0	0	**-K°**	**μ**

triangular lattice

$$A = 2K^o = \frac{\sqrt{3}}{2}k^o$$

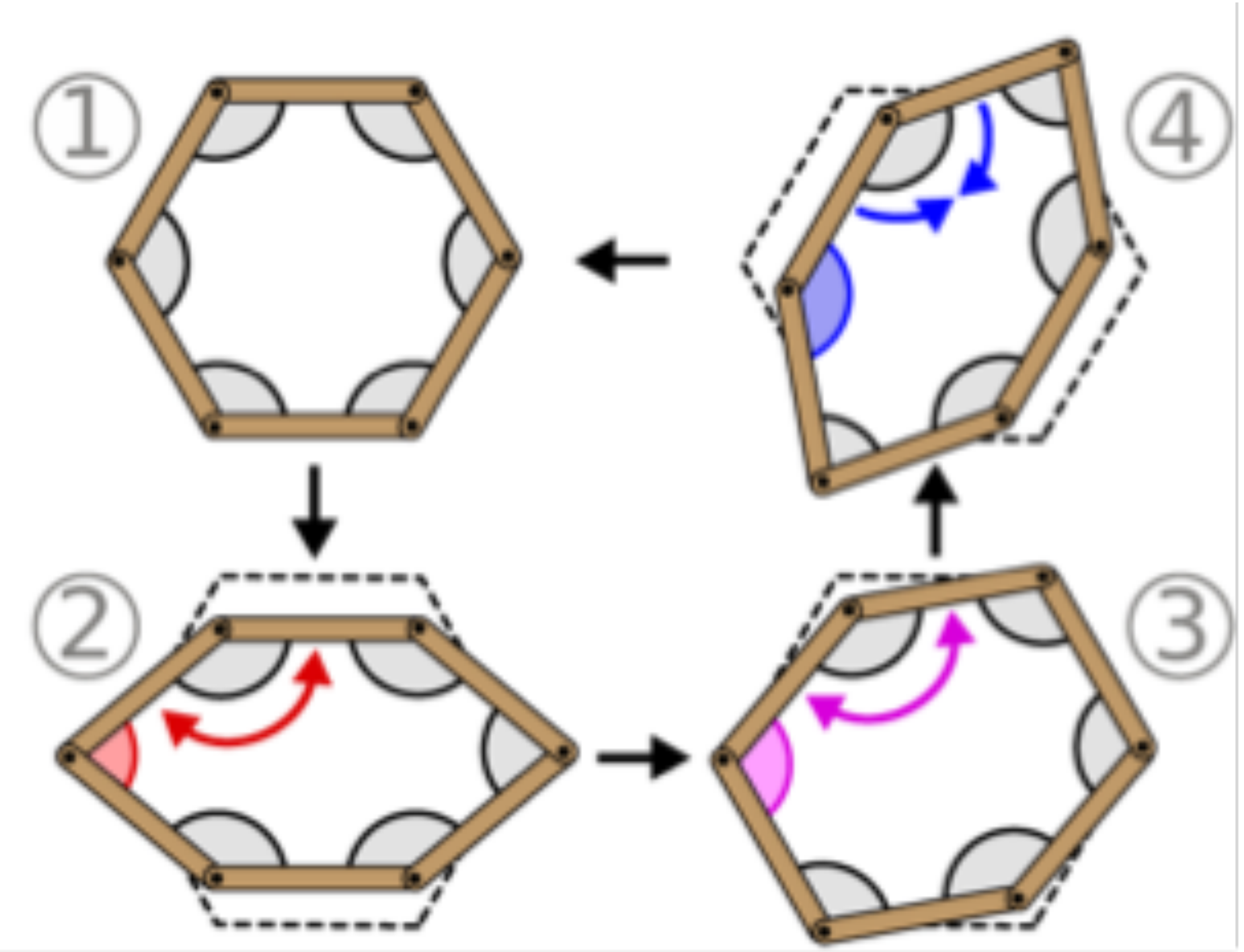
Linear Momentum conserved

Beam violates:

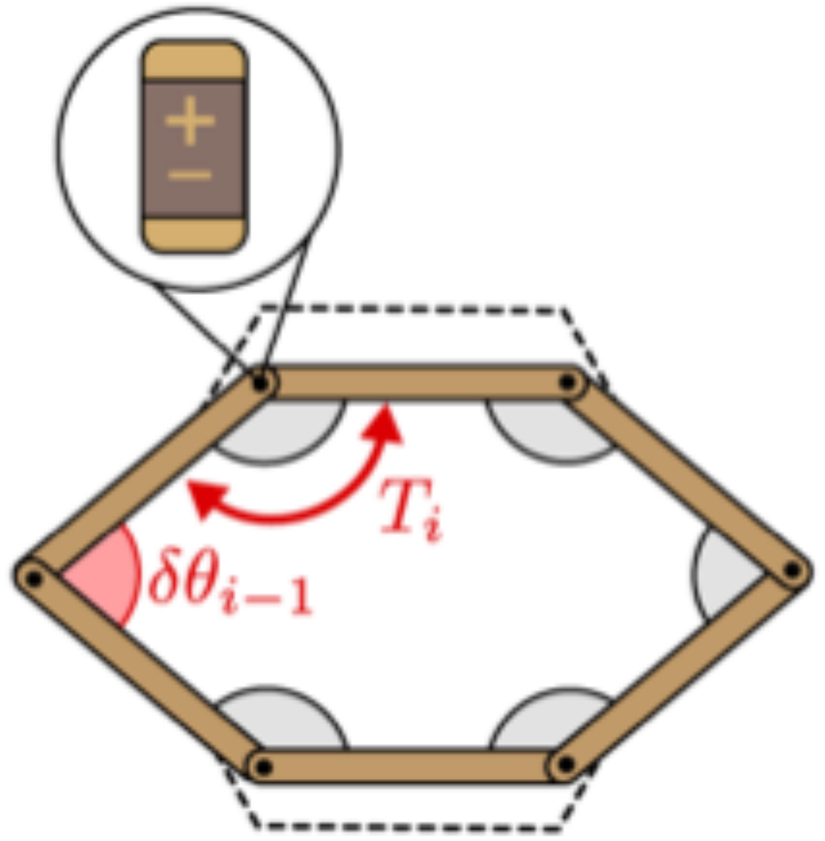
(1) Energy Conservation

(2) Angular momentum Conservation

Microscopic model II: active hinges



A=0 No **net** torques $K^o \propto \kappa_a$



$$T_i = -\kappa\delta\theta_i - \kappa^a\delta\theta_{i-1}$$

Non-reciprocity and chirality

Angular momentum conserved

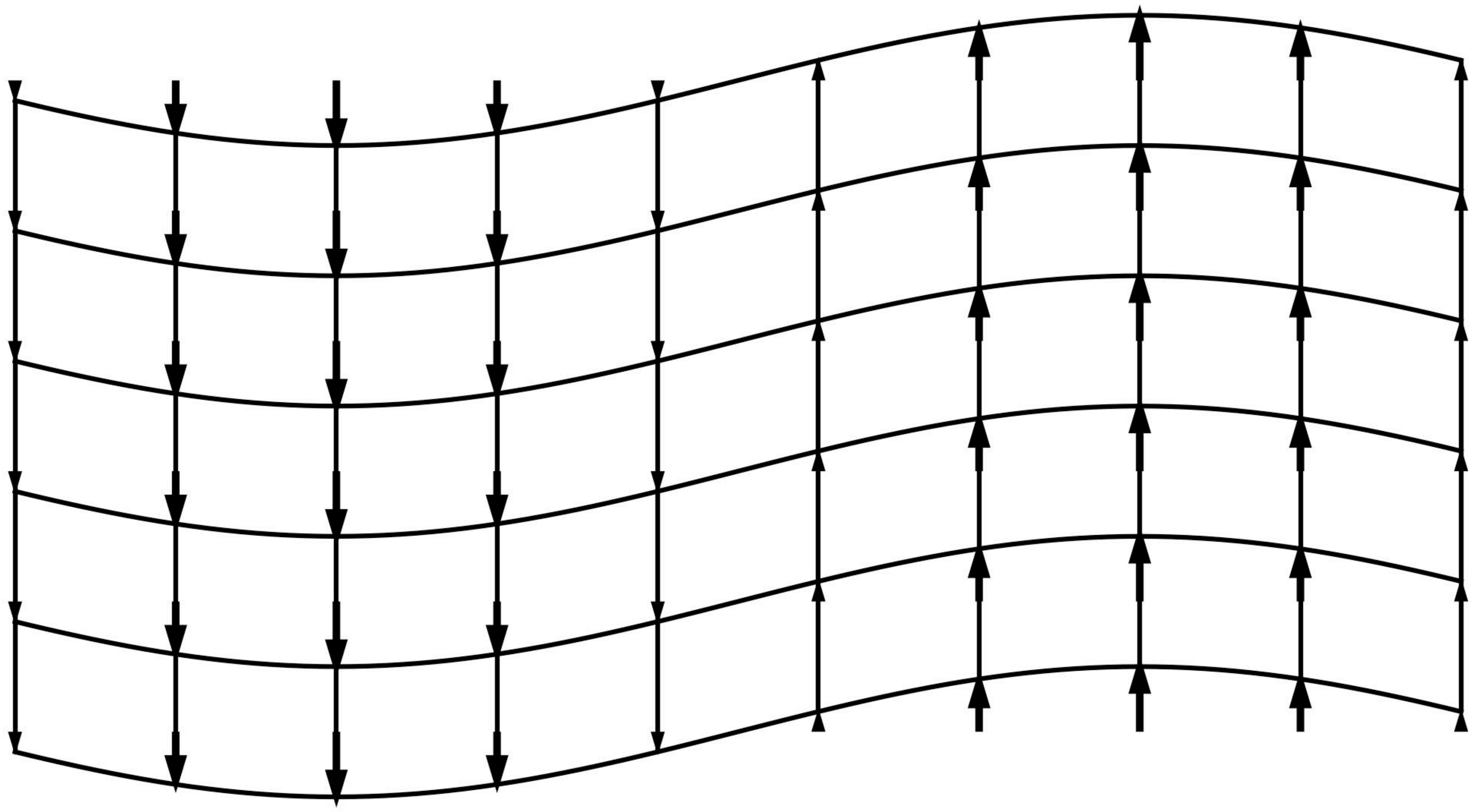
Linear momentum conserved

Hinge violates:
(1) Energy Conservation

Passive Elastodynamics

Inertial

$\overrightarrow{F} = m\overrightarrow{a}$

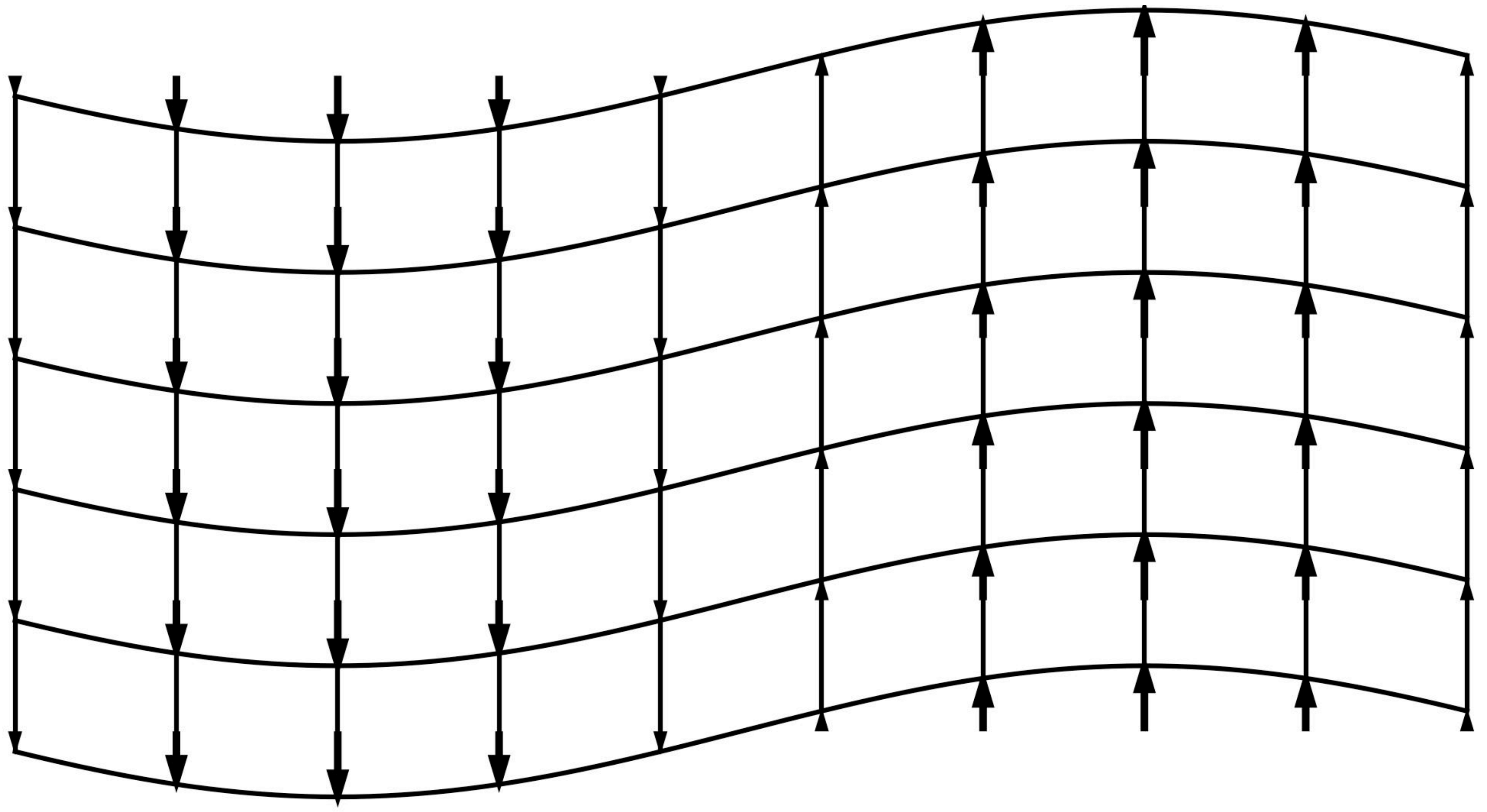


Transverse wave

speed $\sim \sqrt{\mu}$

Over-damped

$\overrightarrow{F} = \eta\overrightarrow{v}$

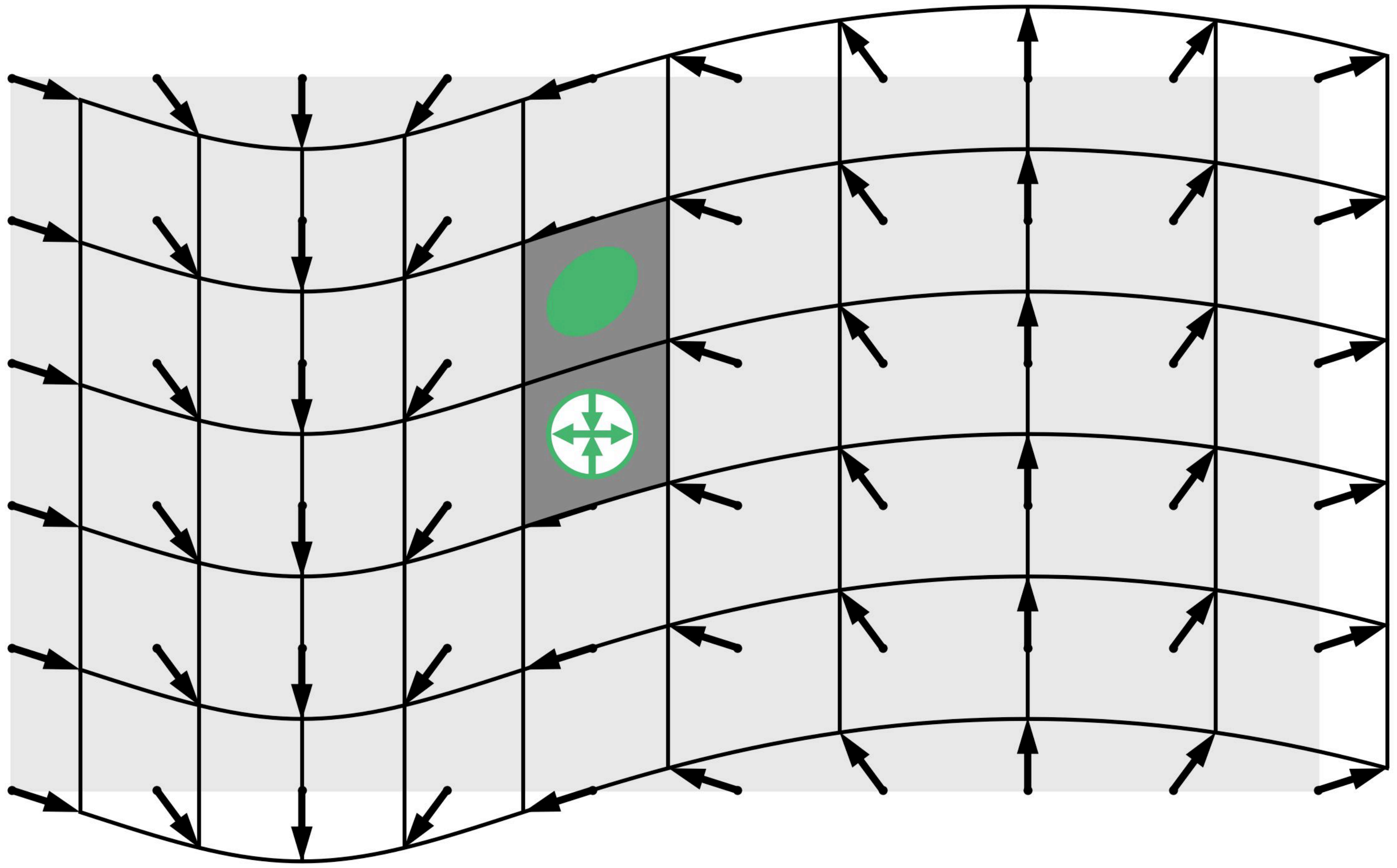


$$F_i = \partial_j \sigma_{ij}$$

$$\sigma_{ij} = K_{ijmn} u_{mn}$$

Odd Elastodynamics

Odd Wave

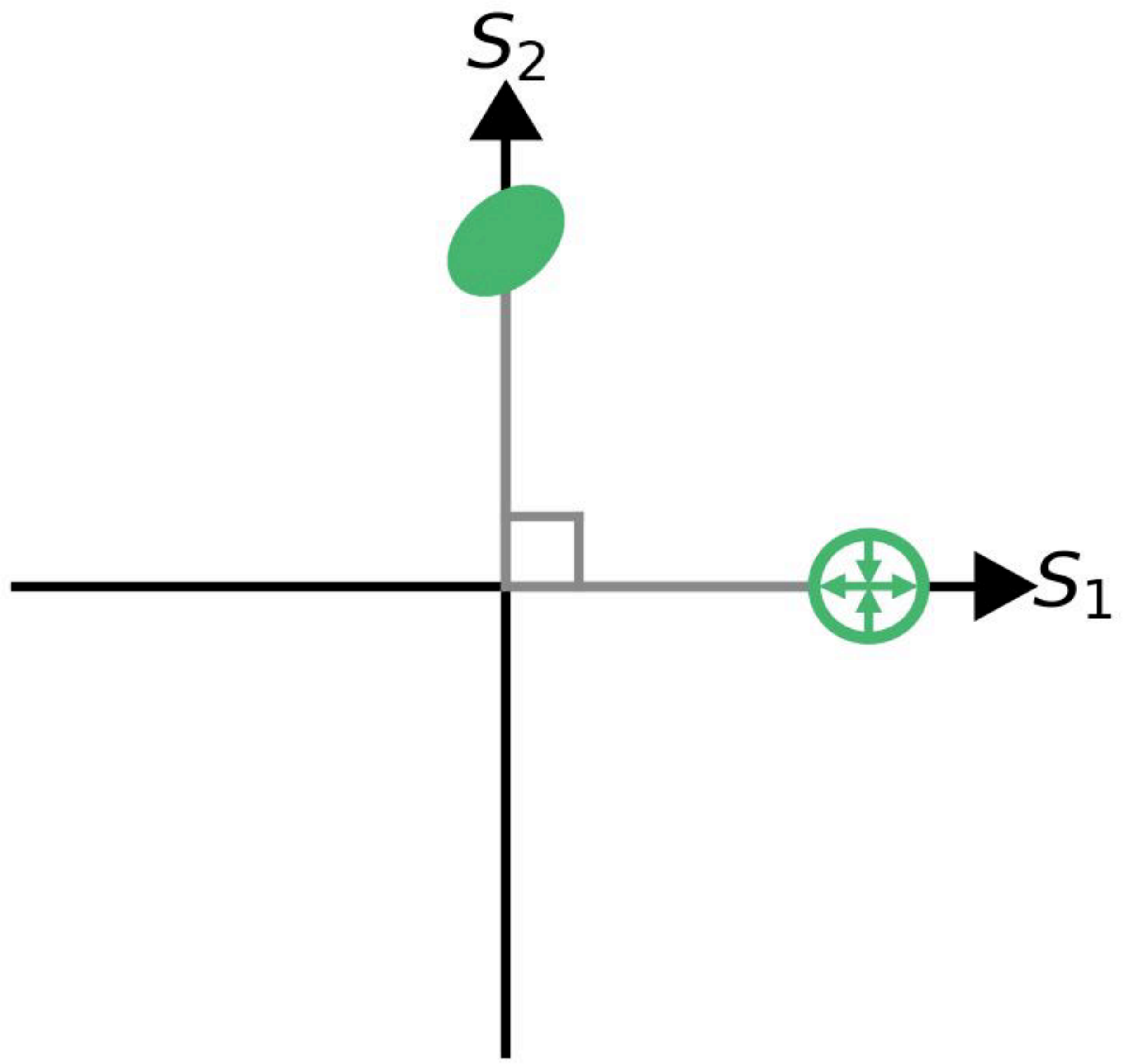


$$\eta \partial_t u_i = \partial_j \sigma_{ij}$$

$$\sigma_{ij} = K^{0}_{ijmn} u_{mn}$$

self-sustained elastic waves propagation in overdamped solids without rigidity!

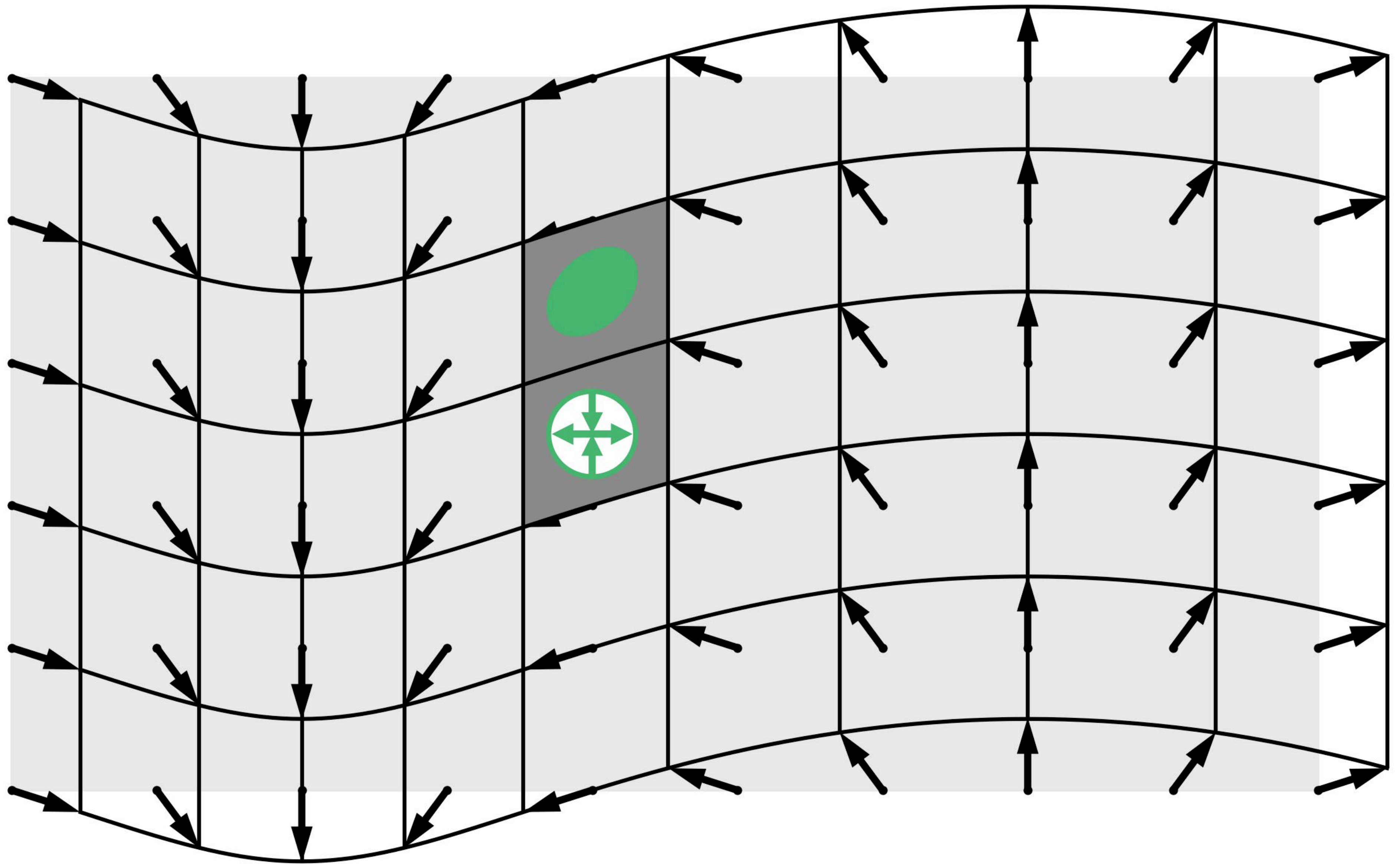
Elastic engine cycle powers the wave



$K^o > 0$
$B = 0$
$\mu = 0$
$A = 0$

Odd Elastodynamics

Odd Wave



$$\eta \partial_t u_i = \partial_j \sigma_{ij}$$

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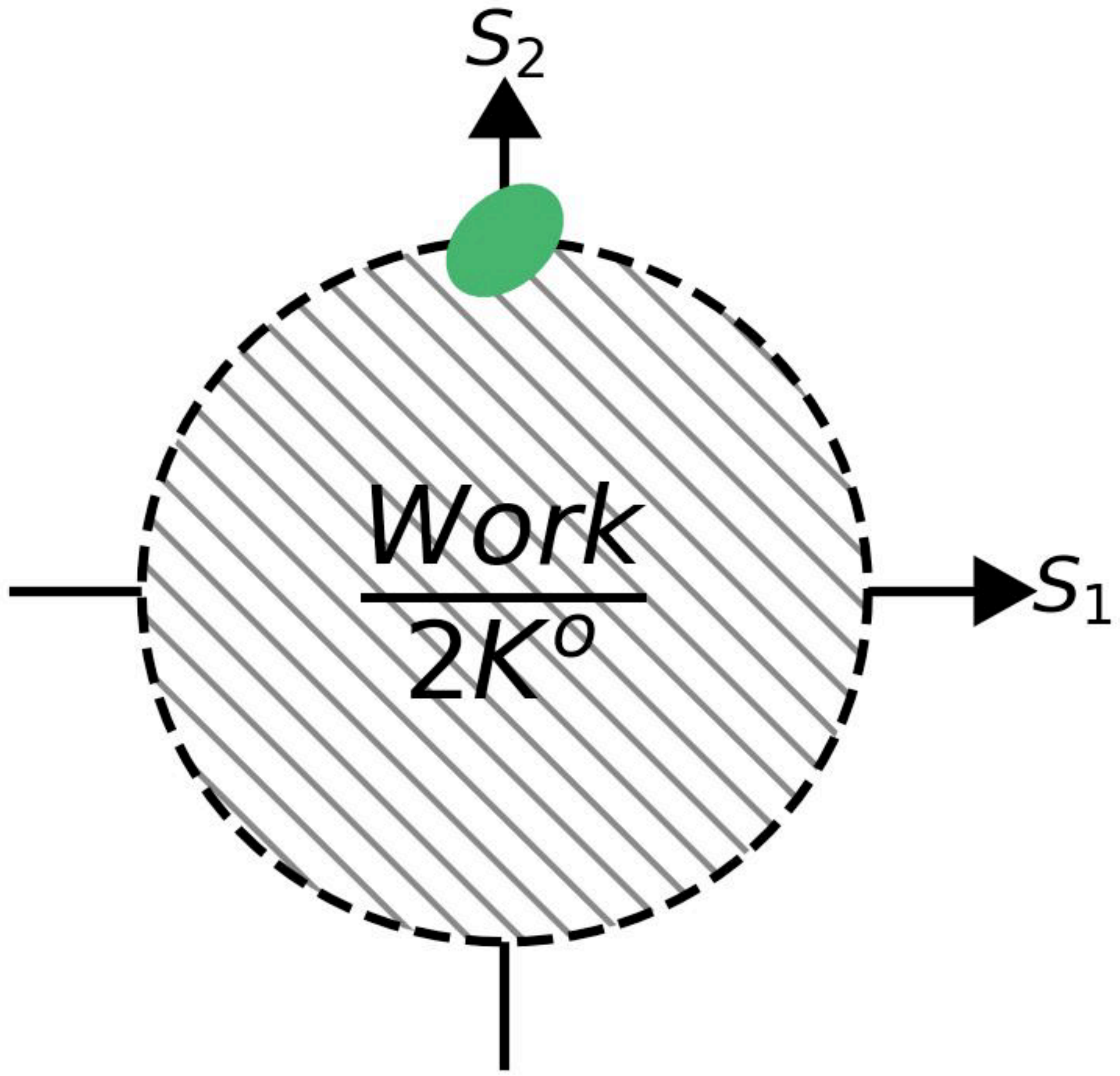
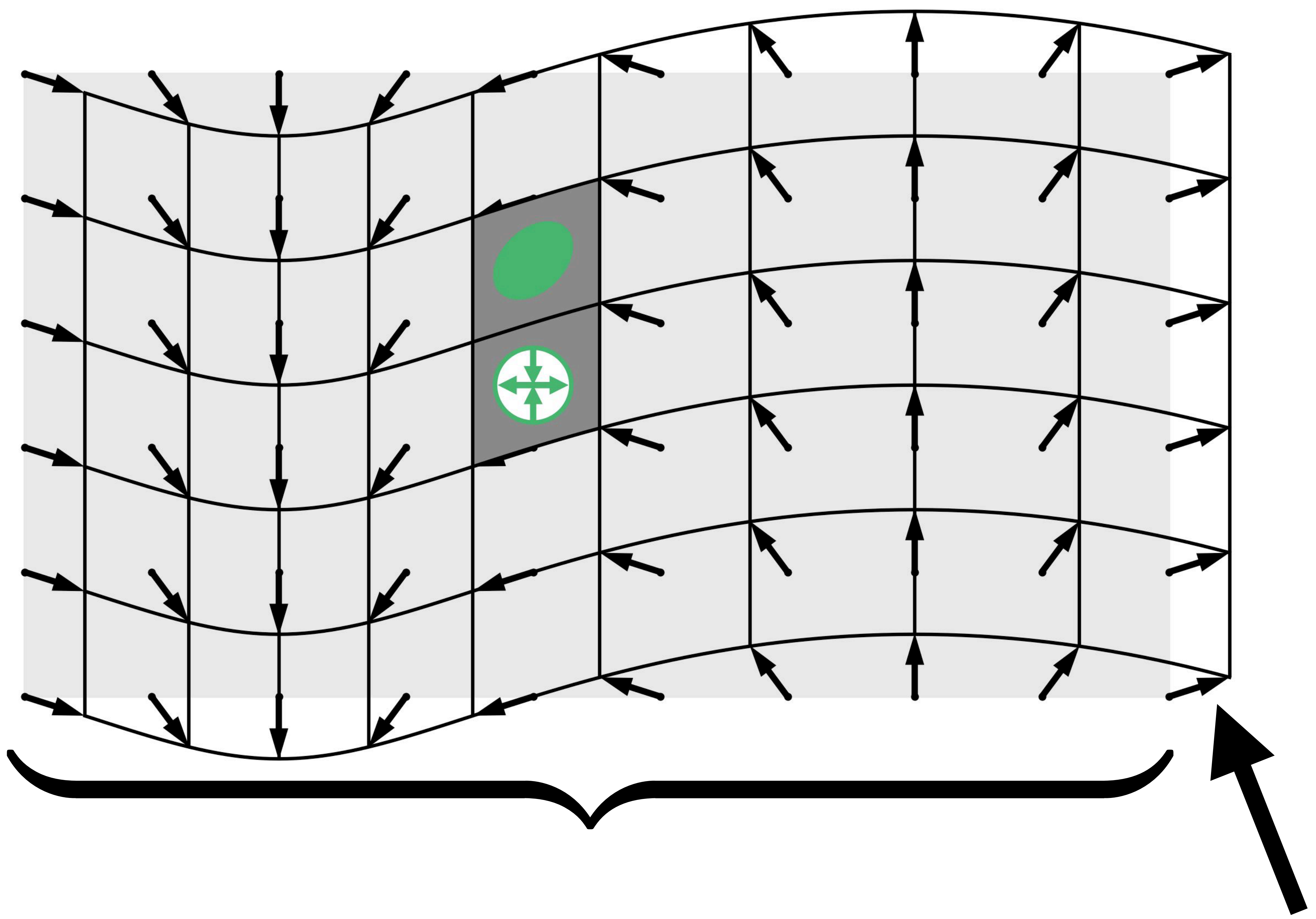
Elastic engine cycle powers the wave



$K^o > 0$
$B = 0$
$\mu = 0$
$A = 0$

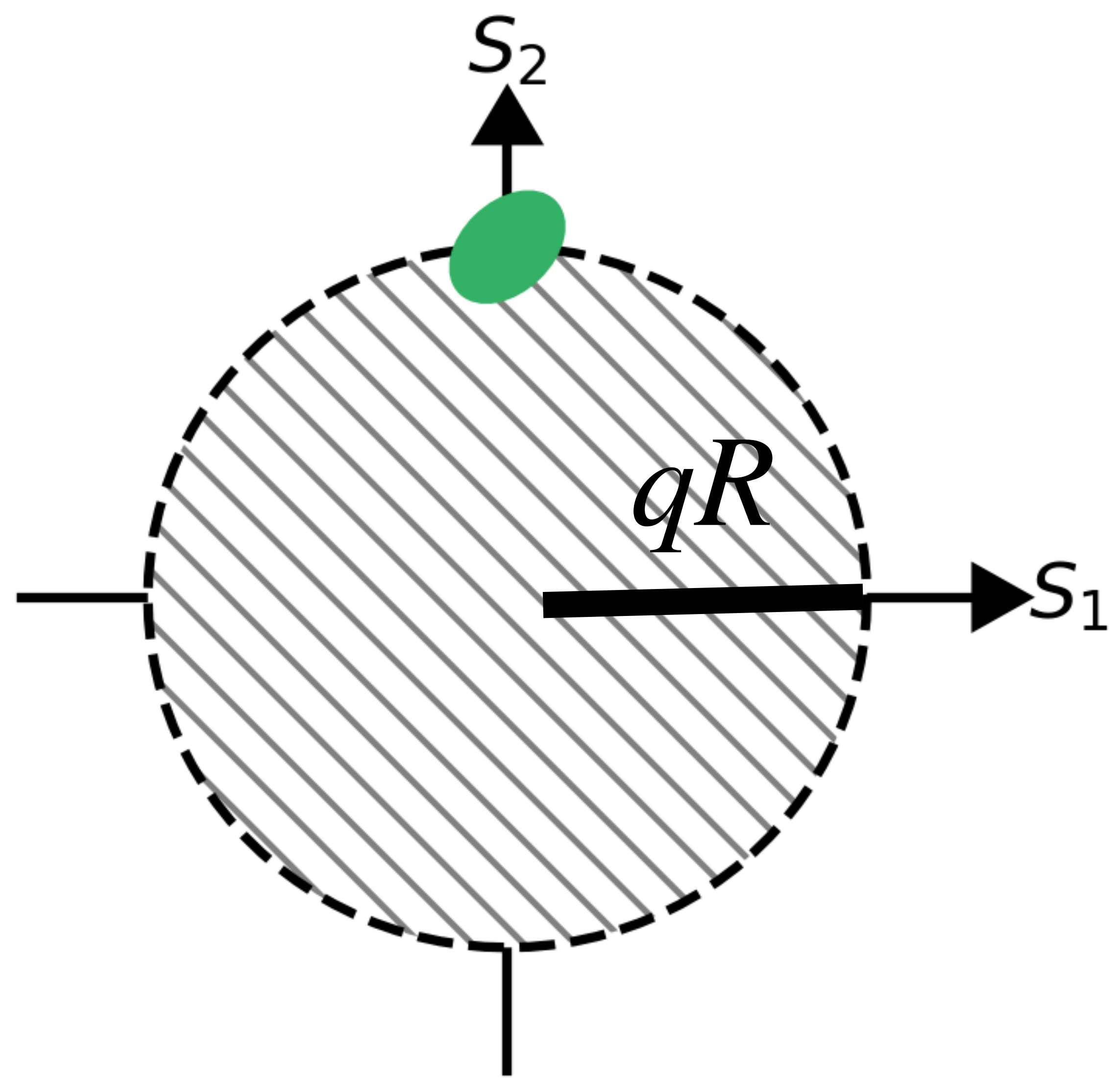
Wave speed from energy balance



Wavenumber q Amplitude R

Period $T = \dfrac{2\pi}{\omega}$ $\quad v = \dfrac{2\pi R}{T}$

Energy out $= \eta v^2 T = \eta 2\pi R^2 \omega$



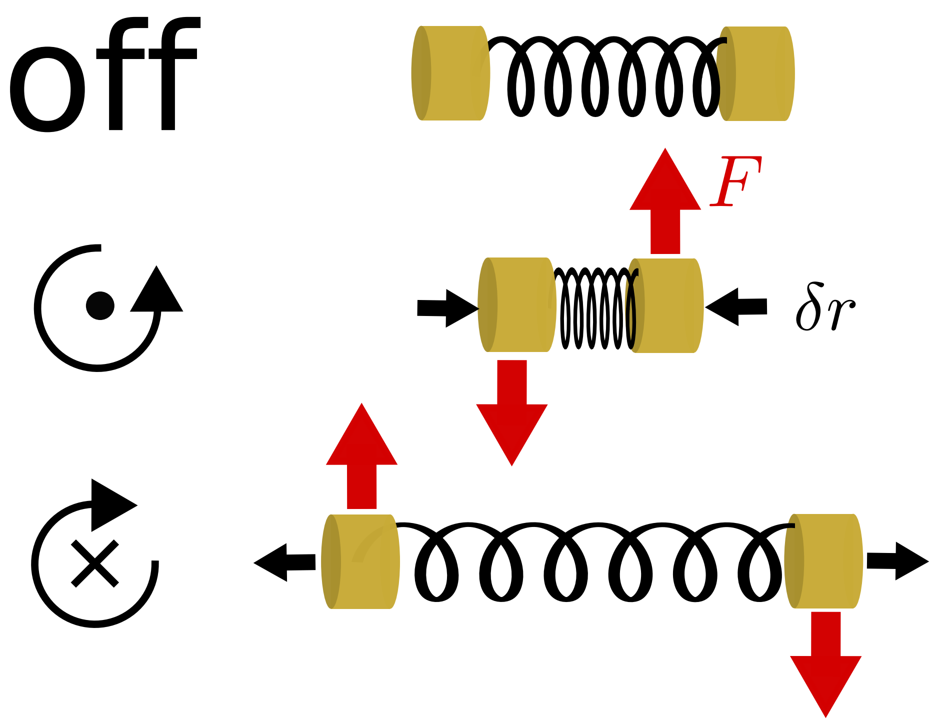
Energy in $= 2K^o \cdot \text{Area} = 2K^o \pi (qR)^2$

Energy in = Energy out

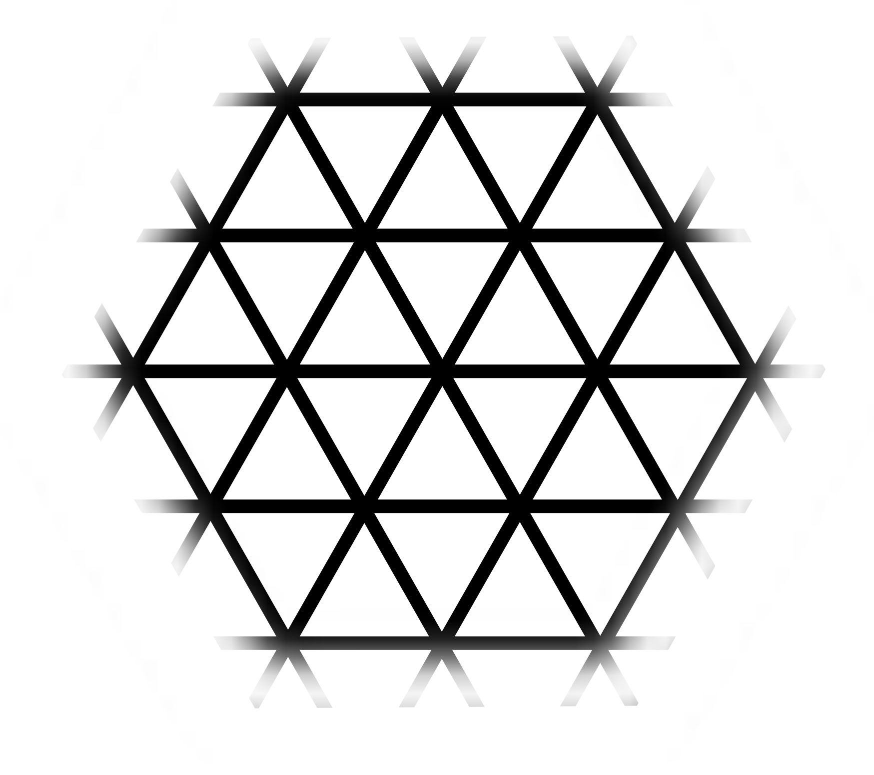
$$\implies \quad \omega = \frac{K^o q^2}{\eta}$$

$$\implies \text{speed} = \frac{d\omega}{dq} = \frac{2K^o}{\eta} q$$

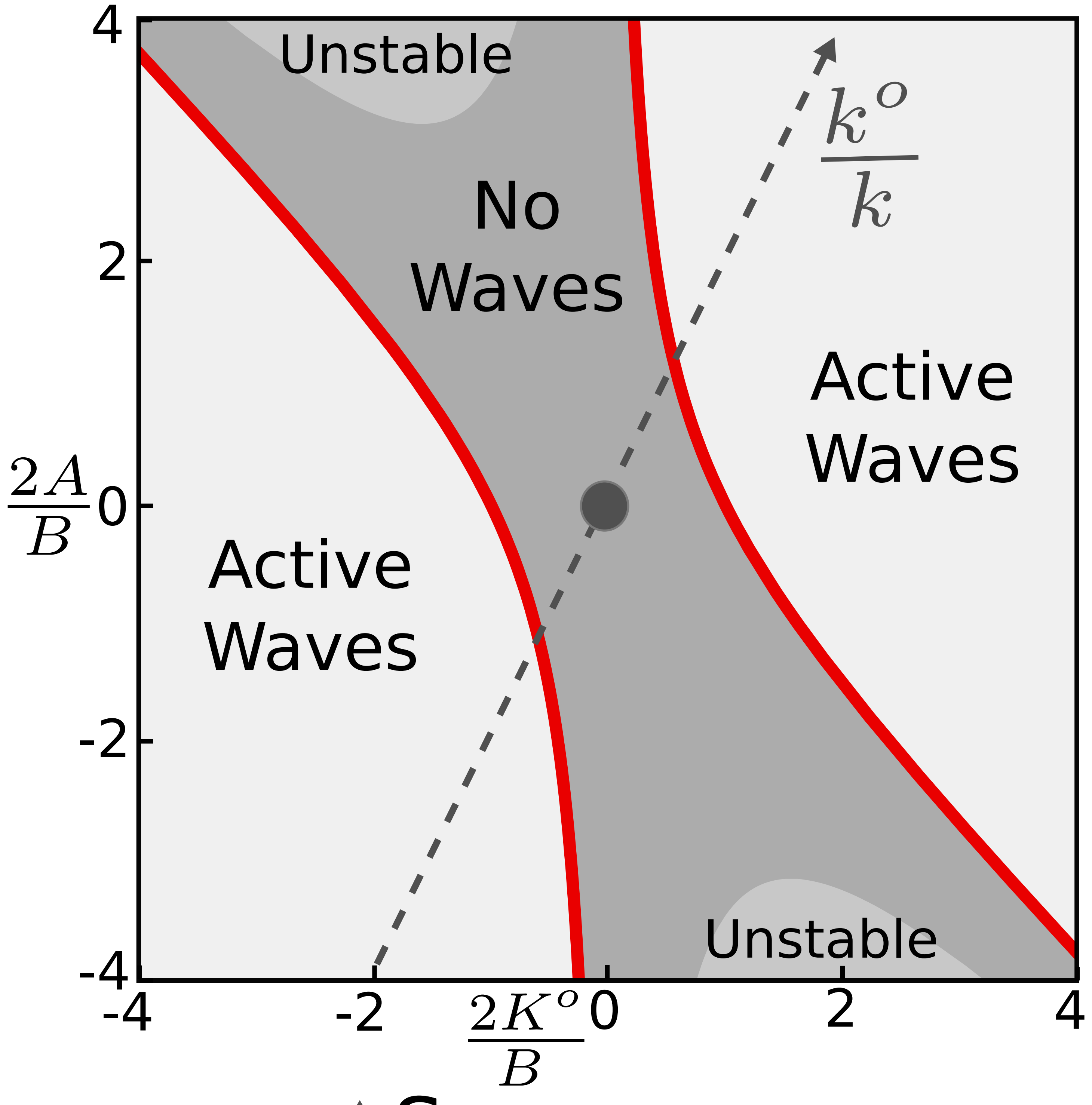
Phase Diagram

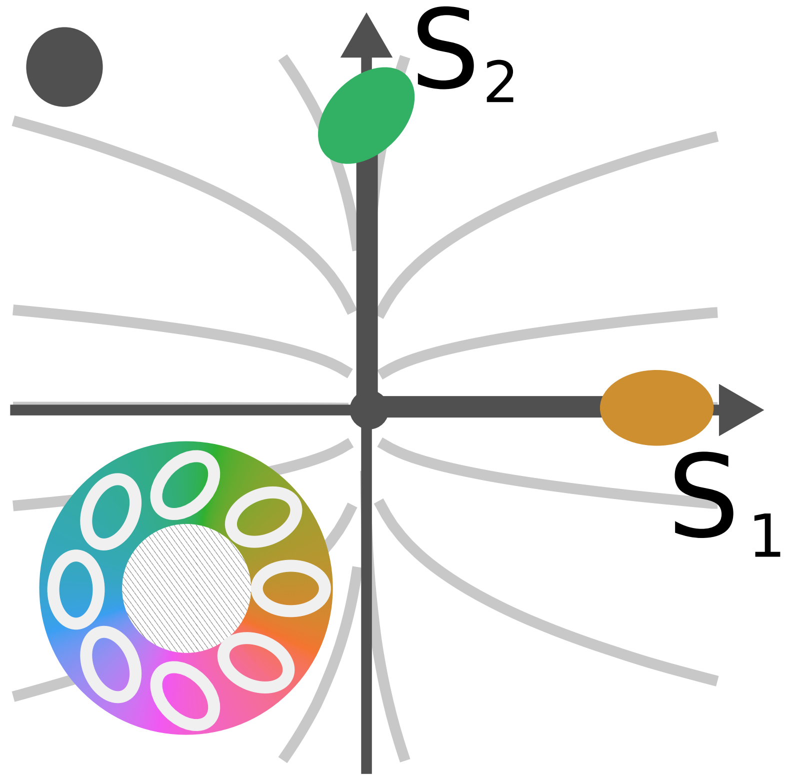


$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$

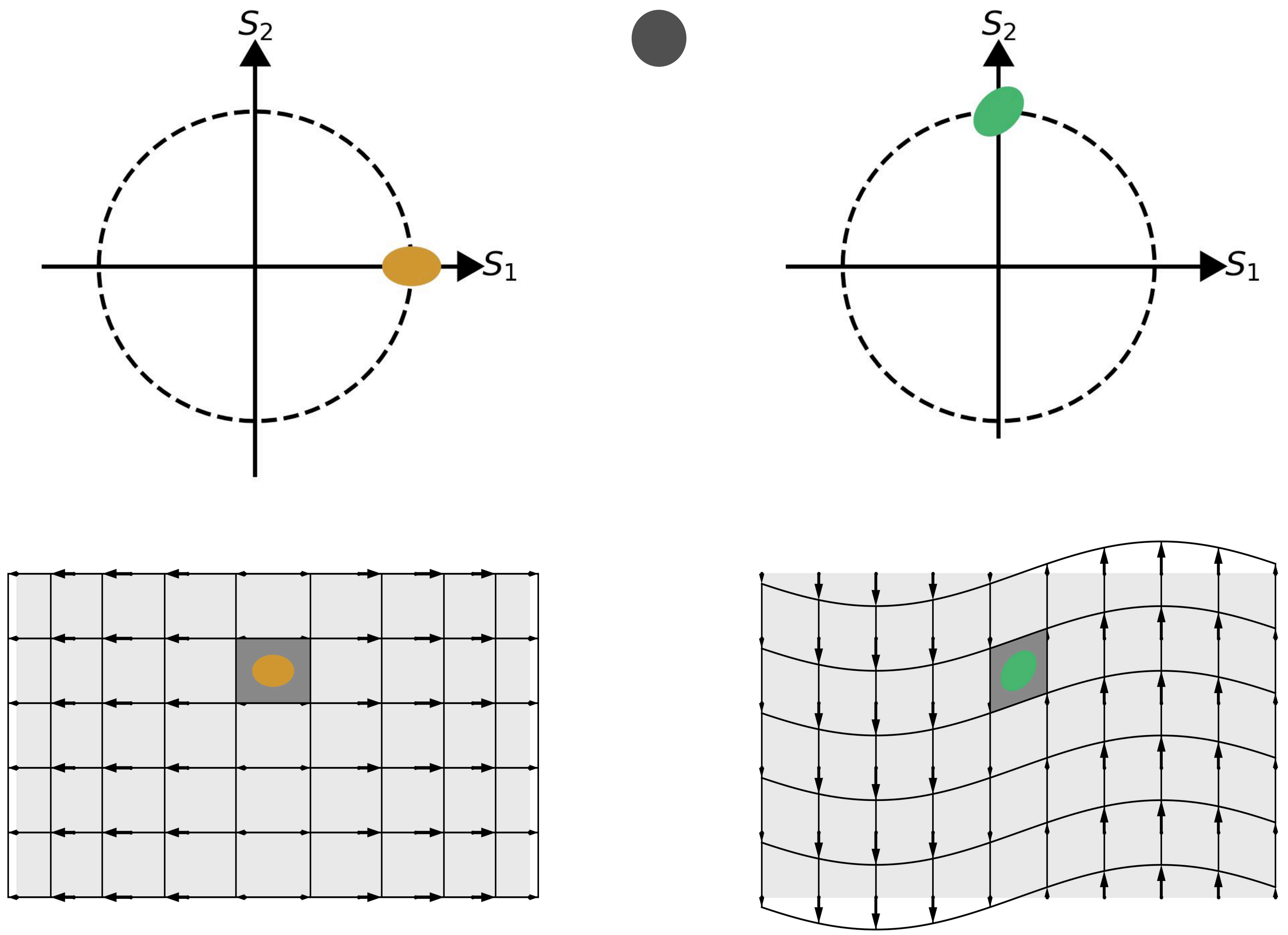


$$A = 2K^o = \frac{\sqrt{3}}{2}k^o$$





Hermitian dynamical matrix

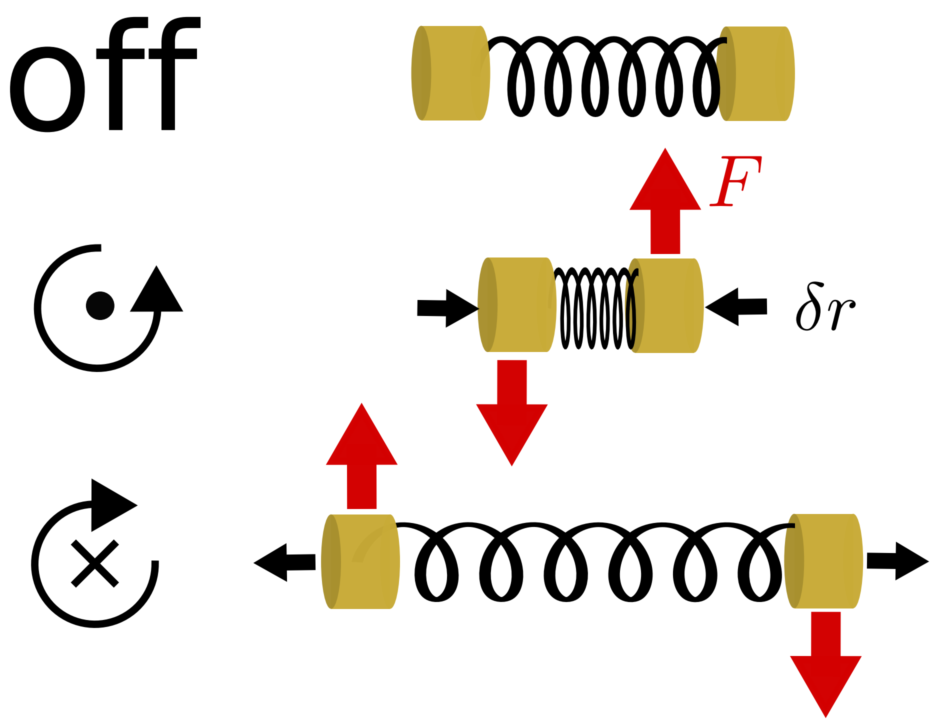


Longitudinal

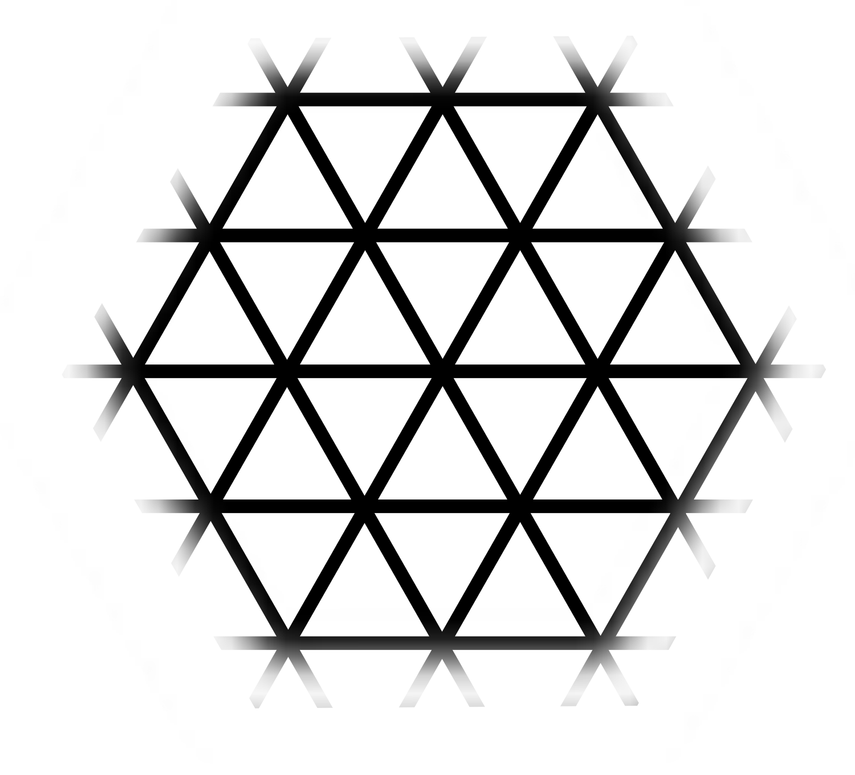
Transverse

$$-i\omega\Gamma\begin{pmatrix}u_\parallel\\u_\perp\end{pmatrix} = -q^2\begin{pmatrix}B+\mu & K^o\\-K^o-A & \mu\end{pmatrix}\begin{pmatrix}u_\parallel\\u_\perp\end{pmatrix}$$

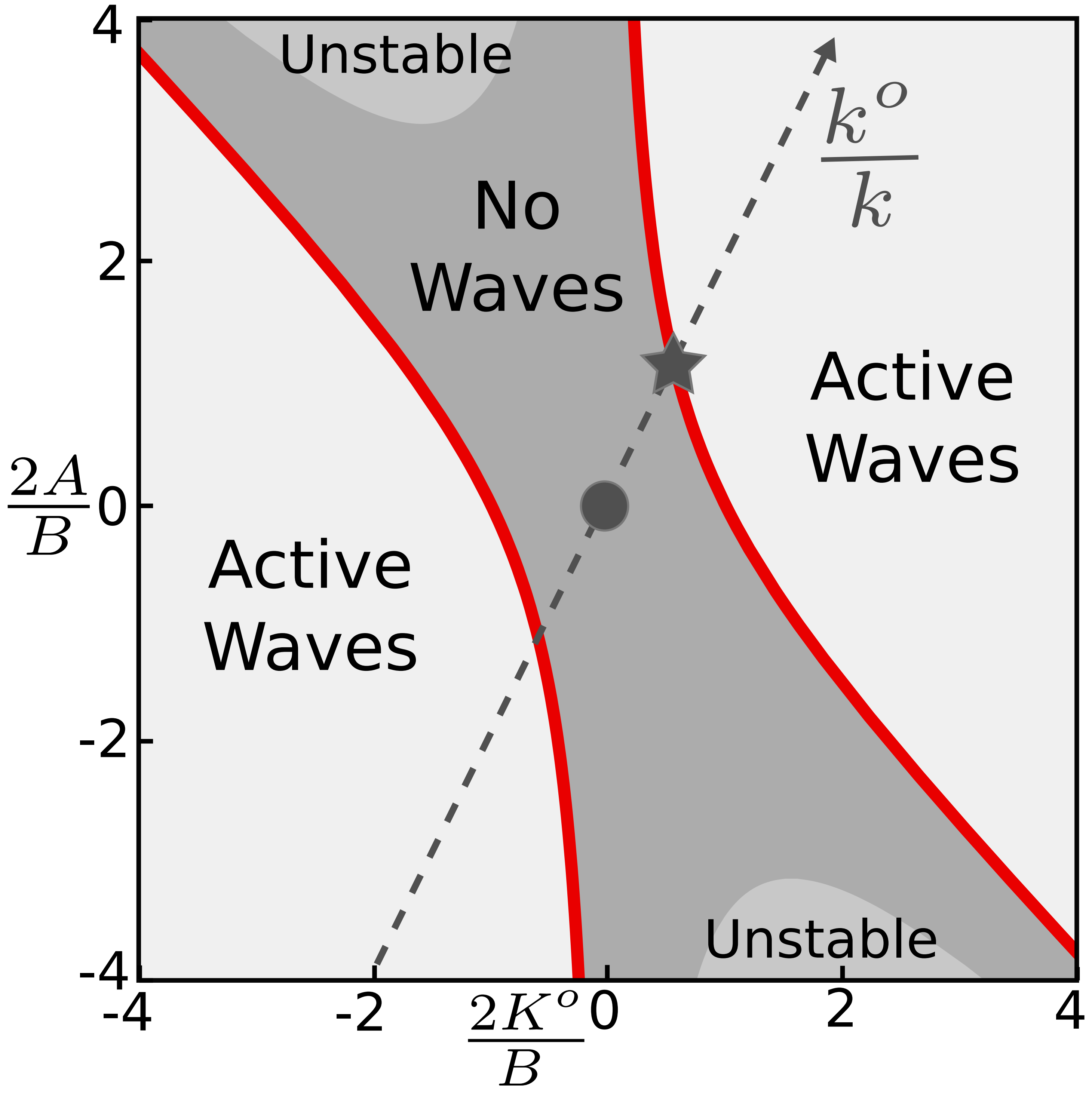
Phase Diagram

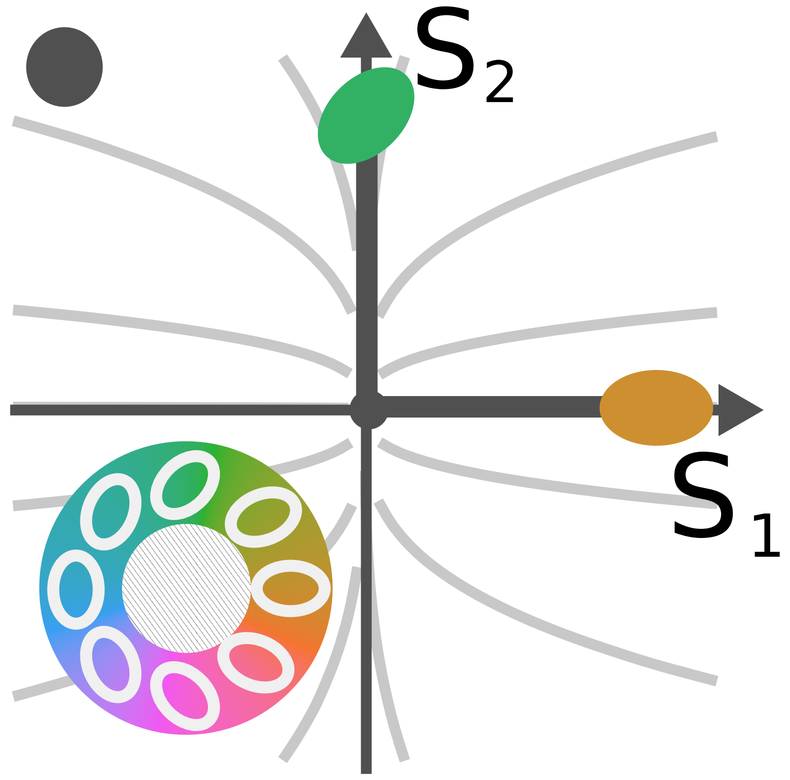


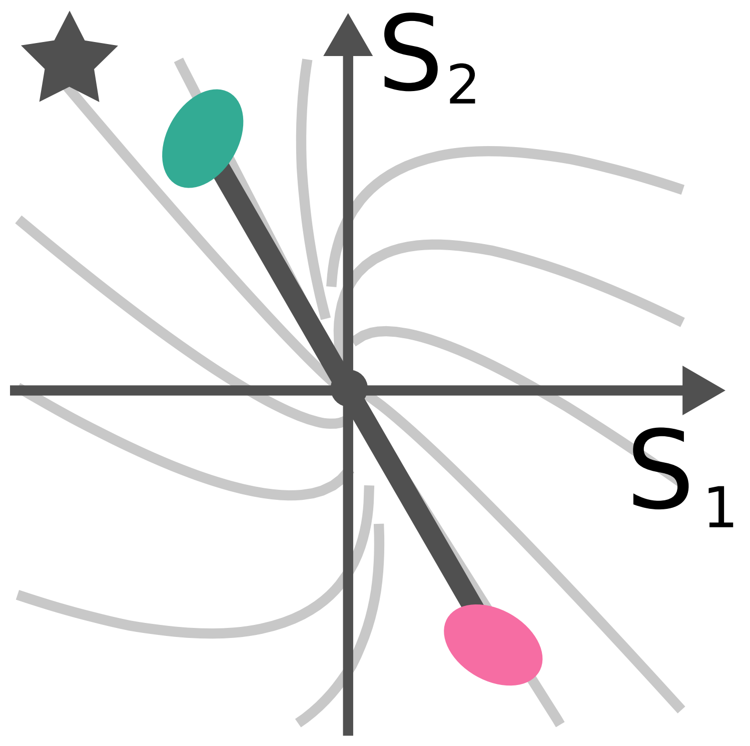
$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$



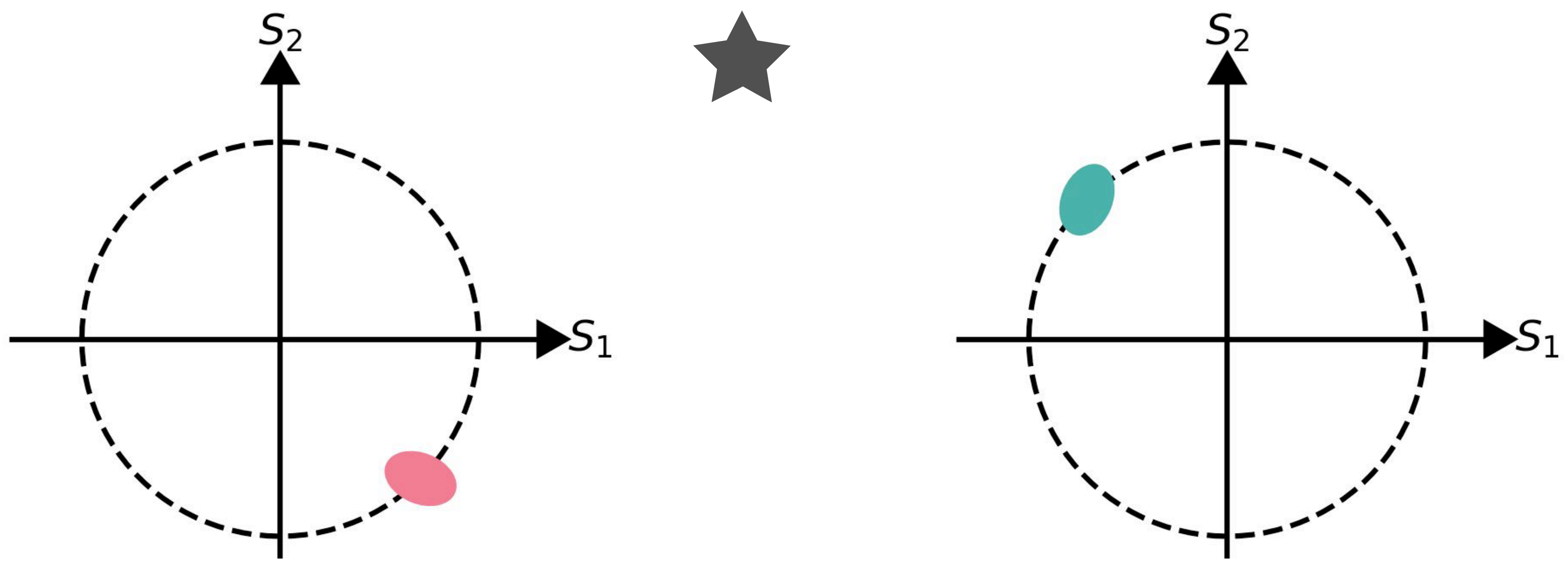
$$A = 2K^o = \frac{\sqrt{3}}{2}k^o$$



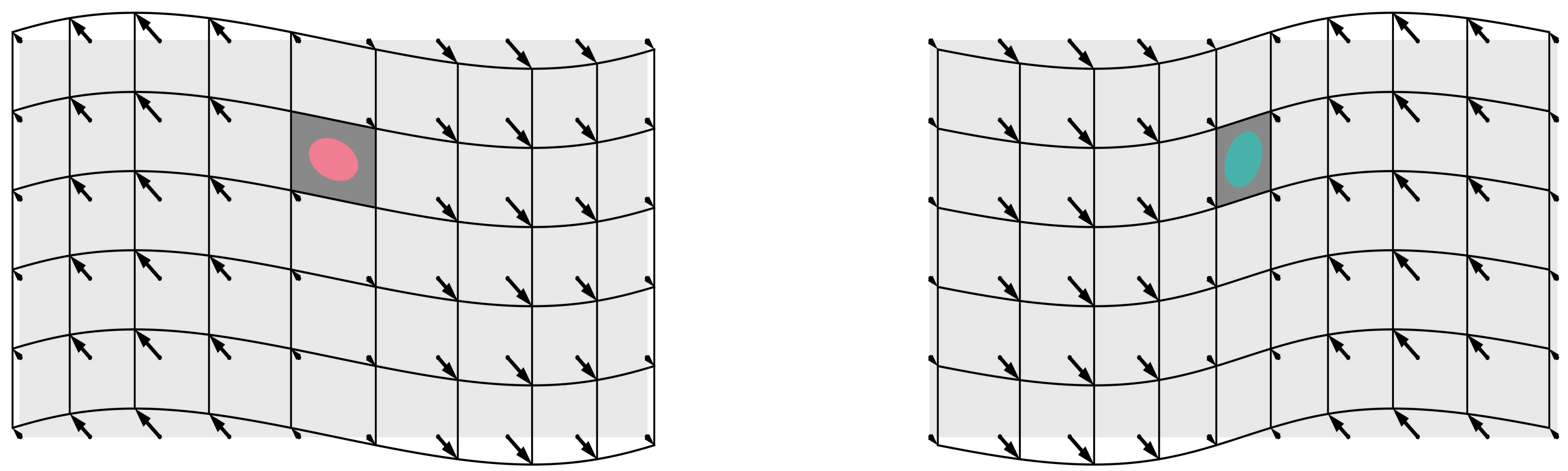




Non-Hermitian dynamical matrix



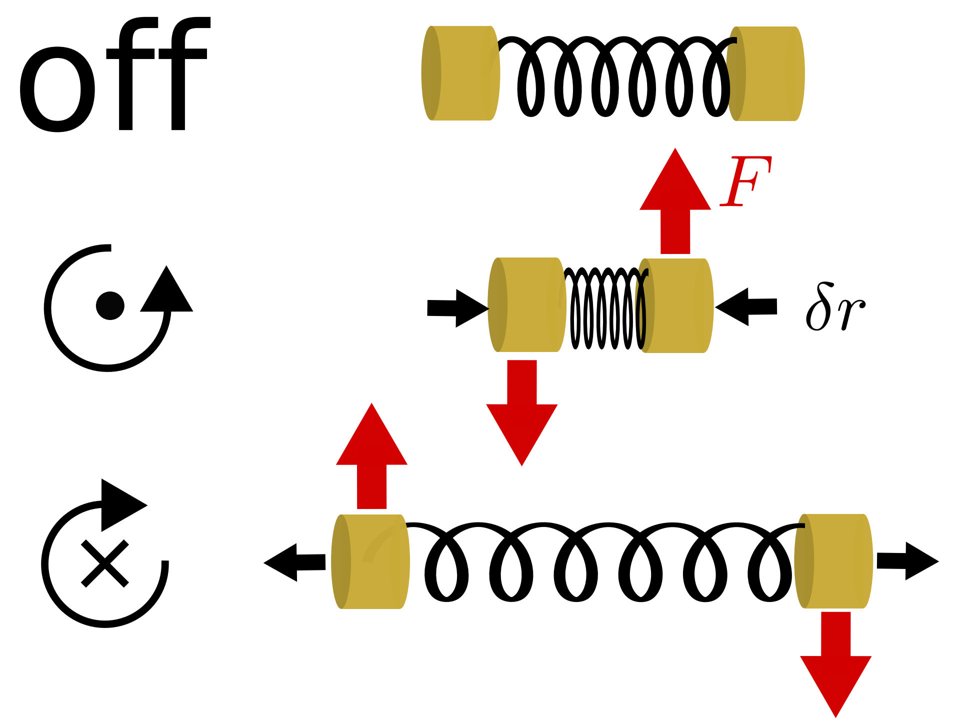
Exceptional Point



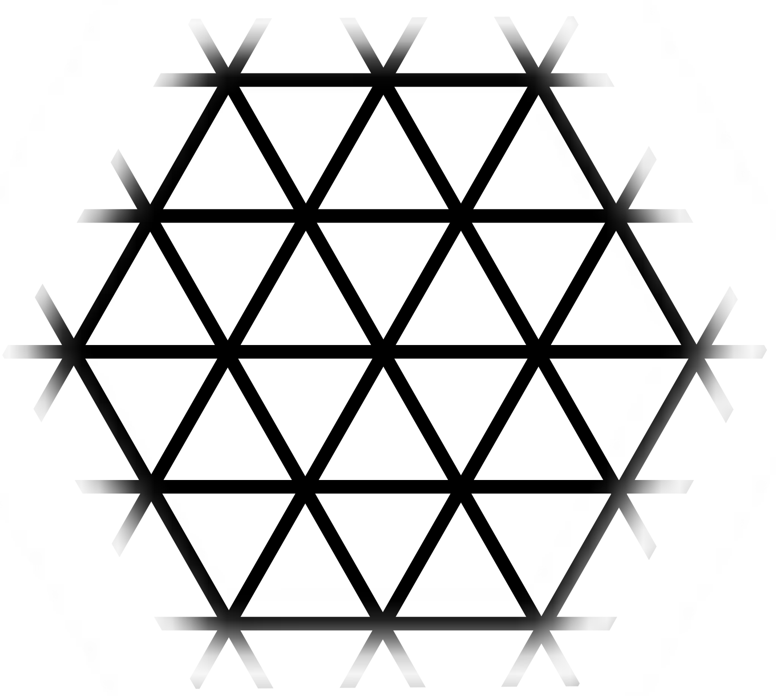
$$-i\omega\Gamma\begin{pmatrix} u_\parallel \\ u_\perp \end{pmatrix} = -q^2\begin{pmatrix} B+\mu & K^o \\ -K^o - A & \mu \end{pmatrix}\begin{pmatrix} u_\parallel \\ u_\perp \end{pmatrix}$$

Phase Diagram

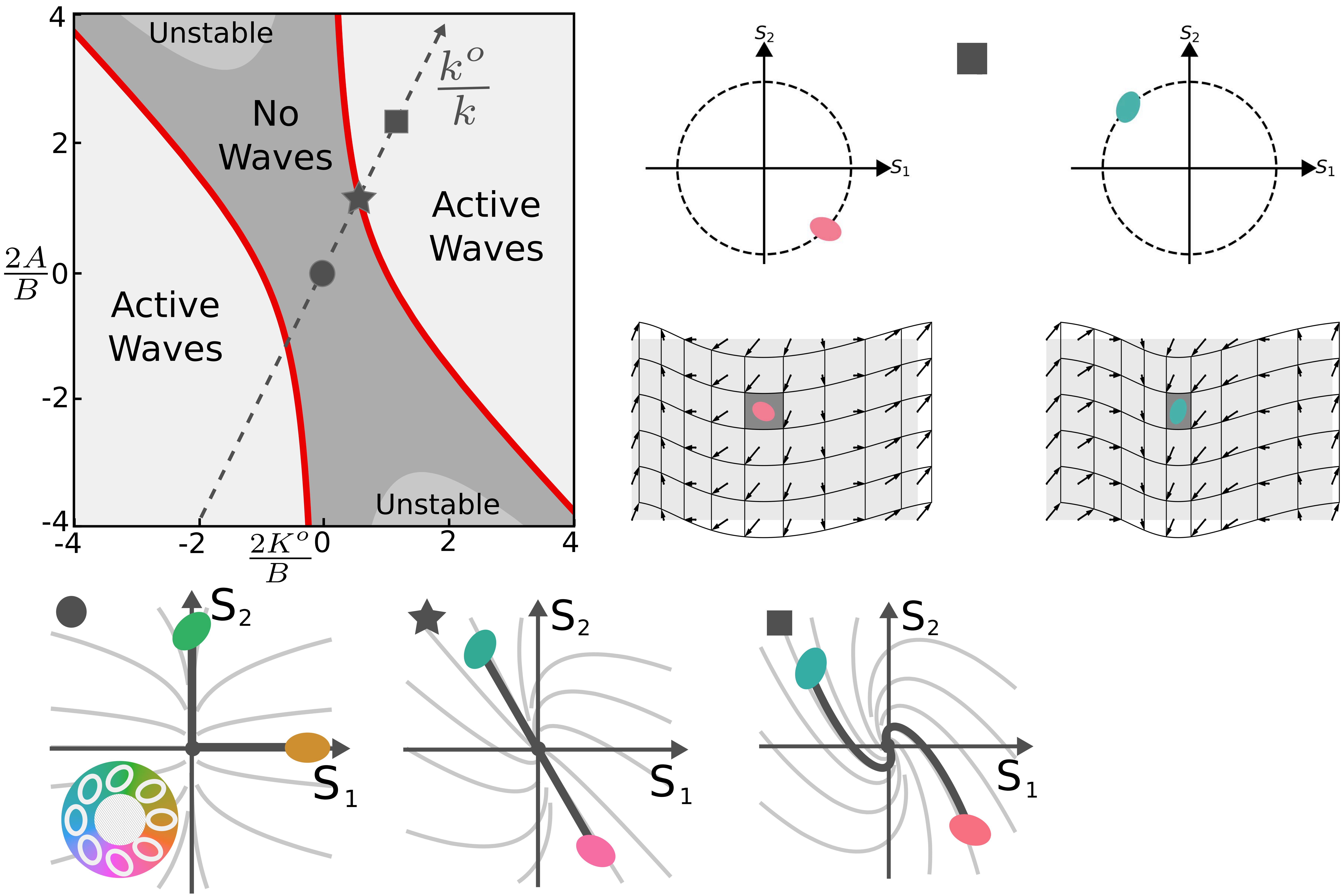
Non-Hermitian dynamical matrix



$$\mathbf{F} = -(k\hat{\mathbf{r}} + k^o\hat{\boldsymbol{\varphi}})\delta r$$

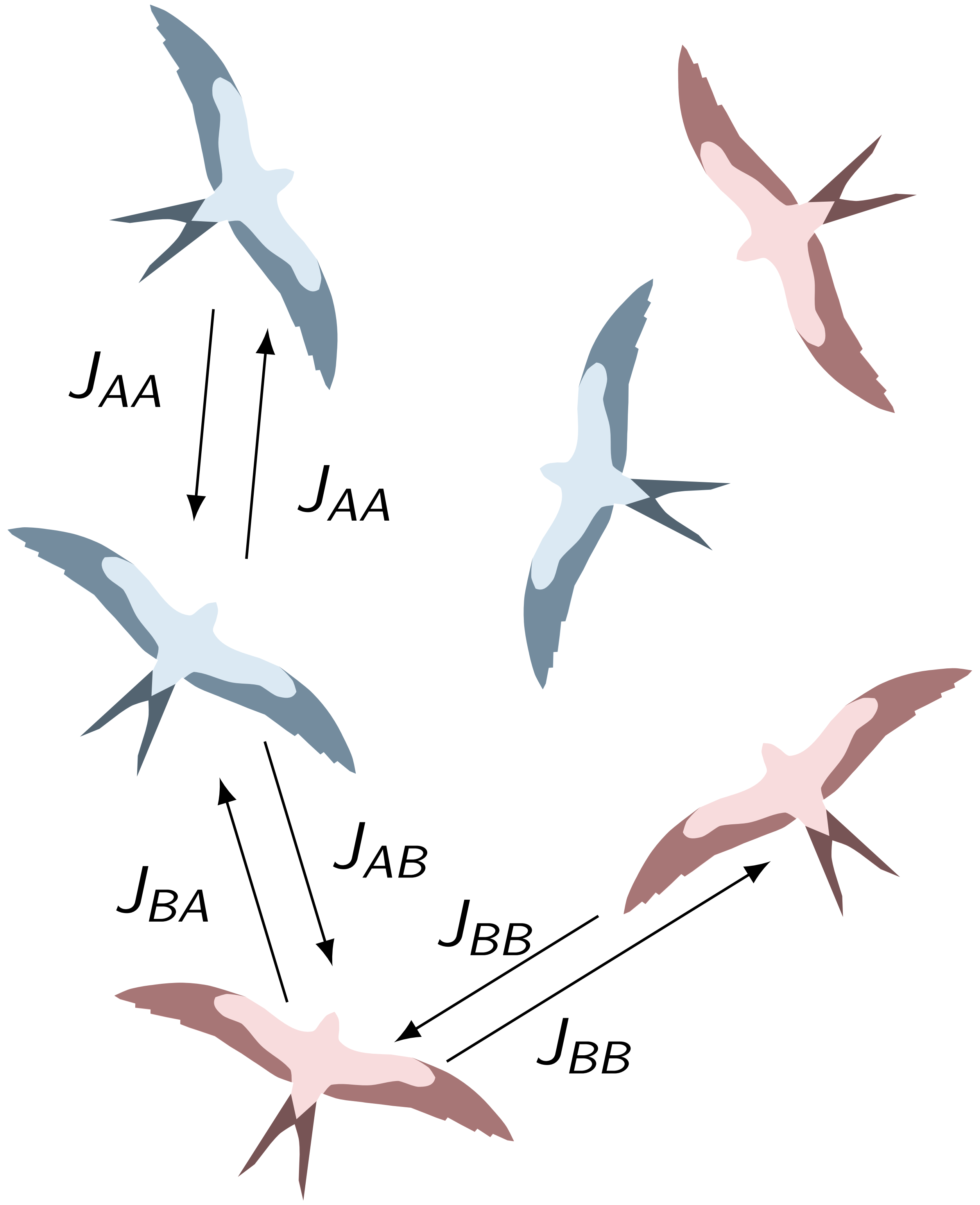


$$A = 2K^o = \frac{\sqrt{3}}{2}k^o$$



Exceptional Point

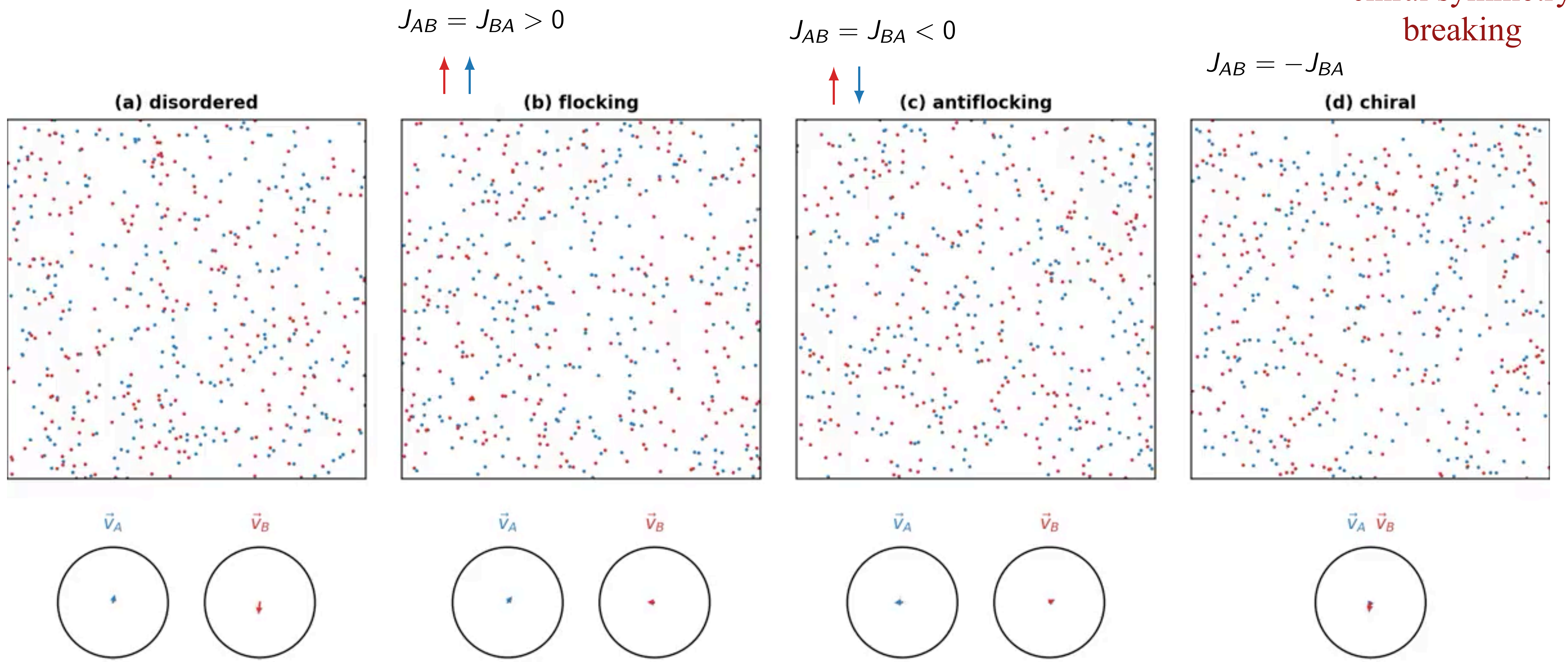
Phase transitions in non-reciprocal active systems



Fruchart, Hanai, Littlewood, Vitelli, arXiv 2003.13176 (2020)

Non-reciprocal binary Vicsek model

spontaneous
chiral symmetry
breaking



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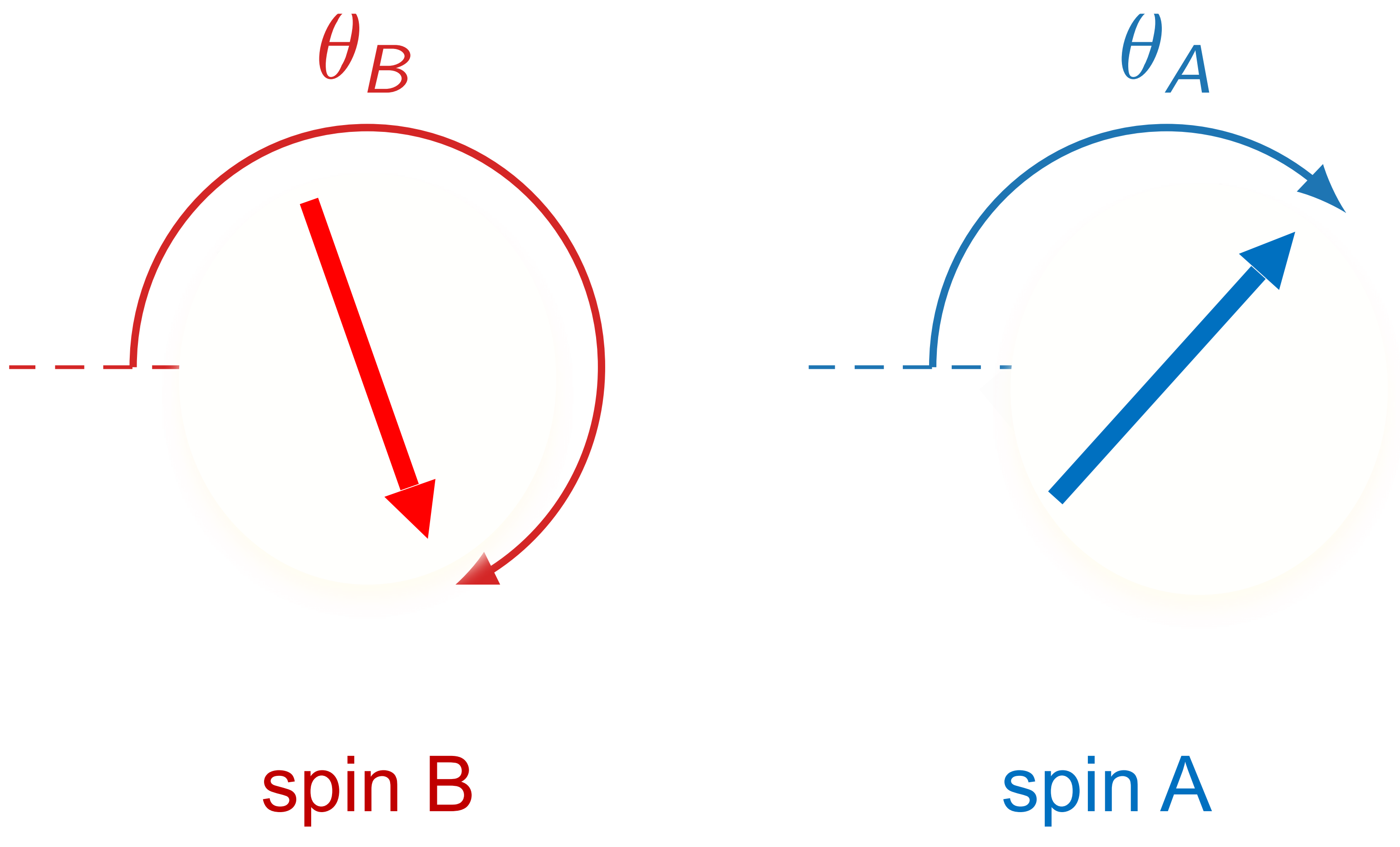
Vicsek deconstructed: XY model



$$J_{ij} = J_{ji}$$

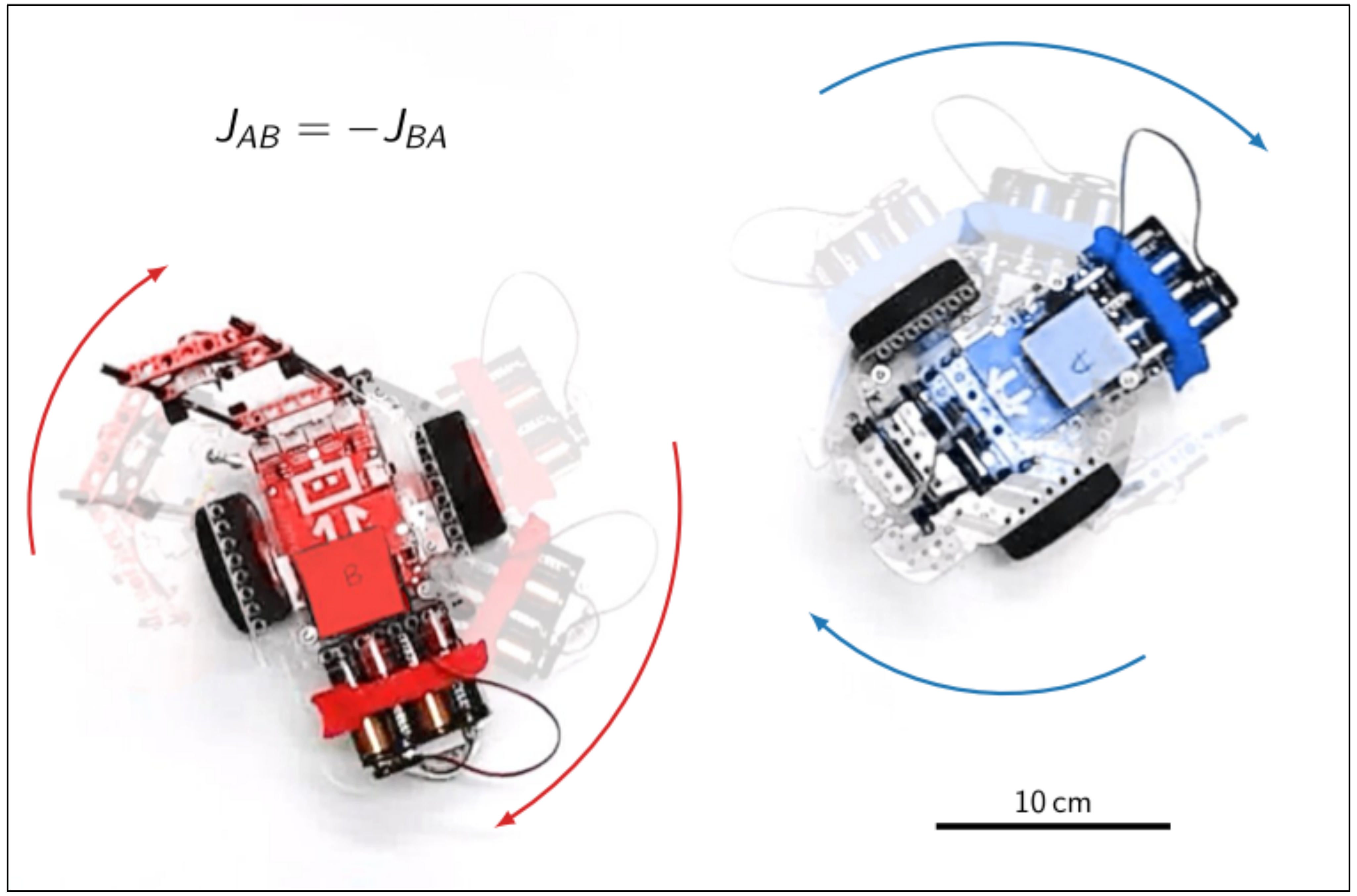
must be symmetric

$$H = -\sum_{(i,j)} J_{ij} \cos(\theta_i - \theta_j)$$

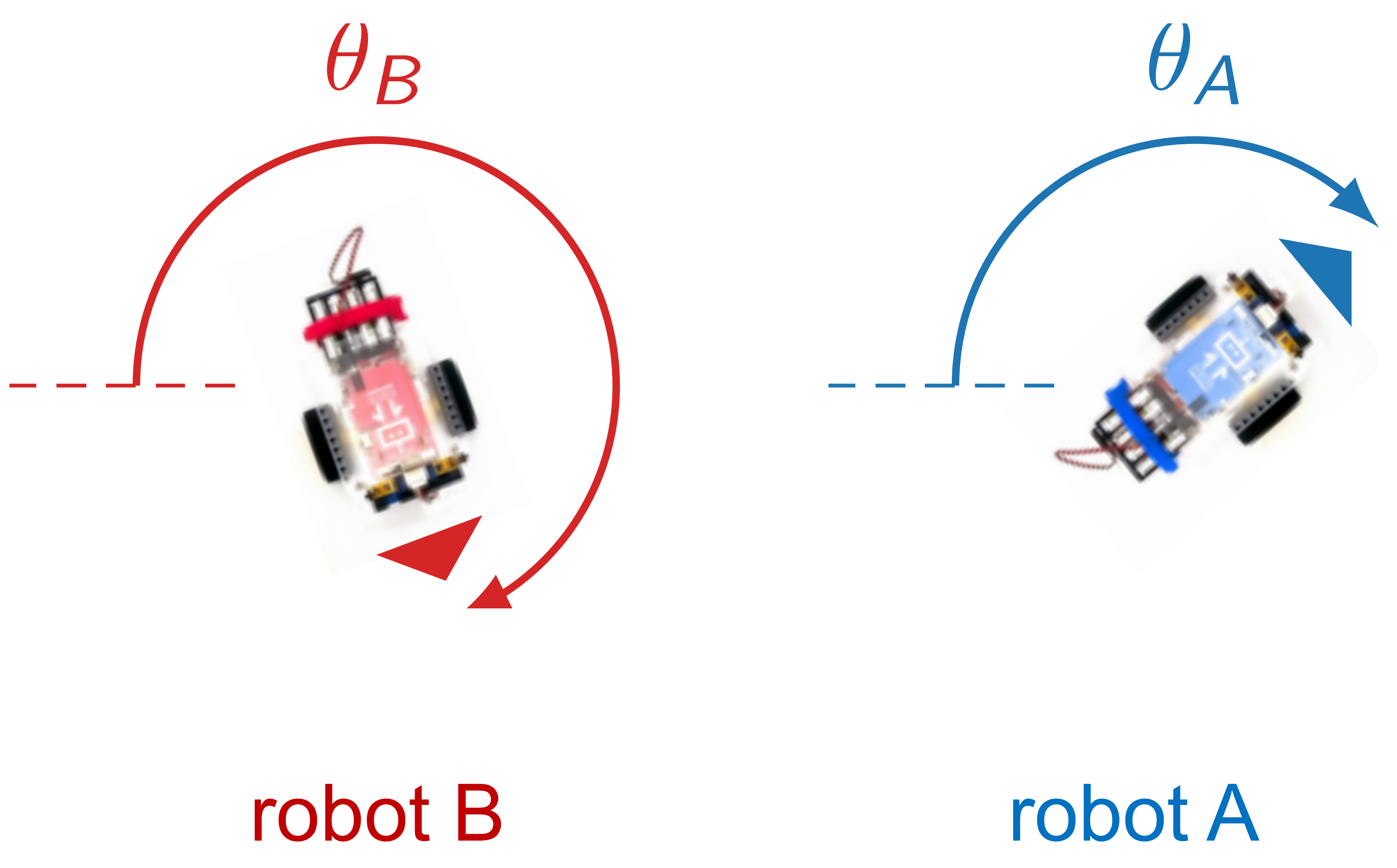
$$F_i = \frac{\partial H}{\partial \theta_i} = \sum_j J_{ij} \sin(\theta_i - \theta_j)$$

Force

Non reciprocal XY model



Fruchart, Hanai, Littlewood, Vitelli, arXiv 2003.13176 (2020)



Programmable robots as non-reciprocal spins

$$J_{ij} = -J_{ji}$$

can be antisymmetric

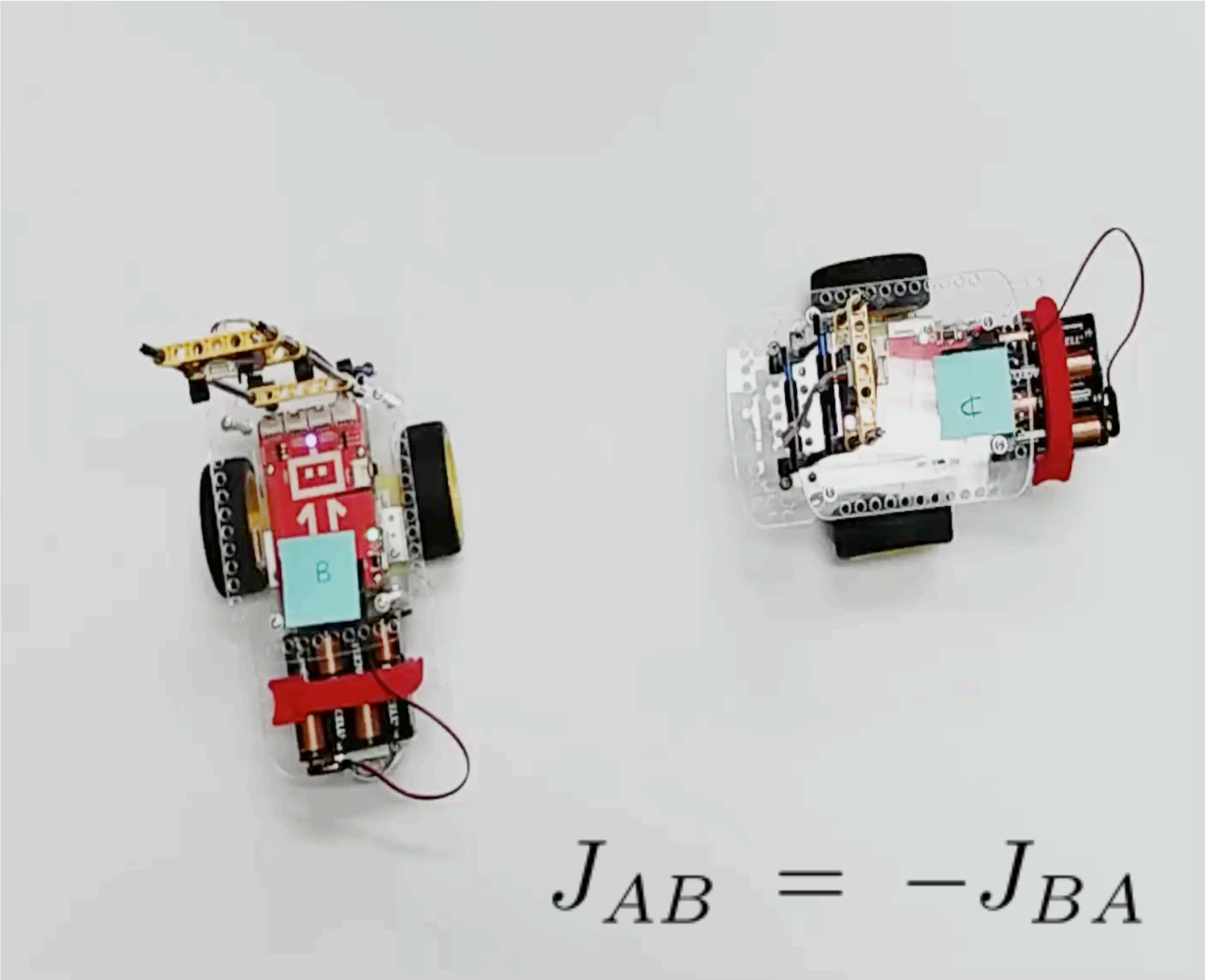
Interactions can be non-conservative

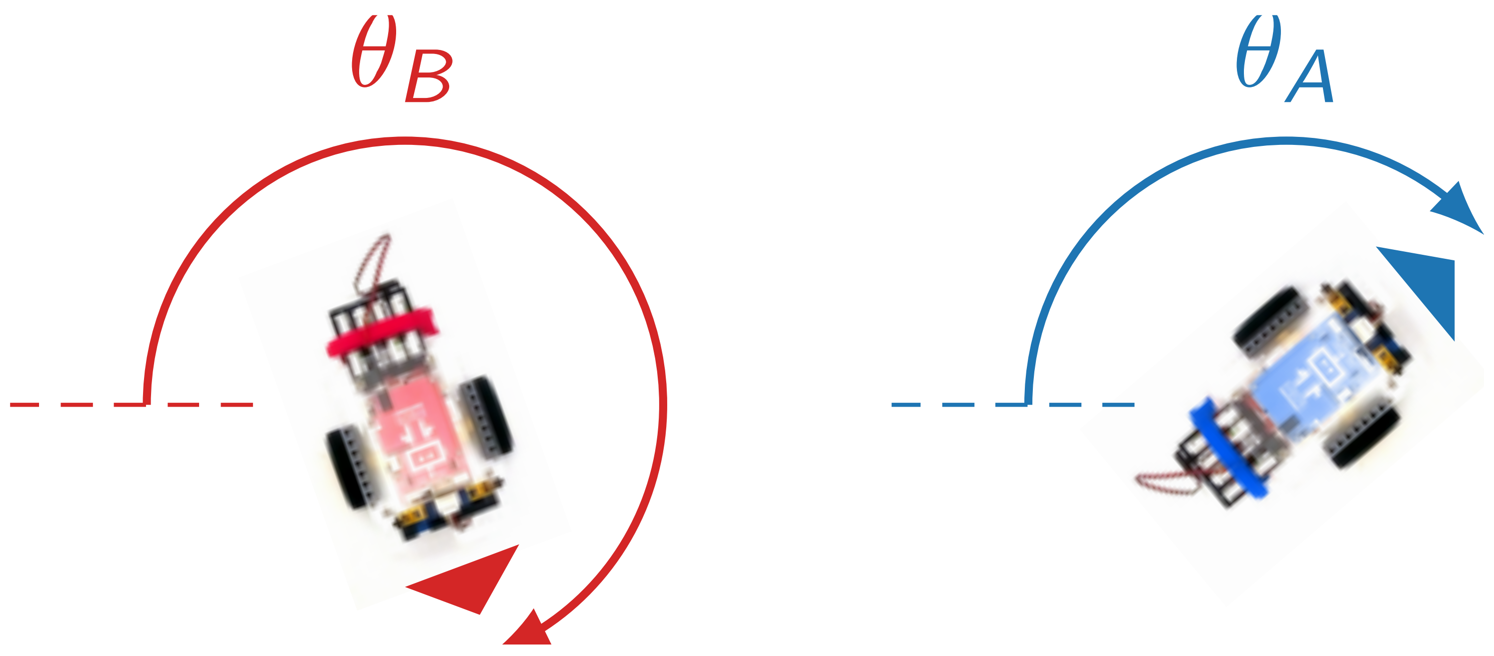
$$H = -\sum_{(i,j)} J_{ij} \cos(\theta_i - \theta_j)$$

$$F_i = \frac{\partial H}{\partial \theta_i} = \sum_j J_{ij} \sin(\theta_i - \theta_j)$$

Force

Non reciprocal XY model





Rotations induced by
non-reciprocal interactions

$$J_{ij} = -J_{ji}$$

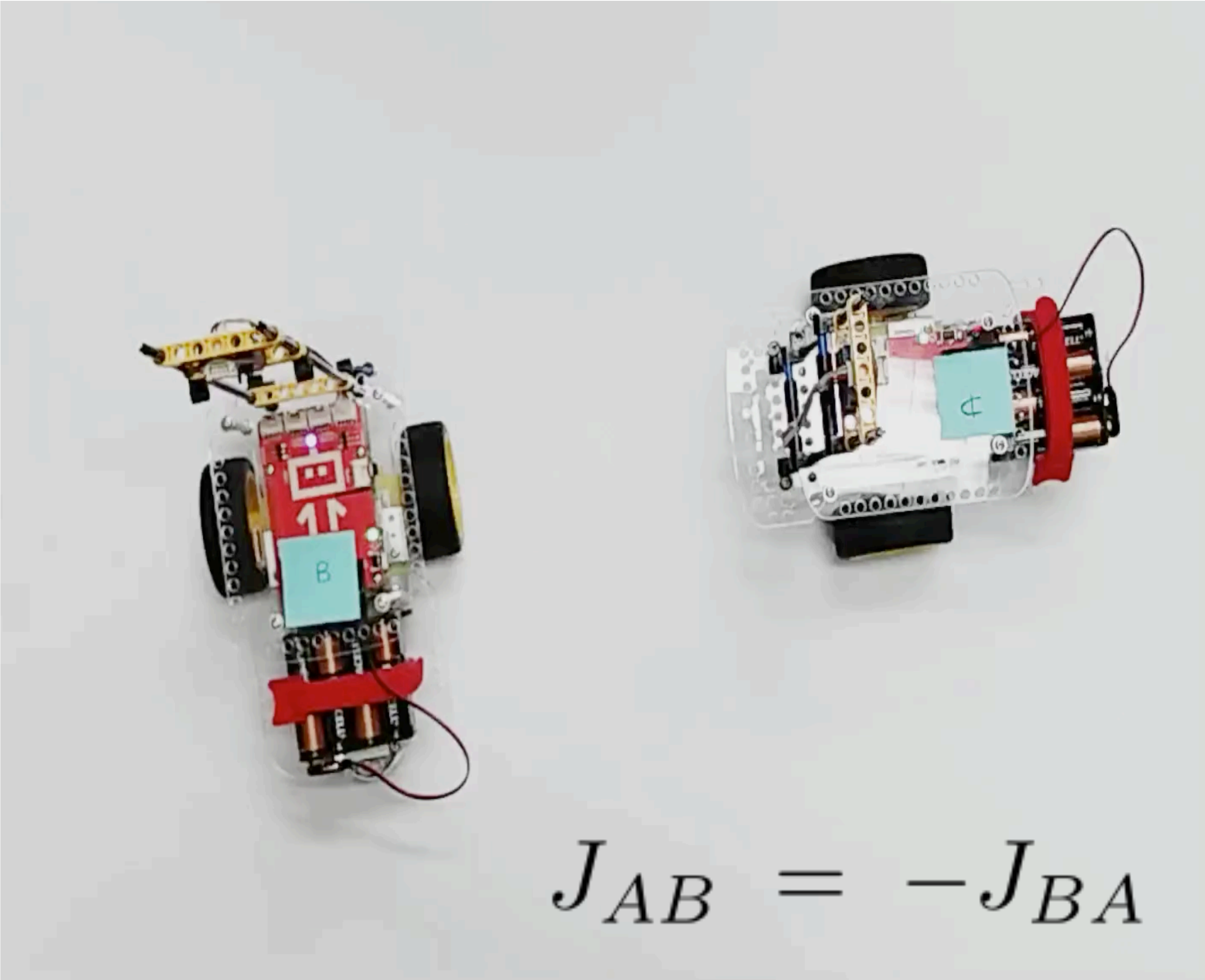
can be
antisymmetric

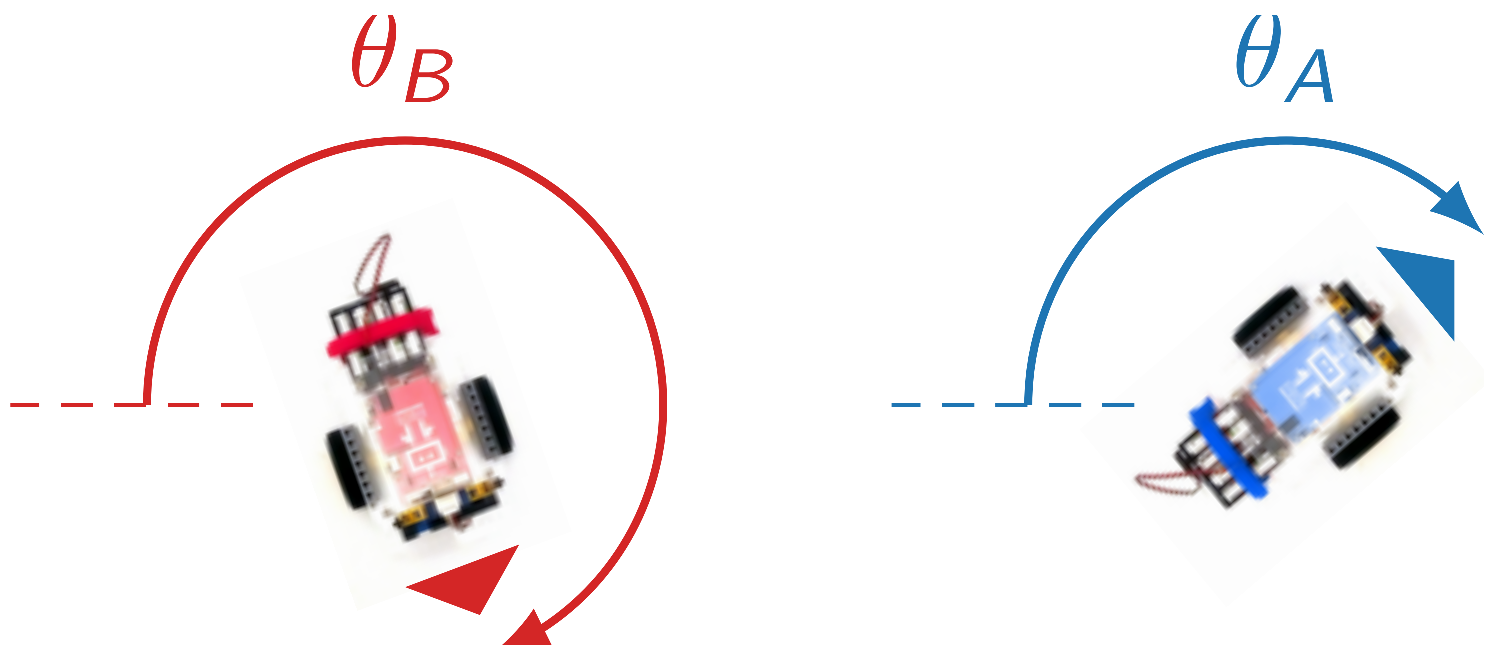
$$\sout{H = -\sum_{(i,j)} J_{ij} \cos(\theta_i - \theta_j)}$$

$$F_i = \cancel{\frac{\partial H}{\partial \theta_i}} = \sum_j J_{ij} \sin(\theta_i - \theta_j)$$

Any infinitesimal amount of reciprocal coupling will destroy the rotation of the two robots !

Non reciprocal XY model





Rotations induced by non-reciprocal interactions

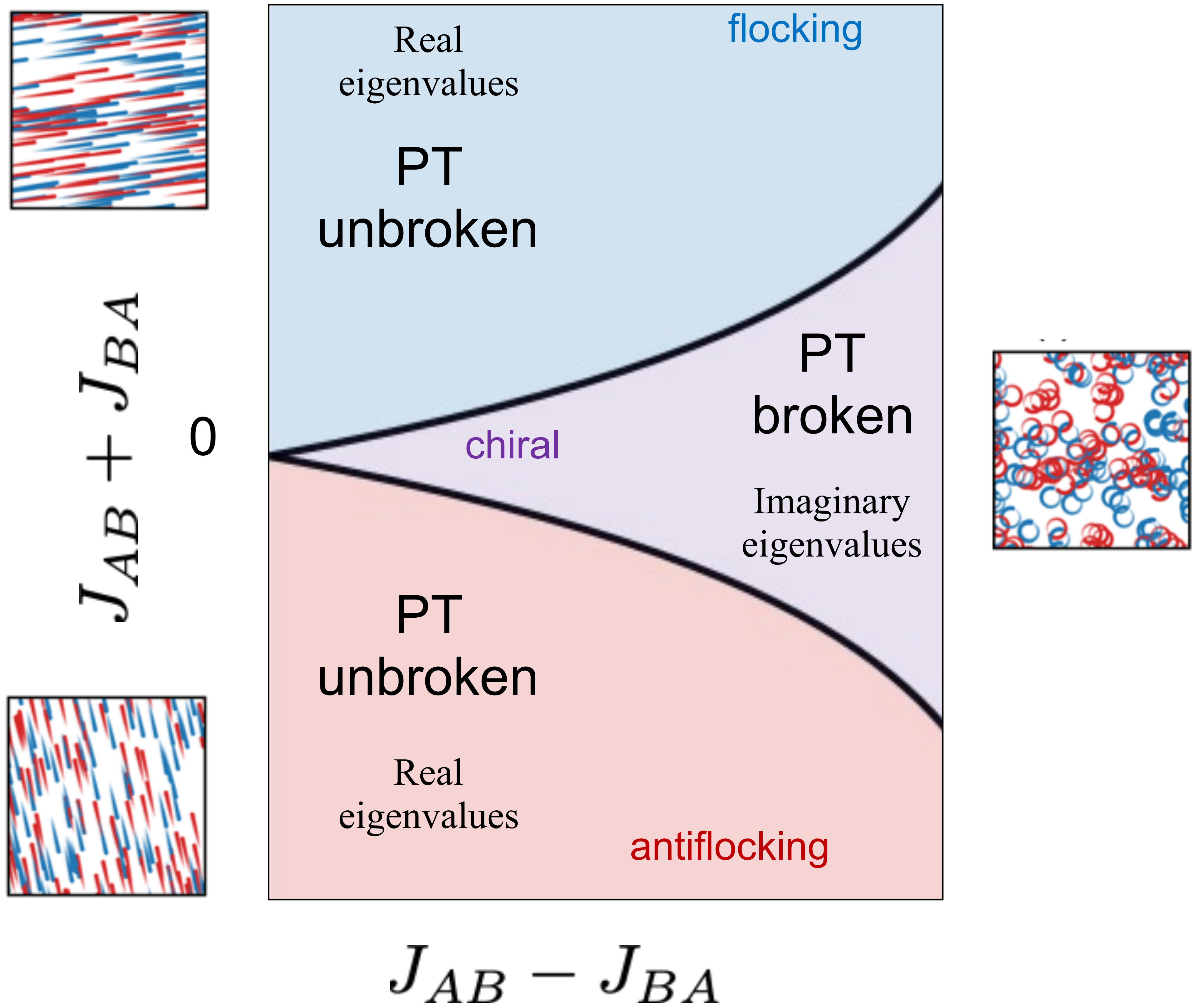
$$J_{ij} = -J_{ji}$$ can be antisymmetric

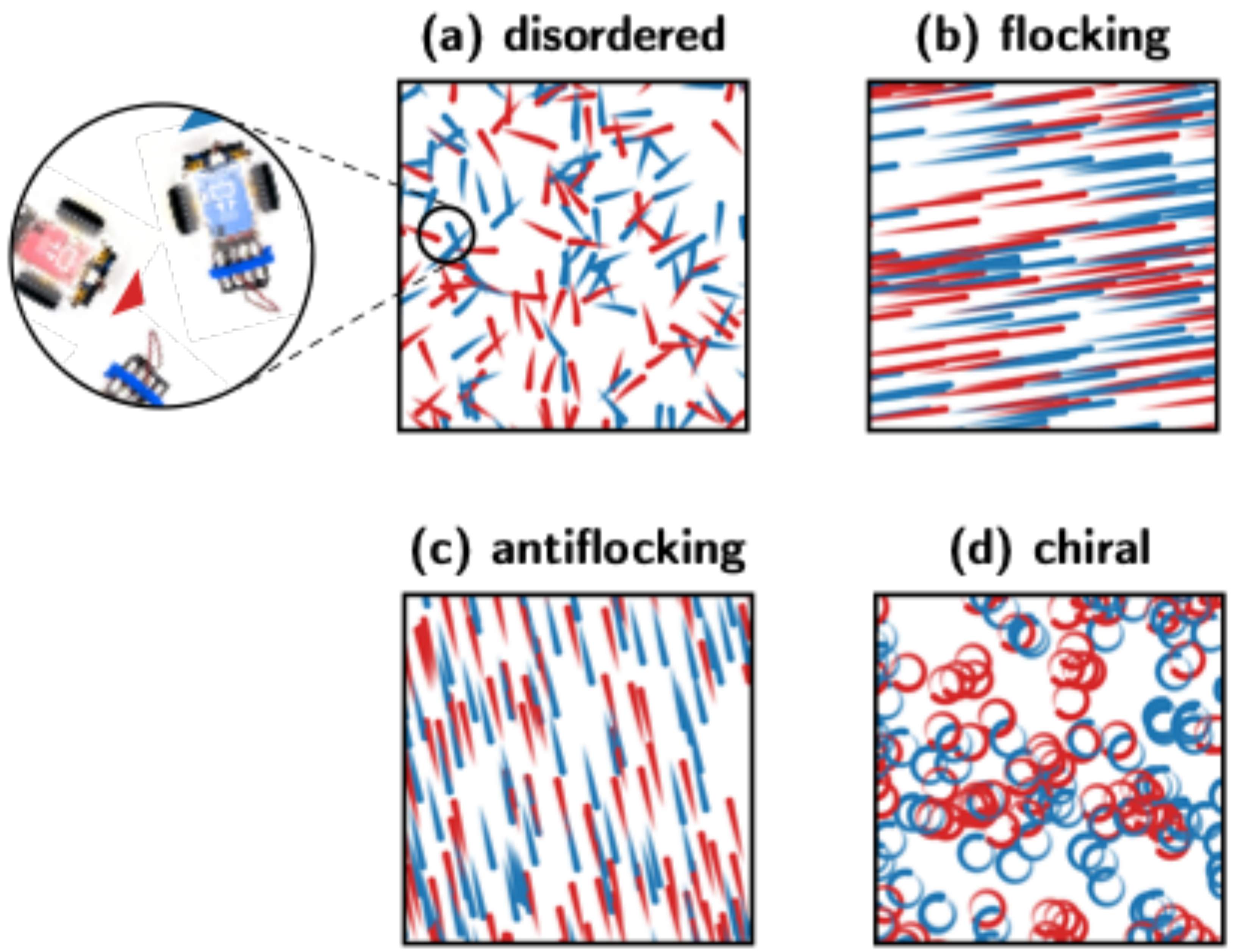
$$H = -\sum_{(i,j)} J_{ij} \cos(\theta_i - \theta_j)$$

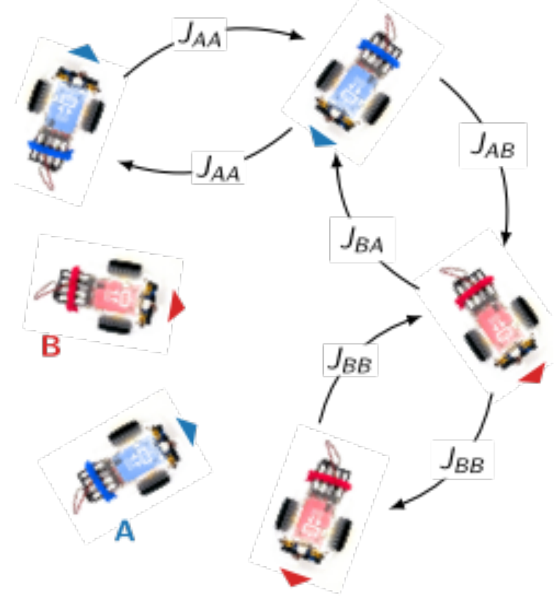
$$F_i = \frac{\partial H}{\partial \theta_i} = \sum_j J_{ij} \sin(\theta_i - \theta_j)$$

Can this motion be stabilized by many body interactions ?

Mean field phase diagram







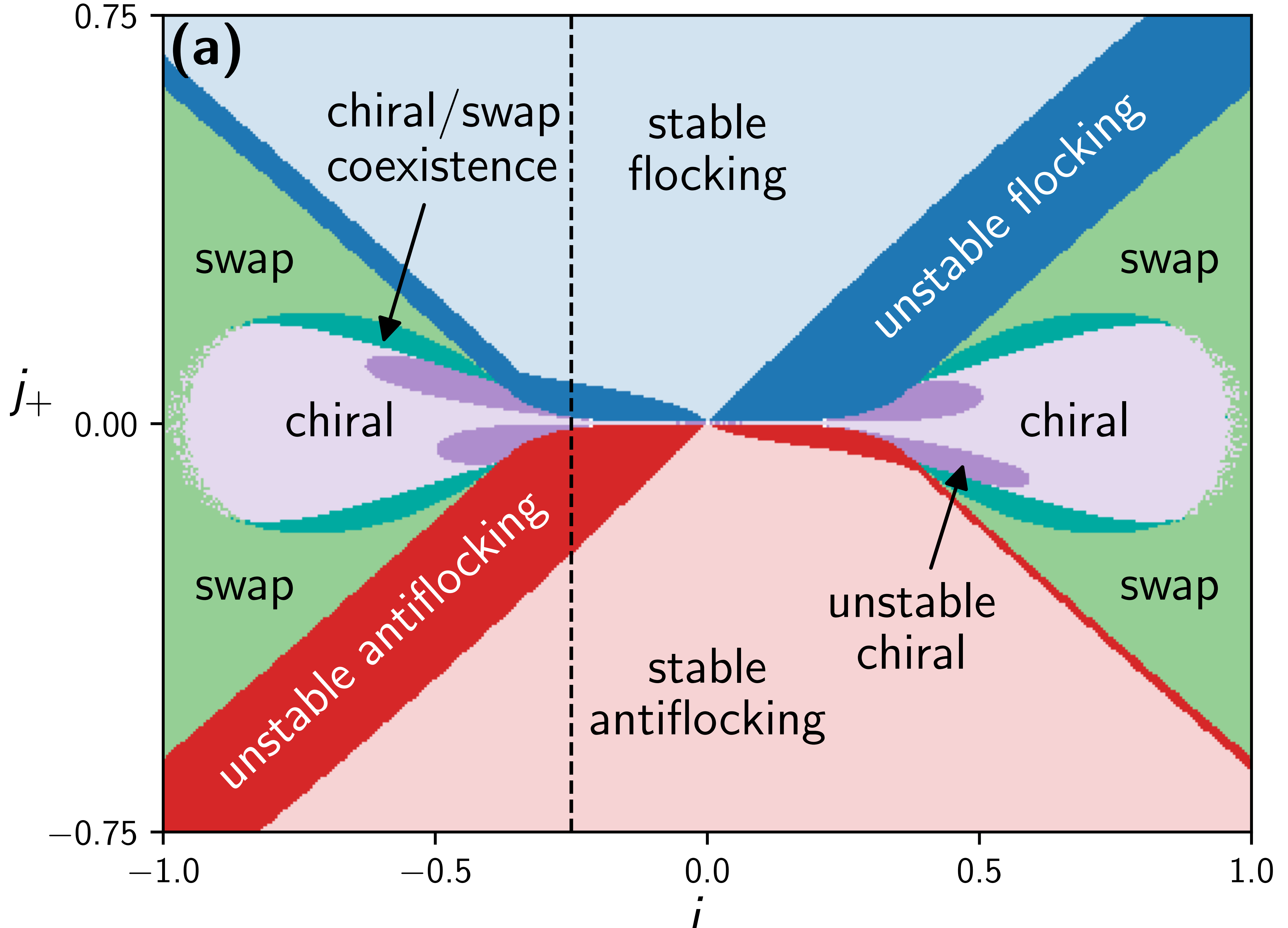
Binary robotic fluid with self-propulsion

$$\partial_t \begin{pmatrix} \vec{v}_A \\ \vec{v}_B \end{pmatrix} = \begin{pmatrix} \alpha_A[\vec{v}_A, \vec{v}_B] & j_{AB} \\ j_{BA} & \alpha_B[\vec{v}_A, \vec{v}_B] \end{pmatrix} \begin{pmatrix} \vec{v}_A \\ \vec{v}_B \end{pmatrix}$$

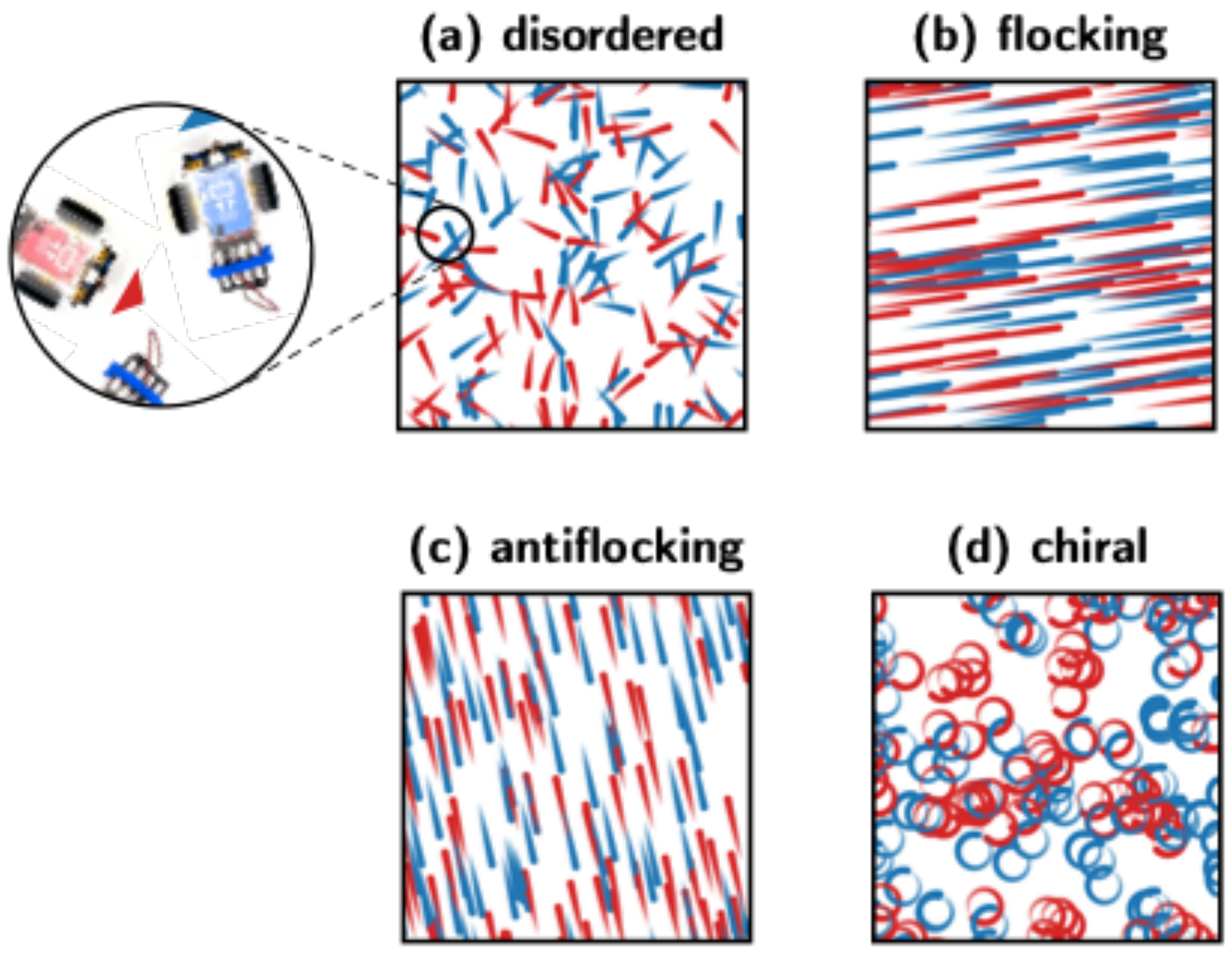
Black boundary is marked by **exceptional points**

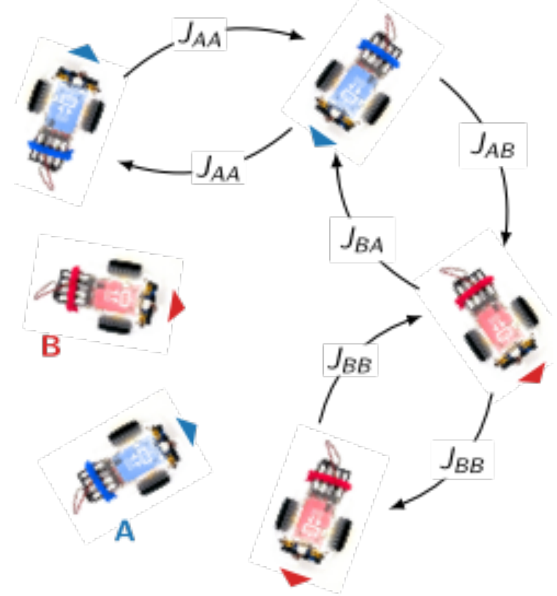
Floquet stability analysis reveals that the chiral phase can be stabilized by many-body interactions

Exceptional point enforced pattern formation



$$j_\pm = [j_{AB} \pm j_{BA}]/2$$





Binary robotic fluid with self-propulsion

$$\partial_t \begin{pmatrix}\vec{v}_A \\ \vec{v}_B\end{pmatrix} = \begin{pmatrix}\alpha_A[\vec{v}_A, \vec{v}_B] & j_{AB} \\ j_{BA} & \alpha_B[\vec{v}_A, \vec{v}_B]\end{pmatrix} \begin{pmatrix}\vec{v}_A \\ \vec{v}_B\end{pmatrix}$$

Floquet stability analysis reveals that the chiral phase can be stabilized by many-body interactions

Exceptional points plus convective terms generate pattern formation near phase boundaries

Fruchart, Hanai, Littlewood, Vitelli, arXiv 2003.13176 (2020)

Thanks for your attention